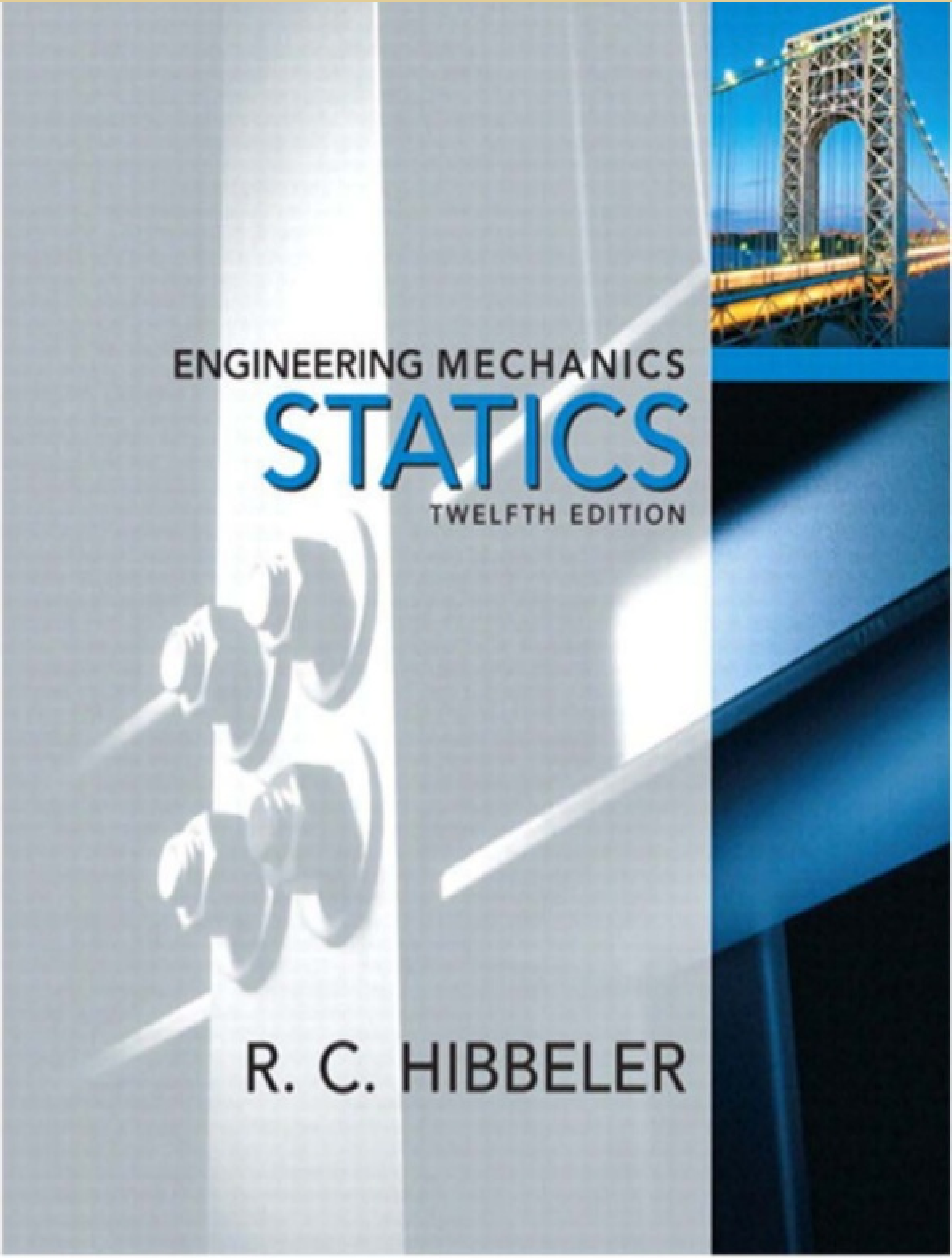


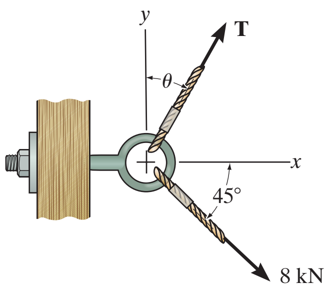
ENGINEERING MECHANICS
STATICS

TWELFTH EDITION

R. C. HIBBELER

Sumário

•2–1. If $\theta = 30^\circ$ and $T = 6$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively. Applying the law of cosines to Fig. *b*,

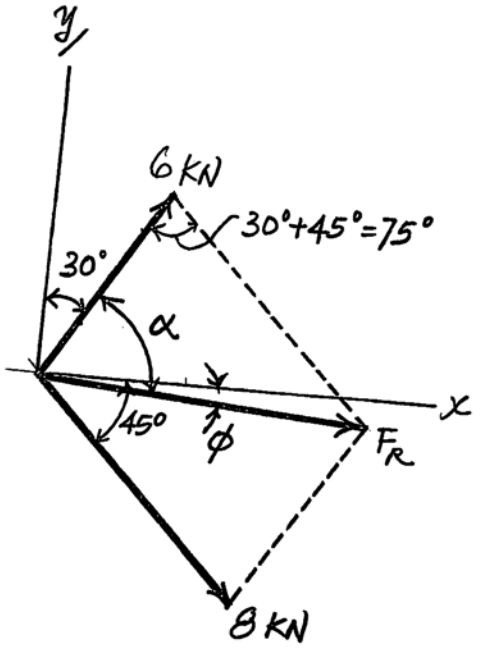
$$F_R = \sqrt{6^2 + 8^2 - 2(6)(8)\cos 75^\circ}$$
$$= 8.669 \text{ kN} = 8.67 \text{ kN} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result , yields

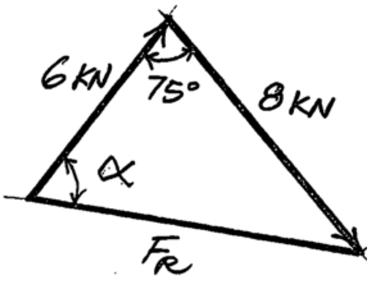
$$\frac{\sin\alpha}{8} = \frac{\sin 75^\circ}{8.669} \qquad \alpha = 63.05^\circ$$

Thus, the direction angle ϕ of $\mathbf{F}_R$ measured clockwise from the positive x axis is

$$\phi = \alpha - 60^\circ = 63.05^\circ - 60^\circ = 3.05^\circ \qquad \textbf{Ans.}$$

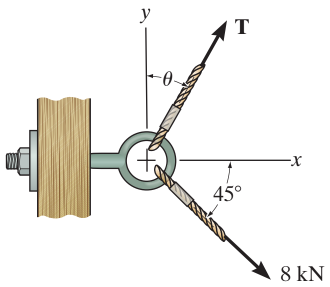


(a)



(b)

2–2. If $\theta = 60^\circ$ and $T = 5$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

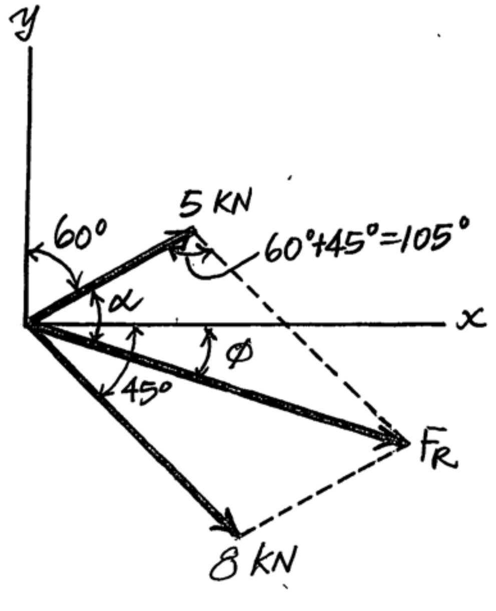
$$F_R = \sqrt{5^2 + 8^2 - 2(5)(8)\cos 105^\circ}$$
$$= 10.47 \text{ kN} = 10.5 \text{ kN} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result, yields

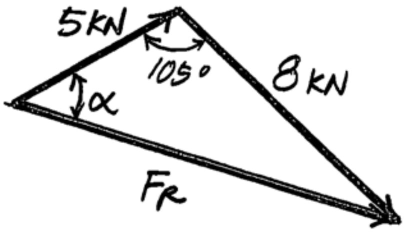
$$\frac{\sin\alpha}{8} = \frac{\sin 105^\circ}{10.47} \qquad \alpha = 47.54^\circ$$

Thus, the direction angle ϕ of $\mathbf{F}_R$ measured clockwise from the positive x axis is

$$\phi = \alpha - 30^\circ = 47.54^\circ - 30^\circ = 17.5^\circ \qquad \textbf{Ans.}$$

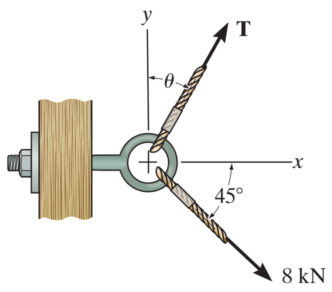


(a)



(b)

2–3. If the magnitude of the resultant force is to be 9 kN directed along the positive x axis, determine the magnitude of force **T** acting on the eyebolt and its angle θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

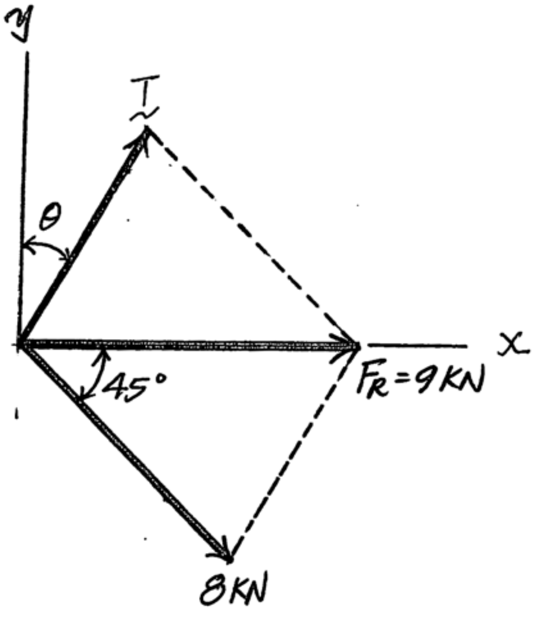
Applying the law of cosines to Fig. *b*,

$$T = \sqrt{8^2 + 9^2 - 2(8)(9)\cos 45^\circ}$$
$$= 6.571 \text{ kN} = 6.57 \text{ kN} \qquad \textbf{Ans.}$$

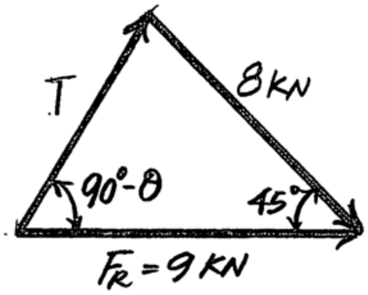
Applying the law of sines to Fig. *b* and using this result, yields

$$\frac{\sin(90^\circ - \theta)}{8} = \frac{\sin 45^\circ}{6.571}$$

$$\theta = 30.6^\circ \qquad \textbf{Ans.}$$

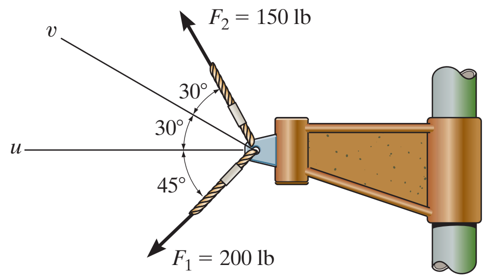


(a)



(b)

***2–4.** Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive u axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

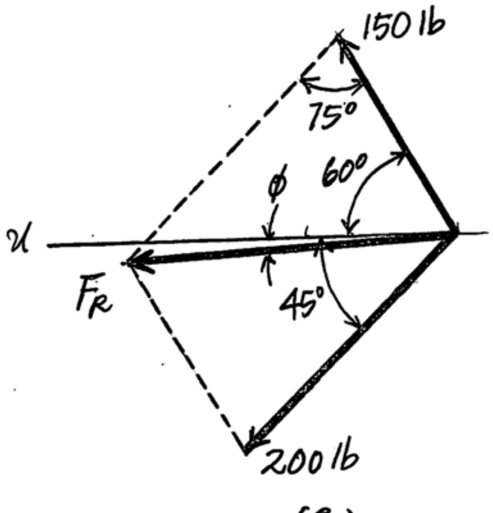
$$F_R = \sqrt{200^2 + 150^2 - 2(200)(150)\cos 75°}$$
$$= 216.72 \text{ lb} = 217 \text{ lb}$$ **Ans.**

Applying the law of sines to Fig. *b* and using this result yields

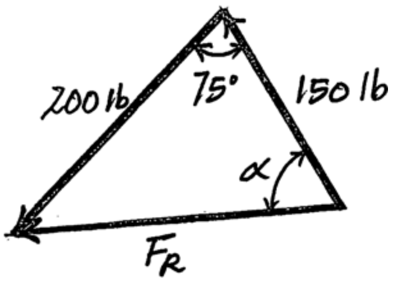
$$\frac{\sin\alpha}{200} = \frac{\sin 75°}{216.72} \qquad \alpha = 63.05°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured counterclockwise from the positive u axis, is

$$\phi = \alpha - 60° = 63.05° - 60° = 3.05°$$ **Ans.**

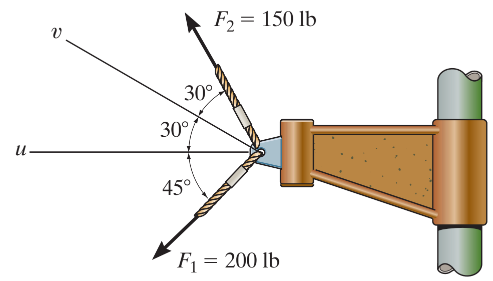


(a)



(b)

•2–5. Resolve $\mathbf{F}_1$ into components along the *u* and *v* axes, and determine the magnitudes of these components.

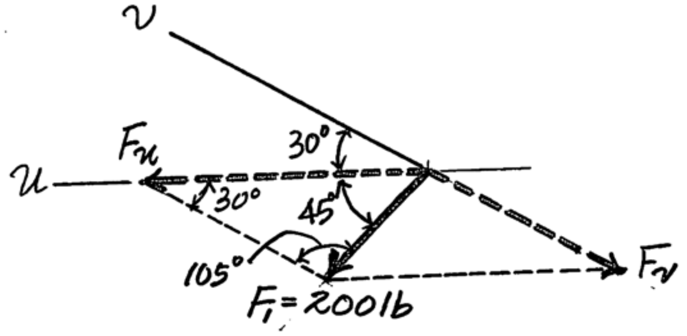


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

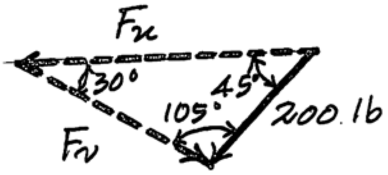
Applying the law of sines to Fig. *b*, yields

$$\frac{F_u}{\sin 105°} = \frac{200}{\sin 30°} \qquad F_u = 386 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 45°} = \frac{200}{\sin 30°} \qquad F_v = 283 \text{ lb} \qquad \textbf{Ans.}$$

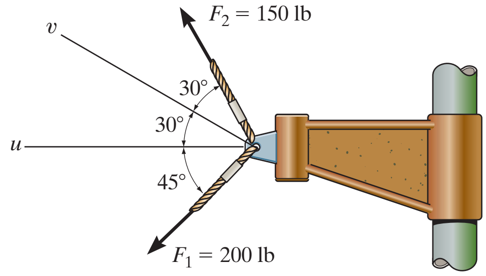


(a)



(b)

2–6. Resolve $\mathbf{F}_2$ into components along the u and v axes, and determine the magnitudes of these components.

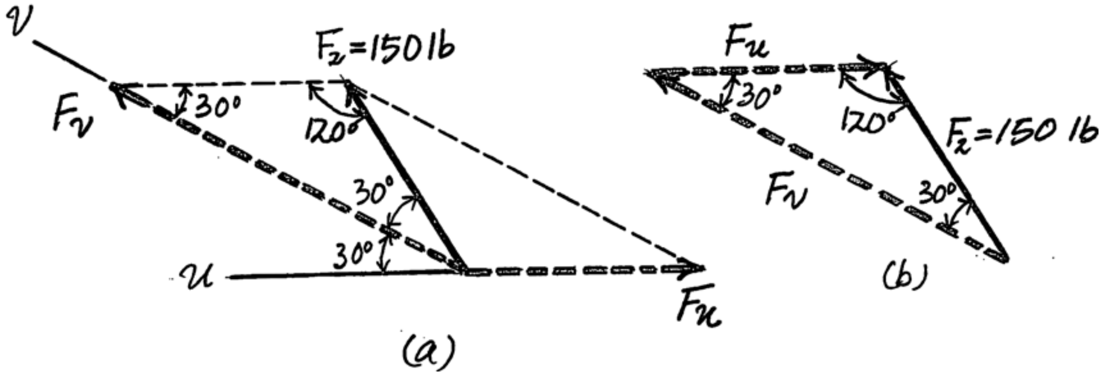


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

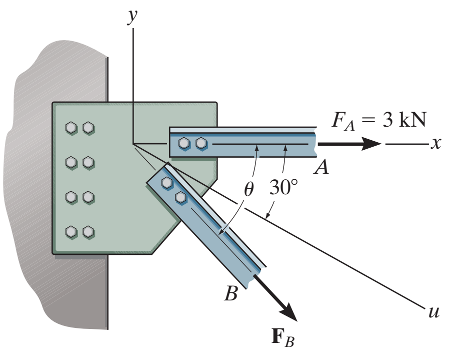
Applying the law of sines to Fig. *b*,

$$\frac{F_u}{\sin 30°} = \frac{150}{\sin 30°} \qquad F_u = 150 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 120°} = \frac{150}{\sin 30°} \qquad F_v = 260 \text{ lb} \qquad \textbf{Ans.}$$



2–7. If $F_B = 2$ kN and the resultant force acts along the positive u axis, determine the magnitude of the resultant force and the angle θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of sines to Fig. *b*, yields

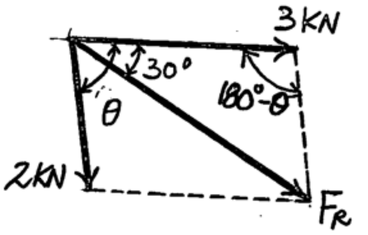
$$\frac{\sin\phi}{3} = \frac{\sin 30°}{2} \qquad \phi = 48.59°$$

Thus,

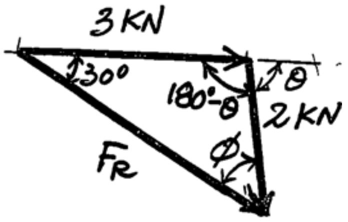
$$\theta = 30° + \phi = 30° + 48.59° = 78.59° = 78.6° \qquad \textbf{Ans.}$$

With the result $\theta = 78.59°$, applying the law of sines to Fig. *b* again, yields

$$\frac{F_R}{\sin(180° - 78.59°)} = \frac{2}{\sin 30°} \qquad F_R = 3.92 \text{ kN} \qquad \textbf{Ans.}$$

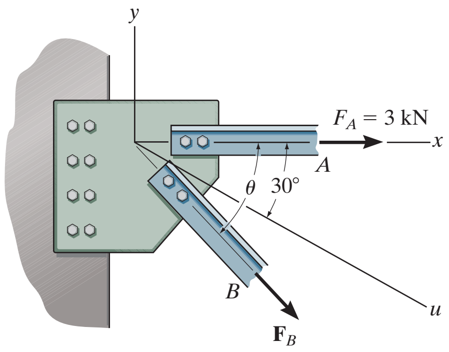


(a)



(b)

***2–8.** If the resultant force is required to act along the positive u axis and have a magnitude of 5 kN, determine the required magnitude of $\mathbf{F}_B$ and its direction θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

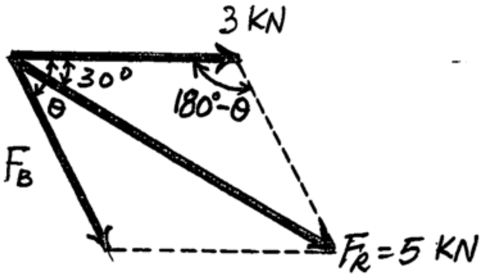
Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{3^2 + 5^2 - 2(3)(5)\cos 30^\circ}$$

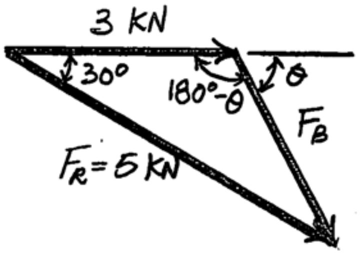
$$= 2.832 \text{ kN} = 2.83 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and realizing that $\sin(180^\circ - \theta) = \sin\theta$, the application of the sine law to Fig. *b*, yields

$$\frac{\sin\theta}{5} = \frac{\sin 30^\circ}{2.832} \qquad \theta = 62.0^\circ \qquad \textbf{Ans.}$$

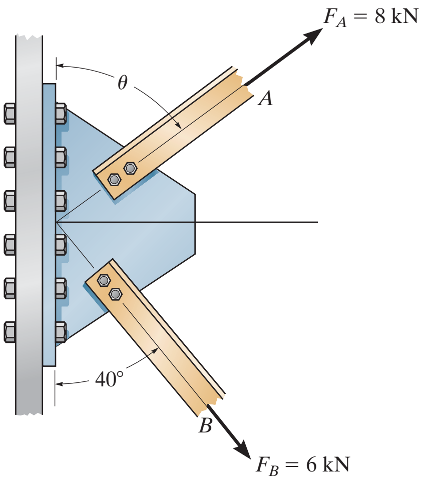


(a)



(b)

•2–9. The plate is subjected to the two forces at A and B as shown. If $\theta = 60^\circ$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of cosines [Fig. (b)], we have

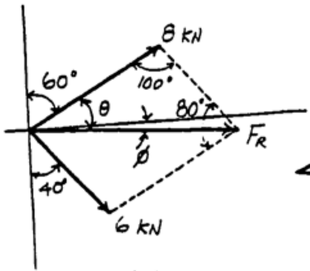
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$
$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans}$$

The angle θ can be determined using law of sines [Fig. (b)].

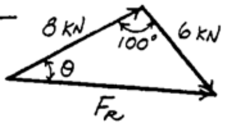
$$\frac{\sin\theta}{6} = \frac{\sin 100^\circ}{10.80}$$
$$\sin\theta = 0.5470$$
$$\theta = 33.16^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ \qquad \textbf{Ans}$$

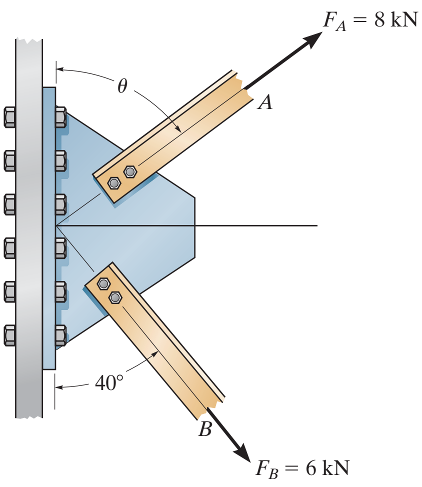


(a)



(b)

2–10. Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

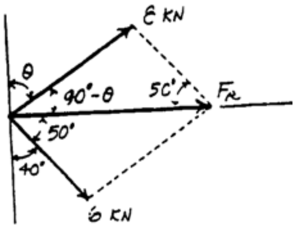
$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

$$\sin(90^\circ - \theta) = 0.5745$$

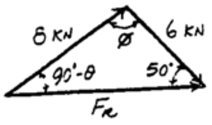
$$\theta = 54.93^\circ = 54.9^\circ \qquad \textbf{Ans}$$

From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93^\circ}$$
$$= 10.4 \text{ kN} \qquad \textbf{Ans}$$

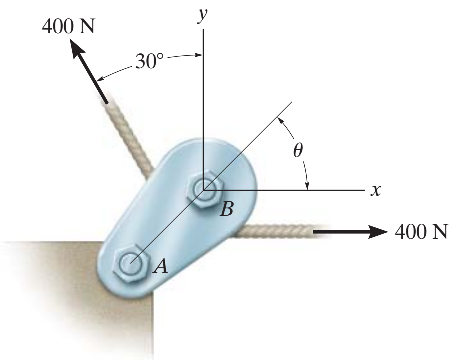


(a)



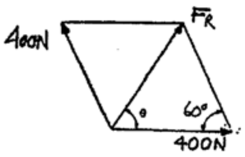
(b)

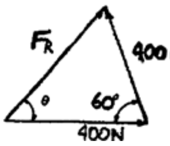
2–11. If the tension in the cable is 400 N, determine the magnitude and direction of the resultant force acting on the pulley. This angle is the same angle θ of line AB on the tailboard block.



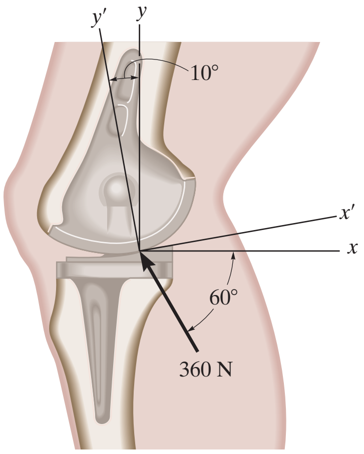
$$F_R = \sqrt{(400)^2 + (400)^2 - 2(400)(400)\cos 60^\circ} = 400 \text{ N} \qquad \textbf{Ans}$$

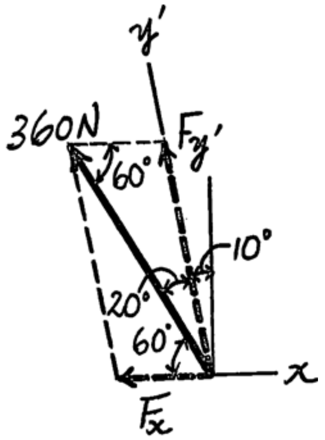
$$\frac{\sin\theta}{400} = \frac{\sin 60^\circ}{400}; \quad \theta = 60^\circ \qquad \textbf{Ans}$$





***2–12.** The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x and y' axes.

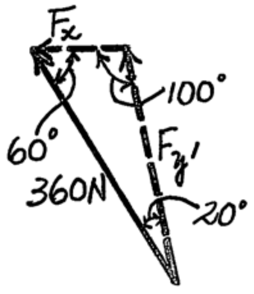




(a)

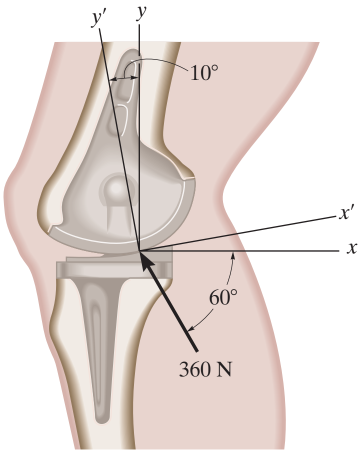
$$\frac{-F_x}{\sin 20^\circ} = \frac{360}{\sin 100^\circ}; \quad F_x = -125 \text{ N} \quad \textbf{Ans}$$

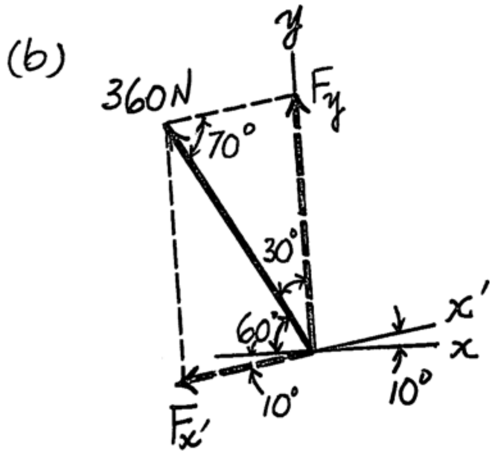
$$\frac{F_{y'}}{\sin 60^\circ} = \frac{360}{\sin 100^\circ}; \quad F_{y'} = 317 \text{ N} \quad \textbf{Ans}$$



(b)

•2–13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x' and y axes.

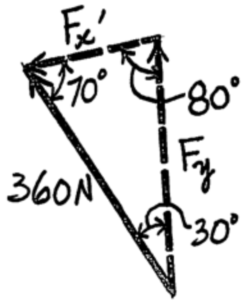




(a)

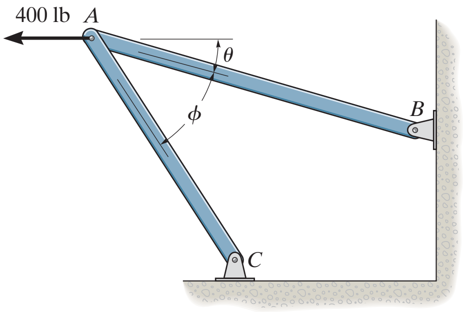
$$\frac{-F_{x'}}{\sin 30^\circ} = \frac{360}{\sin 80^\circ}; \quad F_{x'} = -183\ \text{N} \quad \text{Ans}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{360}{\sin 80^\circ}; \quad F_y = 344\ \text{N} \quad \text{Ans}$$



(b)

2–14. Determine the design angle θ ($0° \leq \theta \leq 90°$) for strut AB so that the 400-lb horizontal force has a component of 500 lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$
$$\sin\theta = 0.8035$$

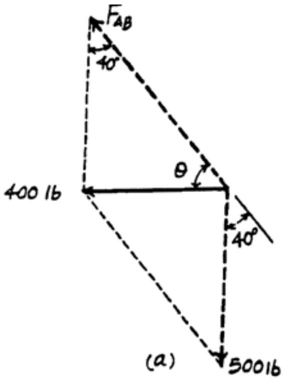
$$\theta = 53.46° = 53.5° \qquad \textbf{Ans}$$

Thus, $\psi = 180° - 40° - 53.46° = 86.54°$

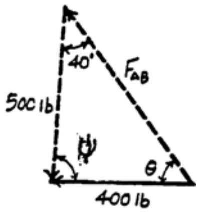
Using law of sines [Fig. (b)]

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621 \text{ lb} \qquad \textbf{Ans}$$

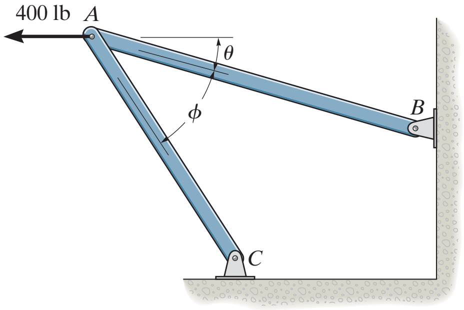


(a)



(b)

2–15. Determine the design angle ϕ ($0° \leq \phi \leq 90°$) between struts AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

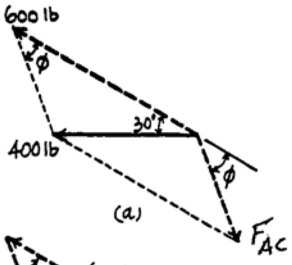
Trigonometry : Using law of cosines [Fig. (b)], we have

$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30°} = 322.97 \text{ lb}$$

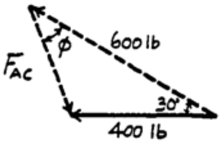
The angle ϕ can be determined using law of sines [Fig. (b)].

$$\frac{\sin\phi}{400} = \frac{\sin 30°}{322.97}$$
$$\sin\phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans}$$

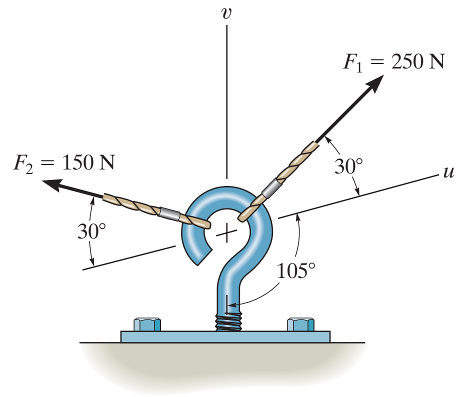


(a)



(b)

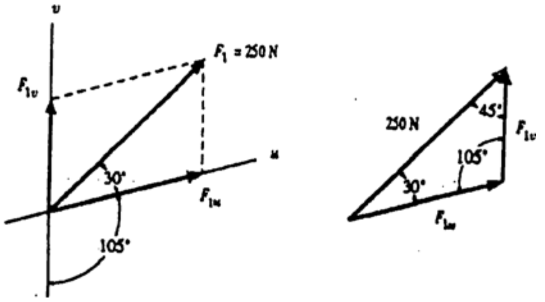
***2–16.** Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.



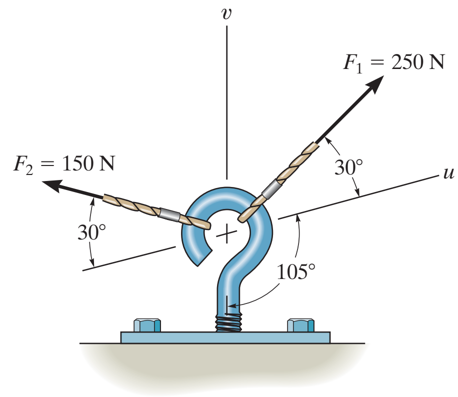
Sine law :

$$\frac{F_{1v}}{\sin 30°} = \frac{250}{\sin 105°} \qquad F_{1v} = 129\text{ N} \qquad \text{Ans}$$

$$\frac{F_{1u}}{\sin 45°} = \frac{250}{\sin 105°} \qquad F_{1u} = 183\text{ N} \qquad \text{Ans}$$



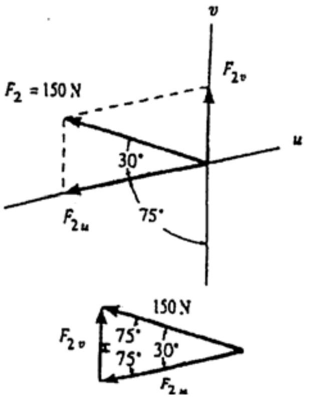
•2–17. Resolve $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of these components.



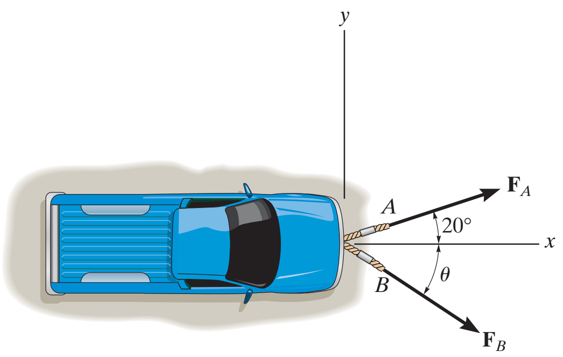
Sine law :

$$\frac{F_{2v}}{\sin 30°} = \frac{150}{\sin 75°} \qquad F_{2v} = 77.6\text{ N} \qquad \text{Ans}$$

$$\frac{F_{2u}}{\sin 75°} = \frac{150}{\sin 75°} \qquad F_{2u} = 150\text{ N} \qquad \text{Ans}$$



2–18. The truck is to be towed using two ropes. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope in order to develop a resultant force of 950 N directed along the positive x axis. Set $\theta = 50°$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

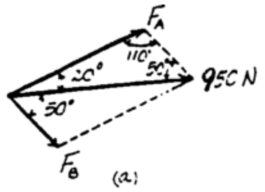
Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 50°} = \frac{950}{\sin 110°}$$

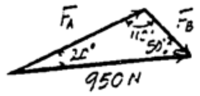
$$F_A = 774\text{ N} \qquad \textbf{Ans}$$

$$\frac{F_B}{\sin 20°} = \frac{950}{\sin 110°}$$

$$F_B = 346\text{ N} \qquad \textbf{Ans}$$

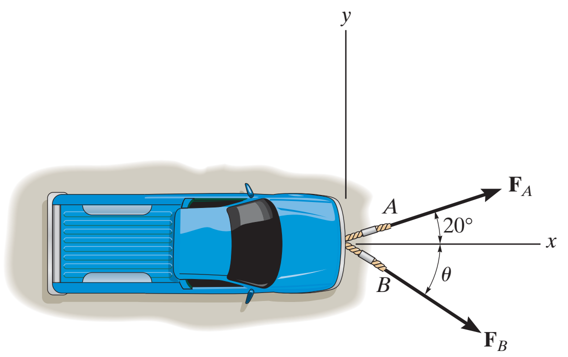


(a)



(b)

2–19. The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive x axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 20° from the x axis as shown.



Parallelogram Law : In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).

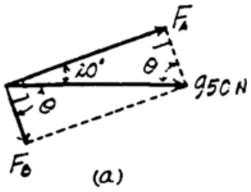
Trigonometry : Fig. (b).

$$F_B = 950\sin 20° = 325\text{ N} \qquad \textbf{Ans}$$

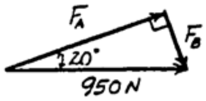
$$F_A = 950\cos 20° = 893\text{ N} \qquad \textbf{Ans}$$

The angle θ is

$$\theta = 90° - 20° = 70.0° \qquad \textbf{Ans}$$

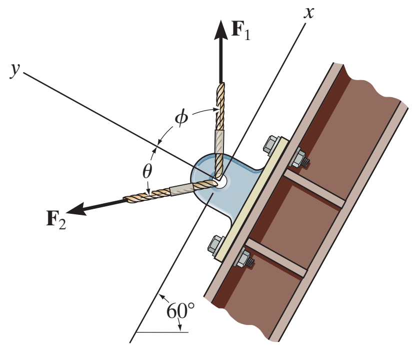


(a)



(b)

***2–20.** If $\phi = 45^\circ$, $F_1 = 5$ kN, and the resultant force is 6 kN directed along the positive y axis, determine the required magnitude of $\mathbf{F}_2$ and its direction θ.



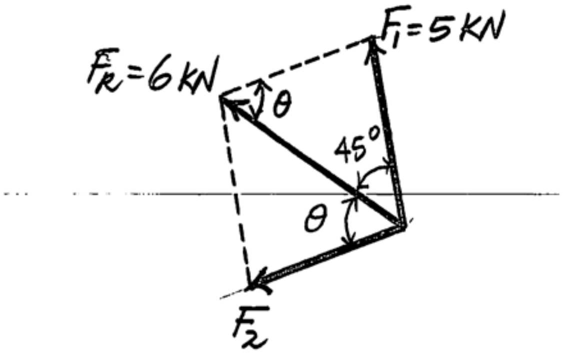
The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

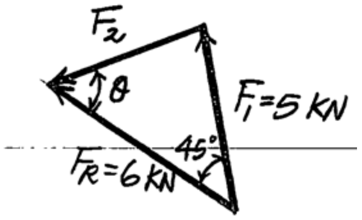
$$F_2 = \sqrt{6^2 + 5^2 - 2(6)(5)\cos 45^\circ}$$
$$= 4.310 \text{ kN} = 4.31 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{5} = \frac{\sin 45^\circ}{4.310} \qquad \theta = 55.1^\circ \quad \textbf{Ans.}$$

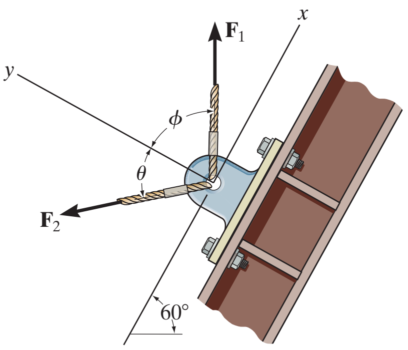


(a)



(b)

•2–21. If $\phi = 30°$ and the resultant force is to be 6 kN directed along the positive y axis, determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle θ if F_2 is required to be a minimum.

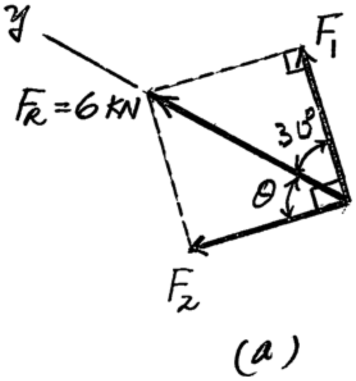


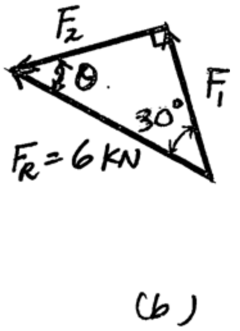
For $\mathbf{F}_2$ to be minimum, it has to be directed perpendicular to F_R.
The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.
By applying simple trigonometry to Fig. *b*,

$$F_1 = 6\cos 30° = 5.20 \text{ kN} \qquad \textbf{Ans.}$$
$$F_2 = 6\sin 30° = 3 \text{ kN} \qquad \textbf{Ans.}$$

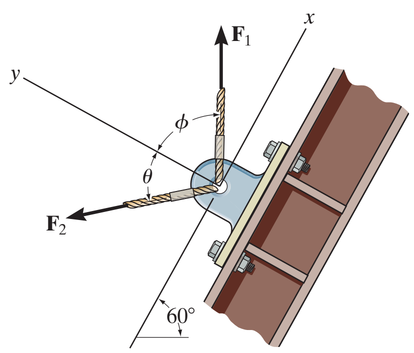
and

$$\theta = 90° - 30° = 60° \quad \textbf{Ans.}$$





2–22. If $\phi = 30°$, $F_1 = 5$ kN, and the resultant force is to be directed along the positive y axis, determine the magnitude of the resultant force if F_2 is to be a minimum. Also, what is F_2 and the angle θ?



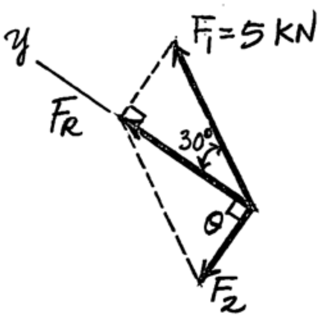
Parallelogram Law and Triangular Rule: The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

For $\mathbf{F}_2$ to be minimum, it must be directed perpendicular to the resultant force. Thus,

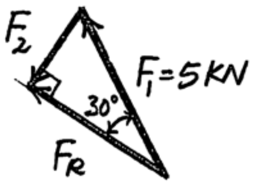
$$\theta = 90° \qquad \textbf{Ans.}$$

By applying simple trigonometry to Fig. *b*,

$$F_2 = 5 \sin 30° = 2.50 \text{ kN} \qquad \textbf{Ans.}$$
$$F_R = 5 \cos 30° = 4.33 \text{ kN} \qquad \textbf{Ans.}$$

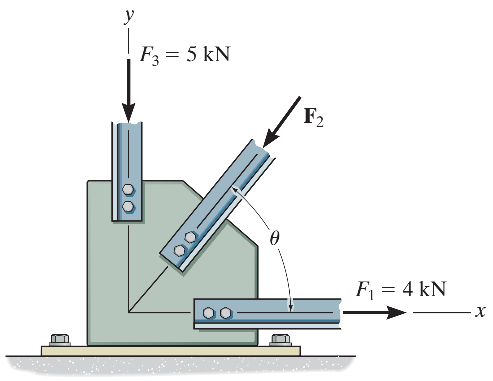


(a)



(b)

2–23. If $\theta = 30°$ and $F_2 = 6$ kN, determine the magnitude of the resultant force acting on the plate and its direction measured clockwise from the positive x axis.



Parallelogram Law and Triangular Rule: This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelogram.

Determination of Unknowns: Referring to Fig. *b*, F' and α can be determined as follows.

$$F' = \sqrt{4^2 + 5^2} = 6.403 \text{ kN}$$

$$\tan\alpha = \frac{5}{4} \qquad \alpha = 51.34°$$

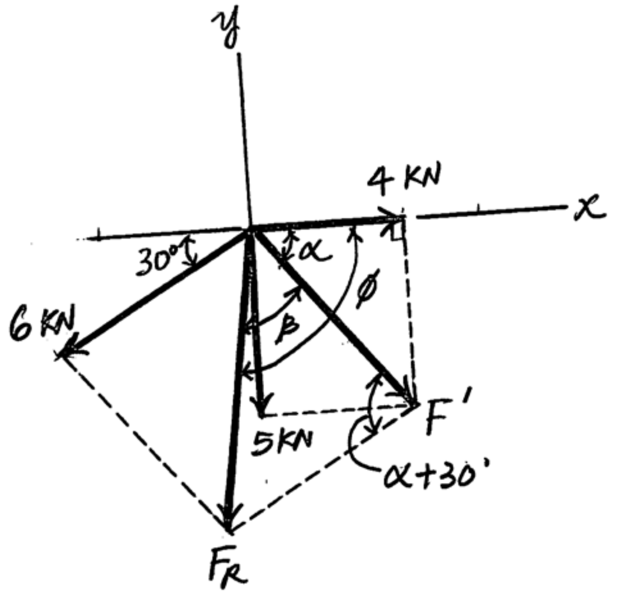
Using the results for F' and α and referring to Fig. *c*, $\mathbf{F}_R$ and β can be determined.

$$F_R = \sqrt{6^2 + 6.403^2 - 2(6)(6.403)\cos(51.34° + 30°)}$$
$$= 8.089 \text{ kN} = 8.09 \text{ kN} \qquad \textbf{Ans.}$$

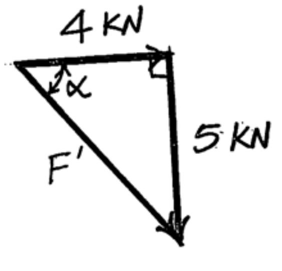
$$\frac{\sin\beta}{6} = \frac{\sin(51.34° + 30°)}{8.089} \qquad \beta = 47.16°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

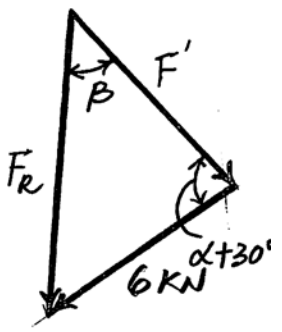
$$\phi = \alpha + \beta = 51.34° + 47.16° = 98.5° \qquad \textbf{Ans.}$$



(a)

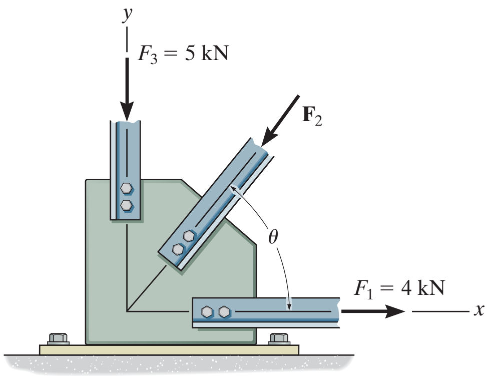


(b)



(c)

***2–24.** If the resultant force $\mathbf{F}_R$ is directed along a line measured 75° clockwise from the positive x axis and the magnitude of $\mathbf{F}_2$ is to be a minimum, determine the magnitudes of $\mathbf{F}_R$ and $\mathbf{F}_2$ and the angle $\theta \leq 90°$.



This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelograms. For $\mathbf{F}_1$ to be minimum, it must be perpendicular to the resultant force's line of action. Thus,

$$\theta = 90° - 75° = 15° \qquad \textbf{Ans.}$$

Referring to Fig. *b*, F' and α can be determined.

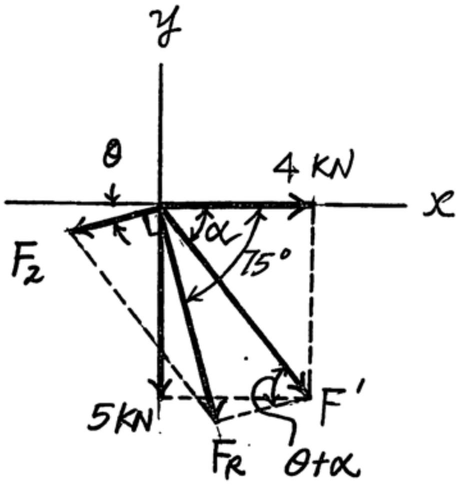
$$F' = \sqrt{4^2 + 5^2} = 6.403\text{ kN}$$

$$\tan\alpha = \frac{5}{4} \qquad \alpha = 51.34°$$

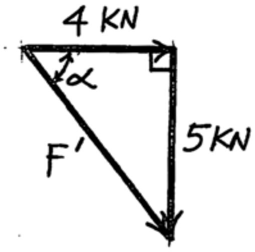
Using the results for θ, α, and F', $\mathbf{F}_R$ and $\mathbf{F}_2$ can be determined by referring to Fig. *c*.

$$F_2 = 6.403\cos(15° + 51.43°) = 2.57\text{ kN} \qquad \textbf{Ans.}$$

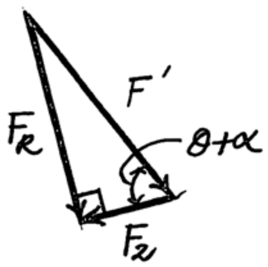
$$F_R = 6.403\sin(15° + 51.43°) = 5.86\text{ kN} \qquad \textbf{Ans.}$$



(a)

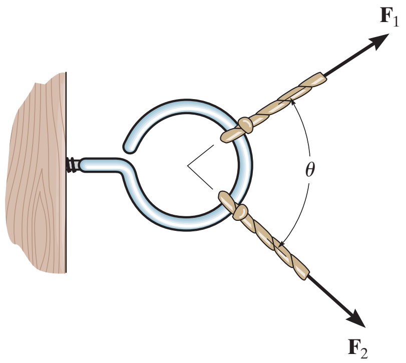


(b)



(c)

•2–25. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



$$\frac{F}{\sin\phi} = \frac{F}{\sin(\theta - \phi)}$$

$$\sin(\theta - \phi) = \sin\phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$$

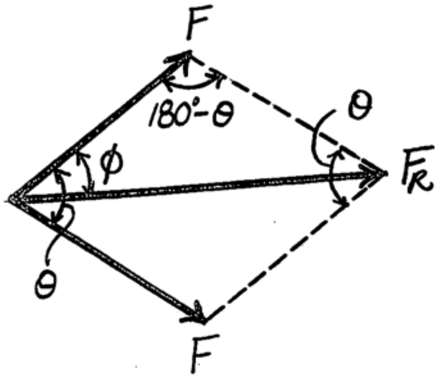
Since $\cos(180^\circ - \theta) = -\cos\theta$

$$F_R = F(\sqrt{2})\sqrt{1 + \cos\theta}$$

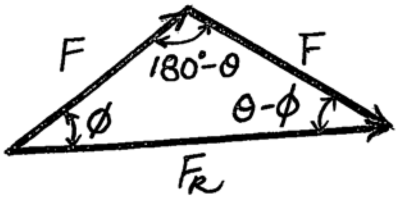
Since $\cos(\frac{\theta}{2}) = \sqrt{\frac{1 + \cos\theta}{2}}$

Then

$$F_R = 2F\cos(\frac{\theta}{2}) \qquad \textbf{Ans}$$

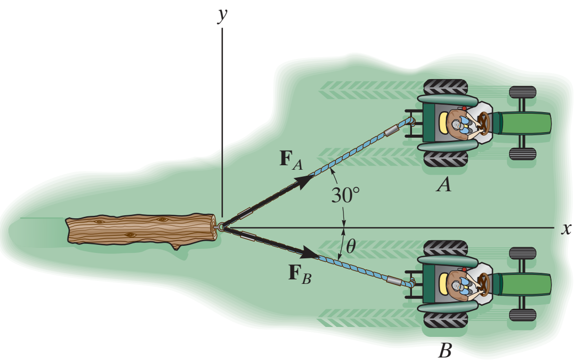


(a)



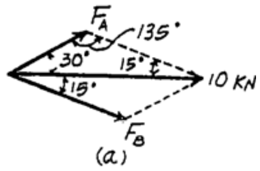
(b)

2–26. The log is being towed by two tractors *A* and *B*. Determine the magnitudes of the two towing forces $\mathbf{F}_A$ and $\mathbf{F}_B$ if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the *x* axis. Set $\theta = 15°$.



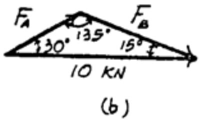
Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have



$$\frac{F_A}{\sin 15°} = \frac{10}{\sin 135°}$$

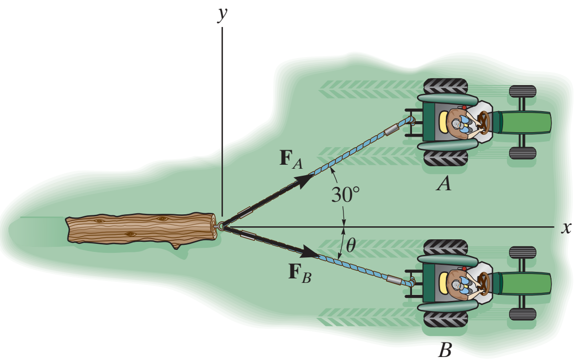
$$F_A = 3.66 \text{ kN} \qquad \textbf{Ans}$$



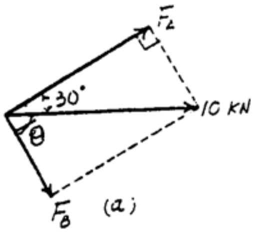
$$\frac{F_B}{\sin 30°} = \frac{10}{\sin 135°}$$

$$F_B = 7.07 \text{ kN} \qquad \textbf{Ans}$$

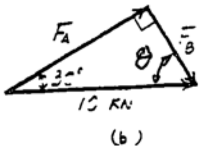
2–27. The resultant $\mathbf{F}_R$ of the two forces acting on the log is to be directed along the positive *x* axis and have a magnitude of 10 kN, determine the angle θ of the cable, attached to *B* such that the magnitude of force $\mathbf{F}_B$ in this cable is a minimum. What is the magnitude of the force in each cable for this situation?



Parallelogram Law : In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).



Trigonometry : Fig. (b).



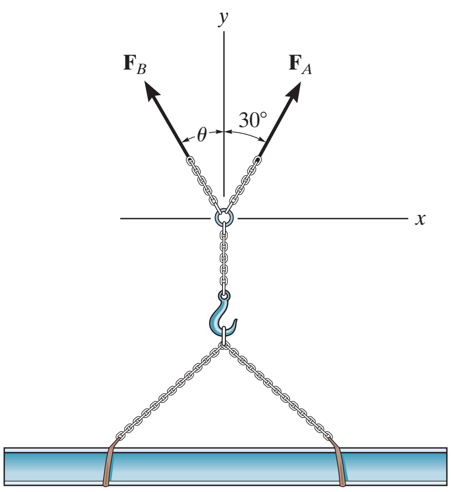
$$F_B = 10 \sin 30° = 5.00 \text{ kN} \qquad \textbf{Ans}$$

$$F_A = 10 \cos 30° = 8.66 \text{ kN} \qquad \textbf{Ans}$$

The angle θ is

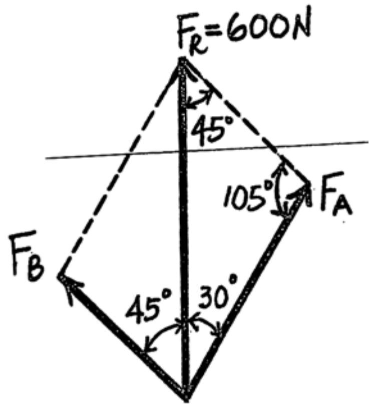
$$\theta = 90° - 30° = 60.0° \qquad \textbf{Ans}$$

***2–28.** The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45°$.

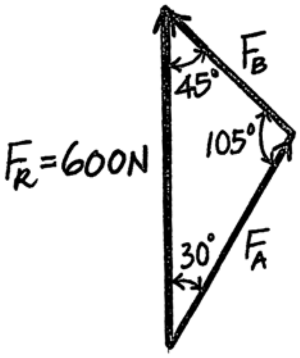


$\dfrac{F_A}{\sin 45°} = \dfrac{600}{\sin 105°};$ $F_A = 439\text{ N}$ Ans

$\dfrac{F_B}{\sin 30°} = \dfrac{600}{\sin 105°};$ $F_B = 311\text{ N}$ Ans

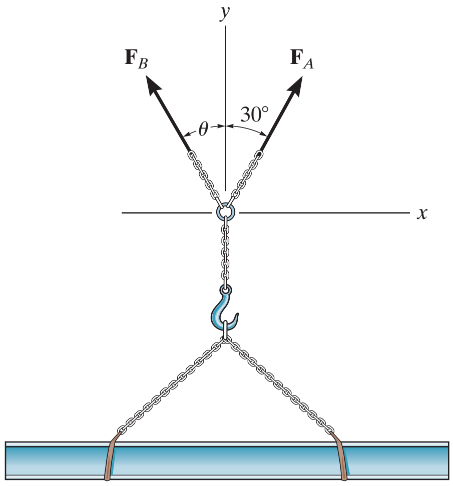


(a)



(b)

•2–29. The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive y axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the y axis, as shown.

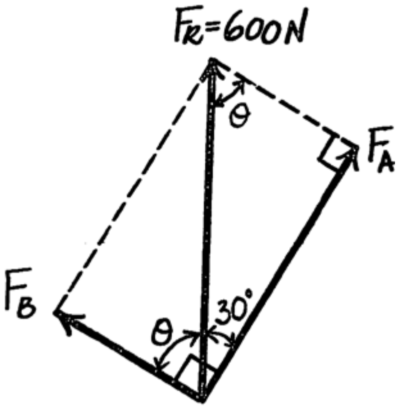


For minimum F_B, require

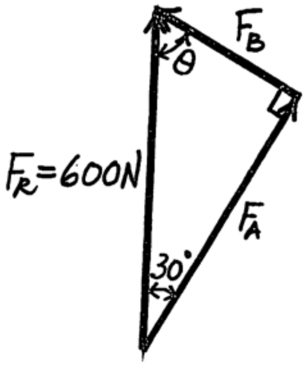
$\theta = 60°$ **Ans**

$F_A = 600 \cos 30° = 520\text{ N}$ **Ans**

$F_B = 600 \sin 30° = 300\text{ N}$ **Ans**

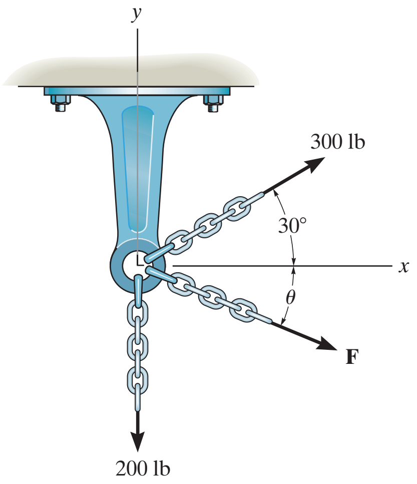


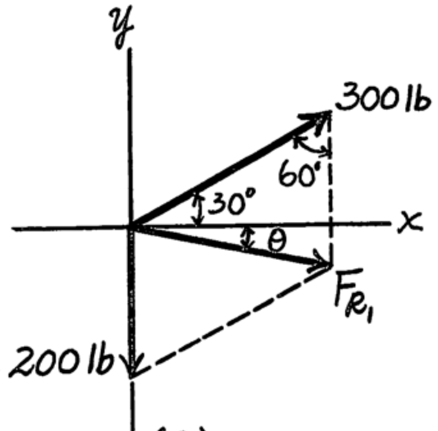
(a)



(b)

2–30. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle θ of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.





(a)

Cosine law :

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$

Sine law :

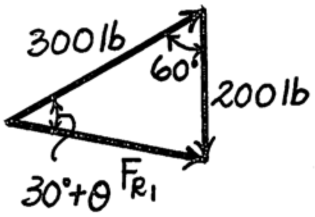
$$\frac{\sin(30^\circ + \theta)}{200} = \frac{\sin 60^\circ}{264.6} \qquad \theta = 10.9^\circ \qquad \textbf{Ans}$$

When **F** is directed along F_{R1} , F will be minimum to create the resultant force.

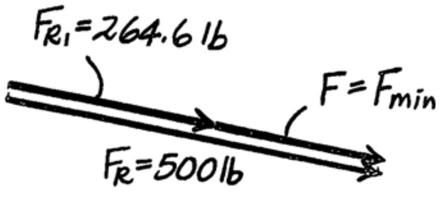
$F_R = F_{R1} + F$

$500 = 264.6 + F_{min}$

$F_{min} = 235 \text{ lb}$ **Ans**

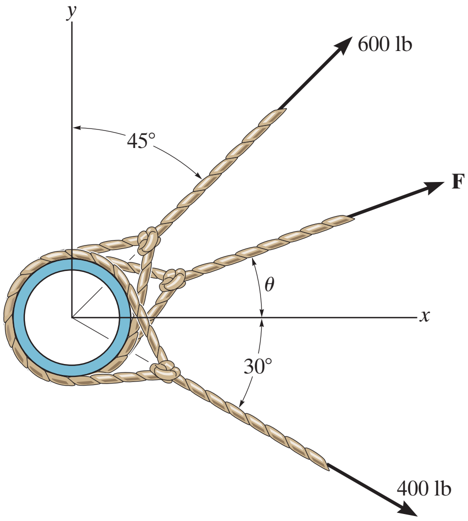


(b)



(c)

2–31. Three cables pull on the pipe such that they create a resultant force having a magnitude of 900 lb. If two of the cables are subjected to known forces, as shown in the figure, determine the angle θ of the third cable so that the magnitude of force **F** in this cable is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces.

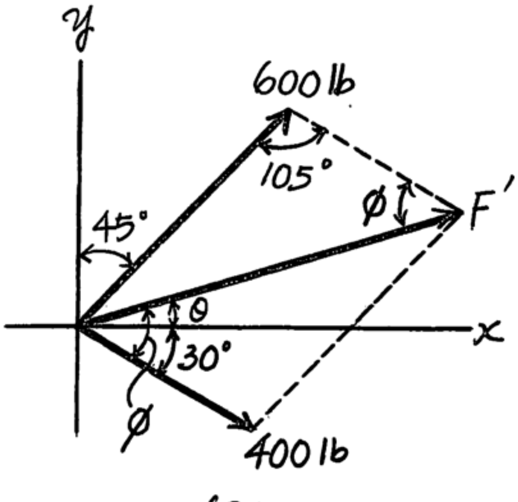


$$F' = \sqrt{(600)^2 + (400)^2 - 2(600)(400)\cos 105^\circ} = 802.64 \text{ lb}$$

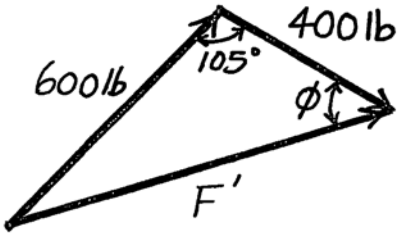
$$F = 900 - 802.64 = 97.4 \text{ lb} \qquad \text{Ans}$$

$$\frac{\sin\phi}{600} = \frac{\sin 105^\circ}{802.64}; \quad \phi = 46.22^\circ$$

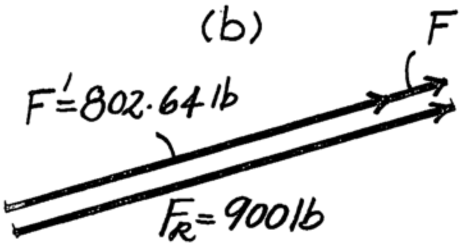
$$\theta = 46.22^\circ - 30^\circ = 16.2^\circ \qquad \text{Ans}$$



(a)

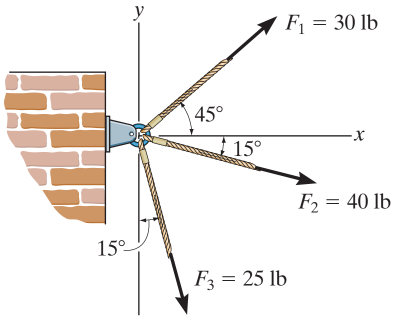


(b)



(c)

***2–32.** Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive *x* axis.



Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 30\cos 45° = 21.21 \text{ lb} \qquad (F_1)_y = 30\sin 45° = 21.21 \text{ lb}$$
$$(F_2)_x = 40\cos 15° = 38.64 \text{ lb} \qquad (F_2)_y = 40\sin 15° = 10.35 \text{ lb}$$
$$(F_3)_x = 25\sin 15° = 6.47 \text{ lb} \qquad (F_3)_y = 25\cos 15° = 24.15 \text{ lb}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

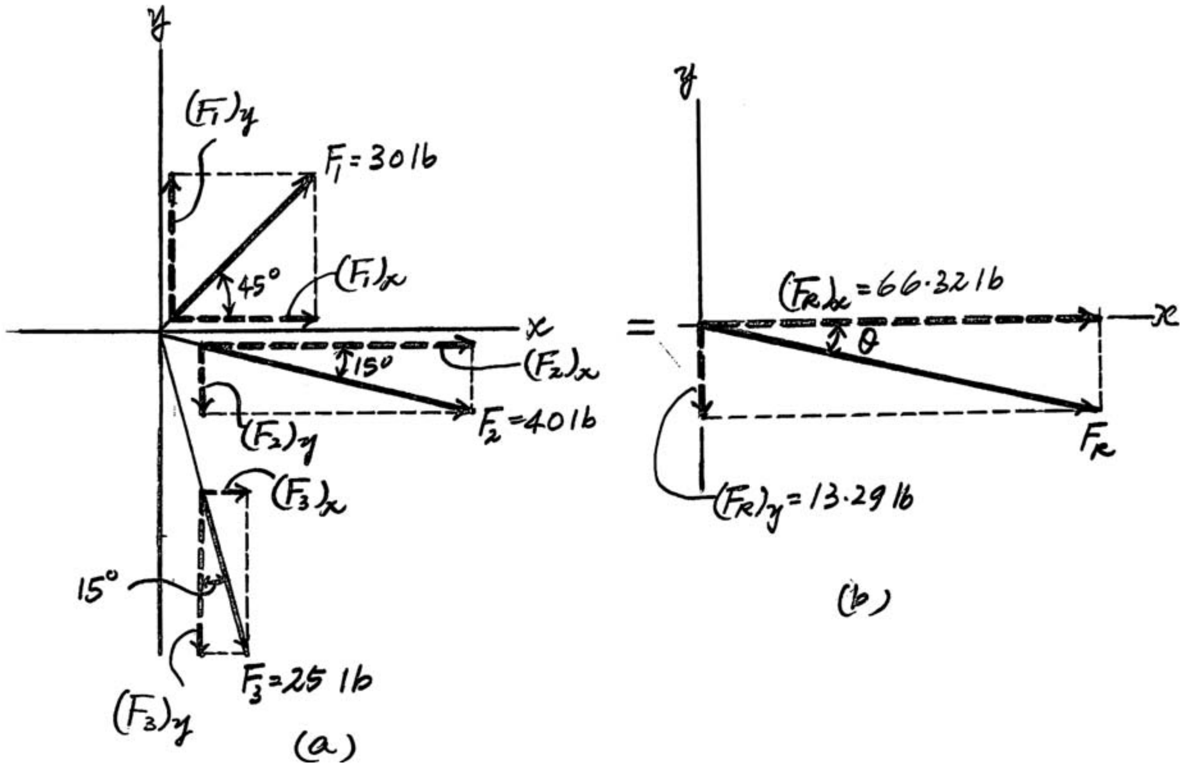
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 21.21 + 38.64 + 6.47 = 66.32 \text{ lb} \rightarrow$$
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 21.21 - 10.35 - 24.15 = -13.29 \text{ lb} = 13.29 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

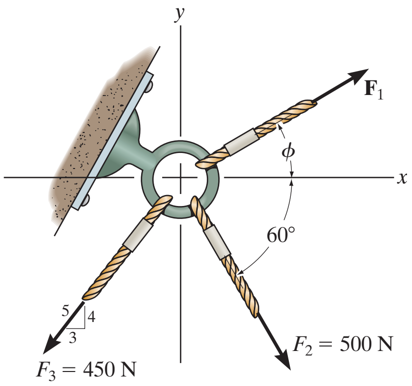
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{66.32^2 + 13.29^2} = 67.6 \text{ lb} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive *x* axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{13.29}{66.32}\right) = 11.3° \qquad \textbf{Ans.}$$



•2–33. If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the x and y components of each force can be written as

$$(F_1)_x = 600\cos 30° = 519.62\text{ N} \qquad (F_1)_y = 600\sin 30° = 300\text{ N}$$
$$(F_2)_x = 500\cos 60° = 250\text{ N} \qquad (F_2)_y = 500\sin 60° = 433.0\text{ N}$$
$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

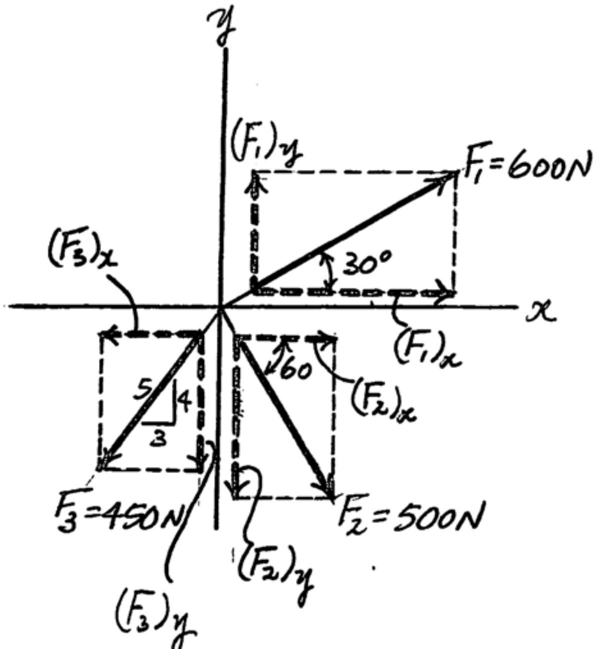
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62\text{ N} \rightarrow$$
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01\text{N} = 493.01\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

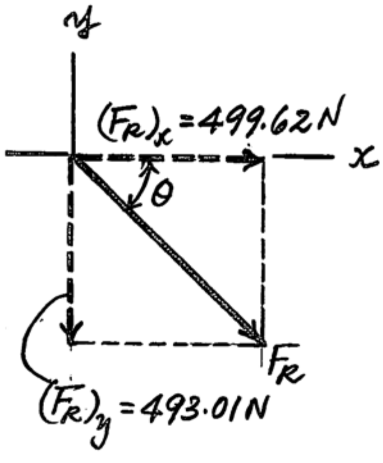
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{499.62^2 + 493.01^2} = 701.91\text{ N} = 702\text{ N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6° \qquad \textbf{Ans.}$$

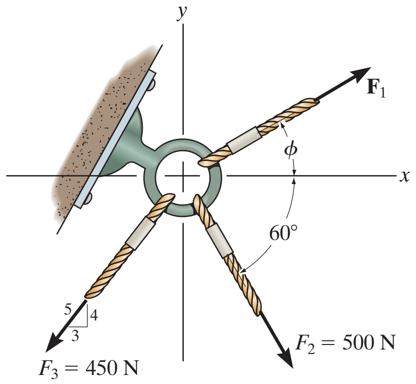


(a)



(b)

2–34. If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle ϕ.



Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1\cos\phi \qquad (F_1)_y = F_1\sin\phi$$

$$(F_2)_x = 500\cos 60° = 250\text{ N} \qquad (F_2)_y = 500\sin 60° = 433.01\text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

$$(F_R)_x = 600\cos 30° = 519.62\text{ N} \qquad (F_R)_y = 600\sin 30° = 300\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 519.62 = F_1\cos\phi + 250 - 270$$

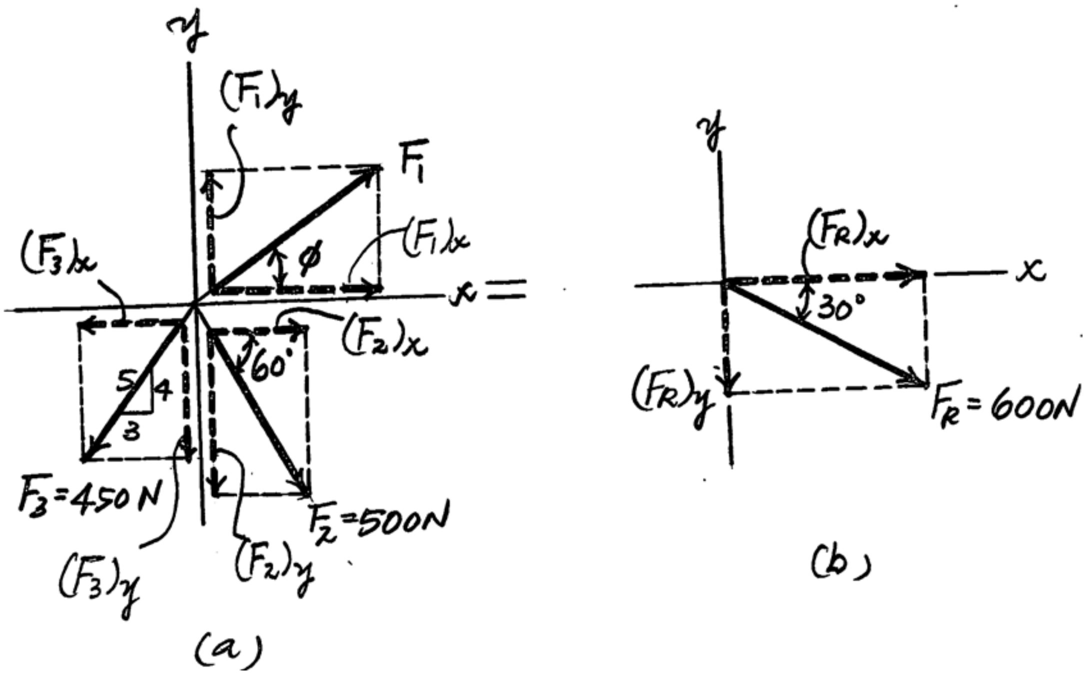
$$F_1\cos\phi = 539.62 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad -300 = F_1\sin\phi - 433.01 - 360$$

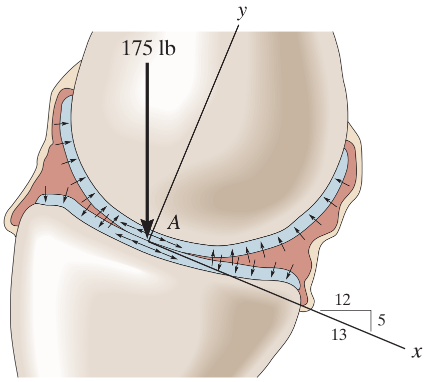
$$F_1\sin\phi = 493.01 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 42.4°$ $\qquad F_1 = 731$ N $\qquad$ **Ans.**

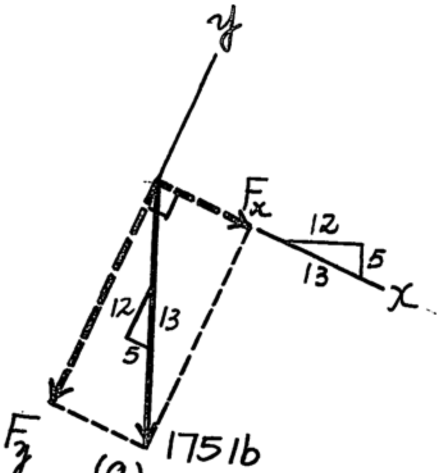


2–35. The contact point between the femur and tibia bones of the leg is at A. If a vertical force of 175 lb is applied at this point, determine the components along the x and y axes. Note that the y component represents the normal force on the load-bearing region of the bones. Both the x and y components of this force cause synovial fluid to be squeezed out of the bearing space.

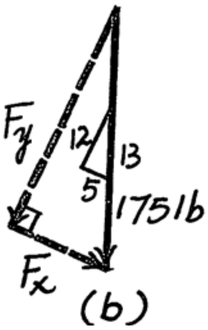


$$F_x = 175\left(\frac{5}{13}\right) = 67.3 \text{ lb} \qquad \text{Ans}$$

$$F_y = -175\left(\frac{12}{13}\right) = -162 \text{ lb} \qquad \text{Ans}$$

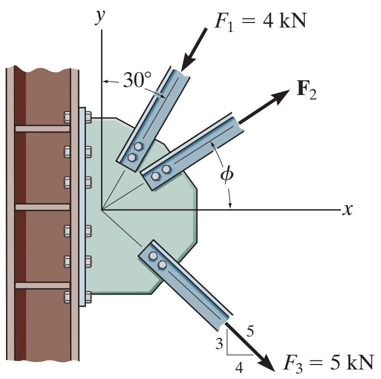


(a)



(b)

***2–36.** If $\phi = 30°$ and $F_2 = 3$ kN, determine the magnitude of the resultant force acting on the plate and its direction θ measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 4\sin 30° = 2\text{ kN} \qquad (F_1)_y = 4\cos 30° = 3.464\text{ kN}$$

$$(F_2)_x = 3\cos 30° = 2.598\text{ kN} \qquad (F_2)_y = 3\sin 30° = 1.50\text{ kN}$$

$$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN} \qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{kN}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = -2 + 2.598 + 4 = 4.598\text{ kN} \rightarrow$$

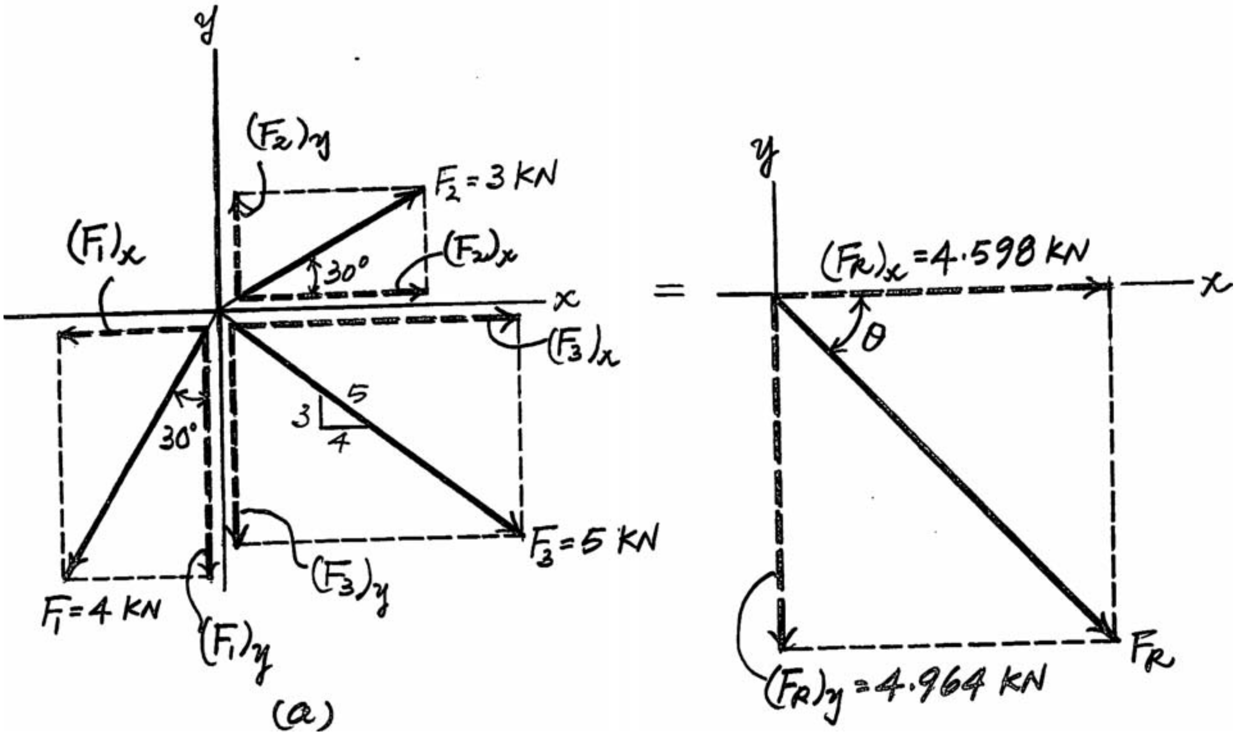
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = -3.464 + 1.50 - 3 = -4.964\text{ kN} = 4.964\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{4.598^2 + 4.964^2} = 6.77\text{ kN} \qquad \textbf{Ans.}$$

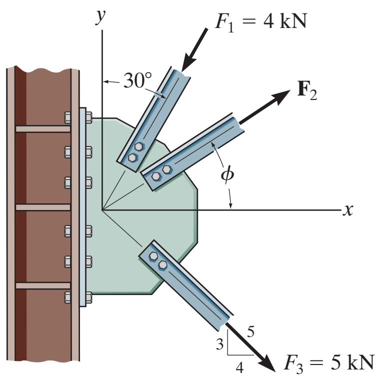
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{4.964}{4.598}\right) = 47.2° \qquad \textbf{Ans.}$$



(a)

•2–37. If the magnitude for the resultant force acting on the plate is required to be 6 kN and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_2$ and its direction ϕ.



Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = 4\sin 30° = 2\text{ kN} \qquad (F_1)_y = 4\cos 30° = 3.464\text{ kN}$$

$$(F_2)_x = F_2\cos\phi \qquad (F_2)_y = F_2\sin\phi$$

$$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN} \qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{ kN}$$

$$(F_R)_x = 6\cos 30° = 5.196\text{ kN} \qquad (F_R)_y = 6\sin 30° = 3\text{ kN}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 5.196 = -2 + F_2\cos\phi + 4$$

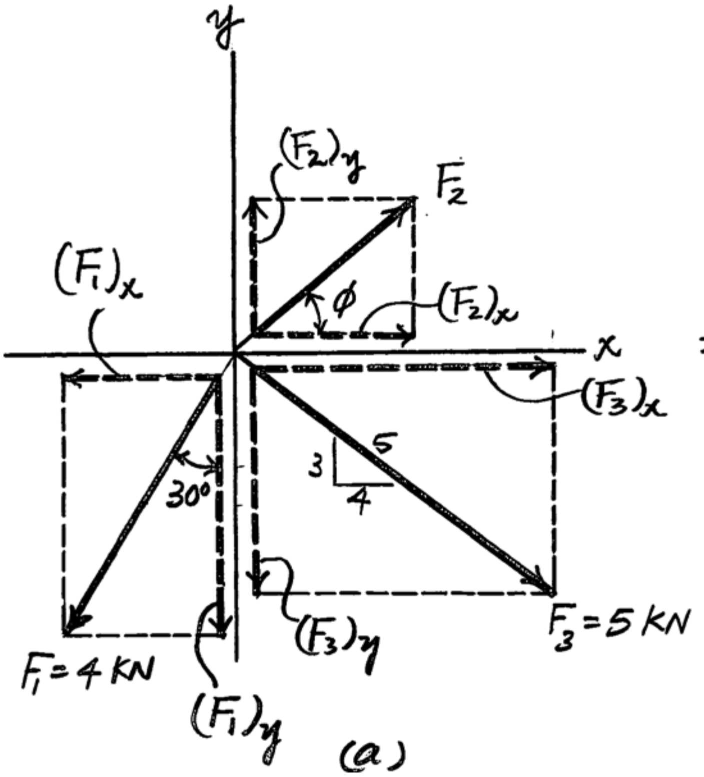
$$F_2\cos\phi = 3.196 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad -3 = -3.464 + F_2\sin\phi - 3$$

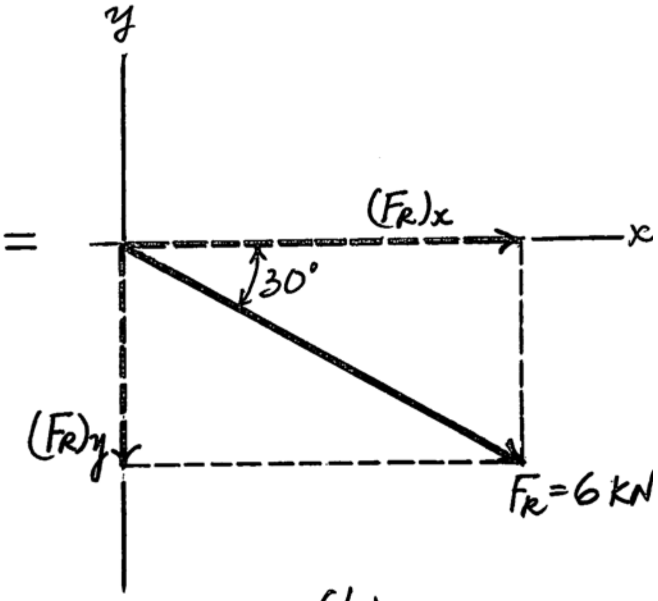
$$F_2\sin\phi = 3.464 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$$\phi = 47.3° \qquad F_2 = 4.71\text{ kN} \qquad \textbf{Ans.}$$

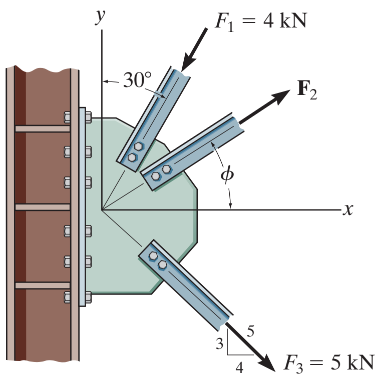


(a)



(b)

2–38. If $\phi = 30°$ and the resultant force acting on the gusset plate is directed along the positive x axis, determine the magnitudes of $\mathbf{F}_2$ and the resultant force.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = 4\sin 30° = 2\text{ kN}$ $\qquad (F_1)_y = 4\cos 30° = 3.464\text{ kN}$

$(F_2)_x = F_2\cos 30° = 0.8660F_2$ $\qquad (F_2)_y = F_2\sin 30° = 0.5F_2$

$(F_3)_x = 5\left(\frac{4}{5}\right) = 4\text{ kN}$ $\qquad (F_3)_y = 5\left(\frac{3}{5}\right) = 3\text{kN}$

$(F_R)_x = F_R$ $\qquad (F_R)_y = 0$

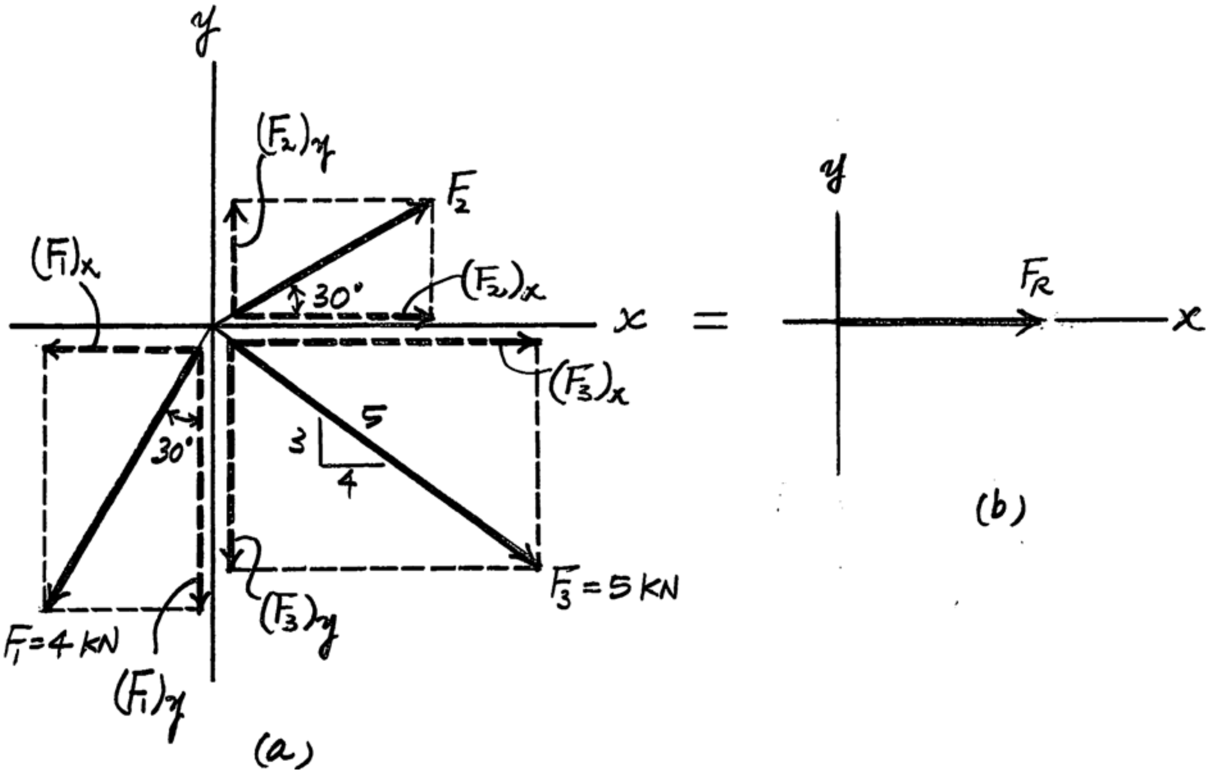
Resultant Force: Summing the force components algebraically along the x and y axes,

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad 0 = -3.464 + 0.5F_2 - 3$$

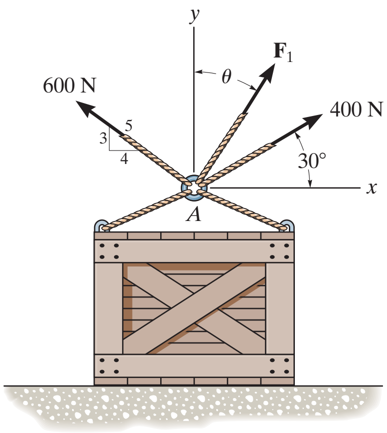
$$F_2 = 12.93\text{ kN} = 12.9\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad F_R = -2 + 0.8660(12.93) + 4$$

$$= 13.2\text{ kN} \qquad \textbf{Ans.}$$



2–39. Determine the magnitude of $\mathbf{F}_1$ and its direction θ so that the resultant force is directed vertically upward and has a magnitude of 800 N.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 0 = F_1 \sin\theta + 400\cos 30° - 600\left(\frac{4}{5}\right)$$

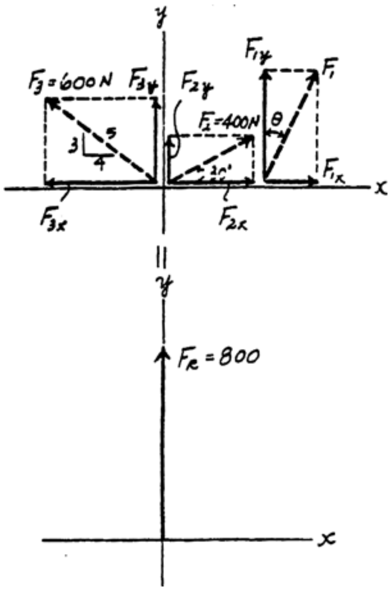
$$F_1 \sin\theta = 133.6 \qquad [1]$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 800 = F_1 \cos\theta + 400\sin 30° + 600\left(\frac{3}{5}\right)$$

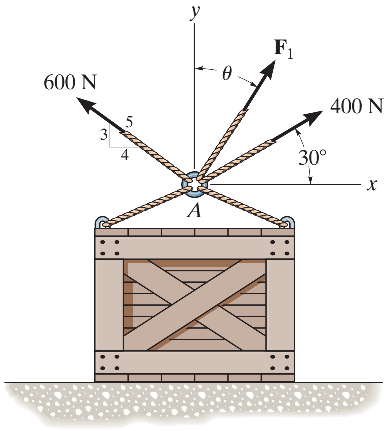
$$F_1 \cos\theta = 240 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 29.1°$ $\quad F_1 = 275$ N $\quad$ **Ans**



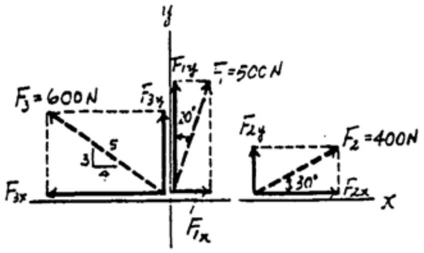
***2–40.** Determine the magnitude and direction measured counterclockwise from the positive *x* axis of the resultant force of the three forces acting on the ring *A*. Take $F_1 = 500$ N and $\theta = 20°$.



Scalar Notation : Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 500\sin 20° + 400\cos 30° - 600\left(\frac{4}{5}\right) = 37.42 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 500\cos 20° + 400\sin 30° + 600\left(\frac{3}{5}\right) = 1029.8 \text{ N} \uparrow$$

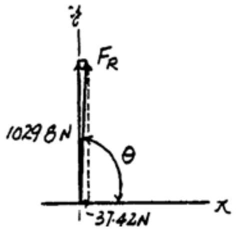


The magnitude of the resultant force $\mathbf{F}_R$ is

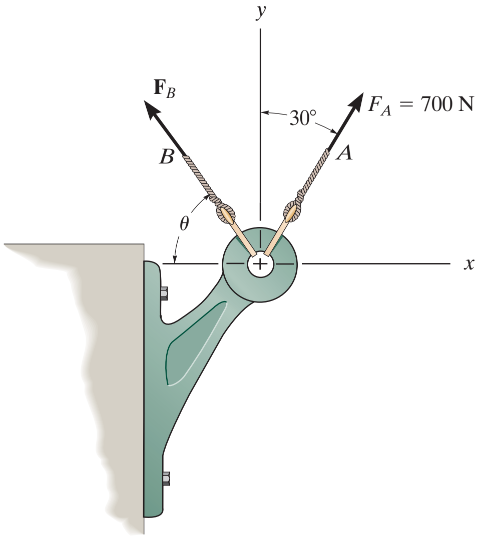
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN} \qquad \textbf{Ans}$$

The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{1029.8}{37.42}\right) = 87.9° \qquad \textbf{Ans}$$



•2–41. Determine the magnitude and direction θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



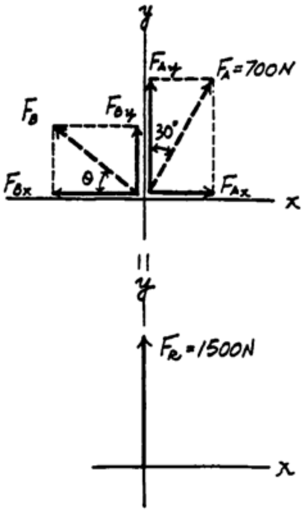
Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad 0 = 700 \sin 30° - F_B \cos\theta$$
$$F_B \cos\theta = 350 \qquad [1]$$

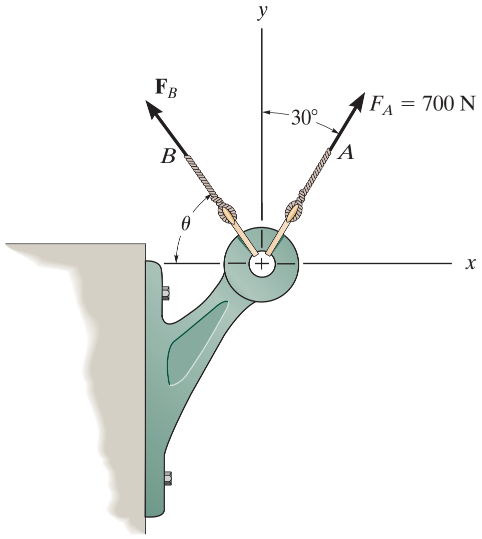
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad 1500 = 700 \cos 30° + F_B \sin\theta$$
$$F_B \sin\theta = 893.8 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 68.6°$ $F_B = 960$ N   **Ans**



2–42. Determine the magnitude and angle measured counterclockwise from the positive y axis of the resultant force acting on the bracket if $F_B = 600$ N and $\theta = 20°$.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 700\sin 30° - 600\cos 20° = -213.8\text{ N} = 213.8\text{ N} \leftarrow$$

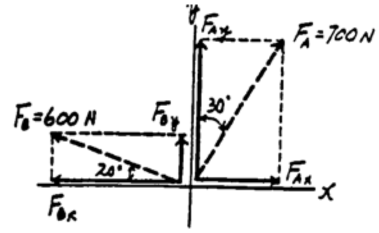
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 700\cos 30° + 600\sin 20° = 811.4\text{ N} \uparrow$$

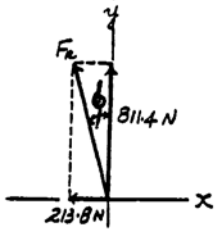
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839\text{ N} \qquad \text{Ans}$$

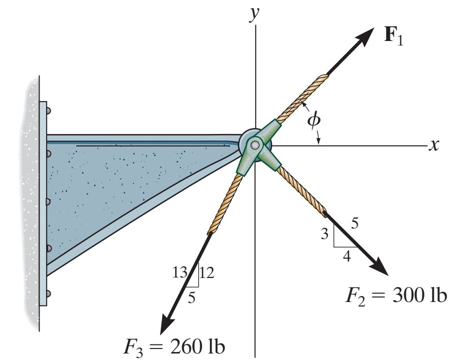
The directional angle θ measured counterclockwise from positive y axis is

$$\phi = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \text{Ans}$$





2–43. If $\phi = 30°$ and $F_1 = 250$ lb, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 250\cos 30° = 216.51\text{ lb} \qquad (F_1)_y = 250\sin 30° = 125\text{ lb}$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240\text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180\text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 216.51 + 240 - 100 = 356.51\text{ lb} \rightarrow$$

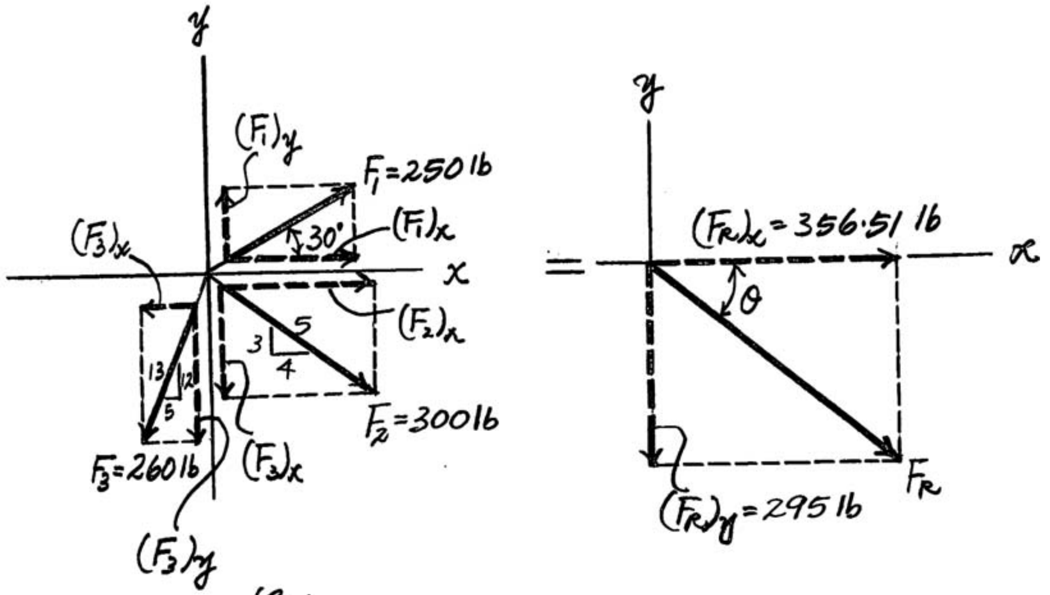
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 125 - 180 - 240 = -295\text{ lb} = 295\text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{356.51^2 + 295^2} = 463\text{ lb} \qquad \textbf{Ans.}$$

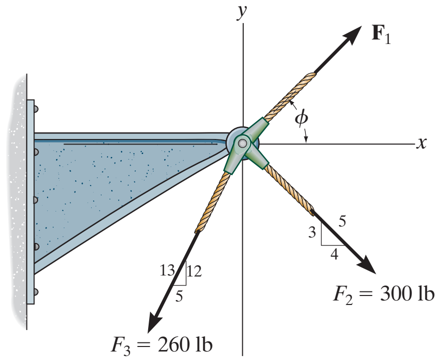
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{295}{356.51}\right) = 39.6° \qquad \textbf{Ans.}$$



(a)

*__2–44.__ If the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive x axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

$$(F_R)_x = 400 \text{ lb} \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 400 = F_1 \cos\phi + 240 - 100$$

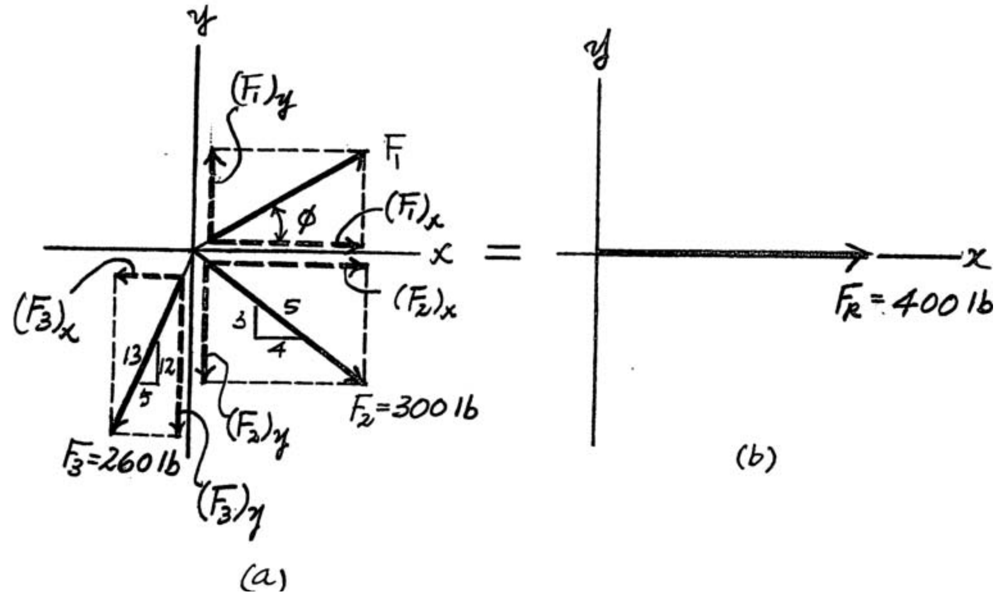
$$F_1 \cos\phi = 260 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1 \sin\phi - 180 - 240$$

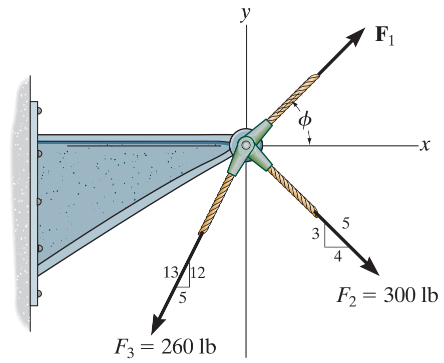
$$F_1 \sin\phi = 420 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 58.2°$ $\qquad F_1 = 494$ lb **Ans.**



•2–45. If the resultant force acting on the bracket is to be directed along the positive *x* axis and the magnitude of $\mathbf{F}_1$ is required to be a minimum, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



Rectangular Components: By referring to Figs. *a* and *b*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1\cos\phi \qquad (F_1)_y = F_1\sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240\text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180\text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

$$(F_R)_x = F_R \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y;\quad 0 = F_1\sin\phi - 180 - 240$$

$$F_1 = \frac{420}{\sin\phi} \qquad (1)$$

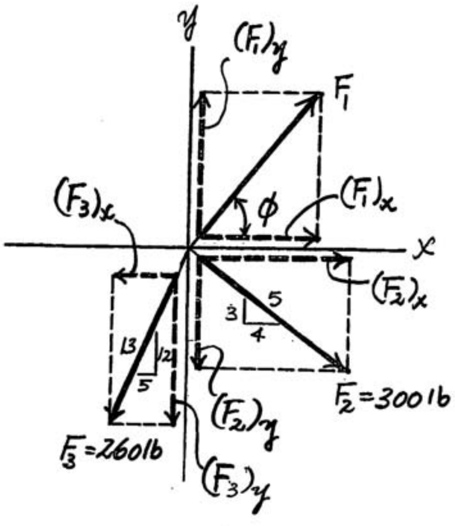
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x;\quad F_R = F_1\cos\phi + 240 - 100 \qquad (2)$$

By inspecting Eq. (1), we realize that F_1 is minimum when $\sin\phi = 1$ or $\phi = 90°$. Thus,

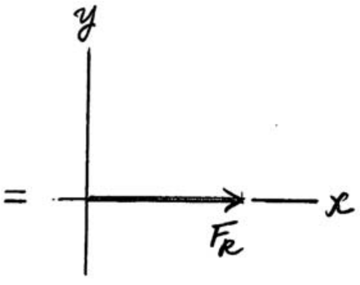
$F_1 = 420$ lb **Ans.**

Substituting these results into Eq. (2), yields

$F_R = 140$ lb **Ans.**

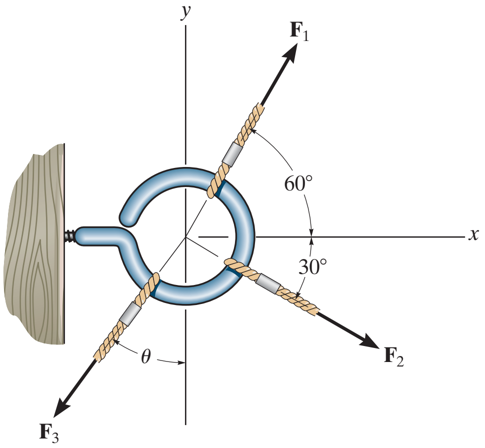


(a)



(b)

2–46. The three concurrent forces acting on the screw eye produce a resultant force $\mathbf{F}_R = 0$. If $F_2 = \frac{2}{3} F_1$ and $\mathbf{F}_1$ is to be 90° from $\mathbf{F}_2$ as shown, determine the required magnitude of $\mathbf{F}_3$ expressed in terms of F_1 and the angle θ.



Cartesian Vector Notation :

$$\mathbf{F}_1 = F_1 \cos 60° \mathbf{i} + F_1 \sin 60° \mathbf{j}$$
$$= 0.50F_1 \mathbf{i} + 0.8660F_1 \mathbf{j}$$

$$\mathbf{F}_2 = \frac{2}{3}F_1 \cos 30° \mathbf{i} - \frac{2}{3}F_1 \sin 30° \mathbf{j}$$
$$= 0.5774F_1 \mathbf{i} - 0.3333F_1 \mathbf{j}$$

$$\mathbf{F}_3 = -F_3 \sin \theta \mathbf{i} - F_3 \cos \theta \mathbf{j}$$

Resultant Force :

$$\mathbf{F}_R = 0 = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$0 = (0.50F_1 + 0.5774F_1 - F_3 \sin \theta)\mathbf{i} + (0.8660F_1 - 0.3333F_1 - F_3 \cos \theta)\mathbf{j}$$

Equating **i** and **j** components, we have

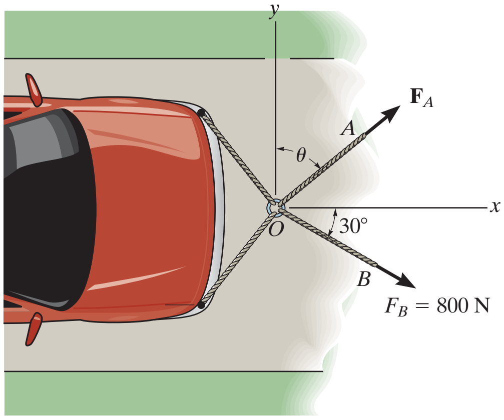
$$0.50F_1 + 0.5774F_1 - F_3 \sin \theta = 0 \qquad [1]$$

$$0.8660F_1 - 0.3333F_1 - F_3 \cos \theta = 0 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 63.7°$ $\quad F_3 = 1.20F_1$ **Ans**

2–47. Determine the magnitude of $\mathbf{F}_A$ and its direction θ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



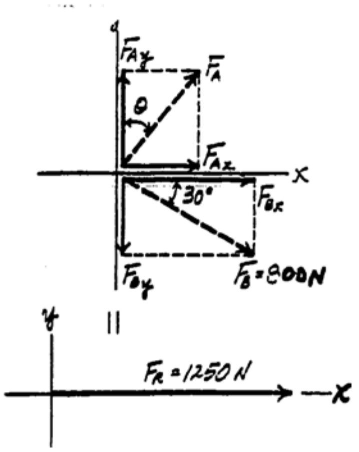
Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad 1250 = F_A \sin \theta + 800 \cos 30°$$
$$F_A \sin \theta = 557.18 \qquad [1]$$

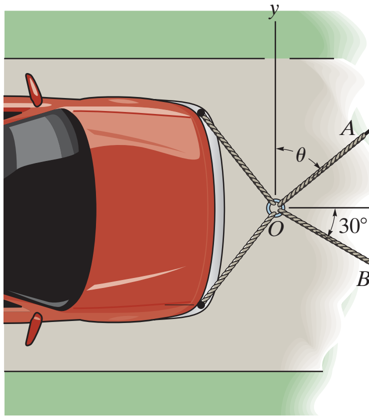
$$+\uparrow F_{R_y} = \Sigma F_y; \quad 0 = F_A \cos \theta - 800 \sin 30°$$
$$F_A \cos \theta = 400 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 54.3°$ $\quad F_A = 686$ N **Ans**



***2–48.** Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force acting on the ring at O if $F_A = 750$ N and $\theta = 45°$.



Scalar Notation : Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 750\sin 45° + 800\cos 30° = 1223.15\text{ N} \rightarrow$$

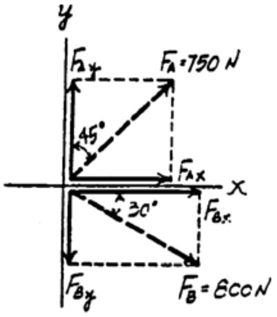
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 750\cos 45° - 800\sin 30° = 130.33\text{ N} \uparrow$$

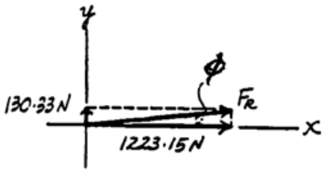
The magnitude of the resultant force F_R is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1223.15^2 + 130.33^2} = 1230\text{ N} = 1.23\text{ kN} \qquad \textbf{Ans}$$

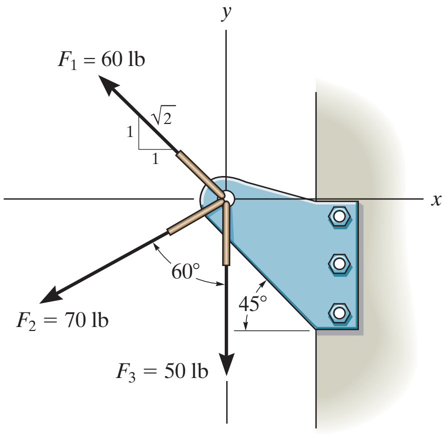
The directional angle θ measured counterclockwise from positive x axis is

$$\phi = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \qquad \textbf{Ans}$$





•2–49. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



$$\mathbf{F}_1 = -60\left(\frac{1}{\sqrt{2}}\right)\mathbf{i} + 60\left(\frac{1}{\sqrt{2}}\right)\mathbf{j} = \{-42.43\mathbf{i} + 42.43\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = -70\sin 60°\mathbf{i} - 70\cos 60°\mathbf{j} = \{-60.62\,\mathbf{i} - 35\,\mathbf{j}\}\text{ lb}$$

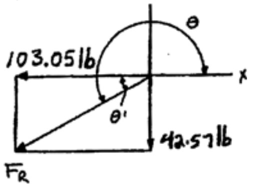
$$\mathbf{F}_3 = \{-50\,\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{-103.05\,\mathbf{i} - 42.57\,\mathbf{j}\}\text{ lb}$$

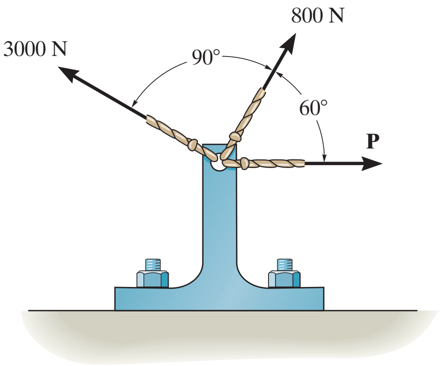
$$F_R = \sqrt{(-103.05)^2 + (-42.57)^2} = 111\text{ lb} \qquad \textbf{Ans}$$

$$\theta' = \tan^{-1}\left(\frac{42.57}{103.05}\right) = 22.4°$$

$$\theta = 180° + 22.4° = 202° \qquad \textbf{Ans}$$



2–50. The three forces are applied to the bracket. Determine the range of values for the magnitude of force **P** so that the resultant of the three forces does not exceed 2400 N.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = P + 800\cos 60^\circ - 3000\cos 30^\circ = P - 2198.08$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 800\sin 60^\circ + 3000\sin 30^\circ = 2192.82$

$$F_R = \sqrt{(P - 2198.08)^2 + (2192.82)^2} \le 2400$$

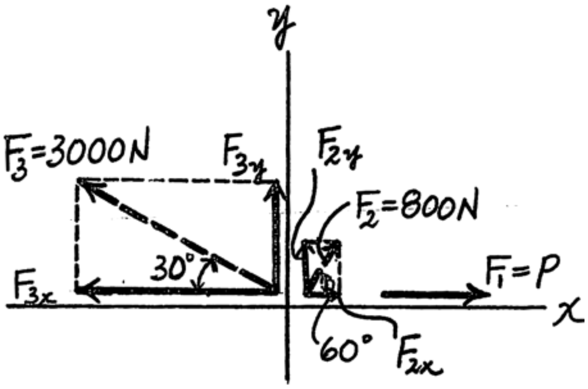
$$(P - 2198.08)^2 + (2192.82)^2 \le (2400)^2$$

$$|(P - 2198.08)| \le 975.47$$

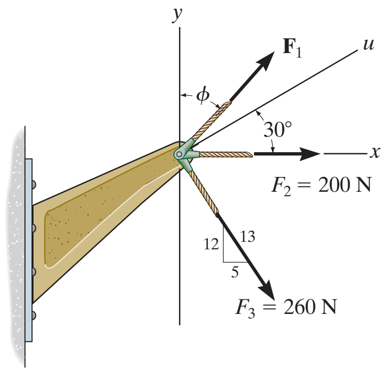
$$-975.47 \le P - 2198.08 \le 975.47$$

$$1222.6\text{ N} \le P \le 3173.5\text{ N}$$

$$1.22\text{ kN} \le P \le 3.17\text{ kN} \qquad \textbf{Ans}$$



2–51. If $F_1 = 150$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 150\sin 30° = 75\text{ N} \qquad (F_1)_y = 150\cos 30° = 129.90\text{ N}$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 75 + 200 + 100 = 375\text{ N} \rightarrow$$

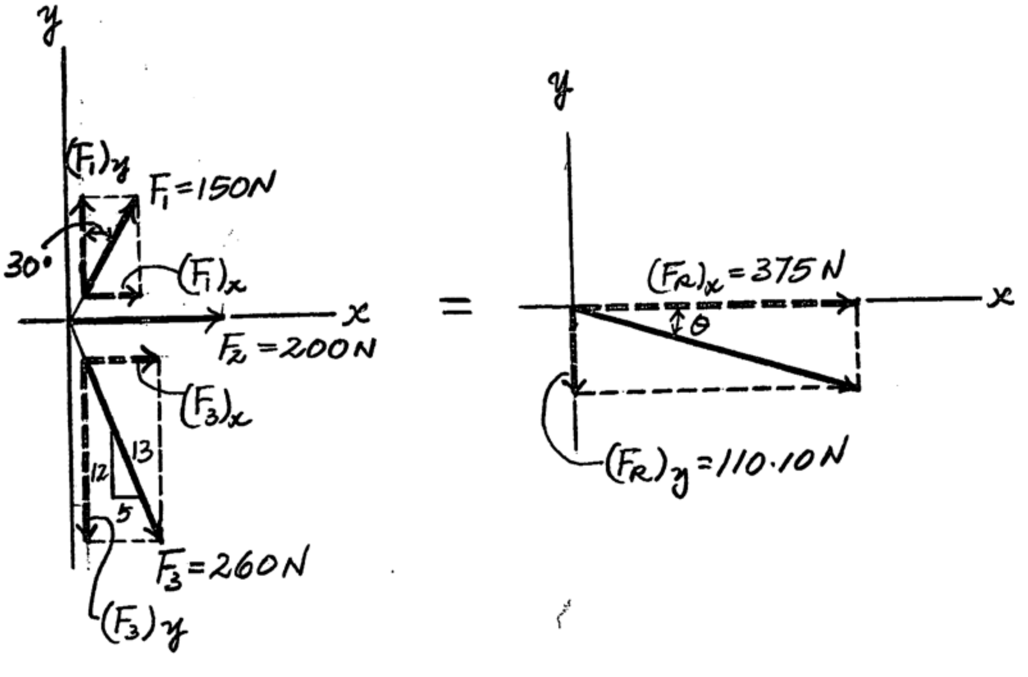
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 129.90 - 240 = -110.10\text{ N} = 110.01\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{375^2 + 110.10^2} = 391\text{N} \qquad \textbf{Ans.}$$

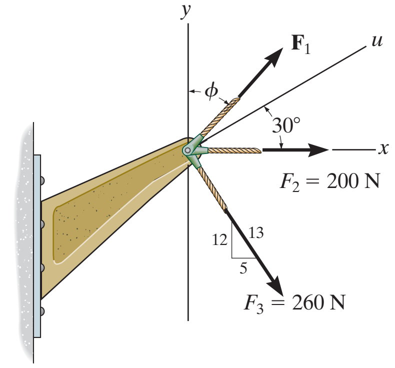
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{110.10}{375}\right) = 16.4° \qquad \textbf{Ans.}$$



(a)

***2–52.** If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \sin\phi \qquad (F_1)_y = F_1 \cos\phi$$

$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ lb}$$

$$(F_R)_x = 450\cos 30° = 389.71\text{N} \qquad (F_R)_y = 450\sin 30° = 225\text{ lb}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 389.71 = F_1\sin\phi + 200 + 100$$

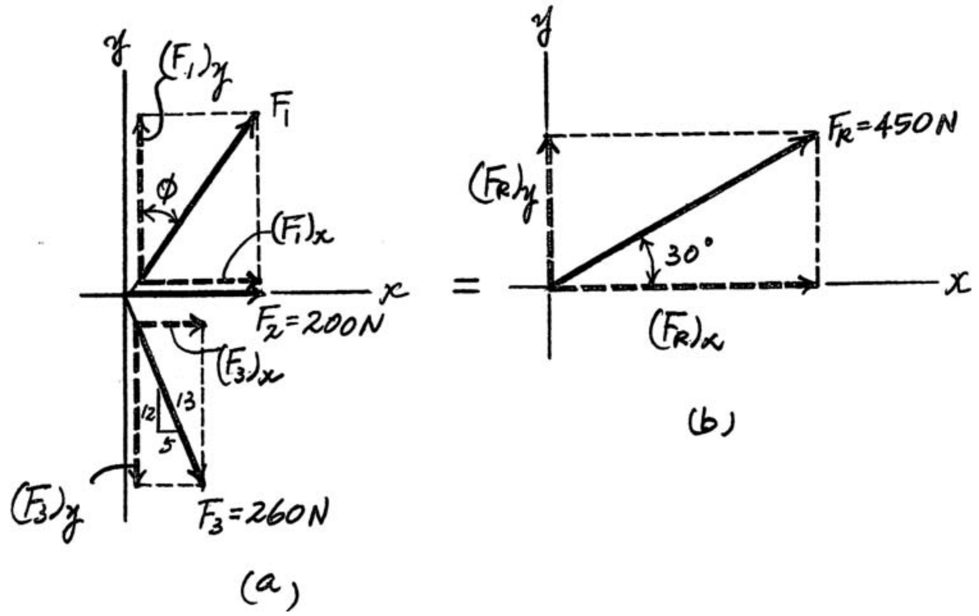
$$F_1 \sin\phi = 89.71 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad 225 = F_1\cos\phi - 240$$

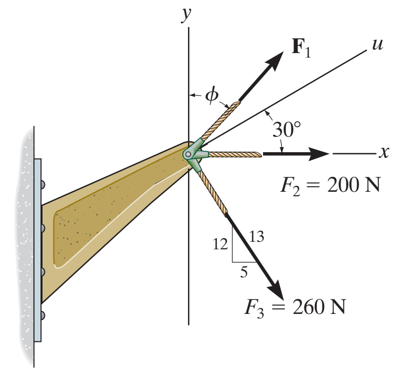
$$F_1\cos\phi = 465 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 10.9°$ $\quad F_1 = 474\text{N}$ **Ans.**



•2–53. If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.

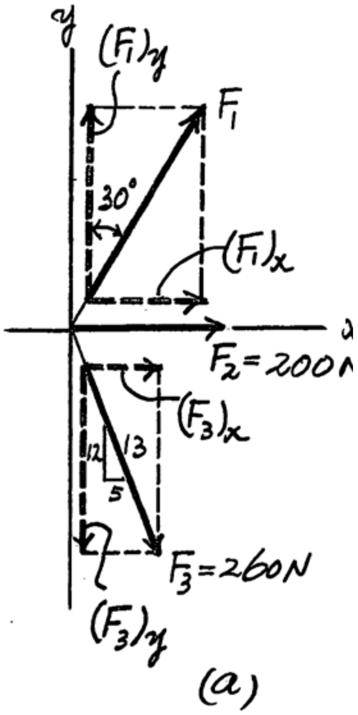


Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$(F_1)_x = F_1 \sin 30° = 0.5F_1 \qquad (F_1)_y = F_1 \cos 30° = 0.8660F_1$

$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$

$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ N}$



Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2} = \sqrt{F_1^2 - 115.69F_1 + 147600} \qquad (1)$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \qquad (2)$$

The first derivative of Eq. (2) is

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad (3)$$

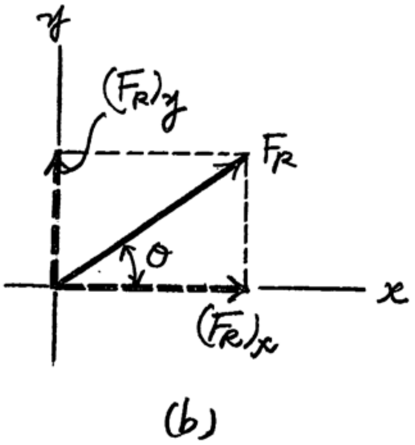
and the second derivative of Eq. (1) is

$$F_R\frac{d^2F_R}{dF_1^2} + \frac{dF_R}{dF_1}\frac{dF_R}{dF_1} = 1 \qquad (4)$$

For $\mathbf{F}_R$ to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

$$F_1 = 57.84\text{ N} = 57.8\text{ N} \qquad \textbf{Ans.}$$



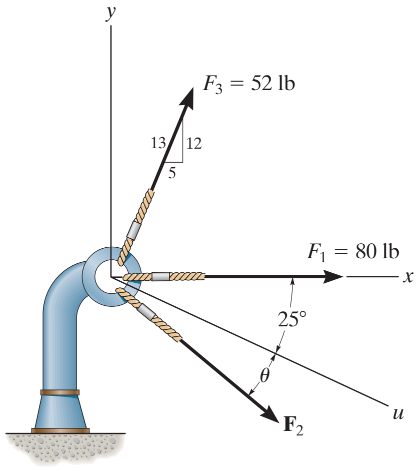
Substituting $F_1 = 57.84$ N and $\frac{dF_R}{dF_1} = 0$ into Eq. (4),

$$\frac{d^2F_R}{dF_1^2} = 0.00263 > 0$$

Thus, $F_1 = 57.84$ N produces a minimum $\mathbf{F}_R$. From Eq. (1),

$$F_R = \sqrt{(57.84)^2 - 115.69(57.84) + 147600} = 380\text{ N} \qquad \textbf{Ans.}$$

2–54. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_2$ so that the resultant force is directed along the positive u axis and has a magnitude of 50 lb.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad 50 \cos 25° = 80 + 52\left(\frac{5}{13}\right) + F_2 \cos (25° + \theta)$$

$$F_2 \cos (25° + \theta) = -54.684 \qquad [1]$$

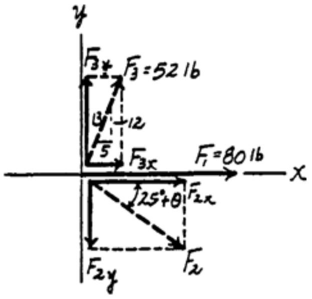
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad -50 \sin 25° = 52\left(\frac{12}{13}\right) - F_2 \sin (25° + \theta)$$

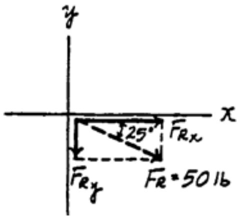
$$F_2 \sin (25° + \theta) = 69.131 \qquad [2]$$

Solving Eq.[1] and [2] yields

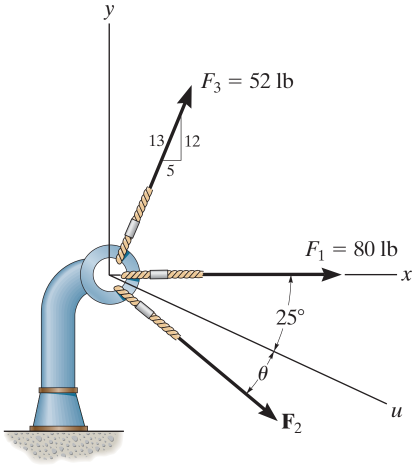
$25° + \theta = 128.35°$ $\quad \theta = 103°$ **Ans**

$F_2 = 88.1$ lb **Ans**





2–55. If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and direction measured clockwise from the positive x axis of the resultant force of the three forces acting on the bracket.



***Scalar Notation*:** Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150\cos 80°$$
$$= 126.05 \text{ lb} \rightarrow$$

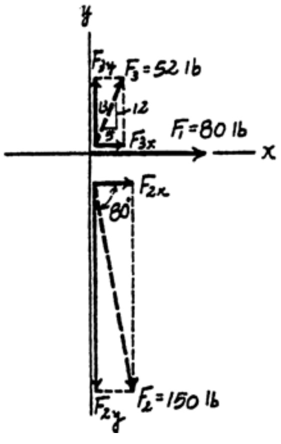
$$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = 52\left(\frac{12}{13}\right) - 150\sin 80°$$
$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

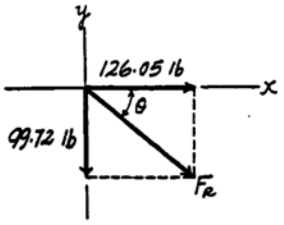
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \qquad \textbf{Ans}$$

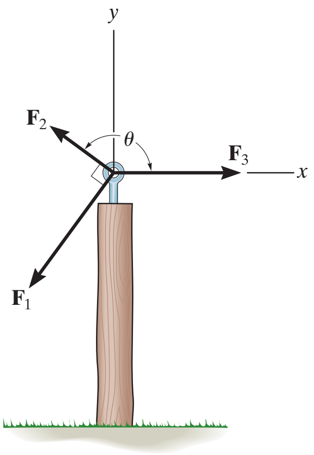
The directional angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3° \qquad \textbf{Ans}$$





***2–56.** The three concurrent forces acting on the post produce a resultant force $\mathbf{F}_R = \mathbf{0}$. If $F_2 = \frac{1}{2}F_1$, and $\mathbf{F}_1$ is to be 90° from $\mathbf{F}_2$ as shown, determine the required magnitude of F_3 expressed in terms of F_1 and the angle θ.



$$\Sigma F_{Rx'} = 0; \qquad F_3\cos(\theta - 90°) = F_1$$

$$\Sigma F_{Ry'} = 0; \qquad F_3\sin(\theta - 90°) = F_2$$

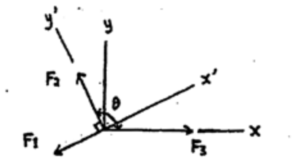
$$\tan(\theta - 90°) = \frac{F_2}{F_1} = \frac{1}{2}$$

$$\theta - 90° = 26.57°$$

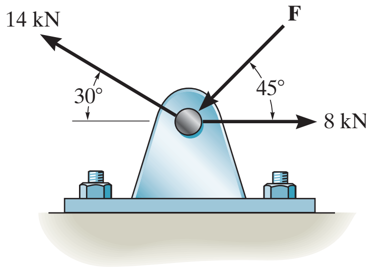
$$\theta = 116.57° = 117° \qquad \textbf{Ans}$$

$$F_3 = \frac{F_1}{\cos(116.57° - 90°)}$$

$$F_3 = 1.12\,F_1 \qquad \textbf{Ans}$$



•**2–57.** Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of this smallest resultant force?



$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = 8 - F\cos 45° - 14\cos 30°$$

$$= -4.1244 - F\cos 45°$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -F\sin 45° + 14\sin 30°$$

$$= 7 - F\sin 45°$$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad (1)$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$$F = 2.03 \text{ kN} \qquad \text{Ans}$$

From Eq. (1); $F_R = 7.87\text{ kN}$ Ans

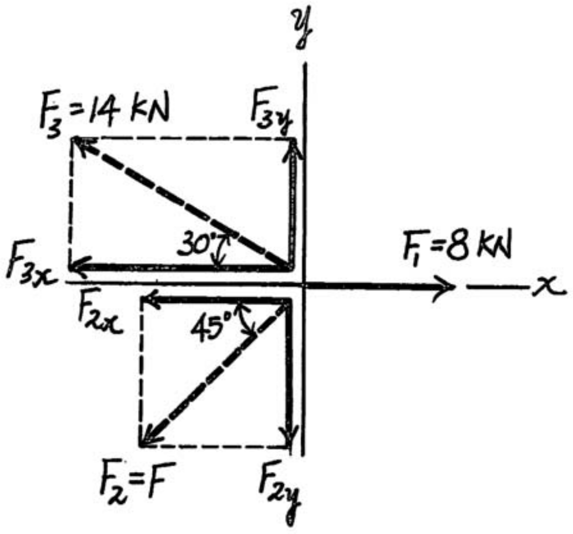
Also, from the figure require

$$(F_R)_{x'} = 0 = \Sigma F_{x'}; \quad F + 14\sin 15° - 8\cos 45° = 0$$

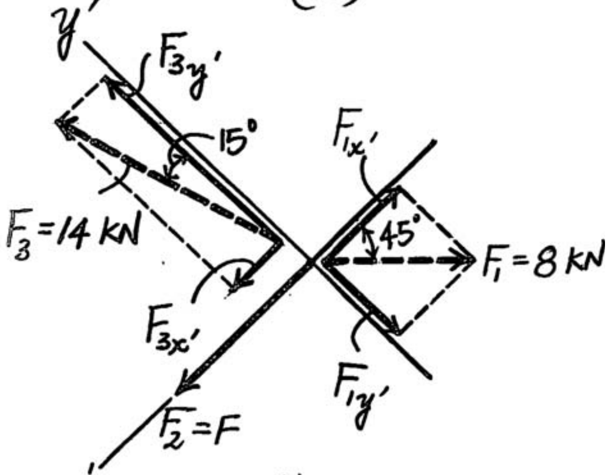
$$F = 2.03 \text{ kN} \qquad \text{Ans}$$

$$(F_R)_{y'} = \Sigma F_{y'}; \quad F_R = 14\cos 15° - 8\sin 45°$$

$$F_R = 7.87 \text{ kN} \qquad \text{Ans}$$

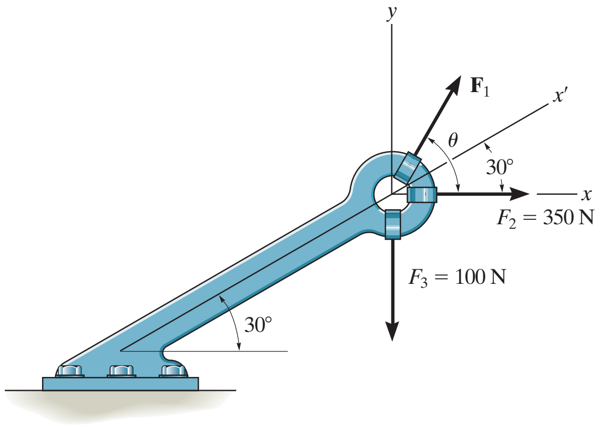


(a)



(b)

2–58. Express each of the three forces acting on the bracket in Cartesian vector form with respect to the *x* and *y* axes. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of $F_R = 600$ N.



$\mathbf{F}_1 = \{F_1 \cos\theta\,\mathbf{i} + F_1 \sin\theta\,\mathbf{j}\}$ N **Ans**

$\mathbf{F}_2 = \{350\mathbf{i}\}$ N **Ans**

$\mathbf{F}_3 = \{-100\mathbf{j}\}$ N **Ans**

Require,

$\mathbf{F}_R = 600 \cos 30°\,\mathbf{i} + 600 \sin 30°\,\mathbf{j}$

$\mathbf{F}_R = \{519.6\mathbf{i} + 300\mathbf{j}\}$ N

$\mathbf{F}_R = \Sigma\mathbf{F}$

Equating the i and j components yields:

$519.6 = F_1 \cos\theta + 350$

$F_1 \cos\theta = 169.6$

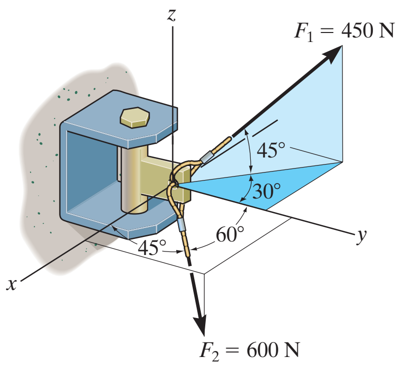
$300 = F_1 \sin\theta - 100$

$F_1 \sin\theta = 400$

$\theta = \tan^{-1}\left[\frac{400}{169.6}\right] = 67.0°$ **Ans**

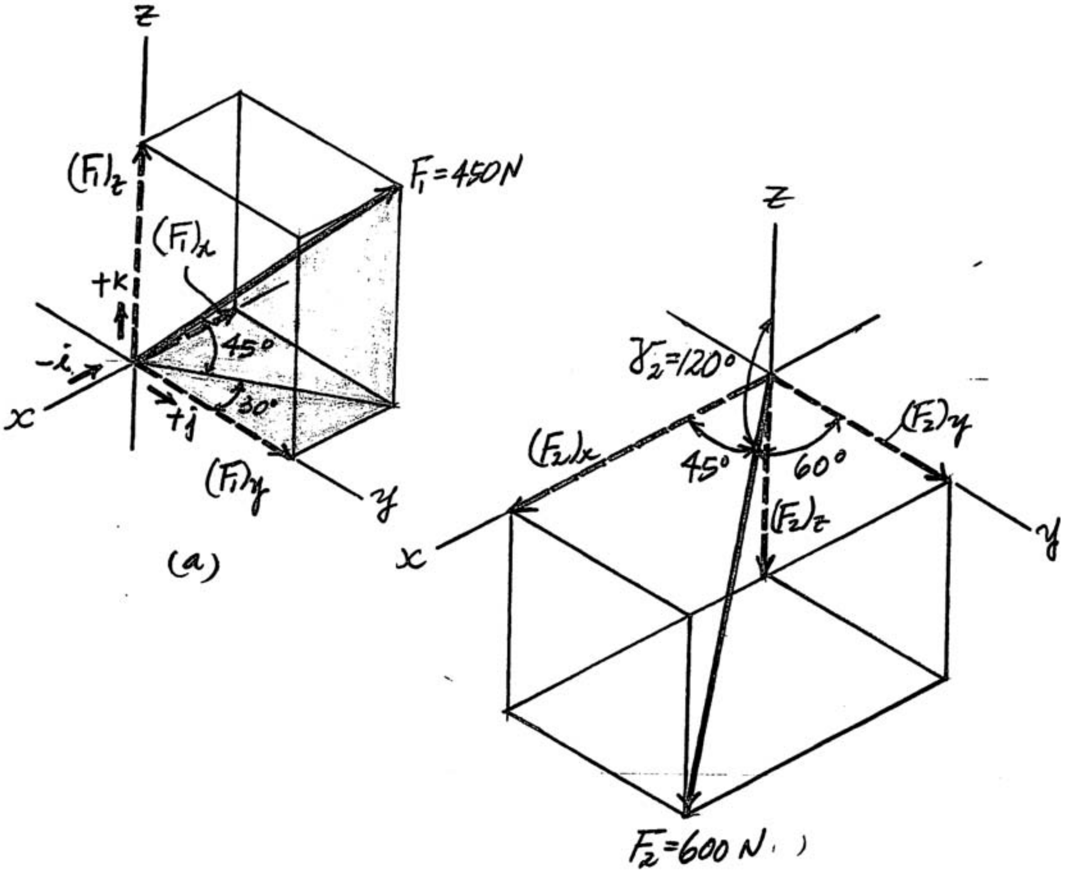
$F_1 = 434$ N **Ans**

2–59. Determine the coordinate angle γ for $\mathbf{F}_2$ and then express each force acting on the bracket as a Cartesian vector.



Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1 - \cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\gamma_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

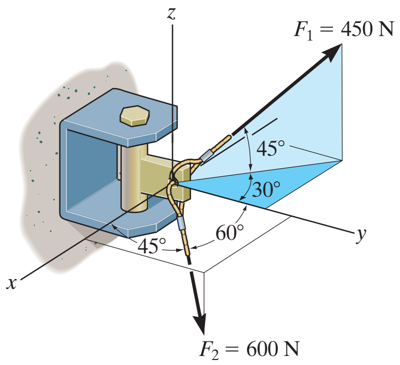
$$\mathbf{F}_1 = 450\cos 45° \sin 30°(-\mathbf{i}) + 450\cos 45° \cos 30°(+\mathbf{j}) + 450 \sin 45°(+\mathbf{k})$$
$$= \{-159\mathbf{i} + 276\mathbf{j} + 318\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$



(a)

(b)

*__2–60.__ Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1-\cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\alpha_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})$$
$$= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$
$$= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97\text{ N} = 634\text{ N} \qquad \textbf{Ans.}$$

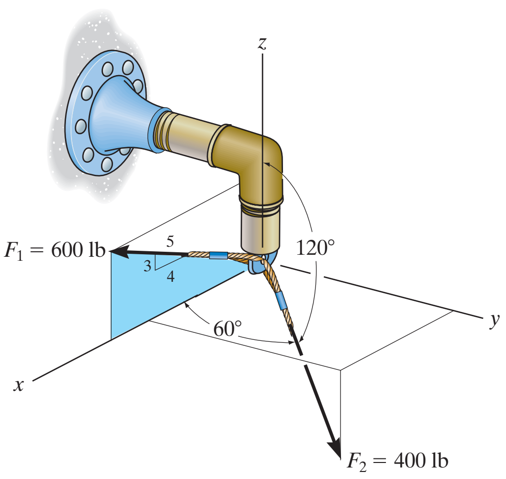
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{265.16}{633.97}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{575.57}{633.97}\right) = 24.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{18.20}{633.97}\right) = 88.4° \qquad \textbf{Ans.}$$

•2–61. Express each force acting on the pipe assembly in Cartesian vector form.



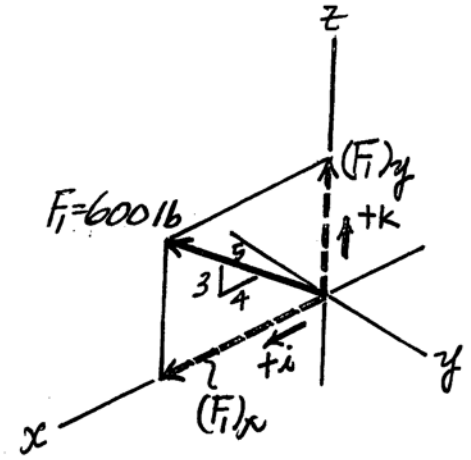
Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\beta_2 = \pm\sqrt{1-\cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 > 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

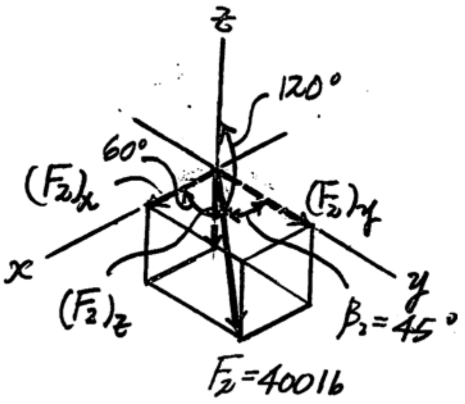
$$= [480\mathbf{i} + 360\mathbf{k}]\text{N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\cos 60°\,\mathbf{i} + 400\cos 45°\,\mathbf{j} + 400\cos 120°\,\mathbf{k}$$

$$= [200\mathbf{i} + 283\mathbf{j} - 200\mathbf{k}]\text{N} \qquad \textbf{Ans.}$$

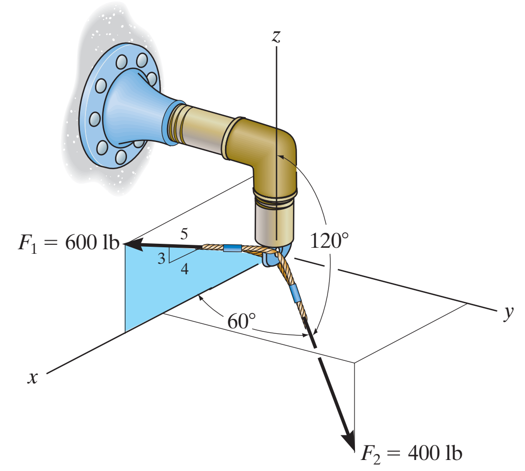


(a)



(b)

2–62. Determine the magnitude and direction of the resultant force acting on the pipe assembly.



Force Vectors: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\gamma_2 < 90°$, thus, $\gamma_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$
$$= \{480\mathbf{i} + 360\mathbf{k}\}\text{ lb}$$
$$\mathbf{F}_2 = 400\cos 60°\,\mathbf{i} + 400\cos 45°\,\mathbf{j} + 400\cos 120°\,\mathbf{k}$$
$$= \{200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k}\}\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (480\mathbf{i} + 360\mathbf{k}) + (200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k})$$
$$= \{680\mathbf{i} + 282.84\mathbf{j} + 160\mathbf{k}\}\text{ lb}$$

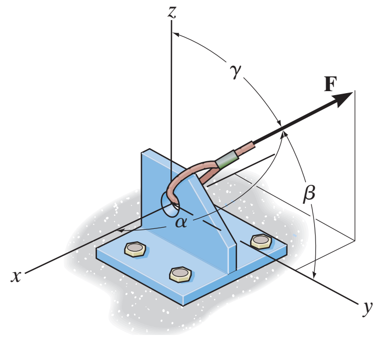
The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{680^2 + 282.84^2 + 160^2} = 753.66\text{ lb} = 754\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{680}{753.66}\right) = 25.5° \qquad \textbf{Ans.}$$
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{282.84}{753.66}\right) = 68.0° \qquad \textbf{Ans.}$$
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{160}{753.66}\right) = 77.7° \qquad \textbf{Ans.}$$

2–63. The force **F** acts on the bracket within the octant shown. If $F = 400$ N, $\beta = 60°$, and $\gamma = 45°$, determine the x, y, z components of **F**.



Coordinate Direction Angles: Since β and γ are known, the third angle α can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2\alpha + \cos^2 60° + \cos^2 45° = 1$$
$$\cos\alpha = \pm 0.5$$

Since **F** is in the octant shown in Fig. *a*, θ_x must be greater than 90°. Thus, $\alpha = \cos^{-1}(-0.5) = 120°$.

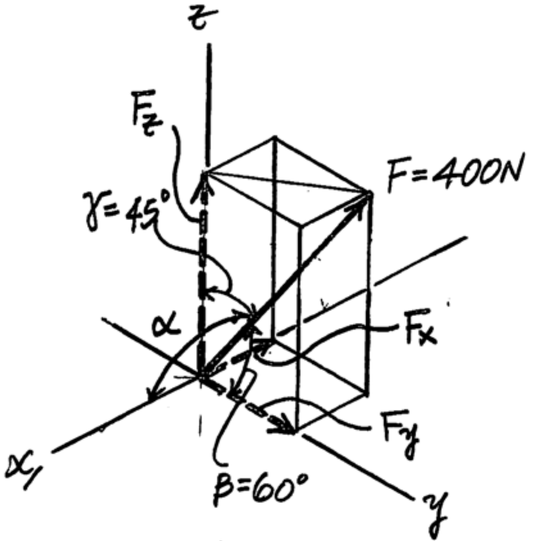
Rectangular Components: By referring to Fig. *a*, the x, y, and z components of **F** can be written as

$F_x = F\cos\alpha = 400\cos 120° = -200\text{ N}$ **Ans.**

$F_y = F\cos\beta = 400\cos 60° = 200\text{ N}$ **Ans.**

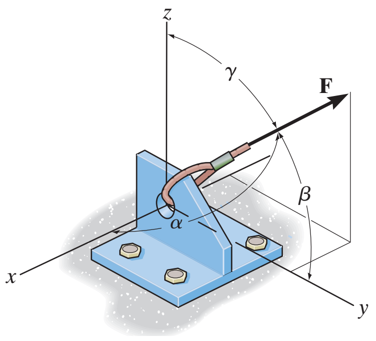
$F_z = F\cos\gamma = 400\cos 45° = 283\text{ N}$ **Ans.**

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis.



(a)

***2–64.** The force **F** acts on the bracket within the octant shown. If the magnitudes of the x and z components of **F** are $F_x = 300$ N and $F_z = 600$ N, respectively, and $\beta = 60°$, determine the magnitude of **F** and its y component. Also, find the coordinate direction angles α and γ.



Rectangular Components: The magnitude of **F** is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{300^2 + F_y^2 + 600^2}$$

$$F^2 = F_y^2 + 450\ 000 \qquad (1)$$

The magnitude of $\mathbf{F}_y$ is given by

$$F_y = F\cos 60° = 0.5F \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F = 774.60\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

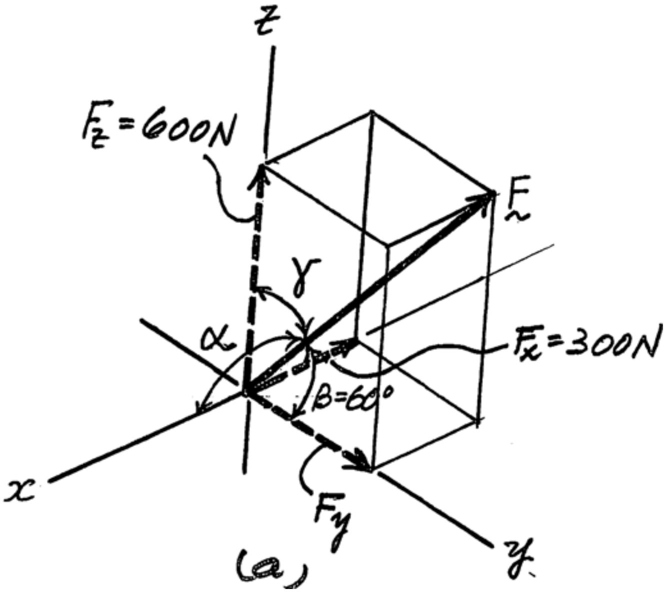
$$F_y = 387\text{ N} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: Since **F** is contained in the octant so that $\mathbf{F}_x$ is directed towards the negative x axis, the coordinate direction angle θ_x is given by

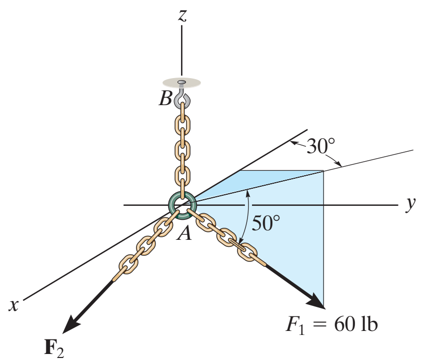
$$\alpha = \cos^{-1}\left(\frac{-F_x}{F}\right) = \cos^{-1}\left(\frac{-300}{774.60}\right) = 113° \qquad \textbf{Ans.}$$

The third coordinate direction angle is

$$\gamma = \cos^{-1}\left(\frac{-F_z}{F}\right) = \cos^{-1}\left(\frac{600}{774.60}\right) = 39.2° \qquad \textbf{Ans.}$$



•2–65. The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.



Cartesian Vector Notation :

$$\mathbf{F}_R = \{-100\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = 60\{-\cos 50°\cos 30°\mathbf{i} + \cos 50°\sin 30°\mathbf{j} - \sin 50°\mathbf{k}\} \text{ lb}$$
$$= \{-33.40\mathbf{i} + 19.28\mathbf{j} - 45.96\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j} + F_{2_z}\mathbf{k}\} \text{ lb}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$-100\mathbf{k} = \{(F_{2_x} - 33.40)\mathbf{i} + (F_{2_y} + 19.28)\mathbf{j} + (F_{2_z} - 45.96)\mathbf{k}\}$$

Equating **i**, **j** and **k** components, we have

$$F_{2_x} - 33.40 = 0 \qquad F_{2_x} = 33.40 \text{ lb}$$
$$F_{2_y} + 19.28 = 0 \qquad F_{2_y} = -19.28 \text{ lb}$$
$$F_{2_z} - 45.96 = -100 \qquad F_{2_z} = -54.04 \text{ lb}$$

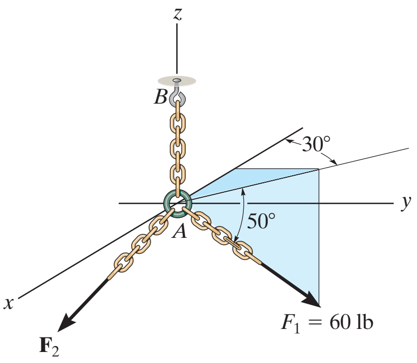
The magnitude of force $\mathbf{F}_2$ is

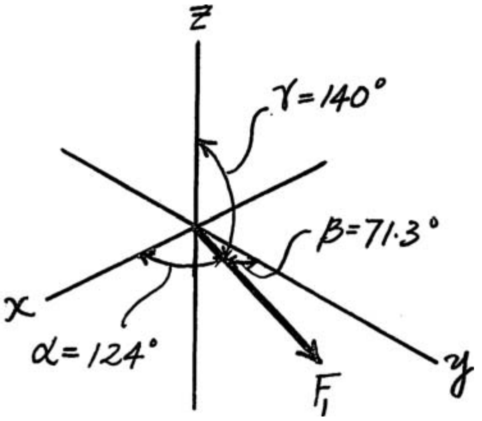
$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2 + F_{2_z}^2}$$
$$= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2}$$
$$= 66.39 \text{ lb} = 66.4 \text{ lb} \qquad \textbf{Ans}$$

The coordinate direction angles for $\mathbf{F}_2$ are

$$\cos\alpha = \frac{F_{2_x}}{F_2} = \frac{33.40}{66.39} \qquad \alpha = 59.8° \qquad \textbf{Ans}$$
$$\cos\beta = \frac{F_{2_y}}{F_2} = \frac{-19.28}{66.39} \qquad \beta = 107° \qquad \textbf{Ans}$$
$$\cos\gamma = \frac{F_{2_z}}{F_2} = \frac{-54.04}{66.39} \qquad \gamma = 144° \qquad \textbf{Ans}$$

2–66. Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.





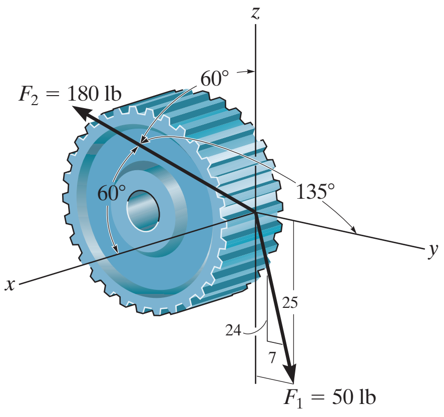
Unit Vector For Foce $\mathbf{F}_1$ **:**

$$\mathbf{u}_{F_1} = -\cos 50° \cos 30° \mathbf{i} + \cos 50° \sin 30° \mathbf{j} - \sin 50° \mathbf{k}$$
$$= -0.5567\mathbf{i} + 0.3214\mathbf{j} - 0.7660\mathbf{k}$$

Coordinate Direction Angles : From the unit vector obtained above, we have

$\cos \alpha = -0.5567$ $\quad \alpha = 124°$ $\quad$ **Ans**
$\cos \beta = 0.3214$ $\quad \beta = 71.3°$ $\quad$ **Ans**
$\cos \gamma = -0.7660$ $\quad \gamma = 140°$ $\quad$ **Ans**

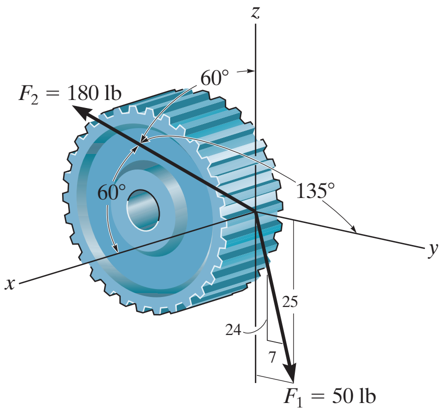
2–67. The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.



$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\,\mathbf{j} - 48.0\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_2 = 180 \cos 60°\mathbf{i} + 180 \cos 135°\mathbf{j} + 180 \cos 60°\mathbf{k}$$
$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

***2–68.** The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



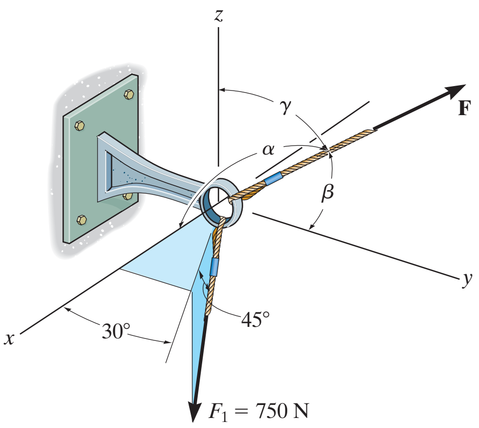
$$F_{Rx} = 180 \cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135° = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

•2–69. If the resultant force acting on the bracket is $\mathbf{F}_R = \{-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k}\}$ N, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Fig. *a*. $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750\cos 45°\cos 30°(+\mathbf{i}) + 750\cos 45°\sin 30°(+\mathbf{j}) + 750\sin 45°(-\mathbf{k})$$
$$= [459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}]\text{N}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$
$$-300\mathbf{i} + 650\mathbf{j} + 250\mathbf{k} = (459.28 + F\cos\theta_x)\mathbf{i} + (265.17 + F\cos\theta_y)\mathbf{j} + (F\cos\theta_z - 530.33)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$-300 = 459.28 + F\cos\alpha$$
$$F\cos\alpha = -759.28 \qquad (1)$$

$$650 = 265.17 + F\cos\beta$$
$$F\cos\beta = 384.83 \qquad (2)$$

$$250 = F\cos\gamma - 530.33$$
$$F\cos\gamma = 780.33 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

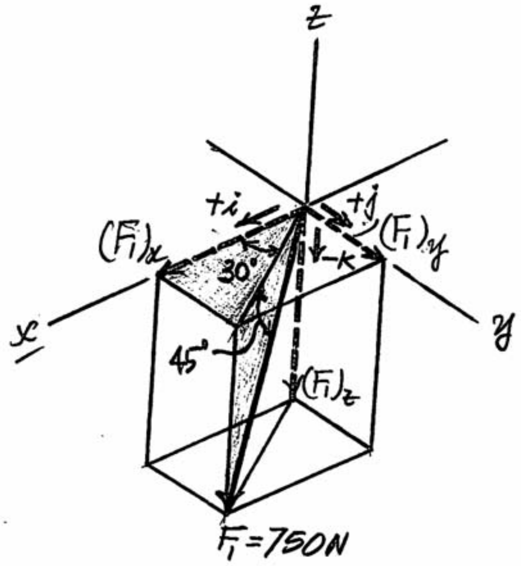
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 1\,333\,518.08 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

$$F = 1154.78\text{ N} = 1.15\text{ kN} \qquad \textbf{Ans.}$$

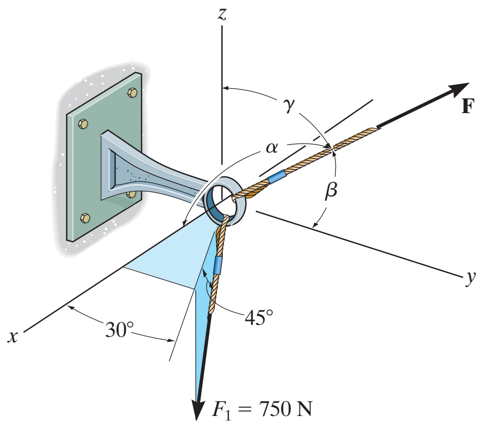
Substituting $F = 1154.78$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 131° \qquad \beta = 70.5° \qquad \gamma = 47.5° \qquad \textbf{Ans.}$$



(a)

2–70. If the resultant force acting on the bracket is to be $\mathbf{F}_R = \{800\mathbf{j}\}$ N, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and **F** into their *x*, *y*, and *z* components, as shown in Figs. *b* and *c*, respectively, $\mathbf{F}_1$ and **F** can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 750\cos 45^\circ \cos 30^\circ(+\mathbf{i}) + 750\cos 45^\circ \sin 30^\circ(+\mathbf{j}) + 750\sin 45^\circ(-\mathbf{k})$$
$$= [459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}]\text{N}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and **F** vectorally, Figs. *a*, *b*, and *c*, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$800\mathbf{j} = (459.28\mathbf{i} + 265.17\mathbf{j} - 530.33\mathbf{k}) + (F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k})$$
$$800\mathbf{j} = (459.28 + F\cos\alpha)\mathbf{i} + (265.17 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 530.33)\mathbf{k}$$

Equating the **i**, **j**, and **k** components, we have

$$0 = 459.28 + F_2\cos\alpha$$
$$F\cos\alpha = -459.28 \qquad (1)$$

$$800 = 265.17 + F\cos\beta$$
$$F\cos\beta = 534.8 \qquad (2)$$

$$0 = F\cos\gamma - 530.33$$
$$F\cos\gamma = 530.33 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

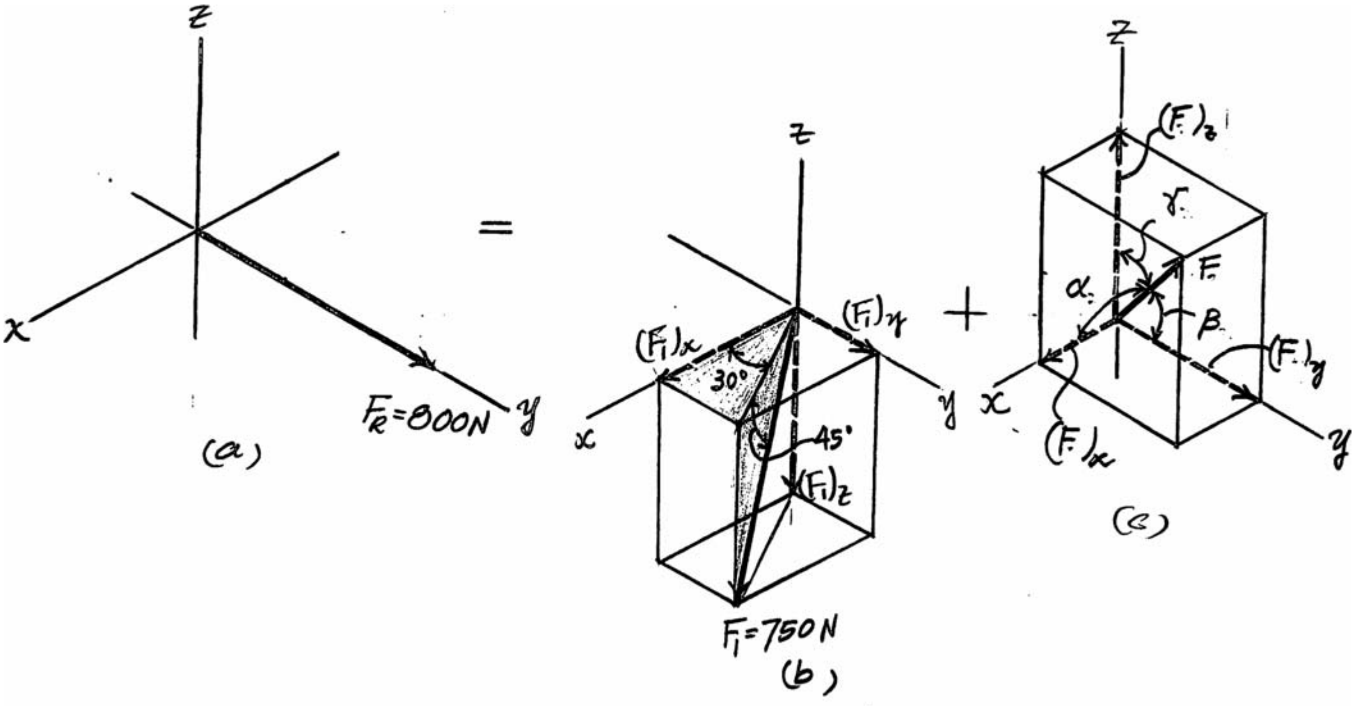
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 778\,235.93 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

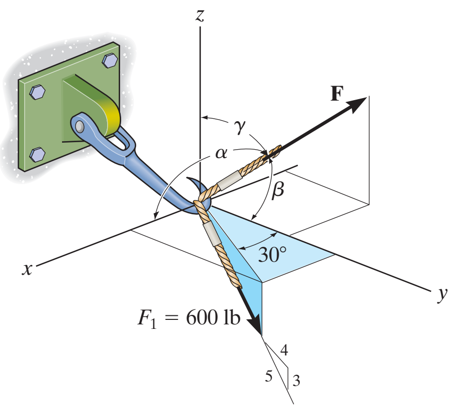
$$F = 882.17\text{ N} = 882\text{ N} \qquad \textbf{Ans.}$$

Substituting $F = 882.17$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 121^\circ \quad \beta = 52.7^\circ \quad \gamma = 53.0^\circ \qquad \textbf{Ans.}$$



2–71. If $\alpha = 120°$, $\beta < 90°$, $\gamma = 60°$, and $F = 400$ lb, determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



Force Vectors: Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, then $\cos\beta = \pm\sqrt{1-\cos^2 120° - \cos^2 60°} = \pm 0.7071$. However, it is required that $\beta < 90°$, thus, $\beta = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$

$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\} \text{ lb}$$

$$\mathbf{F} = 400\cos 120°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 60°\mathbf{k}$$

$$= \{-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k}\} \text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$= (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k})$$

$$= \{40\mathbf{i} + 698.53\mathbf{j} - 160\mathbf{k}\} \text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

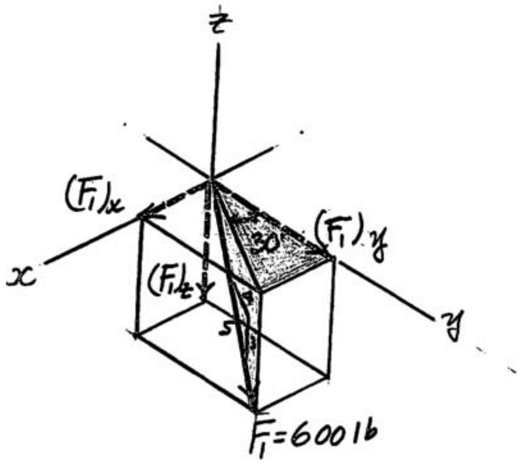
$$= \sqrt{(40)^2 + (698.53)^2 + (-160)^2} = 717.74 \text{ lb} = 718 \text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

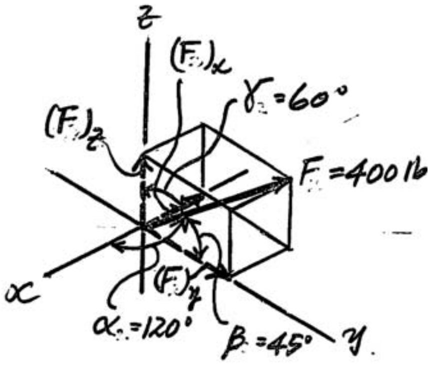
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{40}{717.74}\right) = 86.8° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{698.53}{717.74}\right) = 13.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-160}{717.74}\right) = 103° \qquad \textbf{Ans.}$$

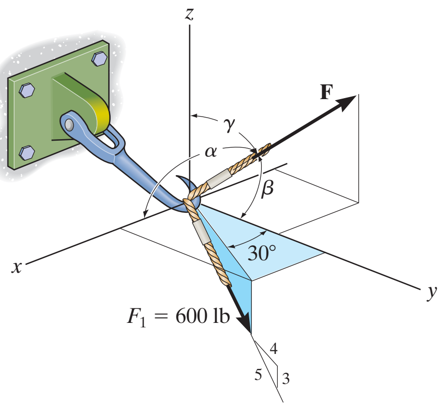


(a)



(b)

***2–72.** If the resultant force acting on the hook is $\mathbf{F}_R = \{-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k}\}$ lb, determine the magnitude and coordinate direction angles of **F**.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$

$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$

$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240 + F\cos\alpha)\mathbf{i} + (415.69 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 360)\mathbf{k}$$

Equating the **i**, **j**, and **k** components, we have

$$-200 = 240 + F\cos\theta_x$$

$$F\cos\alpha = -440 \qquad (1)$$

$$800 = 415.69 + F\cos\beta$$

$$F\cos\beta = 384.31 \qquad (2)$$

$$150 = F\cos\gamma - 360$$

$$F\cos\gamma = 510 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 601392.49 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

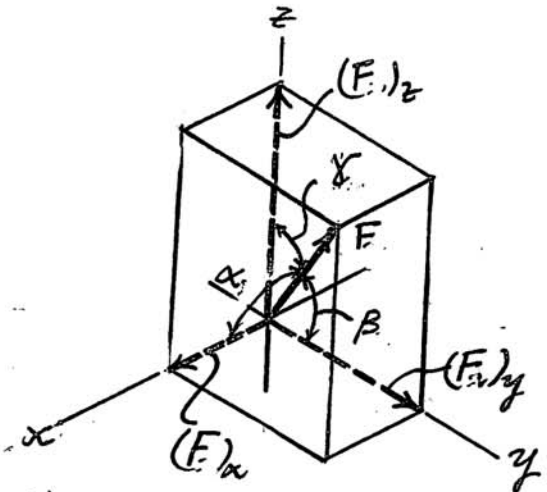
$$F = 775.49\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

Substituting $F = 775.49$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 125° \qquad \beta = 60.3° \qquad \gamma = 48.9° \qquad \textbf{Ans.}$$

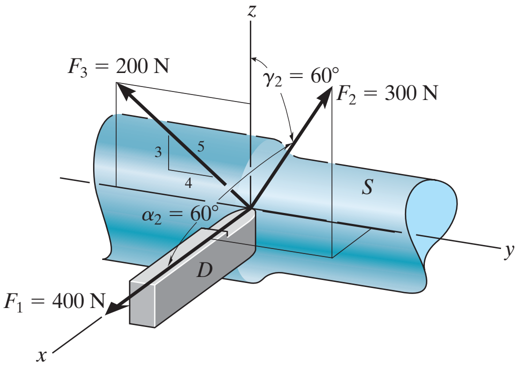
see prob. 2-71a

(a)



(b)

•2–73. The shaft *S* exerts three force components on the die *D*. Find the magnitude and coordinate direction angles of the resultant force. Force $\mathbf{F}_2$ acts within the octant shown.



$\mathbf{F}_1 = 400\,\mathbf{i}$

Since $\cos^2 60° + \cos^2 \beta_2 + \cos^2 60° = 1$

Solving for the positive root, $\beta_2 = 45°$

$\mathbf{F}_2 = 300 \cos 60°\,\mathbf{i} + 300 \cos 45°\,\mathbf{j} + 300 \cos 60°\,\mathbf{k}$

$= 150\,\mathbf{i} + 212.1\,\mathbf{j} + 150\,\mathbf{k}$

$\mathbf{F}_3 = -200\left(\frac{4}{5}\right)\mathbf{j} + 200\left(\frac{3}{5}\right)\mathbf{k}$

$= -160\,\mathbf{j} + 120\,\mathbf{k}$

Then

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 550\,\mathbf{i} + 52.1\,\mathbf{j} + 270\,\mathbf{k}$

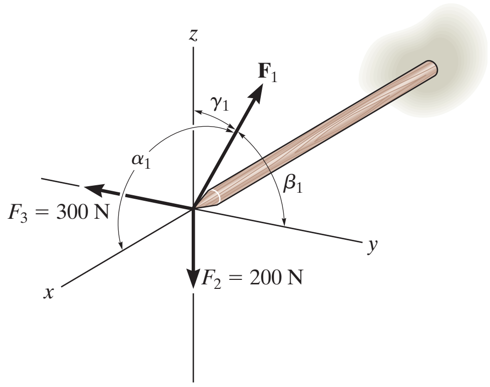
$F_R = \sqrt{(550)^2 + (52.1)^2 + (270)^2} = 614.9 \text{ N} = 615 \text{ N}$ **An**

$\alpha = \cos^{-1}\left(\frac{550}{614.9}\right) = 26.6°$ **Ans**

$\beta = \cos^{-1}\left(\frac{52.1}{614.9}\right) = 85.1°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{270}{614.9}\right) = 64.0°$ **Ans**

2–74. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\mathbf{F}_1$ so that the resultant force acting on the mast is $\mathbf{F}_R = \{350\mathbf{i}\}$ N.



$\mathbf{F}_1 = 500 \cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500 \cos\gamma_1\,\mathbf{k}$

$\mathbf{F}_R = \mathbf{F}_1 + (-300\mathbf{j}) + (-200\mathbf{k})$

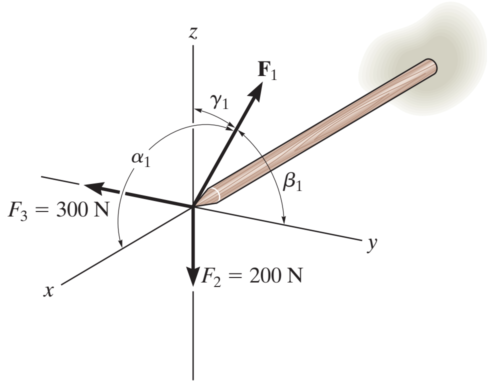
$350\mathbf{i} = 500 \cos\alpha_1\,\mathbf{i} + (500 \cos\beta_1 - 300)\mathbf{j} + (500 \cos\gamma_1 - 200)\mathbf{k}$

$350 = 500 \cos\alpha_1;$ $\alpha_1 = 45.6°$ **Ans**

$0 = 500\cos\beta_1 - 300;$ $\beta_1 = 53.1°$ **Ans**

$0 = 500\cos\gamma_1 - 200;$ $\gamma_1 = 66.4°$ **Ans**

2–75. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\mathbf{F}_1$ so that the resultant force acting on the mast is zero.



$\mathbf{F}_1 = \{500\cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500\cos\gamma_1\,\mathbf{k}\}$ N

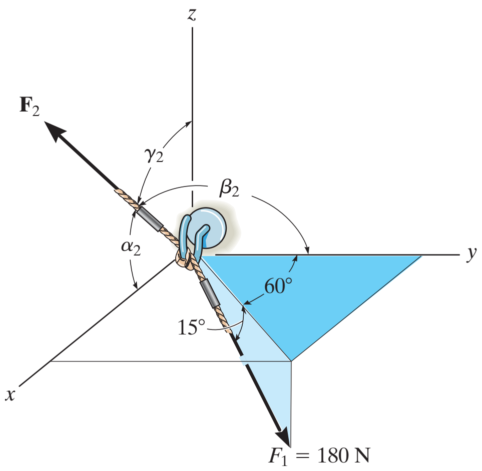
$\mathbf{F}_2 = \{-200\mathbf{k}\}$ N

$\mathbf{F}_3 = \{-300\mathbf{j}\}$ N

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$

$500\cos\alpha_1 = 0;$	$\alpha_1 = 90°$	**Ans**
$500\cos\beta_1 = 300;$	$\beta_1 = 53.1°$	**Ans**
$500\cos\gamma_1 = 200;$	$\gamma_1 = 66.4°$	**Ans**

***2–76.** Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces acts along the positive x axis and has a magnitude of 500 N.



$\mathbf{F}_1 = (180\cos 15°)\sin 60°\,\mathbf{i} + (180\cos 15°)\cos 60°\,\mathbf{j} - 180\sin 15°\,\mathbf{k}$

$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$

$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$

$\mathbf{F}_R = \{500\,\mathbf{i}\}$ N

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

i components :

$500 = 150.57 + F_2\cos\alpha_2$

$F_{2x} = F_2\cos\alpha_2 = 349.43$

j components :

$0 = 86.93 + F_2\cos\beta_2$

$F_{2y} = F_2\cos\beta_2 = -86.93$

k components :

$0 = -46.59 + F_2\cos\gamma_2$

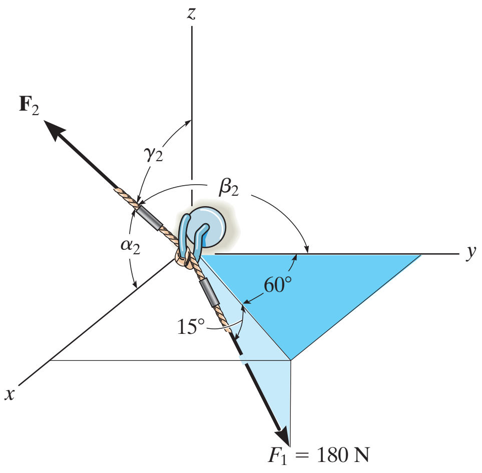
$F_{2z} = F_2\cos\gamma_2 = 46.59$

Thus,

$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$

$F_2 = 363$ N	**Ans**
$\alpha_2 = 15.8°$	**Ans**
$\beta_2 = 104°$	**Ans**
$\gamma_2 = 82.6°$	**Ans**

•2–77. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



$$\mathbf{F}_1 = (180\cos 15^\circ)\sin 60^\circ\,\mathbf{i} + (180\cos 15^\circ)\cos 60^\circ\,\mathbf{j} - 180\sin 15^\circ\,\mathbf{k}$$

$$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$$

$$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$$

$$\mathbf{F}_R = 0$$

i components :

$$0 = 150.57 + F_2\cos\alpha_2$$

$$F_2\cos\alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2\cos\beta_2$$

$$F_2\cos\beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2\cos\gamma_2$$

$$F_2\cos\gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving,

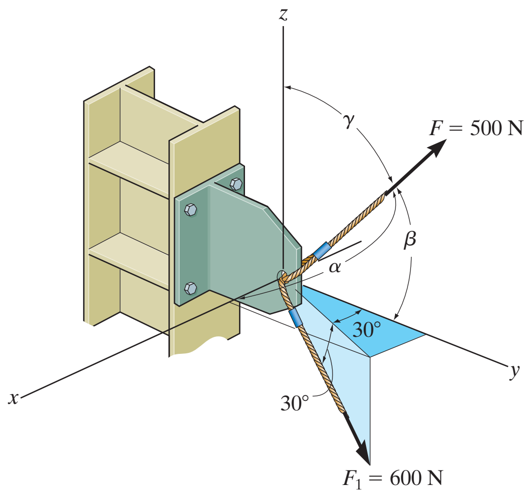
$F_2 = 180$ N **Ans**

$\alpha_2 = 147^\circ$ **Ans**

$\beta_2 = 119^\circ$ **Ans**

$\gamma_2 = 75.0^\circ$ **Ans**

2–78. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta < 90°$.



Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$
$$=\{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\text{ N}$$
$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

$$\mathbf{F}_R = F_R\mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$F_R\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\theta_x\mathbf{i} + 500\cos\theta_y\mathbf{j} + 500\cos\theta_z\mathbf{k})$$
$$F_R\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500\cos\alpha$$
$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$
$$F_R = 450 + 500\cos\beta \qquad (1)$$

$$0 = 500\cos\gamma - 300$$
$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

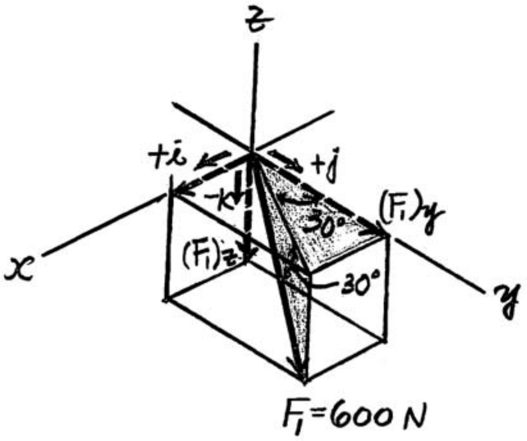
$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

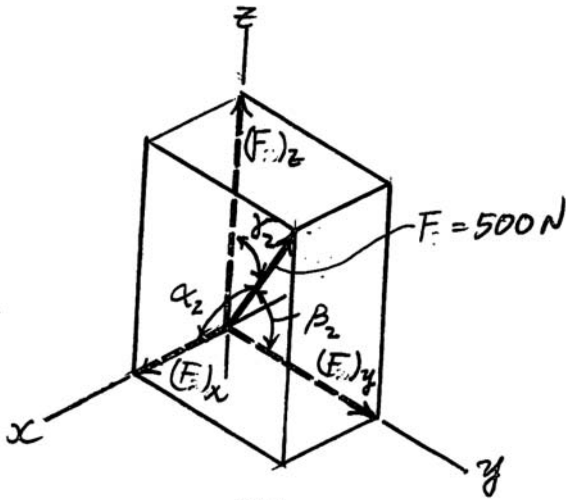
$$F_R = 450 + 500(0.6083) = 754\text{ N} \qquad \textbf{Ans.}$$

and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

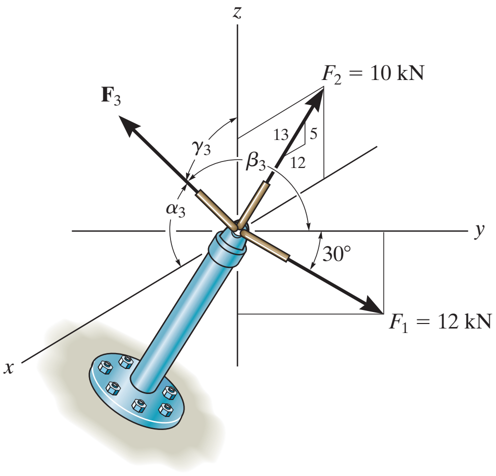


(a)



(b)

2–79. Specify the magnitude of $\mathbf{F}_3$ and its coordinate direction angles $\alpha_3, \beta_3, \gamma_3$ so that the resultant force $\mathbf{F}_R = \{9\mathbf{j}\}$ kN.



$$\mathbf{F}_1 = 12\cos 30^\circ\,\mathbf{j} - 12\sin 30^\circ\,\mathbf{k} = 10.392\,\mathbf{j} - 6\,\mathbf{k}$$

$$\mathbf{F}_2 = -\frac{12}{13}(10)\,\mathbf{i} + \frac{5}{13}(10)\,\mathbf{k} = -9.231\,\mathbf{i} + 3.846\,\mathbf{k}$$

Require

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$9\,\mathbf{j} = 10.392\,\mathbf{j} - 6\,\mathbf{k} - 9.231\,\mathbf{i} + 3.846\,\mathbf{k} + \mathbf{F}_3$$

$$\mathbf{F}_3 = 9.231\,\mathbf{i} - 1.392\,\mathbf{j} + 2.154\,\mathbf{k}$$

Hence,

$$F_3 = \sqrt{(9.231)^2 + (-1.392)^2 + (2.154)^2}$$

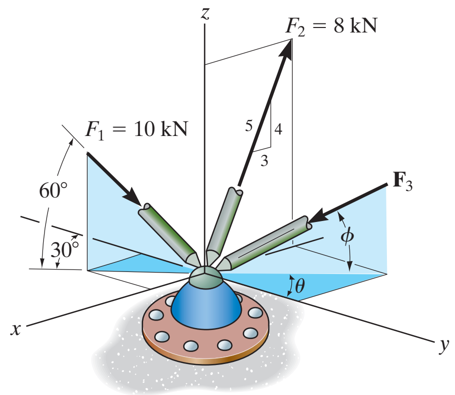
$F_3 = 9.581$ kN $= 9.58$ kN **Ans**

$\alpha_3 = \cos^{-1}\left(\frac{9.231}{9.581}\right) = 15.5^\circ$ **Ans**

$\beta_3 = \cos^{-1}\left(\frac{-1.392}{9.581}\right) = 98.4^\circ$ **Ans**

$\gamma_3 = \cos^{-1}\left(\frac{2.154}{9.581}\right) = 77.0^\circ$ **Ans**

***2–80.** If $F_3 = 9$ kN, $\theta = 30°$, and $\phi = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the ball-and-socket joint.



Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

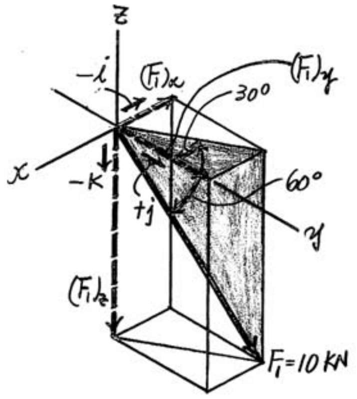
$$\mathbf{F}_1 = 10\cos 60°\sin 30°(-\mathbf{i}) + 10\cos 60°\cos 30°(+\mathbf{j}) + 10\sin 60°(-\mathbf{k})$$
$$= \{-2.5\mathbf{i} + 4.330\mathbf{j} - 8.660\mathbf{k}\}\text{ kN}$$

$$\mathbf{F}_2 = 8\left(\frac{3}{5}\right)(-\mathbf{i}) + 0\mathbf{j} + 8\left(\frac{4}{5}\right)(+\mathbf{k})$$
$$= \{-4.8\mathbf{i} + 6.4\mathbf{k}\}\text{kN}$$

$$\mathbf{F}_3 = 9\cos 45°\sin 30°(+\mathbf{i}) + 9\cos 45°\cos 30°(-\mathbf{j}) + 9\sin 45°(-\mathbf{k})$$
$$= \{3.182\mathbf{i} - 5.511\mathbf{j} - 6.364\mathbf{k}\}\text{ kN}$$

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= (-2.5\mathbf{i} + 4.330\mathbf{j} - 8.660\mathbf{k}) + (-4.8\mathbf{i} + 6.4\mathbf{k}) + (3.182\mathbf{i} - 5.511\mathbf{j} - 6.364\mathbf{k})$$
$$= \{-4.118\mathbf{i} - 1.181\mathbf{j} - 8.624\mathbf{k}\}\text{ kN}$$



(a)

The magnitude of $\mathbf{F}_R$ is

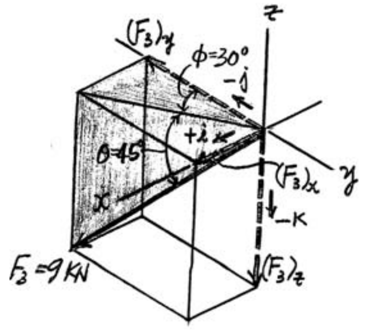
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$
$$= \sqrt{(-4.118)^2 + (-1.181)^2 + (-8.624)^2} = 9.630\text{ kN} = 9.63\text{ kN} \quad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

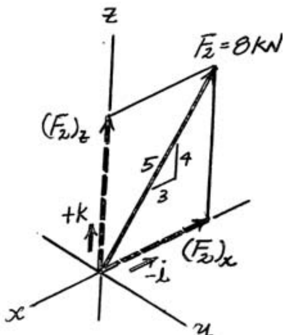
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-4.118}{9.630}\right) = 115° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-1.181}{9.630}\right) = 97.0° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-8.624}{9.630}\right) = 154° \qquad \textbf{Ans.}$$

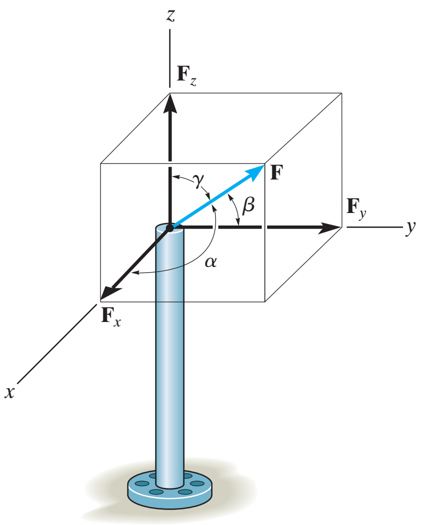


(c)



(b)

•2–81. The pole is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 30° + \cos^2 75° = 1$$

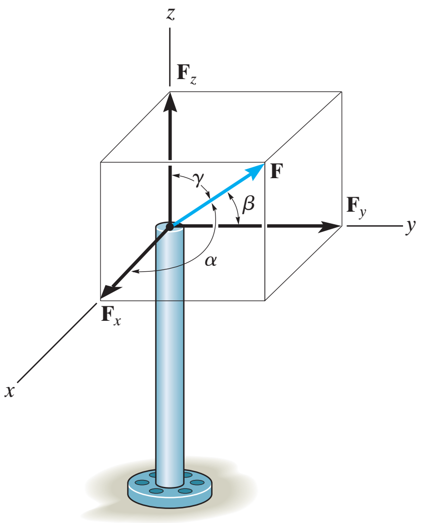
$$\alpha = 64.67°$$

$F_x = 3\cos 64.67° = 1.28 \text{ kN}$ **Ans**

$F_y = 3\cos 30° = 2.60 \text{ kN}$ **Ans**

$F_z = 3\cos 75° = 0.776 \text{ kN}$ **Ans**

2–82. The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



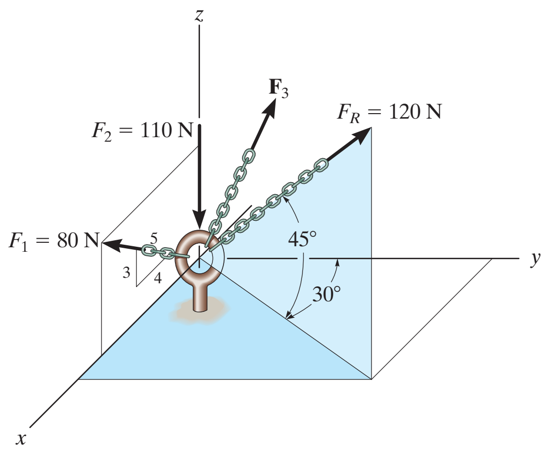
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$F = 2.02 \text{ kN}$ **Ans**

$F_y = 2.02\cos 75° = 0.523 \text{ kN}$ **Ans**

2–83. Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



Cartesian Vector Notation :

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\text{ N}$$
$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\text{N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\text{ N}$$

Resultant Force :

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}$$
$$= \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\text{ N}$$
$$F_{3_y} = 73.48\text{ N}$$
$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\text{ N}$$

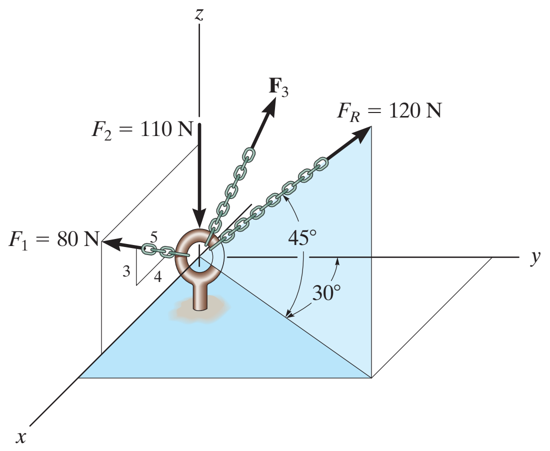
The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$
$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$
$$= 165.62\text{ N} = 166\text{ N} \qquad \text{Ans}$$

The coordinate direction angles for $\mathbf{F}_3$ are

$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5° \qquad \text{Ans}$$
$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7° \qquad \text{Ans}$$
$$\cos\gamma = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5° \qquad \text{Ans}$$

***2–84.** Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



Unit Vector of $\mathbf{F}_1$ and $\mathbf{F}_R$:

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

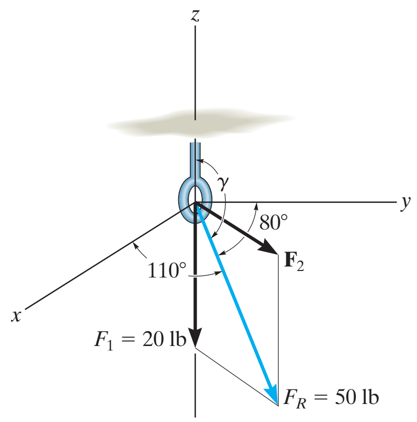
$$\mathbf{u}_R = \cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}$$
$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$$\cos\alpha_{F_1} = 0.8 \qquad \alpha_{F_1} = 36.9° \qquad \text{Ans}$$
$$\cos\beta_{F_1} = 0 \qquad \beta_{F_1} = 90.0° \qquad \text{Ans}$$
$$\cos\gamma_{F_1} = 0.6 \qquad \gamma_{F_1} = 53.1° \qquad \text{Ans}$$

$$\cos\alpha_R = 0.3536 \qquad \alpha_R = 69.3° \qquad \text{Ans}$$
$$\cos\beta_R = 0.6124 \qquad \beta_R = 52.2° \qquad \text{Ans}$$
$$\cos\gamma_R = 0.7071 \qquad \gamma_R = 45.0° \qquad \text{Ans}$$

•2–85. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bolt. If the resultant force $\mathbf{F}_R$ has a magnitude of 50 lb and coordinate direction angles $\alpha = 110^\circ$ and $\beta = 80^\circ$, as shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



$$(1)^2 = \cos^2 110^\circ + \cos^2 80^\circ + \cos^2 \gamma$$

$$\gamma = 157.44^\circ$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$50 \cos 110^\circ = (F_2)_x$$

$$50 \cos 80^\circ = (F_2)_y$$

$$50 \cos 157.44^\circ = (F_2)_z - 20$$

$$(F_2)_x = -17.10$$

$$(F_2)_y = 8.68$$

$$(F_2)_z = -26.17$$

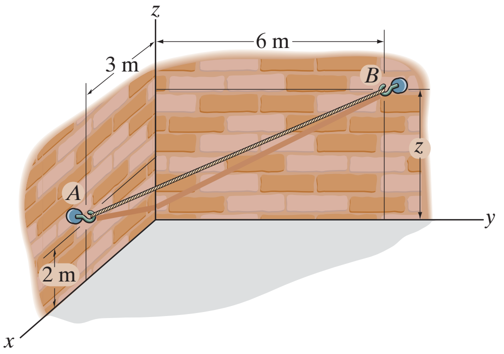
$$F_2 = \sqrt{(-17.10)^2 + (8.68)^2 + (-26.17)^2} = 32.4 \text{ lb} \qquad \textbf{Ans}$$

$$\alpha_2 = \cos^{-1}\left(\frac{-17.10}{32.4}\right) = 122^\circ \qquad \textbf{Ans}$$

$$\beta_2 = \cos^{-1}\left(\frac{8.68}{32.4}\right) = 74.5^\circ \qquad \textbf{Ans}$$

$$\gamma_2 = \cos^{-1}\left(\frac{-26.17}{32.4}\right) = 144^\circ \qquad \textbf{Ans}$$

2–86. Determine the position vector **r** directed from point *A* to point *B* and the length of cord *AB*. Take $z = 4$ m.



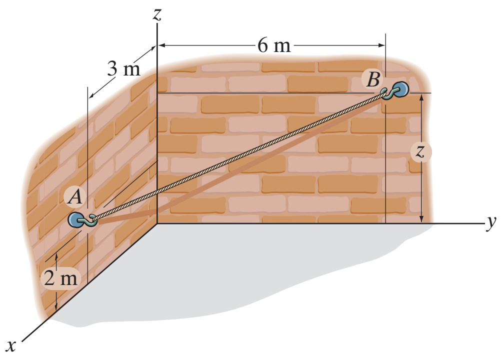
Position Vector: The coordinates for points *A* and *B* are *A*(3, 0, 2) m and *B*(0, 6, 4) m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (4-2)\mathbf{k}$$
$$= \{-3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m} \qquad \textbf{Ans.}$$

The length of cord *AB* is

$$r_{AB} = \sqrt{(-3)^2 + 6^2 + 2^2} = 7 \text{ m} \qquad \textbf{Ans.}$$

2–87. If the cord *AB* is 7.5 m long, determine the coordinate position +*z* of point *B*



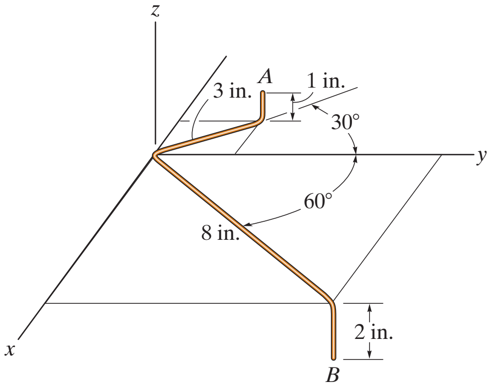
Position Vector: The coordinates for points *A* and *B* are $A(3, 0, 2)$ m and $B(0, 6, z)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i}+(6-0)\mathbf{j}+(z-2)\mathbf{k}$$
$$=\{-3\mathbf{i}+6\mathbf{j}+(z-2)\mathbf{k}\}\text{ m}$$

Since the length of cord is equal to the magnitude of $\mathbf{r}_{AB}$, then

$$r_{AB} = 7.5 = \sqrt{(-3)^2+6^2+(z-2)^2}$$
$$56.25 = 45+(z-2)^2$$
$$z-2 = \pm 3.354$$
$$z = 5.35\text{ m} \qquad \textbf{Ans.}$$

***2–88.** Determine the distance between the end points *A* and *B* on the wire by first formulating a position vector from *A* to *B* and then determining its magnitude.

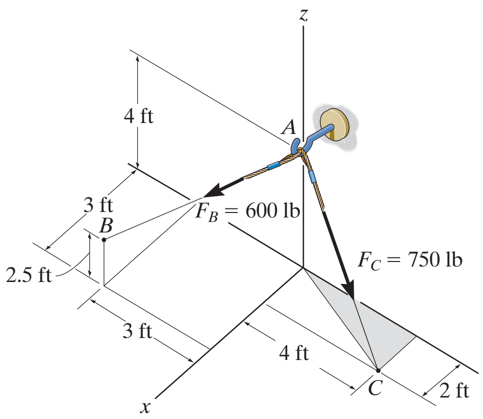


$$\mathbf{r}_{AB} = (8\sin 60° - (-3\sin 30°))\,\mathbf{i} + (8\cos 60° - 3\cos 30°)\,\mathbf{j} + (-2-1)\,\mathbf{k}$$

$$\mathbf{r}_{AB} = \{8.428\,\mathbf{i} + 1.402\,\mathbf{j} - 3\,\mathbf{k}\}\text{ in.}$$

$$r_{AB} = \sqrt{(8.428)^2 + (1.402)^2 + (-3)^2} = 9.06\text{ in.} \qquad \textbf{Ans}$$

•2–89. Determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



Unit Vectors: The coordinate points *A*, *B*, and *C* are shown in Fig. *a*. Thus,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i}+(-3-0)\mathbf{j}+(2.5-4)\mathbf{k}}{\sqrt{(3-0)^2+(-3-0)^2+(2.5-4)^2}} = \frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i}+(4-0)\mathbf{j}+(0-4)\mathbf{k}}{\sqrt{(2-0)^2+(4-0)^2+(0-4)^2}} = \frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

Force Vectors: Multiplying the magnitude of the force with its unit vector, we have

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}\right) = \{400\mathbf{i}-400\mathbf{j}-200\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 750\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right) = \{250\mathbf{i}+500\mathbf{j}-500\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_R = \mathbf{F}_B+\mathbf{F}_C = 400\mathbf{i}-400\mathbf{j}-200\mathbf{k}+250\mathbf{i}+500\mathbf{j}-500\mathbf{k}$$

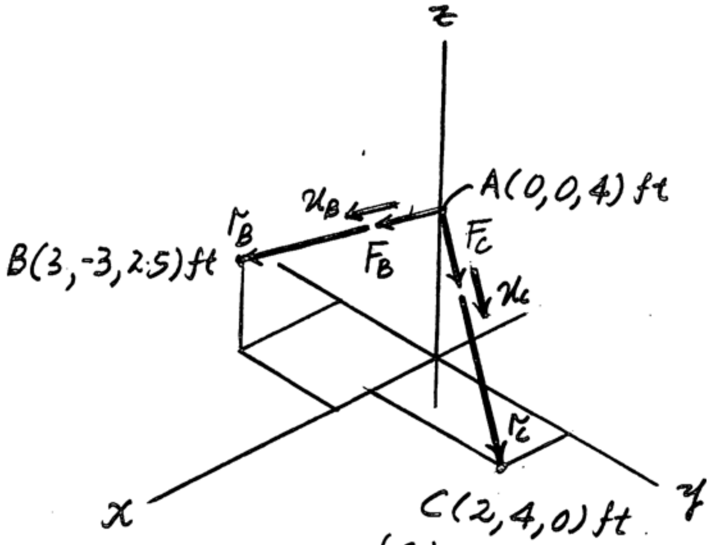
$$\mathbf{F}_R = \{650\mathbf{i}+100\mathbf{j}-700\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{650^2+100^2+(-700)^2} = 960\text{ lb} \qquad \textbf{Ans.}$$

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{650}{960}\right) = 47.4^\circ \qquad \textbf{Ans.}$$

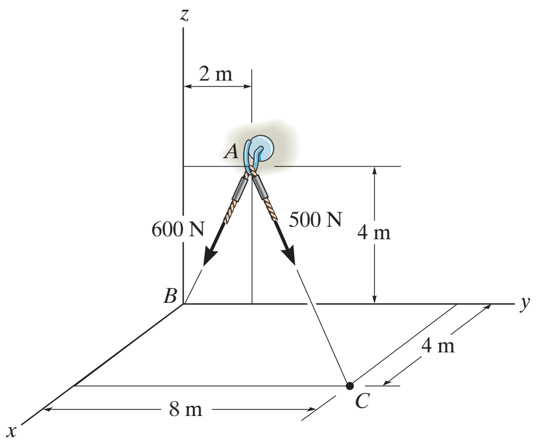
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{100}{960}\right) = 84.0^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-700}{960}\right) = 137^\circ \qquad \textbf{Ans.}$$



(a)

2–90. Determine the magnitude and coordinate direction angles of the resultant force.



$\mathbf{r}_{AB} = \{-2\,\mathbf{j} - 4\,\mathbf{k}\}\text{m}; \quad r_{AB} = 4.472\text{ m}$

$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = -0.447\,\mathbf{j} - 0.894\,\mathbf{k}$

$\mathbf{F}_{AB} = 600\,\mathbf{u}_{AB} = \{-268.33\,\mathbf{j} - 536.66\,\mathbf{k}\}\text{N}$

$\mathbf{r}_{AC} = \{4\,\mathbf{i} + 6\,\mathbf{j} - 4\,\mathbf{k}\}\text{m}; \quad r_{AC} = 8.246\text{ m}$

$\mathbf{u}_{AC} = \left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 0.485\,\mathbf{i} + 0.728\,\mathbf{j} - 0.485\,\mathbf{k}$

$\mathbf{F}_{AC} = 500\,\mathbf{u}_{AC} = \{242.54\,\mathbf{i} + 363.80\,\mathbf{j} - 242.54\,\mathbf{k}\}\text{N}$

$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$

$\mathbf{F}_R = \{242.54\,\mathbf{i} + 95.47\,\mathbf{j} - 779.20\,\mathbf{k}\}$

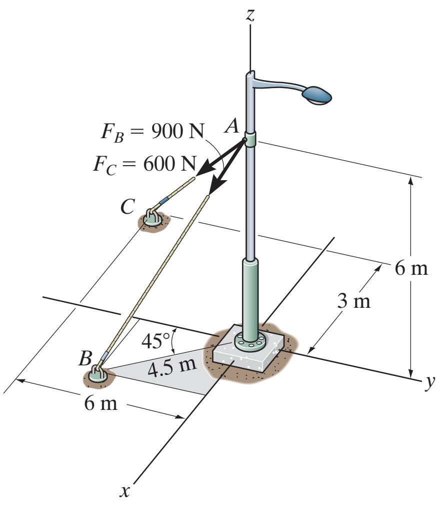
$F_R = \sqrt{(242.54)^2 + (95.47)^2 + (-779.20)^2} = 821.64 = 822\text{ N}$ **Ans**

$\alpha = \cos^{-1}\left(\frac{242.54}{821.64}\right) = 72.8^\circ$ **Ans**

$\beta = \cos^{-1}\left(\frac{95.47}{821.64}\right) = 83.3^\circ$ **Ans**

$\gamma = \cos^{-1}\left(\frac{-779.20}{821.64}\right) = 162^\circ$ **Ans**

2–91. Determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(4.5\sin 45° - 0)\mathbf{i} + (-4.5\cos 45° - 0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(4.5\sin 45° - 0)^2 + (-4.5\cos 45° - 0)^2 + (0-6)^2}}$$
$$= 0.4243\mathbf{i} - 0.4243\mathbf{j} - 0.8\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (-6-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-6-0)^2 + (0-6)^2}}$$
$$= -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 900(0.4243\mathbf{i} - 0.4243\mathbf{j} - 0.8\mathbf{k}) = \{381.84\mathbf{i} - 381.84\mathbf{j} - 720\mathbf{k}\}\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} - 400\mathbf{j} - 400\mathbf{k}\}\text{N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (381.84\mathbf{i} - 381.84\mathbf{j} - 720\mathbf{k}) + (-200\mathbf{i} - 400\mathbf{j} - 400\mathbf{k})$$
$$= \{181.84\mathbf{i} - 781.84\mathbf{j} - 1120\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

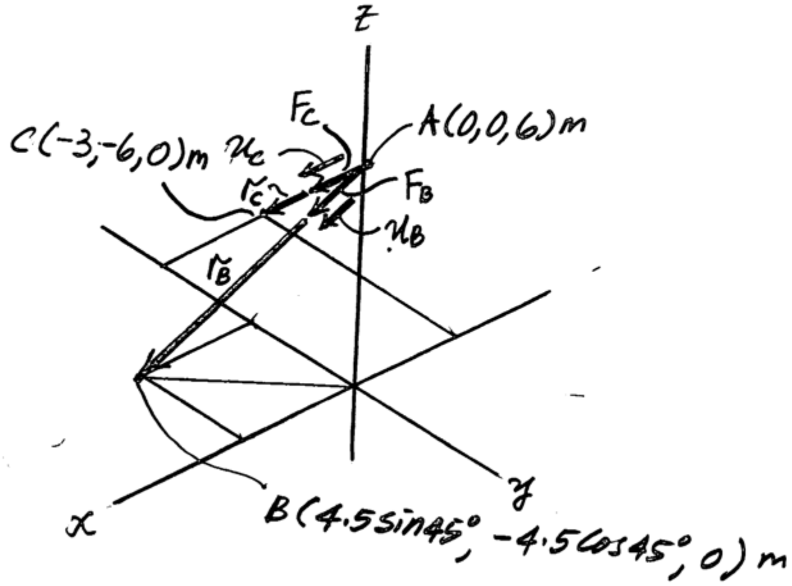
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{(181.84)^2 + (-781.84)^2 + (-1120)^2} = 1377.95\text{ N} = 1.38\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{181.84}{1377.95}\right) = 82.4° \qquad \textbf{Ans.}$$

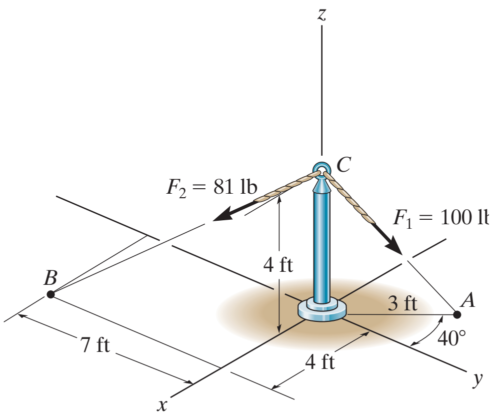
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-781.84}{1377.95}\right) = 125° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1120}{1377.95}\right) = 144° \qquad \textbf{Ans.}$$



(a)

***2–92.** Determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{F}_1 = -100\left(\frac{3}{5}\right)\sin 40°\,\mathbf{i} + 100\left(\frac{3}{5}\right)\cos 40°\,\mathbf{j} - 100\left(\frac{4}{5}\right)\mathbf{k}$$

$$= \{-38.567\,\mathbf{i} + 45.963\,\mathbf{j} - 80\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 81\text{ lb}\left(\frac{4}{9}\,\mathbf{i} - \frac{7}{9}\,\mathbf{j} - \frac{4}{9}\,\mathbf{k}\right)$$

$$= \{36\,\mathbf{i} - 63\,\mathbf{j} - 36\,\mathbf{k}\}\text{lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-2.567\,\mathbf{i} - 17.04\,\mathbf{j} - 116.0\,\mathbf{k}\}\text{ lb}$$

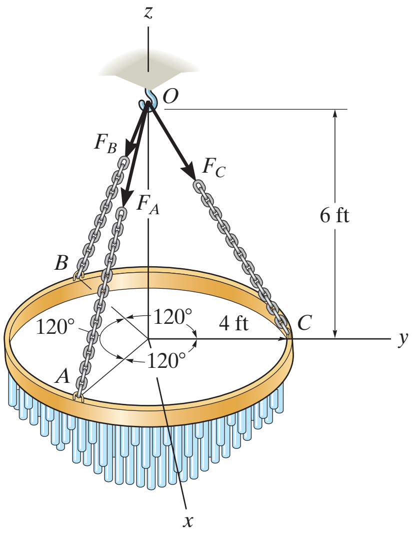
$$F_R = \sqrt{(-2.567)^2 + (-17.04)^2 + (-116.0)^2} = 117.27\text{ lb} = 117\text{ lb} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{-2.567}{117.27}\right) = 91.3° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-17.04}{117.27}\right) = 98.4° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-116.0}{117.27}\right) = 172° \qquad \textbf{Ans}$$

•2–93. The chandelier is supported by three chains which are concurrent at point *O*. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



$$\mathbf{F}_A = 60\,\frac{(4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_B = 60\,\frac{(-4\cos 30°\,\mathbf{i} - 4\sin 30°\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(-4\cos 30°)^2 + (-4\sin 30°)^2 + (-6)^2}}$$

$$= \{-28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = 60\,\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8\,\mathbf{k}\}\text{ lb}$$

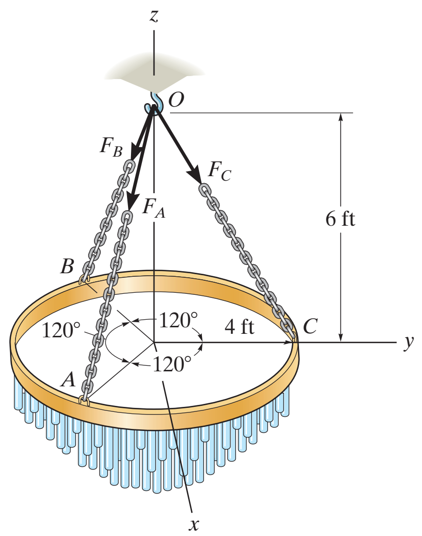
$$F_R = 150\text{ lb} \qquad \textbf{Ans}$$

$$\alpha = 90° \qquad \textbf{Ans}$$

$$\beta = 90° \qquad \textbf{Ans}$$

$$\gamma = 180° \qquad \textbf{Ans}$$

2–94. The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



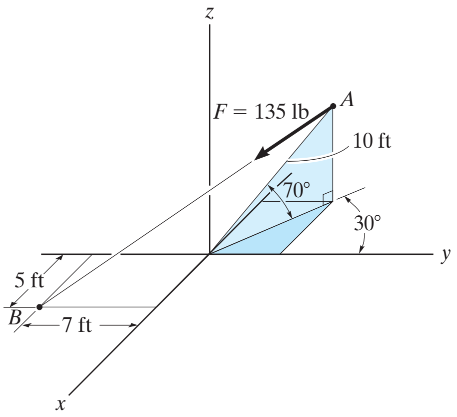
$$\mathbf{F}_C = F\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\,\mathbf{j} - 0.8321\,F\,\mathbf{k}$$

$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

$$F_{R_z} = \Sigma F_z; \qquad 130 = 3(0.8321\,F)$$

$F = 52.1$ lb **Ans**

2–95. Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



Unit Vector : The coordinates of point A are

$$A(-10\cos 70^\circ \sin 30^\circ,\ 10\cos 70^\circ \cos 30^\circ,\ 10\sin 70^\circ)\ \text{ft}$$
$$= A(-1.710,\ 2.962,\ 9.397)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{[5-(-1.710)]\mathbf{i} + (-7-2.962)\mathbf{j} + (0-9.397)\mathbf{k}\}\ \text{ft}$$
$$= \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{6.710^2 + (-9.962)^2 + (-9.397)^2} = 15.250\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}}{15.250}$$
$$= 0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}$$

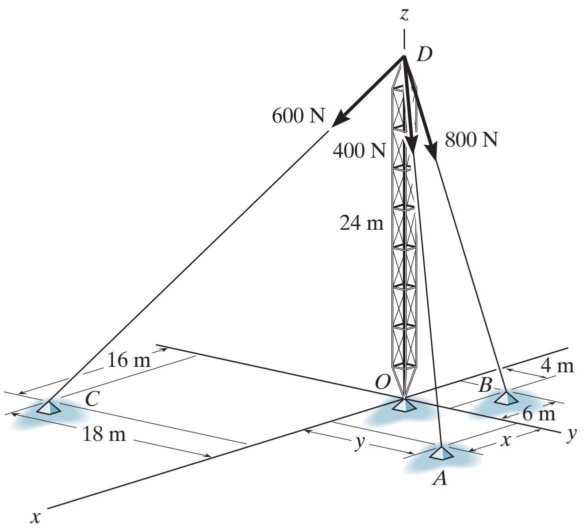
Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 135\{0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k}\}\ \text{lb}$$
$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

Coordinate Direction Angles : From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$\cos\alpha = 0.4400$	$\alpha = 63.9^\circ$	**Ans**
$\cos\beta = -0.6532$	$\beta = 131^\circ$	**Ans**
$\cos\gamma = -0.6162$	$\gamma = 128^\circ$	**Ans**

***2–96.** The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

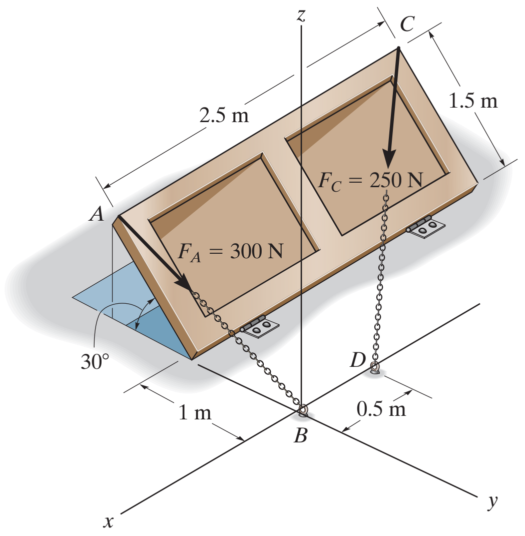
$$= 1501.66\text{ N} = 1.50\text{ kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168^\circ \qquad \textbf{Ans}$$

•2–97. The door is held opened by means of two chains. If the tension in *AB* and *CD* is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



Unit Vector : First determine the position vector $\mathbf{r}_{AB}$ and $\mathbf{r}_{CD}$. The coordinates of points *A* and *C* are

$$A[0,\ -(1+1.5\cos 30°),\ 1.5\sin 30°]\ \text{m} = A(0,\ -2.299,\ 0.750)\ \text{m}$$
$$C[-2.50,\ -(1+1.5\cos 30°),\ 1.5\sin 300°]\ \text{m} = C(-2.50,\ -2.299,\ 0.750)\ \text{m}$$

Then

$$\mathbf{r}_{AB} = \{(0-0)\mathbf{i}+[0-(-2.299)]\mathbf{j}+(0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.299\mathbf{j}-0.750\mathbf{k}\}\ \text{m}$$
$$r_{AB} = \sqrt{2.299^2+(-0.750)^2} = 2.418\ \text{m}$$
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.299\mathbf{j}-0.750\mathbf{k}}{2.418} = 0.9507\mathbf{j}-0.3101\mathbf{k}$$

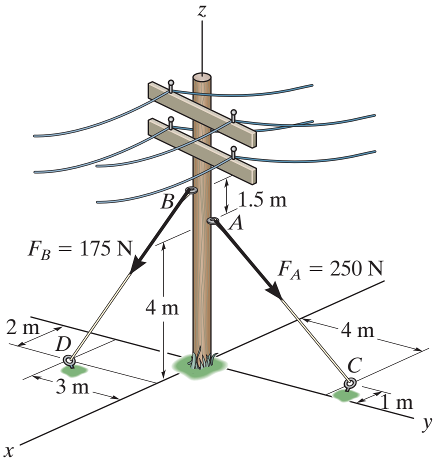
$$\mathbf{r}_{CD} = \{[-0.5-(-2.5)]\mathbf{i}+[0-(-2.299)]\mathbf{j}+(0-0.750)\mathbf{k}\}\ \text{m}$$
$$= \{2.00\mathbf{i}+2.299\mathbf{j}-0.750\mathbf{k}\}\ \text{m}$$
$$r_{CD} = \sqrt{2.00^2+2.299^2+(-0.750)^2} = 3.138\ \text{m}$$
$$\mathbf{u}_{CD} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{2.00\mathbf{i}+2.299\mathbf{j}-0.750\mathbf{k}}{3.138} = 0.6373\mathbf{i}+0.7326\mathbf{j}-0.2390\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{AB} = 300\{0.9507\mathbf{j}-0.3101\mathbf{k}\}\ \text{N}$$
$$= \{285.21\mathbf{j}-93.04\mathbf{k}\}\ \text{N}$$
$$= \{285\mathbf{j}-93.0\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{CD} = 250\{0.6373\mathbf{i}+0.7326\mathbf{j}-0.2390\mathbf{k}\}\ \text{N}$$
$$= \{159.33\mathbf{i}+183.15\mathbf{j}-59.75\mathbf{k}\}\ \text{N}$$
$$= \{159\mathbf{i}+183\mathbf{j}-59.7\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

2–98. The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.



Unit Vector :

$$\mathbf{r}_{AC} = \{(-1-0)\mathbf{i}+(4-0)\mathbf{j}+(0-4)\mathbf{k}\}\ \text{m} = \{-1\mathbf{i}+4\mathbf{j}-4\mathbf{k}\}\ \text{m}$$
$$r_{AC} = \sqrt{(-1)^2+4^2+(-4)^2} = 5.745\ \text{m}$$
$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i}+4\mathbf{j}-4\mathbf{k}}{5.745} = -0.1741\mathbf{i}+0.6963\mathbf{j}-0.6963\mathbf{k}$$

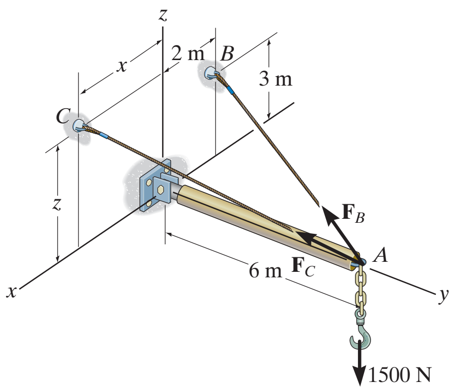
$$\mathbf{r}_{BD} = \{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-5.5)\mathbf{k}\}\ \text{m} = \{2\mathbf{i}-3\mathbf{j}-5.5\mathbf{k}\}\ \text{m}$$
$$r_{BD} = \sqrt{2^2+(-3)^2+(-5.5)^2} = 6.576\ \text{m}$$
$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i}-3\mathbf{j}-5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i}-0.4562\mathbf{j}-0.8363\mathbf{k}$$

Force Vector :

$$\mathbf{F}_A = F_A\mathbf{u}_{AC} = 250\{-0.1741\mathbf{i}+0.6963\mathbf{j}-0.6963\mathbf{k}\}\ \text{N}$$
$$= \{-43.52\mathbf{i}+174.08\mathbf{j}-174.08\mathbf{k}\}\ \text{N}$$
$$= \{-43.5\mathbf{i}+174\mathbf{j}-174\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{F}_B = F_B\mathbf{u}_{BD} = 175\{0.3041\mathbf{i}-0.4562\mathbf{j}-0.8363\mathbf{k}\}\ \text{N}$$
$$= \{53.22\mathbf{i}-79.83\mathbf{j}-146.36\mathbf{k}\}\ \text{N}$$
$$= \{53.2\mathbf{i}-79.8\mathbf{j}-146\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

2–99. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point *A* towards *O*, determine the magnitudes of the resultant force and forces $\mathbf{F}_B$ and $\mathbf{F}_C$. Set $x = 3$ m and $z = 2$ m.



Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i}+(0-6)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(3-0)^2+(0-6)^2+(2-0)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed along the negative *y* axis, and the load **W** is directed along the *z* axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force: The vector addition of $\mathbf{F}_B$, $\mathbf{F}_C$, and **W** is equal to $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}\right)+\left(\frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B+\frac{3}{7}F_C\right)\mathbf{i}+\left(-\frac{6}{7}F_B-\frac{6}{7}F_C\right)\mathbf{j}+\left(\frac{3}{7}F_B+\frac{2}{7}F_C-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -\frac{2}{7}F_B+\frac{3}{7}F_C \qquad (1)$$

$$-F_R = -\frac{6}{7}F_B-\frac{6}{7}F_C \qquad (2)$$

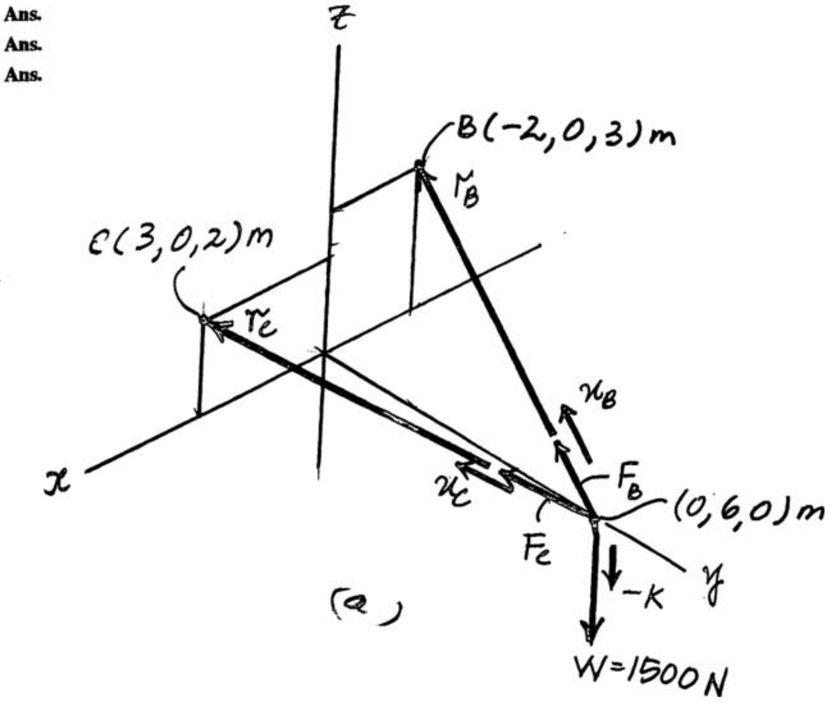
$$0 = \frac{3}{7}F_B+\frac{2}{7}F_C-1500 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

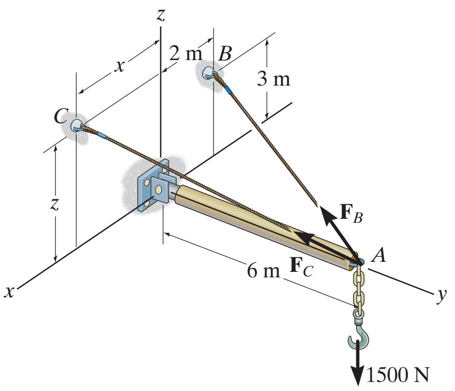
$F_C = 1615.38\text{ N} = 1.62\text{ kN}$ **Ans.**

$F_B = 2423.08\text{ N} = 2.42\text{ kN}$ **Ans.**

$F_R = 3461.53\text{ N} = 3.46\text{ kN}$ **Ans.**



***2–100.** Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point *A* towards *O*, determine the values of *x* and *z* for the coordinates of point *C* and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



Force Vectors: From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

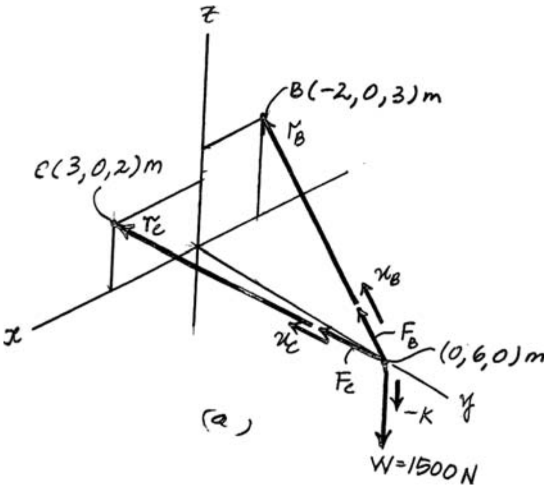
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}$$



Since the resultant force $\mathbf{F}_R$ is directed along the negative *y* axis, and the load is directed along the *z* axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k})+\left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$x = 0.5679z$ (4)

Substituting Eq. (4) into Eq. (1), and solving

$z = 2.197\text{ m} = 2.20\text{ m}$ **Ans.**

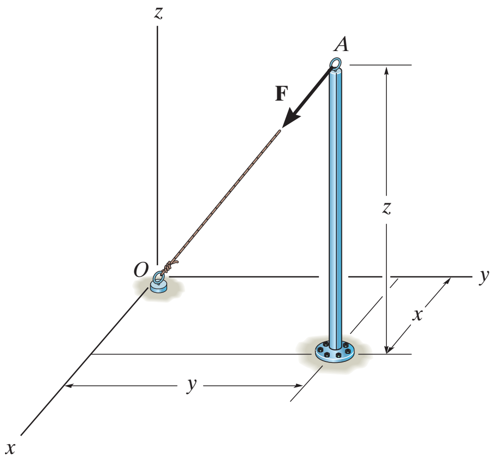
Substituting $z = 2.197$ m into Eq. (4), yields

$x = 1.248\text{ m} = 1.25\text{ m}$ **Ans.**

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$F_R = 3591.85\text{ N} = 3.59\text{ kN}$ **Ans.**

•2–101. The cable *AO* exerts a force on the top of the pole of $\mathbf{F} = \{-120\mathbf{i} - 90\mathbf{j} - 80\mathbf{k}\}$ lb. If the cable has a length of 34 ft, determine the height z of the pole and the location (x, y) of its base.



$$F = \sqrt{(-120)^2 + (-90)^2 + (-80)^2} = 170 \text{ lb}$$

$$\mathbf{u} = \frac{\mathbf{F}}{F} = -\frac{120}{170}\mathbf{i} - \frac{90}{170}\mathbf{j} - \frac{80}{170}\mathbf{k}$$

$$\mathbf{r} = 34\,\mathbf{u} = \{-24\,\mathbf{i} - 18\,\mathbf{j} - 16\,\mathbf{k}\} \text{ ft}$$

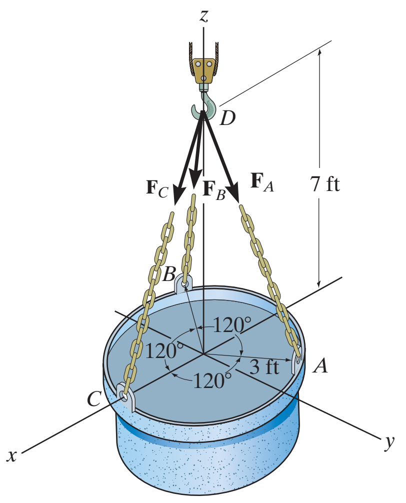
Thus,

$x = 24$ ft **Ans**

$y = 18$ ft **Ans**

$z = 16$ ft **Ans**

2–102. If the force in each chain has a magnitude of 450 lb, determine the magnitude and coordinate direction angles of the resultant force.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, and $\mathbf{u}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{(-3\sin 30^\circ - 0)\mathbf{i} + (3\cos 30^\circ - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30^\circ - 0)^2 + (3\cos 30^\circ - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_B = \frac{(-3\sin 30^\circ - 0)\mathbf{i} + (-3\cos 30^\circ - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30^\circ - 0)^2 + (-3\cos 30^\circ - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_C = \frac{(3-0)\mathbf{i} + (0-0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(3-0)^2 + (0-0)^2 + (0-7)^2}} = 0.3939\mathbf{i} - 0.9191\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 450(-0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}) = \{-88.63\mathbf{i} + 153.51\mathbf{j} - 413.62\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 450(-0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}) = \{-88.63\mathbf{i} - 153.51\mathbf{j} - 413.62\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450(0.3939\mathbf{i} - 0.9191\mathbf{k}) = \{177.26\mathbf{i} - 413.62\mathbf{k}\}\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = (-88.63\mathbf{i} + 153.51\mathbf{j} - 413.62\mathbf{k}) + (-88.63\mathbf{i} - 153.51\mathbf{j} - 413.62\mathbf{k}) + (177.26\mathbf{i} - 413.62\mathbf{k})$$
$$= \{-1240.85\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

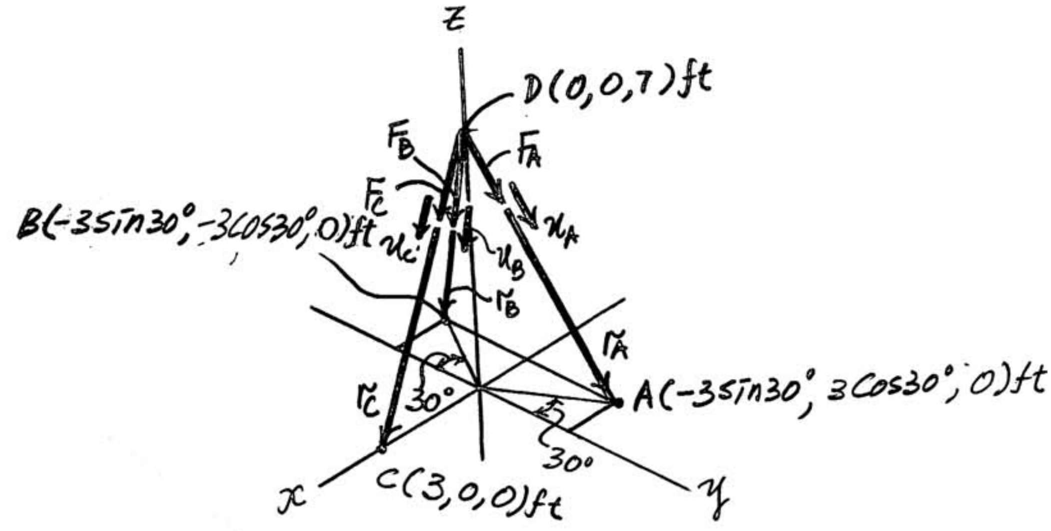
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$
$$= \sqrt{0^2 + 0^2 + (-1240.85)^2} = 1240.85\text{ lb} = 1.24\text{ kip} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{1240.85}\right) = 90^\circ \qquad \textbf{Ans.}$$

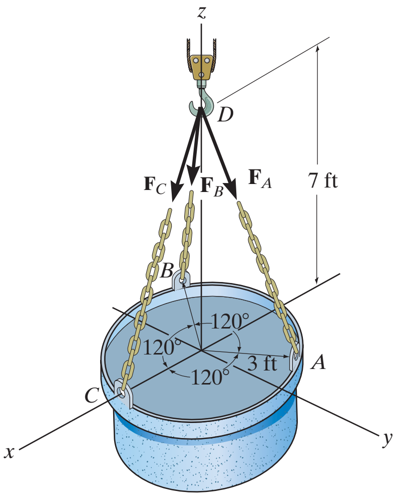
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{1240.85}\right) = 90^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1240.85}{1240.85}\right) = 180^\circ \qquad \textbf{Ans.}$$



(a)

2–103. If the resultant of the three forces is $\mathbf{F}_R = \{-900\mathbf{k}\}$ lb, determine the magnitude of the force in each chain.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, and $\mathbf{u}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{(-3\sin 30° - 0)\mathbf{i} + (3\cos 30° - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30° - 0)^2 + (3\cos 30° - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} + 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_B = \frac{(-3\sin 30° - 0)\mathbf{i} + (-3\cos 30° - 0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(-3\sin 30° - 0)^2 + (-3\cos 30° - 0)^2 + (0-7)^2}} = -0.1970\mathbf{i} - 0.3411\mathbf{j} - 0.9191\mathbf{k}$$

$$\mathbf{u}_C = \frac{(3-0)\mathbf{i} + (0-0)\mathbf{j} + (0-7)\mathbf{k}}{\sqrt{(3-0)^2 + (0-0)^2 + (0-7)^2}} = 0.3939\mathbf{i} - 0.9191\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = -0.1970F_A\mathbf{i} + 0.3411F_A\mathbf{j} - 0.9191F_A\mathbf{k}$$
$$\mathbf{F}_B = F_B\mathbf{u}_B = -0.1970F_B\mathbf{i} - 0.3411F_B\mathbf{j} - 0.9191F_B\mathbf{k}$$
$$\mathbf{F}_C = F_C\mathbf{u}_C = 0.3939F_C\mathbf{i} - 0.9191F_C\mathbf{k}$$

Resultant Force: The vector addition of $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$ is equal to $\mathbf{F}_R$. Thus,

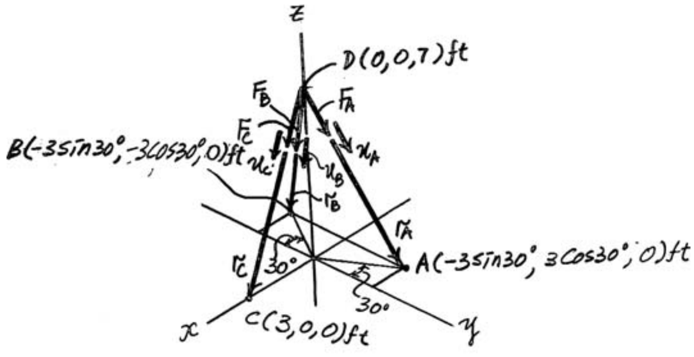
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$
$$-900\mathbf{k} = (-0.1970F_A\mathbf{i} + 0.3411F_A\mathbf{j} - 0.9191F_A\mathbf{k}) + (-0.1970F_B\mathbf{i} - 0.3411F_B\mathbf{j} - 0.9191F_B\mathbf{k}) + (0.3939F_C\mathbf{i} - 0.9191F_C\mathbf{k})$$
$$-900\mathbf{k} = (-0.1970F_A - 0.1970F_B + 0.3939F_C)\mathbf{i} + (0.3411F_A - 0.3411F_B)\mathbf{j} + (-0.9191F_A - 0.9191F_B - 0.9191F_C)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -0.1970F_A - 0.1970F_B + 0.3939F_C \qquad (1)$$
$$0 = 0.3411F_A - 0.3411F_B \qquad (2)$$
$$-900\mathbf{k} = -0.9191F_A - 0.9191F_B - 0.9191F_C \qquad (3)$$

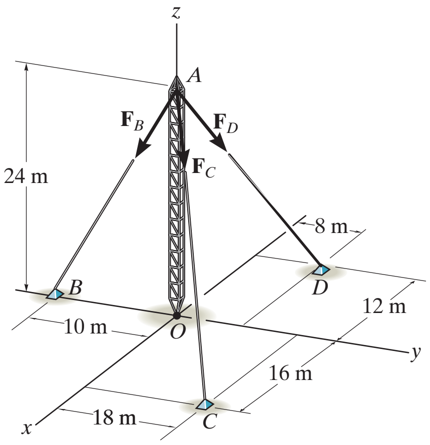
Solving Eqs. (1), (2), and (3), yields

$$F_A = F_B = F_C = 326\text{ lb}$$ **Ans.**



(a)

***2–104.** The antenna tower is supported by three cables. If the forces of these cables acting on the antenna are $F_B = 520$ N, $F_C = 680$ N, and $F_D = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



$$\mathbf{F}_B = 520\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 520\left(-\frac{10}{26}\mathbf{j} - \frac{24}{26}\mathbf{k}\right) = -200\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_C = 680\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 680\left(\frac{16}{34}\mathbf{i} + \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) = 320\,\mathbf{i} + 360\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_D = 560\left(\frac{\mathbf{r}_{AD}}{r_{AD}}\right) = 560\left(-\frac{12}{28}\mathbf{i} + \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right) = -240\,\mathbf{i} + 160\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{80\,\mathbf{i} + 320\,\mathbf{j} - 1440\,\mathbf{k}\}\text{ N}$$

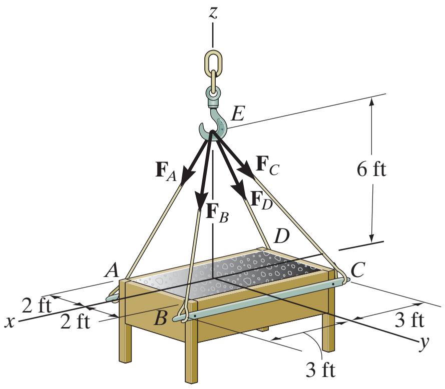
$$F_R = \sqrt{(80)^2 + (320)^2 + (-1440)^2} = 1477.3 = 1.48\text{ kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{80}{1477.3}\right) = 86.9^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{320}{1477.3}\right) = 77.5^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1440}{1477.3}\right) = 167^\circ \qquad \textbf{Ans}$$

•2–105. If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$ and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i}+(-2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(3-0)^2+(-2-0)^2+(0-6)^2}} = \frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i}+(2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(3-0)^2+(2-0)^2+(0-6)^2}} = \frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i}+(2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(-3-0)^2+(2-0)^2+(0-6)^2}} = -\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i}+(-2-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(-3-0)^2+(-2-0)^2+(0-6)^2}} = -\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right) = [30\mathbf{i}-20\mathbf{j}-60\mathbf{k}]\ \text{lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 70\left(\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right) = [30\mathbf{i}+20\mathbf{j}-60\mathbf{k}]\ \text{lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(-\frac{3}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i}+20\mathbf{j}-60\mathbf{k}]\ \text{lb}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 70\left(-\frac{3}{7}\mathbf{i}-\frac{2}{7}\mathbf{j}-\frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i}-20\mathbf{j}-60\mathbf{k}]\ \text{lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A+\mathbf{F}_B+\mathbf{F}_C+\mathbf{F}_D = (30\mathbf{i}-20\mathbf{j}-60\mathbf{k})+(30\mathbf{i}+20\mathbf{j}-60\mathbf{k})+(-30\mathbf{i}+20\mathbf{j}-60\mathbf{k})+(-30\mathbf{i}-20\mathbf{j}-60\mathbf{k})$$
$$=\{-240\mathbf{k}\}\ \text{N}$$

The magnitude of $\mathbf{F}_R$ is

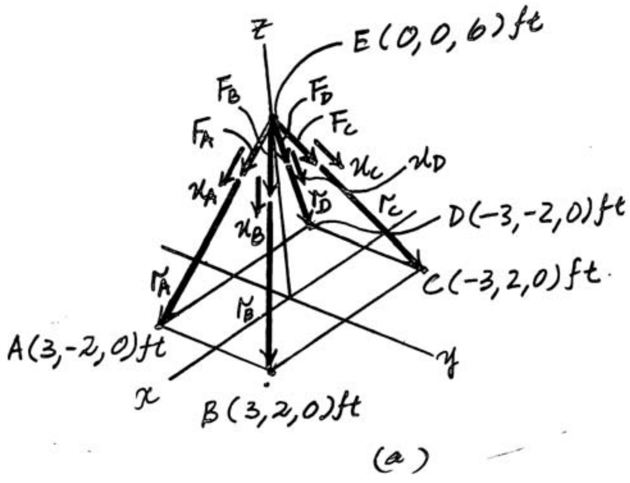
$$F_R = \sqrt{(F_R)_x{}^2+(F_R)_y{}^2+(F_R)_z{}^2}$$
$$= \sqrt{0+0+(-240)^2} = 240\ \text{lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

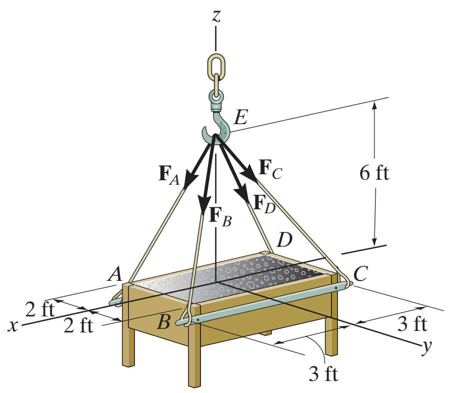
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-240}{240}\right) = 180° \qquad \textbf{Ans.}$$



2–106. If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$ and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Since the magnitudes of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ are the same and denoted as $\mathbf{F}$, they can be written as

$$\mathbf{F}_A = F_A\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

Resultant Force: The vector addition of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$ and $\mathbf{F}_D$ is equal to $\mathbf{F}_R$. Thus,

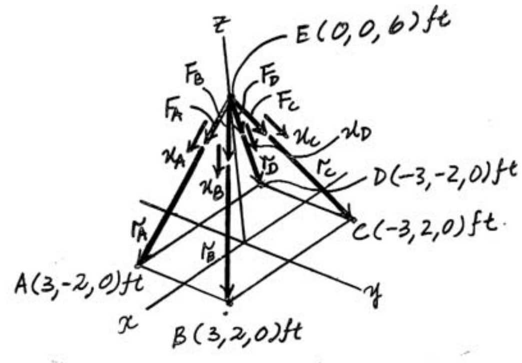
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$\{-360\mathbf{k}\} = \left[F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right]$$

$$-360\mathbf{k} = -\frac{24}{7}\mathbf{k}$$

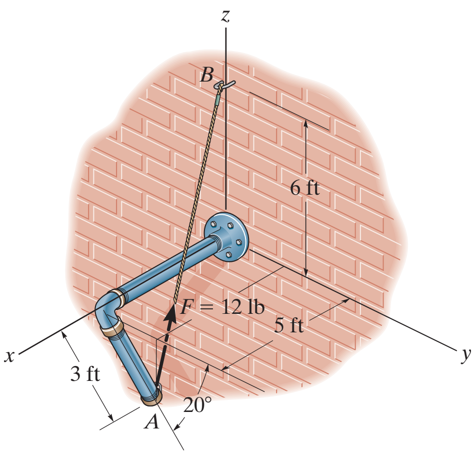
Thus,

$$360 = \frac{24}{7}F \qquad F = 105\text{ lb} \qquad \textbf{Ans.}$$



(a)

2–107. The pipe is supported at its end by a cord *AB*. If the cord exerts a force of $F = 12$ lb on the pipe at *A*, express this force as a Cartesian vector.



Unit Vector : The coordinates of point *A* are

$$A\,(5,\ 3\cos 20°,\ -3\sin 20°)\ \text{ft} = A\,(5.00,\ 2.819,\ -1.026)\ \text{ft}$$

Then

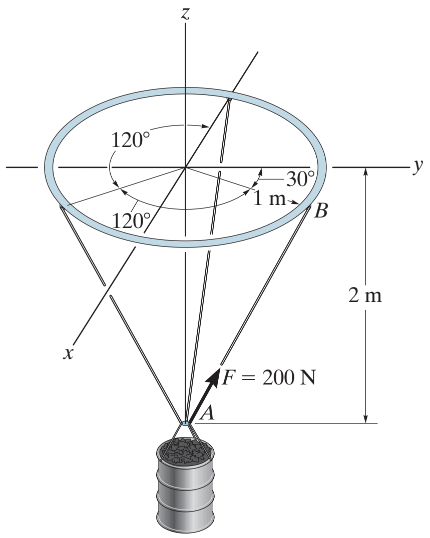
$$\mathbf{r}_{AB} = \{(0-5.00)\,\mathbf{i} + (0-2.819)\,\mathbf{j} + [6-(-1.026)]\,\mathbf{k}\}\ \text{ft}$$
$$= \{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{(-5.00)^2 + (-2.819)^2 + 7.026^2} = 9.073\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5.00\mathbf{i} - 2.819\mathbf{j} + 7.026\mathbf{k}}{9.073}$$
$$= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}$$

Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 12\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\}\ \text{lb}$$
$$= \{-6.61\mathbf{i} - 3.73\mathbf{j} + 9.29\mathbf{k}\}\ \text{lb}$$ **Ans**

***2–108.** The load at *A* creates a force of 200 N in wire *AB*. Express this force as a Cartesian vector, acting on *A* and directed towards *B*.



$$\mathbf{r}_{AB} = (1\sin 30° - 0)\mathbf{i} + (1\cos 30° - 0)\mathbf{j} + (2-0)\mathbf{k}$$

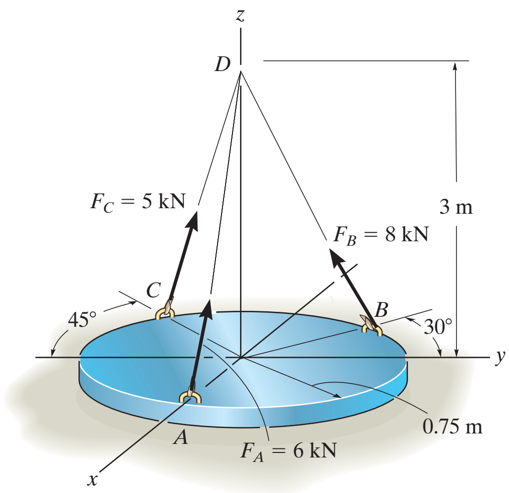
$$= \{0.5\,\mathbf{i} + 0.866\,\mathbf{j} + 2\mathbf{k}\}\text{m}$$

$$r_{AB} = \sqrt{(0.5)^2 + (0.866)^2 + (2)^2} = 2.236\ \text{m}$$

$$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 0.2236\mathbf{i} + 0.3873\,\mathbf{j} + 0.8944\,\mathbf{k}$$

$$\mathbf{F} = 200\,\mathbf{u}_{AB} = \{44.7\,\mathbf{i} + 77.5\,\mathbf{j} + 179\,\mathbf{k}\}\ \text{N}$$. **A**

•2–109. The cylindrical plate is subjected to the three cable forces which are concurrent at point *D*. Express each force which the cables exert on the plate as a Cartesian vector, and determine the magnitude and coordinate direction angles of the resultant force.



$\mathbf{r}_A = (0-0.75)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k} = \{-0.75\,\mathbf{i} + 0\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_A = \sqrt{(-0.75)^2 + 0^2 + 3^2} = 3.0923$ m

$\mathbf{F}_A = F_A\left(\frac{\mathbf{r}_A}{r_A}\right) = 6\left(\frac{-0.75\,\mathbf{i} + 3\,\mathbf{k}}{3.0923}\right)$

$= \{-1.4552\,\mathbf{i} + 5.8209\,\mathbf{k}\}$ kN

$= \{-1.46\,\mathbf{i} + 5.82\,\mathbf{k}\}$ kN Ans

$\mathbf{r}_C = [0-(-0.75 \sin 45°)]\mathbf{i} + [0-(-0.75 \cos 45°)]\mathbf{j} + (3-0)\mathbf{k}$

$= \{0.5303\,\mathbf{i} + 0.5303\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_C = \sqrt{(0.5303)^2 + (0.5303)^2 + 3^2} = 3.0923$ m

$\mathbf{F}_C = F_C\left(\frac{\mathbf{r}_C}{r_C}\right) = 5\left(\frac{0.5303\,\mathbf{i} + 0.5303\,\mathbf{j} + \widehat{3}\,\mathbf{k}}{3.0923}\right)$

$= \{0.8575\,\mathbf{i} + 0.8575\,\mathbf{j} + 4.8507\,\mathbf{k}\}$ kN

$= \{0.857\,\mathbf{i} + 0.857\,\mathbf{j} + 4.85\,\mathbf{k}\}$ kN Ans

$\mathbf{r}_B = [0-(-0.75 \sin 30°)]\mathbf{i} + (0-0.75 \cos 30°)\mathbf{j} + (3-0)\mathbf{k}$

$= \{0.375\,\mathbf{i} - 0.6495\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_B = \sqrt{(0.375)^2 + (-0.6495)^2 + 3^2} = 3.0923$ m

$\mathbf{F}_B = F_B\left(\frac{\mathbf{r}_B}{r_B}\right) = 8\left(\frac{0.375\,\mathbf{i} - 0.6495\,\mathbf{j} + 3\,\mathbf{k}}{3.0923}\right)$

$= \{0.9701\mathbf{i} - 1.6803\mathbf{j} + 7.7611\mathbf{k}\}$ kN

$= \{0.970\mathbf{i} - 1.68\mathbf{j} + 7.76\mathbf{k}\}$ kN Ans

$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$

$= \{-1.4552\,\mathbf{i} + 5.8209\,\mathbf{k}\} + \{0.9701\,\mathbf{i} - 1.6803\,\mathbf{j} + 7.7611\mathbf{k}\}$

$+ \{0.8575\,\mathbf{i} + 0.8575\,\mathbf{j} + 4.8507\,\mathbf{k}\}$

$= \{0.3724\,\mathbf{i} - 0.8228\,\mathbf{j} + 18.4327\,\mathbf{k}\}$ kN

$F_R = \sqrt{(0.3724)^2 + (-0.8228)^2 + (18.4327)^2}$

$= 18.4548$ kN $= 18.5$ kN Ans

$\mathbf{u}_R = \frac{\mathbf{F}_R}{F_R} = \frac{0.3724\mathbf{i} - 0.8228\mathbf{j} + 18.4327\mathbf{k}}{18.4548}$

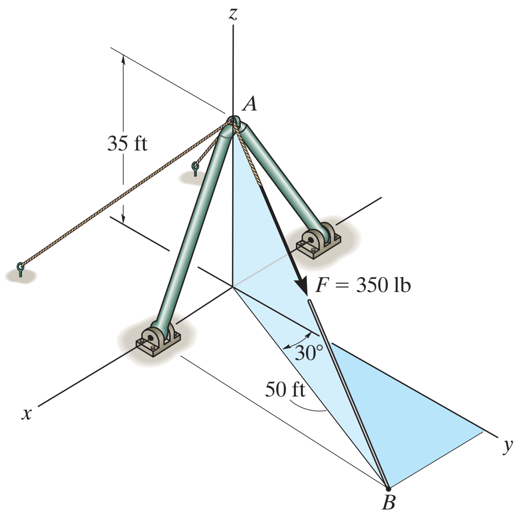
$= 0.02018\mathbf{i} - 0.04459\mathbf{j} + 0.9988\mathbf{k}$

$\cos\alpha = 0.02018$ $\alpha = 88.8°$ Ans

$\cos\beta = -0.04458$ $\beta = 92.6°$ Ans

$\cos\gamma = 0.9988$ $\gamma = 2.81°$ Ans

2–110. The cable attached to the shear-leg derrick exerts a force on the derrick of $F = 350$ lb. Express this force as a Cartesian vector.



Unit Vector : The coordinates of point B are

$$B(50\sin 30°,\ 50\cos 30°,\ 0)\ \text{ft} = B(25.0,\ 43.301,\ 0)\ \text{ft}$$

Then

$$\mathbf{r}_{AB} = \{(25.0-0)\mathbf{i} + (43.301-0)\mathbf{j} + (0-35)\mathbf{k}\}\ \text{ft}$$
$$= \{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}\}\ \text{ft}$$
$$r_{AB} = \sqrt{25.0^2 + 43.301^2 + (-35.0)^2} = 61.033\ \text{ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{25.0\mathbf{i} + 43.301\mathbf{j} - 35.0\mathbf{k}}{61.033}$$
$$= 0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}$$

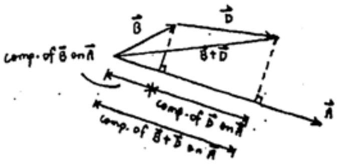
Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 350\{0.4096\mathbf{i} + 0.7094\mathbf{j} - 0.5735\mathbf{k}\}\ \text{lb}$$
$$= \{143\mathbf{i} + 248\mathbf{j} - 201\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans}$$

2–111. Given the three vectors A, B, and D, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.

Since the component of $(\mathbf{B} + \mathbf{D})$ is equal to the sum of the components of $\mathbf{B}$ and $\mathbf{D}$, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$


$\vec{B}$
$\vec{D}$
$\vec{B}+\vec{D}$
comp. of $\vec{B}$ on $\vec{A}$
comp. of $\vec{D}$ on $\vec{A}$
comp. of $\vec{B}+\vec{D}$ on $\vec{A}$
$\vec{A}$

Also,

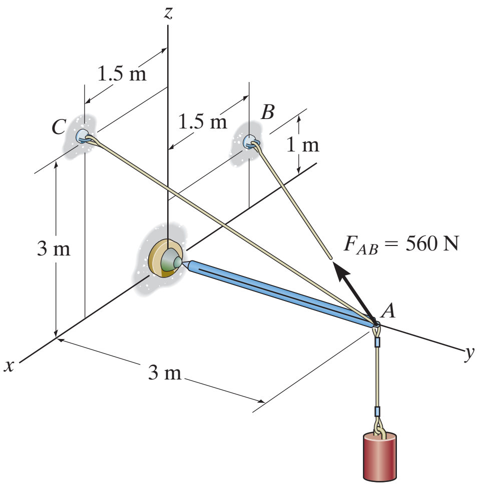
$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}$$

***2–112.** Determine the projected component of the force $F_{AB} = 560$ N acting along cable AC. Express the result as a Cartesian vector.



Force Vectors: The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(-1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-1.5-0)^2+(0-3)^2+(1-0)^2}} = -\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(1.5-0)^2+(0-3)^2+(3-0)^2}} = \frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

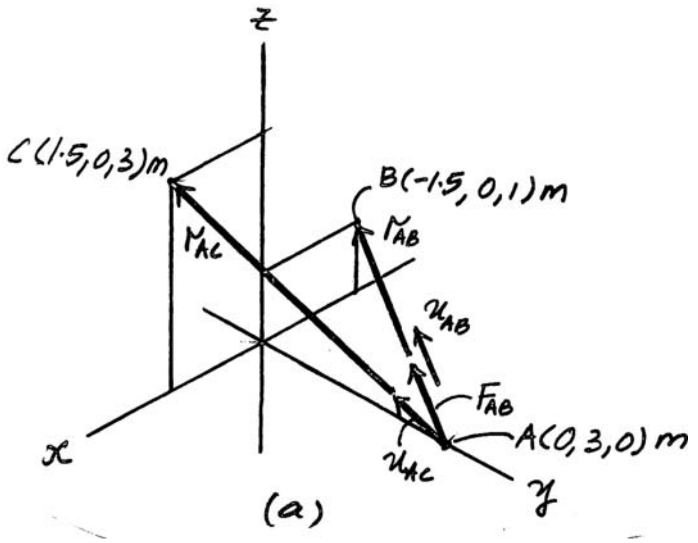
$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 560\left(-\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}\right) = [-240\mathbf{i}-480\mathbf{j}+160\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AB}$ is

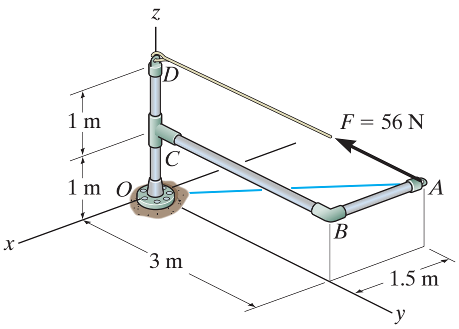
$$\begin{aligned}(F_{AB})_{AC} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} &= (-240\mathbf{i}-480\mathbf{j}+160\mathbf{k})\cdot\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)\\ &= (-240)\left(\frac{1}{3}\right)+(-480)\left(-\frac{2}{3}\right)+160\left(\frac{2}{3}\right)\\ &= 346.67\text{ N}\end{aligned}$$

Thus, $(\mathbf{F}_{AB})_{AC}$ expressed in Cartesian vector form is

$$\begin{aligned}(\mathbf{F}_{AB})_{AC} = (F_{AB})_{AC}\,\mathbf{u}_{AC} &= 346.67\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)\\ &= [116\mathbf{i}-231\mathbf{j}+231\mathbf{k}]\text{N}\end{aligned}$$ **Ans.**



•2–113. Determine the magnitudes of the components of force $F = 56$ N acting along and perpendicular to line *AO*.



Unit Vectors: The unit vectors $\mathbf{u}_{AD}$ and $\mathbf{u}_{AO}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{r_{AD}} = \frac{[0-(-1.5)]\mathbf{i} + (0-3)\mathbf{j} + (2-1)\mathbf{k}}{\sqrt{[0-(-1.5)]^2 + (0-3)^2 + (2-1)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AO} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{[0-(-1.5)]\mathbf{i} + (0-3)\mathbf{j} + (0-1)\mathbf{k}}{\sqrt{[0-(-1.5)]^2 + (0-3)^2 + (0-1)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{AD} = 56\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = [24\mathbf{i} - 48\mathbf{j} + 16\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to line *AO* is

$$(\mathbf{F}_{AO})_{\text{paral}} = \mathbf{F} \cdot \mathbf{u}_{AO} = (24\mathbf{i} - 48\mathbf{j} + 16\mathbf{k}) \cdot \left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}\right)$$

$$= (24)\left(\frac{3}{7}\right) + (-48)\left(-\frac{6}{7}\right) + (16)\left(-\frac{2}{7}\right)$$

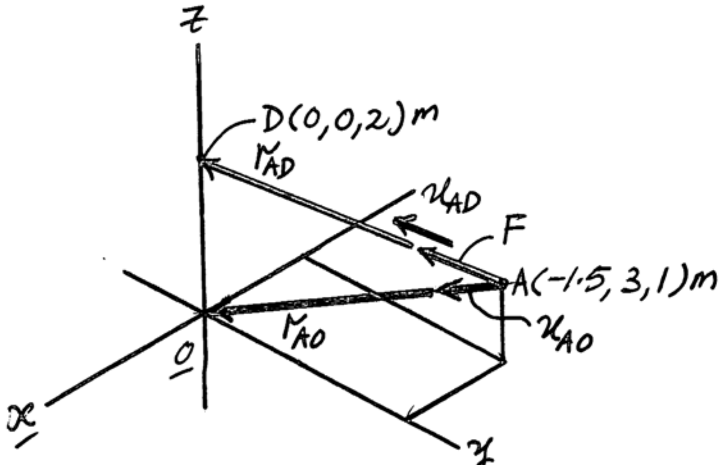
$$= 46.86\text{ N} = 46.9\text{ N} \qquad \textbf{Ans.}$$

The component of **F** perpendicular to line *AO* is

$$(\mathbf{F}_{AO})_{\text{per}} = \sqrt{F^2 - (F_{AO})_{\text{paral}}}$$

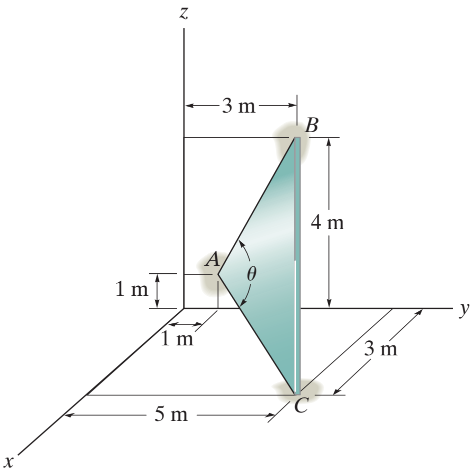
$$= \sqrt{56^2 - 46.86^2}$$

$$= 30.7\text{ N} \qquad \textbf{Ans.}$$



(a)

2–114. Determine the length of side *BC* of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding θ, r_{AB}, and r_{AC} and then using the cosine law.



$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}$ m

$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39$ m **Ans**

Also,

$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990$ m

$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056$ m

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$

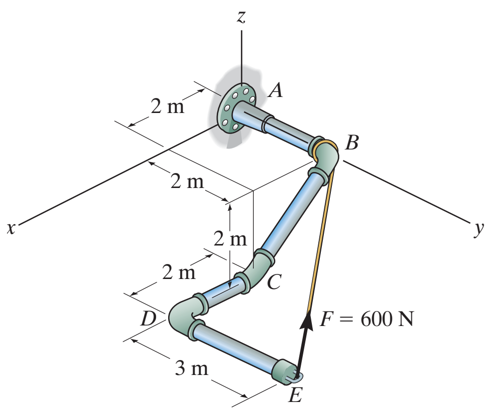
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219°$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219°}$$

$r_{BC} = 5.39$ m **Ans**

2–115. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

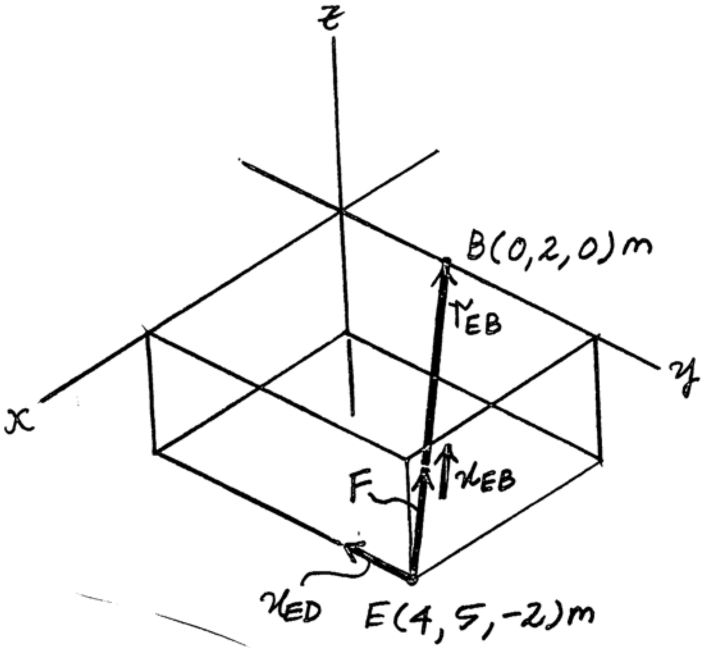
$$\mathbf{F} = F\mathbf{u}_{EB} = 600(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the component of **F** *parallel* to segment *DE* of the pipe assembly is

$$\begin{aligned}(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} &= (-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k})\cdot(-\mathbf{j})\\ &= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)\\ &= 334.25 = 334\text{ N}\end{aligned}$$ **Ans.**

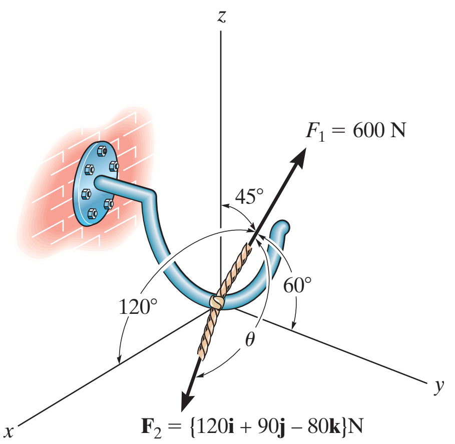
The component of **F** perpendicular to segment *DE* of the pipe assembly is

$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498\text{ N}$$ **Ans.**



(a)

***2–116.** Two forces act on the hook. Determine the angle θ between them. Also, what are the projections of $\mathbf{F}_1$ and $\mathbf{F}_2$ along the y axis?



$$\mathbf{F}_1 = 600\cos 120°\,\mathbf{i} + 600\cos 60°\,\mathbf{j} + 600\cos 45°\,\mathbf{k}$$

$$= -300\,\mathbf{i} + 300\,\mathbf{j} + 424.3\,\mathbf{k}; \quad F_1 = 600\text{ N}$$

$$\mathbf{F}_2 = 120\,\mathbf{i} + 90\,\mathbf{j} - 80\,\mathbf{k}; \quad F_2 = 170\text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

$$\theta = \cos^{-1}\left(\frac{-42\,944}{(170)(600)}\right) = 115° \qquad \textbf{Ans}$$

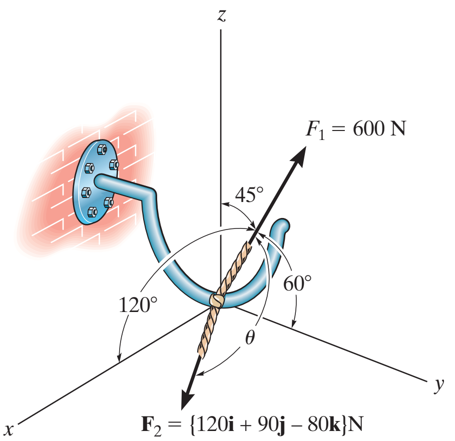
$$\mathbf{u} = \mathbf{j}$$

So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300\text{ N} \qquad \textbf{Ans}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90\text{ N} \qquad \textbf{Ans}$$

•2–117. Two forces act on the hook. Determine the magnitude of the projection of $\mathbf{F}_2$ along $\mathbf{F}_1$.

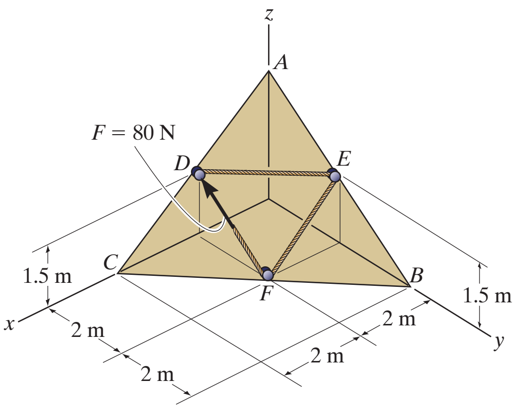


$$\mathbf{u}_1 = \cos 120°\,\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\,\mathbf{k}$$

$$\text{Proj } F_2 = \mathbf{F}_2 \cdot \mathbf{u}_1 = (120)(\cos 120°) + (90)(\cos 60°) + (-80)(\cos 45°)$$

$$|\text{Proj } F_2| = 71.6\text{ N} \qquad \textbf{Ans}$$

2–118. Determine the projection of force $F = 80$ N along line BC. Express the result as a Cartesian vector.



Unit Vectors: The unit vectors $\mathbf{u}_{FD}$ and $\mathbf{u}_{FC}$ must be determined first. From Fig. a,

$$\mathbf{u}_{FD} = \frac{\mathbf{r}_{FD}}{r_{FD}} = \frac{(2-2)\mathbf{i} + (0-2)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(2-2)^2 + (0-2)^2 + (1.5-0)^2}} = -\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

$$\mathbf{u}_{FC} = \frac{\mathbf{r}_{FC}}{r_{FC}} = \frac{(4-2)\mathbf{i} + (0-2)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(4-2)^2 + (0-2)^2 + (0-0)^2}} = 0.7071\mathbf{i} - 0.7071\mathbf{j}$$

Thus, the force vector **F** is given by

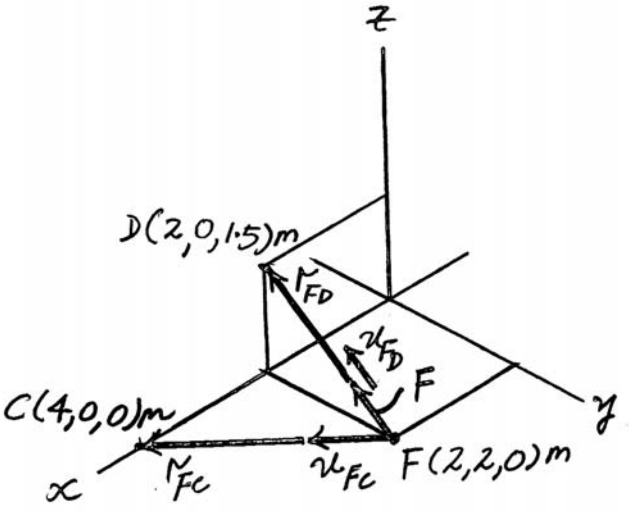
$$\mathbf{F} = F\mathbf{u}_{FD} = 80\left(-\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = [-64\mathbf{j} + 48\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the projected component of **F** along line BC is

$$\begin{aligned} F_{BC} = \mathbf{F}\cdot\mathbf{u}_{FC} &= (-64\mathbf{j} + 48\mathbf{k})\cdot(0.7071\mathbf{i} - 0.7071\mathbf{j}) \\ &= (0)(0.7071) + (-64)(-0.7071) + 48(0) \\ &= 45.25 = 45.2\text{ N} \end{aligned}$$ **Ans.**

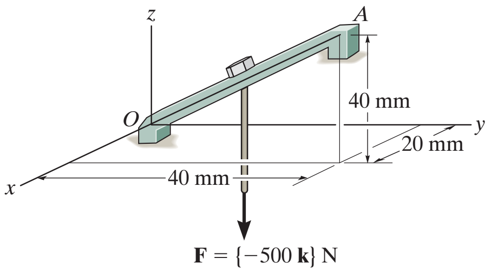
The component of $\mathbf{F}_{BC}$ can be expressed in Cartesian vector form as

$$\begin{aligned} \mathbf{F}_{BC} = F_{BC}(\mathbf{u}_{FC}) &= 45.25(0.7071\mathbf{i} - 0.7071\mathbf{j}) \\ &= \{32\mathbf{i} - 32\mathbf{j}\}\text{ N} \end{aligned}$$ **Ans.**



(a)

2–119. The clamp is used on a jig. If the vertical force acting on the bolt is $\mathbf{F} = \{-500\mathbf{k}\}$ N, determine the magnitudes of its components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the *OA* axis and perpendicular to it.



Unit Vector : The unit vector along *OA* axis is

$$\mathbf{u}_{AO} = \frac{(0-20)\mathbf{i} + (0-40)\mathbf{j} + (0-40)\mathbf{k}}{\sqrt{(0-20)^2 + (0-40)^2 + (0-40)^2}} = -\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

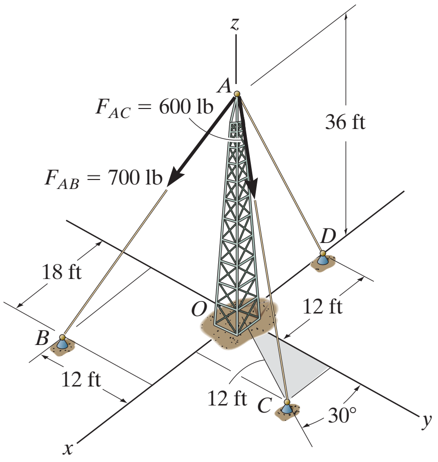
Projected Component of* F *Along OA Axis :

$$\begin{aligned} F_1 = \mathbf{F} \cdot \mathbf{u}_{AO} &= (-500\mathbf{k}) \cdot \left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) \\ &= (0)\left(-\frac{1}{3}\right) + (0)\left(-\frac{2}{3}\right) + (-500)\left(-\frac{2}{3}\right) \\ &= 333.33 \text{ N} = 333 \text{ N} \qquad \textbf{Ans} \end{aligned}$$

Component of* F *Perpendicular to OA Axis : Since the magnitude of force **F** is $F = 500$ N so that

$$F_2 = \sqrt{F^2 - F_1^2} = \sqrt{500^2 - 333.33^2} = 373 \text{ N} \qquad \textbf{Ans}$$

***2–120.** Determine the magnitude of the projected component of force $\mathbf{F}_{AB}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(18-0)\mathbf{i} + (-12-0)\mathbf{j} + (0-36)\mathbf{k}}{\sqrt{(18-0)^2 + (-12-0)^2 + (0-36)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

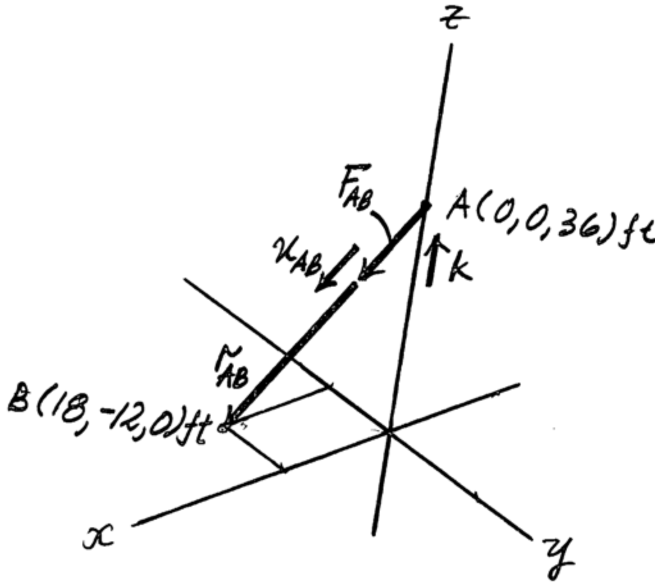
$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 700\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k}\}\text{ lb}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AB}$ along the z axis is

$$(F_{AB})_z = \mathbf{F}_{AB}\cdot\mathbf{k} = (300\mathbf{i} - 200\mathbf{j} - 600\mathbf{k})\cdot\mathbf{k}$$
$$= -600\text{ lb}$$

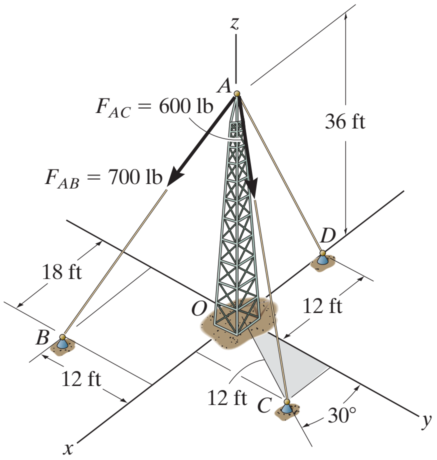
The negative sign indicates that $(\mathbf{F}_{AB})_z$ is directed towards the negative z axis. Thus

$(F_{AB})_z = 600$ lb **Ans.**



(a)

•**2–121.** Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC}=\frac{\mathbf{r}_{AC}}{r_{AC}}=\frac{(12\sin 30°-0)\mathbf{i}+(12\cos 30°-0)\mathbf{j}+(0-36)\mathbf{k}}{\sqrt{(12\sin 30°-0)^2+(12\cos 30°-0)^2+(0-36)^2}}=0.1581\mathbf{i}+0.2739\mathbf{j}-0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

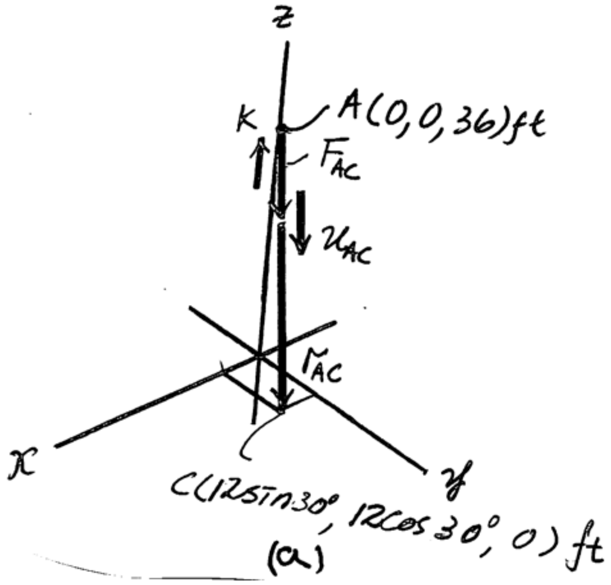
$$\mathbf{F}_{AC}=F_{AC}\mathbf{u}_{AC}=600(0.1581\mathbf{i}+0.2739\mathbf{j}-0.9487\mathbf{k})=\{94.87\mathbf{i}+164.32\mathbf{j}-569.21\mathbf{k}\}\text{N}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

$$(F_{AC})_z=\mathbf{F}_{AC}\cdot\mathbf{k}=(94.87\mathbf{i}+164.32\mathbf{j}-569.21\mathbf{k})\cdot\mathbf{k}$$
$$=-569\text{ lb}$$

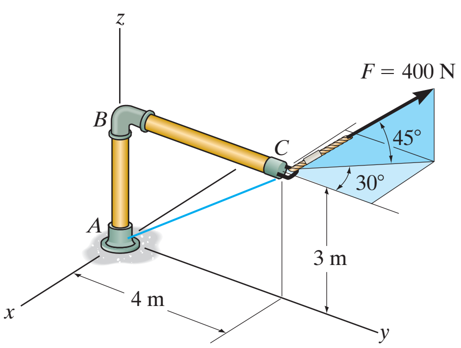
The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative z axis. Thus

$(F_{AC})_z = 569$ lb **Ans.**



(a)

2–122. Determine the projection of force $F = 400$ N acting along line AC of the pipe assembly. Express the result as a Cartesian vector.



Force and unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. (a)

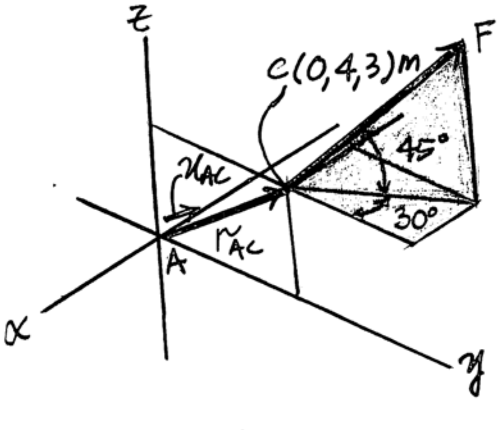
$$\mathbf{F} = 400(-\cos 45°\sin 30°\,\mathbf{i} + \cos 45°\cos 30°\,\mathbf{j} + \sin 45°\,\mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}$$
$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line AC is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k})\cdot\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right)$$
$$= (-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right)$$
$$= 365.66\ \text{lb} \qquad \textbf{Ans.}$$

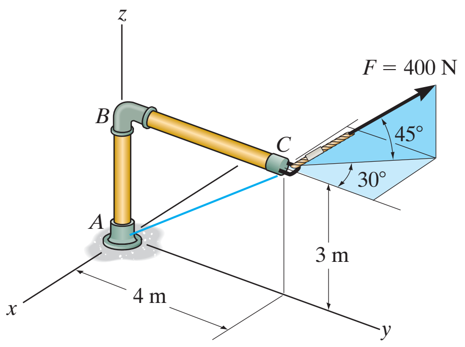
Thus, $\mathbf{F}_{AC}$ written in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 365.66\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\}\ \text{lb} \qquad \textbf{Ans.}$$



(a)

2–123. Determine the magnitudes of the components of force $F = 400$ N acting parallel and perpendicular to segment BC of the pipe assembly.



Force Vector: The force vector **F** must be determined first. From Fig. *a*,

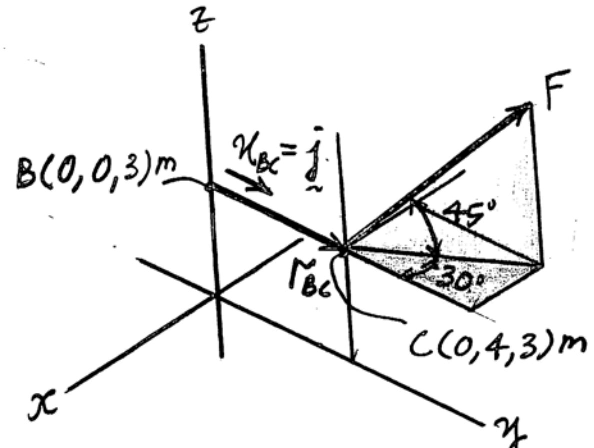
$$\mathbf{F} = 400(-\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}\text{ N}$$

Vector Dot Product: By inspecting Fig. (a) we notice that $u_{BC} = \mathbf{j}$. Thus, the magnitude of the component of **F** parallel to segment BC of the pipe assembly is

$$(F_{BC})_{\text{paral}} = \mathbf{F}\cdot\mathbf{j} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k})\cdot\mathbf{j}$$
$$= -141.42(0) + 244.95(1) + 282.84(0)$$
$$= 244.95\text{ lb} = 245\text{ N} \qquad \textbf{Ans.}$$

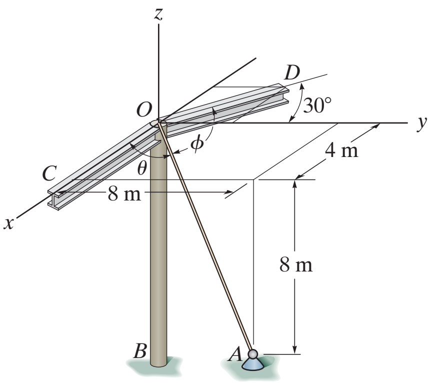
The magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{\text{per}} = \sqrt{F^2 - (F_{BC})_{\text{paral}}} = \sqrt{400^2 - 244.95^2} = 316\text{ N} \qquad \textbf{Ans.}$$



(a)

***2–124.** Cable *OA* is used to support column *OB*. Determine the angle θ it makes with beam *OC*.



Unit Vector :

$$\mathbf{u}_{OC} = 1\mathbf{i}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\mathbf{i}+(8-0)\mathbf{j}+(-8-0)\mathbf{k}}{\sqrt{(4-0)^2+(8-0)^2+(-8-0)^2}} = \frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

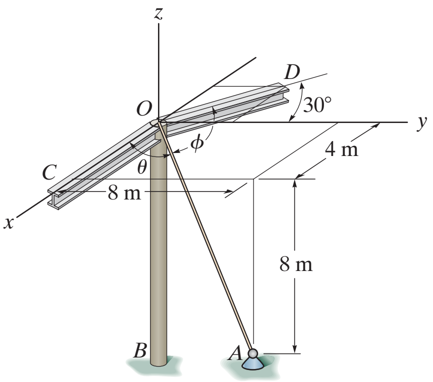
The Angle Between Two Vectors θ :

$$\mathbf{u}_{OC}\cdot\mathbf{u}_{OA} = (1\mathbf{i})\cdot\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right) = 1\left(\frac{1}{3}\right)+(0)\left(\frac{2}{3}\right)+0\left(-\frac{2}{3}\right) = \frac{1}{3}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{OC}\cdot\mathbf{u}_{OA}) = \cos^{-1}\frac{1}{3} = 70.5^\circ \qquad \textbf{Ans}$$

•2–125. Cable *OA* is used to support column *OB*. Determine the angle ϕ it makes with beam *OD*.



Unit Vector :

$$\mathbf{u}_{OD} = -\sin 30^\circ\mathbf{i}+\cos 30^\circ\mathbf{j} = -0.5\mathbf{i}+0.8660\mathbf{j}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\mathbf{i}+(8-0)\mathbf{j}+(-8-0)\mathbf{k}}{\sqrt{(4-0)^2+(8-0)^2+(-8-0)^2}} = \frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

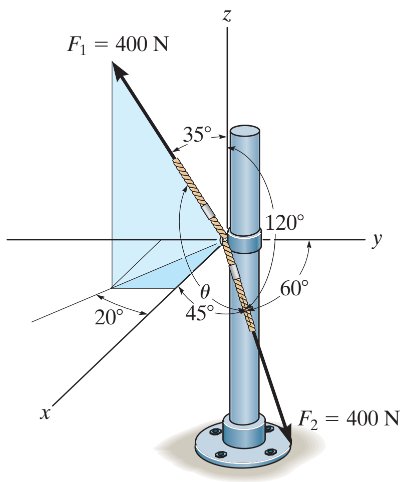
The Angles Between Two Vectors ϕ :

$$\begin{aligned}\mathbf{u}_{OD}\cdot\mathbf{u}_{OA} &= (-0.5\mathbf{i}+0.8660\mathbf{j})\cdot\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right)\\ &= (-0.5)\left(\frac{1}{3}\right)+(0.8660)\left(\frac{2}{3}\right)+0\left(-\frac{2}{3}\right)\\ &= 0.4107\end{aligned}$$

Then,

$$\phi = \cos^{-1}(\mathbf{u}_{OD}\cdot\mathbf{u}_{OA}) = \cos^{-1}0.4107 = 65.8^\circ \qquad \textbf{Ans}$$

2–126. The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector :

$$\mathbf{u}_{F_1} = \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k}$$
$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1 \mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \text{ N}$$
$$= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\} \text{ N}$$

Unit Vector : The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k}$$
$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

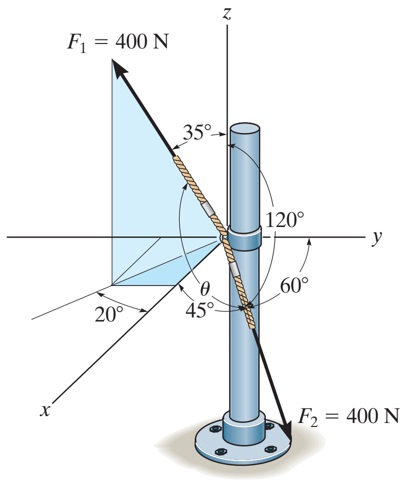
Projected Component of $\mathbf{F}_1$ Along Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$
$$= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5)$$
$$= -50.6 \text{ N}$$

Negative sign indicates that the force component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

thus the magnitude is $(\mathbf{F}_1)_{F_2} = 50.6$ N **Ans**

2–127. Determine the angle θ between the two cables attached to the post.



Unit Vector :

$$\mathbf{u}_{F_1} = \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k}$$
$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k}$$
$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

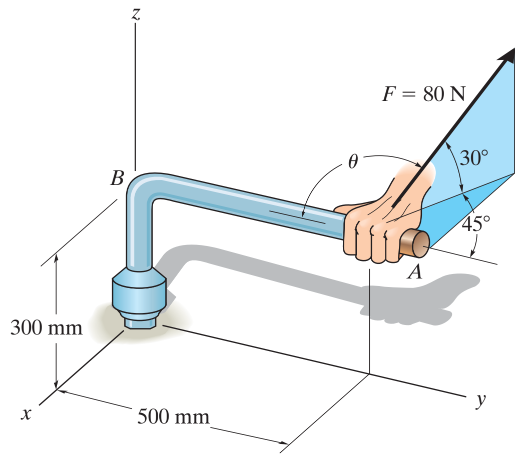
The Angle Between Two Vectors θ : The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$
$$= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5)$$
$$= -0.1265$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1265) = 97.3^\circ$$ **Ans**

***2–128.** A force of $F = 80$ N is applied to the handle of the wrench. Determine the angle θ between the tail of the force and the handle AB.



$$\mathbf{u}_F = -\cos 30°\sin 45°\mathbf{i} + \cos 30°\cos 45°\mathbf{j} + \sin 30°\mathbf{k}$$

$$= -0.6124\,\mathbf{i} + 0.6124\,\mathbf{j} + 0.5\,\mathbf{k}$$

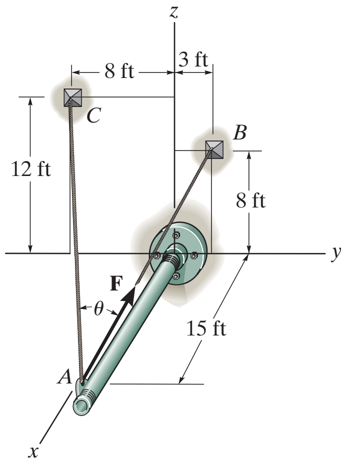
$$\mathbf{u}_{AB} = -\mathbf{j}$$

$$\cos\theta = \mathbf{u}_F \cdot \mathbf{u}_{AB} = (-0.6124\,\mathbf{i} + 0.6124\,\mathbf{j} + 0.5\,\mathbf{k}) \cdot (-\mathbf{j})$$

$$= -0.6124$$

$\theta = 128°$ **Ans**

•2–129. Determine the angle θ between cables AB and AC.



Position Vector :

$$\mathbf{r}_{AB} = \{(0-15)\mathbf{i} + (3-0)\mathbf{j} + (8-0)\mathbf{k}\}\ \text{ft}$$
$$= \{-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}\}\ \text{ft}$$

$$\mathbf{r}_{AC} = \{(0-15)\mathbf{i} + (-8-0)\mathbf{j} + (12-0)\mathbf{k}\}\ \text{ft}$$
$$= \{-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k}\}\ \text{ft}$$

The magnitudes of the postion vectors are

$$r_{AB} = \sqrt{(-15)^2 + 3^2 + 8^2} = 17.263\ \text{ft}$$
$$r_{AC} = \sqrt{(-15)^2 + (-8)^2 + 12^2} = 20.809\ \text{ft}$$

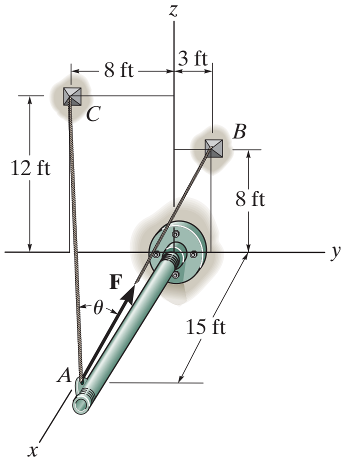
The Angle Between Two Vectors θ:

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (-15\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) \cdot (-15\mathbf{i} - 8\mathbf{j} + 12\mathbf{k})$$
$$= (-15)(-15) + (3)(-8) + 8(12)$$
$$= 297\ \text{ft}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{r_{AB}\,r_{AC}}\right) = \cos^{-1}\left[\frac{297}{17.263(20.809)}\right] = 34.2° \qquad \textbf{Ans}$$

2–130. If **F** has a magnitude of 55 lb, determine the magnitude of its projected components acting along the *x* axis and along cable *AC*.



Force Vector:

$$\mathbf{u}_{AB}=\frac{(0-15)\mathbf{i}+(3-0)\mathbf{j}+(8-0)\mathbf{k}}{\sqrt{(0-15)^2+(3-0)^2+(8-0)^2}}$$
$$=-0.8689\mathbf{i}+0.1738\mathbf{j}+0.4634\mathbf{k}$$

$$\mathbf{F}=F\mathbf{u}_{AB}=55(-0.8689\mathbf{i}+0.1738\mathbf{j}+0.4634\mathbf{k})\text{ lb}$$
$$=\{-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k}\}\text{ lb}$$

Unit Vector: The unit vector along negative *x* axis and *AC* are

$$\mathbf{u}_x=-1\mathbf{i}$$

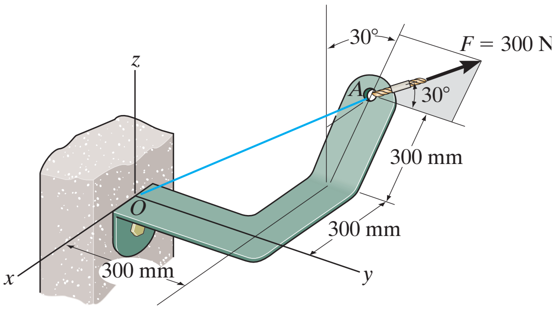
$$\mathbf{u}_{AC}=\frac{(0-15)\mathbf{i}+(-8-0)\mathbf{j}+(12-0)\mathbf{k}}{\sqrt{(0-15)^2+(-8-0)^2+(12-0)^2}}$$
$$=-0.7209\mathbf{i}-0.3845\mathbf{j}+0.5767\mathbf{k}$$

Projected Component of F:

$$F_x=\mathbf{F}\cdot\mathbf{u}_x=(-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k})\cdot(-1\mathbf{i})$$
$$=(-47.791)(-1)+9.558(0)+25.489(0)$$
$$=47.8\text{ lb}\qquad\textbf{Ans}$$

$$F_{AC}=\mathbf{F}\cdot\mathbf{u}_{AC}=(-47.791\mathbf{i}+9.558\mathbf{j}+25.489\mathbf{k})\cdot(-0.7209\mathbf{i}-0.3845\mathbf{j}+0.5767\mathbf{k})$$
$$=(-47.791)(-0.7209)+(9.558)(-0.3845)+(25.489)(0.5767)$$
$$=45.5\text{ lb}\qquad\textbf{Ans}$$

2–131. Determine the magnitudes of the projected components of the force $F = 300$ N acting along the *x* and *y* axes.



Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F}=-300\sin30^\circ\sin30^\circ\mathbf{i}+300\cos30^\circ\mathbf{j}+300\sin30^\circ\cos30^\circ\mathbf{k}$$
$$=[-75\mathbf{i}+259.81\mathbf{j}+129.90\mathbf{k}]\text{N}$$

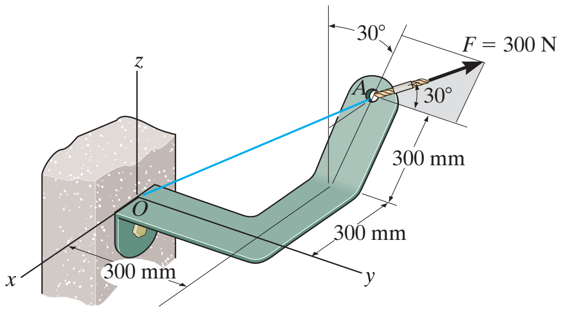
Vector Dot Product: The magnitudes of the projected component of **F** along the *x* and *y* axes are

$$F_x=\mathbf{F}\cdot\mathbf{i}=(-75\mathbf{i}+259.81\mathbf{j}+129.90\mathbf{k})\cdot\mathbf{i}$$
$$=-75(1)+259.81(0)+129.90(0)$$
$$=-75\text{ N}$$
$$F_y=\mathbf{F}\cdot\mathbf{j}=(-75\mathbf{i}+259.81\mathbf{j}+129.90\mathbf{k})\cdot\mathbf{j}$$
$$=-75(0)+259.81(1)+129.90(0)$$
$$=260\text{ N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative *x* axis. Thus

$F_x = 75$ N, $\quad F_y = 260$ N **Ans.**

***2–132.** Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



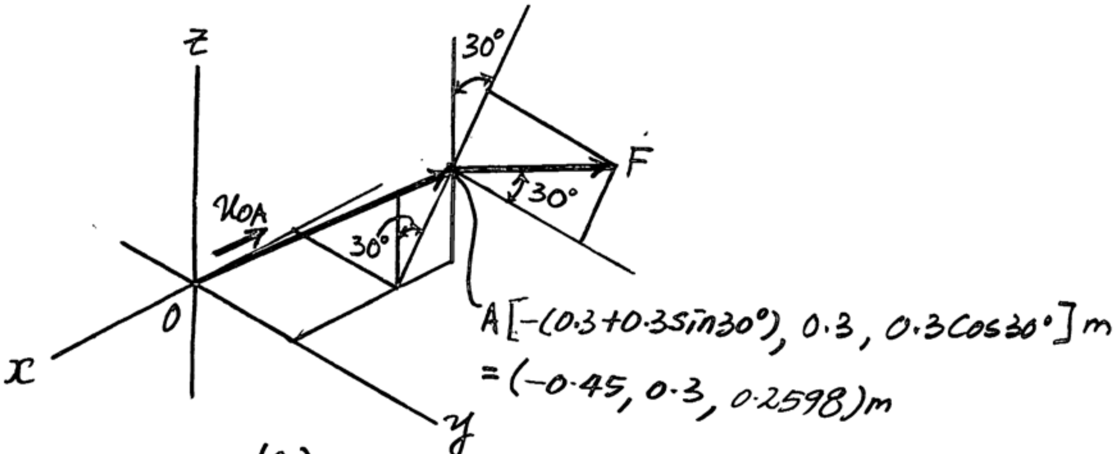
Force and unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. (*a*)

$$\mathbf{F} = -300\sin 30^\circ \sin 30^\circ \mathbf{i} + 300\cos 30^\circ \mathbf{j} + 300\sin 30^\circ \cos 30^\circ \mathbf{k})$$
$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\}\text{N}$$

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45-0)\mathbf{i} + (0.3-0)\mathbf{j} + (0.2598-0)\mathbf{k}}{\sqrt{(-0.45-0)^2 + (0.3-0)^2 + (0.2598-0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

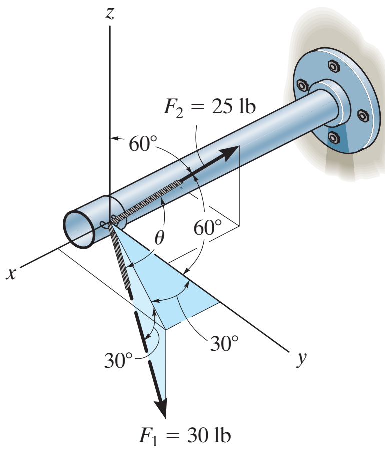
Vector Dot Product: The magnitude of the projected component of **F** along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$
$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$
$$= 242\text{ N} \qquad \textbf{Ans.}$$



(a)

•2–133. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



Force Vector:

$$\mathbf{u}_{F_1} = \cos 30^\circ \sin 30^\circ \mathbf{i} + \cos 30^\circ \cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}$$
$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_1 \mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \text{ lb}$$
$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\} \text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135^\circ$ for $\mathbf{F}_2$ using Eq.2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60^\circ$ and $\gamma = 60^\circ$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 60^\circ \mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

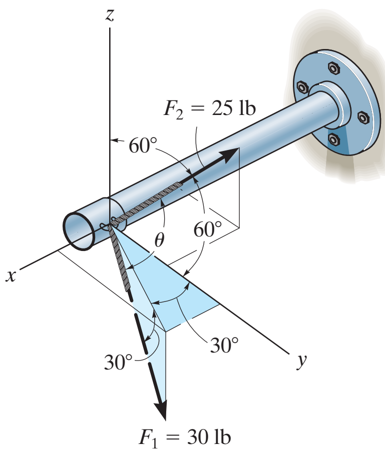
Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$
$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$
$$= -5.44 \text{ lb}$$

Negative sign indicates that the projected component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(\mathbf{F}_1)_{F_2} = 5.44$ lb. **Ans**

2–134. Determine the angle θ between the two cables attached to the pipe.



The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$
$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$
$$= -0.1812$$

Then,

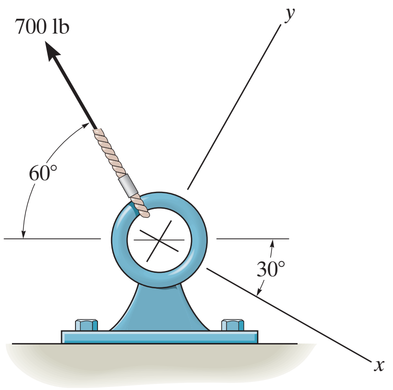
$$\theta = \cos^{-1}(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}) = \cos^{-1}(-0.1812) = 100^\circ \qquad \textbf{Ans}$$

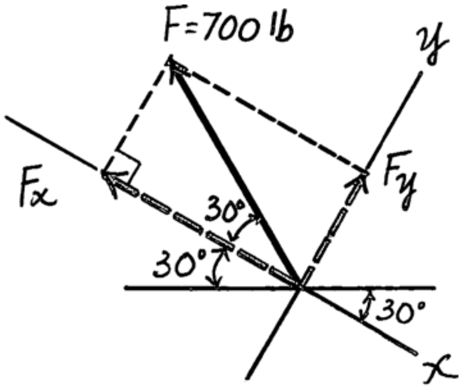
Unit Vector:

$$\mathbf{u}_{F_1} = \cos 30^\circ \sin 30^\circ \mathbf{i} + \cos 30^\circ \cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}$$
$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}$$
$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

2–135. Determine the *x* and *y* components of the 700-lb force.

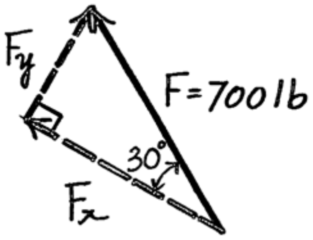




(a)

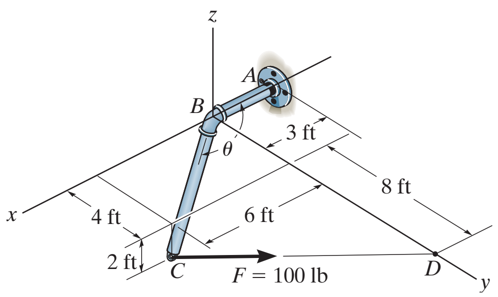
$$F_x = -700 \cos 30° = -606 \text{ lb} \quad \textbf{Ans}$$

$$F_y = 700 \sin 30° = 350 \text{ lb} \quad \textbf{Ans}$$



(b)

***2–136.** Determine the magnitude of the projected component of the 100-lb force acting along the axis *BC* of the pipe.



Force Vector :

$$\mathbf{u}_{CD} = \frac{(0-6)\mathbf{i} + (12-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (12-4)^2 + [0-(-2)]^2}}$$
$$= -0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{CD} = 100(-0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k})$$
$$= \{-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}\}\ \text{lb}$$

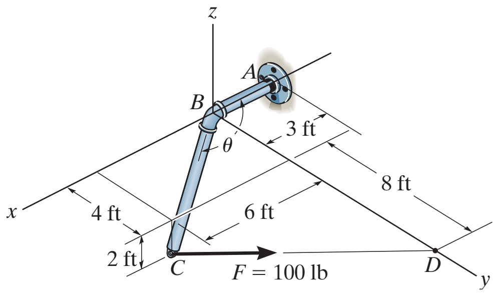
Unit Vector : The unit vector along *CB* is

$$\mathbf{u}_{CB} = \frac{(0-6)\mathbf{i} + (0-4)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-6)^2 + (0-4)^2 + [0-(-2)]^2}}$$
$$= -0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k}$$

Projected Component of F Along CB :

$$F_{CB} = \mathbf{F} \cdot \mathbf{u}_{CB} = (-58.835\mathbf{i} + 78.446\mathbf{j} + 19.612\mathbf{k}) \cdot (-0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k})$$
$$= (-58.835)(-0.8018) + (78.446)(-0.5345) + (19.612)(0.2673)$$
$$= 10.5\ \text{lb} \qquad \textbf{Ans}$$

•2–137. Determine the angle θ between pipe segments *BA* and *BC*.



Position Vector :

$$\mathbf{r}_{BA} = \{-3\mathbf{i}\}\ \text{ft}$$

$$\mathbf{r}_{BC} = \{(6-0)\mathbf{i} + (4-0)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft}$$
$$= \{6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

The magnitudes of the position vectors are

$$r_{BA} = 3.00\ \text{ft} \qquad r_{BC} = \sqrt{6^2 + 4^2 + (-2)^2} = 7.483\ \text{ft}$$

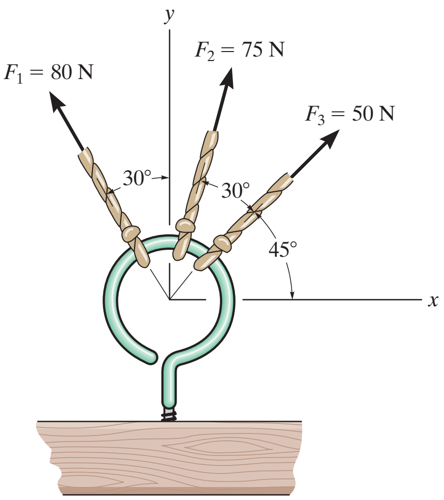
The Angle Between Two Vectors θ :

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-3\mathbf{i}) \cdot (6\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$
$$= (-3)(6) + (0)(4) + 0(-2)$$
$$= -18.0\ \text{ft}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}}\right) = \cos^{-1}\left[\frac{-18.0}{3.00(7.483)}\right] = 143° \qquad \textbf{Ans}$$

2–138. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.



$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105^\circ} = 104.7\text{ N}$$

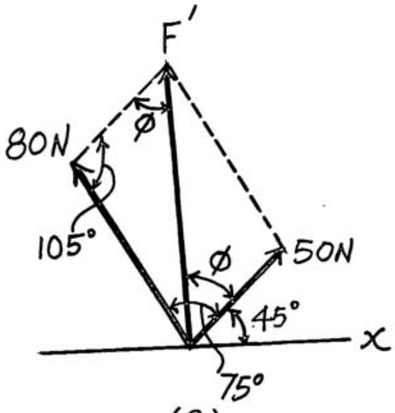
$$\frac{\sin\phi}{80} = \frac{\sin 105^\circ}{104.7}; \quad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46^\circ}$$

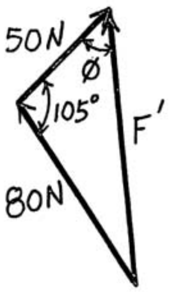
$$F_R = 177.7 = 178\text{ N} \qquad \text{Ans}$$

$$\frac{\sin\beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \quad \beta = 10.23^\circ$$

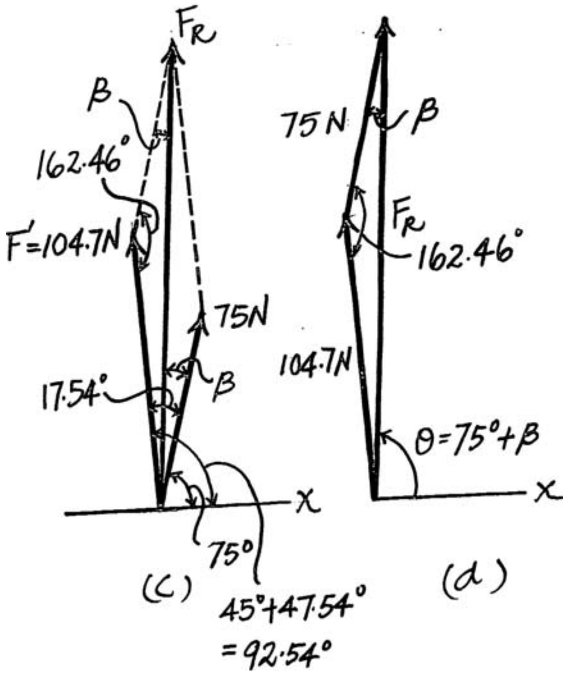
$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ \qquad \text{Ans}$$



(a)

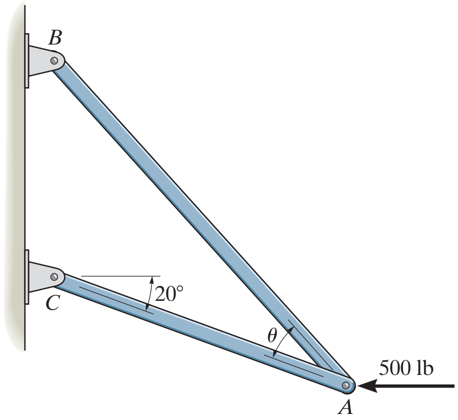


(b)



(c) (d)

2–139. Determine the design angle θ ($\theta < 90°$) between the two struts so that the 500-lb horizontal force has a component of 600 lb directed from A toward C. What is the component of force acting along member BA?



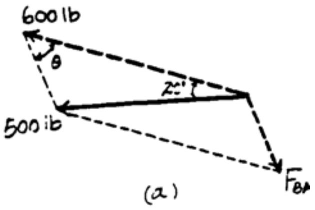
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*.

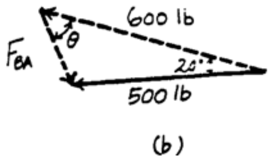
Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{500^2 + 600^2 - 2(500)(600)\cos 20°}$$
$$= 214.91 \text{ lb} = 215 \text{ lb}$$ **Ans.**

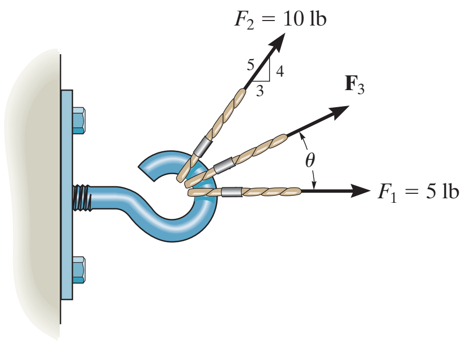
Applying the law of sines to Fig. *b* and using this result yields

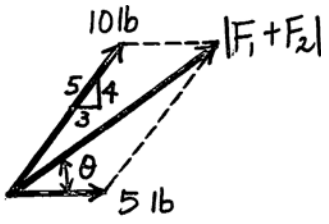
$$\frac{\sin\theta}{500} = \frac{\sin 20°}{214.91} \qquad \theta = 52.7°$$ **Ans.**



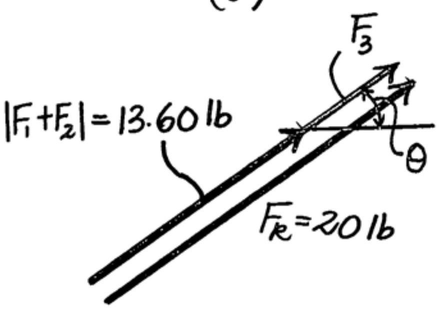


***2–140.** Determine the magnitude and direction of the *smallest* force $\mathbf{F}_3$ so that the resultant force of all three forces has a magnitude of 20 lb.





(a)



(b)

F_3 is minimum :

$$F_3 = 20 - |F_1 + F_2|$$

$$F_1 + F_2 = \left(5 + 10\left(\frac{3}{5}\right)\right)\mathbf{i} + \left(10\left(\frac{4}{5}\right)\right)\mathbf{j} = 11\,\mathbf{i} + 8\,\mathbf{j}$$

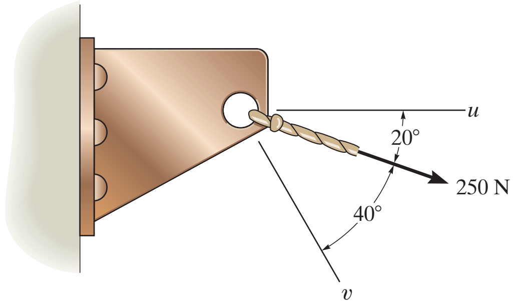
$$|F_1 + F_2| = \sqrt{11^2 + 8^2} = 13.601 \text{ lb}$$

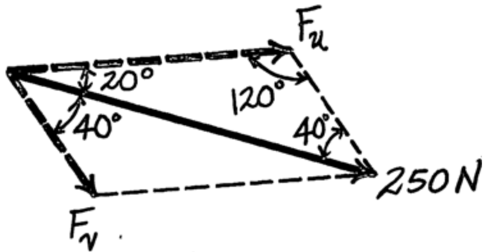
$$\theta = \tan^{-1}\left(\frac{8}{11}\right) = 36.0^\circ \qquad \textbf{Ans}$$

Thus

$$(F_3)_{\min} = 20 - 13.601 = 6.40 \text{ lb} \qquad \textbf{Ans}$$

•2–141. Resolve the 250-N force into components acting along the u and v axes and determine the magnitudes of these components.

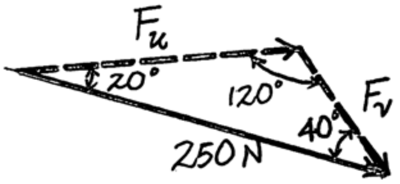




(a)

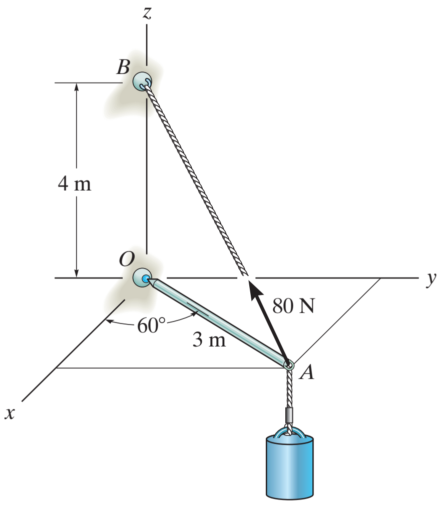
$$\frac{250}{\sin 120^\circ} = \frac{F_u}{\sin 40^\circ}; \qquad F_u = 186\text{ N} \qquad \text{Ans}$$

$$\frac{250}{\sin 120^\circ} = \frac{F_v}{\sin 20^\circ}; \qquad F_v = 98.7\text{ N} \qquad \text{Ans}$$



(b)

2–142. Cable AB exerts a force of 80 N on the end of the 3-m-long boom OA. Determine the magnitude of the projection of this force along the boom.



Vector Analysis :

$$\mathbf{F} = 80\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 80\left(-\frac{3\cos 60^\circ}{5}\mathbf{i} - \frac{3\sin 60^\circ}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}\right)$$

$$= -24\mathbf{i} - 41.57\mathbf{j} + 64\mathbf{k}$$

$$\mathbf{u}_{AO} = -\cos 60^\circ\,\mathbf{i} - \sin 60^\circ\,\mathbf{j} = -0.5\mathbf{i} - 0.866\mathbf{j}$$

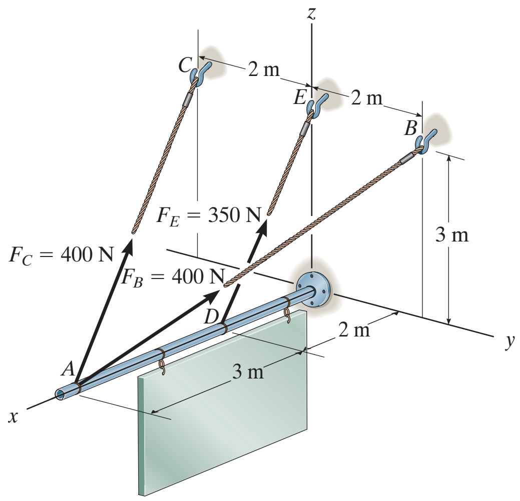
$$\text{Proj}\,F = \mathbf{F}\cdot\mathbf{u}_{AO} = (-24)(-0.5) + (-41.57)(-0.866) + (64)0 = 48.0\text{ N} \qquad \text{Ans}$$

Scalar Anaysis :

$$\text{Angle } OAB = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$\text{Proj}\,F = 80\cos 53.13^\circ = 48.0\text{ N} \qquad \text{Ans}$$

2–143. The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.



Unit Vector:

$$\mathbf{r}_{AB} = \{(0-5)\mathbf{i} + (2-0)\mathbf{j} + (3-0)\mathbf{k}\}\text{ m} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(-5)^2 + 2^2 + 3^2} = 6.164\text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{6.164} = -0.8111\mathbf{i} + 0.3244\mathbf{j} + 0.4867\mathbf{k}$$

$$\mathbf{r}_{AC} = \{(0-5)\mathbf{i} + (-2-0)\mathbf{j} + (3-0)\mathbf{k}\}\text{ m} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(-5)^2 + (-2)^2 + 3^2} = 6.164\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{6.164} = -0.8111\mathbf{i} - 0.3244\mathbf{j} + 0.4867\mathbf{k}$$

$$\mathbf{r}_{DE} = \{(0-2)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k}\}\text{ m} = \{-2\mathbf{i} + 3\mathbf{k}\}\text{ m}$$

$$r_{DE} = \sqrt{(-2)^2 + 3^2} = 3.605\text{ m}$$

$$\mathbf{u}_{DE} = \frac{\mathbf{r}_{DE}}{r_{DE}} = \frac{-2\mathbf{i} + 3\mathbf{k}}{3.605} = -0.5547\mathbf{i} + 0.8321\mathbf{k}$$

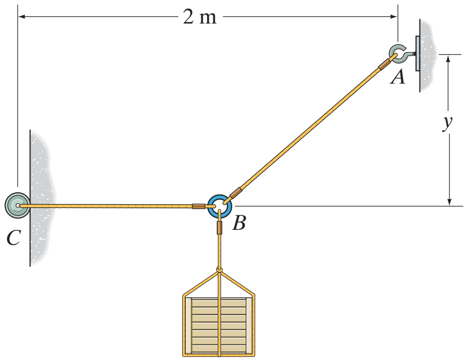
Force Vector:

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 400\{-0.8111\mathbf{i} + 0.3244\mathbf{j} + 0.4867\mathbf{k}\}\text{ N}$$
$$= \{-324.44\mathbf{i} + 129.78\mathbf{j} + 194.67\mathbf{k}\}\text{ N}$$
$$= \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}\text{ N} \qquad \textbf{Ans}$$

$$\mathbf{F}_C = F_C\mathbf{u}_{AB} = 400\{-0.8111\mathbf{i} - 0.3244\mathbf{j} + 0.4867\mathbf{k}\}\text{ N}$$
$$= \{-324.44\mathbf{i} - 129.78\mathbf{j} + 194.67\mathbf{k}\}\text{ N}$$
$$= \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}\text{ N} \qquad \textbf{Ans}$$

$$\mathbf{F}_E = F_E\mathbf{u}_{DE} = 350\{-0.5547\mathbf{i} + 0.8321\mathbf{k}\}\text{ N}$$
$$= \{-194.15\mathbf{i} + 291.22\mathbf{k}\}\text{ N}$$
$$= \{-194\mathbf{i} + 291\mathbf{k}\}\text{ N} \qquad \textbf{Ans}$$

•3–1. Determine the force in each cord for equilibrium of the 200-kg crate. Cord *BC* remains horizontal due to the roller at *C*, and *AB* has a length of 1.5 m. Set $y = 0.75$ m.



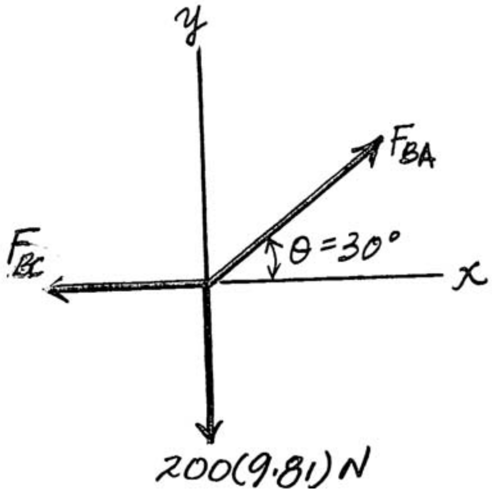
Geometry: From the geometry of the figure,

$$\theta = \sin^{-1}\left(\frac{0.75}{1.5}\right) = 30^\circ$$

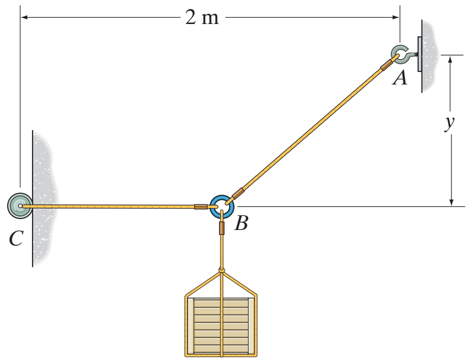
Equations of Equilibrium: Applying the equations of equilibrium to the free - body diagram in Fig. (*a*),

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BA}\sin 30^\circ - 200(9.81) = 0 \quad F_{BA} = 3924\text{ N} = 3.92\text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 3924\cos 30^\circ - F_{BC} = 0 \quad F_{BC} = 3398.28\text{ N} = 3.40\text{ kN}$ **Ans.**



3–2. If the 1.5-m-long cord AB can withstand a maximum force of 3500 N, determine the force in cord BC and the distance y so that the 200-kg crate can be supported.

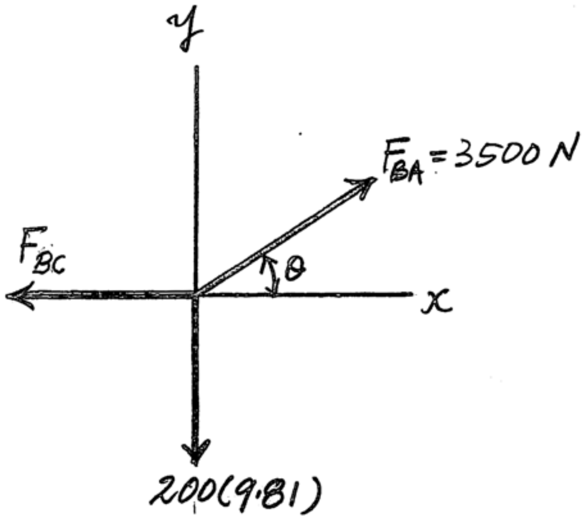


Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free - body diagram in Fig. (a),

$$+\uparrow \Sigma F_y = 0; \qquad 3500\sin\theta - 200(9.81) = 0 \qquad \theta = 34.10^\circ$$

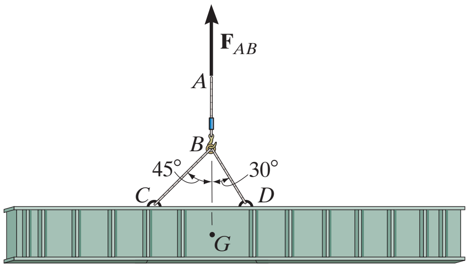
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 3500\cos 34.10^\circ - F_{BC} = 0 \qquad F_{BC} = 2898.37\text{ N} = 2.90\text{ kN} \qquad \textbf{Ans.}$$

$$y = 1.5\sin 34.10^\circ = 0.841\text{ m} = 841\text{ mm} \quad \textbf{Ans.}$$



(a)

3–3. If the mass of the girder is 3 Mg and its center of mass is located at point G, determine the tension developed in cables AB, BC, and BD for equilibrium.



Equations of Equilibrium: The girder is suspended from cable AB. In order to meet the conditions of equilibrium the tensile force developed in cable AB must be equal to the weight of the girder. Thus,

$F_{AB} = 3000(9.81) = 29\,430\text{ N} = 29.43\text{ kN} = 29.4\text{ kN}$ **Ans.**

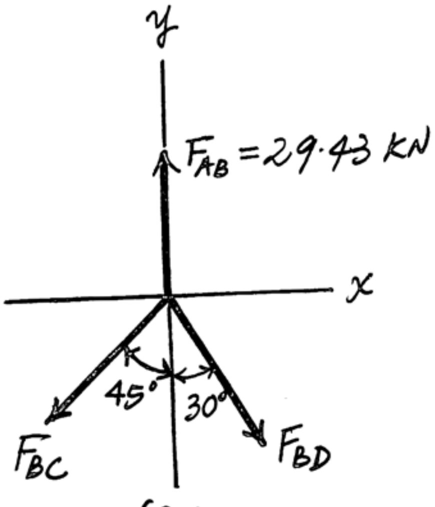
Applying the equations of equilibrium along the x and y axes to the free - body diagram in Fig. (a),

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $F_{BD}\sin 30° - F_{BC}\sin 45° = 0$ (1)

$+\uparrow \Sigma F_y = 0;$ $29.43 - F_{BD}\cos 30° - F_{BC}\cos 45° = 0$ (2)

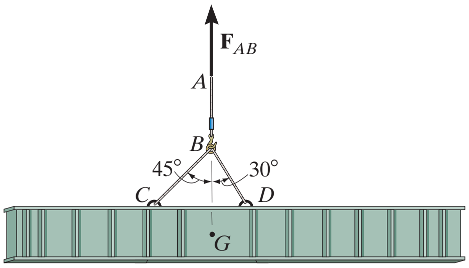
Solving Eqs. (1) and (2), yields

$F_{BC} = 15.2\text{ kN}$ $F_{BD} = 21.5\text{ kN}$ **Ans.**



(a)

***3–4.** If cables *BD* and *BC* can withstand a maximum tensile force of 20 kN, determine the maximum mass of the girder that can be suspended from cable *AB* so that neither cable will fail. The center of mass of the girder is located at point *G*.



Equations of Equilibrium: The girder is suspended from cable *AB*. In order to meet the conditions of equilibrium the tensile force developed in cable *AB* must be equal to the weight of the girder. Thus,

$F_{AB} = m(9.81) = 9.81m$ **Ans.**

Applying the equations of equilibrium along the x and y axes to the free - body diagram in Fig. (a),

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad F_{BD}\sin 30° - F_{BC}\sin 45° = 0$

$F_{BD} = 1.4142 F_{BC}$ (1)

$+\uparrow \Sigma F_y = 0;$ $\quad 9.81m - F_{BD}\cos 30° - F_{BC}\cos 45° = 0$ (2)

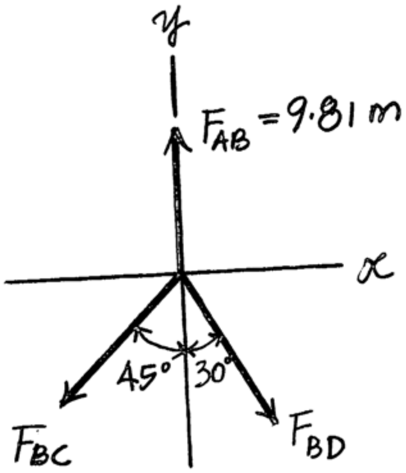
Since $F_{BD} > F_{BC}$, cable *BD* will break before cable *BC*. Substituting $F_{BD} = 20\ 000$ N into Eq. (1),

$F_{BC} = 14\,142.14\text{ N}$

Substituting this result into Eq. (2), yields

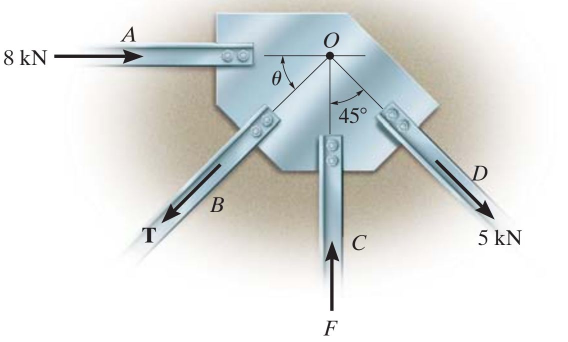
$9.81m - 20\,000\cos 30° - 14\,142.14\cos 45° = 0$

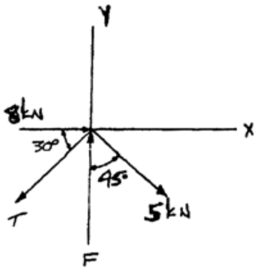
$m = 2\,785\text{ kg} = 2.78\text{ Mg}$ **Ans.**



(a)

•3–5. The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of **F** and **T** for equilibrium. Take $\theta = 30°$.





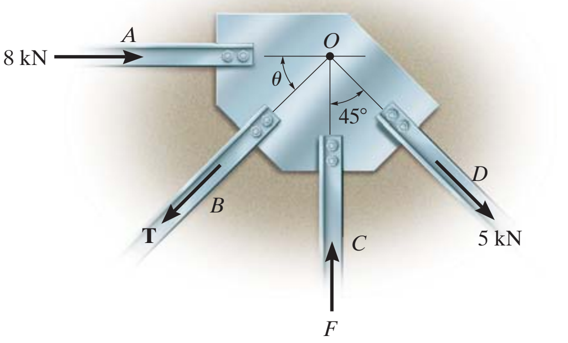
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -T\cos 30° + 8 + 5\sin 45° = 0$

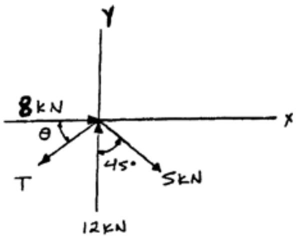
$T = 13.32 = 13.3\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad F - 13.32\sin 30° - 5\cos 45° = 0$

$F = 10.2\text{ kN}$ **Ans**

3–6. The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O. Take $F = 12$ kN.





$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 8 - T\cos\theta + 5\sin 45° = 0$

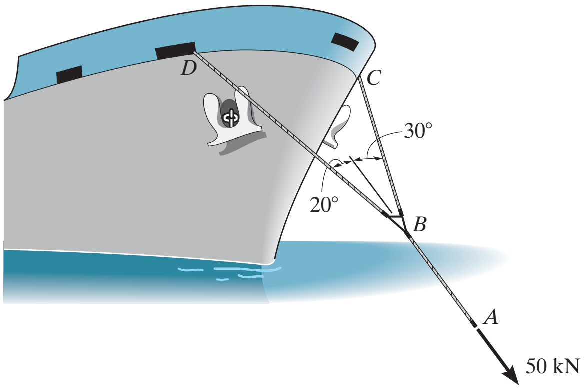
$+\uparrow \Sigma F_y = 0; \quad 12 - T\sin\theta - 5\cos 45° = 0$

Solving,

$T = 14.3\text{ kN}$ **Ans**

$\theta = 36.3°$ **Ans**

3–7. The towing pendant *AB* is subjected to the force of 50 kN exerted by a tugboat. Determine the force in each of the bridles, *BC* and *BD*, if the ship is moving forward with constant velocity.



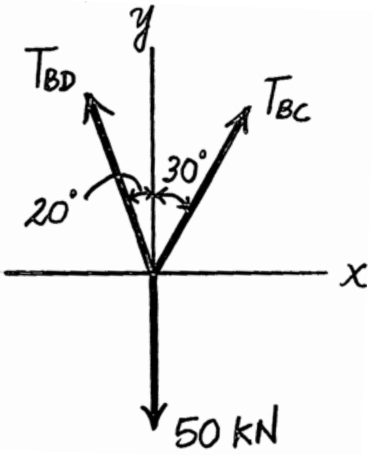
$\xrightarrow{+} \Sigma F_x = 0;\quad T_{BC} \sin 30° - T_{BD} \sin 20° = 0$

$+\uparrow \Sigma F_y = 0;\quad T_{BC} \cos 30° + T_{BD} \cos 20° - 50 = 0$

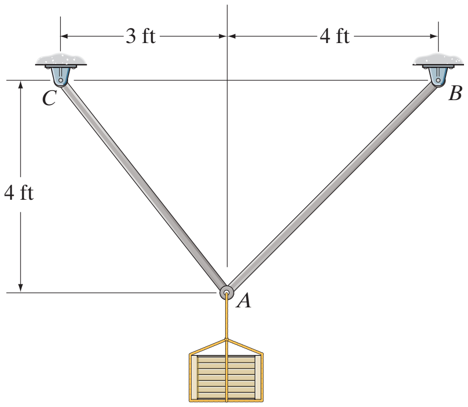
Solving,

$T_{BC} = 22.3 \text{ kN}$ **Ans**

$T_{BD} = 32.6 \text{ kN}$ **Ans**



***3–8.** Members *AC* and *AB* support the 300-lb crate. Determine the tensile force developed in each member.



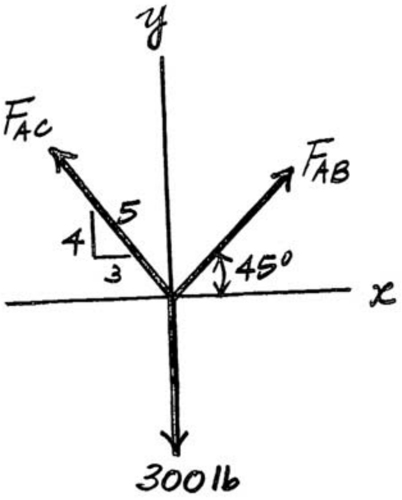
Equations of Equilibrium: Applying the equations of equilibrium along the *x* and *y* axes to the free - body diagram in Fig. (*a*),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB}\cos 45° - F_{AC}\left(\frac{3}{5}\right) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 45° + F_{AC}\left(\frac{4}{5}\right) - 300 = 0 \qquad (2)$$

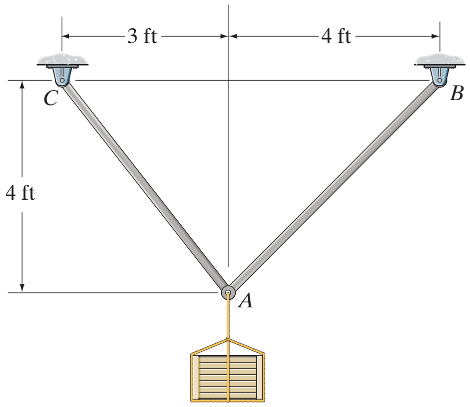
Solving Eqs. (1) and (2), yields

$F_{AC} = 214\text{ lb}$ $\qquad F_{AB} = 182\text{ lb}$ **Ans.**



(a)

•3–9. If members *AC* and *AB* can support a maximum tension of 300 lb and 250 lb, respectively, determine the largest weight of the crate that can be safely supported.



Equations of Equilibrium: Applying the equations of equilibrium along the *x* and *y* axes to the free - body diagram in Fig. (*a*),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB}\cos 45° - F_{AC}\left(\frac{3}{5}\right) = 0 \qquad (1)$$

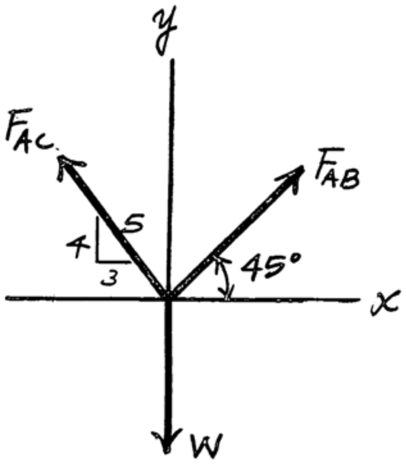
$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 45° + F_{AC}\left(\frac{4}{5}\right) - W = 0 \qquad (2)$$

Assuming that rod *AB* will break first, $F_{AB} = 250$ lb. Substituting this value into Eqs. (1) and (2),

$F_{AC} = 294.63$ lb

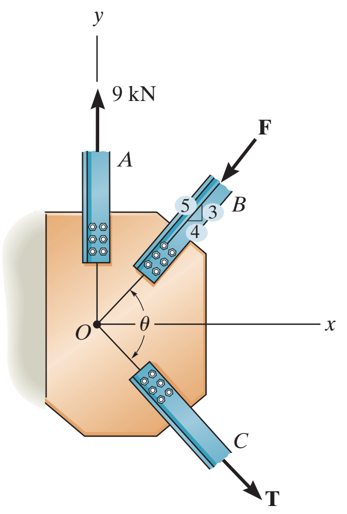
$W = 412$ lb **Ans.**

Since $F_{AC} = 294.63\text{ lb} < 300$ lb, rod *AC* will not break as assumed.



(a)

3–10. The members of a truss are connected to the gusset plate. If the forces are concurrent at point *O*, determine the magnitudes of **F** and **T** for equilibrium. Take $\theta = 90^\circ$.



$\phi = 90^\circ - \tan^{-1}\left(\frac{3}{4}\right) = 53.13^\circ$

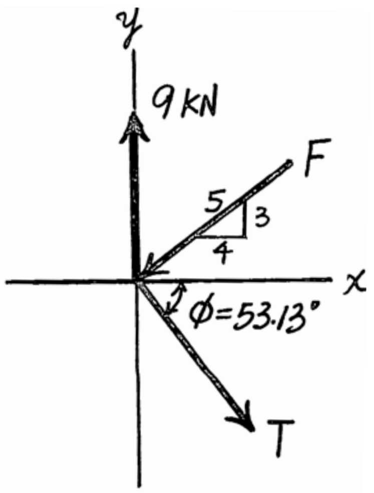
$\xrightarrow{+} \Sigma F_x = 0;\quad T\cos 53.13^\circ - F\left(\frac{4}{5}\right) = 0$

$+\uparrow \Sigma F_y = 0;\quad 9 - T\sin 53.13^\circ - F\left(\frac{3}{5}\right) = 0$

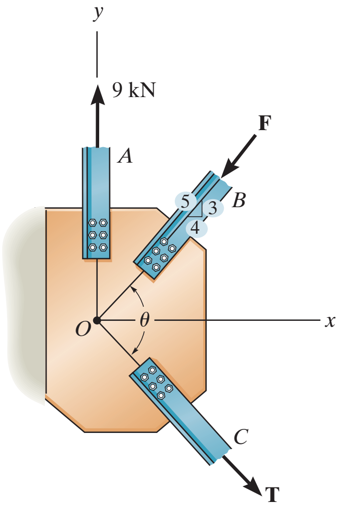
Solving,

$T = 7.20\text{ kN}$ Ans

$F = 5.40\text{ kN}$ Ans



3–11. The gusset plate is subjected to the forces of three members. Determine the tension force in member *C* and its angle θ for equilibrium. The forces are concurrent at point *O*. Take $F = 8$ kN.



$\xrightarrow{+} \Sigma F_x = 0;\quad T\cos\phi - 8\left(\frac{4}{5}\right) = 0 \quad (1)$

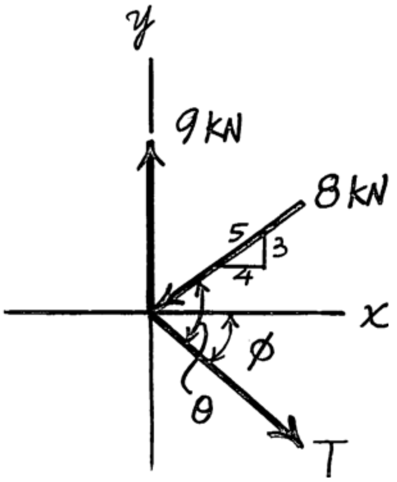
$+\uparrow \Sigma F_y = 0;\quad 9 - 8\left(\frac{3}{5}\right) - T\sin\phi = 0 \quad (2)$

Rearrange then divide Eq. (1) into Eq. (2):

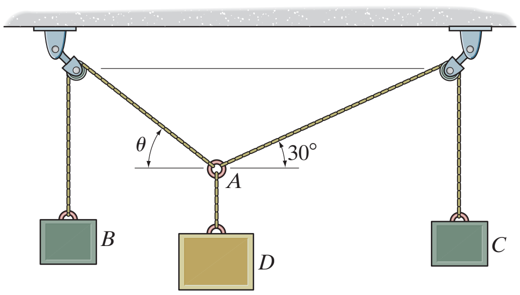
$\tan\phi = 0.656,\quad \phi = 33.27^\circ$

$T = 7.66\text{ kN}$ Ans

$\theta = \phi + \tan^{-1}\left(\frac{3}{4}\right) = 70.1^\circ$ Ans



***3–12.** If block *B* weighs 200 lb and block *C* weighs 100 lb, determine the required weight of block *D* and the angle θ for equilibrium.



Equations of Equilibrium: Applying the equations of equilibrium along the *x* and *y* axes to the free-body diagram shown in Fig. (*a*),

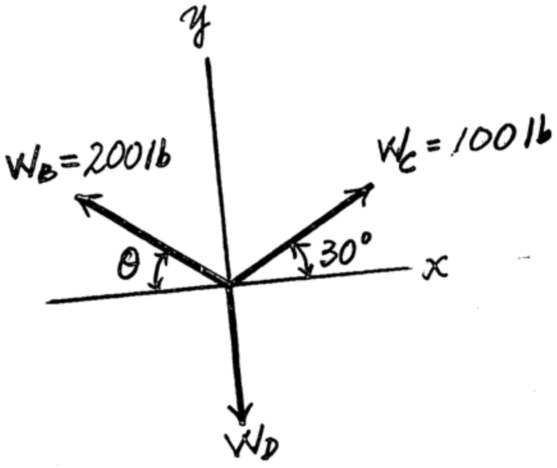
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 100\cos 30° - 200\cos\theta = 0$

$\theta = 64.34° = 64.3°$ **Ans.**

Using this result and writing the equation of equilibrium along the *y* axis, yields

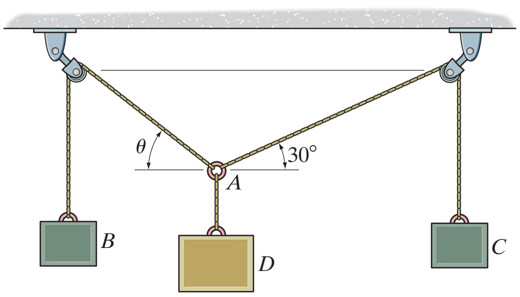
$+\uparrow \Sigma F_y = 0;$ $\quad 100\sin 30° + 200\sin 64.34° - W_D = 0$

$W_D = 230 \text{ lb}$ **Ans.**



(a)

•3–13. If block *D* weighs 300 lb and block *B* weighs 275 lb, determine the required weight of block *C* and the angle θ for equilibrium.



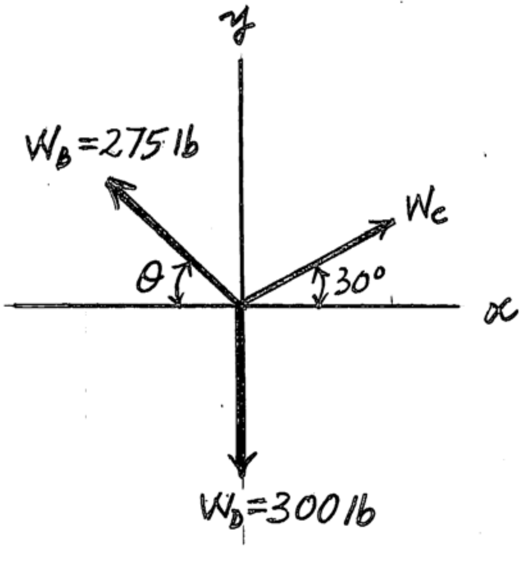
Equations of Equilibrium: Applying the equations of equilibrium along the *x* and *y* axes to the free-body diagram shown in Fig. (*a*),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad W_C \cos 30° - 275 \cos\theta = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad W_C \sin 30° + 275 \sin\theta - 300 = 0 \qquad (2)$$

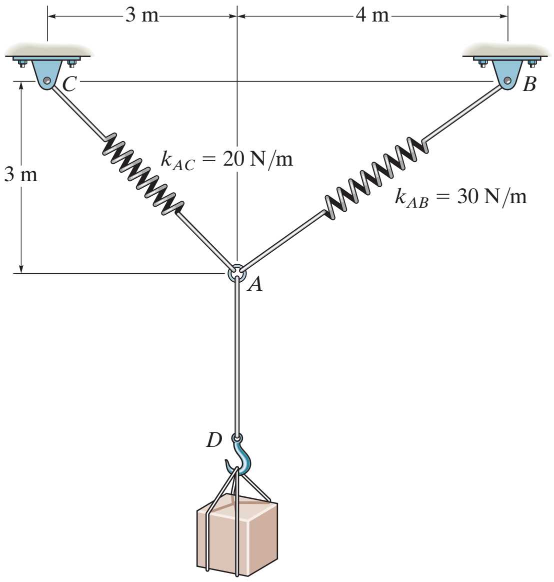
Solving Eqs. (1) and (2), yields

$\theta = 40.9°$ $\qquad W_C = 240$ lb **Ans.**



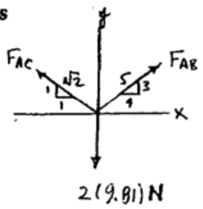
(a)

3–14. Determine the stretch in springs *AC* and *AB* for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.



$F_{AD} = 2(9.81) = x_{AD}(40)$

$x_{AD} = 0.4905$ m **Ans**



$\xrightarrow{+}\Sigma F_x = 0;$ $F_{AB}\left(\frac{4}{5}\right) - F_{AC}\left(\frac{1}{\sqrt{2}}\right) = 0$

$+\uparrow \Sigma F_y = 0;$ $F_{AC}\left(\frac{1}{\sqrt{2}}\right) + F_{AB}\left(\frac{3}{5}\right) - 2(9.81) = 0$

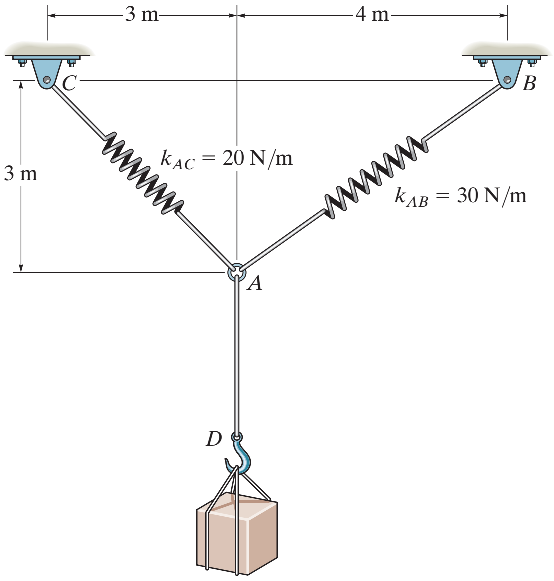
$F_{AC} = 15.86$ N

$x_{AC} = \frac{15.86}{20} = 0.793$ m **Ans**

$F_{AB} = 14.01$ N

$x_{AB} = \frac{14.01}{30} = 0.467$ m **Ans**

3–15. The unstretched length of spring *AB* is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at *D*.



$F = kx = 30(5 - 3) = 60$ N

$\xrightarrow{+}\Sigma F_x = 0;$ $T\cos 45° - 60\left(\frac{4}{5}\right) = 0$

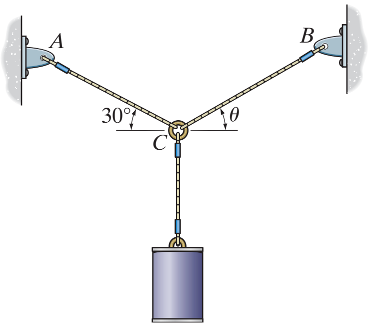
$T = 67.88$ N

$+\uparrow \Sigma F_y = 0;$ $-W + 67.88\sin 45° + 60\left(\frac{3}{5}\right) = 0$

$W = 84$ N

$m = \frac{84}{9.81} = 8.56$ kg **Ans.**

***3–16.** Determine the tension developed in wires *CA* and *CB* required for equilibrium of the 10-kg cylinder. Take $\theta = 40°$.



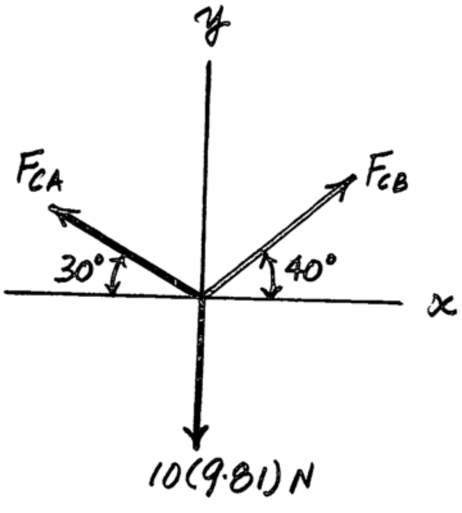
Equations of Equilibrium: Applying the equations of equilibrium along the *x* and *y* axes to the free-body diagram shown in Fig. (*a*),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CB}\cos 40° - F_{CA}\cos 30° = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB}\sin 40° + F_{CA}\sin 30° - 10(9.81) = 0 \qquad (2)$$

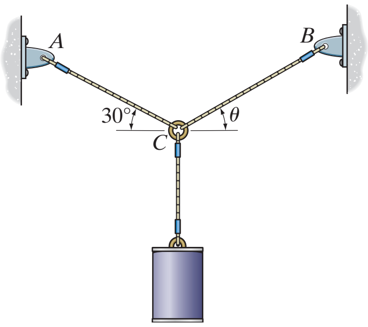
Solving Eqs. (1) and (2), yields

$F_{CA} = 80.0\text{ N}$ $\qquad F_{CB} = 90.4\text{ N}$ **Ans.**



(a)

•3–17. If cable CB is subjected to a tension that is twice that of cable CA, determine the angle θ for equilibrium of the 10-kg cylinder. Also, what are the tensions in wires CA and CB?



Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CB}\cos\theta - F_{CA}\cos 30° = 0 \qquad (1)$$

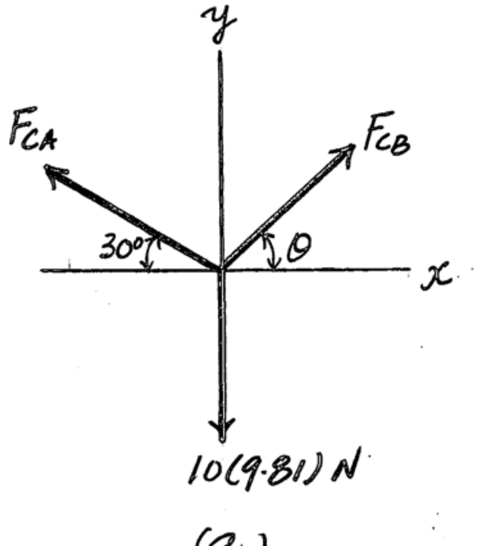
$$+\uparrow \Sigma F_y = 0; \qquad F_{CB}\sin\theta + F_{CA}\sin 30° - 10(9.81) = 0 \qquad (2)$$

However, it is required that

$$F_{CB} = 2F_{CA} \qquad (3)$$

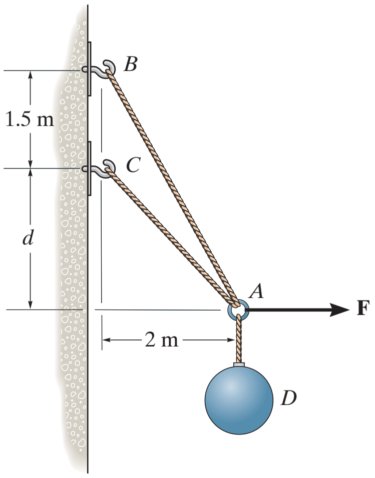
Solving Eqs. (1) and (2), yields

$\theta = 64.3°$ $\qquad F_{CB} = 85.2\text{ N}$ $\qquad F_{CA} = 42.6\text{ N}$ **Ans.**



(a)

3–18. Determine the forces in cables *AC* and *AB* needed to hold the 20-kg ball *D* in equilibrium. Take $F = 300$ N and $d = 1$ m.



Equations of Equilibrium :

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 300 - F_{AB}\left(\frac{4}{\sqrt{41}}\right) - F_{AC}\left(\frac{2}{\sqrt{5}}\right) = 0$$

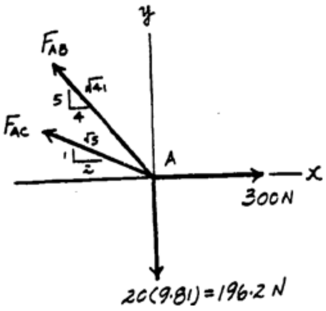
$$0.6247F_{AB} + 0.8944F_{AC} = 300 \qquad [1]$$

$$+\uparrow\Sigma F_y = 0; \quad F_{AB}\left(\frac{5}{\sqrt{41}}\right) + F_{AC}\left(\frac{1}{\sqrt{5}}\right) - 196.2 = 0$$

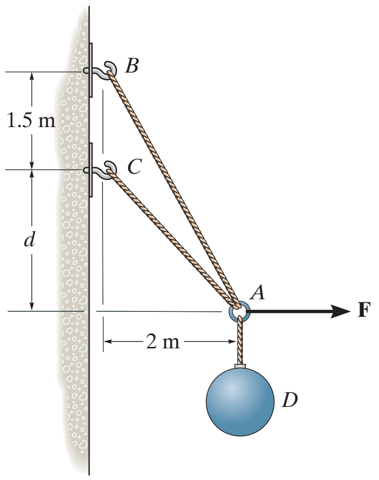
$$0.7809F_{AB} + 0.4472F_{AC} = 196.2 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$F_{AB} = 98.6$ N $\quad F_{AC} = 267$ N $\qquad$ **Ans**



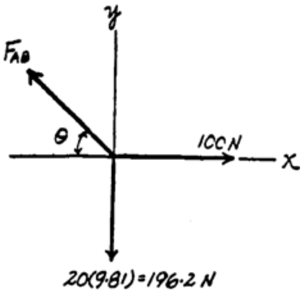
3–19. The ball D has a mass of 20 kg. If a force of $F = 100$ N is applied horizontally to the ring at A, determine the dimension d so that the force in cable AC is zero.



Equations of Equilibrium :

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $100 - F_{AB}\cos\theta = 0$ $F_{AB}\cos\theta = 100$ [1]

$+\uparrow \Sigma F_y = 0;$ $F_{AB}\sin\theta - 196.2 = 0$ $F_{AB}\sin\theta = 196.2$ [2]

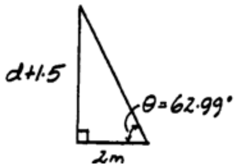


Solving Eqs. [1] and [2] yields

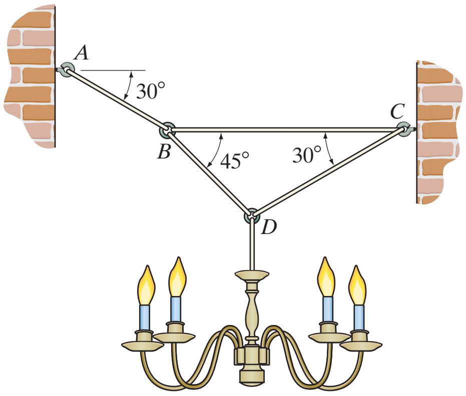
$\theta = 62.99°$ $F_{AB} = 220.21$ N

From the geometry,

$d + 1.5 = 2\tan 62.99°$
$d = 2.42$ m **Ans**



***3–20.** Determine the tension developed in each wire used to support the 50-kg chandelier.



Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. (a).

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad F_{CD}\cos 30° - F_{BD}\cos 45° = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD}\sin 30° + F_{BD}\sin 45° - 50(9.81) = 0 \quad (2)$

Solving Eqs. (1) and (2), yields

$F_{CD} = 359\text{ N} \qquad F_{BD} = 439.77\text{ N} = 440\text{ N}$ **Ans.**

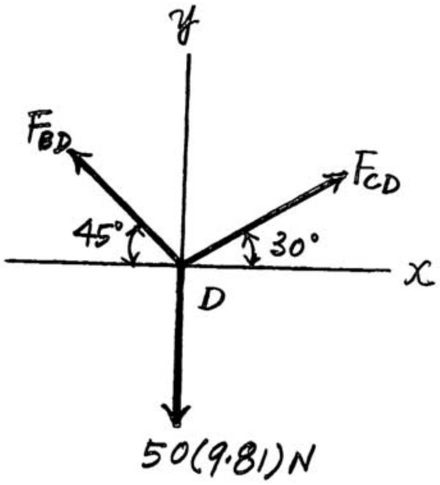
Using the result $F_{BD} = 439.77$ N and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint B shown in Fig. (b),

$+\uparrow \Sigma F_y = 0;$ $\quad F_{AB}\sin 30° - 439.77\sin 45° = 0$

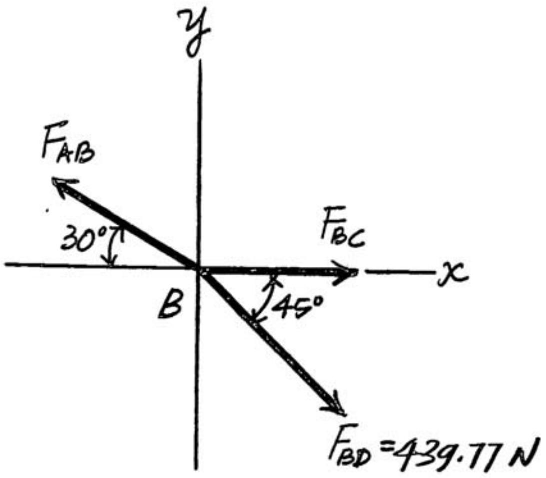
$F_{AB} = 621.93\text{N} = 622\text{ N}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad F_{BC} + 439.77\cos 45° - 621.93\cos 30° = 0$

$F_{BC} = 228\text{ N}$ **Ans.**

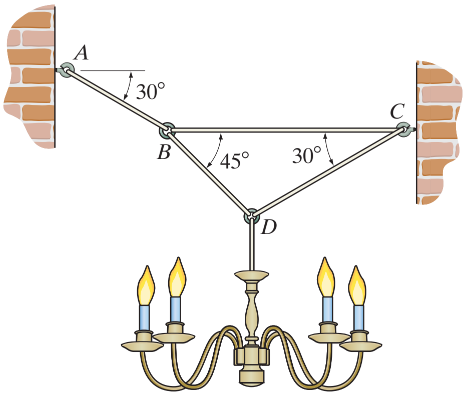


(a)



(b)

•3–21. If the tension developed in each of the four wires is not allowed to exceed 600 N, determine the maximum mass of the chandelier that can be supported.



Equations of Equilibrium: First, we will apply the equation of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. (a).

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CD}\cos 30° - F_{BD}\cos 45° = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\sin 30° + F_{BD}\sin 45° - m(9.81) = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$$F_{CD} = 7.1814m \qquad F_{BD} = 8.7954m$$

Using the result $F_{BD} = 8.7954m$ and applying the equation of equilibrium along the x and y axes to the free-body diagram of joint B shown in Fig. (b),

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 30° - 8.7954m\sin 45° = 0$$

$$F_{AB} = 12.4386m$$

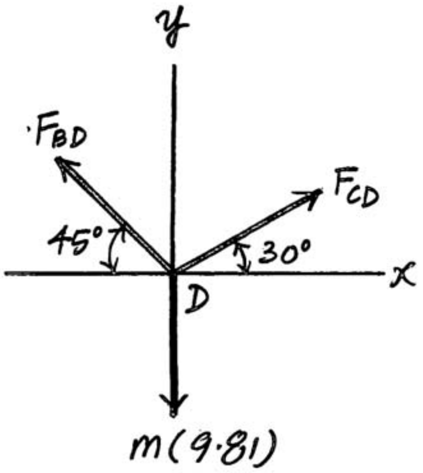
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC} + 8.7954m\cos 45° - 12.4386m\cos 30° = 0$$

$$F_{BC} = 4.5528m$$

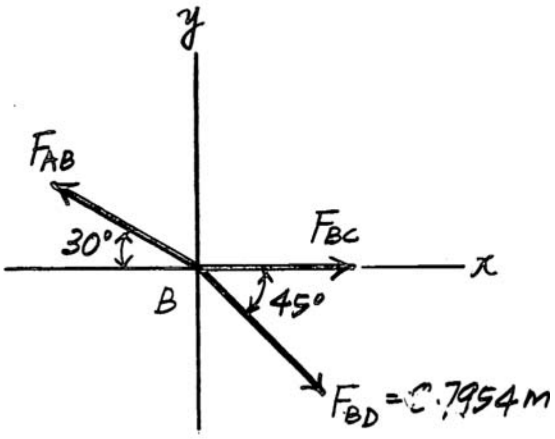
From this result, notice that cable AB is subjected to the greatest tensile force. Thus, it will achieve the maximum allowable tensile force first.

$$F_{AB} = 600 = 12.4386m$$

$$m = 48.2 \text{ kg} \qquad \textbf{Ans.}$$

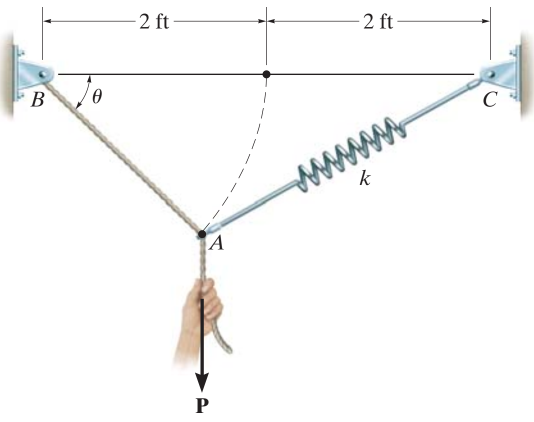


(a)



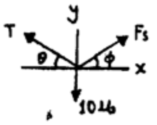
(b)

■**3–22.** A vertical force $P = 10$ lb is applied to the ends of the 2-ft cord AB and spring AC. If the spring has an unstretched length of 2 ft, determine the angle θ for equilibrium. Take $k = 15$ lb/ft.

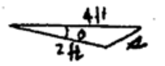


$\xrightarrow{+}\Sigma F_x = 0;\qquad F_s\cos\phi - T\cos\theta = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0;\qquad T\sin\theta + F_s\sin\phi - 10 = 0 \qquad (2)$

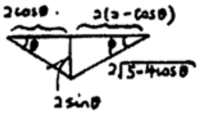


$s = \sqrt{(4)^2 + (2)^2 - 2(4)(2)\cos\theta} - 2 = 2\sqrt{5-4\cos\theta} - 2$



$F_s = ks = 2k(\sqrt{5-4\cos\theta} - 1)$

From Eq. (1): $\quad T = F_s\left(\dfrac{\cos\phi}{\cos\theta}\right)$



$T = 2k(\sqrt{5-4\cos\theta} - 1)\left(\dfrac{2-\cos\theta}{\sqrt{5-4\cos\theta}}\right)\left(\dfrac{1}{\cos\theta}\right)$

From Eq. (2):

$$\frac{2k\left(\sqrt{5-4\cos\theta} - 1\right)(2-\cos\theta)}{\sqrt{5-4\cos\theta}}\tan\theta + \frac{2k\left(\sqrt{5-4\cos\theta} - 1\right)2\sin\theta}{2\sqrt{5-4\cos\theta}} = 10$$

$$\frac{\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}}(2\tan\theta - \sin\theta + \sin\theta) = \frac{10}{2k}$$

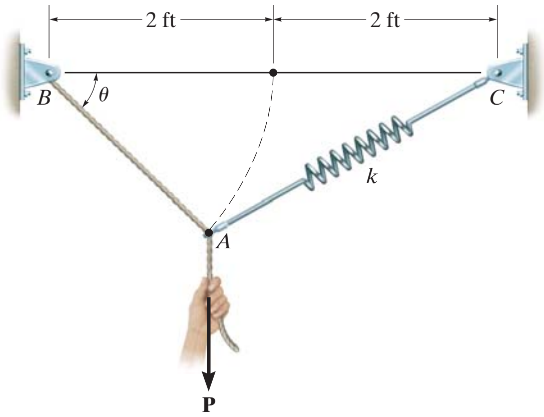
$$\frac{\tan\theta\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}} = \frac{10}{4k}$$

Set $k = 15$ lb/ft

Solving for θ by trial and error,

$\theta = 35.0°$ **Ans**

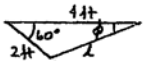
3–23. Determine the unstretched length of spring *AC* if a force $P = 80$ lb causes the angle $\theta = 60°$ for equilibrium. Cord *AB* is 2 ft long. Take $k = 50$ lb/ft.



$$l = \sqrt{4^2 + 2^2 - 2(2)(4)\cos 60°}$$

$$l = \sqrt{12}$$

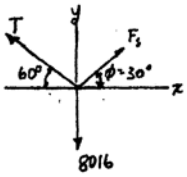
$$\frac{\sqrt{12}}{\sin 60°} = \frac{2}{\sin\phi}$$



$$\phi = \sin^{-1}\left(\frac{2\sin 60°}{\sqrt{12}}\right) = 30°$$

$+\uparrow \Sigma F_y = 0;\quad T\sin 60° + F_s\sin 30° - 80 = 0$

$\xrightarrow{+} \Sigma F_x = 0;\quad -T\cos 60° + F_s\cos 30° = 0$



Solving for F_s,

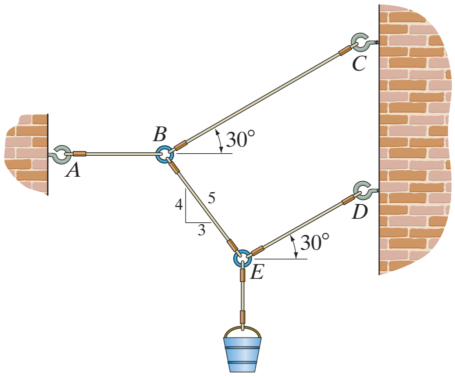
$F_s = 40$ lb

$F_s = kx$

$40 = 50(\sqrt{12} - l')$

$$l = \sqrt{12} - \frac{40}{50} = 2.66\text{ft}$$ **Ans.**

***3–24.** If the bucket weighs 50 lb, determine the tension developed in each of the wires.



Equations of Equilibrium: First, we will apply the equation of equilibrium along the x and y axes to the free-body diagram of joint E shown in Fig. (a).

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{ED}\cos 30° - F_{EB}\left(\frac{3}{5}\right) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{ED}\sin 30° + F_{EB}\left(\frac{4}{5}\right) - 50 = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$F_{ED} = 30.2$ lb $\qquad F_{EB} = 43.61$ lb $= 43.6$ lb **Ans.**

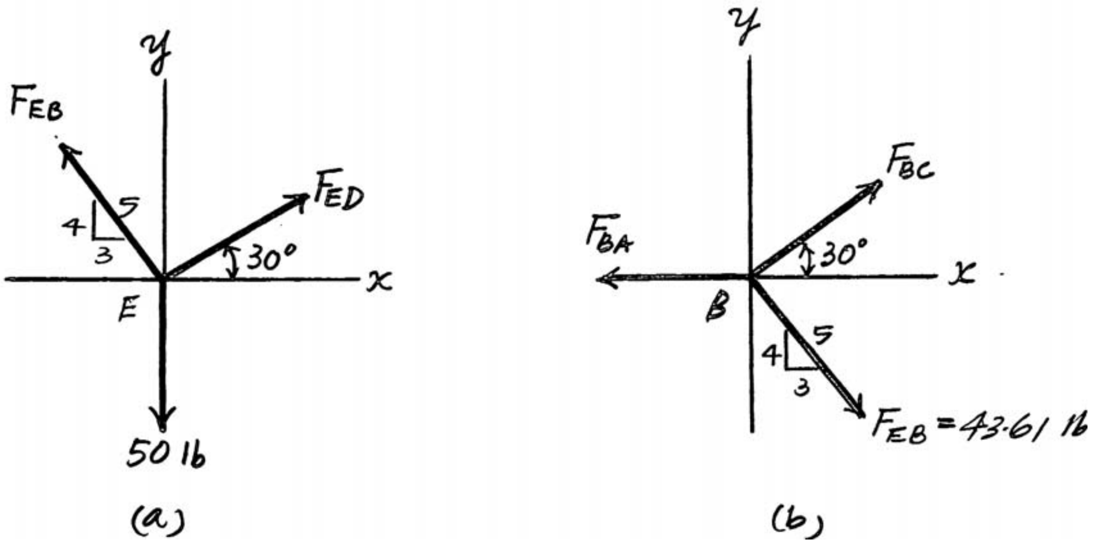
Using the result $F_{EB} = 43.61$ lb and applying the equation of equilibrium to the free-body diagram of joint B shown in Fig. (b),

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin 30° - 43.61\left(\frac{4}{5}\right) = 0$$

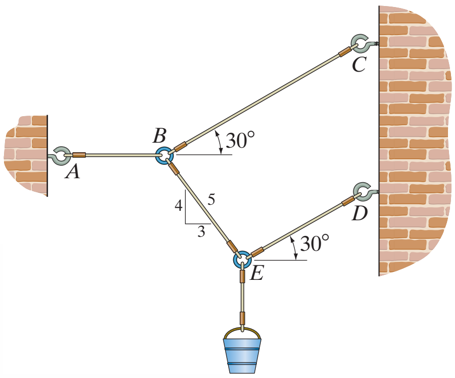
$F_{BC} = 69.78$ lb $= 69.8$ lb **Ans.**

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 69.78\cos 30° + 43.61\left(\frac{3}{5}\right) - F_{BA} = 0$$

$F_{BA} = 86.6$ lb **Ans.**



•3–25. Determine the maximum weight of the bucket that the wire system can support so that no single wire develops a tension exceeding 100 lb.



Equations of Equilibrium: First, we will apply the equations of equilibrium along the *x* and *y* axes to the free-body diagram of joint *E* shown in Fig. (*a*).

$$\xrightarrow{+}\Sigma F_x = 0; \qquad F_{ED}\cos 30° - F_{EB}\left(\frac{3}{5}\right) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{ED}\sin 30° + F_{EB}\left(\frac{4}{5}\right) - W = 0 \qquad (2)$$

Solving,

$$F_{EB} = 0.8723W \qquad F_{ED} = 0.6043W$$

Using the result $F_{EB} = 0.8723W$ and applying the equations of equilibrium to the free-body diagram of joint *B* shown in Fig. (*b*),

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin 30° - 0.8723W\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 1.3957W$$

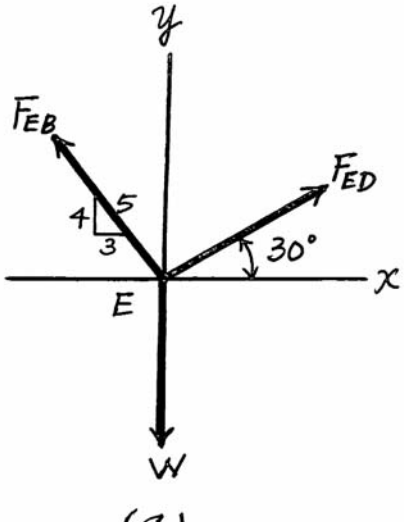
$$\xrightarrow{+}\Sigma F_x = 0; \qquad 1.3957W\cos 30° + 0.8723W\left(\frac{3}{5}\right) - F_{BA} = 0$$

$$F_{BA} = 1.7320W$$

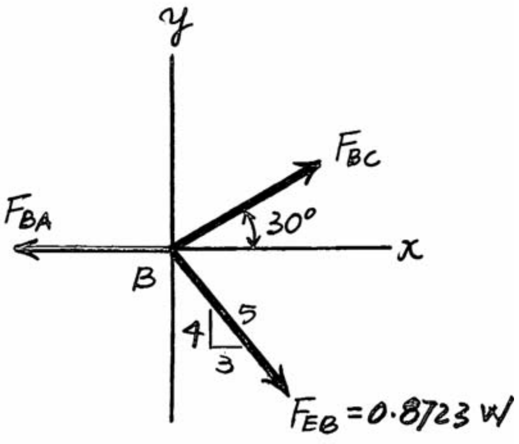
From these results, notice that wire *BA* is subjected to the greatest tensile force. Thus, it will achieve the maximum allowable tensile force first.

$$F_{BA} = 100 = 1.7320W$$

$$W = 57.7\text{ lb} \qquad \textbf{Ans.}$$

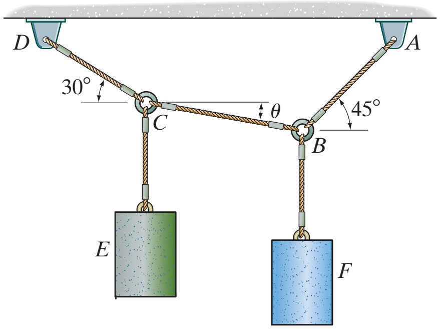


(a)



(b)

3–26. Determine the tensions developed in wires CD, CB, and BA and the angle θ required for equilibrium of the 30-lb cylinder E and the 60-lb cylinder F.



Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free-body diagram of joint C shown in Fig. (a),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC}\cos\theta - F_{CD}\cos 30^\circ = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad -F_{BC}\sin\theta + F_{CD}\sin 30^\circ - 30 = 0 \qquad (2)$$

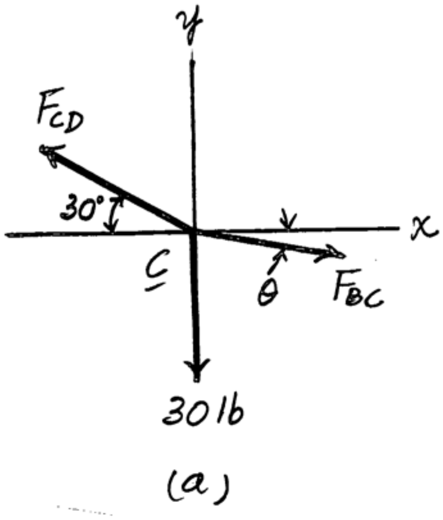
By referring to the free-body diagram of joint B in Fig. (b),

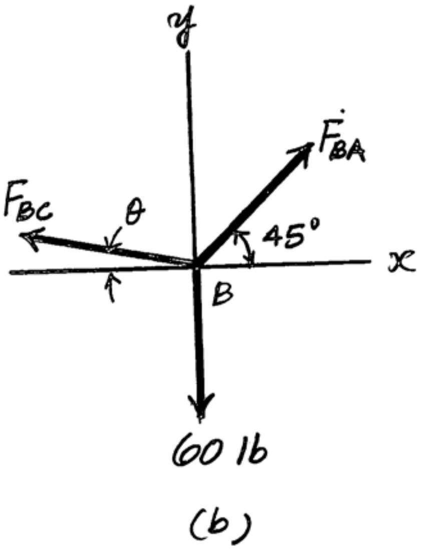
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BA}\cos 45^\circ - F_{BC}\cos\theta = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BA}\sin 45^\circ + F_{BC}\sin\theta - 60 = 0 \qquad (4)$$

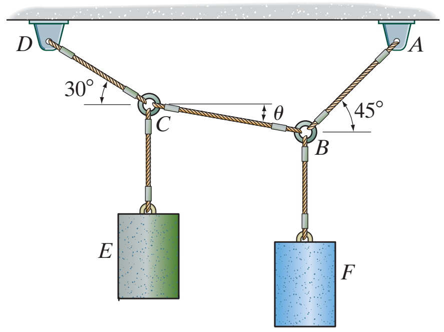
Solving Eqs. (1) through (4), yields

$F_{BA} = 80.7$ lb **Ans.**
$F_{CD} = 65.9$ lb **Ans.**
$F_{BC} = 57.1$ lb **Ans.**
$\theta = 2.95^\circ$ **Ans.**





3–27. If cylinder E weighs 30 lb and $\theta = 15°$, determine the weight of cylinder F.



Equations of Equilibrium: First, we will apply the equation of equilibrium along the x and y axes to the free-body diagram of joint C shown in Fig. (a).

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC}\cos 15° - F_{CD}\cos 30° = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\sin 30° - F_{BC}\sin 15° - 30 = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$$F_{BC} = 100.38 \text{ lb} \qquad F_{CD} = 111.96 \text{ lb} \qquad \textbf{Ans.}$$

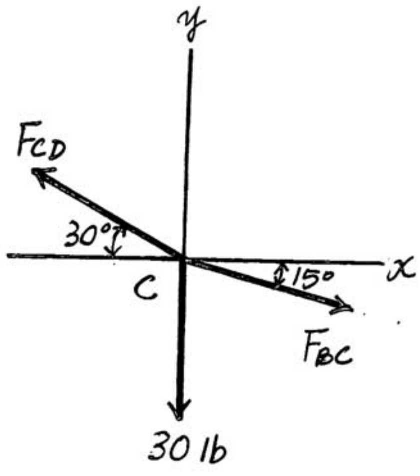
Using the result $F_{BC} = 100.38$ lb and applying the equation of equilibrium along the x and y axes to the free-body diagram of joint B shown in Fig. (b),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BA}\cos 45° - 100.38\cos 15° = 0$$

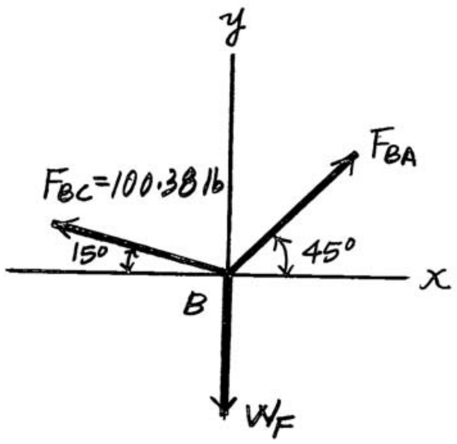
$$F_{BA} = 137.12 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 137.12\sin 45° + 100.38\sin 15° - W_F = 0$$

$$W_F = 123 \text{ lb} \qquad \textbf{Ans.}$$

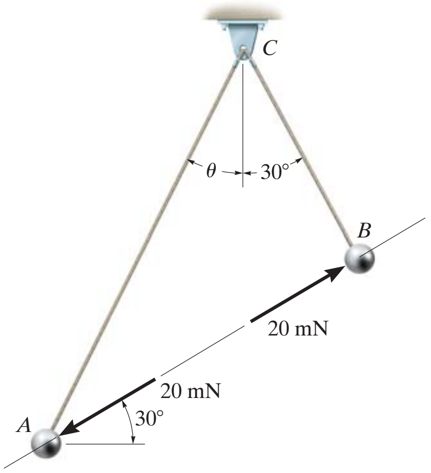


(a)



(b)

***3–28.** Two spheres *A* and *B* have an equal mass and are electrostatically charged such that the repulsive force acting between them has a magnitude of 20 mN and is directed along line *AB*. Determine the angle θ, the tension in cords *AC* and *BC*, and the mass *m* of each sphere.



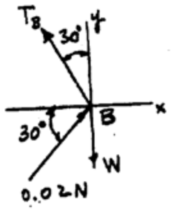
For *B* :

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.02\cos 30° - T_B \sin 30° = 0$

$+\uparrow \Sigma F_y = 0;\quad 0.02\sin 30° + T_B \cos 30° - W = 0$

$T_B = 0.0346\text{ N} = 34.6\text{ mN}$ **Ans**

$W = 0.04\text{ N}$



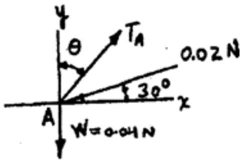
For *A* :

$\xrightarrow{+} \Sigma F_x = 0;\quad T_A \sin\theta - 0.02\cos 30° = 0$

$+\uparrow \Sigma F_y = 0;\quad T_A \cos\theta - 0.02\sin 30° - 0.04 = 0$

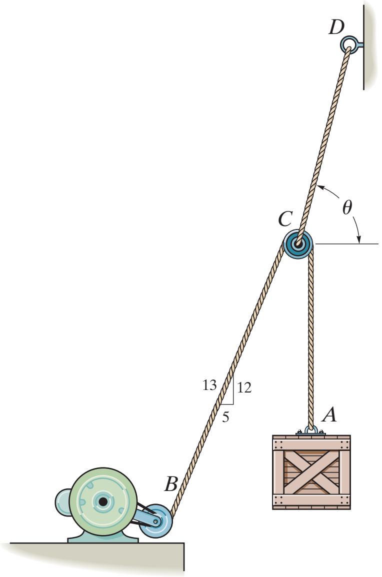
$T_A = 0.0529\text{ N} = 52.9\text{ mN}$ **Ans**

$\theta = 19.1°$ **Ans**



$m = \dfrac{W}{g} = \dfrac{0.04}{9.81} = 4.08\,(10^{-3})\text{ kg} = 4.08\text{ g}$ **Ans**

•3–29. The cords *BCA* and *CD* can each support a maximum load of 100 lb. Determine the maximum weight of the crate that can be hoisted at constant velocity and the angle θ for equilibrium. Neglect the size of the smooth pulley at *C*.

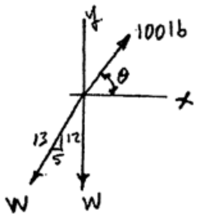


$\xrightarrow{+} \Sigma F_x = 0;\quad 100\cos\theta = W\left(\frac{5}{13}\right)$

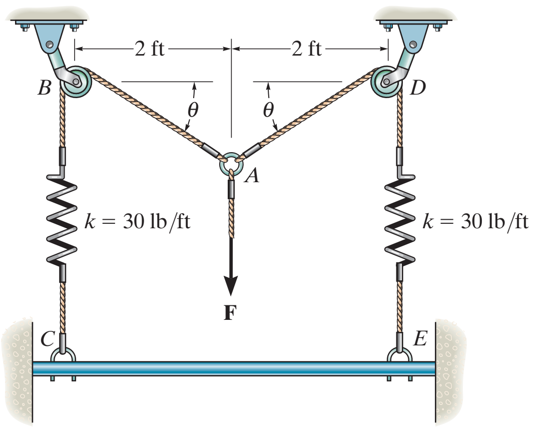
$+\uparrow \Sigma F_y = 0;\quad 100\sin\theta = W\left(\frac{12}{13}\right) + W$

$\theta = 78.7°$ **Ans**

$W = 51.0\text{ lb}$ **Ans**

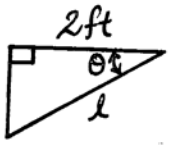


3–30. The springs on the rope assembly are originally unstretched when $\theta = 0°$. Determine the tension in each rope when $F = 90$ lb. Neglect the size of the pulleys at B and D.



$$l = \frac{2}{\cos\theta}$$

$$T = kx = k(l - l_0) = 30\left(\frac{2}{\cos\theta} - 2\right) = 60\left(\frac{1}{\cos\theta} - 1\right) \qquad (1)$$



$+\uparrow \Sigma F_y = 0;\quad 2T\sin\theta - 90 = 0 \qquad (2)$

Substituting Eq. (1) into (2) yields:

$120(\tan\theta - \sin\theta) - 90 = 0$

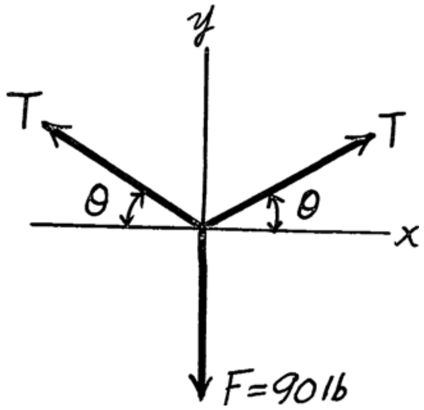
$\tan\theta - \sin\theta = 0.75$

By trial and error:

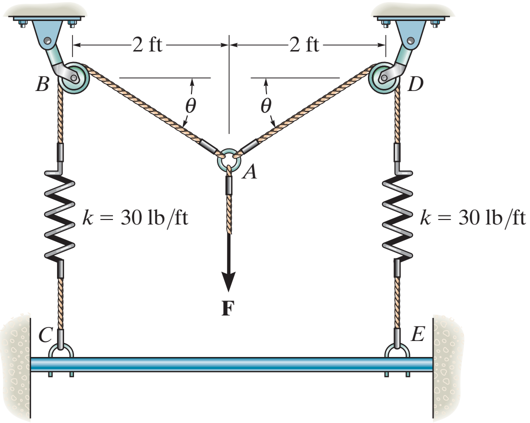
$\theta = 57.957°$

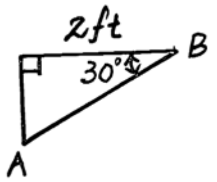
From Eq. (1),

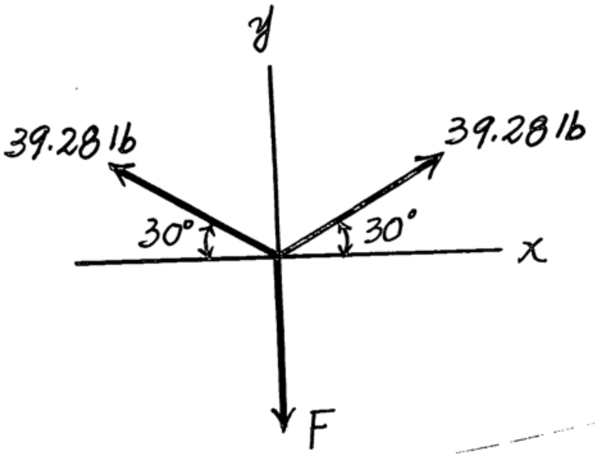
$$T = 60\left(\frac{1}{\cos 57.957°} - 1\right) = 53.1 \text{ lb} \qquad \text{Ans}$$



3–31. The springs on the rope assembly are originally stretched 1 ft when $\theta = 0°$. Determine the vertical force F that must be applied so that $\theta = 30°$.







$$BA = \frac{2}{\cos 30°} = 2.3094 \text{ ft}$$

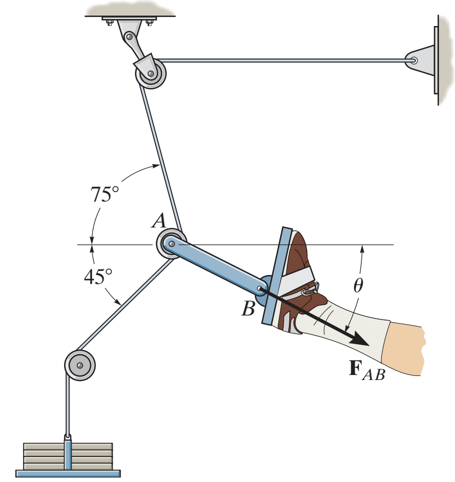
When $\theta = 30°$, the springs are stretched 1 ft + (2.3094 − 2) ft = 1.3094 ft

$F_s = kx = 30(1.3094) = 39.28$ lb

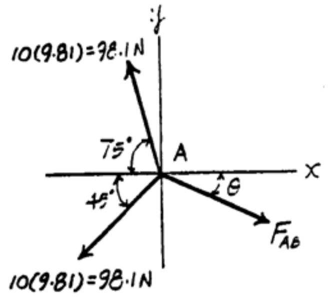
$+\uparrow \Sigma F_y = 0; \quad 2(39.28)\sin 30° - F = 0$

$F = 39.3$ lb **Ans**

***3–32.** Determine the magnitude and direction θ of the equilibrium force F_{AB} exerted along link AB by the tractive apparatus shown. The suspended mass is 10 kg. Neglect the size of the pulley at A.



Free Body Diagram : The tension in the cord is the same throughout the cord, that is $10(9.81) = 9.81$ N.



Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB}\cos\theta - 98.1\cos 75° - 98.1\cos 45° = 0$$

$$F_{AB}\cos\theta = 94.757 \qquad [1]$$

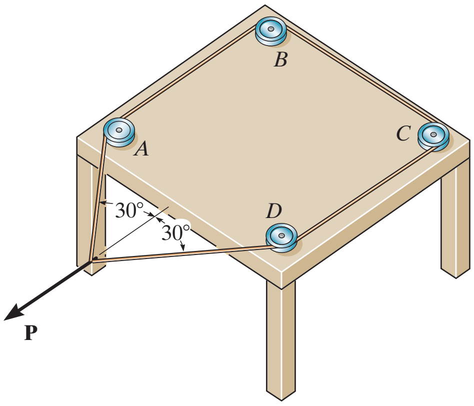
$$+\uparrow \Sigma F_y = 0; \quad 98.1\sin 75° - 98.1\sin 45° - F_{AB}\sin\theta = 0$$

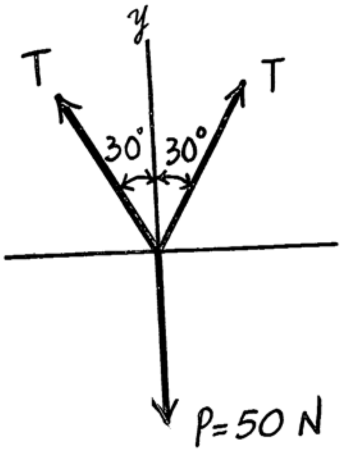
$$F_{AB}\sin\theta = 25.390 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\theta = 15.0° \qquad F_{AB} = 98.1\text{ N} \qquad \textbf{Ans}$$

•3–33. The wire forms a loop and passes over the small pulleys at A, B, C, and D. If its end is subjected to a force of $P = 50$ N, determine the force in the wire and the magnitude of the resultant force that the wire exerts on each of the pulleys.





$+\uparrow \Sigma F_y = 0;\quad 2\,(T\cos 30^\circ) - 50 = 0$

$T = 28.868 = 28.9\text{ N}$ **Ans**

For A and D:

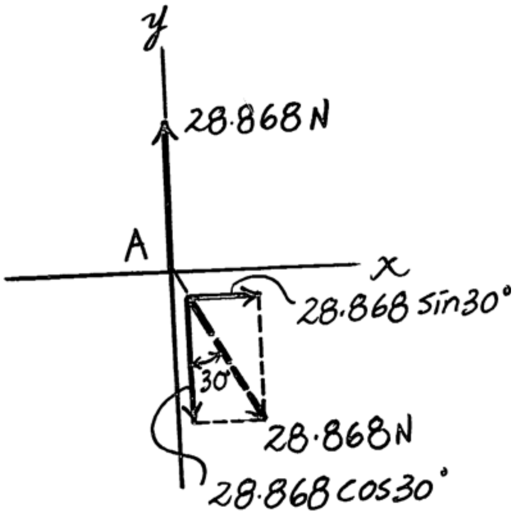
$F_{Rx} = \Sigma F_x;\quad F_{Rx} = 28.868 \sin 30^\circ = 14.43\text{ N}$

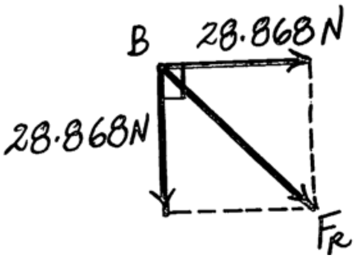
$F_{Rx} = \Sigma F_y;\quad F_{Ry} = 28.868 - 28.868 \cos 30^\circ = 3.868\text{ N}$

$F_R = \sqrt{(14.43)^2 + (3.868)^2} = 14.9\text{ N}$ (A and D) **Ans**

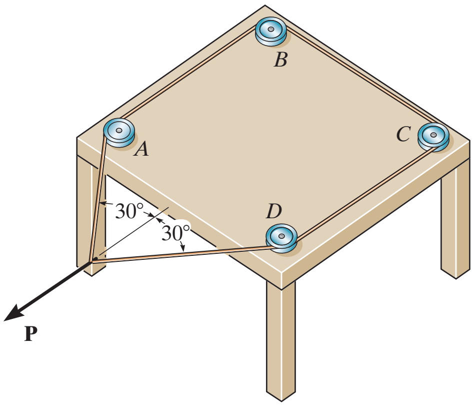
For B and C:

$F_R = \sqrt{(28.868)^2 + (28.868)^2} = 40.8\text{ N}$ (B and C) **Ans**





3–34. The wire forms a loop and passes over the small pulleys at A, B, C, and D. If the maximum *resultant force* that the wire can exert on each pulley is 120 N, determine the greatest force P that can be applied to the wire as shown.



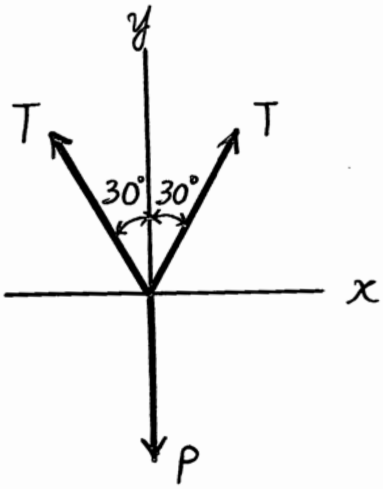
$$+\uparrow \Sigma F_y = 0; \quad 2T\cos 30° - P = 0; \quad T = 0.57735\,P$$

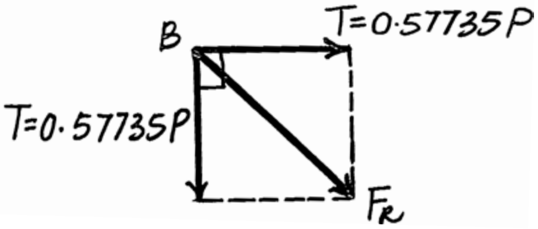
Maximum resultant force is resisted by pulleys B and C.

$$F_R = \sqrt{(0.57735\,P)^2 + (0.57735\,P)^2}$$

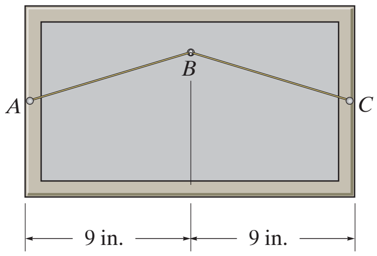
$$F_R = 0.8165\,P = 120$$

$$P = 147\text{ N} \qquad \textbf{Ans}$$





3–35. The picture has a weight of 10 lb and is to be hung over the smooth pin B. If a string is attached to the frame at points A and C, and the maximum force the string can support is 15 lb, determine the shortest string that can be safely used.



Free Body Diagram : Since the pin is smooth, the tension force in the cord is the same throughout the cord.

Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad T\cos\theta - T\cos\theta = 0 \quad (\textit{Satisfied!})$$

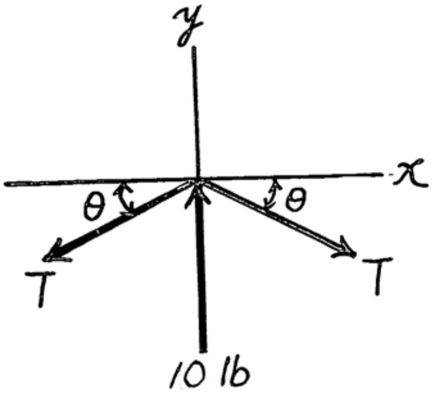
$$+\uparrow \Sigma F_y = 0; \quad 10 - 2T\sin\theta = 0 \quad T = \frac{5}{\sin\theta}$$

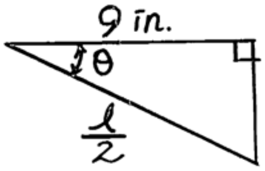
If tension in the cord cannot exceed 15 lb, then

$$\frac{5}{\sin\theta} = 15$$
$$\theta = 19.47^\circ$$

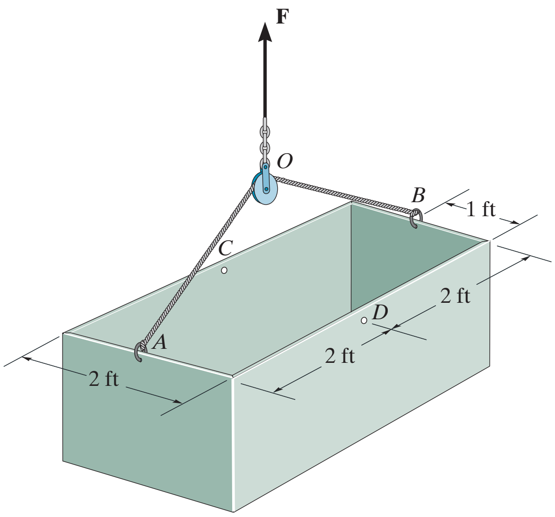
From the geometry, $\frac{l}{2} = \frac{9}{\cos\theta}$ and $\theta = 19.47^\circ$. Therefore

$$l = \frac{18}{\cos 19.47^\circ} = 19.1 \text{ in.} \qquad \textbf{Ans}$$





***3–36.** The 200-lb uniform tank is suspended by means of a 6-ft-long cable, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B or C and D, determine which attachment produces the least amount of tension in the cable. What is this tension?

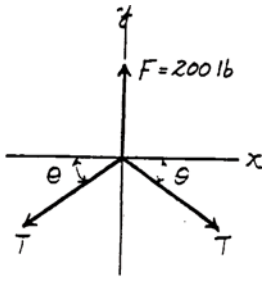


Free Body Diagram : By observation, the force **F** has to support the entire weight of the tank. Thus, $F = 200$ lb. The tension in cable is the same throughout the cable.

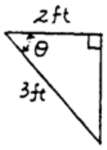
Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T\cos\theta - T\cos\theta = 0 \qquad (\textit{Satisfied!})$$

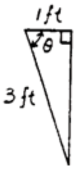
$$+\uparrow \Sigma F_y = 0; \qquad 200 - 2T\sin\theta = 0 \qquad T = \frac{100}{\sin\theta} \qquad [1]$$



From the function obtained above, one realizes that in order to produce the least amount of tension in the cable, $\sin\theta$ hence θ must be as great as possible. Since the attachment of the cable to point C and D produces a greater θ $\left(\theta = \cos^{-1}\frac{1}{3} = 70.53^\circ\right)$ as compared to the attachment of the cable to points A and B $\left(\theta = \cos^{-1}\frac{2}{3} = 48.19^\circ\right)$,



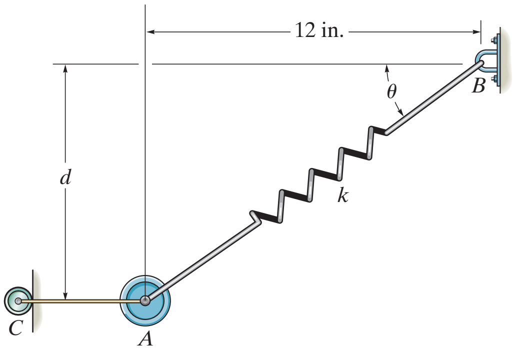
The attachment of the cable to point C and D will produce the least amount of tension in the cable. **Ans**

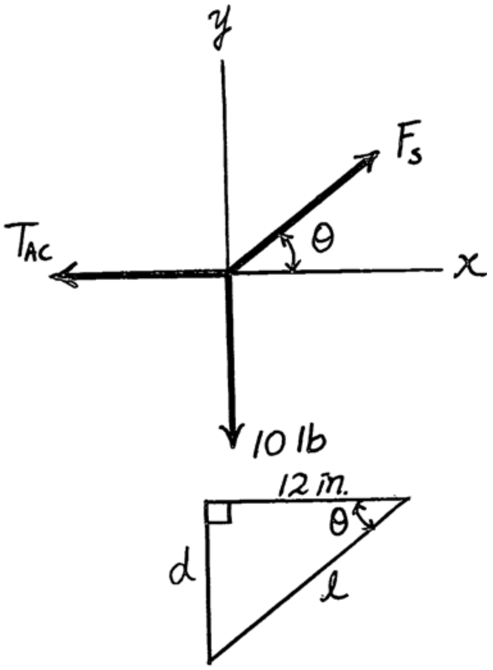


Thus,

$$T = \frac{100}{\sin 70.53^\circ} = 106 \text{ lb} \qquad \textbf{Ans}$$

•■3–37. The 10-lb weight is supported by the cord *AC* and roller and by the spring that has a stiffness of $k = 10$ lb/in. and an unstretched length of 12 in. Determine the distance d to where the weight is located when it is in equilibrium.





$$\xrightarrow{+} \Sigma F_x = 0; \quad -T_{AC} + F_s \cos\theta = 0$$

$$+\uparrow \Sigma F_y = 0; \quad F_s \sin\theta - 10 = 0$$

$$F_s = kx; \quad F_s = 10\left(\frac{12}{\cos\theta} - 12\right)$$

$$= 120(\sec\theta - 1)$$

Thus,

$$120(\sec\theta - 1)\sin\theta = 10$$

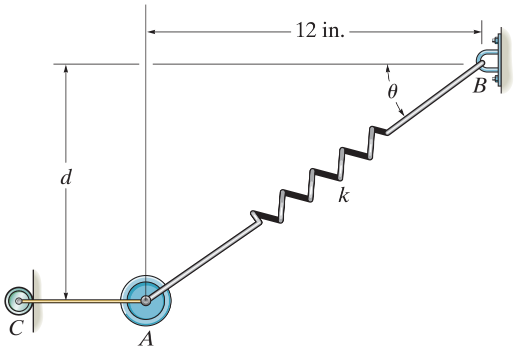
$$(\tan\theta - \sin\theta) = \frac{1}{12}$$

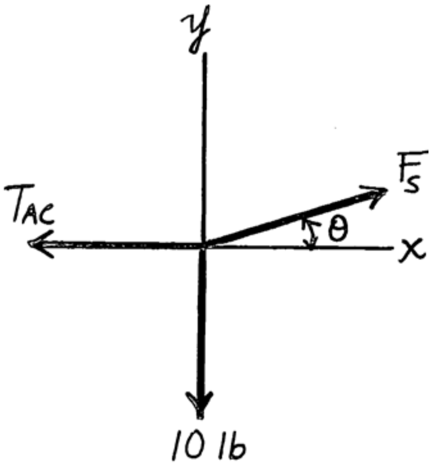
Solving,

$$\theta = 30.71^\circ$$

$$d = 12\tan 30.71^\circ = 7.13 \text{ in.} \qquad \text{Ans}$$

3–38. The 10-lb weight is supported by the cord *AC* and roller and by a spring. If the spring has an unstretched length of 8 in. and the weight is in equilibrium when $d = 4$ in., determine the stiffness k of the spring.





$+\uparrow \Sigma F_y = 0; \quad F_s \sin\theta - 10 = 0$

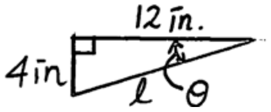
$F_s = kx; \quad F_s = k\left(\dfrac{12}{\cos\theta} - 8\right)$

$\tan\theta = \dfrac{4}{12}; \quad \theta = 18.435°$

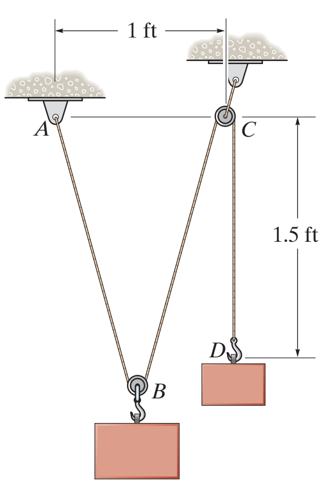
Thus,

$k\left(\dfrac{12}{\cos 18.435°} - 8\right)\sin 18.435° = 10$

$k = 6.80$ lb/in. **Ans**

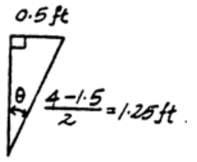


•3–39. A "scale" is constructed with a 4-ft-long cord and the 10-lb block D. The cord is fixed to a pin at A and passes over two *small* pulleys at B and C. Determine the weight of the suspended block at B if the system is in equilibrium.



Free Body Diagram : The tension force in the cord is the same throughout the cord, that is 10 lb. From the geometry,

$\theta = \sin^{-1}\left(\frac{0.5}{1.25}\right) = 23.58°$.

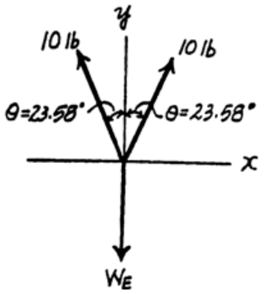


Equations of Equilibrium :

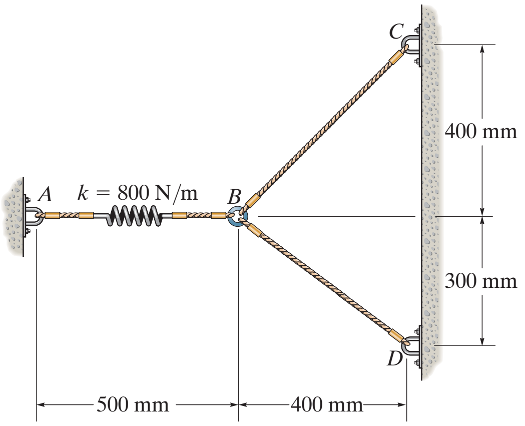
$\xrightarrow{+} \Sigma F_x = 0;\quad 10\sin 23.58° - 10\sin 23.58° = 0$ ***(Satisfied!)***

$+\uparrow \Sigma F_y = 0;\quad 2(10)\cos 23.58° - W_E = 0$

$W_E = 18.3\text{ lb}$ **Ans**



•*3–40. The spring has a stiffness of $k = 800$ N/m and an unstretched length of 200 mm. Determine the force in cables BC and BD when the spring is held in the position shown.



The Force in The Spring : The spring stretches $s = l - l_0 = 0.5 - 0.2 = 0.3$ m. Applying Eq. 3 – 2, we have

$$F_{sp} = ks = 800(0.3) = 240\text{ N}$$

Equations of Equilibrium :

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{BC}\cos 45° + F_{BD}\left(\frac{4}{5}\right) - 240 = 0$

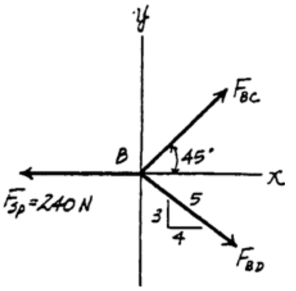
$0.7071F_{BC} + 0.8F_{BD} = 240$ **[1]**

$+\uparrow \Sigma F_y = 0;\quad F_{BC}\sin 45° - F_{BD}\left(\frac{3}{5}\right) = 0$

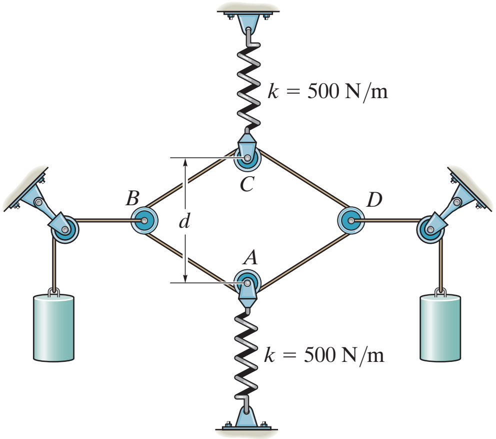
$F_{BC} = 0.8485F_{BD}$ **[2]**

Solving Eqs. [1] and [2] yields,

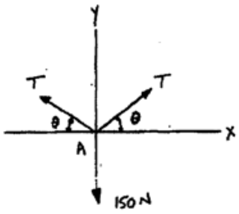
$F_{BD} = 171\text{ N}\quad F_{BC} = 145\text{ N}$ **Ans**



•3–41. A continuous cable of total length 4 m is wrapped around the *small* pulleys at *A*, *B*, *C*, and *D*. If each spring is stretched 300 mm, determine the mass *m* of each block. Neglect the weight of the pulleys and cords. The springs are unstretched when $d = 2$ m.



$F_s = kx;\quad F_s = 500(0.3) = 150\text{ N}$

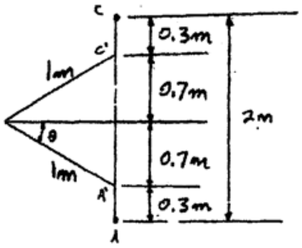


At A:

$+\uparrow \Sigma F_y = 0;\quad -150 + 2T\sin\theta = 0$

$$T = \frac{75}{\sin\theta} \qquad (1)$$

Note that when $\theta = 90°$, the springs are unstretched and the tension in the cord is zero. When the springs are stretched 300 mm = 0.3 m, then $d = (2 - 2(0.3)) = 1.4$ m



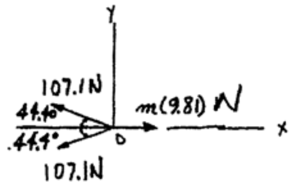
$$\theta = \sin^{-1}\left(\frac{0.7}{1}\right) = 44.4°$$

From Eq. (1), $T = 107.1$ N

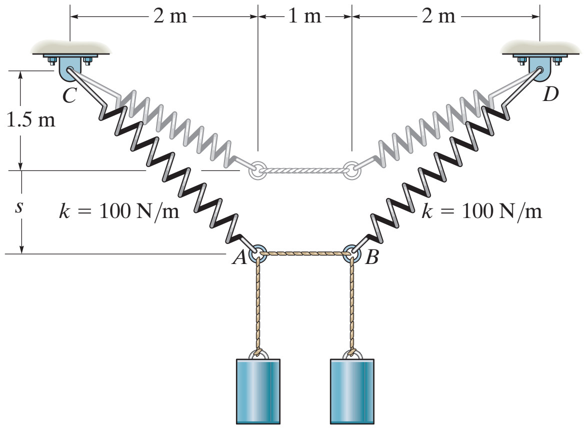
At D:

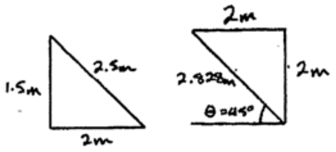
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad -2(107.1)\cos 44.4° + m(9.81) = 0$

$m = 15.6$ kg **Ans**



3–42. Determine the mass of each of the two cylinders if they cause a sag of $s = 0.5$ m when suspended from the rings at *A* and *B*. Note that $s = 0$ when the cylinders are removed.

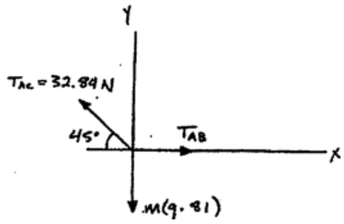




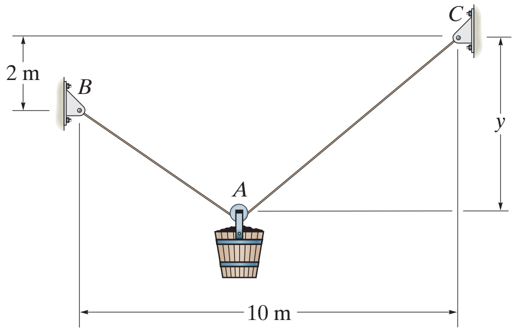
$T_{AC} = 100\text{ N/m}(2.828 - 2.5) = 32.84\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad 32.84\sin 45° - m(9.81) = 0$

$m = 2.37$ kg **Ans**



•3–43. The pail and its contents have a mass of 60 kg. If the cable *BAC* is 15 m long, determine the distance *y* to the pulley at *A* for equilibrium. Neglect the size of the pulley.



Free Body Diagram : Since the pulley is smooth, the tension in the cable is the same throughout the cable.

Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T\sin\theta - T\sin\phi = 0 \qquad \theta = \phi$$

Geometry :

$$l_1 = \sqrt{(10-x)^2 + (y-2)^2} \qquad l_2 = \sqrt{x^2+y^2}$$

Since $\theta = \phi$, two triangles are similar.

$$\frac{10-x}{x} = \frac{y-2}{y} = \frac{\sqrt{(10-x)^2+(y-2)^2}}{\sqrt{x^2+y^2}} \qquad [1]$$

Also,

$$l_1 + l_2 = 15$$

$$\sqrt{(10-x)^2+(y-2)^2} + \sqrt{x^2+y^2} = 15$$

$$\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\right)\sqrt{(10-x)^2+(y-2)^2} + \sqrt{x^2+y^2} = 15 \qquad [2]$$

However, from Eq. [1] $\dfrac{\sqrt{(10-x)^2+(y-2)^2}}{\sqrt{x^2+y^2}} = \dfrac{10-x}{x}$, Eq. [2] becomes

$$\sqrt{x^2+y^2}\left(\frac{10-x}{x}\right) + \sqrt{x^2+y^2} = 15 \qquad [3]$$

Dividing both sides of Eq. [3] by $\sqrt{x^2+y^2}$ yields

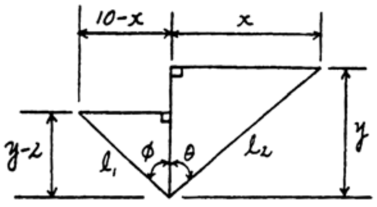
$$\frac{10}{x} = \frac{15}{\sqrt{x^2+y^2}} \qquad x = \sqrt{0.8}y \qquad [4]$$

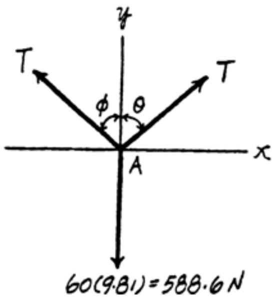
From Eq. [1] $\dfrac{10-x}{x} = \dfrac{y-2}{y} \qquad x = \dfrac{5y}{y-1} \qquad [5]$

Equating Eq. [4] and [5] yields

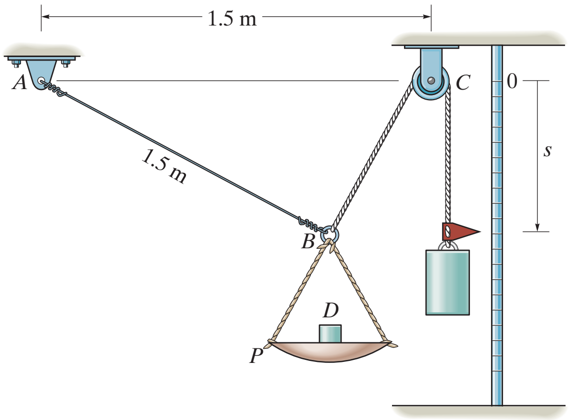
$$\sqrt{0.8}y = \frac{5y}{y-1}$$

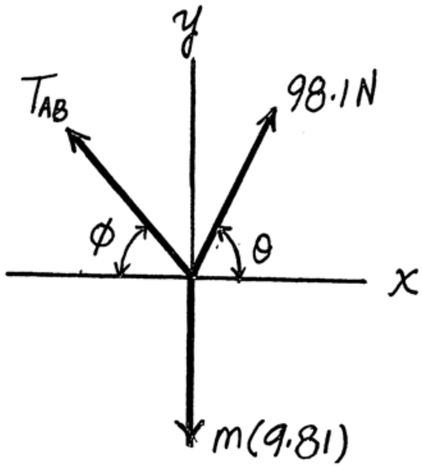
$$y = 6.59 \text{ m} \qquad \textbf{Ans}$$

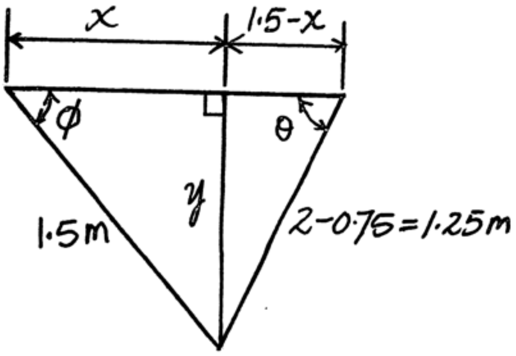




•***3–44.** A scale is constructed using the 10-kg mass, the 2-kg pan *P*, and the pulley and cord arrangement. Cord *BCA* is 2 m long. If $s = 0.75$ m, determine the mass *D* in the pan. Neglect the size of the pulley.







$\xrightarrow{+} \Sigma F_x = 0;\quad 98.1\cos\theta - T_{AB}\cos\phi = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0;\quad T_{AB}\sin\phi + 98.1\sin\theta - m(9.81) = 0 \qquad (2)$

$$(1.5)^2 = x^2 + y^2$$

$$(1.25)^2 = (1.5 - x)^2 + y^2$$

$$(1.25)^2 = (1.5 - x)^2 + (1.5)^2 - x^2$$

$$-3x + 2.9375 = 0$$

$$x = 0.9792 \text{ m}$$

$$y = 1.1363 \text{ m}$$

Thus,

$$\phi = \sin^{-1}\left(\frac{1.1363}{1.5}\right) = 49.25^\circ$$

$$\theta = \sin^{-1}\left(\frac{1.1363}{1.25}\right) = 65.38^\circ$$

Solving Eq. (1) and (2),

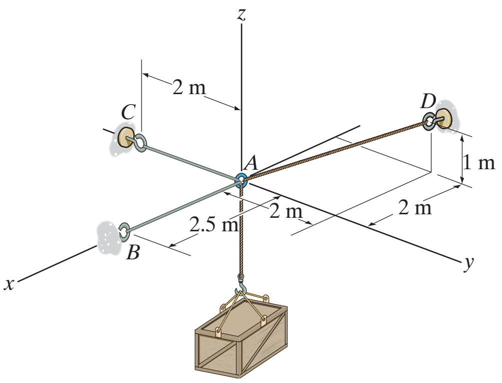
$$T_{AB} = 62.62 \text{ N}$$

$$m = 13.9 \text{ kg}$$

Therefore,

$$m_D = 13.9 \text{ kg} - 2 \text{ kg} = 11.9 \text{ kg} \qquad \text{Ans}$$

•3–45. Determine the tension in the cables in order to support the 100-kg crate in the equilibrium position shown.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{i}$$

$$\mathbf{F}_{AC} = -F_{AC}\,\mathbf{j}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}$$

$$\mathbf{W} = [-100(9.81)\mathbf{k}]\,\mathrm{N} = [-981\,\mathbf{k}]\,\mathrm{N}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{i} + (-F_{AC}\,\mathbf{j}) + \left(-\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}\right) + (-981\mathbf{k}) = \mathbf{0}$$

$$\left(F_{AB}-\frac{2}{3}F_{AD}\right)\mathbf{i}+\left(-F_{AC}+\frac{2}{3}F_{AD}\right)\mathbf{j}+\left(\frac{1}{3}F_{AD}-981\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$F_{AB}-\frac{2}{3}F_{AD} = 0 \qquad (1)$$

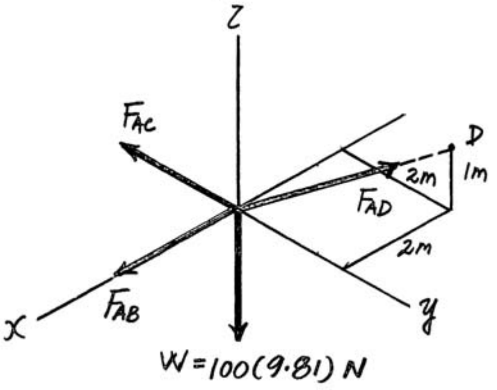
$$-F_{AC}+\frac{2}{3}F_{AD} = 0 \qquad (2)$$

$$\frac{1}{3}F_{AD}-981 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

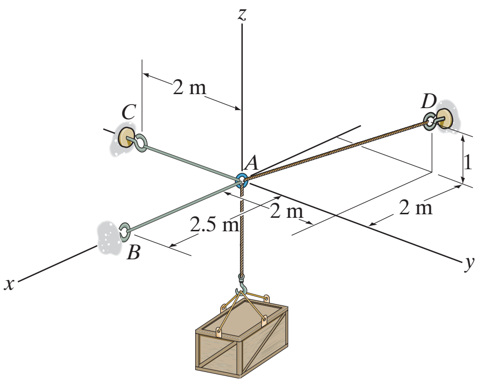
$F_{AD} = 2943\ \mathrm{N} = 2.94\ \mathrm{kN}$ **Ans.**

$F_{AB} = F_{AC} = 1962\ \mathrm{N} = 1.96\ \mathrm{kN}$ **Ans.**



(a)

3–46. Determine the maximum mass of the crate so that the tension developed in any cable does not exceeded 3 kN.



Force Vectors: We can express each of the forces on the free - body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{i}$$

$$\mathbf{F}_{AC} = -F_{AC}\,\mathbf{j}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}$$

$$\mathbf{W} = [-m(9.81)\mathbf{k}]$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{i}+(-F_{AC}\,\mathbf{j})+\left(-\frac{2}{3}F_{AD}\mathbf{i}+\frac{2}{3}F_{AD}\mathbf{j}+\frac{1}{3}F_{AD}\mathbf{k}\right)+[-m(9.81)\mathbf{k}] = \mathbf{0}$$

$$\left(F_{AB}-\frac{2}{3}F_{AD}\right)\mathbf{i}+\left(-F_{AC}+\frac{2}{3}F_{AD}\right)\mathbf{j}+\left(\frac{1}{3}F_{AD}-9.81m\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$F_{AB}-\frac{2}{3}F_{AD} = 0 \qquad (1)$$

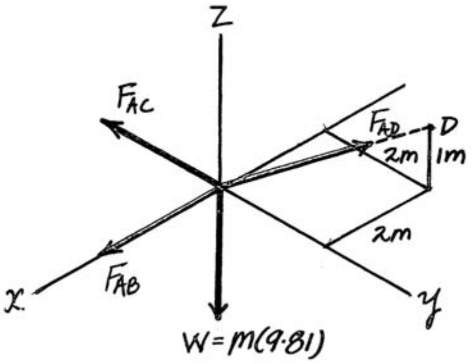
$$-F_{AC}+\frac{2}{3}F_{AD} = 0 \qquad (2)$$

$$\frac{1}{3}F_{AD}-9.81m = 0 \qquad (3)$$

When cable *AD* is subjected to maximum tension, $F_{AD} = 3000$ N. Thus, by substituting this value into Eqs. (1) through (3), we have

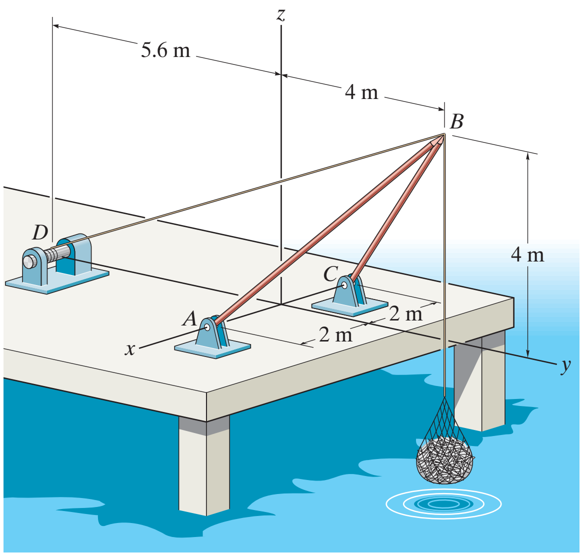
$$F_{AB} = F_{AC} = 2000\text{ N}$$

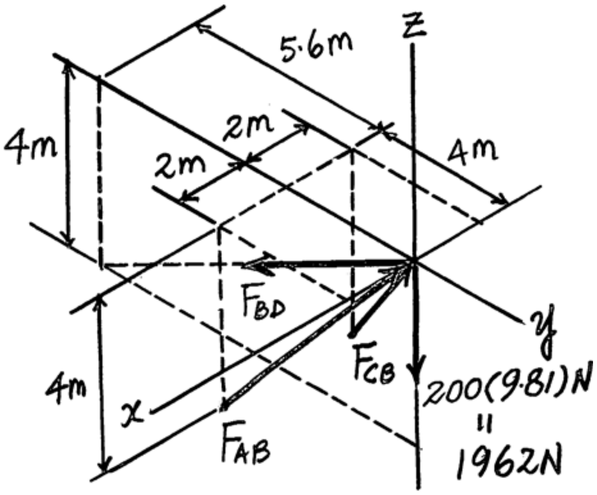
$$m = 102\text{ kg} \qquad \textbf{Ans.}$$



(a)

3–47. The shear leg derrick is used to haul the 200-kg net of fish onto the dock. Determine the compressive force along each of the legs *AB* and *CB* and the tension in the winch cable *DB*. Assume the force in each leg acts along its axis.





$$\mathbf{F}_{AB} = F_{AB}\left(-\frac{2}{6}\mathbf{i} + \frac{4}{6}\mathbf{j} + \frac{4}{6}\mathbf{k}\right)$$

$$= -0.3333\,F_{AB}\,\mathbf{i} + 0.6667\,F_{AB}\,\mathbf{j} + 0.6667\,F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{CB} = F_{CB}\left(\frac{2}{6}\mathbf{i} + \frac{4}{6}\mathbf{j} + \frac{4}{6}\mathbf{k}\right)$$

$$= 0.3333\,F_{CB}\,\mathbf{i} + 0.6667\,F_{CB}\,\mathbf{j} + 0.6667\,F_{CB}\,\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}\left(-\frac{9.6}{10.4}\mathbf{j} - \frac{4}{10.4}\mathbf{k}\right)$$

$$= -0.9231\,F_{BD}\,\mathbf{j} - 0.3846\,F_{BD}\,\mathbf{k}$$

$$\mathbf{W} = -1962\,\mathbf{k}$$

$\Sigma F_x = 0;$ $-0.3333\,F_{AB} + 0.3333\,F_{CB} = 0$

$\Sigma F_y = 0;$ $0.6667\,F_{AB} + 0.6667\,F_{CB} - 0.9231\,F_{BD} = 0$

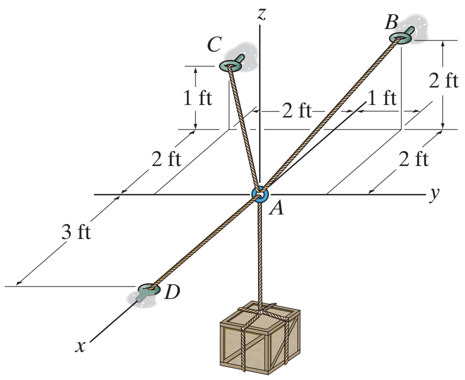
$\Sigma F_z = 0;$ $0.6667\,F_{AB} + 0.6667\,F_{CB} - 0.3846\,F_{BD} - 1962 = 0$

$F_{AB} = 2.52$ kN **Ans**

$F_{CB} = 2.52$ kN **Ans**

$F_{BD} = 3.64$ kN **Ans**

***3–48.** Determine the tension developed in cables *AB*, *AC*, and *AD* required for equilibrium of the 300-lb crate.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i}+(1-0)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(-2-0)^2+(1-0)^2+(2-0)^2}}\right] = -\frac{2}{3}F_{AB}\mathbf{i}+\frac{1}{3}F_{AB}\mathbf{j}+\frac{2}{3}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-2-0)\mathbf{i}+(-2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(-2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{1}{3}F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\mathbf{i}$$

$$\mathbf{W} = [-300\mathbf{k}]\text{ lb}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}\mathbf{i}+\frac{1}{3}F_{AB}\mathbf{j}+\frac{2}{3}F_{AB}\mathbf{k}\right)+\left(-\frac{2}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{1}{3}F_{AC}\mathbf{k}\right)+F_{AD}\mathbf{i}+(-300\mathbf{k}) = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}-\frac{2}{3}F_{AC}+F_{AD}\right)\mathbf{i}+\left(\frac{1}{3}F_{AB}-\frac{2}{3}F_{AC}\right)\mathbf{j}+\left(\frac{2}{3}F_{AB}+\frac{1}{3}F_{AC}-300\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{2}{3}F_{AB}-\frac{2}{3}F_{AC}+F_{AD} = 0 \qquad (1)$$

$$\frac{1}{3}F_{AB}-\frac{2}{3}F_{AC} = 0 \qquad (2)$$

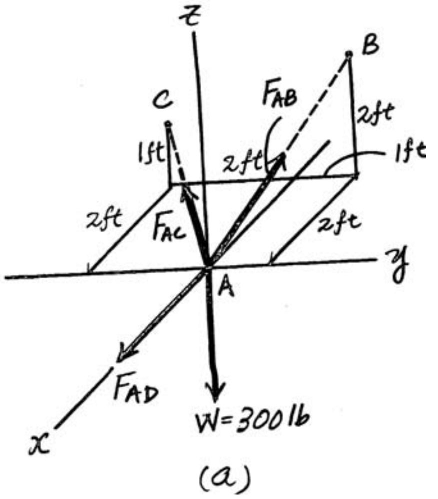
$$\frac{2}{3}F_{AB}+\frac{1}{3}F_{AC}-300 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 360$ lb **Ans.**

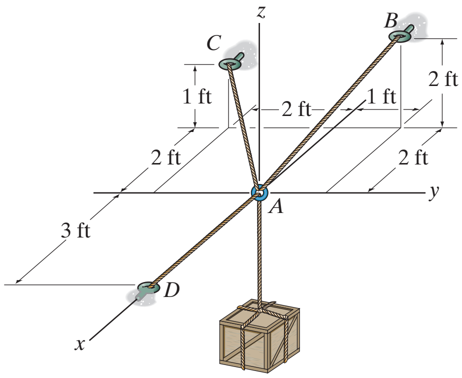
$F_{AC} = 180$ lb **Ans.**

$F_{AD} = 360$ lb **Ans.**



(a)

•3–49. Determine the maximum weight of the crate so that the tension developed in any cable does not exceed 450 lb.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i}+(1-0)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(-2-0)^2+(1-0)^2+(2-0)^2}}\right] = -\frac{2}{3}F_{AB}\mathbf{i}+\frac{1}{3}F_{AB}\mathbf{j}+\frac{2}{3}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-2-0)\mathbf{i}+(-2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(-2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{1}{3}F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\mathbf{i}$$

$$\mathbf{W} = -W\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}\mathbf{i}+\frac{1}{3}F_{AB}\mathbf{j}+\frac{2}{3}F_{AB}\mathbf{k}\right)+\left(-\frac{2}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{1}{3}F_{AC}\mathbf{k}\right)+F_{AD}\mathbf{i}+(-W\mathbf{k}) = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}-\frac{2}{3}F_{AC}+F_{AD}\right)\mathbf{i}+\left(\frac{1}{3}F_{AB}-\frac{2}{3}F_{AC}\right)\mathbf{j}+\left(\frac{2}{3}F_{AB}+\frac{1}{3}F_{AC}-W\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{2}{3}F_{AB}-\frac{2}{3}F_{AC}+F_{AD} = 0 \qquad (1)$$

$$\frac{1}{3}F_{AB}-\frac{2}{3}F_{AC} = 0 \qquad (2)$$

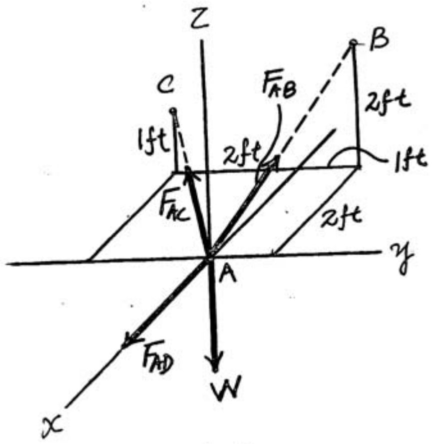
$$\frac{2}{3}F_{AB}+\frac{1}{3}F_{AC}-W = 0 \qquad (3)$$

Let us assume that cable *AB* achieves maximum tension first. Substituting $F_{AB} = 450$ lb into Eqs. (1) through (3) and solving, yields

$F_{AC} = 225$ lb $\qquad$ $F_{AD} = 450$ lb

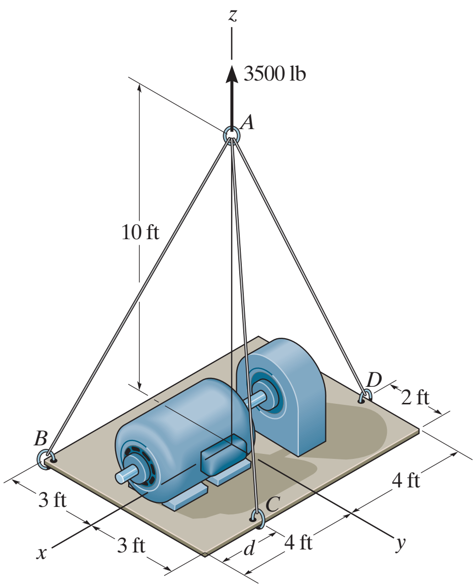
$W = 375$ lb $\qquad$ **Ans.**

Since $F_{AC} = 225 \text{ lb} < 450 \text{ lb}$, our assumption is correct.



(a)

3–50. Determine the force in each cable needed to support the 3500-lb platform. Set $d = 2$ ft.



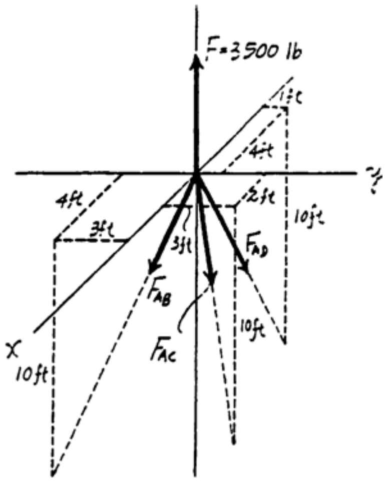
Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{4\mathbf{i} - 3\mathbf{j} - 10\mathbf{k}}{\sqrt{4^2 + (-3)^2 + (-10)^2}}\right) = 0.3578F_{AB}\mathbf{i} - 0.2683F_{AB}\mathbf{j} - 0.8944F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{2\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}}{\sqrt{2^2 + 3^2 + (-10)^2}}\right) = 0.1881F_{AC}\mathbf{i} + 0.2822F_{AC}\mathbf{j} - 0.9407F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-4\mathbf{i} + 1\mathbf{j} - 10\mathbf{k}}{\sqrt{(-4)^2 + 1^2 + (-10)^2}}\right) = -0.3698F_{AD}\mathbf{i} + 0.09245F_{AD}\mathbf{j} - 0.9245F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{3500\mathbf{k}\}\ \text{lb}$$



Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$(0.3578F_{AB} + 0.1881F_{AC} - 0.3698F_{AD})\mathbf{i} + (-0.2683F_{AB} + 0.2822F_{AC} + 0.09245F_{AD})\mathbf{j}$$
$$+(-0.8944F_{AB} - 0.9407F_{AC} - 0.9245F_{AD} + 3500)\mathbf{k} = \mathbf{0}$$

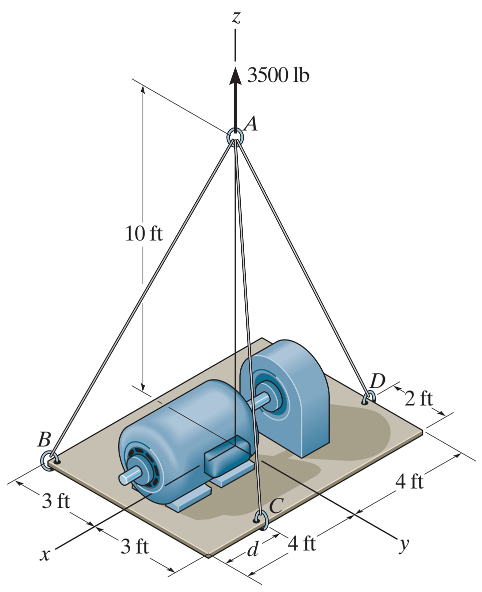
Equating **i**, **j** and **k** components, we have

$$0.3578F_{AB} + 0.1881F_{AC} - 0.3698F_{AD} = 0 \qquad [1]$$
$$-0.2683F_{AB} + 0.2822F_{AC} + 0.09245F_{AD} = 0 \qquad [2]$$
$$-0.8944F_{AB} - 0.9407F_{AC} - 0.9245F_{AD} + 3500 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$F_{AB} = 1369.59\ \text{lb} = 1.37\ \text{kip}$ $\qquad F_{AC} = 744.11\ \text{lb} = 0.744\ \text{kip}$ **Ans**
$F_{AD} = 1703.62\ \text{lb} = 1.70\ \text{kip}$ **Ans**

3–51. Determine the force in each cable needed to support the 3500-lb platform. Set $d = 4$ ft.



Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{4\mathbf{i} - 3\mathbf{j} - 10\mathbf{k}}{\sqrt{4^2 + (-3)^2 + (-10)^2}}\right) = 0.3578F_{AB}\mathbf{i} - 0.2683F_{AB}\mathbf{j} - 0.8944F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{3\mathbf{j} - 10\mathbf{k}}{\sqrt{3^2 + (-10)^2}}\right) = 0.2873F_{AC}\mathbf{j} - 0.9578F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-4\mathbf{i} + 1\mathbf{j} - 10\mathbf{k}}{\sqrt{(-4)^2 + 1^2 + (-10)^2}}\right) = -0.3698F_{AD}\mathbf{i} + 0.09245F_{AD}\mathbf{j} - 0.9245F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{3500\mathbf{k}\}\ \text{lb}$$

Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

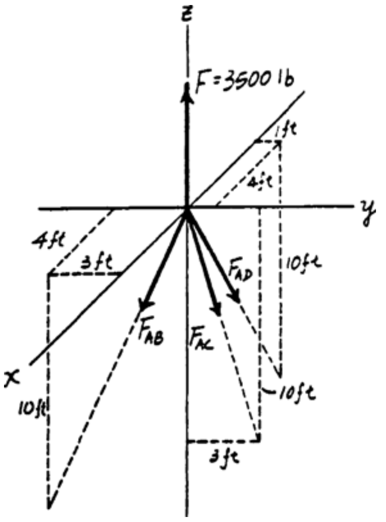
$$(0.3578F_{AB} - 0.3698F_{AD})\mathbf{i} + (-0.2683F_{AB} + 0.2873F_{AC} + 0.09245F_{AD})\mathbf{j}$$
$$+ (-0.8944F_{AB} - 0.9578F_{AC} - 0.9245F_{AD} + 3500)\,\mathbf{k} = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

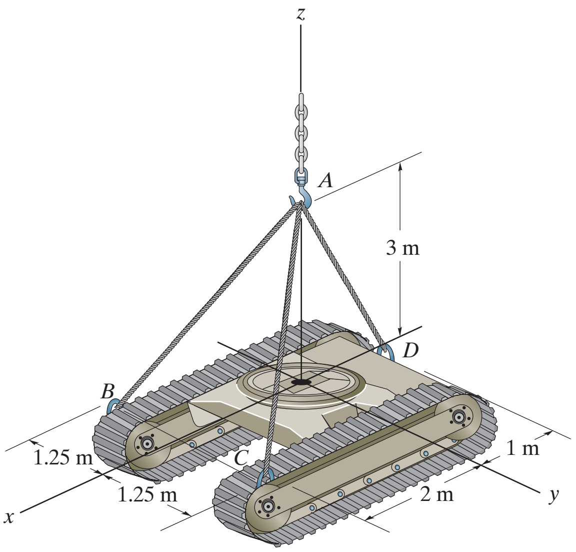
$$0.3578F_{AB} - 0.3698F_{AD} = 0 \qquad [1]$$
$$-0.2683F_{AB} + 0.2873F_{AC} + 0.09245F_{AD} = 0 \qquad [2]$$
$$-0.8944F_{AB} - 0.9578F_{AC} - 0.9245F_{AD} + 3500 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$F_{AB} = 1467.42$ lb $= 1.47$ kip $F_{AC} = 913.53$ lb $= 0.914$ kip **Ans**
$F_{AD} = 1419.69$ lb $= 1.42$ kip **Ans**



***3–52.** Determine the force in each of the three cables needed to lift the tractor which has a mass of 8 Mg.



Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{2\mathbf{i} - 1.25\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + (-1.25)^2 + (-3)^2}}\right) = 0.5241F_{AB}\mathbf{i} - 0.3276F_{AB}\mathbf{j} - 0.7861F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{2\mathbf{i} + 1.25\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 1.25^2 + (-3)^2}}\right) = 0.5241F_{AC}\mathbf{i} + 0.3276F_{AC}\mathbf{j} - 0.7861F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-1\mathbf{i} - 3\mathbf{k}}{\sqrt{(-1)^2 + (-3)^2}}\right) = -0.3162F_{AD}\mathbf{i} - 0.9487F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{78.48\mathbf{k}\}\ \text{kN}$$

Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = 0$$

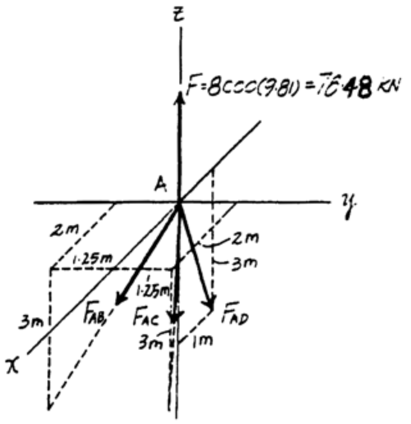
$$(0.5241F_{AB} + 0.5241F_{AC} - 0.3162F_{AD})\mathbf{i} + (-0.3276F_{AB} + 0.3276F_{AC})\mathbf{j}$$
$$+ (-0.7861F_{AB} - 0.7861F_{AC} - 0.9487F_{AD} + 78.48)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components, we have

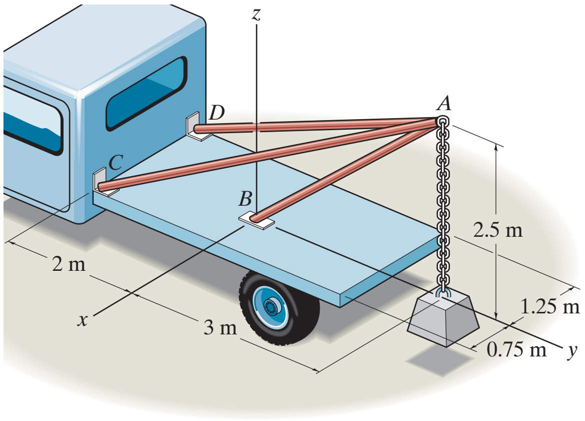
$$0.5241F_{AB} + 0.5241F_{AC} - 0.3162F_{AD} = 0 \qquad [1]$$
$$-0.3276F_{AB} + 0.3276F_{AC} = 0 \qquad [2]$$
$$-0.7861F_{AB} - 0.7861F_{AC} - 0.9487F_{AD} + 78.48 = 0 \qquad [3]$$

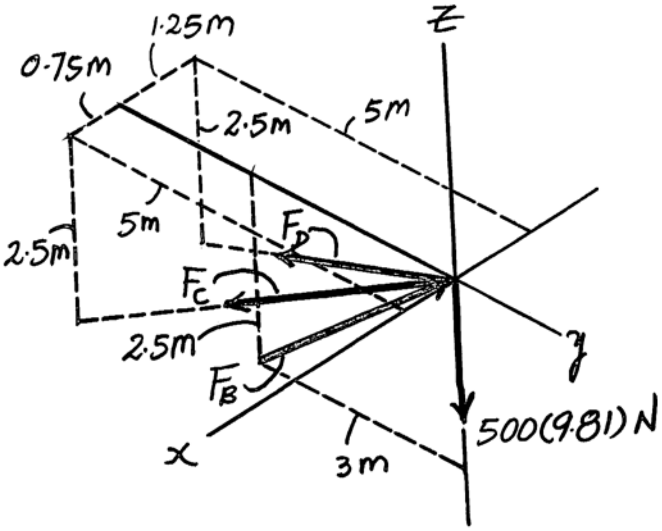
Solving Eqs. [1], [2] and [3] yields

$$F_{AB} = F_{AC} = 16.6\ \text{kN} \qquad F_{AD} = 55.2\ \text{kN} \qquad \textbf{Ans}$$



•3–53. Determine the force acting along the axis of each of the three struts needed to support the 500-kg block.





$$\mathbf{F}_B = F_B\left(\frac{3\,\mathbf{j} + 2.5\,\mathbf{k}}{3.905}\right)$$

$$= 0.7682\,F_B\,\mathbf{j} + 0.6402\,F_B\,\mathbf{k}$$

$$\mathbf{F}_C = F_C\left(\frac{0.75\,\mathbf{i} - 5\,\mathbf{j} - 2.5\,\mathbf{k}}{5.640}\right)$$

$$= 0.1330\,F_C\,\mathbf{i} - 0.8865\,F_C\,\mathbf{j} - 0.4432\,F_C\,\mathbf{k}$$

$$\mathbf{F}_D = F_D\left(\frac{-1.25\,\mathbf{i} - 5\,\mathbf{j} - 2.5\,\mathbf{k}}{5.728}\right)$$

$$= -0.2182\,F_D\,\mathbf{i} - 0.8729\,F_D\,\mathbf{j} - 0.4364\,F_D\,\mathbf{k}$$

$$\mathbf{W} = -500(9.81)\,\mathbf{k} = -4905\,\mathbf{k}$$

$\Sigma \mathbf{F} = 0;\quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = 0$

$\Sigma F_x = 0;\quad 0.1330\,F_C - 0.2182\,F_D = 0$

$\Sigma F_y = 0;\quad 0.7682\,F_B - 0.8865\,F_C - 0.8729\,F_D = 0$

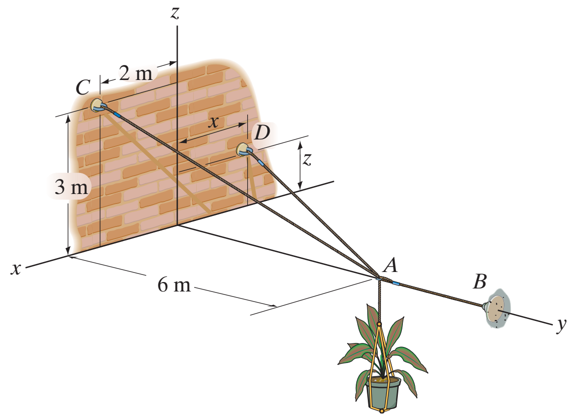
$\Sigma F_z = 0;\quad 0.6402\,F_B - 0.4432\,F_C - 0.4364\,F_D - 4905 = 0$

$F_B = 19.2\text{ kN}$ **Ans**

$F_C = 10.4\text{ kN}$ **Ans**

$F_D = 6.32\text{ kN}$ **Ans**

3–54. If the mass of the flowerpot is 50 kg, determine the tension developed in each wire for equilibrium. Set $x = 1.5$ m and $z = 2$ m.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{j}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i}+(-6-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(2-0)^2+(-6-0)^2+(3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i}-\frac{6}{7}F_{AC}\,\mathbf{j}+\frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-1.5-0)\mathbf{i}+(-6-0)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(-1.5-0)^2+(-6-0)^2+(2-0)^2}}\right] = -\frac{3}{13}F_{AD}\,\mathbf{i}-\frac{12}{13}F_{AD}\,\mathbf{j}+\frac{4}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = [-50(9.81)\mathbf{k}]\,\mathrm{N} = [-490.5\,\mathbf{k}]\,\mathrm{N}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{j}+\left(\frac{2}{7}F_{AC}\,\mathbf{i}-\frac{6}{7}F_{AC}\,\mathbf{j}+\frac{3}{7}F_{AC}\,\mathbf{k}\right)+\left(-\frac{3}{13}F_{AD}\,\mathbf{i}-\frac{12}{13}F_{AD}\,\mathbf{j}+\frac{4}{13}F_{AD}\,\mathbf{k}\right)+(-490.5\mathbf{k}) = \mathbf{0}$$

$$\left(\frac{2}{7}F_{AC}-\frac{3}{13}F_{AD}\right)\mathbf{i}+\left(F_{AB}-\frac{6}{7}F_{AC}-\frac{12}{13}F_{AD}\right)\mathbf{j}+\left(\frac{3}{7}F_{AC}+\frac{4}{13}F_{AD}-490.5\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$\frac{2}{7}F_{AC}-\frac{3}{13}F_{AD} = 0 \qquad (1)$$

$$F_{AB}-\frac{6}{7}F_{AC}-\frac{12}{13}F_{AD} = 0 \qquad (2)$$

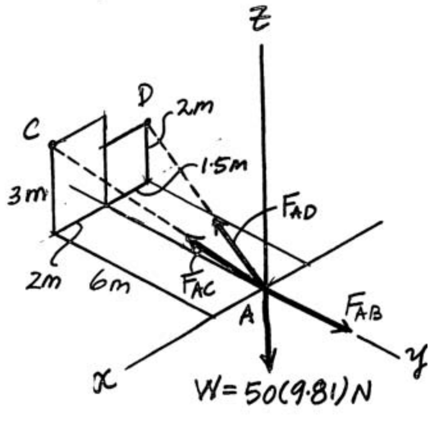
$$\frac{3}{7}F_{AC}+\frac{4}{13}F_{AD}-490.5 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 1211.82\text{ N} = 1.21\text{ kN}$ **Ans.**

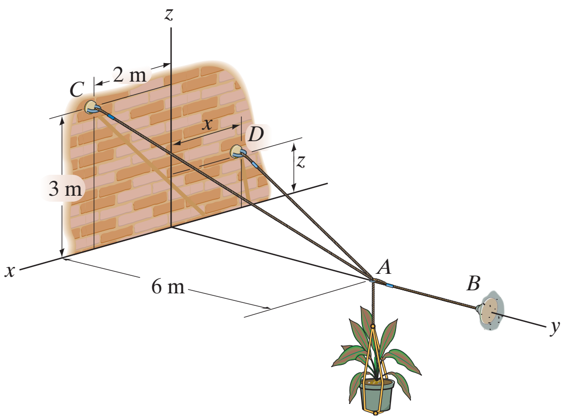
$F_{AC} = 606\text{ N}$ **Ans.**

$F_{AD} = 750\text{ N}$ **Ans.**



(*a*)

3–55. If the mass of the flowerpot is 50 kg, determine the tension developed in each wire for equilibrium. Set $x = 2$ m and $z = 1.5$ m.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{j}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i}+(-6-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(2-0)^2+(-6-0)^2+(3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i}-\frac{6}{7}F_{AC}\,\mathbf{j}+\frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(-6-0)\mathbf{j}+(1.5-0)\mathbf{k}}{\sqrt{(-2-0)^2+(-6-0)^2+(1.5-0)^2}}\right] = -\frac{4}{13}F_{AD}\,\mathbf{i}-\frac{12}{13}F_{AD}\,\mathbf{j}+\frac{3}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = [-50(9.81)\mathbf{k}]\,\mathrm{N} = [-490.5\mathbf{k}]\,\mathrm{N}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{j}+\left(\frac{2}{7}F_{AC}\,\mathbf{i}-\frac{6}{7}F_{AC}\,\mathbf{j}+\frac{3}{7}F_{AC}\,\mathbf{k}\right)+\left(-\frac{4}{13}F_{AD}\,\mathbf{i}-\frac{12}{13}F_{AD}\,\mathbf{j}+\frac{3}{13}F_{AD}\,\mathbf{k}\right)+(-490.5\mathbf{k}) = \mathbf{0}$$

$$\left(\frac{2}{7}F_{AC}-\frac{4}{13}F_{AD}\right)\mathbf{i}+\left(F_{AB}-\frac{6}{7}F_{AC}-\frac{12}{13}F_{AD}\right)\mathbf{j}+\left(\frac{3}{7}F_{AC}+\frac{3}{13}F_{AD}-490.5\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$\frac{2}{7}F_{AC}-\frac{4}{13}F_{AD} = 0 \qquad (1)$$

$$F_{AB}-\frac{6}{7}F_{AC}-\frac{12}{13}F_{AD} = 0 \qquad (2)$$

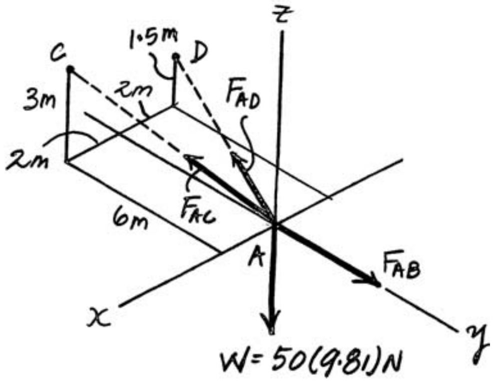
$$\frac{3}{7}F_{AC}+\frac{3}{13}F_{AD}-490.5 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 1308\ \mathrm{N} = 1.31\ \mathrm{kN}$ **Ans.**

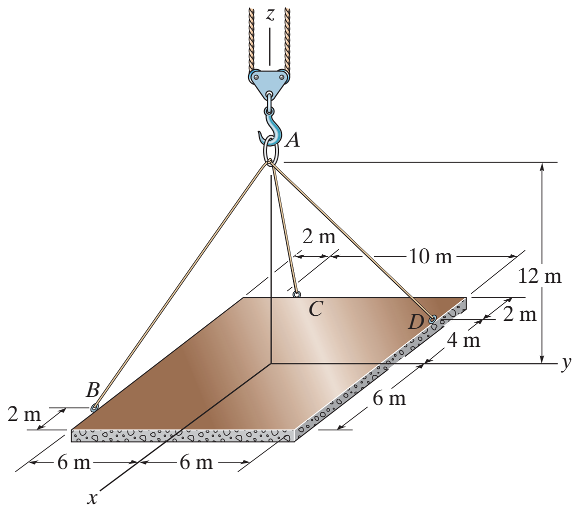
$F_{AC} = 763\ \mathrm{N}$ **Ans.**

$F_{AD} = 708.5\ \mathrm{N}$ **Ans.**



(a)

***3–56.** The ends of the three cables are attached to a ring at *A* and to the edge of a uniform 150-kg plate. Determine the tension in each of the cables for equilibrium.

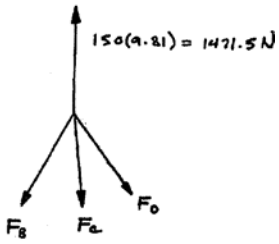


$$\mathbf{P} = 150(9.81)\mathbf{k} = 1471.5\,\mathbf{k}$$

$$\mathbf{F}_B = \frac{4}{14}F_B\mathbf{i} - \frac{6}{14}F_B\mathbf{j} - \frac{12}{14}F_B\mathbf{k}$$

$$\mathbf{F}_C = -\frac{6}{14}F_C\mathbf{i} - \frac{4}{14}F_C\mathbf{j} - \frac{12}{14}F_C\mathbf{k}$$

$$\mathbf{F}_D = -\frac{4}{14}F_D\mathbf{i} + \frac{6}{14}F_D\mathbf{j} - \frac{12}{14}F_D\mathbf{k}$$



$$\Sigma F_x = 0; \quad \frac{4}{14}F_B - \frac{6}{14}F_C - \frac{4}{14}F_D = 0$$

$$\Sigma F_y = 0; \quad -\frac{6}{14}F_B - \frac{4}{14}F_C + \frac{6}{14}F_D = 0$$

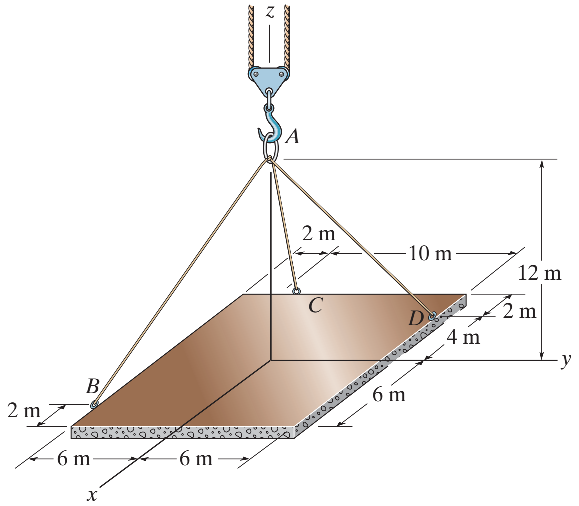
$$\Sigma F_z = 0; \quad -\frac{12}{14}F_B - \frac{12}{14}F_C - \frac{12}{14}F_D + 1471.5 = 0$$

$F_B = 858$ N **Ans**

$F_C = 0$ **Ans**

$F_D = 858$ N **Ans**

•3–57. The ends of the three cables are attached to a ring at *A* and to the edge of the uniform plate. Determine the largest mass the plate can have if each cable can support a maximum tension of 15 kN.

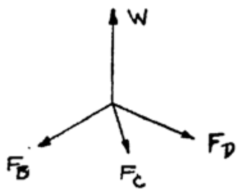


$$\mathbf{W} = W\mathbf{k}$$

$$\mathbf{F}_B = F_B\left(\frac{4}{14}\mathbf{i} - \frac{6}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\left(-\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\left(-\frac{4}{14}\mathbf{i} + \frac{6}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$



$$\Sigma F_x = 0; \quad \frac{4}{14}F_B - \frac{6}{14}F_C - \frac{4}{14}F_D = 0$$

$$\Sigma F_y = 0; \quad -\frac{6}{14}F_B - \frac{4}{14}F_C + \frac{6}{14}F_D = 0$$

$$\Sigma F_z = 0; \quad -\frac{12}{14}F_B - \frac{12}{14}F_C - \frac{12}{14}F_D + W = 0$$

Assume $F_B = 15$ kN. Solving,

$F_C = 0 < 15$ kN (OK)

$F_D = 15$ kN (OK)

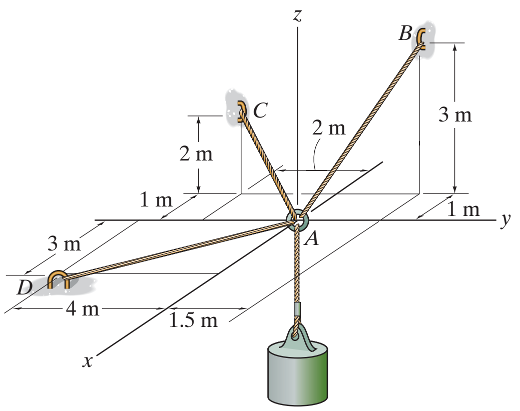
Thus,

$$-\frac{12}{14}(15) - 0 - \frac{12}{14}(15) + W = 0$$

$W = 25.714$ kN

$$m = \frac{W}{g} = \frac{25.714}{9.81} = 2.62 \text{ Mg}$$ **Ans**

3–58. Determine the tension developed in cables AB, AC, and AD required for equilibrium of the 75-kg cylinder.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-1-0)\mathbf{i}+(1.5-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-1-0)^2+(1.5-0)^2+(3-0)^2}}\right] = -\frac{2}{7}F_{AB}\mathbf{i}+\frac{3}{7}F_{AB}\mathbf{j}+\frac{6}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-1-0)\mathbf{i}+(-2-0)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(-1-0)^2+(-2-0)^2+(2-0)^2}}\right] = -\frac{1}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{2}{3}F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(3-0)\mathbf{i}+(-4-0)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(3-0)^2+(-4-0)^2+(0-0)^2}}\right] = \frac{3}{5}F_{AD}\mathbf{i}-\frac{4}{5}F_{AD}\mathbf{j}$$

$$\mathbf{W} = [-75(9.81)\mathbf{k}]\text{N} = [-735.75\mathbf{k}]\text{N}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{7}F_{AB}\mathbf{i}+\frac{3}{7}F_{AB}\mathbf{j}+\frac{6}{7}F_{AB}\mathbf{k}\right)+\left(-\frac{1}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{2}{3}F_{AC}\mathbf{k}\right)+\left(\frac{3}{5}F_{AD}\mathbf{i}-\frac{4}{5}F_{AD}\mathbf{j}\right)+(-735.75\mathbf{k}) = \mathbf{0}$$

$$\left(-\frac{2}{7}F_{AB}-\frac{1}{3}F_{AC}+\frac{3}{5}F_{AD}\right)\mathbf{i}+\left(\frac{3}{7}F_{AB}-\frac{2}{3}F_{AC}-\frac{4}{5}F_{AD}\right)\mathbf{j}+\left(\frac{6}{7}F_{AB}+\frac{2}{3}F_{AC}-735.75\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{2}{7}F_{AB}-\frac{1}{3}F_{AC}+\frac{3}{5}F_{AD} = 0 \quad (1)$$

$$\frac{3}{7}F_{AB}-\frac{2}{3}F_{AC}-\frac{4}{5}F_{AD} = 0 \quad (2)$$

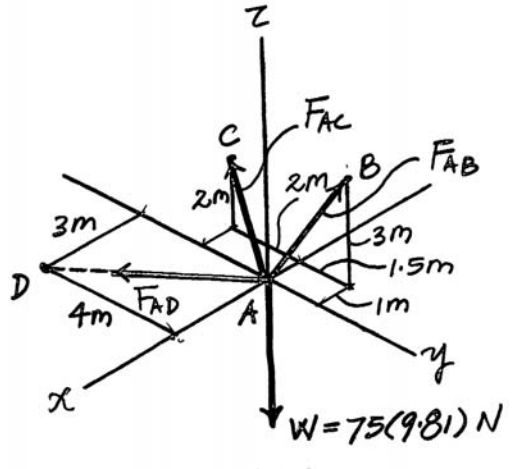
$$\frac{6}{7}F_{AB}+\frac{2}{3}F_{AC}-735.75 = 0 \quad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 831\text{ N}$ **Ans.**

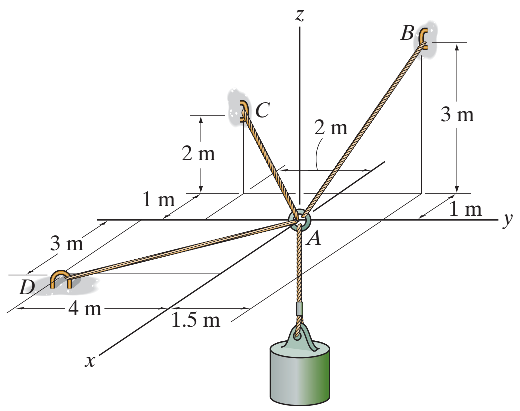
$F_{AC} = 35.6\text{ N}$ **Ans.**

$F_{AD} = 415\text{ N}$ **Ans.**



(a)

3–59. If each cable can withstand a maximum tension of 1000 N, determine the largest mass of the cylinder for equilibrium.



Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-1-0)\mathbf{i}+(1.5-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-1-0)^2+(1.5-0)^2+(3-0)^2}}\right] = -\frac{2}{7}F_{AB}\,\mathbf{i}+\frac{3}{7}F_{AB}\,\mathbf{j}+\frac{6}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-1-0)\mathbf{i}+(-2-0)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(-1-0)^2+(-2-0)^2+(2-0)^2}}\right] = -\frac{1}{3}F_{AC}\,\mathbf{i}-\frac{2}{3}F_{AC}\,\mathbf{j}+\frac{2}{3}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(3-0)\mathbf{i}+(-4-0)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(3-0)^2+(-4-0)^2+(0-0)^2}}\right] = \frac{3}{5}F_{AD}\,\mathbf{i}-\frac{4}{5}F_{AD}\,\mathbf{j}$$

$$\mathbf{W} = -m(9.81)\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AD}+\mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{7}F_{AB}\mathbf{i}+\frac{3}{7}F_{AB}\mathbf{j}+\frac{6}{7}F_{AB}\mathbf{k}\right)+\left(-\frac{1}{3}F_{AC}\mathbf{i}-\frac{2}{3}F_{AC}\mathbf{j}+\frac{2}{3}F_{AC}\mathbf{k}\right)+\left(\frac{3}{5}F_{AD}\mathbf{i}-\frac{4}{5}F_{AD}\mathbf{j}\right)+[-m(9.81)\mathbf{k}] = \mathbf{0}$$

$$\left(-\frac{2}{7}F_{AB}-\frac{1}{3}F_{AC}+\frac{3}{5}F_{AD}\right)\mathbf{i}+\left(\frac{3}{7}F_{AB}-\frac{2}{3}F_{AC}-\frac{4}{5}F_{AD}\right)\mathbf{j}+\left(\frac{6}{7}F_{AB}+\frac{2}{3}F_{AC}-m(9.81)\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{2}{7}F_{AB}-\frac{1}{3}F_{AC}+\frac{3}{5}F_{AD} = 0 \qquad (1)$$

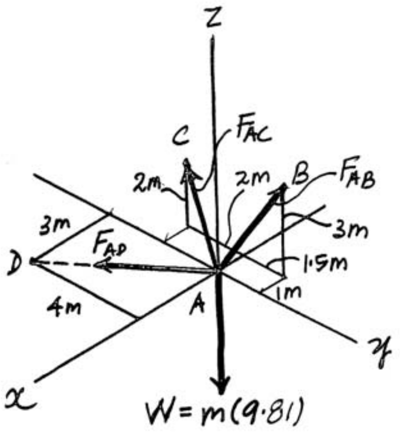
$$\frac{3}{7}F_{AB}-\frac{2}{3}F_{AC}-\frac{4}{5}F_{AD} = 0 \qquad (2)$$

$$\frac{6}{7}F_{AB}+\frac{2}{3}F_{AC}-m(9.81) = 0 \qquad (3)$$

Let us assume that cable *AB* achieves maximum tension first. Substituting $F_{AB} = 1000$ N into Eqs. (1) through (3) and solving, yields

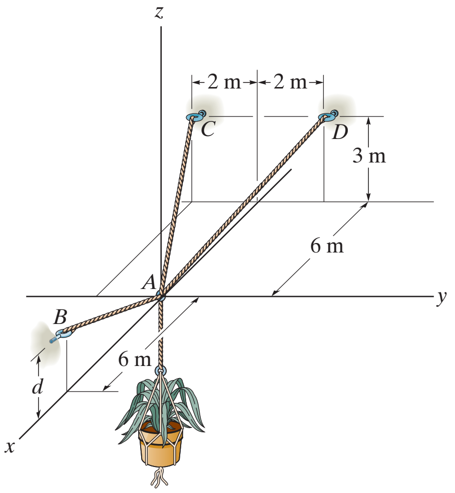
$F_{AD} = 500$ N $\qquad F_{AC} = 42.86$ N

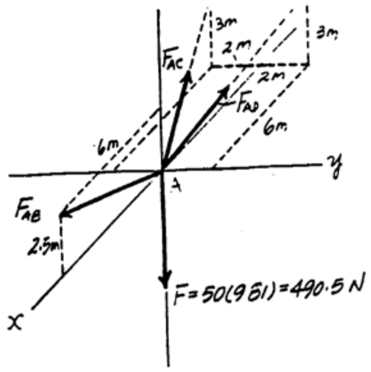
$m = 90.3$ kg **Ans.**



(a)

*__3–60.__ The 50-kg pot is supported from A by the three cables. Determine the force acting in each cable for equilibrium. Take $d = 2.5$ m.





Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{6\mathbf{i} + 2.5\mathbf{k}}{\sqrt{6^2 + 2.5^2}}\right) = \frac{12}{13}F_{AB}\mathbf{i} + \frac{5}{13}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{-6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + (-2)^2 + 3^2}}\right) = -\frac{6}{7}F_{AC}\mathbf{i} - \frac{2}{7}F_{AC}\mathbf{j} + \frac{3}{7}F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + 2^2 + 3^2}}\right) = -\frac{6}{7}F_{AD}\mathbf{i} + \frac{2}{7}F_{AD}\mathbf{j} + \frac{3}{7}F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{-490.5\mathbf{k}\}\ \text{N}$$

Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$\left(\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{6}{7}F_{AD}\right)\mathbf{i} + \left(-\frac{2}{7}F_{AC} + \frac{2}{7}F_{AD}\right)\mathbf{j} + \left(\frac{5}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{3}{7}F_{AD} - 490.5\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components, we have

$$\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{6}{7}F_{AD} = 0 \qquad [1]$$

$$-\frac{2}{7}F_{AC} + \frac{2}{7}F_{AD} = 0 \qquad [2]$$

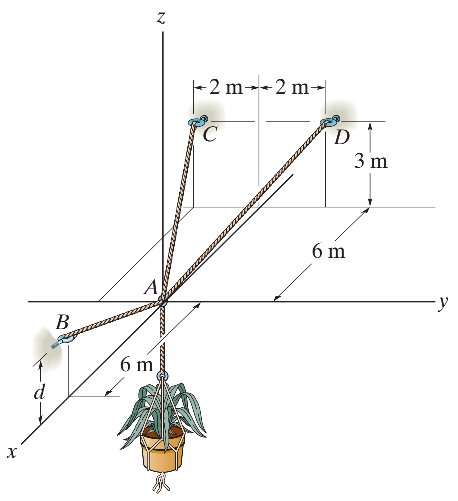
$$\frac{5}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{3}{7}F_{AD} - 490.5 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AC} = F_{AD} = 312\ \text{N}$$

$$F_{AB} = 580\ \text{N} \qquad \textbf{Ans}$$

•**3–61.** Determine the height *d* of cable *AB* so that the force in cables *AD* and *AC* is one-half as great as the force in cable *AB*. What is the force in each cable for this case? The flower pot has a mass of 50 kg.



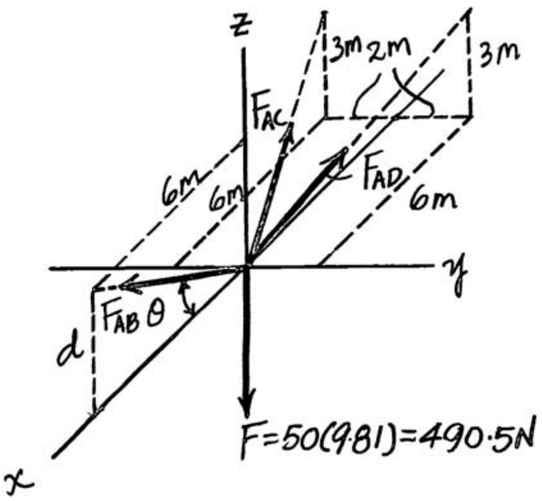
Cartesian Vector Notation :

$$\mathbf{F}_{AB} = (F_{AB})_x\,\mathbf{i} + (F_{AB})_z\,\mathbf{k}$$

$$\mathbf{F}_{AC} = \frac{F_{AB}}{2}\left(\frac{-6\mathbf{i}-2\mathbf{j}+3\mathbf{k}}{\sqrt{(-6)^2+(-2)^2+3^2}}\right) = -\frac{3}{7}F_{AB}\mathbf{i} - \frac{1}{7}F_{AB}\mathbf{j} + \frac{3}{14}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AD} = \frac{F_{AB}}{2}\left(\frac{-6\mathbf{i}+2\mathbf{j}+3\mathbf{k}}{\sqrt{(-6)^2+2^2+3^2}}\right) = -\frac{3}{7}F_{AB}\mathbf{i} + \frac{1}{7}F_{AB}\mathbf{j} + \frac{3}{14}F_{AB}\mathbf{k}$$

$$\mathbf{F} = \{-490.5\mathbf{k}\}\ \text{N}$$



Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = 0$$

$$\left((F_{AB})_x - \frac{3}{7}F_{AB} - \frac{3}{7}F_{AB}\right)\mathbf{i} + \left(-\frac{1}{7}F_{AB} + \frac{1}{7}F_{AB}\right)\mathbf{j} + \left((F_{AB})_z + \frac{3}{14}F_{AB} + \frac{3}{14}F_{AB} - 490.5\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components, we have

$$(F_{AB})_x - \frac{3}{7}F_{AB} - \frac{3}{7}F_{AB} = 0 \qquad (F_{AB})_x = \frac{6}{7}F_{AB} \qquad [1]$$

$$-\frac{1}{7}F_{AB} + \frac{1}{7}F_{AB} = 0 \qquad \textit{(Satisfied!)}$$

$$(F_{AB})_z + \frac{3}{14}F_{AB} + \frac{3}{14}F_{AB} - 490.5 = 0 \qquad (F_{AB})_z = 490.5 - \frac{3}{7}F_{AB} \qquad [2]$$

However, $F_{AB}^2 = (F_{AB})_x^2 + (F_{AB})_z^2$, then substitute Eqs. [1] and [2] into this expression yields

$$F_{AB}^2 = \left(\frac{6}{7}F_{AB}\right)^2 + \left(490.5 - \frac{3}{7}F_{AB}\right)^2$$

Solving for positive root, we have

$$F_{AB} = 519.79\ \text{N} = 520\ \text{N} \qquad \textbf{Ans}$$

Thus, $$F_{AC} = F_{AD} = \frac{1}{2}(519.79) = 260\ \text{N} \qquad \textbf{Ans}$$

Also,

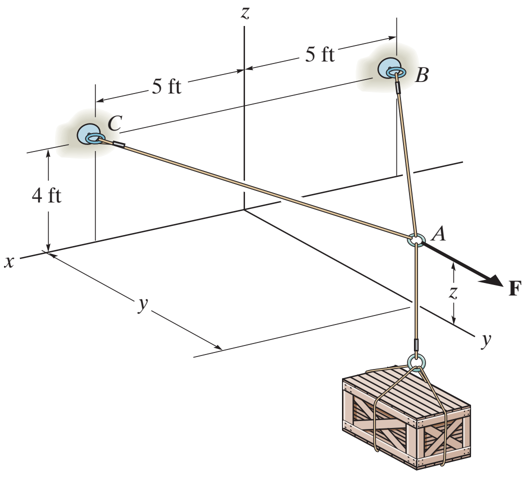
$$(F_{AB})_x = \frac{6}{7}(519.79) = 445.53\ \text{N}$$

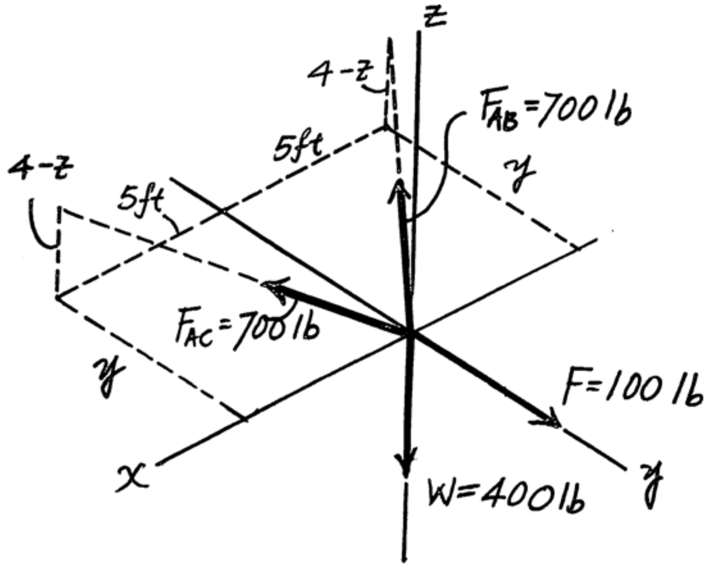
$$(F_{AB})_z = 490.5 - \frac{3}{7}(519.79) = 267.73\ \text{N}$$

then, $$\theta = \tan^{-1}\left[\frac{(F_{AB})_z}{(F_{AB})_x}\right] = \tan^{-1}\left(\frac{267.73}{445.53}\right) = 31.00^\circ$$

$$d = 6\tan\theta = 6\tan 31.00^\circ = 3.61\ \text{m} \qquad \textbf{Ans}$$

3–62. A force of $F = 100$ lb holds the 400-lb crate in equilibrium. Determine the coordinates $(0, y, z)$ of point A if the tension in cords AC and AB is 700 lb each.





$$\mathbf{F}_{AC} = 700\left(\frac{5\,\mathbf{i} - y\,\mathbf{j} + (4-z)\mathbf{k}}{\sqrt{5^2 + (-y)^2 + (4-z)^2}}\right)$$

$$\mathbf{F}_{AB} = 700\left(\frac{-5\,\mathbf{i} - y\,\mathbf{j} + (4-z)\mathbf{k}}{\sqrt{(-5)^2 + (-y)^2 + (4-z)^2}}\right)$$

$\mathbf{F} = \{100\,\mathbf{j}\}$ lb $\quad \mathbf{W} = \{-400\,\mathbf{k}\}$ lb

$$\Sigma F_x = 0; \quad \frac{3500}{\sqrt{25 + y^2 + (4-z)^2}} + \frac{-3500}{\sqrt{25 + y^2 + (4-z)^2}} = 0$$

$$\Sigma F_y = 0; \quad \frac{-700y}{\sqrt{25 + y^2 + (4-z)^2}} + \frac{-700y}{\sqrt{25 + y^2 + (4-z)^2}} + 100 = 0 \qquad (1)$$

$$\Sigma F_z = 0; \quad \frac{700(4-z)}{\sqrt{25 + y^2 + (4-z)^2}} + \frac{700(4-z)}{\sqrt{25 + y^2 + (4-z)^2}} - 400 = 0 \qquad (2)$$

$$1400\,y = 100\sqrt{25 + y^2 + (4-z)^2}$$

$$1400(4-z) = 400\sqrt{25 + y^2 + (4-z)^2}$$

$$\frac{y}{4-z} = \frac{1}{4} \qquad 4\,y = 4 - z$$

Thus,

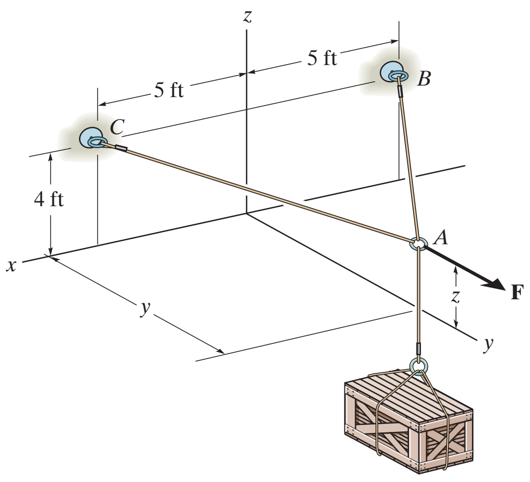
$$1400\,y = 100\sqrt{25 + y^2 + 16\,y^2}$$

$$196\,y^2 = 25 + 17\,y^2$$

$y = 0.3737$ ft $= 0.374$ ft **Ans**

$4(0.3737) = 4 - z; \quad z = 2.51$ ft **Ans**

3–63. If the maximum allowable tension in cables *AB* and *AC* is 500 lb, determine the maximum height z to which the 200-lb crate can be lifted. What horizontal force F must be applied? Take $y = 8$ ft.



$\Sigma F_y = 0;\quad -2\left[500\left(\frac{8}{\sqrt{5^2+8^2+(4-z)^2}}\right)\right] + F = 0 \qquad (1)$

$\Sigma F_z = 0;\quad 2\left[500\left(\frac{4-z}{\sqrt{5^2+8^2+(4-z)^2}}\right)\right] - 200 = 0 \qquad (2)$

Dividing Eq. (2) by Eq. (1),

$$\frac{4-z}{8} = \frac{200}{F}$$

$$(4-z) = \frac{1600}{F}$$

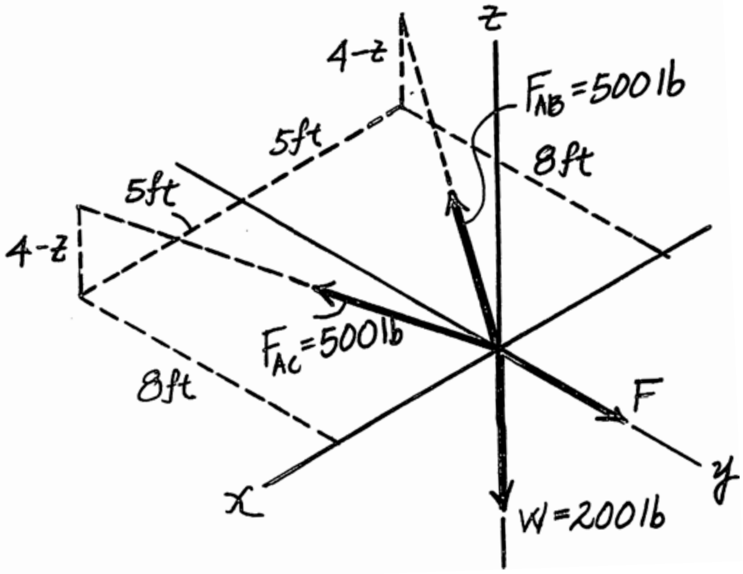
From Eq. (1):

$$\frac{8000}{F} = \sqrt{89 + \left(\frac{1600}{F}\right)^2}$$

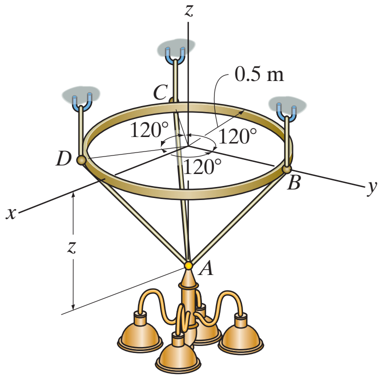
$$\left(\frac{8000}{F}\right)^2 = 89 + \left(\frac{1600}{F}\right)^2$$

$F = 831$ lb Ans

$z = 2.07$ ft Ans



***3–64.** The thin ring can be adjusted vertically between three equally long cables from which the 100-kg chandelier is suspended. If the ring remains in the horizontal plane and $z = 600$ mm, determine the tension in each cable.



Geometry: Referring to the geometry of the free - body diagram shown in Fig. (*a*), the lengths of cables *AB*, *AC*, and *AD* are all $l = \sqrt{0.5^2 + 0.6^2} = \sqrt{0.61}$ m.

Equations of Equilibrium: Equilibrium requires

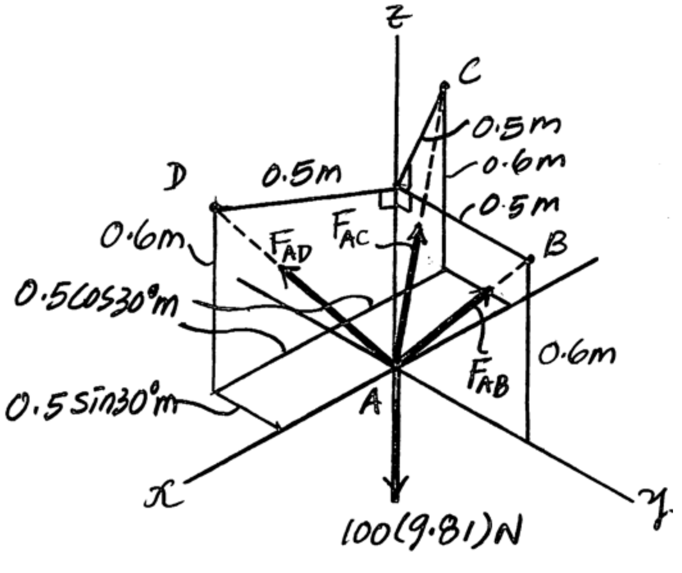
$$\Sigma F_x = 0; \quad F_{AD}\left(\frac{0.5\cos 30^\circ}{\sqrt{0.61}}\right) - F_{AC}\left(\frac{0.5\cos 30^\circ}{\sqrt{0.61}}\right) = 0 \quad F_{AD} = F_{AC} = F$$

$$\Sigma F_y = 0; \quad F_{AB}\left(\frac{0.5}{\sqrt{0.61}}\right) - 2\left[F\left(\frac{0.5\sin 30^\circ}{\sqrt{0.61}}\right)\right] = 0 \qquad F_{AB} = F$$

Thus, cables *AB*, *AC*, and *AD* all develop the same tension.

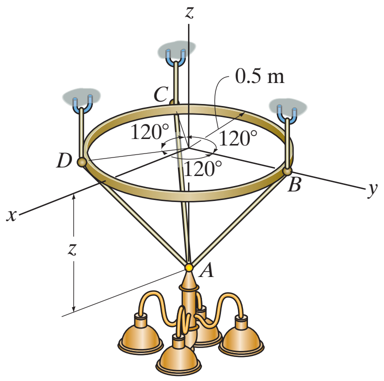
$$\Sigma F_z = 0; \quad 3F\left(\frac{0.6}{\sqrt{0.61}}\right) - 100(9.81) = 0$$

$$F_{AB} = F_{AC} = F_{AD} = 426 \text{ N} \qquad \textbf{Ans.}$$



(a)

•3–65. The thin ring can be adjusted vertically between three equally long cables from which the 100-kg chandelier is suspended. If the ring remains in the horizontal plane and the tension in each cable is not allowed to exceed 1 kN, determine the smallest allowable distance *z* required for equilibrium.



Geometry: Referring to the geometry of the free - body diagram shown in Fig. (a), the lengths of cables AB, AC, and AD are all $l = \sqrt{0.5^2 + z^2}$.

Equations of Equilibrium: Equilibrium requires

$$\Sigma F_x = 0; \quad F_{AD}\left(\frac{0.5\cos 30°}{\sqrt{0.5^2+z^2}}\right) - F_{AC}\left(\frac{0.5\cos 30°}{\sqrt{0.5^2+z^2}}\right) = 0 \qquad F_{AD} = F_{AC} = F$$

$$\Sigma F_y = 0; \quad F_{AB}\left(\frac{0.5}{\sqrt{0.5^2+z^2}}\right) - 2\left[F\left(\frac{0.5\sin 30°}{\sqrt{0.5^2+z^2}}\right)\right] = 0 \qquad F_{AB} = F$$

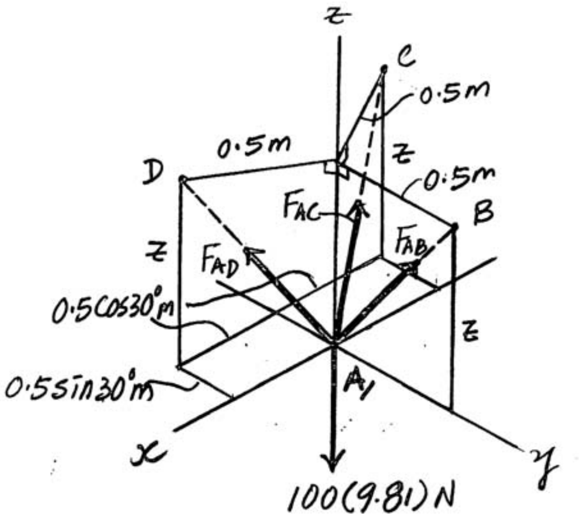
Thus, cables AB, AC, and AD all develop the same tension.

$$\Sigma F_z = 0; \quad 3F\left(\frac{z}{\sqrt{0.5^2+z^2}}\right) - 100(9.81) = 0$$

Cables AB, AC, and AD will also achieve maximum tension simultaneously. Substituting $F = 1000$ N, we obtain

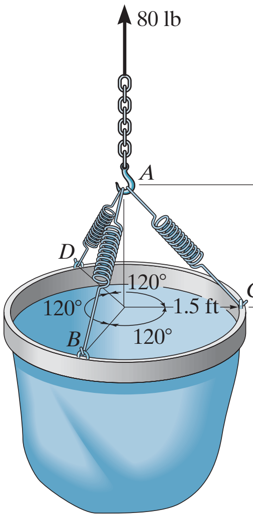
$$3(1000)\left(\frac{z}{\sqrt{0.5^2+z^2}}\right) - 100(9.81) = 0$$

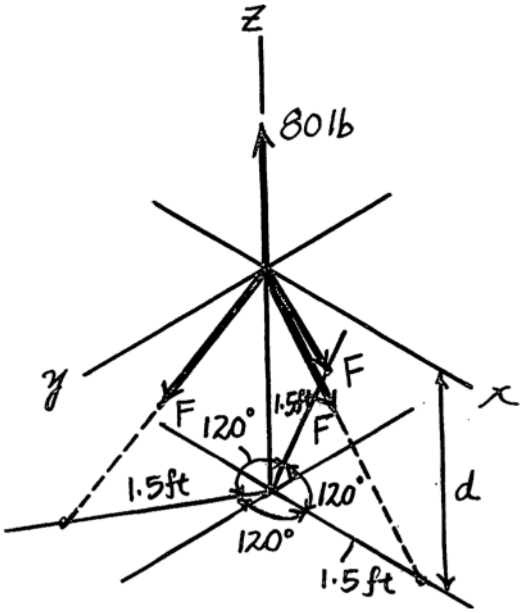
$$z = 0.1730 \text{ m} = 173 \text{ mm} \qquad \textbf{Ans.}$$



(a)

3–66. The bucket has a weight of 80 lb and is being hoisted using three springs, each having an unstretched length of $l_0 = 1.5$ ft and stiffness of $k = 50$ lb/ft. Determine the vertical distance d from the rim to point A for equilibrium.





$\Sigma F_z = 0;$
$$80 - \left(\frac{3\,d}{\sqrt{d^2 + (1.5)^2}}\right) F = 0$$

$$80 - \frac{3\,d}{\sqrt{d^2 + (1.5)^2}}\left[50\left(\sqrt{d^2 + (1.5)^2} - 1.5\right)\right] = 0$$

$$\frac{d}{\sqrt{d^2 + (1.5)^2}}\left(\sqrt{d^2 + (1.5)^2} - 1.5\right) = 0.5333$$

$$d\sqrt{d^2 + (1.5)^2} - 1.5\,d = 0.5333\sqrt{d^2 + (1.5)^2}$$

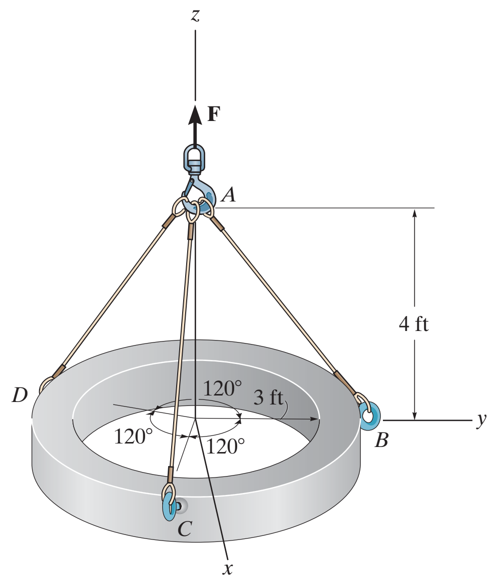
$$\sqrt{d^2 + (1.5)^2}\,(d - 0.5333) = 1.5\,d$$

$$\left[d^2 + (1.5)^2\right]\left[d^2 - 2\,d\,(0.5333) + (0.5333)^2\right] = (1.5)^2\,d^2$$

$$d^4 - 1.067\,d^3 + 0.284\,d^2 - 2.4\,d + 0.64 = 0$$

$$d = 1.64 \text{ ft} \qquad \textbf{Ans}$$

3–67. Three cables are used to support a 900-lb ring. Determine the tension in each cable for equilibrium.



Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{3\mathbf{j} - 4\mathbf{k}}{\sqrt{3^2 + (-4)^2}}\right) = 0.6F_{AB}\mathbf{j} - 0.8F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{3\cos 30°\mathbf{i} - 3\sin 30°\mathbf{j} - 4\mathbf{k}}{\sqrt{(3\cos 30°)^2 + (-3\sin 30°)^2 + (-4)^2}}\right)$$
$$= 0.5196F_{AC}\mathbf{i} - 0.3F_{AC}\mathbf{j} - 0.8F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-3\cos 30°\mathbf{i} - 3\sin 30°\mathbf{j} - 4\mathbf{k}}{\sqrt{(-3\cos 30°)^2 + (-3\sin 30°)^2 + (-4)^2}}\right)$$
$$= -0.5196F_{AD}\mathbf{i} - 0.3F_{AD}\mathbf{j} - 0.8F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{900\mathbf{k}\}\ \text{lb}$$

Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$(0.5196F_{AC} - 0.5196F_{AD})\mathbf{i} + (0.6F_{AB} - 0.3F_{AC} - 0.3F_{AD})\mathbf{j}$$
$$+ (-0.8F_{AB} - 0.8F_{AC} - 0.8F_{AD} + 900)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

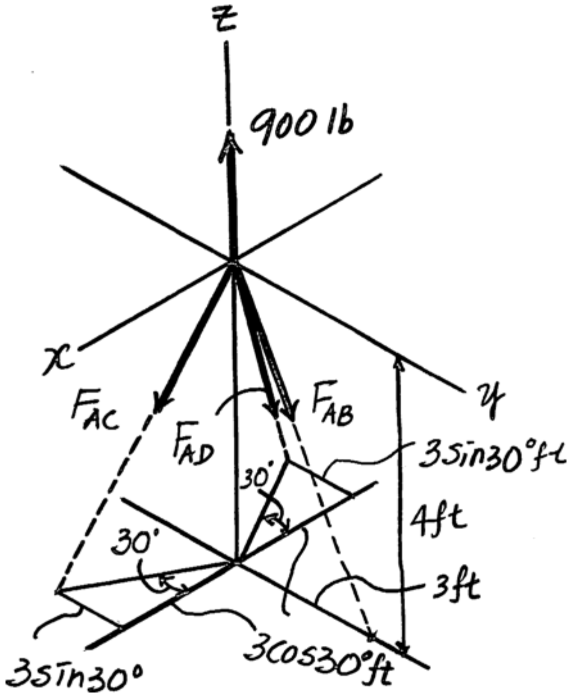
$$0.5196F_{AC} - 0.5196F_{AD} = 0 \qquad [1$$
$$0.6F_{AB} - 0.3F_{AC} - 0.3F_{AD} = 0 \qquad [2$$
$$-0.8F_{AB} - 0.8F_{AC} - 0.8F_{AD} + 900 = 0 \qquad [3$$

Solving Eqs. [1], [2] and [3] yields

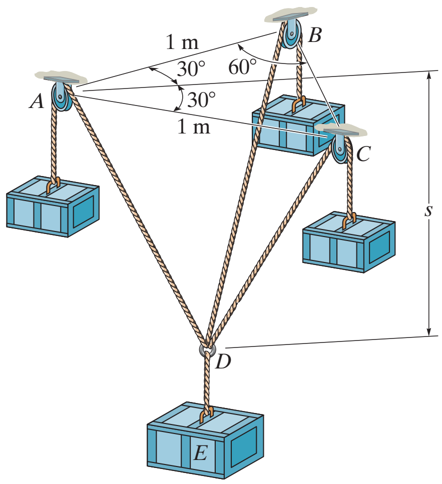
$$F_{AB} = F_{AC} = F_{AD} = 375\ \text{lb} \qquad \textbf{Ans}$$

This problem also can be easily solved if one realizes that due to symmetry all cables are subjected to a same tensile force, that is $F_{AB} = F_{AC} = F_{AD} = F$. Summing forces along z axis yields

$$\Sigma F_z = 0; \qquad 900 - 3F\left(\frac{4}{5}\right) = 0 \qquad F = 375\ \text{lb}$$



***3–68.** The three outer blocks each have a mass of 2 kg, and the central block *E* has a mass of 3 kg. Determine the sag *s* for equilibrium of the system.



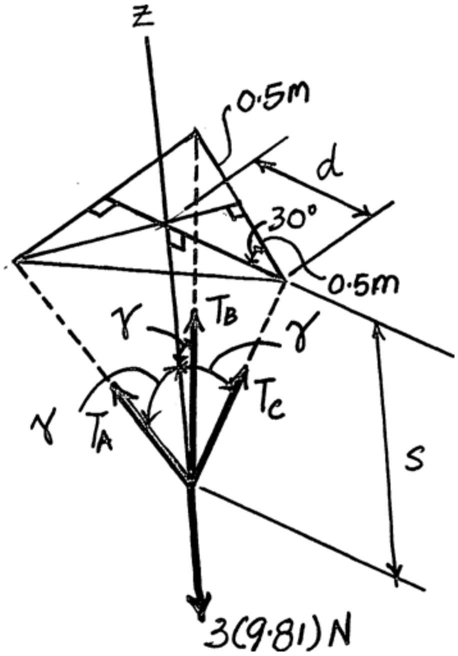
$T_A = T_B = T_C = 2\,(9.81)$

$\Sigma F_z = 0; \quad 3\,(2\,(9.81))\cos\gamma - 3\,(9.81) = 0$

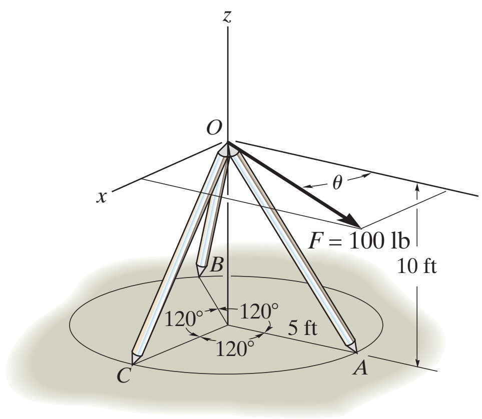
$\cos\gamma = 0.5; \quad \gamma = 60°$

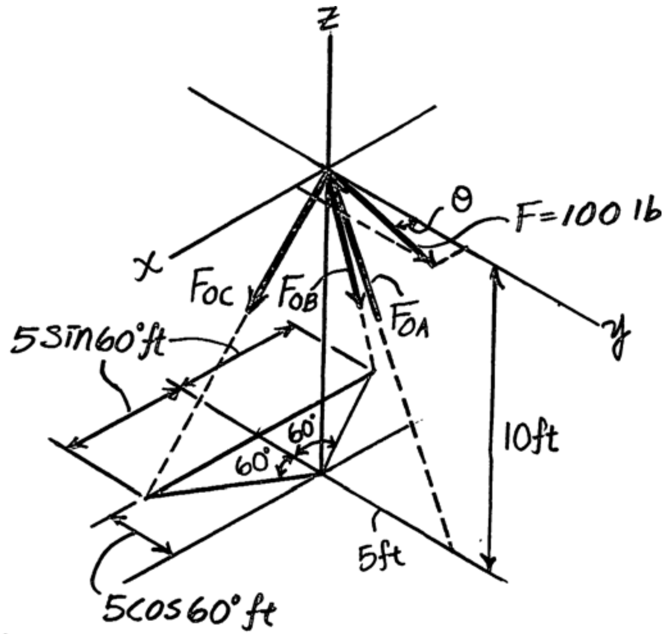
$d = \dfrac{0.5}{\cos 30°} = 0.577\text{ m}$

$s = \dfrac{0.577}{\tan 60°} = 0.333\text{ m} = 333\text{ mm} \qquad$ Ans



•3–69. Determine the angle θ such that an equal force is developed in legs OB and OC. What is the force in each leg if the force is directed along the axis of each leg? The force $\mathbf{F}$ lies in the $x-y$ plane. The supports at A, B, C can exert forces in either direction along the attached legs.





$$\mathbf{F}_{OA} = F_{OA}\left(-\frac{5}{11.180}\,\mathbf{j} + \frac{10}{11.180}\,\mathbf{k}\right)$$

$$= F_{OA}\,(-0.4472\,\mathbf{j} + 0.89443\,\mathbf{k})$$

$$\mathbf{F}_{OB} = F_{OB}\left(-\frac{5\sin 60^\circ}{11.18}\,\mathbf{i} - \frac{5\cos 60^\circ}{11.18}\,\mathbf{j} - \frac{10}{11.18}\,\mathbf{k}\right)$$

$$= F_{OB}\,(-\,0.3873\,\mathbf{i} - 0.2236\,\mathbf{j} - 0.8944\,\mathbf{k})$$

$$\mathbf{F}_{OC} = F_{OC}\left(\frac{5\sin 60^\circ}{11.18}\,\mathbf{i} - \frac{5\cos 60^\circ}{11.18}\,\mathbf{j} - \frac{10}{11.18}\,\mathbf{k}\right)$$

$$= F_{OC}\,(\,0.3873\,\mathbf{i} - 0.2236\,\mathbf{j} - 0.8944\,\mathbf{k})$$

$$\mathbf{F} = 100\,(\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$$

$\Sigma F_x = 0;\quad -\,0.3873\,F_{OB} + 0.3873\,F_{OC} + 100\sin\theta = 0$

If $F_{OC} = F_{OB}$, then $100\sin\theta = 0;\quad \theta = 0^\circ$ **Ans**

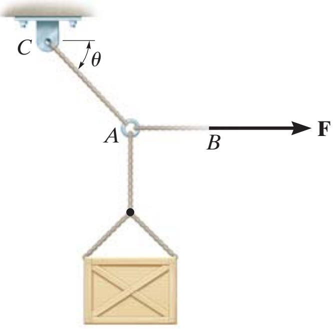
$\Sigma F_y = 0;\quad -\,0.4472\,F_{OA} - 0.2236\,F_{OB} - 0.2236\,F_{OC} + 100 = 0$

$\Sigma F_z = 0;\quad 0.8944\,F_{OA} - 0.8944\,F_{OB} - 0.8944\,F_{OC} = 0$

$F_{OA} = 149\text{ lb}$ **Ans**

$F_{OB} = F_{OC} = 74.5\text{ lb}$ **Ans**

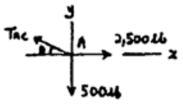
3–70. The 500-lb crate is hoisted using the ropes AB and AC. Each rope can withstand a maximum tension of 2500 lb before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted.



Case 1 : Assume $T_{AB} = 2500$ lb

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 2500 - T_{AC}\cos\theta = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad T_{AC}\sin\theta - 500 = 0$



Solving,

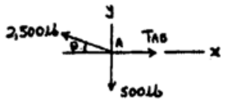
$\theta = 11.31°$

$T_{AC} = 2549.5\text{ lb} > 2500\text{ lb}$ (N.G!)

Case 2 : Assume $T_{AC} = 2500$ lb

$+\uparrow \Sigma F_y = 0;$ $\quad 2500\sin\theta - 500 = 0$

$\theta = 11.54°$

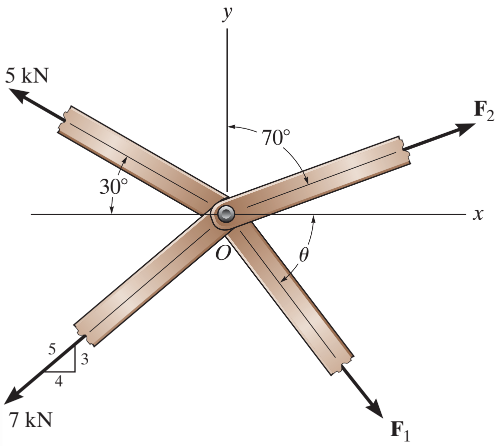


$\xrightarrow{+} \Sigma F_x = 0;$ $\quad T_{AB} - 2500\cos 11.54° = 0$

$T_{AB} = 2449.49\text{ lb} < 2500\text{ lb}$

Thus, the smallest angle is $\theta = 11.5°$ **Ans**

3–71. The members of a truss are pin connected at joint O. Determine the magnitude of $\mathbf{F}_1$ and its angle θ for equilibrium. Set $F_2 = 6$ kN.

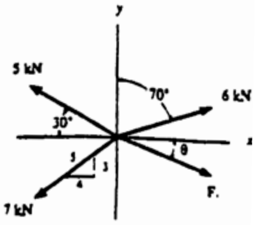


$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 6\sin 70° + F_1\cos\theta - 5\cos 30° - \frac{4}{5}(7) = 0$

$F_1\cos\theta = 4.2920$

$+\uparrow \Sigma F_y = 0;$ $\quad 6\cos 70° + 5\sin 30° - F_1\sin\theta - \frac{3}{5}(7) = 0$

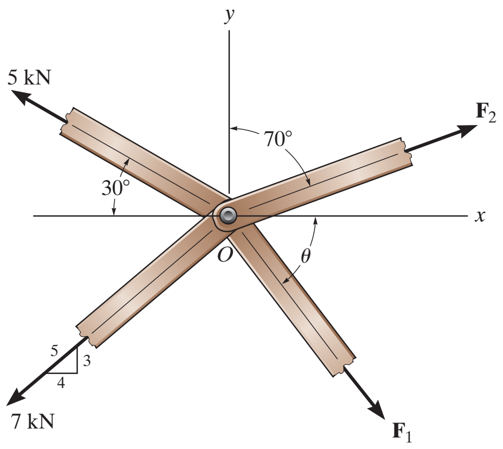
$F_1\sin\theta = 0.3521$



Solving:

$\theta = 4.69°$ **Ans** $\quad F_1 = 4.31$ kN **Ans**

***3–72.** The members of a truss are pin connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60°$.

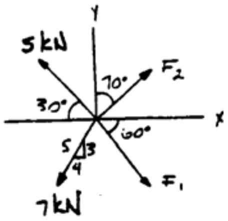


$\xrightarrow{+} \Sigma F_x = 0;\quad F_2 \sin 70° + F_1 \cos 60° - 5 \cos 30° - \frac{4}{5}(7) = 0$

$0.9397F_2 + 0.5F_1 = 9.930$

$+\uparrow \Sigma F_y = 0;\quad F_2 \cos 70° + 5 \sin 30° - F_1 \sin 60° - \frac{3}{5}(7) = 0$

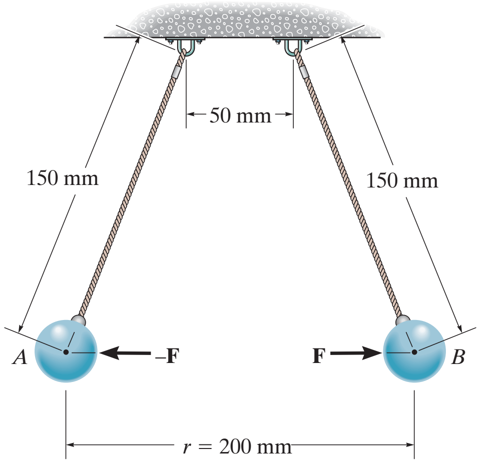
$0.3420F_2 - 0.8660F_1 = 1.7$

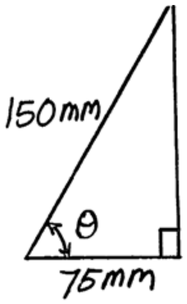


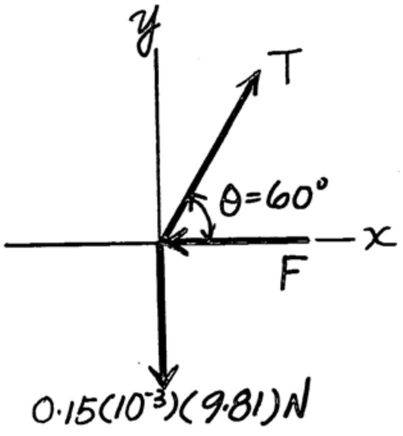
Solving:

$F_2 = 9.60$ kN **Ans** $F_1 = 1.83$ kN **Ans**

•3–73. Two electrically charged pith balls, each having a mass of 0.15 g, are suspended from light threads of equal length. Determine the magnitude of the horizontal repulsive force, F, acting on each ball if the measured distance between them is $r = 200$ mm.







$\cos\theta = \frac{75}{150} \qquad \theta = 60°$

$+\uparrow \Sigma F_y = 0; \quad T \sin 60° - 0.15(10)^{-3}(9.81) = 0$

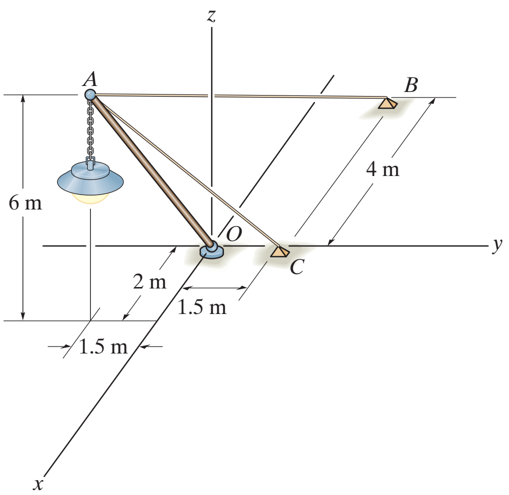
$T = 1.699(10)^{-3}$ N

$\xrightarrow{+} \Sigma F_x = 0; \quad 1.699(10)^{-3} \cos 60° - F = 0$

$F = 0.850(10)^{-3}$ N

$= 0.850$ mN **Ans**

3–74. The lamp has a mass of 15 kg and is supported by a pole *AO* and cables *AB* and *AC*. If the force in the pole acts along its axis, determine the forces in *AO*, *AB*, and *AC* for equilibrium.



Cartesian Vector Notation :

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{-6\mathbf{i}+3\mathbf{j}-6\mathbf{k}}{\sqrt{(-6)^2+3^2+(-6)^2}}\right) = -\frac{2}{3}F_{AB}\mathbf{i}+\frac{1}{3}F_{AB}\mathbf{j}-\frac{2}{3}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{-2\mathbf{i}+3\mathbf{j}-6\mathbf{k}}{\sqrt{(-2)^2+3^2+(-6)^2}}\right) = -\frac{2}{7}F_{AC}\mathbf{i}+\frac{3}{7}F_{AC}\mathbf{j}-\frac{6}{7}F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AO} = F_{AO}\left(\frac{2\mathbf{i}-1.5\mathbf{j}+6\mathbf{k}}{\sqrt{2^2+(-1.5)^2+6^2}}\right) = \frac{4}{13}F_{AO}\mathbf{i}-\frac{3}{13}F_{AO}\mathbf{j}+\frac{12}{13}F_{AO}\mathbf{k}$$

$$\mathbf{F} = \{-147.15\mathbf{k}\}\ \text{N}$$

Equations of Equilibrium :

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB}+\mathbf{F}_{AC}+\mathbf{F}_{AO}+\mathbf{F} = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}-\frac{2}{7}F_{AC}+\frac{4}{13}F_{AO}\right)\mathbf{i}+\left(\frac{1}{3}F_{AB}+\frac{3}{7}F_{AC}-\frac{3}{13}F_{AO}\right)\mathbf{j} + \left(-\frac{2}{3}F_{AB}-\frac{6}{7}F_{AC}+\frac{12}{13}F_{AO}-147.15\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j** and **k** components, we have

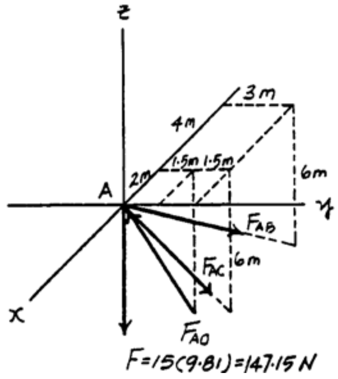
$$-\frac{2}{3}F_{AB}-\frac{2}{7}F_{AC}+\frac{4}{13}F_{AO} = 0 \qquad [1]$$

$$\frac{1}{3}F_{AB}+\frac{3}{7}F_{AC}-\frac{3}{13}F_{AO} = 0 \qquad [2]$$

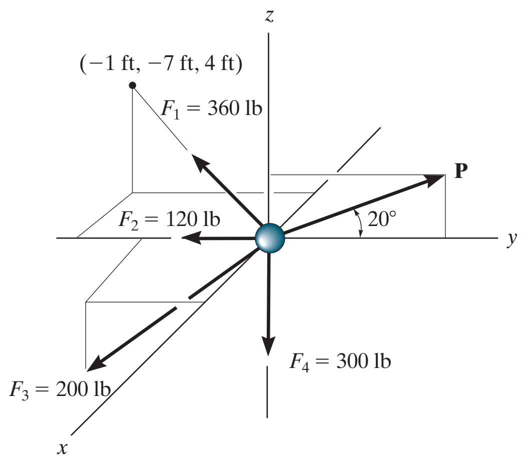
$$-\frac{2}{3}F_{AB}-\frac{6}{7}F_{AC}+\frac{12}{13}F_{AO}-147.15 = 0 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AB} = 110\ \text{N} \qquad F_{AC} = 85.8\ \text{N} \qquad F_{AO} = 319\ \text{N} \qquad \textbf{Ans}$$

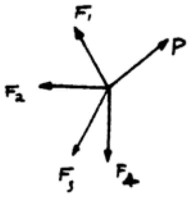


3–75. Determine the magnitude of **P** and the coordinate direction angles of $\mathbf{F}_3$ required for equilibrium of the particle. Note that $\mathbf{F}_3$ acts in the octant shown.



$$\mathbf{F}_1 = 360\left(-\frac{1}{\sqrt{66}}\mathbf{i} - \frac{7}{\sqrt{66}}\mathbf{j} + \frac{4}{\sqrt{66}}\mathbf{k}\right)$$

$$= -44.313\,\mathbf{i} - 310.191\,\mathbf{j} + 177.252\,\mathbf{k}$$



$$\mathbf{F}_2 = -120\,\mathbf{j}$$

$$\mathbf{F}_4 = -300\,\mathbf{k}$$

$$\mathbf{F}_3 = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k} \qquad (1)$$

$$\mathbf{P} = P\cos 20°\,\mathbf{j} + P\sin 20°\,\mathbf{k}$$

$\Sigma F_x = 0;\qquad -44.313 + F_x = 0$

$$F_x = 44.313 \text{ lb}$$

$\Sigma F_y = 0;\qquad -310.191 - 120 + F_y + 0.9397\,P = 0$

$\Sigma F_z = 0;\qquad 177.252 - 300 + F_z + 0.3420\,P = 0$

From Eq. (1), require

$$200 = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$(200)^2 = (44.313)^2 + (430.191 - 0.9397P)^2 + (122.748 - 0.3420P)^2$$

$$P^2 - 892.459P + 162\,095 = 0$$

Solving,

$P = 638.65$ lb and $P = 253.81$ lb

Thus, with $P = 638.65$ lb, $F_y = -169.95$ lb.
With $P = 253.81$ lb, $F_y = 191.69$ lb.
In order for F_3 to be within the octant shown, choose

$P = 639$ lb **Ans**

so that

$F_z = -95.672$

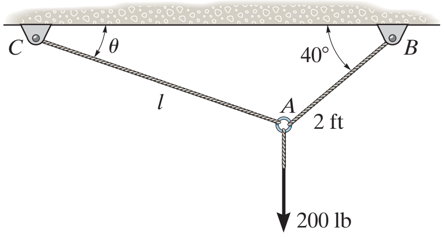
Thus, the direction of F_3 is :

$\alpha_3 = \cos^{-1}\left(\frac{44.313}{200}\right) = 77.2°$ **Ans**

$\beta_3 = \cos^{-1}\left(\frac{-169.95}{200}\right) = 148°$ **Ans**

$\gamma_3 = \cos^{-1}\left(\frac{-95.672}{200}\right) = 119°$ **Ans**

***3–76.** The ring of negligible size is subjected to a vertical force of 200 lb. Determine the longest length l of cord AC such that the tension acting in AC is 160 lb. Also, what is the force acting in cord AB? *Hint:* Use the equilibrium condition to determine the required angle θ for attachment, then determine l using trigonometry applied to ΔABC.



Equations of Equilibrium :

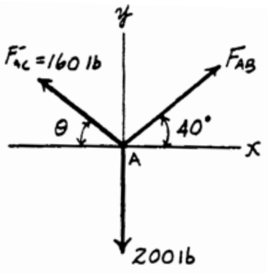
$\xrightarrow{+} \Sigma F_x = 0;\quad F_{AB}\cos 40° - 160\cos\theta = 0$ [1]

$+\uparrow \Sigma F_y = 0;\quad F_{AB}\sin 40° + 160\sin\theta - 200 = 0$ [2]

Solving Eqs. [1] and [2] yields

$$\theta = 33.25°$$

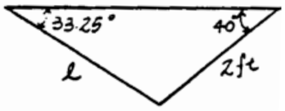
$$F_{AB} = 175 \text{ lb}$$ **Ans**



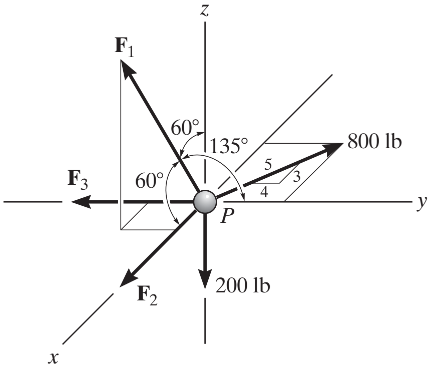
Geometry : Applying law of sines, we have

$$\frac{l}{\sin 40°} = \frac{2}{\sin 33.25°}$$

$$l = 2.34 \text{ ft}$$ **Ans**



•3–77. Determine the magnitudes of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ for equilibrium of the particle.



$\Sigma F_x = 0;\quad F_2 + F_1\cos 60° - 800\left(\frac{3}{5}\right) = 0$

$\Sigma F_y = 0;\quad 800\left(\frac{4}{5}\right) + F_1\cos 135° - F_3 = 0$

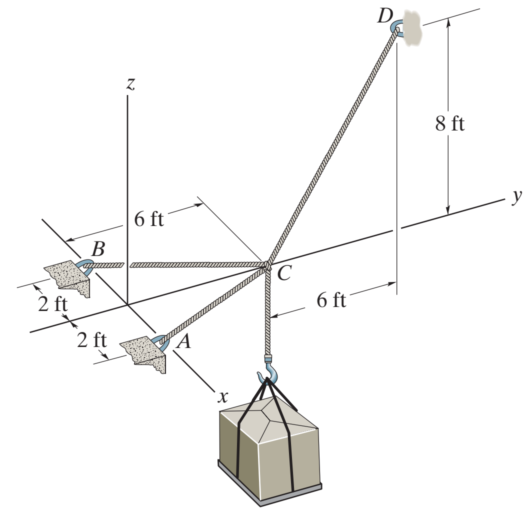
$\Sigma F_z = 0;\quad F_1\cos 60° - 200 = 0$

$F_1 = 400 \text{ lb}$ **Ans**

$F_2 = 280 \text{ lb}$ **Ans**

$F_3 = 357 \text{ lb}$ **Ans**

3–78. Determine the force in each cable needed to support the 500-lb load.



Equation of Equilibrium :

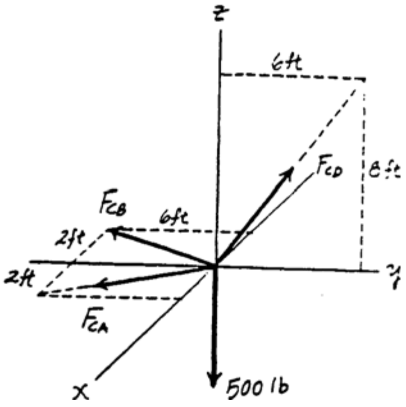
$\Sigma F_z = 0;\quad F_{CD}\left(\frac{4}{5}\right) - 500 = 0 \qquad F_{CD} = 625 \text{ lb}$ **Ans**

Using the results $F_{CD} = 625$ lb and then summing forces along x and y axes we have

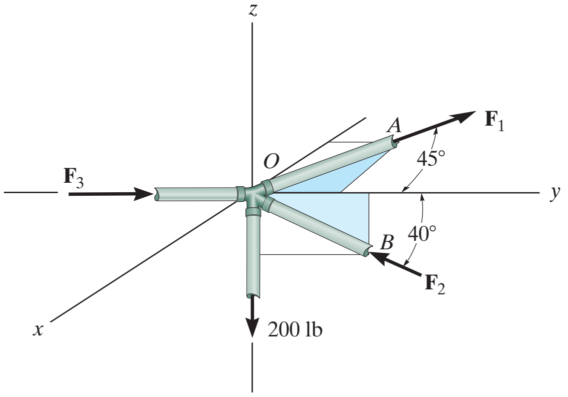
$\Sigma F_x = 0;\quad F_{CA}\left(\frac{2}{\sqrt{40}}\right) - F_{CB}\left(\frac{2}{\sqrt{40}}\right) = 0 \qquad F_{CA} = F_{CB} = F$

$\Sigma F_y = 0;\quad 2F\left(\frac{6}{\sqrt{40}}\right) - 625\left(\frac{3}{5}\right) = 0$

$F_{CA} = F_{CB} = F = 198 \text{ lb}$ **Ans**



3–79. The joint of a space frame is subjected to four member forces. Member OA lies in the x–y plane and member OB lies in the y–z plane. Determine the forces acting in each of the members required for equilibrium of the joint.



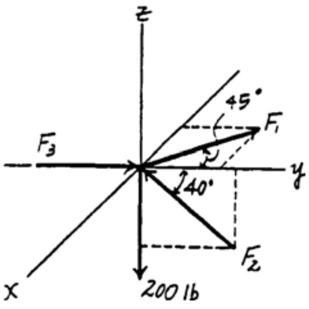
Equation of Equilibrium :

$\Sigma F_x = 0;\quad F_1 \sin 45° = 0 \qquad F_1 = 0$ **Ans**

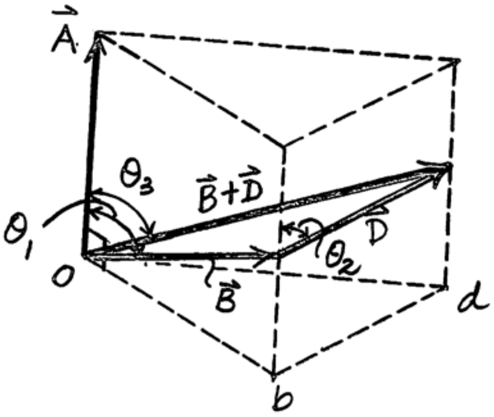
$\Sigma F_z = 0;\quad F_2 \sin 40° - 200 = 0 \qquad F_2 = 311.14 \text{ lb} = 311 \text{ lb}$ **Ans**

Using the results $F_1 = 0$ and $F_2 = 311.14$ lb and then summing forces along the y axis, we have

$\Sigma F_y = 0;\quad F_3 - 311.14\cos 40° = 0 \qquad F_3 = 238 \text{ lb}$ **Ans**



•4–1. If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.



Consider the three vectors; with **A** vertical.

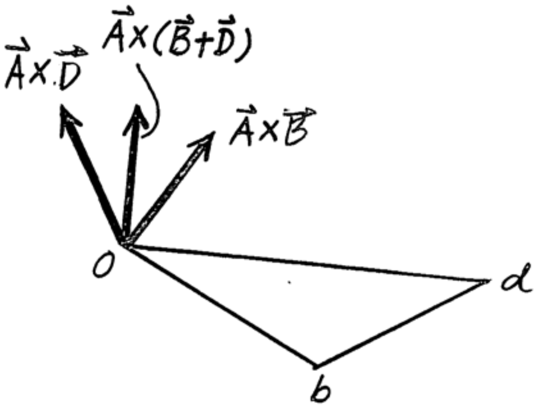
Note *obd* is perpendicular to **A**.

$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}|(|\mathbf{B} + \mathbf{D}|) \sin\theta_3$

$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta_1$

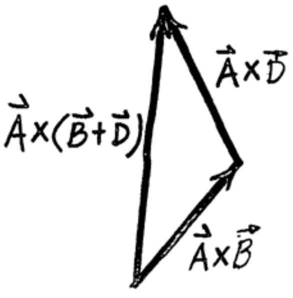
$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin\theta_2$

Also, these three cross products all lie in the plane *obd* since they are all perpendicular to **A**. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.



The three vector cross - products also form a closed triangle *o′b′d′* which is similar to triangle *obd*. Thus from the figure,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{D} \qquad \text{(QED)}$$



Note also,

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\mathbf{D} = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$\begin{aligned} &= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i} \\ &\quad -[A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j} \\ &\quad +[A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k} \\ &= [(A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}] \\ &\quad + [(A_yD_z - A_zD_y)\mathbf{i} - (A_xD_z - A_zD_x)\mathbf{j} + (A_xD_y - A_yD_x)\mathbf{k}] \end{aligned}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \text{(QED)}$$

4–2. Prove the triple scalar product identity $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C}$.

As shown in the figure

$$\text{Area} = B(C\sin\theta) = |\mathbf{B}\times\mathbf{C}|$$

Thus,

Volume of parallelepiped is $|\mathbf{B}\times\mathbf{C}||h|$

But,

$$|h| = |\mathbf{A}\cdot\mathbf{u}_{(\mathbf{B}\times\mathbf{C})}| = \left|\mathbf{A}\cdot\left(\frac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}\right)\right|$$

Thus,

$$\text{Volume} = |\mathbf{A}\cdot\mathbf{B}\times\mathbf{C}|$$

Since $|\mathbf{A}\times\mathbf{B}\cdot\mathbf{C}|$ represents this same volume then

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C} \qquad \text{(QED)}$$

Also,

$$LHS = \mathbf{A}\cdot\mathbf{B}\times\mathbf{C}$$

$$= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix}$$

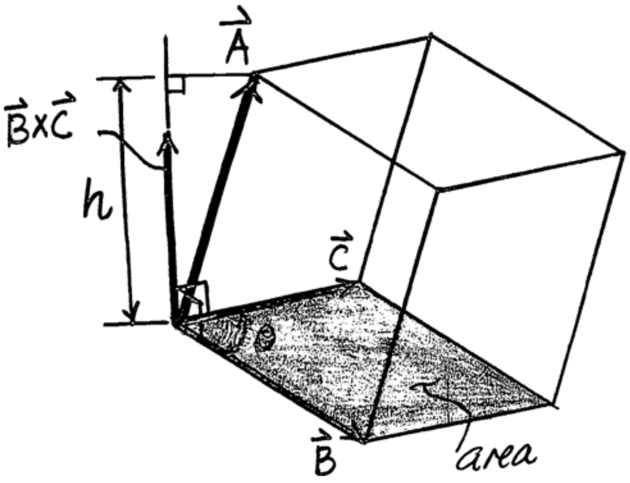
$$= A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - B_zC_x) + A_z(B_xC_y - B_yC_x)$$
$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

$$RHS = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C}$$

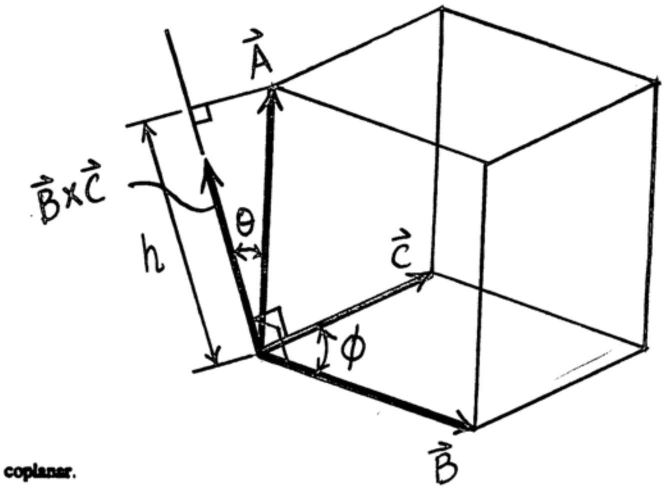
$$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}\cdot(C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$
$$= C_x(A_yB_z - A_zB_y) - C_y(A_xB_z - A_zB_x) + C_z(A_xB_y - A_yB_x)$$
$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

Thus, $LHS = RHS$

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C} \qquad \text{(QED)}$$



4–3. Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, the three vectors *must* lie in the same plane.



Consider,

$$|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| = |\mathbf{A}||\mathbf{B} \times \mathbf{C}| \cos\theta$$

$$= (|\mathbf{A}| \cos\theta)|\mathbf{B} \times \mathbf{C}|$$

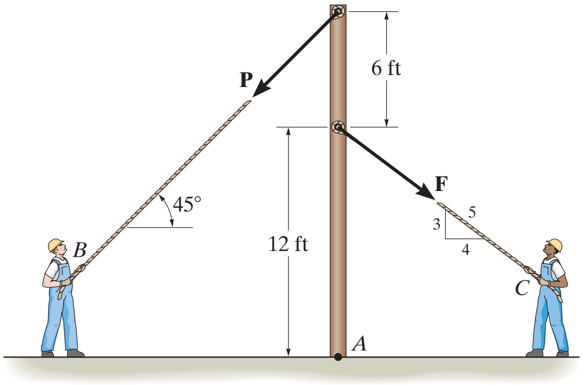
$$= |h||\mathbf{B} \times \mathbf{C}|$$

$$= BC|h| \sin\phi$$

= volume of parallelepiped.

If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, then the volume equals zero, so that **A**, **B**, and **C** are coplanar.

***4–4.** Two men exert forces of $F = 80$ lb and $P = 50$ lb on the ropes. Determine the moment of each force about A. Which way will the pole rotate, clockwise or counterclockwise?



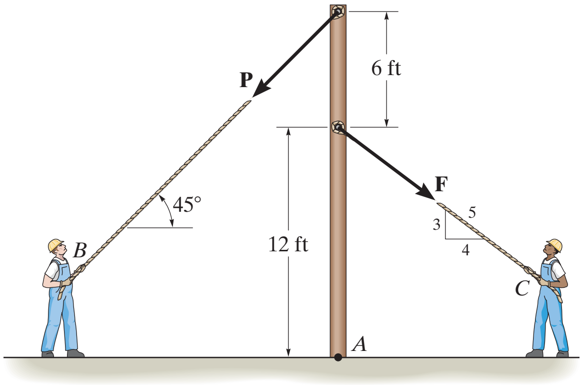
$$\curvearrowright + \ (M_A)_C = 80\left(\frac{4}{5}\right)(12) = 768 \text{ lb} \cdot \text{ft} \curvearrowright \quad \text{Ans}$$

$$\curvearrowleft + \ (M_A)_B = 50(\cos 45^\circ)(18) = 636 \text{ lb} \cdot \text{ft} \curvearrowleft \quad \text{Ans}$$

Since $(M_A)_C > (M_A)_B$

Clockwise **Ans**

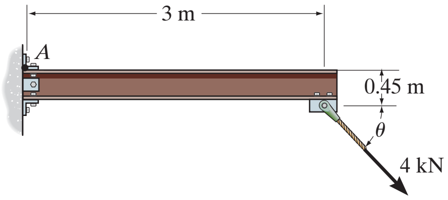
•**4–5.** If the man at B exerts a force of $P = 30$ lb on his rope, determine the magnitude of the force **F** the man at C must exert to prevent the pole from rotating, i.e., so the resultant moment about A of both forces is zero.



$$\curvearrowleft + \quad 30(\cos 45^\circ)(18) - F\left(\frac{4}{5}\right)(12) = 0$$

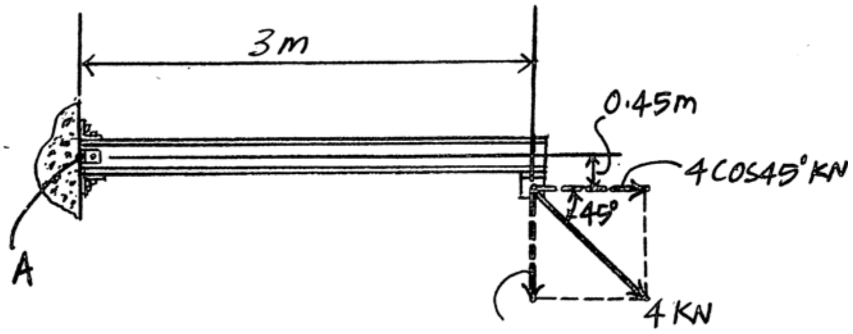
$$F = 39.8 \text{ lb} \quad \text{Ans}$$

4–6. If $\theta = 45°$, determine the moment produced by the 4-kN force about point A.



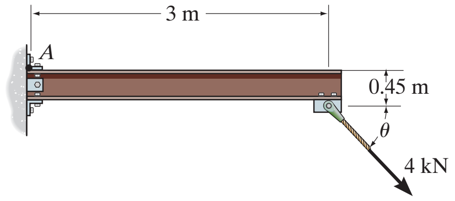
Resolving the 4 - kN force into its horizontal and vertical components, Fig. a, and applying the principle of moments,

$$\curvearrowleft +M_A = 4\cos 45°(0.45) - 4\sin 45°(3)$$
$$= -7.21\ \text{kN}\cdot\text{m} = 7.21\ \text{kN}\cdot\text{m}\quad\text{(clockwise)}\qquad\textbf{Ans.}$$



(a)

4–7. If the moment produced by the 4-kN force about point A is $10\ \text{kN}\cdot\text{m}$ clockwise, determine the angle θ, where $0° \le \theta \le 90°$.



Resolving the 4 - kN force into its horizontal and vertical components, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft +M_A = -10 = 4\cos\theta(0.45) - 4\sin\theta(3)$$

$$12\sin\theta - 1.8\cos\theta = 10 \qquad (1)$$

Referring to the geometry of Fig. *a*,

$$\cos\phi = \frac{12}{\sqrt{147.24}} \qquad \sin\phi = \frac{1.8}{\sqrt{147.24}} \qquad (2)$$

Dividing Eq. (1) by $\sqrt{147.24}$ yields

$$\frac{12}{\sqrt{147.24}}\sin\theta - \frac{1.8}{\sqrt{147.24}}\cos\theta = \frac{10}{\sqrt{147.24}} \qquad (3)$$

Substituting Eq. (2) into (3) yields

$$\sin\theta\cos\phi - \cos\theta\sin\phi = \frac{10}{\sqrt{147.24}}$$

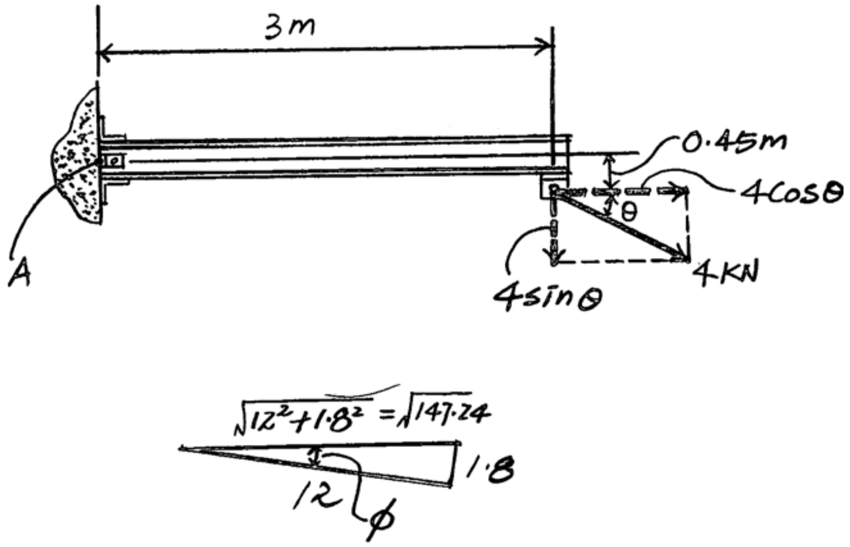
$$\sin(\theta - \phi) = \frac{10}{\sqrt{147.24}}$$

However, $\phi = \tan^{-1}\left(\frac{1.8}{12}\right) = 8.531°$. Thus,

$$\sin(\theta - 8.531°) = \frac{10}{\sqrt{147.24}}$$

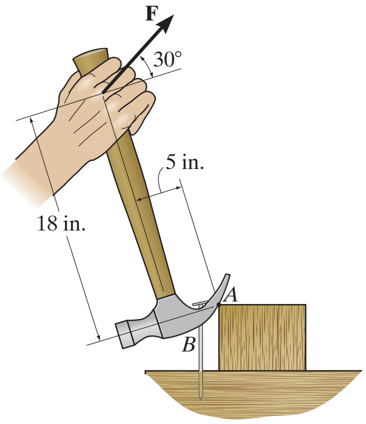
$$\theta - 8.531° = 55.50°$$

$$\theta = 64.0° \qquad \textbf{Ans.}$$



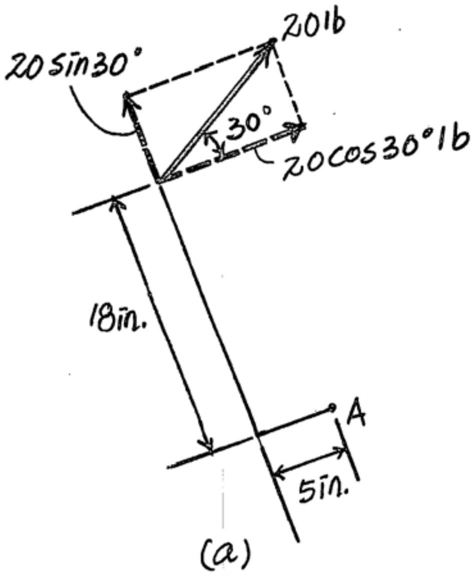
(a)

***4–8.** The handle of the hammer is subjected to the force of $F = 20$ lb. Determine the moment of this force about the point A.

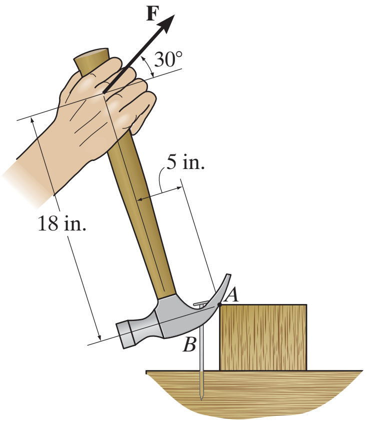


Resolving the 20 - lb force into components parallel and perpendicular to the hammer, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft + M_A = -20\cos 30°(18) - 20\sin 30°(5)$$
$$= -361.77 \text{ lb}\cdot\text{in} = 362 \text{ lb}\cdot\text{in} \quad \text{(clockwise)}$$ **Ans.**



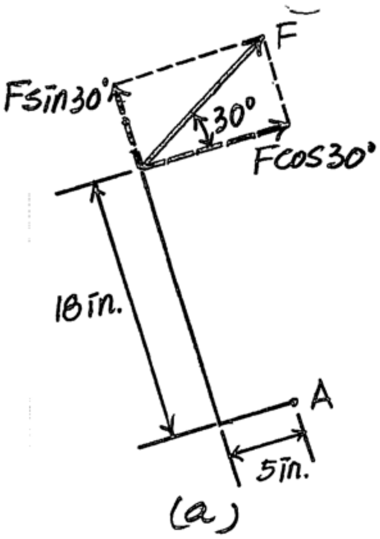
•4–9. In order to pull out the nail at B, the force **F** exerted on the handle of the hammer must produce a clockwise moment of $500\ \text{lb}\cdot\text{in.}$ about point A. Determine the required magnitude of force **F**.



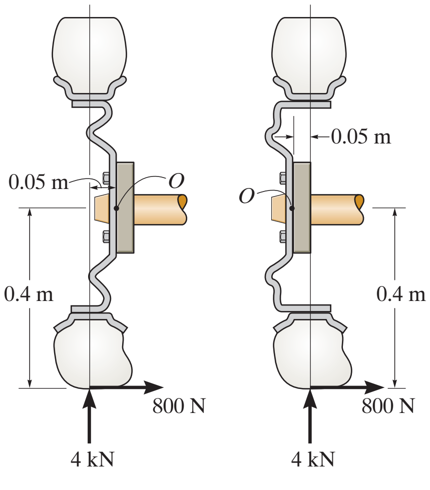
Resolving force **F** into components parallel and perpendicular to the hammer, Fig. *a*, and applying the principle of moments,

$$\circlearrowleft +M_A = -500 = -F\cos 30°(18) - F\sin 30°(5)$$

$$F = 27.6\ \text{lb} \qquad \textbf{Ans.}$$



4–10. The hub of the wheel can be attached to the axle either with negative offset (left) or with positive offset (right). If the tire is subjected to both a normal and radial load as shown, determine the resultant moment of these loads about point *O* on the axle for both cases.



Case 1

Case 2

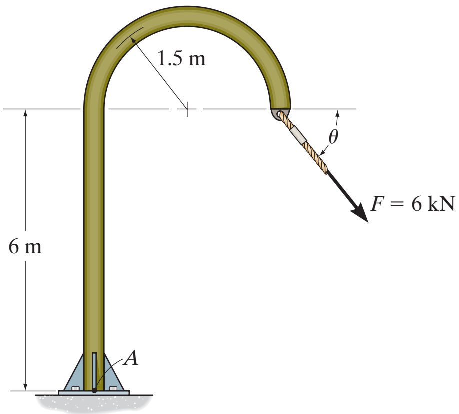
For case 1 with negative offset, we have

$$\circlearrowleft + \; M_O = 800(0.4) - 4000(0.05)$$
$$= 120\ \text{N}\cdot\text{m} \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

For case 2 with positive offset, we have

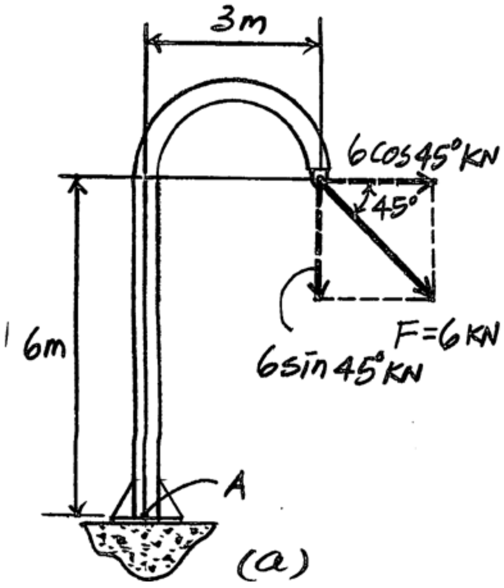
$$\circlearrowleft + \; M_O = 800(0.4) + 4000(0.05)$$
$$= 520\ \text{N}\cdot\text{m} \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

4–11. The member is subjected to a force of $F = 6$ kN. If $\theta = 45°$, determine the moment produced by **F** about point A.



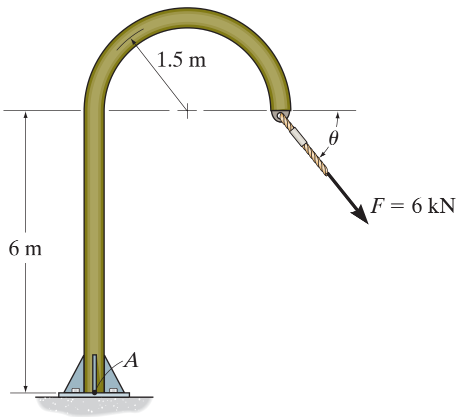
Resolving force **F** into horizontal and vertical components, Fig. a, and applying the principle of moments,

$$\circlearrowleft +M_A = -6\cos 45°(6) - 6\sin 45°(3)$$
$$= -38.18\ \text{kN}\cdot\text{m} = 38.2\ \text{kN}\cdot\text{m}\ \text{(clockwise)} \qquad \textbf{Ans.}$$



(a)

***4–12.** Determine the angle $\theta\,(0^\circ \le \theta \le 180^\circ)$ of the force **F** so that it produces a maximum moment and a minimum moment about point A. Also, what are the magnitudes of these maximum and minimum moments?



In order to produce the maximum moment about point A, force **F** must act perpendicular to line AB, Fig. a. From the geometry of this diagram,

$$\phi = \tan^{-1}\left(\frac{6}{3}\right) = 63.43^\circ$$

$$\theta = 90^\circ - \phi = 90^\circ - 63.43^\circ = 26.6^\circ \qquad \textbf{Ans.}$$

Also,

$$d = \sqrt{6^2 + 3^2} = \sqrt{45}\ \text{m}$$

The maximum moment of **F** about point A is given by

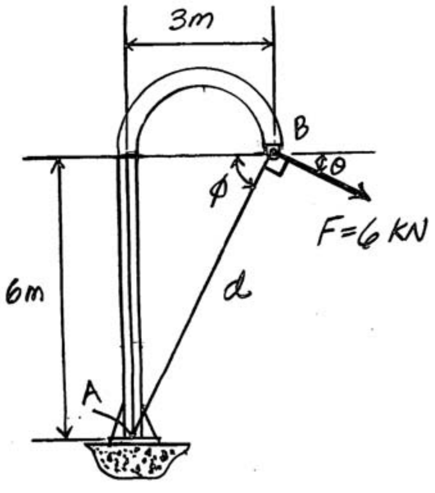
$$(M_A)_{\max} = Fd = 6\left(\sqrt{45}\right) = 40.2\ \text{kN}\cdot\text{m}$$

The minimum moment of **F** about point A occurs when the line of action of **F** passes through point A. Referring to Fig. b,

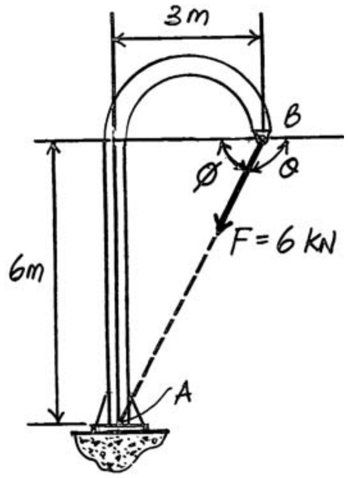
$$\theta = 180^\circ - \phi = 180^\circ - 63.43^\circ = 117^\circ \qquad \textbf{Ans.}$$

and

$$(M_A)_{\min} = Fd = 6(0) = 0 \qquad \textbf{Ans.}$$

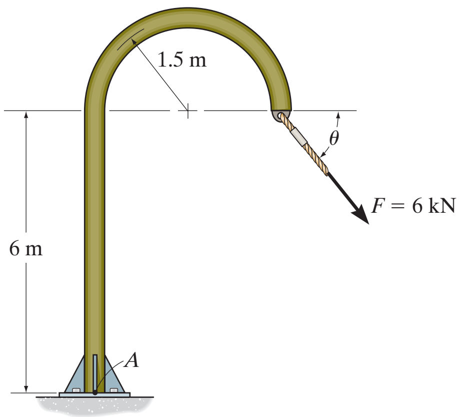


(a)



(b)

•4–13. Determine the moment produced by the force **F** about point A in terms of the angle θ. Plot the graph of M_A versus θ, where $0° \leq \theta \leq 180°$.



Moment Function: Resolving force **F** into horizontal and vertical components, Fig. *a*, and applying the principle of moments,

$$\begin{aligned}\curvearrowleft +M_A &= -6\cos\theta(6) - 6\sin\theta(3)\\ &= -(36\cos\theta + 18\sin\theta)\ \text{kN}\cdot\text{m}\\ &= (36\cos\theta + 18\sin\theta)\,\text{kN}\cdot\text{m}\ \text{(clockwise)}\end{aligned}$$

The maximum moment occurs when $\dfrac{dM_A}{d\theta} = 0$.

$$\frac{dM_A}{d\theta} = -36\sin\theta + 18\cos\theta = 0$$

$$\theta = 26.6°$$

The maximum moment of **F** about point *A* is given by

$$(M_A)_{\text{max}} = 36\cos 26.57° + 18\sin 26.57° = 40.2\ \text{kN}\cdot\text{m}$$

Also,

$$M_A\big|_{\theta=0°} = 36\cos 0° + 18\sin 0° = 36\ \text{kN}\cdot\text{m}$$

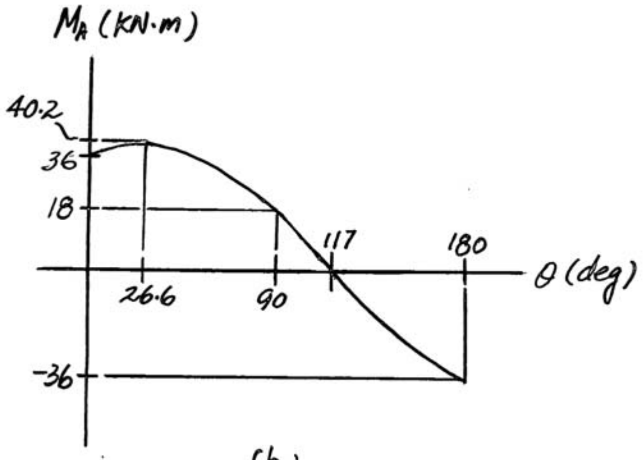
$$M_A\big|_{\theta=90°} = 36\cos 90° + 18\sin 90° = 18\ \text{kN}\cdot\text{m}$$

$$M_A\big|_{\theta=180°} = 36\cos 180° + 18\sin 180° = -36\ \text{kN}\cdot\text{m}$$

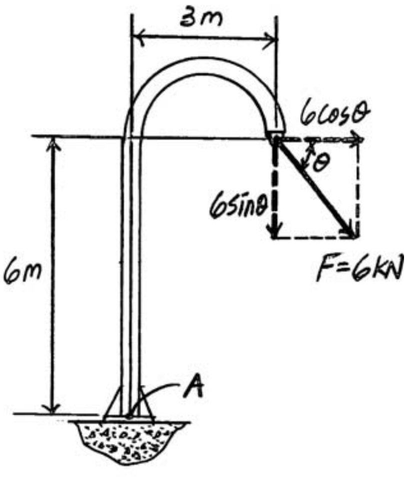
When $M_A = 0$,

$$0 = 36\cos\theta + 18\sin\theta \qquad \theta = 117°$$

The plot of M_A versus θ is shown in Fig. *b*.

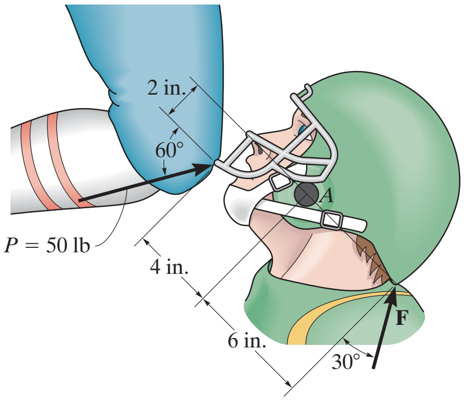


(b)



(a)

4–14. Serious neck injuries can occur when a football player is struck in the face guard of his helmet in the manner shown, giving rise to a guillotine mechanism. Determine the moment of the knee force $P = 50$ lb about point A. What would be the magnitude of the neck force $\mathbf{F}$ so that it gives the counterbalancing moment about A?

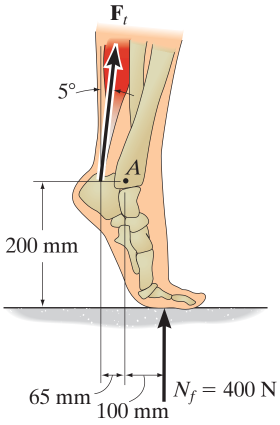


$$\circlearrowleft + M_A = 50 \sin 60° (4) - 50 \cos 60° (2) = 123.2 = 123 \text{ lb} \cdot \text{in.} \circlearrowright \quad \textbf{Ans}$$

$$123.2 = F \cos 30° (6)$$

$$F = 23.7 \text{ lb} \qquad \textbf{Ans}$$

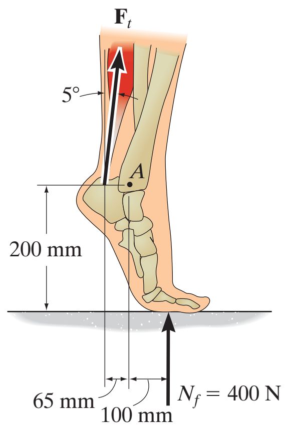
4–15. The Achilles tendon force of $F_t = 650$ N is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of $N_f = 400$ N. Determine the resultant moment of $\mathbf{F}_t$ and $\mathbf{N}_f$ about the ankle joint A.



Referring to Fig. *a*,

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad (M_R)_A = 400(0.1) - 650(0.65)\cos 5°$$
$$= -2.09 \text{ N} \cdot \text{m} = 2.09 \text{ N} \cdot \text{m} \text{ (clockwise)} \qquad \textbf{Ans.}$$

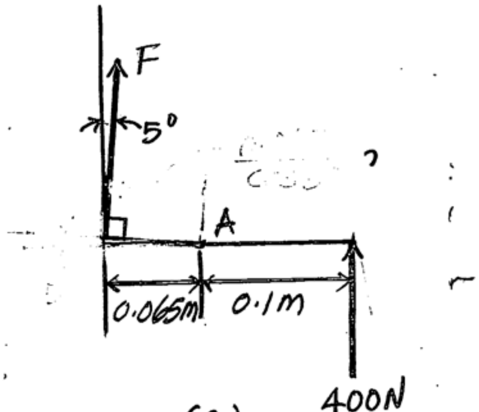
***4–16.** The Achilles tendon force $\mathbf{F}_t$ is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of $N_t = 400$ N. If the resultant moment produced by forces $\mathbf{F}_t$ and $\mathbf{N}_t$ about the ankle joint A is required to be zero, determine the magnitude of $\mathbf{F}_t$.



Referring to Fig. *a*,

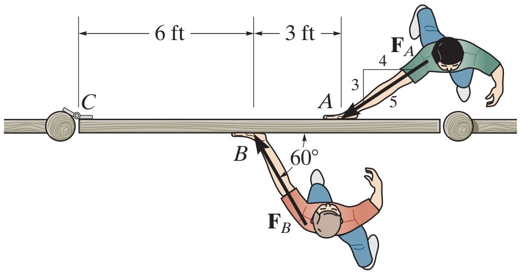
$\circlearrowleft +(M_R)_A = \Sigma Fd;$ $\quad 0 = 400(0.1) - F\cos 5°(0.065)$

$F = 618$ N **Ans.**



(a)

•4–17. The two boys push on the gate with forces of $F_A = 30$ lb and as shown. Determine the moment of each force about C. Which way will the gate rotate, clockwise or counterclockwise? Neglect the thickness of the gate.



$\circlearrowleft + \ (M_{F_A})_C = -30\left(\frac{3}{5}\right)(9)$

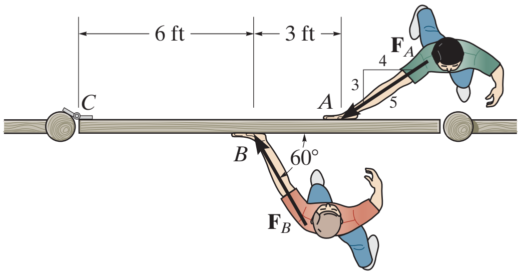
$= -162 \text{ lb}\cdot\text{ft} = 162 \text{ lb}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

$\circlearrowleft + \ (M_{F_B})_C = 50(\sin 60°)(6)$

$= 260 \text{ lb}\cdot\text{ft}$ ***(Counterclockwise)*** **Ans**

Since $(M_{F_B})_C > (M_{F_A})_C$, the gate will rotate ***Counterclockwise.*** **Ans**

4–18. Two boys push on the gate as shown. If the boy at B exerts a force of $F_B = 30$ lb, determine the magnitude of the force $\mathbf{F}_A$ the boy at A must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

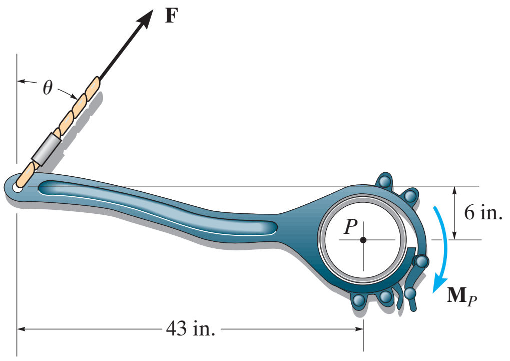


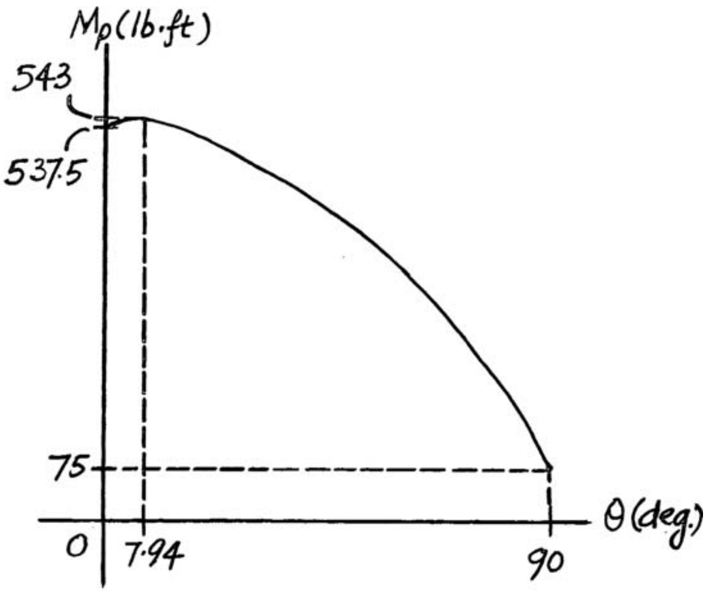
In order to prevent the gate from turning, the resultant moment about point C must be equal to zero.

$\circlearrowleft + M_{R_C} = \Sigma Fd;$ $\quad M_{R_C} = 0 = 30\sin 60°(6) - F_A\left(\frac{3}{5}\right)(9)$

$F_A = 28.9$ lb **Ans**

4–19. The tongs are used to grip the ends of the drilling pipe P. Determine the torque (moment) M_P that the applied force $F = 150$ lb exerts on the pipe about point P as a function of θ. Plot this moment M_P versus θ for $0 \leq \theta \leq 90°$.





$$M_P = 150\cos\theta(43) + 150\sin\theta(6)$$

$$= (6450\cos\theta + 900\sin\theta)\ \text{lb}\cdot\text{in.}$$

$$= (537.5\cos\theta + 75\sin\theta)\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

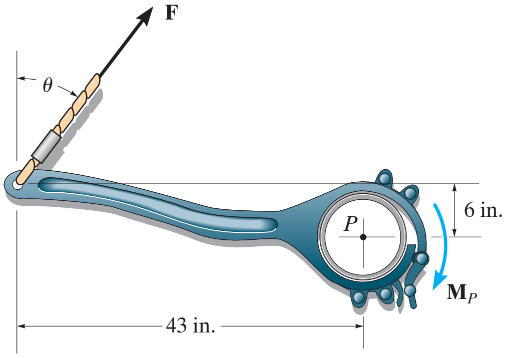
$$\frac{dM_P}{d\theta} = -537.5\sin\theta + 75\cos\theta = 0 \qquad \tan\theta = \frac{75}{537.5} \qquad \theta = 7.943°$$

At $\theta = 7.943°$, M_P is maximum.

$$(M_P)_{max} = 538\cos 7.943° + 75\sin 7.943° = 543\ \text{lb}\cdot\text{ft}$$

Also $(M_P)_{max} = 150\ \text{lb}\left(\left(\frac{43}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right)^{\frac{1}{2}} = 543\ \text{lb}\cdot\text{ft}$

***4–20.** The tongs are used to grip the ends of the drilling pipe P. If a torque (moment) of $M_P = 800$ lb · ft is needed at P to turn the pipe, determine the cable force F that must be applied to the tongs. Set $\theta = 30°$.

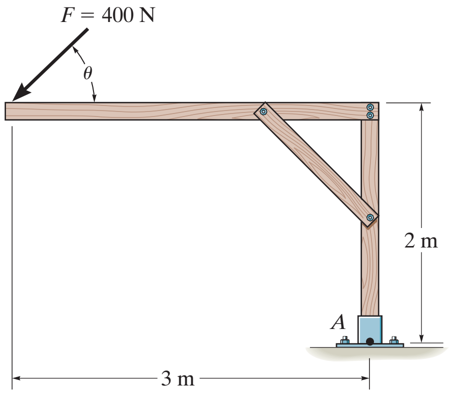


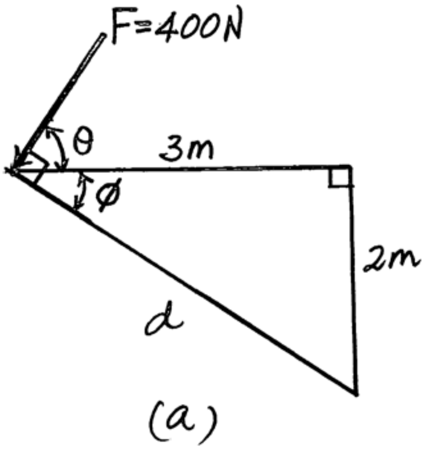
$$M_P = F\cos 30°(43) + F\sin 30°(6) \qquad \text{Set } M_P = 800(12)\ \text{lb}\cdot\text{in.}$$

$$800(12) = F\cos 30°(43) + F\sin 30°(6)$$

$$F = 239\ \text{lb} \qquad \textbf{Ans}$$

•4–21. Determine the direction θ for $0° \leq \theta \leq 180°$ of the force **F** so that it produces the maximum moment about point A. Calculate this moment.



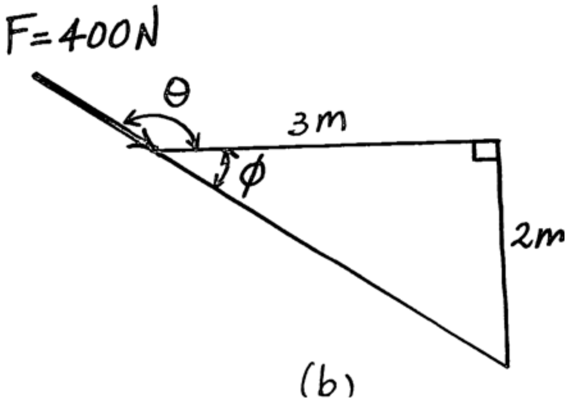


$$\curvearrowleft +M_A = 400\sqrt{(3)^2+(2)^2} = 1442\ \text{N}\cdot\text{m}$$

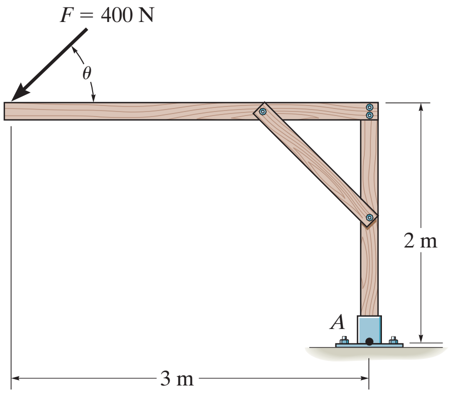
$M_A = 1.44\ \text{kN}\cdot\text{m}$ **Ans**

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$$

$\theta = 90° - 33.69° = 56.3°$ **Ans**



4–22. Determine the moment of the force **F** about point *A* as a function of θ. Plot the results of M (ordinate) versus θ (abscissa) for $0° \le \theta \le 180°$.



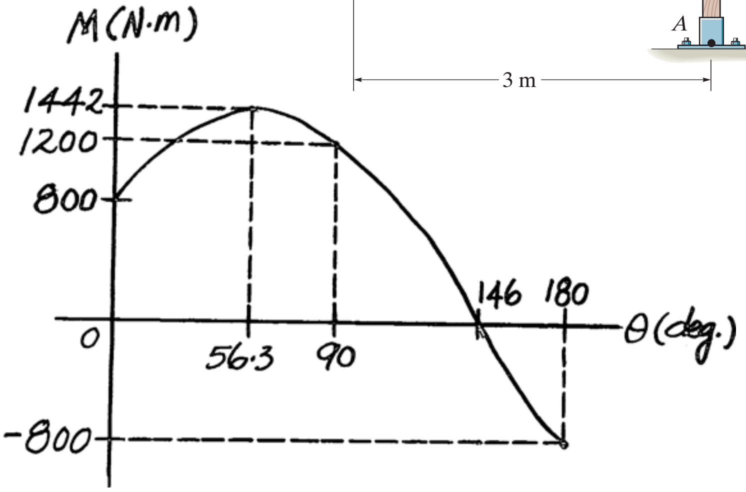
$\circlearrowleft + M_A = 400 \sin\theta(3) + 400 \cos\theta(2)$

$= 1200 \sin\theta + 800 \cos\theta$ **Ans**

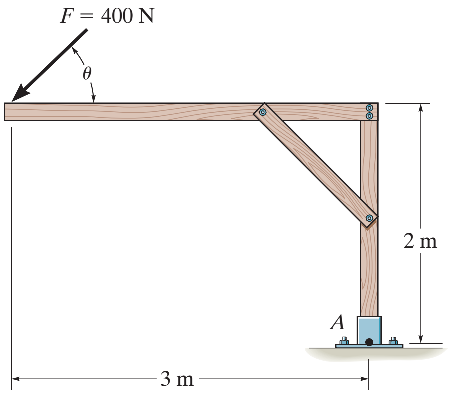
$\frac{dM_A}{d\theta} = 1200 \cos\theta - 800 \sin\theta = 0$

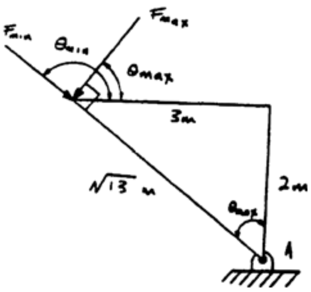
$\theta = \tan^{-1}\left(\frac{1200}{800}\right) = 56.3°$

$(M_A)_{max} = 1200 \sin 56.3° + 800 \cos 56.3° = 1442 \text{ N} \cdot \text{m}$



4–23. Determine the minimum moment produced by the force **F** about point *A*. Specify the angle θ ($0° \le \theta \le 180°$).

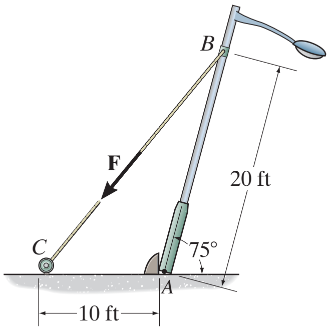




$M_{min} = 400(0) = 0$ **Ans**

$\theta_{min} = 90° + 56.3° = 146°$ **Ans**

***4–24.** In order to raise the lamp post from the position shown, force **F** is applied to the cable. If $F = 200$ lb, determine the moment produced by **F** about point A.



Geometry: Applying the law of cosines to Fig. *a*,

$$BC^2 = 10^2 + 20^2 - 2(10)(20)\cos 105^\circ$$

$$BC = 24.57 \text{ ft}$$

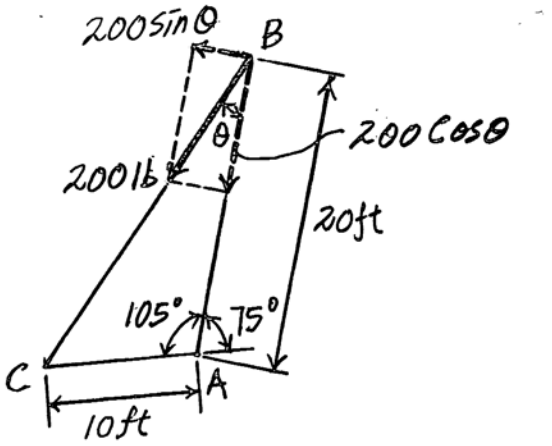
Then, applying the law of sines,

$$\frac{\sin\theta}{10} = \frac{\sin 105^\circ}{24.57} \qquad \theta = 23.15^\circ$$

Moment About Point A: By resolving force **F** into components parallel and perpendicular to the lamp pole, Fig. *a*, and applying the principle of moments,

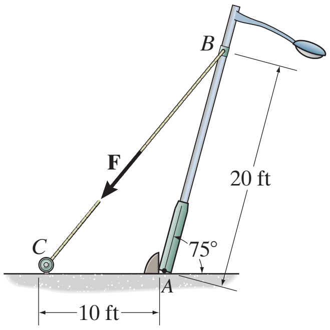
$$\curvearrowleft + (M_R)_A = \Sigma Fd; \qquad M_A = 200\sin 23.15^\circ(20) + 200\cos 23.15^\circ(0)$$

$$= 1572.73 \text{ lb}\cdot\text{ft} = 1.57 \text{ kip}\cdot\text{ft} \text{ (counterclockwise)} \qquad \textbf{Ans.}$$



(a)

•4–25. In order to raise the lamp post from the position shown, the force **F** on the cable must create a counterclockwise moment of 1500 lb · ft about point A. Determine the magnitude of **F** that must be applied to the cable.



Geometry: Applying the law of cosines to Fig. a,

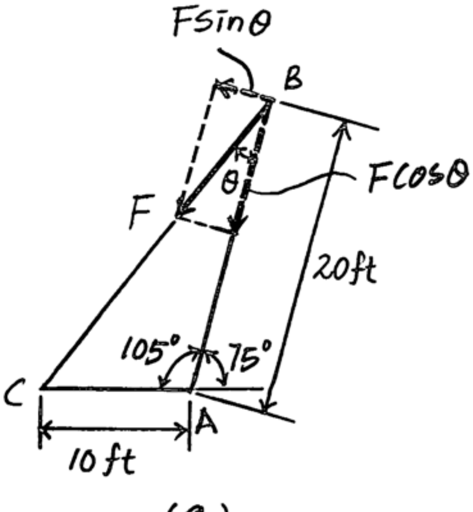
$$BC^2 = 10^2 + 20^2 - 2(10)(20)\cos 105°$$
$$BC = 24.57 \text{ ft}$$

Then, applying the law of sines,

$$\frac{\sin\theta}{10} = \frac{\sin 105°}{24.57} \qquad \theta = 23.15°$$

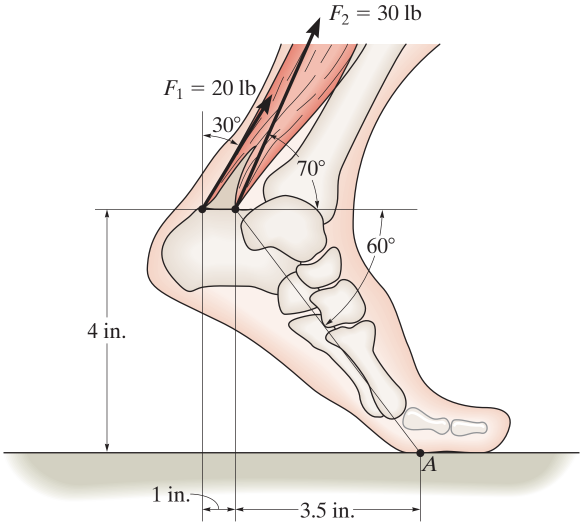
Moment About Point A: By resolving force **F** into components parallel and perpendicular to the lamp pole, Fig. a, and applying the principle of moments,

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad 1500 = F\sin 23.15°(20)$$
$$F = 191 \text{ lb}$$ **Ans.**



(a)

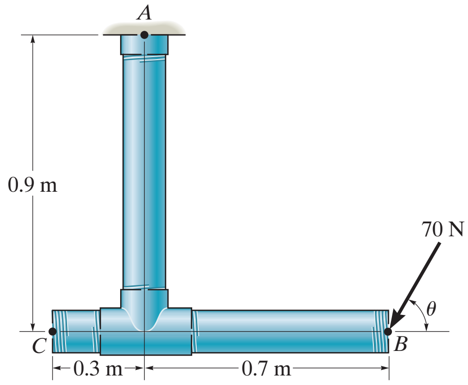
4–26. The foot segment is subjected to the pull of the two plantarflexor muscles. Determine the moment of each force about the point of contact A on the ground.



$(M_A)_1 = 20 \cos 30°(4.5) + 20 \sin 30°(4) = 118 \text{ lb} \cdot \text{in.} \curvearrowright$ **Ans**

$(M_A)_2 = 30 \cos 70°(4) + 30 \sin 70°(3.5) = 140 \text{ lb} \cdot \text{in.} \curvearrowright$ **Ans**

4–27. The 70-N force acts on the end of the pipe at B. Determine (a) the moment of this force about point A, and (b) the magnitude and direction of a horizontal force, applied at C, which produces the same moment. Take $\theta = 60°$.



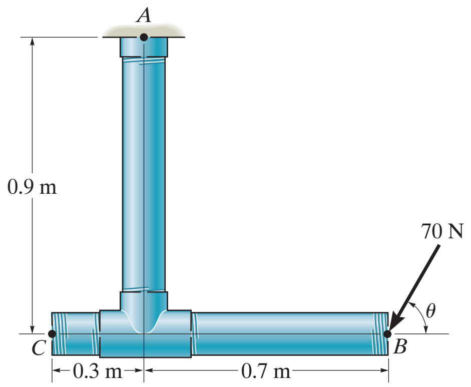
(a) $\circlearrowleft + M_A = 70 \sin 60°(0.7) + 70 \cos 60°(0.9)$

$M_A = 73.94 = 73.9 \text{ N} \cdot \text{m} \curvearrowright$ **Ans**

(b) $F_C(0.9) = 73.94$

$F_C = 82.2 \text{ N} \leftarrow$ **Ans**

***4–28.** The 70-N force acts on the end of the pipe at B. Determine the angles θ $(0^\circ \leq \theta \leq 180^\circ)$ of the force that will produce maximum and minimum moments about point A. What are the magnitudes of these moments?



$$\circlearrowleft + M_A = 70\sin\theta(0.7) + 70\cos\theta(0.9)$$

$$M_A = 49\sin\theta + 63\cos\theta$$

For maximum moment $\dfrac{dM_A}{d\theta} = 0$

$$\frac{dM_A}{d\theta} = 0; \qquad 49\cos\theta - 63\sin\theta = 0$$

$$\theta = \tan^{-1}\left(\frac{49}{63}\right) = 37.9^\circ \qquad \textbf{Ans}$$

$$(M_A)_{max} = 49\sin 37.9^\circ + 63\cos 37.9^\circ$$

$$= 79.8\ \mathrm{N\cdot m} \qquad \textbf{Ans}$$

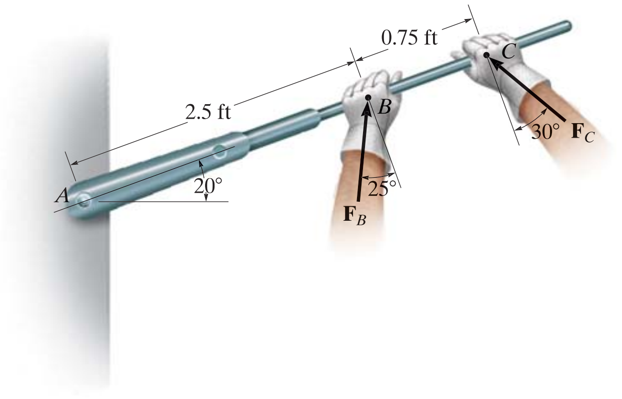
For minimum moment $M_A = 0$

$$M_A = 0; \qquad 49\sin\theta + 63\cos\theta = 0$$

$$\theta = 180^\circ + \tan^{-1}\left(\frac{-63}{49}\right) = 128^\circ \qquad \textbf{Ans}$$

$$(M_A)_{min} = 49\sin 128^\circ + 63\cos 128^\circ = 0 \qquad \textbf{Ans}$$

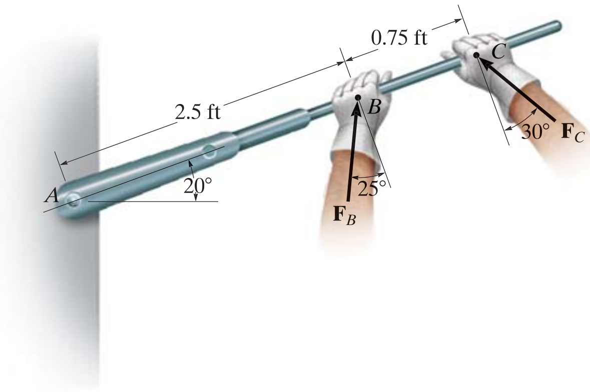
•4–29. Determine the moment of each force about the bolt located at A. Take $F_B = 40$ lb, $F_C = 50$ lb.



$$\circlearrowleft + M_B = 40\cos 25^\circ(2.5) = 90.6\ \mathrm{lb\cdot ft} \circlearrowleft \qquad \textbf{Ans}$$

$$\circlearrowleft + M_C = 50\cos 30^\circ(3.25) = 141\ \mathrm{lb\cdot ft} \circlearrowleft \qquad \textbf{Ans}$$

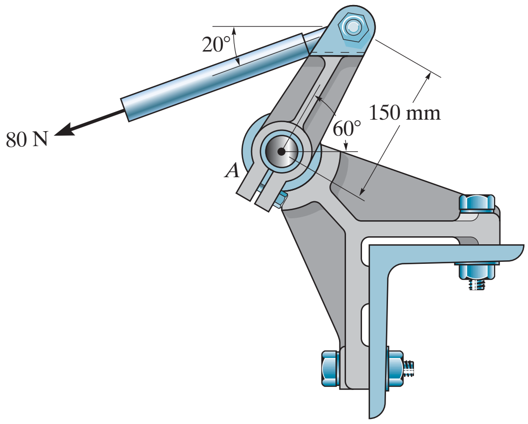
4–30. If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at A.



$$\circlearrowleft + M_A = 30\cos 25°(2.5) + 45\cos 30°(3.25)$$

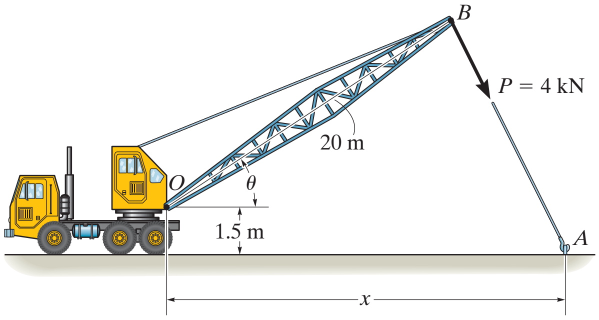
$$= 195 \text{ lb}\cdot\text{ft} \circlearrowleft \qquad \textbf{Ans}$$

4–31. The rod on the power control mechanism for a business jet is subjected to a force of 80 N. Determine the moment of this force about the bearing at A.



$$\circlearrowleft + M_A = 80\cos 20°\,(0.15\sin 60°) - 80\sin 20°\,(0.15\cos 60°) = 7.71\text{ N}\cdot\text{m} \circlearrowleft \qquad \textbf{Ans}$$

***4–32.** The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at A so that this force creates a maximum moment about point O. What is this moment?



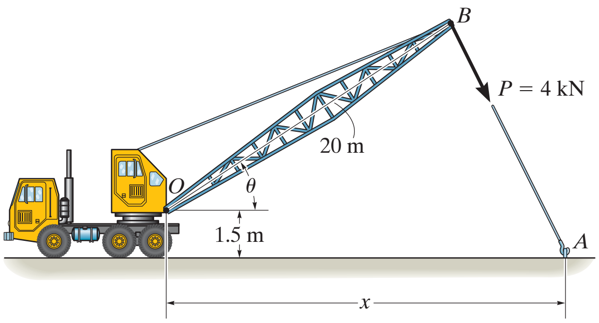
Maximum moment, $OB \perp BA$

$$\circlearrowright + (M_O)_{max} = 4\text{ kN}(20) = 80\text{ kN}\cdot\text{m} \circlearrowright \qquad \textbf{Ans}$$

$$4\text{ kN}\sin 60°(x) - 4\text{ kN}\cos 60°(1.5) = 80\text{ kN}\cdot\text{m}$$

$$x = 24.0\text{ m} \qquad \textbf{Ans}$$

•4–33. The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $x = 25$ m, determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?



Maximum moment, $OB \perp BA$

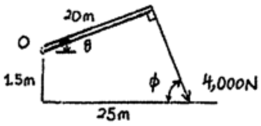
$\circlearrowleft + (M_O)_{max} = 4000(20) = 80\,000 \text{ N}\cdot\text{m} = 80.0 \text{ kN}\cdot\text{m}$ **Ans**

$4000 \sin\phi(25) - 4000 \cos\phi(1.5) = 80\,000$

$25 \sin\phi - 1.5 \cos\phi = 20$

$\phi = 56.43°$

$\theta = 90° - 56.43° = 33.6°$ **Ans**



Also,

$(1.5)^2 + z^2 = y^2$

$2.25 + z^2 = y^2$

Similar triangles

$$\frac{20+y}{z} = \frac{25+z}{y}$$

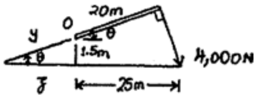
$20y + y^2 = 25z + z^2$

$20(\sqrt{2.25 + z^2}) + 2.25 + z^2 = 25z + z^2$

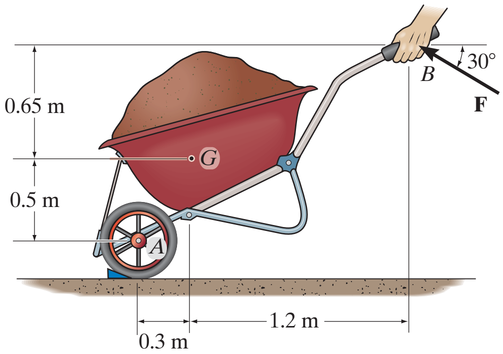
$z = 2.259$ m

$y = 2.712$ m

$\theta = \cos^{-1}\left(\frac{2.259}{2.712}\right) = 33.6°$ **Ans**



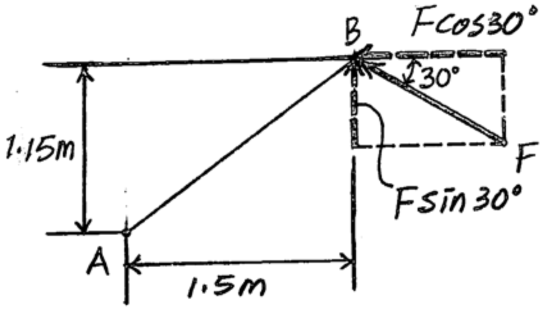
4–34. In order to hold the wheelbarrow in the position shown, force **F** must produce a counterclockwise moment of 200 N · m about the axle at *A*. Determine the required magnitude of force **F**.



Resolving force **F** into its horzontal and vertical components, Fig. *a*, and applying the principle of moments,

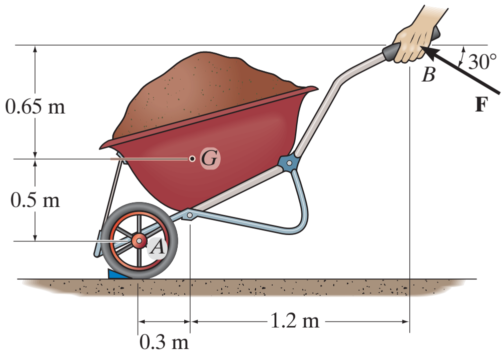
$$\circlearrowleft + M_A = 200 = F\sin 30°(1.5) + F\cos 30°(1.15)$$

$$F = 115\,\text{N}$$ **Ans.**



(a)

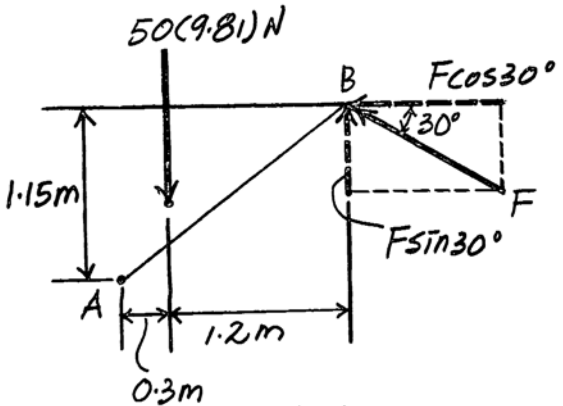
4–35. The wheelbarrow and its contents have a mass of 50 kg and a center of mass at G. If the resultant moment produced by force **F** and the weight about point A is to be zero, determine the required magnitude of force **F**.



Resolving force **F** into its horzontal and vertical components, Fig. a, and applying the principle of moments,

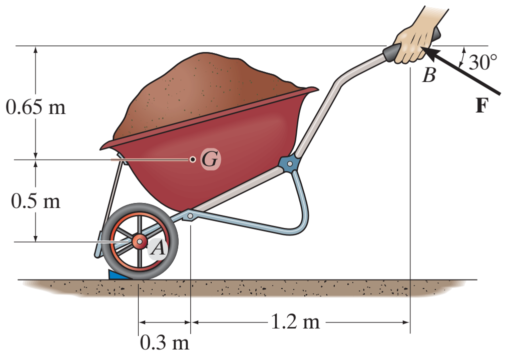
$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad 0 = F\sin 30^\circ(1.5) + F\cos 30^\circ(1.15) - 50(9.81)(0.3)$$

$$F = 84.3\ \text{N} \qquad \textbf{Ans.}$$



(a)

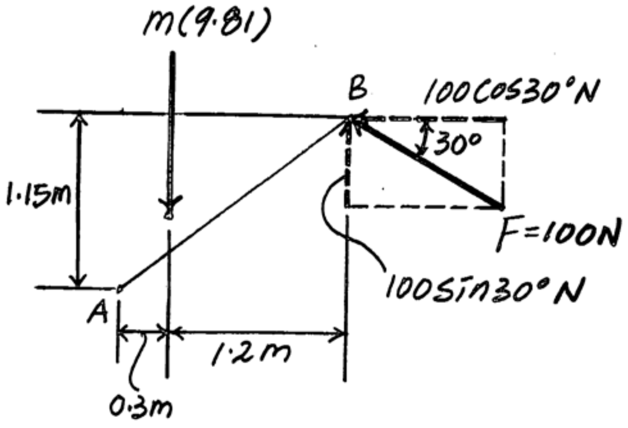
***4–36.** The wheelbarrow and its contents have a center of mass at *G*. If $F = 100$ N and the resultant moment produced by force **F** and the weight about the axle at *A* is zero, determine the mass of the wheelbarrow and its contents.



Resolving force **F** into its horzontal and vertical components, Fig. *a*, and applying the principle of moments,

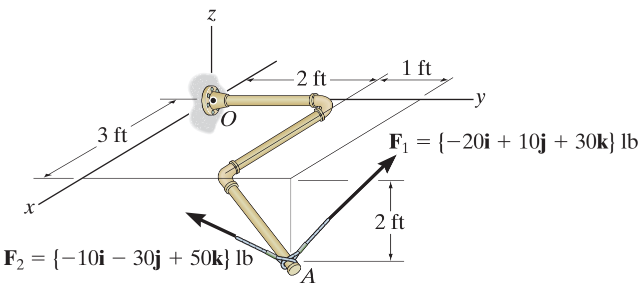
$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad 0 = 100\cos 30°(1.15) + 100\sin 30°(1.5) - M(9.81)(0.3)$$

$$M = 59.3 \text{ kg}$$ **Ans.**



(a)

•4–37. Determine the moment produced by $\mathbf{F}_1$ about point O. Express the result as a Cartesian vector.

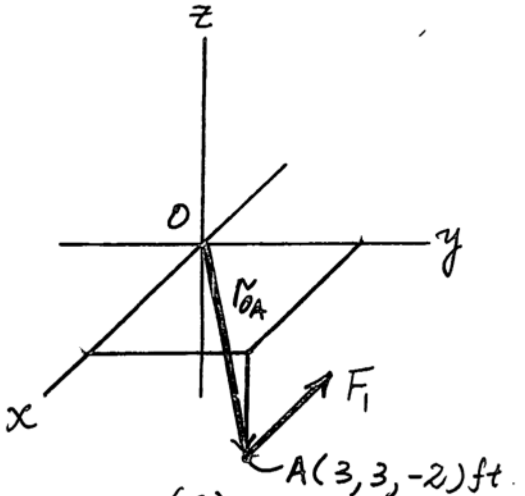

z
2 ft
1 ft
y
O
3 ft
$\mathbf{F}_1 = \{-20\mathbf{i} + 10\mathbf{j} + 30\mathbf{k}\}$ lb
x
2 ft
$\mathbf{F}_2 = \{-10\mathbf{i} - 30\mathbf{j} + 50\mathbf{k}\}$ lb
A

Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = (3-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k} = [3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]\,\text{ft}$$

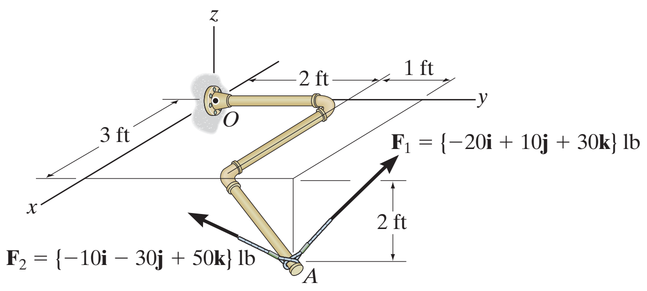
Vector Cross Product: The moment of $\mathbf{F}_1$ about point O is

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -20 & 10 & 30 \end{vmatrix} = [110\mathbf{i} - 50\mathbf{j} + 90\mathbf{k}]\,\text{lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



(a)

4–38. Determine the moment produced by $\mathbf{F}_2$ about point *O*. Express the result as a Cartesian vector.

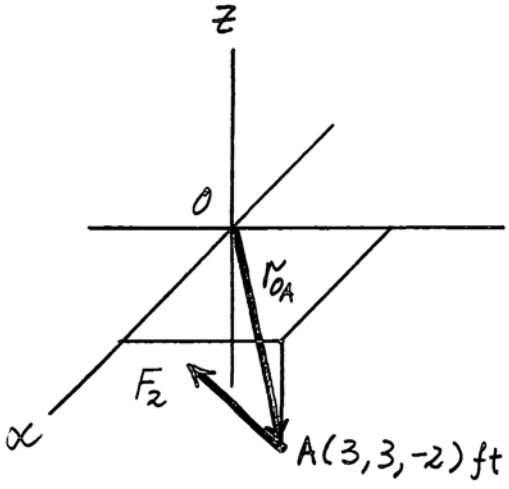

z
2 ft
1 ft
y
O
3 ft
$\mathbf{F}_1 = \{-20\mathbf{i} + 10\mathbf{j} + 30\mathbf{k}\}$ lb
2 ft
x
$\mathbf{F}_2 = \{-10\mathbf{i} - 30\mathbf{j} + 50\mathbf{k}\}$ lb
A

Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = (3-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k} = [3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]\,\text{ft}$$

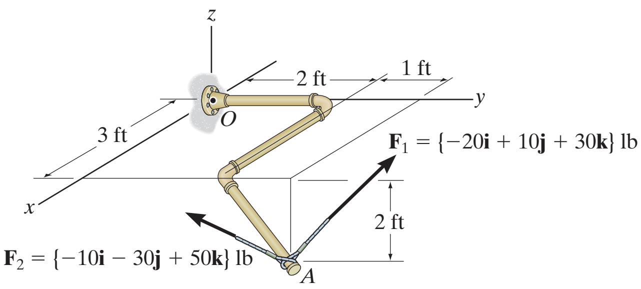
Vector Cross Product: The moment of $\mathbf{F}_2$ about point *O* is

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -10 & -30 & 50 \end{vmatrix} = [90\mathbf{i} - 130\mathbf{j} - 60\mathbf{k}]\,\text{lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



(a)

4–39. Determine the resultant moment produced by the two forces about point O. Express the result as a Cartesian vector.


z
2 ft
1 ft
y
O
3 ft
x
$\mathbf{F}_1 = \{-20\mathbf{i} + 10\mathbf{j} + 30\mathbf{k}\}$ lb
2 ft
$\mathbf{F}_2 = \{-10\mathbf{i} - 30\mathbf{j} + 50\mathbf{k}\}$ lb
A

Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. a, must be determined first.

$$\mathbf{r}_{OA} = (3-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k} = [3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]\text{ft}$$

Resultant Moment: The resultant moment of $\mathbf{F}_1$ and $\mathbf{F}_2$ about point O can be determined by

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OA} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -20 & 10 & 30 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -10 & -30 & 50 \end{vmatrix}$$

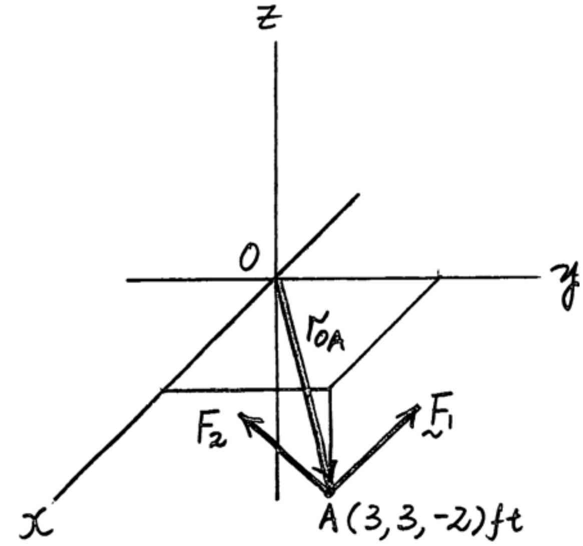
$$= [200\mathbf{i} - 180\mathbf{j} + 30\mathbf{k}]\ \text{lb}\cdot\text{ft}$$ **Ans.**

Or we can apply the principle of moments which gives

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times (\mathbf{F}_1 + \mathbf{F}_2)$$

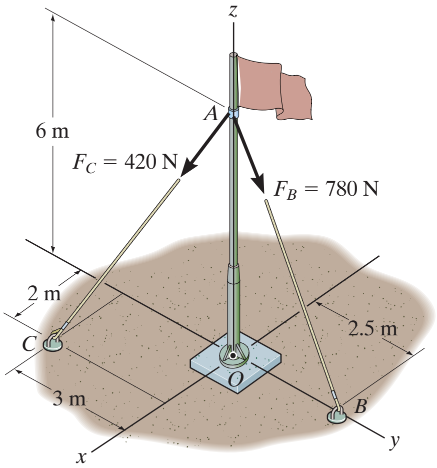
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & -2 \\ -30 & -20 & 80 \end{vmatrix}$$

$$= \{200\mathbf{i} - 180\mathbf{j} + 30\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$ **Ans.**



(a)

***4–40.** Determine the moment produced by force $\mathbf{F}_B$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OB}$ can be used to determine the moment of $\mathbf{F}_B$ about point O.

$\mathbf{r}_{OA} = \{6\mathbf{k}\}$ m $\qquad \mathbf{r}_{OB} = \{2.5\mathbf{j}\}$ m

The force vector $\mathbf{F}_B$ is given by

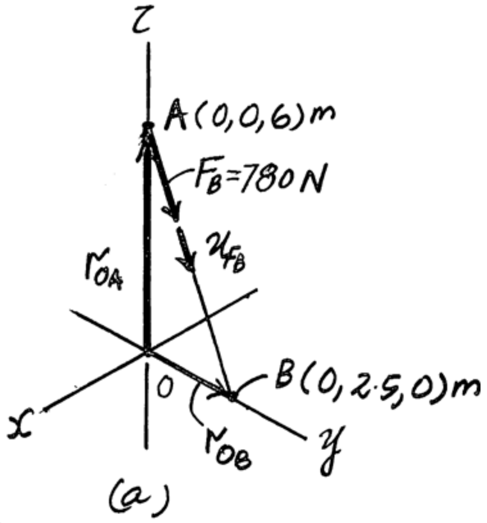
$$\mathbf{F}_B = F_B\mathbf{u}_{FB} = 780\left[\frac{(0-0)\mathbf{i}+(2.5-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(2.5-0)^2+(0-6)^2}}\right] = \{300\mathbf{j}-720\mathbf{k}\}\text{N}$$

Vector Cross Product: The moment of $\mathbf{F}_B$ about point O is given by

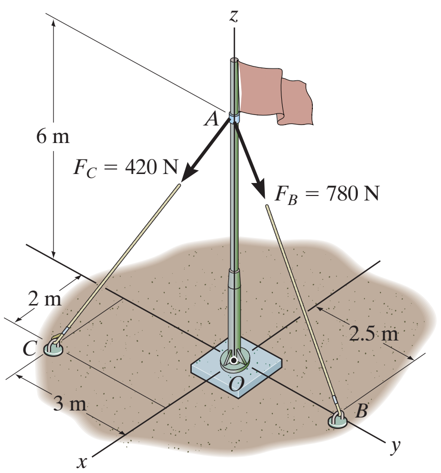
$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}_B = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 0 & 300 & -720\end{vmatrix} = \{-1800\mathbf{i}\}\text{ N}\cdot\text{m} = \{-1.80\mathbf{i}\}\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

or

$$\mathbf{M}_O = \mathbf{r}_{OB}\times\mathbf{F}_B = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 2.5 & 0\\ 0 & 300 & -720\end{vmatrix} = \{-1800\mathbf{i}\}\text{ N}\cdot\text{m} = \{-1.80\mathbf{i}\}\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



•4–41. Determine the moment produced by force $\mathbf{F}_C$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: Either position vector $\mathbf{r}_{OA}$ or $\mathbf{r}_{OC}$ can be used to determine the moment of $\mathbf{F}_C$ about point O.

$\mathbf{r}_{OA} = \{6\mathbf{k}\}$ m

$\mathbf{r}_{OC} = (2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-0)\mathbf{k} = [2\mathbf{i} - 3\mathbf{j}]$ m

The force vector $\mathbf{F}_C$ is given by

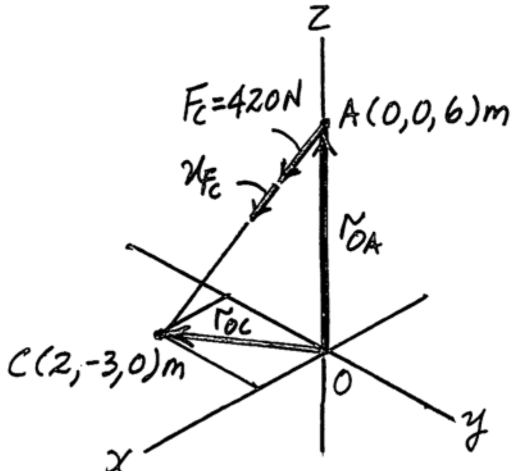
$$\mathbf{F}_C = F_C \mathbf{u}_{FC} = 420\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}}\right] = [120\mathbf{i} - 180\mathbf{j} - 360\mathbf{k}]\text{N}$$

Vector Cross Product: The moment of $\mathbf{F}_C$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 120 & -180 & -360 \end{vmatrix} = [1080\mathbf{i} + 720\mathbf{j}]\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

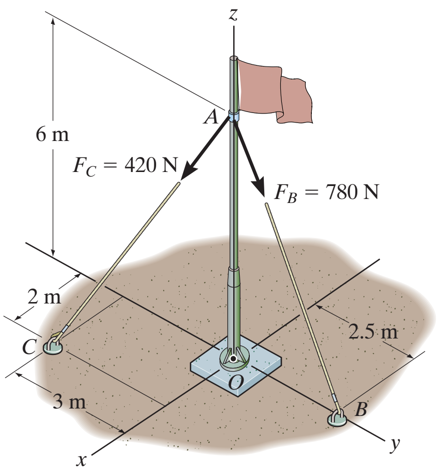
or

$$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 0 \\ 120 & -180 & -360 \end{vmatrix} = [1080\mathbf{i} + 720\mathbf{j}]\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–42. Determine the resultant moment produced by forces $\mathbf{F}_B$ and $\mathbf{F}_C$ about point O. Express the result as a Cartesian vector.



Position Vector and Force Vectors: The position vector $\mathbf{r}_{OA}$ and force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = \{6\mathbf{k}\}\text{ m}$$

$$\mathbf{F}_B = F_B\mathbf{u}_{FB} = 780\left[\frac{(0-0)\mathbf{i}+(2.5-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(0-0)^2+(2.5-0)^2+(0-6)^2}}\right] = [300\mathbf{j}-720\mathbf{k}]\text{N}$$

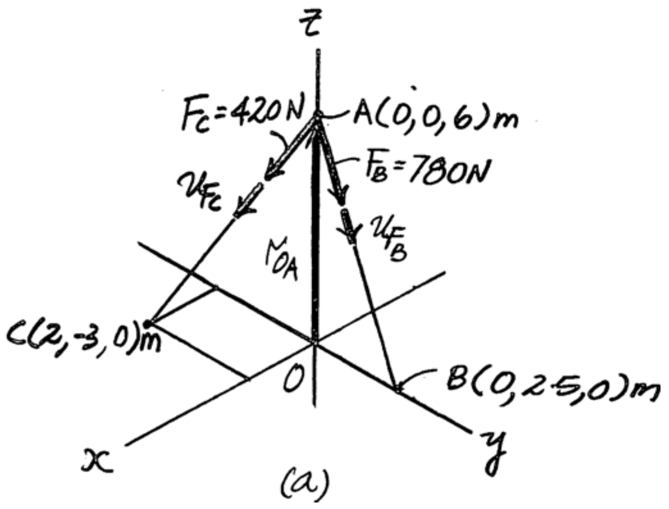
$$\mathbf{F}_C = F_C\mathbf{u}_{FC} = 420\left[\frac{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-6)\mathbf{k}}{\sqrt{(2-0)^2+(-3-0)^2+(0-6)^2}}\right] = [120\mathbf{i}-180\mathbf{j}-360\mathbf{k}]\text{N}$$

Resultant Moment: The resultant moment of $\mathbf{F}_B$ and $\mathbf{F}_C$ about point O is given by

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}_B + \mathbf{r}_{OA}\times\mathbf{F}_C$$

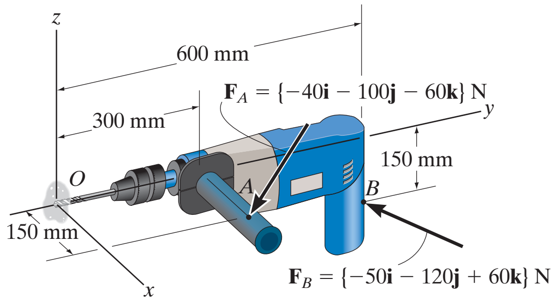
$$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 0 & 300 & -720\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 120 & -180 & -360\end{vmatrix}$$

$$= [-720\mathbf{i} + 720\mathbf{j}]\ \text{N}\cdot\text{m}$$ **Ans.**



(a)

4–43. Determine the moment produced by each force about point *O* located on the drill bit. Express the results as Cartesian vectors.


z
600 mm
$\mathbf{F}_A = \{-40\mathbf{i} - 100\mathbf{j} - 60\mathbf{k}\}$ N
y
300 mm
150 mm
O
A
B
150 mm
x
$\mathbf{F}_B = \{-50\mathbf{i} - 120\mathbf{j} + 60\mathbf{k}\}$ N

Position Vector: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = (0.15-0)\mathbf{i} + (0.3-0)\mathbf{j} + (0-0)\mathbf{k} = [0.15\mathbf{i} + 0.3\mathbf{j}]\ \text{m}$$

$$\mathbf{r}_{OB} = (0-0)\mathbf{i} + (0.6-0)\mathbf{j} + (-0.15-0)\mathbf{k} = [0.6\mathbf{j} - 0.15\mathbf{k}]\ \text{m}$$

Vector Cross Product: The moment of $\mathbf{F}_A$ about point *O* is

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.3 & 0 \\ -40 & -100 & -60 \end{vmatrix}$$

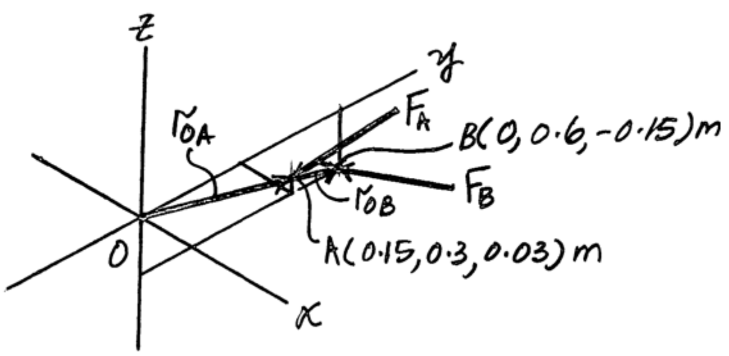
$$= [-18\mathbf{i} + 9\mathbf{j} - 3\mathbf{k}]\ \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$

The moment of $\mathbf{F}_B$ about point *O* is

$$(\mathbf{M}_R)_O = \mathbf{r}_{OB} \times \mathbf{F}_B$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.6 & -0.15 \\ -50 & -120 & 60 \end{vmatrix}$$

$$= [18\mathbf{i} + 7.5\mathbf{j} + 30\mathbf{k}]\ \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$



(a)

***4–44.** A force of $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produces a moment of $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m about the origin of coordinates, point O. If the force acts at a point having an x coordinate of $x = 1$ m, determine the y and z coordinates.

$$\mathbf{M}_{RO} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix} = [4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}]\ \text{kN}\cdot\text{m}$$

$y + 2z = 4$

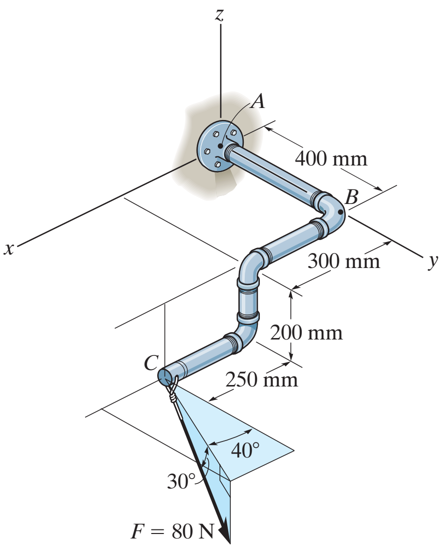
$-1 + 6z = 5$

$-2 - 6y = -14$

$y = 2$ m **Ans.**

$z = 1$ m **Ans.**

•4–45. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.



Position Vector And Force Vector :

$$\mathbf{r}_{AC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\ \text{m}$$
$$= \{0.55\mathbf{i} + 0.4\mathbf{j} - 0.2\mathbf{k}\}\ \text{m}$$

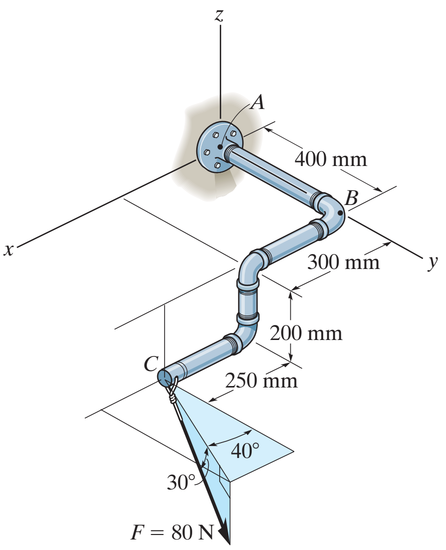
$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k})\ \text{N}$$
$$= \{44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\ \text{N}$$

Moment of Force* F *About Point A : Applying Eq. 4 – 7, we have

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans**

4–46. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point *B*.



Position Vector And Force Vector :

$$\mathbf{r}_{BC} = \{(0.55-0)\mathbf{i}+(0.4-0.4)\mathbf{j}+(-0.2-0)\mathbf{k}\}\text{ m}$$
$$= \{0.55\mathbf{i}-0.2\mathbf{k}\}\text{ m}$$

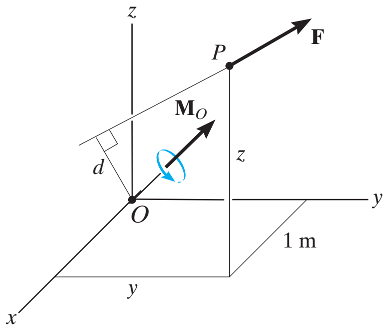
$$\mathbf{F} = 80(\cos 30^\circ\sin 40^\circ\mathbf{i}+\cos 30^\circ\cos 40^\circ\mathbf{j}-\sin 30^\circ\mathbf{k})\text{ N}$$
$$= \{44.53\mathbf{i}+53.07\mathbf{j}-40.0\mathbf{k}\}\text{ N}$$

Moment of Force F About Point B : Applying Eq. 4 – 7, we have

$$\mathbf{M}_B = \mathbf{r}_{BC}\times\mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{10.6\mathbf{i}+13.1\mathbf{j}+29.2\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

4–47. The force **F** = {6**i** + 8**j** + 10**k**} N creates a moment about point *O* of $\mathbf{M}_O$ = {−14**i** + 8**j** + 2**k**} N·m. If the force passes through a point having an *x* coordinate of 1 m, determine the *y* and *z* coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance *d* from point *O* to the line of action of **F**.



$$-14\mathbf{i}+8\mathbf{j}+2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$$-14 = 10y - 8z$$

$$8 = -10 + 6z$$

$$2 = 8 - 6y$$

$$y = 1\text{ m} \qquad \textbf{Ans}$$

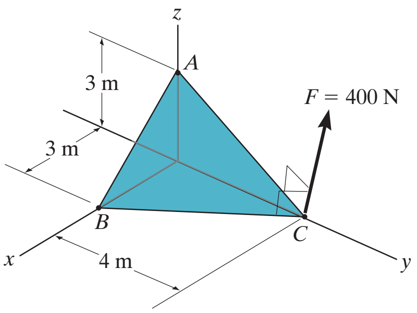
$$z = 3\text{ m} \qquad \textbf{Ans}$$

$$M_O = \sqrt{(-14)^2+(8)^2+(2)^2} = 16.25\text{ N}\cdot\text{m}$$

$$F = \sqrt{(6)^2+(8)^2+(10)^2} = 14.14\text{ N}$$

$$d = \frac{16.25}{14.14} = 1.15\text{ m} \qquad \textbf{Ans}$$

***4–48.** Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point *A*. Express the result as a Cartesian vector.



Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. *a*. Here

$$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]\ \text{m}$$

$$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}]\ \text{m}$$

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$= [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\ \text{m}^2$$

Then,

$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

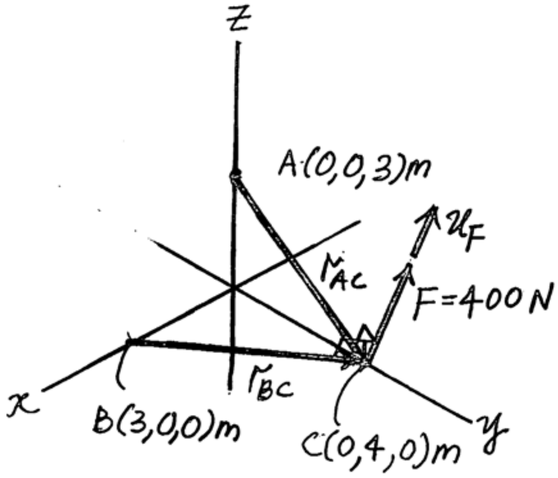
$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$

$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\ \text{N}$$

Vector Cross Product: The moment of **F** about point *A* is

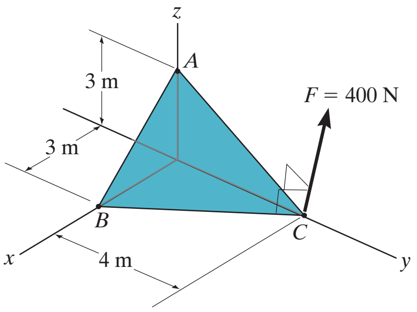
$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$

$$= [1.56\mathbf{i} - 0.750\mathbf{j} - 1\mathbf{k}]\ \text{kN} \cdot \text{m} \qquad \textbf{Ans.}$$



(a)

•4–49. Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point B. Express the result as a Cartesian vector.



Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. a. Here

$$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}]\text{ m}$$

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix} = [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\text{ m}^2$$

Then,

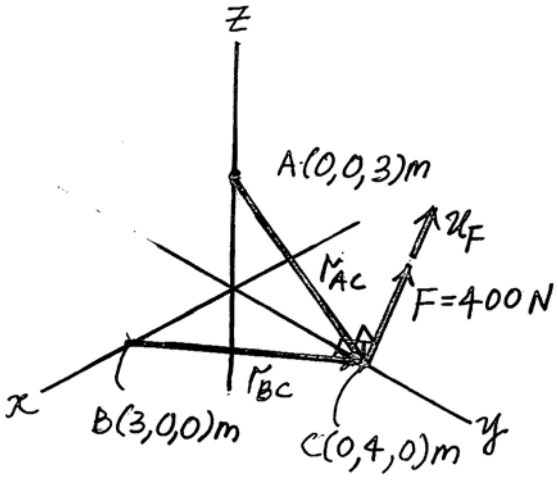
$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$
$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\text{ N}$$

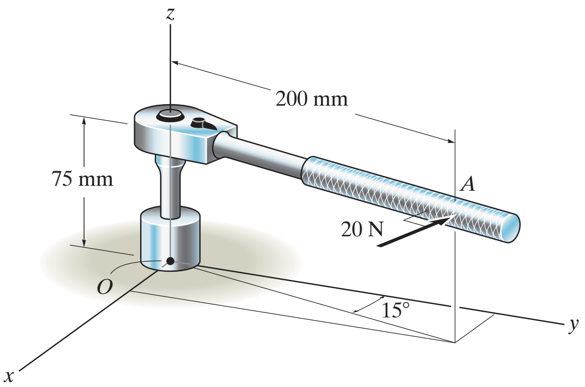
Vector Cross Product: The moment of **F** about point B is

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$
$$= [1\mathbf{i} + 0.750\mathbf{j} - 1.56\mathbf{k}]\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–50. A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and the coordinate direction angles of the moment created by this force about point *O*.



$$\mathbf{r}_A = 0.2 \sin 15^\circ\,\mathbf{i} + 0.2 \cos 15^\circ\,\mathbf{j} + 0.075\,\mathbf{k}$$

$$= 0.05176\,\mathbf{i} + 0.1932\,\mathbf{j} + 0.075\,\mathbf{k}$$

$$\mathbf{F} = -20 \cos 15^\circ\,\mathbf{i} + 20 \sin 15^\circ\,\mathbf{j}$$

$$= -19.32\,\mathbf{i} + 5.176\,\mathbf{j}$$

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.05176 & 0.1932 & 0.075 \\ -19.32 & 5.176 & 0 \end{vmatrix}$$

$$= \{-0.3882\,\mathbf{i} - 1.449\,\mathbf{j} + 4.00\,\mathbf{k}\}\ \text{N}\cdot\text{m}$$

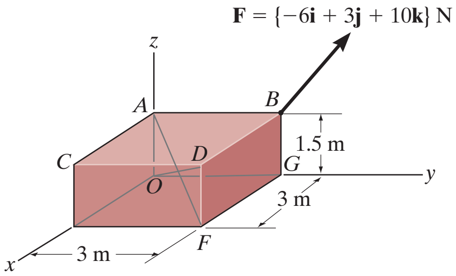
$M_O = 4.272 = 4.27\ \text{N}\cdot\text{m}$ **Ans**

$\alpha = \cos^{-1}\left(\frac{-0.3882}{4.272}\right) = 95.2^\circ$ **Ans**

$\beta = \cos^{-1}\left(\frac{-1.449}{4.272}\right) = 110^\circ$ **Ans**

$\gamma = \cos^{-1}\left(\frac{4}{4.272}\right) = 20.6^\circ$ **Ans**

4–51. Determine the moment produced by force **F** about the diagonal *AF* of the rectangular block. Express the result as a Cartesian vector.



Moment About Diagonal AF: Either position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{FB}$, Fig. *a*, can be used to find the moment of **F** about diagonal *AF*.

$$\mathbf{r}_{AB} = (0-0)\mathbf{i}+(3-0)\mathbf{j}+(1.5-1.5)\mathbf{k} = [3\mathbf{j}]\ \text{m}$$

$$\mathbf{r}_{FB} = (0-3)\mathbf{i}+(3-3)\mathbf{j}+(1.5-0)\mathbf{k} = [-3\mathbf{i}+1.5\mathbf{k}]\ \text{m}$$

The unit vector $\mathbf{u}_{AF}$, Fig. *a*, that specifies the direction of diagonal *AF* is given by

$$\mathbf{u}_{AF} = \frac{(3-0)\mathbf{i}+(3-0)\mathbf{j}+(0-1.5)\mathbf{k}}{\sqrt{(3-0)^2+(3-0)^2+(0-1.5)^2}} = \frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}$$

The magnitude of the moment of **F** about diagonal *AF* axis is

$$M_{AF} = \mathbf{u}_{AF}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 3 & 0 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[3(10)-(3)(0)] - \frac{2}{3}[0(10)-(-6)(0)] + \left(-\frac{1}{3}\right)[0(3)-(-6)(3)]$$

$$= 14\ \text{N}\cdot\text{m}$$

or

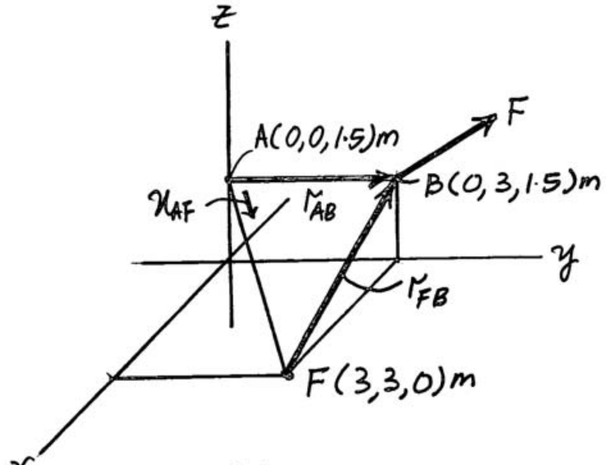
$$M_{AF} = \mathbf{u}_{AF}\cdot\mathbf{r}_{FB}\times\mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -3 & 0 & 1.5 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[0(10)-(3)(1.5)] - \frac{2}{3}[(-3)(10)-(-6)(1.5)] + \left(-\frac{1}{3}\right)[(-3)(3)-(-6)(0)]$$

$$= 14\ \text{N}\cdot\text{m}$$

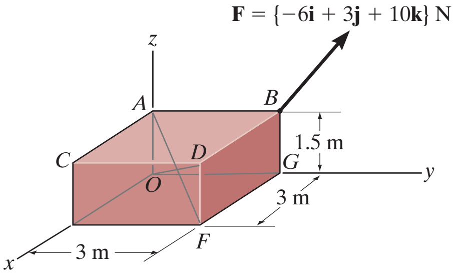
Thus, $\mathbf{M}_{AF}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{AF} = M_{AF}\,\mathbf{u}_{AF} = 14\left(\frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}\right) = [9.33\mathbf{i}+9.33\mathbf{j}-4.67\mathbf{k}]\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

***4–52.** Determine the moment produced by force **F** about the diagonal *OD* of the rectangular block. Express the result as a Cartesian vector.



Moment About Diagonal OD: Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{DB}$, Fig. *a*, can be used to find the moment of **F** about diagonal *OD*.

$$\mathbf{r}_{OB} = (0-0)\mathbf{i} + (3-0)\mathbf{j} + (1.5-0)\mathbf{k} = [3\mathbf{j} + 1.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{DB} = (0-3)\mathbf{i} + (3-3)\mathbf{j} + (1.5-1.5)\mathbf{k} = [-3\mathbf{i}]\text{ m}$$

The unit vector $\mathbf{u}_{OD}$, Fig. *a*, that specifies the direction of diagonal *OD* is given by

$$\mathbf{u}_{AF} = \frac{(3-0)\mathbf{i} + (3-0)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(3-0)^2 + (3-0)^2 + (0-1.5)^2}} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

The magnitude of the moment of **F** about diagonal *OD* is

$$M_{OD} = \mathbf{u}_{OD} \cdot \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 3 & 1.5 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[3(10) - (3)(1.5)] - \frac{2}{3}[0(10) - (-6)(1.5)] + \frac{1}{3}[0(3) - (-6)(3)]$$

$$= 17\text{ N}\cdot\text{m}$$

or

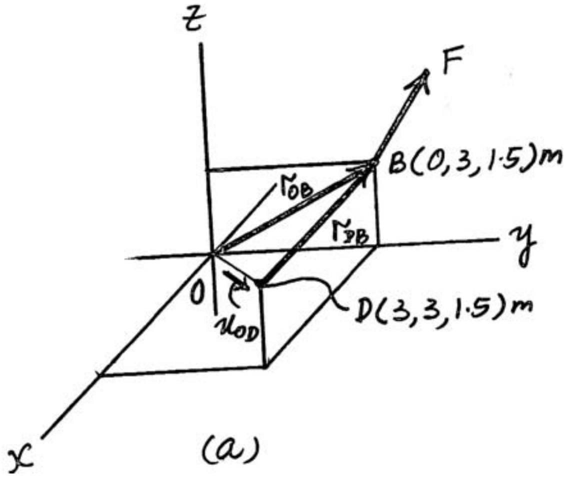
$$M_{OD} = \mathbf{u}_{OD} \cdot \mathbf{r}_{DB} \times \mathbf{F} = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -3 & 0 & 0 \\ -6 & 3 & 10 \end{vmatrix}$$

$$= \frac{2}{3}[0(10) - (3)(0)] - \frac{2}{3}[-3(10) - (-6)(0)] + \frac{1}{3}[-3(3) - (-6)(0)]$$

$$= 17\text{ N}\cdot\text{m}$$

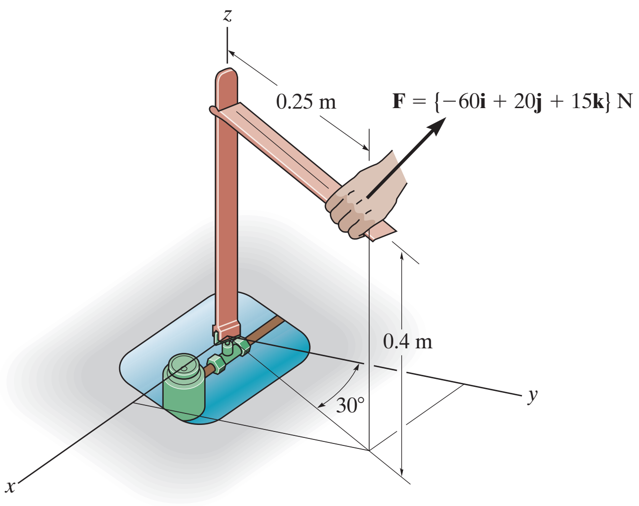
Thus, $\mathbf{M}_{OD}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{OD} = M_{OD}\mathbf{u}_{OD} = 17\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right) = [11.3\mathbf{i} + 11.3\mathbf{j} + 5.67\mathbf{k}]\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

•4–53. The tool is used to shut off gas valves that are difficult to access. If the force **F** is applied to the handle, determine the component of the moment created about the z axis of the valve.



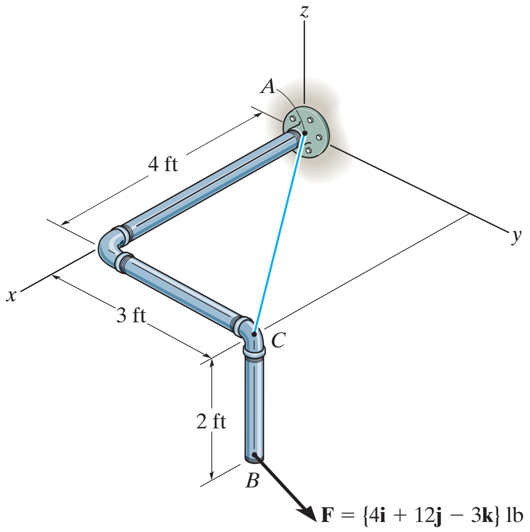
$\mathbf{u} = \mathbf{k}$

$\mathbf{r} = 0.25 \sin 30° \,\mathbf{i} + 0.25 \cos 30° \,\mathbf{j}$

$= 0.125\,\mathbf{i} + 0.2165\,\mathbf{j}$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix} = 15.5 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

4–54. Determine the magnitude of the moments of the force **F** about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



a) ***Vector Analysis***

Position Vector :

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

Moment of Force* F *About x, y and z Axes : The unit vectors along x, y and z axes are **i**, **j** and **k** respectively. Applying Eq. 4–11, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB} \times \mathbf{F})$$
$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB} \times \mathbf{F})$$
$$= \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB} \times \mathbf{F})$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - 4(3)] = 36.0 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

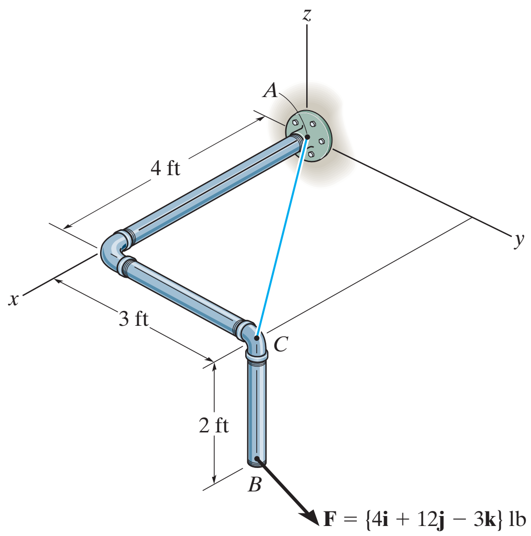
b) ***Scalar Analysis***

$M_x = \Sigma M_x;\quad M_x = 12(2) - 3(3) = 15.0 \text{ lb}\cdot\text{ft} \qquad$ **Ans**

$M_y = \Sigma M_y;\quad M_y = -4(2) + 3(4) = 4.00 \text{ lb}\cdot\text{ft} \qquad$ **Ans**

$M_z = \Sigma M_z;\quad M_z = -4(3) + 12(4) = 36.0 \text{ lb}\cdot\text{ft} \qquad$ **Ans**

4–55. Determine the moment of the force **F** about an axis extending between *A* and *C*. Express the result as a Cartesian vector.



Position Vector :

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\ \text{ft}$$
$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i}+(3-0)\mathbf{j}+(-2-0)\mathbf{k}\}\ \text{ft} = \{4\mathbf{i}+3\mathbf{j}-2\mathbf{k}\}\ \text{ft}$$

Unit Vector Along AC Axis :

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i}+(3-0)\mathbf{j}}{\sqrt{(4-0)^2+(3-0)^2}} = 0.8\mathbf{i}+0.6\mathbf{j}$$

Moment of Force F About AC Axis : With $\mathbf{F} = \{4\mathbf{i}+12\mathbf{j}-3\mathbf{k}\}$ lb. applying Eq. 4–11, we have

$$M_{AC} = \mathbf{u}_{AC}\cdot(\mathbf{r}_{CB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$
$$= 0.8[(0)(-3)-12(-2)]-0.6[0(-3)-4(-2)]+0$$

$$= 14.4\ \text{lb}\cdot\text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

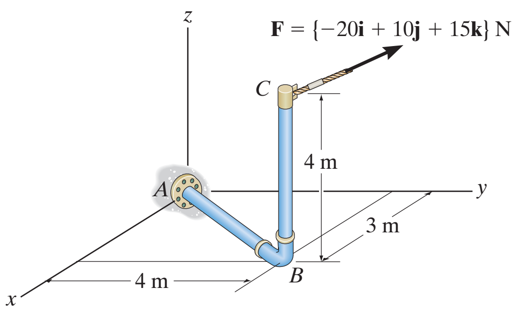
$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$
$$= 0.8[(3)(-3)-12(-2)]-0.6[4(-3)-4(-2)]+0$$

$$= 14.4\ \text{lb}\cdot\text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

$$\mathbf{M}_{AC} = M_{AC}\mathbf{u}_{AC}$$
$$= 14.4(0.8\mathbf{i}+0.6\mathbf{j})$$
$$= \{11.5\mathbf{i}+8.64\mathbf{j}\}\ \text{lb}\cdot\text{ft}$$ **Ans**

***4–56.** Determine the moment produced by force **F** about segment *AB* of the pipe assembly. Express the result as a Cartesian vector.



Moment About Line AB: Either position vector $\mathbf{r}_{AC}$ or $\mathbf{r}_{BC}$ can be conveniently used to determine the moment of **F** about line *AB*.

$$\mathbf{r}_{AC} = (3-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k} = [3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (3-3)\mathbf{i} + (4-4)\mathbf{j} + (4-0)\mathbf{k} = [4\mathbf{k}]\text{ m}$$

The unit vector $\mathbf{u}_{AB}$, Fig. *a*, that specifies the direction of line *AB* is given by

$$\mathbf{u}_{AB} = \frac{(3-0)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (0-0)^2}} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$$

Thus, the magnitude of the moment of **F** about line *AB* is given by

$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 3 & 4 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[4(15) - 10(4)] - \frac{4}{5}[3(15) - (-20)(4)] + 0$$

$$= -88\text{ N}\cdot\text{m}$$

or

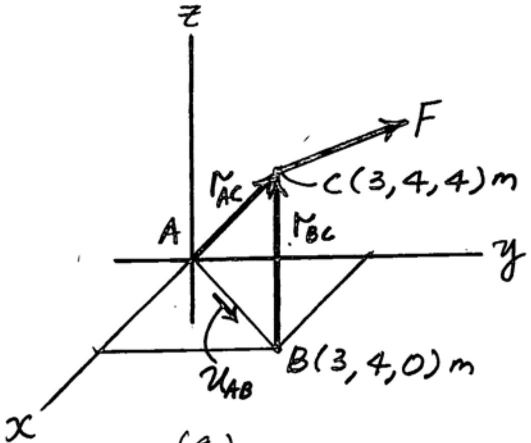
$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 0 & 0 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[0(15) - 10(4)] - \frac{4}{5}[0(15) - (-20)(4)] + 0$$

$$= -88\text{ N}\cdot\text{m}$$

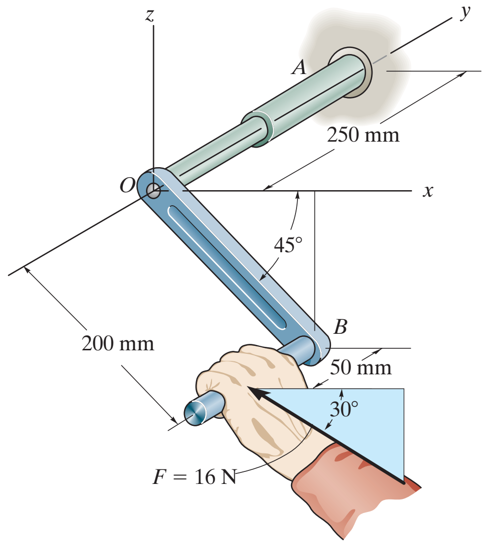
Thus, $\mathbf{M}_{AB}$ can be expressed in Cartesian vector form as

$$\mathbf{M}_{AB} = M_{AB}\mathbf{u}_{AB} = -88\left(\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}\right) = [-52.8\mathbf{i} - 70.4\mathbf{j}]\text{ N}\cdot\text{m}$$ **Ans.**



(a)

•4–57. Determine the magnitude of the moment that the force **F** exerts about the *y* axis of the shaft. Solve the problem using a Cartesian vector approach and using a scalar approach.



a) *Vector Analysis*

Position Vector and Force Vector :

$$\mathbf{r}_{OB} = \{0.2\cos 45°\mathbf{i} - 0.2\sin 45°\mathbf{k}\}\ \text{m} = \{0.1414\mathbf{i} - 0.1414\mathbf{k}\}\ \text{m}$$

$$\mathbf{F} = 16\{-\cos 30°\mathbf{i} + \sin 30°\mathbf{k}\}\ \text{N} = \{-13.856\mathbf{i} + 8.00\mathbf{k}\}\ \text{N}$$

Moment of Force* F *About y Axis : The unit vector along the *y* axis is **j**. Applying Eq. 4 – 11, we have

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{OB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 0.1414 & 0 & -0.1414 \\ -13.856 & 0 & 8 \end{vmatrix}$$

$$= 0 - 1[0.1414(8) - (-13.856)(-0.1414)] + 0$$

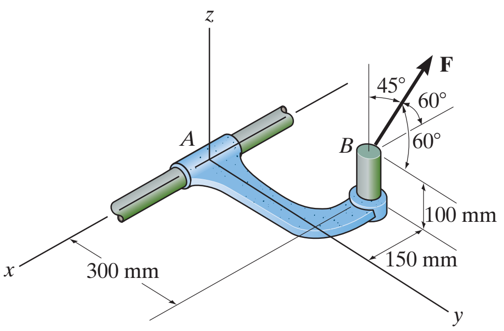
$$= 0.828\ \text{N}\cdot\text{m}$$ **Ans**

b) *Scalar Analysis*

$$M_y = \Sigma M_y; \quad M_y = 16\cos 30°(0.2\sin 45°) - 16\sin 30°(0.2\cos 45°)$$

$$= 0.828\ \text{N}\cdot\text{m}$$ **Ans**

4–58. If $F = 450$ N, determine the magnitude of the moment produced by this force about the x axis.



Moment About the x axis: Either position vector $\mathbf{r}_{AB}$ or $\mathbf{r}_{CB}$ can be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{AB} = (-0.15-0)\mathbf{i}+(0.3-0)\mathbf{j}+(0.1-0)\mathbf{k} = [-0.15\mathbf{i}+0.3\mathbf{j}+0.1\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{CB} = [(-1.5-(-0.15))]\mathbf{i}+(0.3-0)\mathbf{j}+(0.1-0)\mathbf{k} = [0.3\mathbf{j}+0.1\mathbf{k}]\text{ m}$$

The force vector **F** is given by

$$\mathbf{F} = 450(-\cos 60°\mathbf{i}+\cos 60°\,\mathbf{j}+\cos 45°\mathbf{k}) = [-225\mathbf{i}+225\mathbf{j}+318.20\mathbf{k}]\text{N}$$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

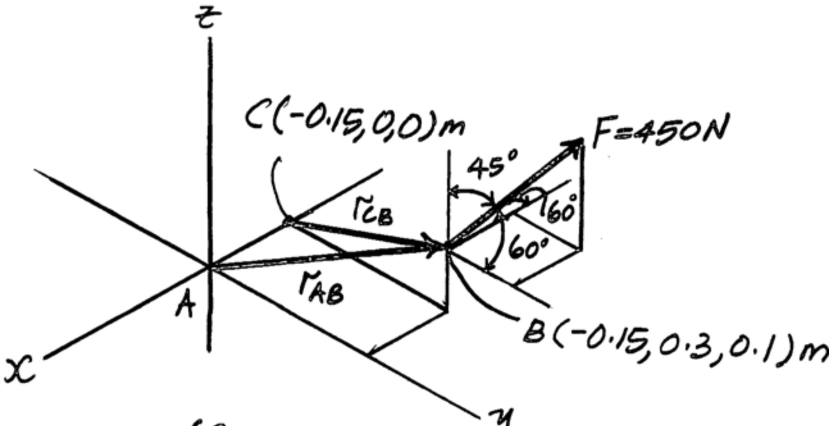
$$M_x = \mathbf{i}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ -0.15 & 0.3 & 0.1 \\ -225 & 225 & 318.20 \end{vmatrix}$$

$$= 1[0.3(318.20)-(225)(0.1)]+0+0 = 73.0\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

or

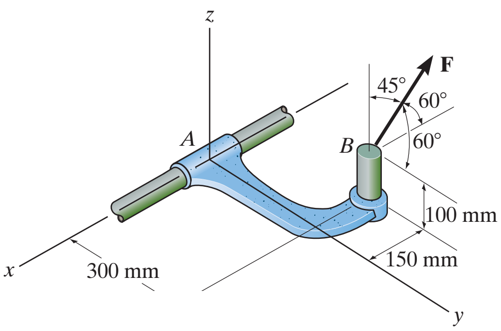
$$M_x = \mathbf{i}\cdot\mathbf{r}_{CB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.1 \\ -225 & 225 & 318.20 \end{vmatrix}$$

$$= 1[0.3(318.20)-(225)(0.1)]+0+0 = 73.0\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

4–59. The friction at sleeve *A* can provide a maximum resisting moment of 125 N · m about the *x* axis. Determine the largest magnitude of force **F** that can be applied to the bracket so that the bracket will not turn.



Moment About the x axis: The position vector $\mathbf{r}_{AB}$, Fig. *a*, will be used to determine the moment of **F** about the *x* axis.

$$\mathbf{r}_{AB} = (-0.15-0)\mathbf{i}+(0.3-0)\mathbf{j}+(0.1-0)\mathbf{k} = [-0.15\mathbf{i}+0.3\mathbf{j}+0.1\mathbf{k}]\text{ m}$$

The force vector **F** is given by

$$\mathbf{F} = F(-\cos 60^\circ\,\mathbf{i}+\cos 60^\circ\,\mathbf{j}+\cos 45^\circ\,\mathbf{k}) = -0.5F\mathbf{i}+0.5F\mathbf{j}+0.7071F\mathbf{k}$$

Knowing that the unit vector of the *x* axis is **i**, the magnitude of the moment of **F** about the *x* axis is given by

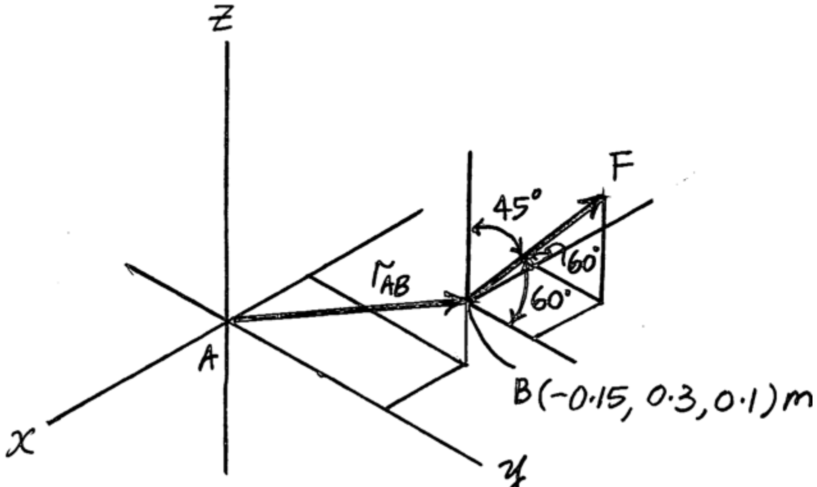
$$M_x = \mathbf{i}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ -0.15 & 0.3 & 0.1 \\ -0.5F & 0.5F & 0.7071F \end{vmatrix}$$

$$= 1[0.3(0.7071F) - 0.5F(0.1)] + 0 + 0 = 0.1621F$$

Since the friction at sleeve *A* can resist a moment of $M_x = 125\text{ N}\cdot\text{m}$, the maximum allowable magnitude of **F** is given by

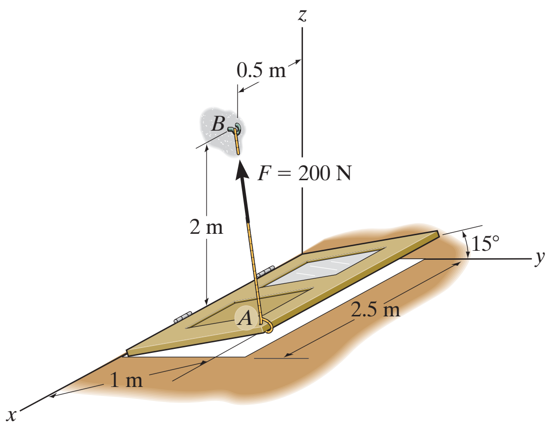
$$125 = 0.1621F$$

$$F = 771\text{ N} \qquad \textbf{Ans.}$$



(a)

***4–60.** Determine the magnitude of the moment produced by the force of $F = 200$ N about the hinged axis (the x axis) of the door.



Moment About the x axis: Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{CA}$ can be used to determine the moment of $\mathbf{F}$ about the x axis.

$$\mathbf{r}_{CA} = (2.5 - 2.5)\mathbf{i} + (0.9659 - 0)\mathbf{j} + (0.2588 - 0)\mathbf{k} = [0.9659\mathbf{j} + 0.2588\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{OB} = (0.5 - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = [0.5\mathbf{i} + 2\mathbf{k}]\text{ m}$$

The force vector $\mathbf{F}$ is given by

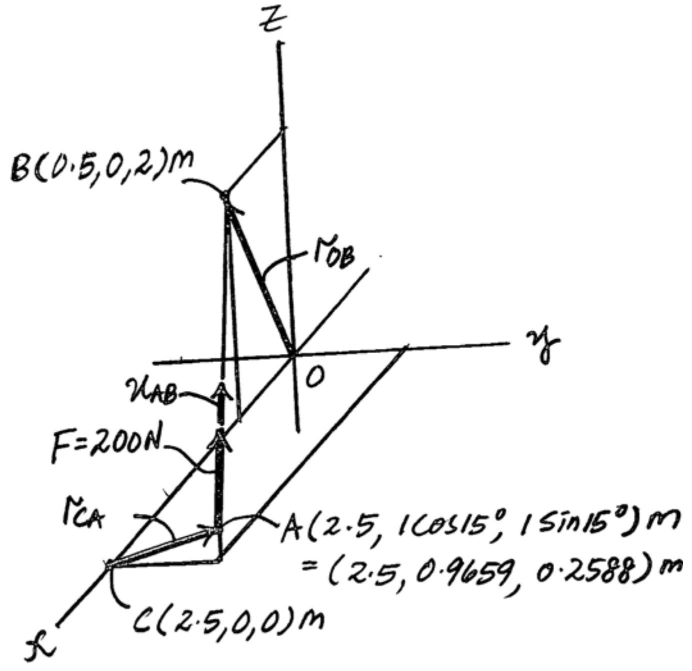
$$\mathbf{F} = F\mathbf{u}_{AB} = 200\left[\frac{(0.5 - 2.5)\mathbf{i} + (0 - 0.9659)\mathbf{j} + (2 - 0.2588)\mathbf{k}}{\sqrt{(0.5 - 2.5)^2 + (0 - 0.9659)^2 + (2 - 0.2588)^2}}\right] = [-141.73\mathbf{i} - 68.45\mathbf{j} + 123.39\mathbf{k}]\text{ N}$$

Knowing that the unit vector of the x axis is $\mathbf{i}$, the magnitude of the moment of $\mathbf{F}$ about the x axis is given by

$$M_x = \mathbf{i} \cdot \mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.9659 & 0.2588 \\ -141.73 & -68.45 & 123.39 \end{vmatrix} = 137\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

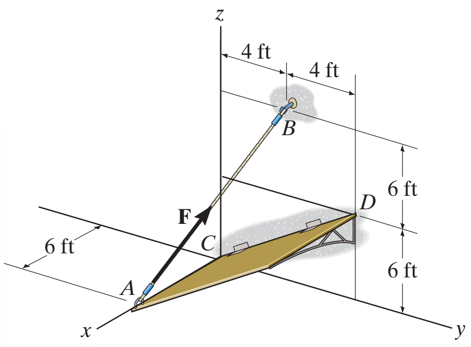
or

$$M_x = \mathbf{i} \cdot \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0.5 & 0 & 2 \\ -141.73 & -68.45 & 123.39 \end{vmatrix} = 137\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

•4–61. If the tension in the cable is $F = 140$ lb, determine the magnitude of the moment produced by this force about the hinged axis, CD, of the panel.



Moment About the CD axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{DB}$, Fig. *a*, can be used to determine the moment of **F** about the *CD* axis.

$$\mathbf{r}_{CA} = (6-0)\mathbf{i} + (0-0)\mathbf{j} + (0-0)\mathbf{k} = [6\mathbf{i}]\text{ ft}$$

$$\mathbf{r}_{DB} = (0-0)\mathbf{i} + (4-8)\mathbf{j} + (12-6)\mathbf{k} = [-4\mathbf{j} + 6\mathbf{k}]\text{ ft}$$

Referring to Fig. *a*, the force vector **F** can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = 140\left[\frac{(0-6)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k}}{\sqrt{(0-6)^2 + (4-0)^2 + (12-0)^2}}\right] = [-60\mathbf{i} + 40\mathbf{j} + 120\mathbf{k}]\text{ lb}$$

The unit vector $\mathbf{u}_{CD}$, Fig. *a*, that specifies the direction of the *CD* axis is given by

$$\mathbf{u}_{CD} = \frac{(0-0)\mathbf{i} + (8-0)\mathbf{j} + (6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (8-0)^2 + (6-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the *CD* axis is given by

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[6(120) - (-60)(0)] + \frac{3}{5}[6(40) - (-60)(0)]$$

$$= -432 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

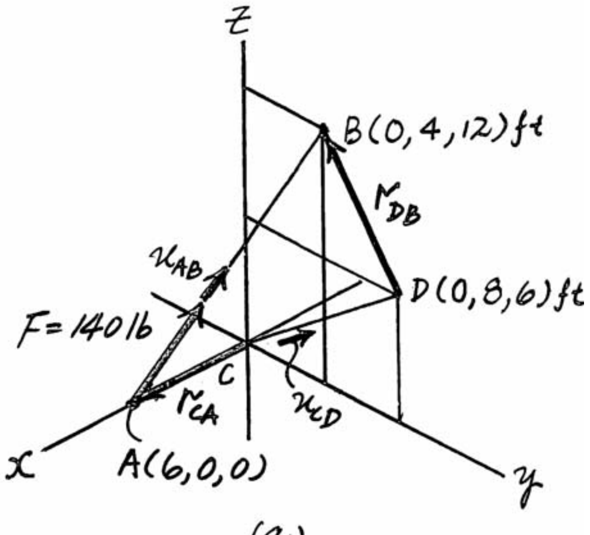
or

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{DB} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & -4 & 6 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[0(120) - (-60)(6)] + \frac{3}{5}[0(40) - (-60)(-4)]$$

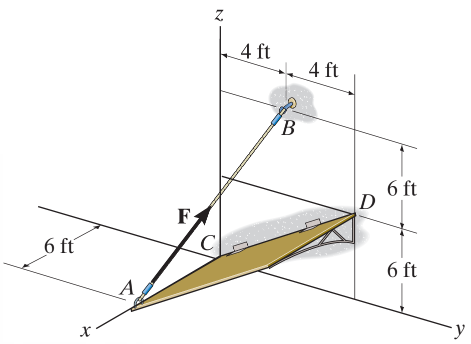
$$= -432 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{M}_{CD}$ acts in the opposite sense to that of $\mathbf{u}_{CD}$.



(a)

4–62. Determine the magnitude of force **F** in cable *AB* in order to produce a moment of 500 lb · ft about the hinged axis *CD*, which is needed to hold the panel in the position shown.



Moment About the CD axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{CB}$, Fig. *a*, can be used to determine the moment of **F** about the *CD* axis.

$$\mathbf{r}_{CA} = (6-0)\mathbf{i} + (0-0)\mathbf{j} + (0-0)\mathbf{k} = [6\mathbf{i}]\text{ft}$$

$$\mathbf{r}_{CB} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k} = [4\mathbf{j} + 12\mathbf{k}]\text{ft}$$

Referring to Fig. *a*, the force vector **F** can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = F\left[\frac{(0-6)\mathbf{i} + (4-0)\mathbf{j} + (12-0)\mathbf{k}}{\sqrt{(0-6)^2 + (4-0)^2 + (12-0)^2}}\right] = -\frac{3}{7}F\mathbf{i} + \frac{2}{7}F\mathbf{j} + \frac{6}{7}F\mathbf{k}$$

The unit vector $\mathbf{u}_{CD}$, Fig. *a*, that specifies the direction of the *CD* axis is given by

$$\mathbf{u}_{CD} = \frac{(0-0)\mathbf{i} + (8-0)\mathbf{j} + (6-0)\mathbf{k}}{\sqrt{(0-0)^2 + (8-0)^2 + (6-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the *CD* axis is required to be $\mathbf{M}_{CD} = |500|$ lb · ft. Thus,

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F}$$

$$|500| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix}$$

$$-500 = 0 - \frac{4}{5}\left[6\left(\frac{6}{7}F\right) - \left(-\frac{3}{7}F\right)(0)\right] + \frac{3}{5}\left[6\left(\frac{2}{7}F\right) - \left(-\frac{3}{7}F\right)(0)\right]$$

$F = 162$ lb **Ans.**

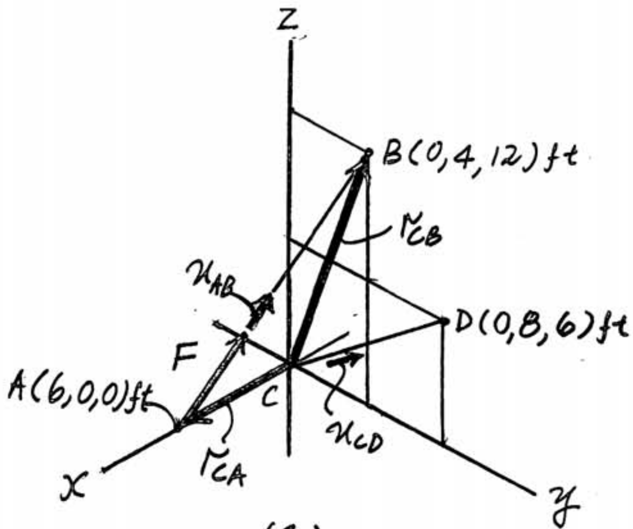
or

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CB} \times \mathbf{F}$$

$$|500| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & 4 & 12 \\ -\frac{3}{7}F & \frac{2}{7}F & \frac{6}{7}F \end{vmatrix}$$

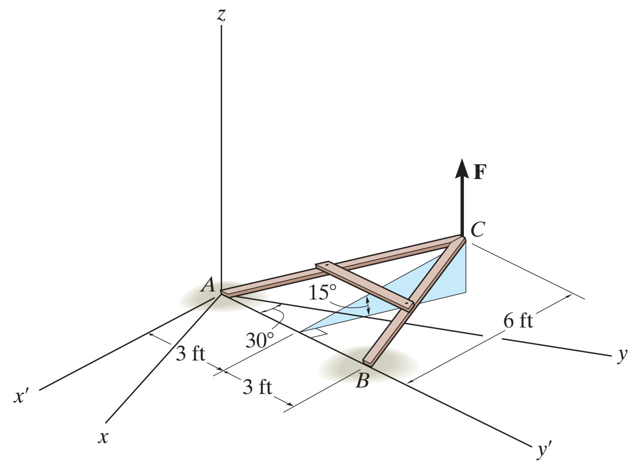
$$-500 = 0 - \frac{4}{5}\left[0\left(\frac{6}{7}F\right) - \left(-\frac{3}{7}F\right)(12)\right] + \frac{3}{5}\left[0\left(\frac{2}{7}F\right) - \left(-\frac{3}{7}F\right)(4)\right]$$

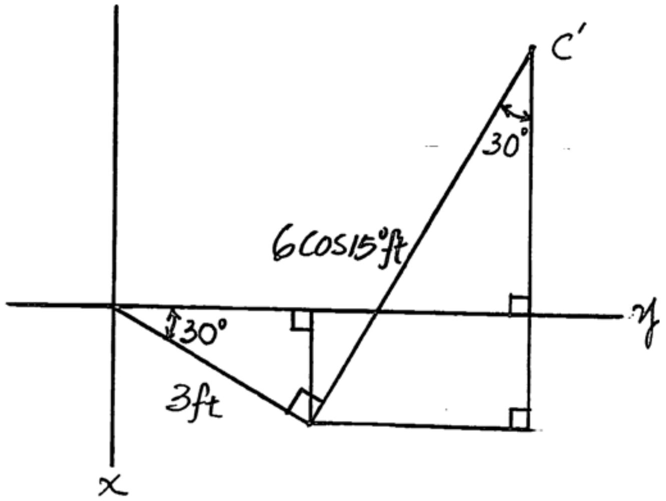
$F = 162$ lb **Ans.**



(a)

4–63. The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y' axis passing through points A and B when the frame is in the position shown.





Scalar analysis :

$M_{y'} = 80\,(6\cos 15°) = 464\ \text{lb}\cdot\text{ft}$ **Ans**

Vector analysis :

$\mathbf{u}_{AB} = \sin 30°\,\mathbf{i} + \cos 30°\,\mathbf{j}$

Coordinates of point C :

$x = 3\sin 30° - 6\cos 15°\cos 30° = -3.52$ ft

$y = 3\cos 30° + 6\cos 15°\sin 30° = 5.50$ ft

$z = 6\sin 15° = 1.55$ ft

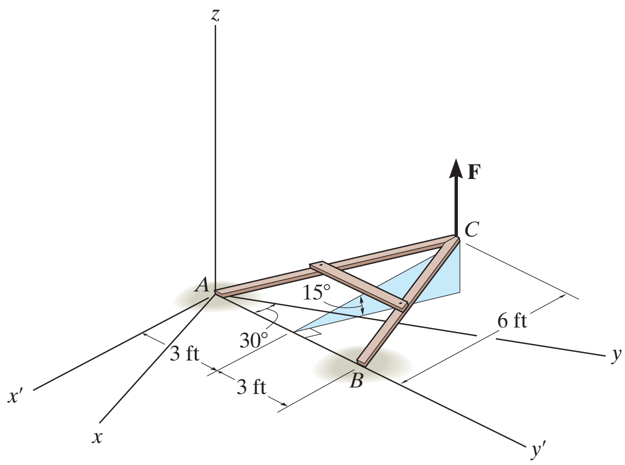
$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$

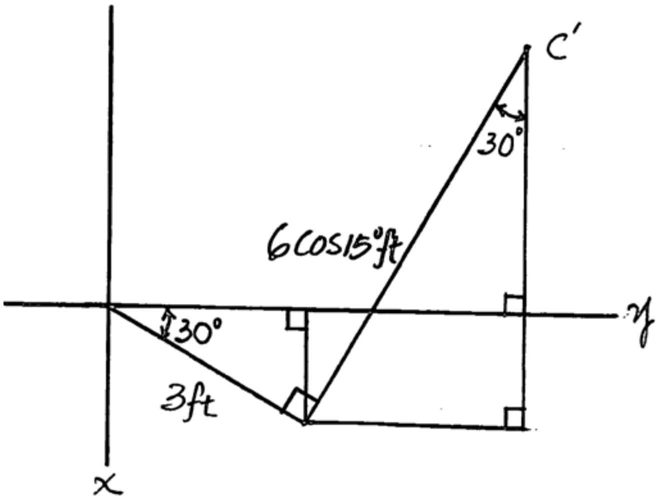
$\mathbf{F} = 80\,\mathbf{k}$

$$M_{y'} = \begin{vmatrix} \sin 30° & \cos 30° & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix}$$

$M_{y'} = 464\ \text{lb}\cdot\text{ft}$ **Ans**

***4–64.** The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the x axis when the frame is in the position shown.





Using x', y', z :

$\mathbf{u}_x = \cos 30° \, \mathbf{i}' + \sin 30° \, \mathbf{j}'$

$\mathbf{r}_{AC} = -6 \cos 15° \, \mathbf{i}' + 3 \, \mathbf{j}' + 6 \sin 15° \, \mathbf{k}$

$\mathbf{F} = 80 \, \mathbf{k}$

$$M_x = \begin{vmatrix} \cos 30° & \sin 30° & 0 \\ -6 \cos 15° & 3 & 6 \sin 15° \\ 0 & 0 & 80 \end{vmatrix} = 207.85 + 231.82 + 0$$

$M_x = 440$ lb · ft   Ans

Also, using x, y, z.

Coordinates of point C :

$x = 3 \sin 30° - 6 \cos 15° \cos 30° = -3.52$ ft

$y = 3 \cos 30° + 6 \cos 15° \sin 30° = 5.50$ ft

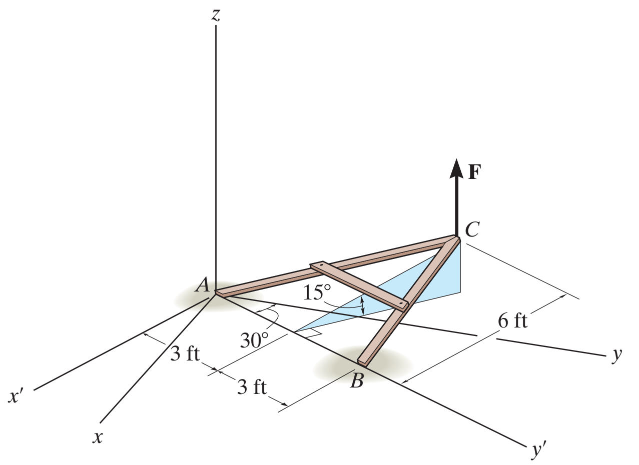
$z = 6 \sin 15° = 1.55$ ft

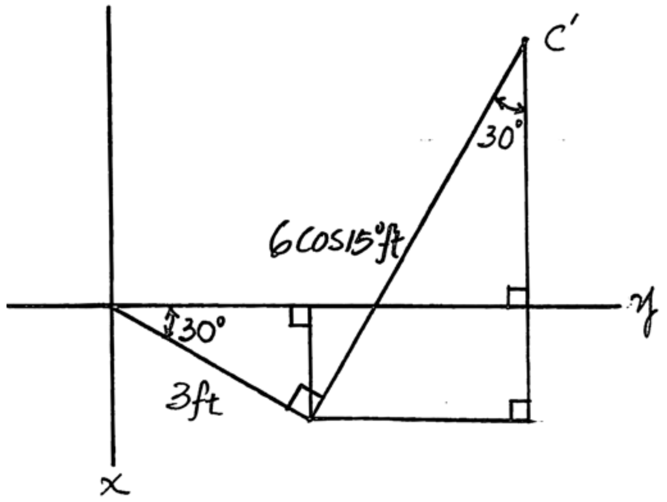
$\mathbf{r}_{AC} = -3.52 \, \mathbf{i} + 5.50 \, \mathbf{j} + 1.55 \, \mathbf{k}$

$\mathbf{F} = 80 \, \mathbf{k}$

$$M_x = \begin{vmatrix} 1 & 0 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 440 \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

•4–65. The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y axis when the frame is in the position shown.





Using x', y', z:

$\mathbf{u}_{y} = -\sin 30°\,\mathbf{i}' + \cos 30°\,\mathbf{j}'$

$\mathbf{r}_{AC} = -6\cos 15°\,\mathbf{i}' + 3\,\mathbf{j}' + 6\sin 15°\,\mathbf{k}$

$\mathbf{F} = 80\,\mathbf{k}$

$$M_y = \begin{vmatrix} -\sin 30° & \cos 30° & 0 \\ -6\cos 15° & 3 & 6\sin 15° \\ 0 & 0 & 80 \end{vmatrix} = -120 + 401.52 + 0$$

$M_y = 282$ lb · ft Ans

Also, using x, y, z:

Coordinates of point C:

$x = 3\sin 30° - 6\cos 15°\cos 30° = -3.52$ ft

$y = 3\cos 30° + 6\cos 15°\sin 30° = 5.50$ ft

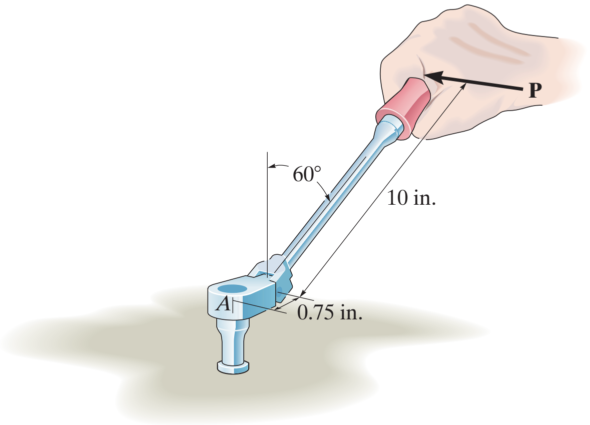
$z = 6\sin 15° = 1.55$ ft

$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$

$\mathbf{F} = 80\,\mathbf{k}$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 282 \text{ lb} \cdot \text{ft}$$ Ans

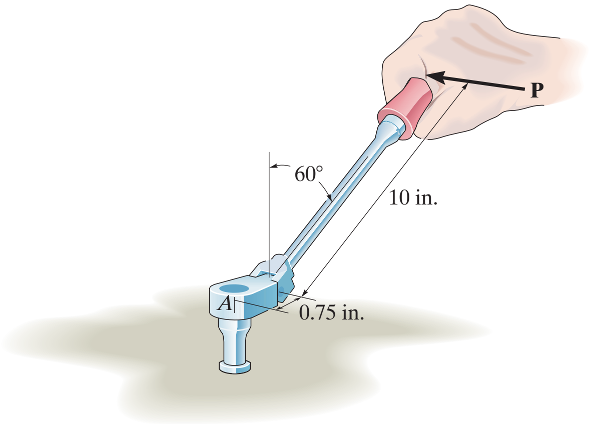
4–66. The flex-headed ratchet wrench is subjected to a force of $P = 16$ lb, applied perpendicular to the handle as shown. Determine the moment or torque this imparts along the vertical axis of the bolt at A.



$$M = 16(0.75 + 10\sin 60°)$$

$$M = 151 \text{ lb}\cdot\text{in.} \qquad \textbf{Ans}$$

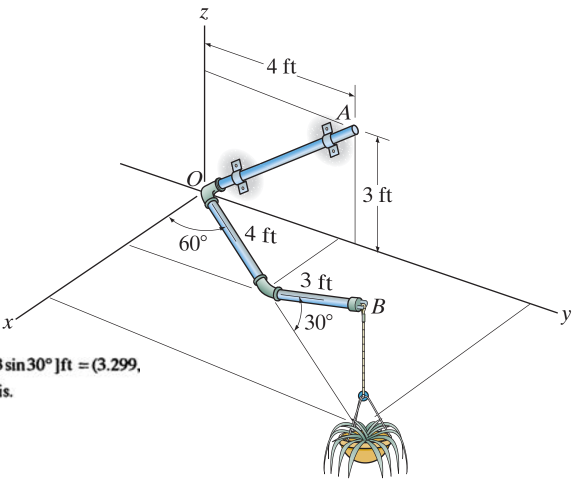
4–67. If a torque or moment of 80 lb · in. is required to loosen the bolt at A, determine the force P that must be applied perpendicular to the handle of the flex-headed ratchet wrench.



$$80 = P(0.75 + 10\sin 60°)$$

$$P = \frac{80}{9.41} = 8.50 \text{ lb} \qquad \textbf{Ans}$$

***4–68.** The pipe assembly is secured on the wall by the two brackets. If the flower pot has a weight of 50 lb, determine the magnitude of the moment produced by the weight about the *OA* axis.



Moment About the OA axis: The coordinates of point B are $[(4+3\cos 30°)\cos 60°, (4+3\cos 30°)\sin 60°, 3\sin 30°]$ ft $= (3.299, 5.714, 1.5)$ ft. Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{AB}$ can be used to determine the moment of $\mathbf{W}$ about the OA axis.

$$\mathbf{r}_{OB} = (3.299-0)\mathbf{i} + (5.714-0)\mathbf{j} + (1.5-0)\mathbf{k} = [3.299\mathbf{i} + 5.714\mathbf{j} + 1.5\mathbf{k}]\text{ft}$$

$$\mathbf{r}_{AB} = (3.299-0)\mathbf{i} + (5.714-4)\mathbf{j} + (1.5-3)\mathbf{k} = [3.299\mathbf{i} + 1.714\mathbf{j} - 1.5\mathbf{k}]\text{ft}$$

Since $\mathbf{W}$ is directed towards the negative z axis, we can write

$$\mathbf{W} = [-50\mathbf{k}]\text{ lb}$$

The unit vector $\mathbf{u}_{OA}$, Fig. *a*, that specifies the direction of the OA axis is given by

$$\mathbf{u}_{OA} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

The magnitude of the moment of $\mathbf{W}$ about the OA axis is given by

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{OB} \times \mathbf{W} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 1.5 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[3.299(-50) - 0(1.5)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

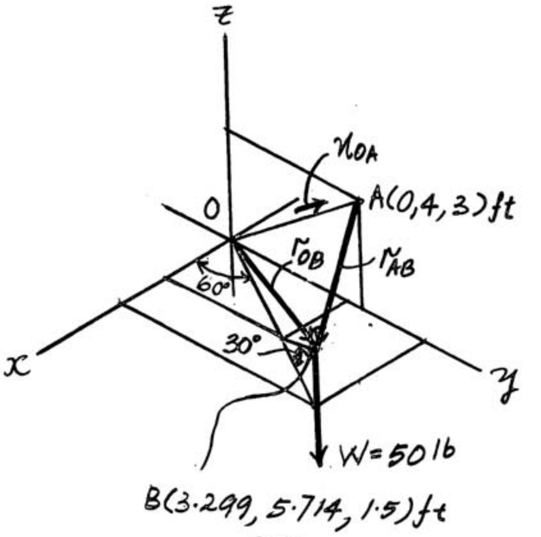
$$= 132 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

or

$$M_{OA} = \mathbf{u}_{OA} \cdot \mathbf{r}_{AB} \times \mathbf{W} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 1.714 & -1.5 \\ 0 & 0 & -50 \end{vmatrix}$$

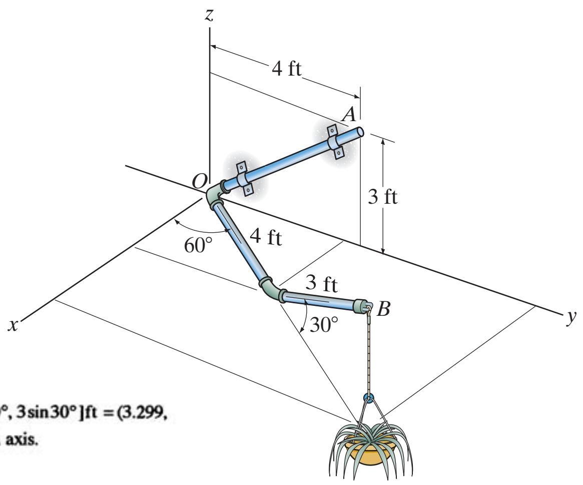
$$= 0 - \frac{4}{5}[3.299(-50) - 0(-1.5)] + \frac{3}{5}[3.299(0) - 0(1.714)]$$

$$= 132 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



(a)

•4–69. The pipe assembly is secured on the wall by the two brackets. If the frictional force of both brackets can resist a maximum moment of 150 lb · ft, determine the largest weight of the flower pot that can be supported by the assembly without causing it to rotate about the *OA* axis.



Moment About the OA axis: The coordinates of point *B* are $[(4+3\cos 30°)\cos 60°, (4+3\cos 30°)\sin 60°, 3\sin 30°]$ ft $= (3.299, 5.714, 1.5)$ ft. Either position vector $\mathbf{r}_{OB}$ or $\mathbf{r}_{OC}$ can be used to determine the moment of **W** about the *OA* axis.

$$\mathbf{r}_{OB} = (3.299-0)\mathbf{i}+(5.714-0)\mathbf{j}+(1.5-0)\mathbf{k} = [3.299\mathbf{i}+5.714\mathbf{j}+1.5\mathbf{k}]\text{ft}$$

$$\mathbf{r}_{AB} = (3.299-0)\mathbf{i}+(5.714-4)\mathbf{j}+(1.5-3)\mathbf{k} = [3.299\mathbf{i}+1.714\mathbf{j}-1.5\mathbf{k}]\text{ft}$$

Since **W** is directed towards the negative *z* axis, we can write

$$\mathbf{W} = -W\mathbf{k}$$

The unit vector $\mathbf{u}_{OA}$, Fig. *a*, that specifies the direction of the *OA* axis is given by

$$\mathbf{u}_{OA} = \frac{(0-0)\mathbf{i}+(4-0)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(0-0)^2+(4-0)^2+(3-0)^2}} = \frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$$

Since it is required that the magnitude of the moment of **W** about the *OA* axis not exceed 150 ft · lb, we can write

$$M_{OA} = \mathbf{u}_{OA}\cdot\mathbf{r}_{OB}\times\mathbf{W}$$

$$|150| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 1.5 \\ 0 & 0 & -W \end{vmatrix}$$

$$150 = 0 - \frac{4}{5}[3.299(-W) - 0(1.5)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

$W = 56.8$ lb **Ans.**

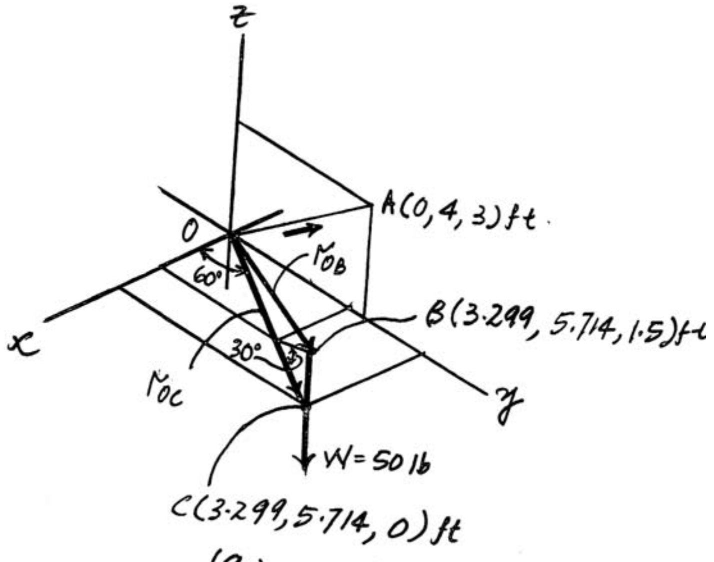
or

$$M_{OA} = \mathbf{u}_{OA}\cdot\mathbf{r}_{OB}\times\mathbf{W}$$

$$|150| = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 3.299 & 5.714 & 0 \\ 0 & 0 & -W \end{vmatrix}$$

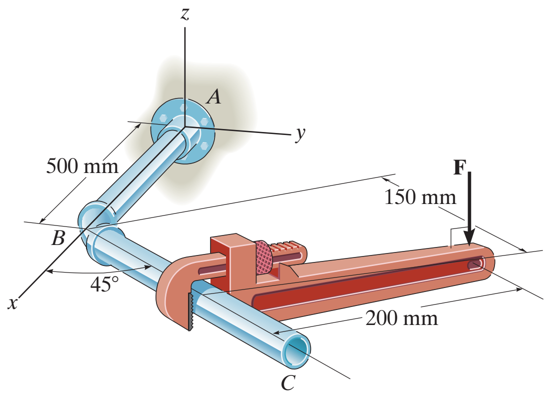
$$150 = 0 - \frac{4}{5}[3.299(-W) - 0(0)] + \frac{3}{5}[3.299(0) - 0(5.714)]$$

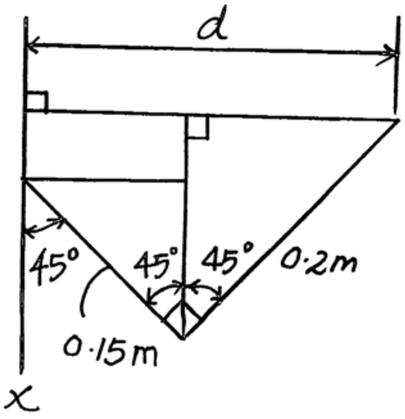
$W = 56.8$ lb **Ans.**



(a)

4–70. A vertical force of $F = 60$ N is applied to the handle of the pipe wrench. Determine the moment that this force exerts along the axis AB (x axis) of the pipe assembly. Both the wrench and pipe assembly ABC lie in the $x-y$ plane. *Suggestion:* Use a scalar analysis.





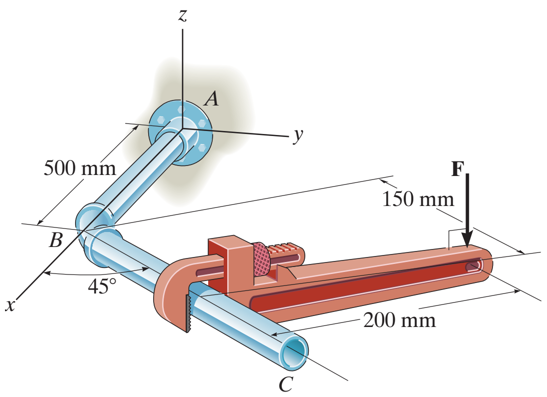
Scalar Analysis : From the geometry, the perpendicular distance from x axis to force $\mathbf{F}$ is $d = 0.15\sin 45° + 0.2\sin 45° = 0.2475$ m.

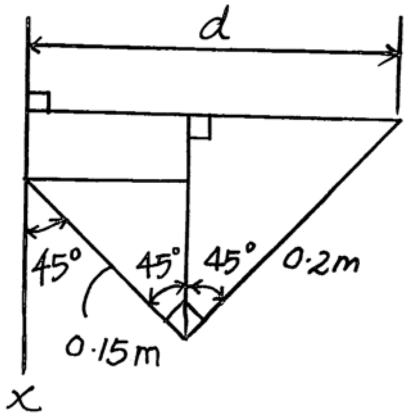
$$M_x = \Sigma M_x; \qquad M_x = -Fd = -60(0.2475) = -14.8\ \text{N}\cdot\text{m}$$

Negative sign indicates that $\mathbf{M}_x$ is directed toward negative x axis.

$\mathbf{M}_x = 14.8\ \text{N}\cdot\text{m}$ **Ans**

4–71. Determine the magnitude of the vertical force **F** acting on the handle of the wrench so that this force produces a component of moment along the *AB* axis (*x* axis) of the pipe assembly of $(M_A)_x = \{-5\mathbf{i}\}$ N · m. Both the pipe assembly *ABC* and the wrench lie in the *x*−*y* plane. *Suggestion:* Use a scalar analysis.





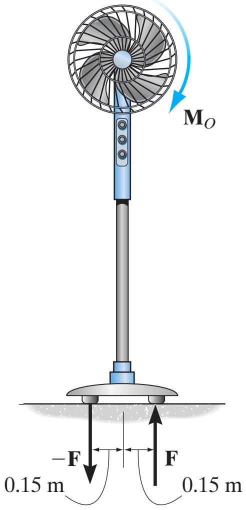
Scalar Analysis : From the geometry, the perpendicular distance from x axis to **F** is $d = 0.15\sin 45° + 0.2\sin 45° = 0.2475$ m.

$$M_x = \Sigma M_x; \qquad -5 = -F(0.2475)$$

$$F = 20.2 \text{ N} \qquad \textbf{Ans}$$

---

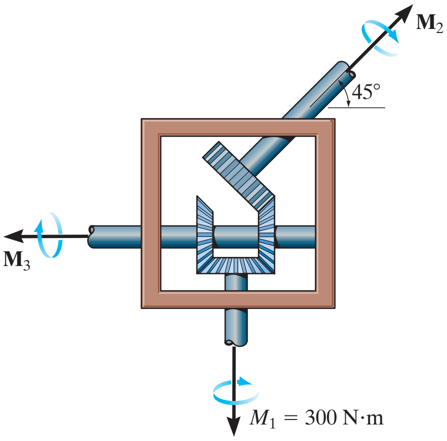
*__4–72.__ The frictional effects of the air on the blades of the standing fan creates a couple moment of $M_O = 6$ N · m on the blades. Determine the magnitude of the couple forces at the base of the fan so that the resultant couple moment on the fan is zero.



Couple Moment: The couple moment of **F** produces a counterclockwise moment of $M_C = F(0.15 + 0.15) = 0.3F$. Since the resultant couple moment about the axis perpendicular to the page is required to be zero,

$$\circlearrowleft +(M_c)_R = \Sigma M; \qquad 0 = 0.3F - 6 \qquad F = 20 \text{ N} \qquad \textbf{Ans.}$$

•4–73. Determine the required magnitude of the couple moments $\mathbf{M}_2$ and $\mathbf{M}_3$ so that the resultant couple moment is zero.



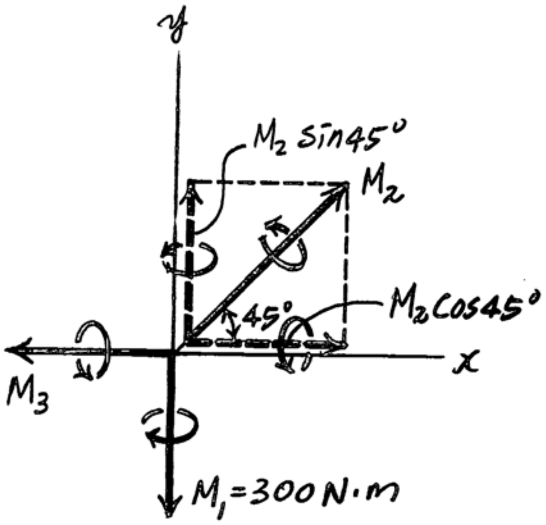
Since the couple moment is the free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ can be simplified as shown in Fig. *a*. Since the resultant of $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ is required to be zero,

$(M_R)_y = \Sigma M_y;$ $\quad 0 = M_2 \sin 45° - 300$

$M_2 = 424.26 \text{ N}\cdot\text{m} = 424 \text{ N}\cdot\text{m}$ **Ans.**

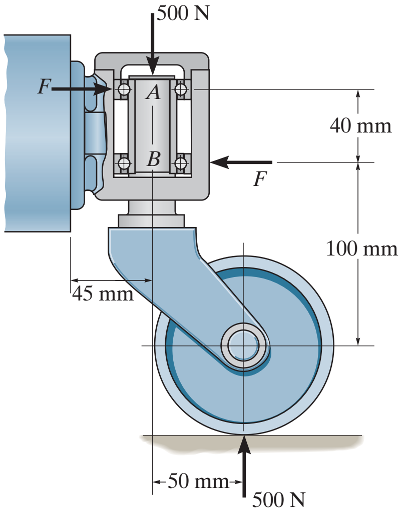
$(M_R)_x = \Sigma M_x;$ $\quad 0 = 424.26 \cos 45° - M_3$

$M_3 = 300 \text{ N}\cdot\text{m}$ **Ans.**



(a)

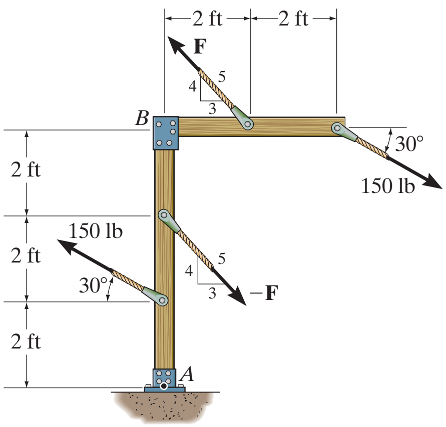
4–74. The caster wheel is subjected to the two couples. Determine the forces F that the bearings exert on the shaft so that the resultant couple moment on the caster is zero.



$\circlearrowleft + \Sigma M_A = 0; \quad 500(50) - F(40) = 0$

$F = 625\text{ N}$ **Ans**

4–75. If $F = 200$ lb, determine the resultant couple moment.



a) By resolving the 150 - lb and 200 - lb couples into their x and y components, Fig. a, the couple moments $(M_C)_1$ and $(M_C)_2$ produced by the 150 - lb and 200 - lb couples, respectively, are given by

$$\circlearrowleft +(M_c)_1 = -150\cos 30°(4) - 150\sin 30°(4) = -819.62\text{ lb}\cdot\text{ft} = 819.62\text{ lb}\cdot\text{ft}$$

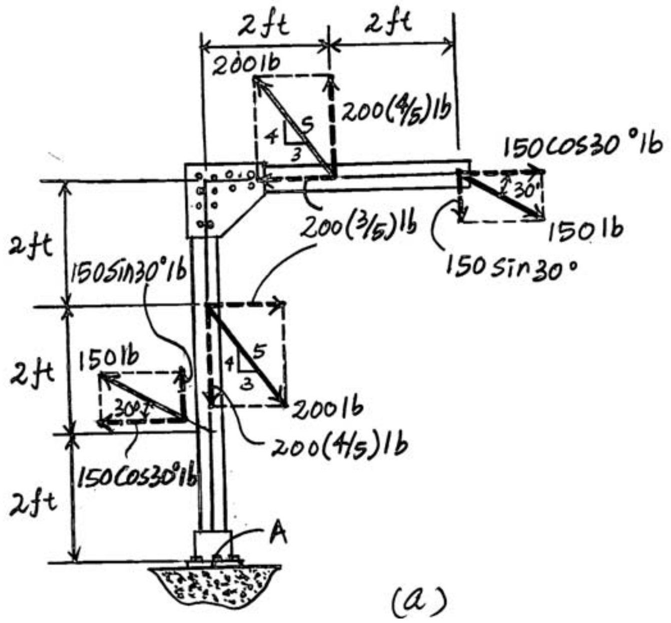
$$\circlearrowleft +(M_c)_2 = 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(2) = 560\text{ lb}\cdot\text{ft}$$

Thus, the resultant couple moment can be determined from

$$\circlearrowleft +(M_c)_R = (M_c)_1 + (M_c)_2$$
$$= -819.62 + 560 = -259.62\text{ lb}\cdot\text{ft} = 260\text{ lb}\cdot\text{ft (clockwise)} \qquad \textbf{Ans.}$$

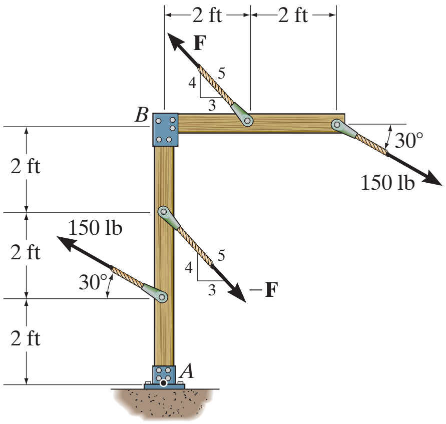
b) By resolving the 150 - lb and 200 - lb couples into their x and y components, Fig. a, and summing the moments of these force components algebraically about point A,

$$\circlearrowleft +(M_C)_R = \Sigma M_A;\ (M_C)_R = -150\sin 30°(4) - 150\cos 30°(6) + 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(6)$$
$$-200\left(\frac{3}{5}\right)(4) + 200\left(\frac{4}{5}\right)(0) + 150\cos 30°(2) + 150\sin 30°(0)$$
$$= -259.62\text{ lb}\cdot\text{ft} = 260\text{ lb}\cdot\text{ft (clockwise)} \qquad \textbf{Ans.}$$



(a)

***4–76.** Determine the required magnitude of force **F** if the resultant couple moment on the frame is 200 lb · ft, clockwise.



By resolving **F** and the 150 - lb couple into their x and y components, Fig. a, the couple moments $(M_C)_1$ and $(M_C)_2$ produced by **F** and the 5 - kN couple, respectively, are given by

$$\circlearrowleft +(M_c)_1 = F\left(\frac{4}{5}\right)(2) + F\left(\frac{3}{5}\right)(2) = 2.8F$$

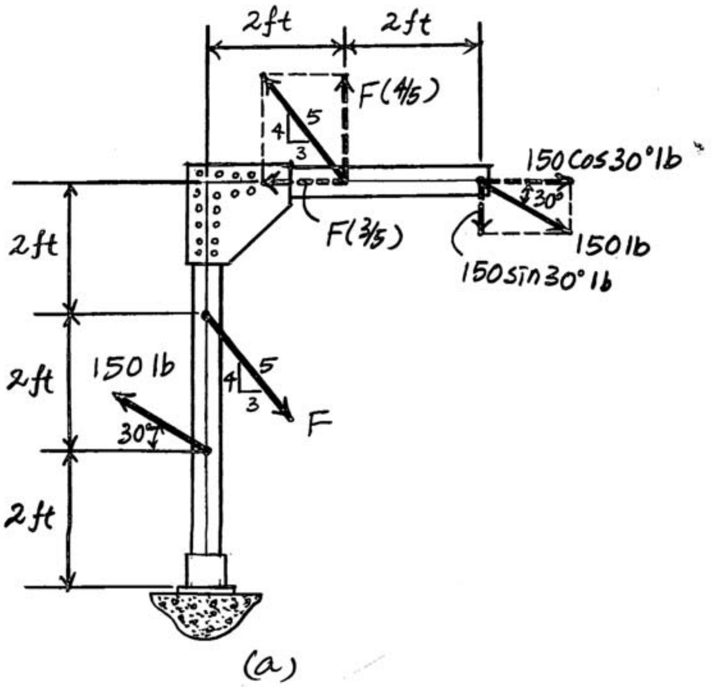
$$\circlearrowleft +(M_c)_2 = -150\cos 30°(4) - 150\sin 30°(4) = -819.62\ \text{lb}\cdot\text{ft} = 819.62\ \text{lb}\cdot\text{ft} \circlearrowright$$

The resultant couple moment acting on the beam is required to be 200 lb·ft, clockwise. Thus,

$$\circlearrowleft +(M_c)_R = (M_c)_1 + (M_c)_2$$

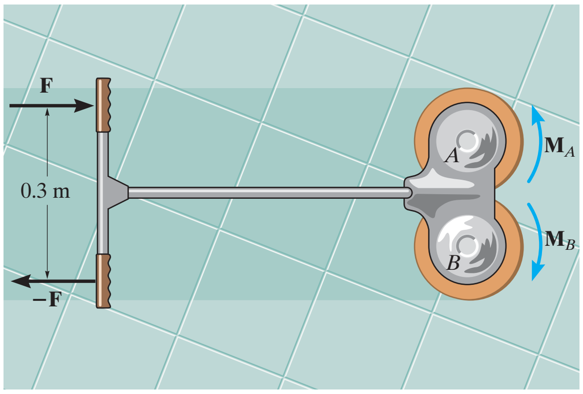
$$-200 = 2.8F - 819.62$$

$$F = 221\ \text{lb}$$ **Ans.**



(a)

•4–77. The floor causes a couple moment of $M_A = 40\ \text{N}\cdot\text{m}$ and $M_B = 30\ \text{N}\cdot\text{m}$ on the brushes of the polishing machine. Determine the magnitude of the couple forces that must be developed by the operator on the handles so that the resultant couple moment on the polisher is zero. What is the magnitude of these forces if the brush at B suddenly stops so that $M_B = 0$?



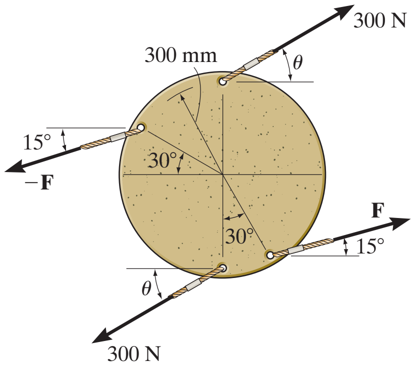
$$\curvearrowleft + \; M_R = 40 - 30 - F'(0.3) = 0$$

$$F' = 33.3\ \text{N} \qquad \textbf{Ans}$$

$$\curvearrowleft + \; M_R = 40 - F(0.3) = 0$$

$$F = 133\ \text{N} \qquad \textbf{Ans}$$

4–78. If $\theta = 30°$, determine the magnitude of force **F** so that the resultant couple moment is $100\ \text{N}\cdot\text{m}$, clockwise.

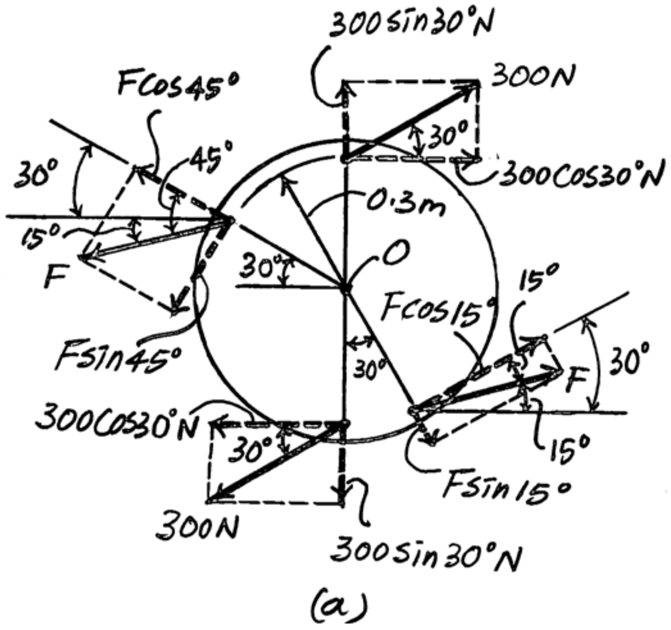


By resolving **F** and the 300 -N couple into their radial and tangential components, Fig. *a*, and summing the moment of these two force components about point O,

$$\curvearrowright +(M_c)_R = \Sigma M_O; \qquad -100 = F\sin 45°(0.3) + F\cos 15°(0.3) - 2(300\cos 30°)(0.3)$$

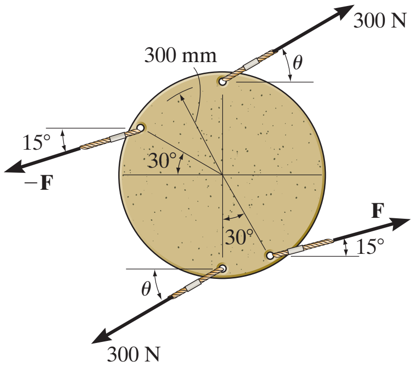
$$F = 111\ \text{N} \qquad \textbf{Ans.}$$

Note: Since the line of action of the radial component of the forces pass through point O, no moment is produced about this point.



(a)

4–79. If $F = 200$ N, determine the required angle θ so that the resultant couple moment is zero.

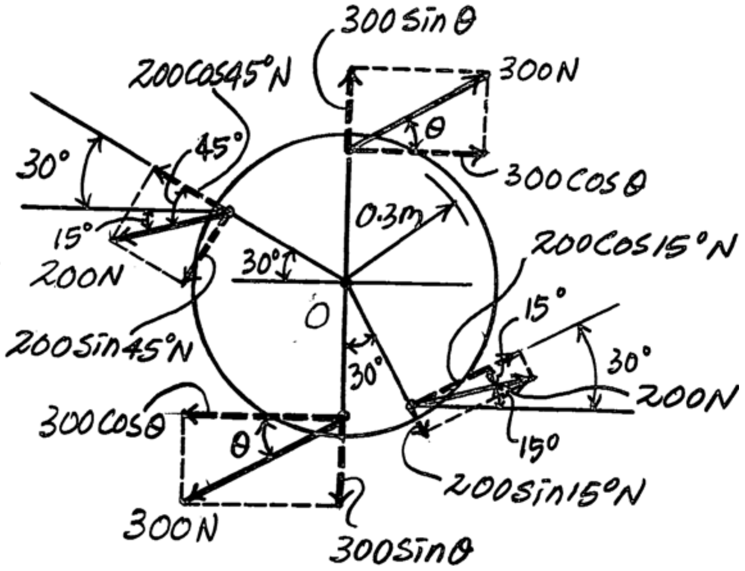


By resolving the 300 - N and 200 -N couples into their radial and tangential components, Fig. *a*, and summing the moment of these two force components about point *O*,

$$\circlearrowleft +(M_c)_R = \Sigma M_O; \qquad 0 = 200\sin 45°(0.3) + 200\cos 15°(0.3) - 300\cos\theta(0.3) - 300\cos\theta(0.3)$$

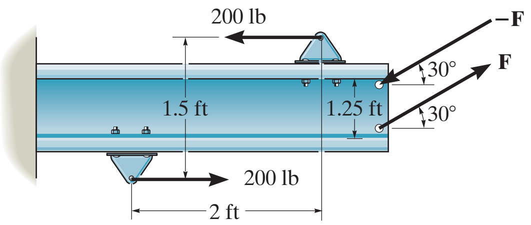
$$\theta = 56.1° \quad \textbf{Ans.}$$

Note: Since the line of action of the radial component of the forces pass through point *O*, no moment is produced about this point.



(a)

***4–80.** Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?

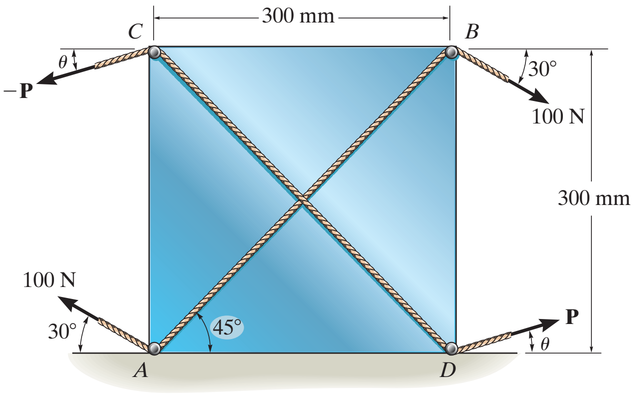


$$\circlearrowleft +M_R = \Sigma M; \qquad 450 = 200(1.5) + F\cos 30°(1.25)$$

$$F = 139 \text{ lb} \qquad \text{Ans}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

•4–81. The cord passing over the two small pegs *A* and *B* of the square board is subjected to a tension of 100 N. Determine the required tension *P* acting on the cord that passes over pegs *C* and *D* so that the resultant couple produced by the two couples is 15 N · m acting clockwise. Take $\theta = 15°$.



$$\circlearrowright + M_R = 100\cos 30°(0.3) + 100\sin 30°(0.3) - P\sin 15°(0.3) - P\cos 15°(0.3) = 15$$

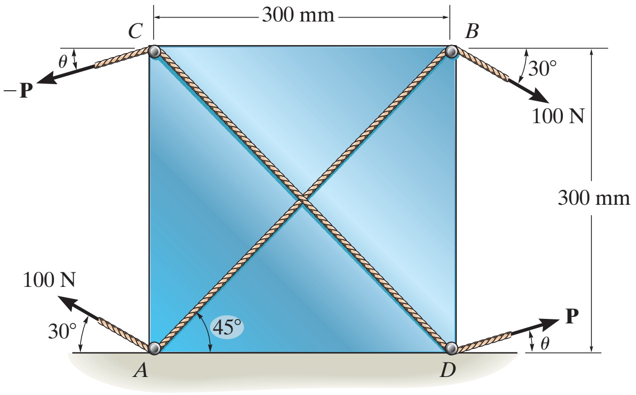
$P = 70.7\text{ N}$ **Ans**

4–82. The cord passing over the two small pegs *A* and *B* of the board is subjected to a tension of 100 N. Determine the *minimum* tension *P* and the orientation θ of the cord passing over pegs *C* and *D*, so that the resultant couple moment produced by the two cords is 20 N · m, clockwise.

For minimum P require $\theta = 45°$ **Ans**

$$\circlearrowright + M_R = 100\cos 30°(0.3) + 100\sin 30°(0.3) - P\left(\frac{0.3}{\cos 45°}\right) = 20$$

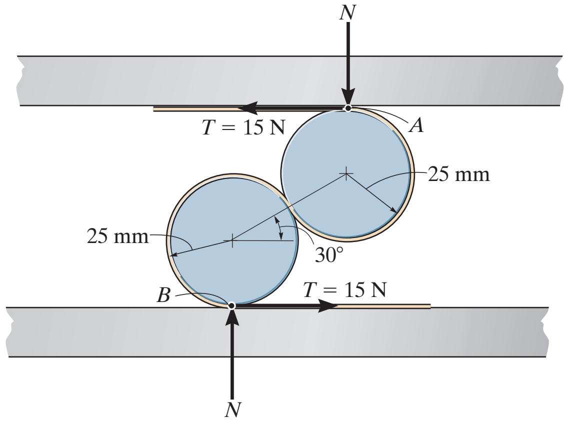
$P = 49.5\text{ N}$ **Ans**



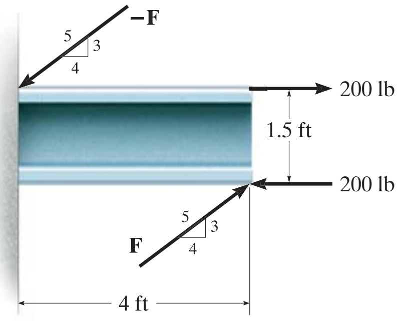
4–83. A device called a rolamite is used in various ways to replace slipping motion with rolling motion. If the belt, which wraps between the rollers, is subjected to a tension of 15 N, determine the reactive forces *N* of the top and bottom plates on the rollers so that the resultant couple acting on the rollers is equal to zero.

$$\circlearrowright + \Sigma M_A = 0; \quad 15(50 + 50\sin 30°) - N(50\cos 30°) = 0$$

$N = 26.0\text{ N}$ **Ans**



***4–84.** Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is 300 lb · ft counterclockwise. Where on the beam does the resultant couple act?

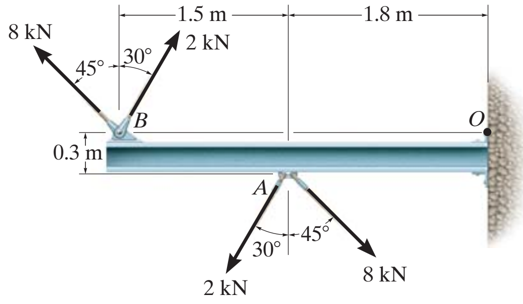


$$\circlearrowleft + (M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$$

$F = 167$ lb **Ans**

Resultant couple can act anywhere. **Ans**

•4–85. Determine the resultant couple moment acting on the beam. Solve the problem two ways: (a) sum moments about point O; and (b) sum moments about point A.



(a)

$$\circlearrowleft + \; M_R = \Sigma M_O; \quad M_R = 8\cos 45^\circ(1.8) + 8\sin 45^\circ(0.3) + 2\cos 30^\circ(1.8)$$

$$- 2\sin 30^\circ(0.3) - 2\cos 30^\circ(3.3) - 8\cos 45^\circ(3.3)$$

$$M_R = -9.69 \text{ kN}\cdot\text{m} = 9.69 \text{ kN}\cdot\text{m} \circlearrowright$$ **Ans**

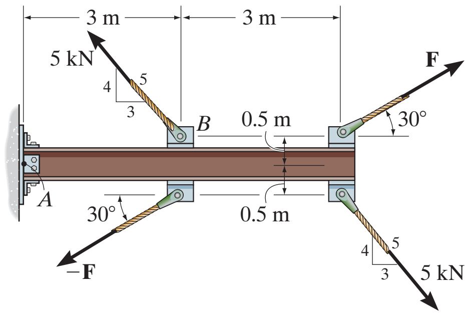
(b)

$$\circlearrowleft + \; M_R = \Sigma M_A; \quad M_R = 8\sin 45^\circ(0.3) - 8\cos 45^\circ(1.5)$$

$$- 2\cos 30^\circ(1.5) - 2\sin 30^\circ(0.3)$$

$$= -9.69 \text{ kN}\cdot\text{m} = 9.69 \text{ kN}\cdot\text{m} \circlearrowright$$ **Ans**

4–86. Two couples act on the cantilever beam. If $F = 6$ kN, determine the resultant couple moment.



a) By resolving the 6 - kN and 5 - kN couples into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by the 6- kN and 5 - kN couples, respectively, are given by

$$\circlearrowleft +(M_c)_1 = 6\sin 30°(3) - 6\cos 30°(0.5+0.5) = 3.804 \text{ kN}\cdot\text{m}$$

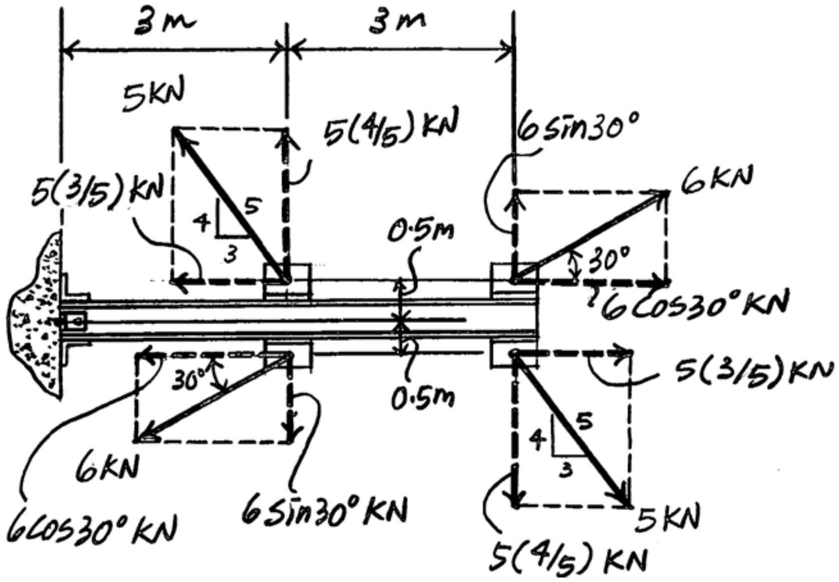
$$\circlearrowleft +(M_c)_2 = 5\left(\frac{3}{5}\right)(0.5+0.5) - 5\left(\frac{4}{5}\right)(3) = -9 \text{ kN}\cdot\text{m}$$

Thus, the resultant couple moment can be determined from

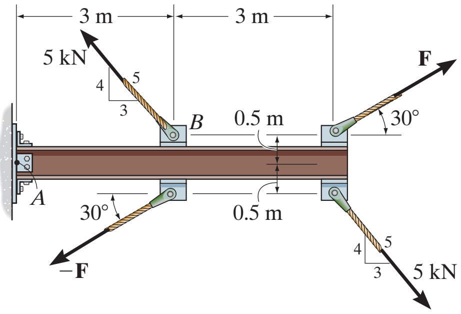
$$(M_c)_R = (M_c)_1 + (M_c)_2$$
$$= 3.804 - 9 = -5.196 \text{ kN}\cdot\text{m} = 5.20 \text{ kN}\cdot\text{m} \text{ (clockwise)} \qquad \textbf{Ans.}$$

b) By resolving the 6 - kN and 5 - kN couples into their x and y components, Fig. a, and summing the moments of these force components about point A, we can write

$$\circlearrowleft +(M_c)_R = \Sigma M_A; \qquad (M_c)_R = 5\left(\frac{3}{5}\right)(0.5) + 5\left(\frac{4}{5}\right)(3) - 6\cos 30°(0.5) - 6\sin 30°(3)$$
$$+ 6\sin 30°(6) - 6\cos 30°(0.5) + 5\left(\frac{3}{5}\right)(0.5) - 5\left(\frac{4}{5}\right)(6)$$
$$= -5.196 \text{ kN}\cdot\text{m} = 5.20 \text{ kN}\cdot\text{m} \text{ (clockwise)} \qquad \textbf{Ans.}$$



4–87. Determine the required magnitude of force **F**, if the resultant couple moment on the beam is to be zero.



By resolving **F** and the 5 - kN couple into their x and y components, Fig. a, the couple moments $(M_c)_1$ and $(M_c)_2$ produced by **F** and the 5 - kN couple, respectively, are given by

$$\circlearrowleft +(M_c)_1 = F\sin 30°(3) - F\cos 30°(1) = 0.6340F$$

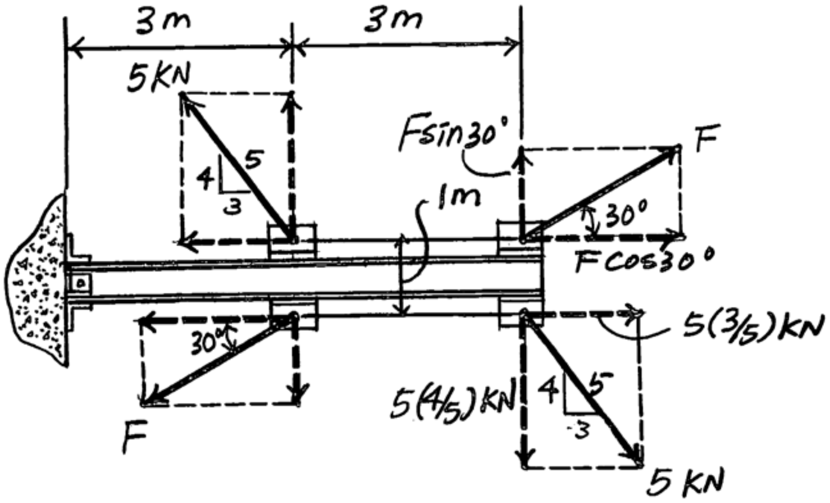
$$\circlearrowleft +(M_c)_2 = 5\left(\frac{3}{5}\right)(1) - 5\left(\frac{4}{5}\right)(3) = -9\text{ kN}\cdot\text{m}$$

The resultant couple moment acting on the beam is required to be zero. Thus,

$$(M_c)_R = (M_c)_1 + (M_c)_2$$

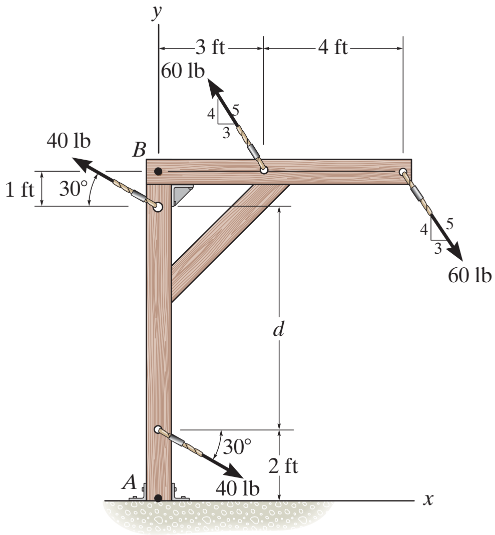
$$0 = 0.6340F - 9$$

$$F = 14.2\text{ kN}\cdot\text{m}$$ **Ans.**



(a)

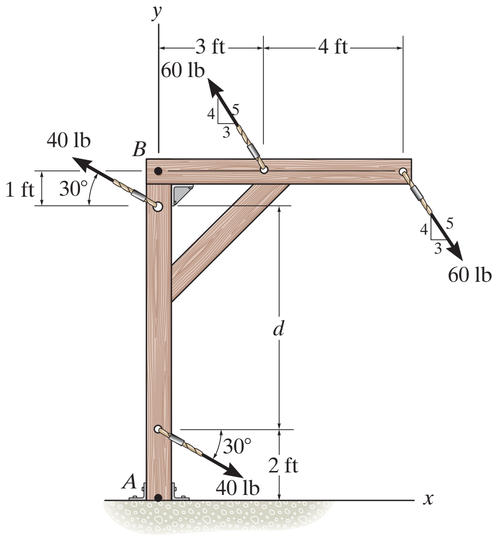
***4–88.** Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the 40-lb couple forces.



$$\curvearrowleft + M_C = 0 = 40\cos 30^\circ(d) - 60\left(\frac{4}{5}\right)(4)$$

$d = 5.54$ ft **Ans**

•4–89. Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point A.



(a)

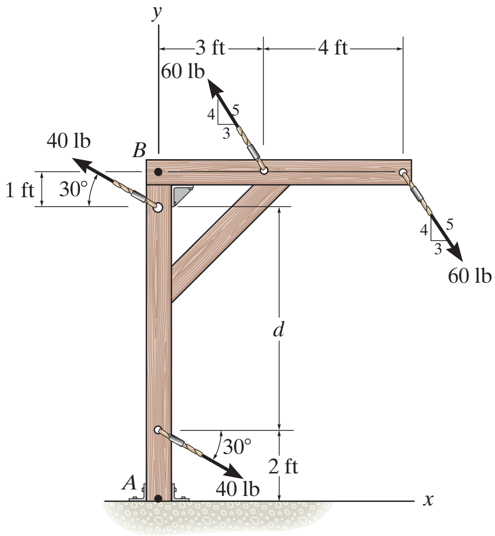
$$\curvearrowleft + M_C = 40\cos 30^\circ(4) - 60\left(\frac{4}{5}\right)(4) = -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \curvearrowright$$ **Ans**

(b)

$$\curvearrowleft + M_C = -40\cos 30^\circ(2) + 40\cos 30^\circ(6) + 60\left(\frac{4}{5}\right)(3) + 60\left(\frac{3}{5}\right)(7) - 60\left(\frac{4}{5}\right)(7) - 60\left(\frac{3}{5}\right)(7)$$

$$= -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \curvearrowright$$ **Ans**

4–90. Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point B.



(a)

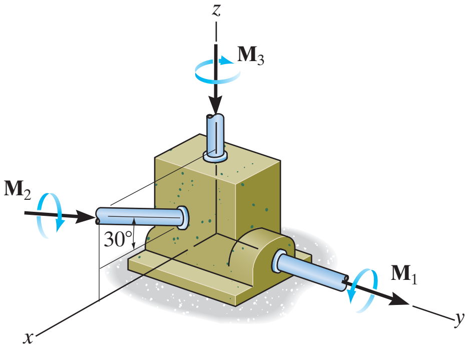
$$\curvearrowleft + M_C = 40\cos 30^\circ(4) - 60\left(\frac{4}{5}\right)(4) = -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \curvearrowright$$ **Ans**

(b)

$$\curvearrowleft + M_C = 40\cos 30^\circ(5) - 40\cos 30^\circ(1) + 60\left(\frac{4}{5}\right)(3) - 60\left(\frac{4}{5}\right)(7)$$

$$= -53.4 \text{ lb}\cdot\text{ft} = 53.4 \text{ lb}\cdot\text{ft} \curvearrowright$$ **Ans**

4–91. If $M_1 = 500$ N·m, $M_2 = 600$ N·m, and $M_3 = 450$ N·m, determine the magnitude and coordinate direction angles of the resultant couple moment.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$\mathbf{M}_1 = [500\mathbf{j}]\ \text{N}\cdot\text{m}$

$\mathbf{M}_2 = 600(-\cos 30°\mathbf{i} - \sin 30°\mathbf{k}) = \{-519.62\mathbf{i} - 300\mathbf{k}\}\ \text{N}\cdot\text{m}$

$\mathbf{M}_3 = [-450\mathbf{k}]\ \text{N}\cdot\text{m}$

The resultant couple moment is given by

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$ $\qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

$$= 500\mathbf{j} + (-519.62\mathbf{i} - 300\mathbf{k}) + (-450\mathbf{k})$$

$$= [-519.62\mathbf{i} + 500\mathbf{j} - 750\mathbf{k}]\ \text{N}\cdot\text{m}$$

The magnitude of $(\mathbf{M}_c)_R$ is

$$(M_c)_R = \sqrt{[(M_c)_R]_x^2 + [(M_c)_R]_y^2 + [(M_c)_R]_z^2}$$

$$= \sqrt{(-519.62)^2 + 500^2 + (-750)^2}$$

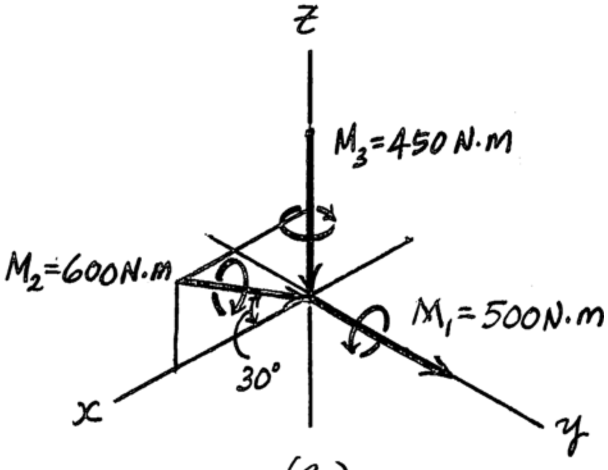
$$= 1040.43\ \text{N}\cdot\text{m} = 1.04\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}_C)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_C)_R]_x}{(M_C)_R}\right) = \cos\left(\frac{-519.62}{1040.43}\right) = 120° \qquad \textbf{Ans.}$$

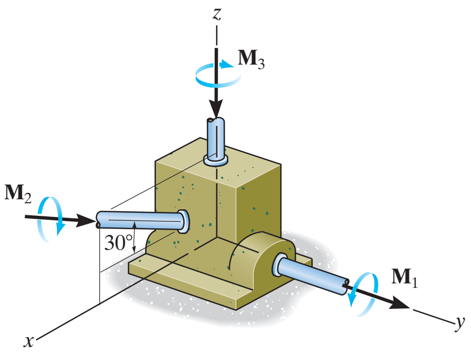
$$\beta = \cos^{-1}\left(\frac{[(M_C)_R]_y}{(M_C)_R}\right) = \cos\left(\frac{500}{1040.43}\right) = 61.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_C)_R]_z}{(M_C)_R}\right) = \cos\left(\frac{-750}{1040.43}\right) = 136° \qquad \textbf{Ans.}$$



(a)

***4–92.** Determine the required magnitude of couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ so that the resultant couple moment is $\mathbf{M}_R = \{-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k}\}\ \mathrm{N\cdot m}$.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$\mathbf{M}_1 = M_1\mathbf{j}$

$\mathbf{M}_2 = M_2(-\cos 30°\,\mathbf{i} - \sin 30°\,\mathbf{k}) = -0.8660M_2\mathbf{i} - 0.5M_2\mathbf{k}$

$\mathbf{M}_3 = -M_3\mathbf{k}$

The resultant couple moment is given by

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};\qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

$$(-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k}) = M_1\mathbf{j} + (-0.8660M_2\mathbf{i} - 0.5M_2\mathbf{k}) + (-M_3\mathbf{k})$$

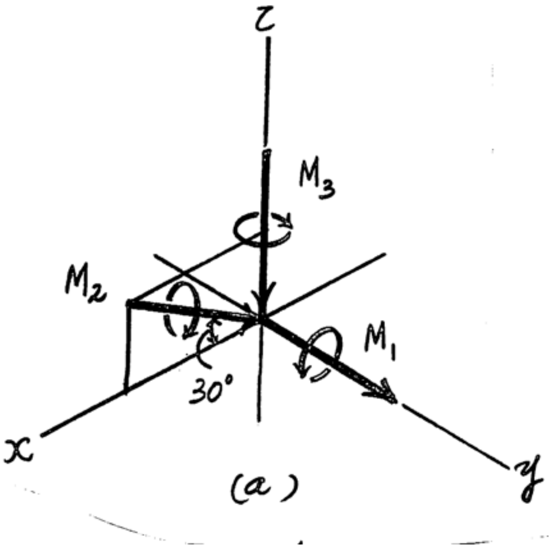
$$-300\mathbf{i} + 450\mathbf{j} - 600\mathbf{k} = -0.8660M_2\mathbf{i} + M_1\mathbf{j} - (0.5M_2 + M_3)\mathbf{k}$$

Equating the **i**, **j**, and **k** components yields

$-300 = -0.8660M_2 \qquad M_2 = 346.41\ \mathrm{N\cdot m} = 346\ \mathrm{N\cdot m}$ **Ans.**

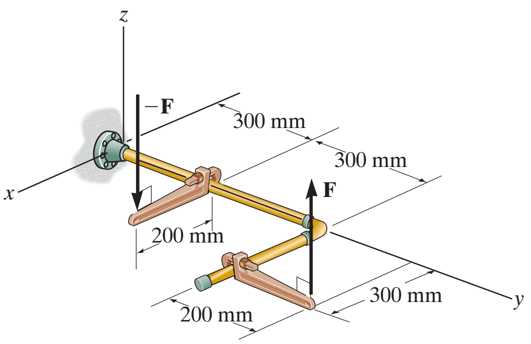
$M_1 = 450\ \mathrm{N\cdot m}$ **Ans.**

$600 = -0.5(346.41) + M_3 \qquad M_3 = 427\ \mathrm{N\cdot m}$ **Ans.**



(a)

•4–93. If $F = 80\,\text{N}$, determine the magnitude and coordinate direction angles of the couple moment. The pipe assembly lies in the x–y plane.



It is easiest to find the couple moment of **F** by taking the moment of **F** or **−F** about point *A* or *B*, respectively, Fig. *a*. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\,\text{m}$$
$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\,\text{m}$$

The force vectors **F** and **−F** can be written as

$\mathbf{F} = \{80\mathbf{k}\}\,\text{N}$ and $-\mathbf{F} = [-80\mathbf{k}]\,\text{N}$

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\,\text{N}\cdot\text{m}$$

or

$$\mathbf{M}_c = \mathbf{r}_{BA} \times -\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\,\text{N}\cdot\text{m}$$

The magnitude of M_c is given by

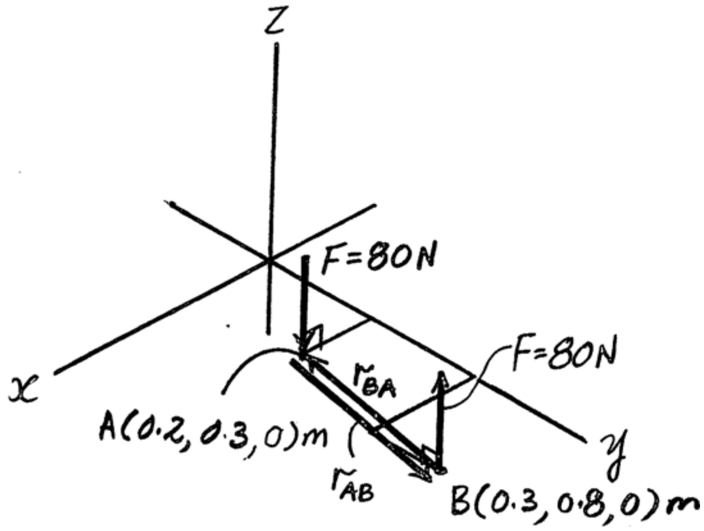
$$M_c = \sqrt{{M_x}^2 + {M_y}^2 + {M_z}^2} = \sqrt{40^2 + (-8)^2 + 0^2} = 40.79\ \text{N}\cdot\text{m} = 40.8\,\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $\mathbf{M}_c$ are

$$\alpha = \cos^{-1}\left(\frac{M_x}{M}\right) = \cos\left(\frac{40}{40.79}\right) = 11.3° \qquad \textbf{Ans.}$$

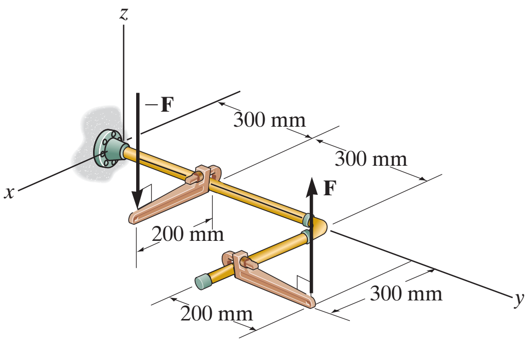
$$\beta = \cos^{-1}\left(\frac{M_y}{M}\right) = \cos\left(\frac{-8}{40.79}\right) = 101° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{M_z}{M}\right) = \cos\left(\frac{0}{40.79}\right) = 90° \qquad \textbf{Ans.}$$



(a)

4–94. If the magnitude of the couple moment acting on the pipe assembly is 50 N · m, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the x–y plane.



It is easiest to find the couple moment of **F** by taking the moment of either **F** or **−F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3-0.2)\mathbf{i}+(0.8-0.3)\mathbf{j}+(0-0)\mathbf{k} = [0.1\mathbf{i}+0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2-0.3)\mathbf{i}+(0.3-0.8)\mathbf{j}+(0-0)\mathbf{k} = [-0.1\mathbf{i}-0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and **−F** can be written as

$$\mathbf{F} = \{F\mathbf{k}\}\text{ N and } -\mathbf{F} = [-F\mathbf{k}]\text{ N}$$

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = 0.5F\mathbf{i}-0.1F\mathbf{j}$$

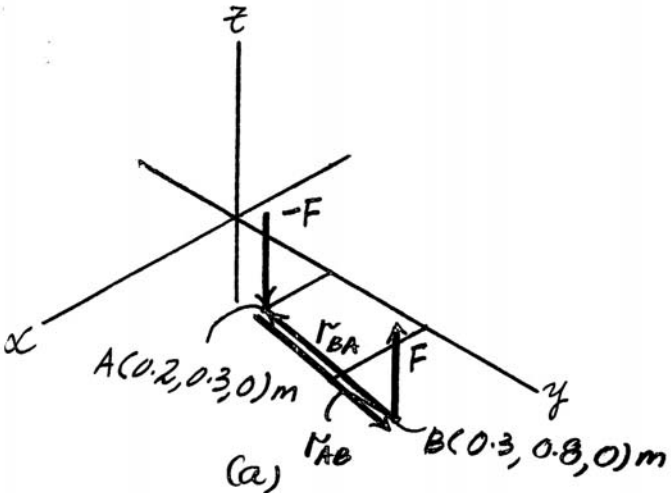
The magnitude of M_c is given by

$$M_c = \sqrt{M_x{}^2+M_y{}^2+M_z{}^2} = \sqrt{(0.5F)^2+(0.1F)^2+0^2} = 0.5099F$$

Since M_c is required to equal 50 N · m,

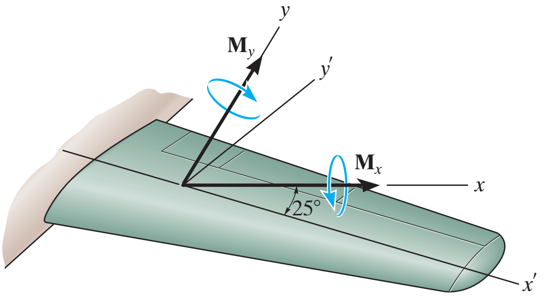
$50 = 0.5099F$

$F = 98.1\text{ N}$ **Ans.**



(a)

4–95. From load calculations it is determined that the wing is subjected to couple moments $M_x = 17$ kip · ft and $M_y = 25$ kip · ft. Determine the resultant couple moments created about the x' and y' axes. The axes all lie in the same horizontal plane.

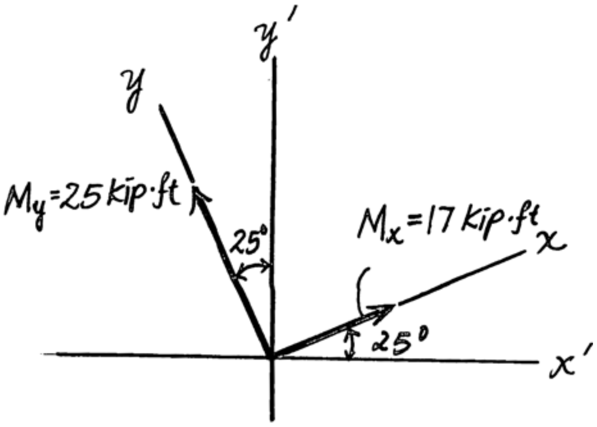


$(M_R)_{x'} = \Sigma M_{x'}$; $(M_R)_{x'} = 17\cos 25° - 25\sin 25°$

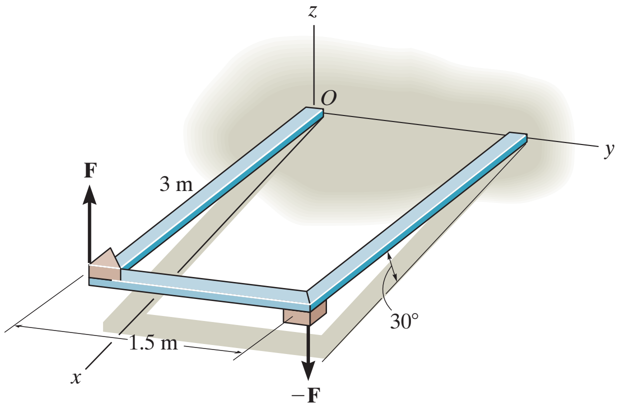
$= 4.84$ kip · ft Ans

$(M_R)_{y'} = \Sigma M_{y'}$; $(M_R)_{y'} = 17\sin 25° + 25\cos 25°$

$= 29.8$ kip · ft Ans



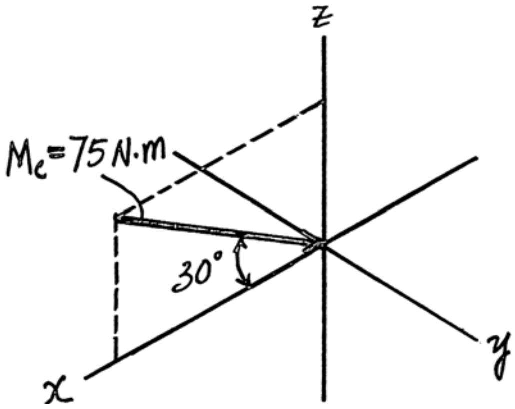
***4–96.** Express the moment of the couple acting on the frame in Cartesian vector form. The forces are applied perpendicular to the frame. What is the magnitude of the couple moment? Take $F = 50$ N.



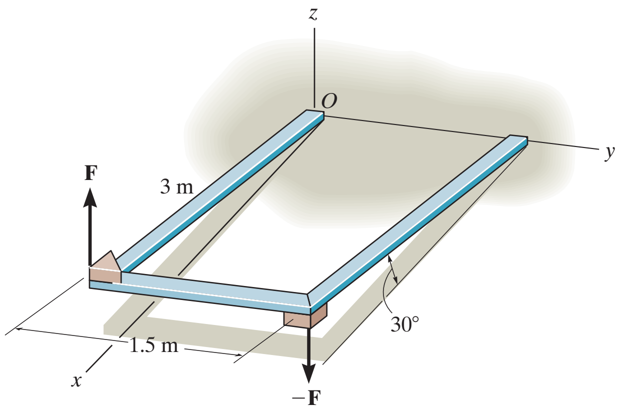
$M_C = 50(1.5) = 75$ N · m Ans

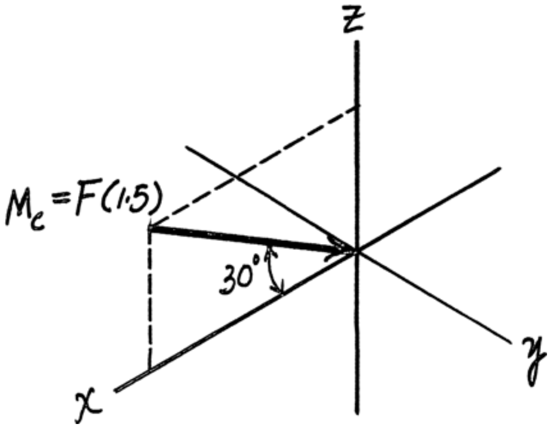
$\mathbf{M}_C = -75(\cos 30° \mathbf{i} + \cos 60° \mathbf{k})$

$= \{-65.0\mathbf{i} - 37.5\mathbf{k}\}$ N · m Ans



•4–97. In order to turn over the frame, a couple moment is applied as shown. If the component of this couple moment along the x axis is $\mathbf{M}_x = \{-20\mathbf{i}\}\ \mathrm{N \cdot m}$, determine the magnitude F of the couple forces.





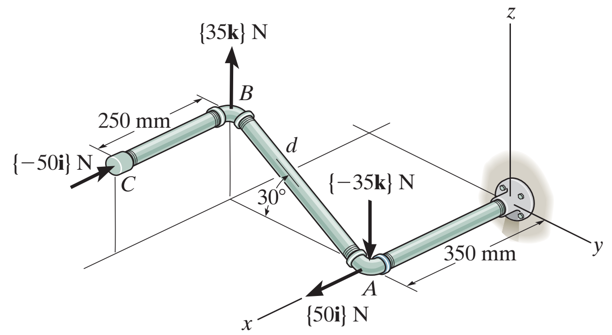
$M_C = F(1.5)$

Thus

$20 = F(1.5)\cos 30°$

$F = 15.4\ \mathrm{N}$ **Ans**

4–98. Determine the resultant couple moment of the two couples that act on the pipe assembly. The distance from A to B is $d = 400$ mm. Express the result as a Cartesian vector.



Vector Analysis

Position Vector :

$$\mathbf{r}_{AB} = \{(0.35 - 0.35)\mathbf{i} + (-0.4\cos 30° - 0)\mathbf{j} + (0.4\sin 30° - 0)\mathbf{k}\}\ \mathrm{m}$$
$$= \{-0.3464\mathbf{j} + 0.20\mathbf{k}\}\ \mathrm{m}$$

Couple Moments : With $\mathbf{F}_1 = \{35\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-50\mathbf{i}\}$ N, applying Eq. 4 – 15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ 0 & 0 & 35 \end{vmatrix} = \{-12.12\mathbf{i}\}\ \mathrm{N \cdot m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ -50 & 0 & 0 \end{vmatrix} = \{-10.0\mathbf{j} - 17.32\mathbf{k}\}\ \mathrm{N \cdot m}$$

Resultant Couple Moment :

$$\mathbf{M}_R = \Sigma \mathbf{M}; \quad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2 = \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\}\ \mathrm{N \cdot m} \qquad \textbf{Ans}$$

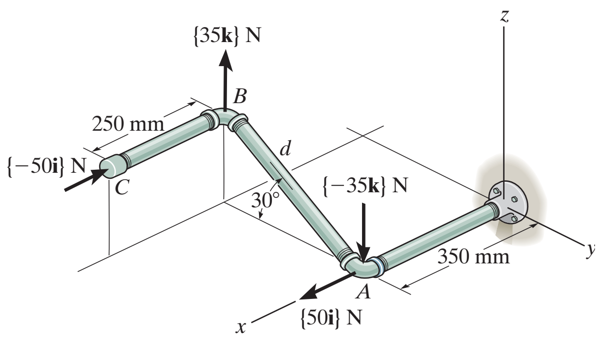
Scalar Analysis : Summing moments about x, y and z axes, we have

$$(M_R)_x = \Sigma M_x; \quad (M_R)_x = -35(0.4\cos 30°) = -12.12\ \mathrm{N \cdot m}$$
$$(M_R)_y = \Sigma M_y; \quad (M_R)_y = -50(0.4\sin 30°) = -10.0\ \mathrm{N \cdot m}$$
$$(M_R)_z = \Sigma M_z; \quad (M_R)_z = -50(0.4\cos 30°) = -17.32\ \mathrm{N \cdot m}$$

Express $\mathbf{M}_R$ as a Cartesian vector, we have

$$\mathbf{M}_R = \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\}\ \mathrm{N \cdot m}$$

4–99. Determine the distance d between A and B so that the resultant couple moment has a magnitude of $M_R = 20\ \text{N}\cdot\text{m}$.



Position Vector :

$$\mathbf{r}_{AB} = \{(0.35 - 0.35)\,\mathbf{i} + (-d\cos 30^\circ - 0)\,\mathbf{j} + (d\sin 30^\circ - 0)\,\mathbf{k}\}\ \text{m}$$
$$= \{-0.8660d\ \mathbf{j} + 0.50d\ \mathbf{k}\}\ \text{m}$$

Couple Moments : With $\mathbf{F}_1 = \{35\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-50\mathbf{i}\}$ N, applying Eq. 4 – 15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ 0 & 0 & 35 \end{vmatrix} = \{-30.31d\ \mathbf{i}\}\ \text{N}\cdot\text{m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ -50 & 0 & 0 \end{vmatrix} = \{-25.0d\ \mathbf{j} - 43.30d\ \mathbf{k}\}\ \text{N}\cdot\text{m}$$

Resultant Couple Moment :

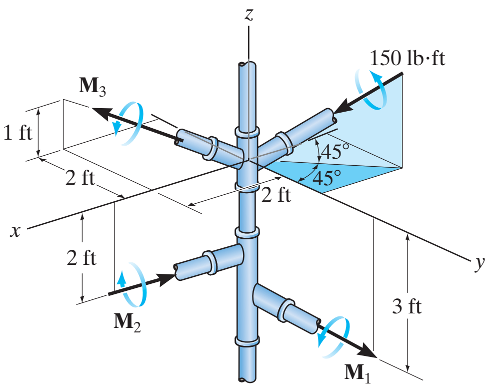
$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2 = \{-30.31d\ \mathbf{i} - 25.0d\ \mathbf{j} - 43.30d\ \mathbf{k}\}\ \text{N}\cdot\text{m}$$

The magnitude of $\mathbf{M}_R$ is 20 N · m thus

$$20 = \sqrt{(-30.31d)^2 + (-25.0d)^2 + (43.30d)^2}$$

$$d = 0.3421\ \text{m} = 342\ \text{mm} \qquad \textbf{Ans}$$

***4–100.** If $M_1 = 180$ lb·ft, $M_2 = 90$ lb·ft, and $M_3 = 120$ lb·ft, determine the magnitude and coordinate direction angles of the resultant couple moment.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, and $\mathbf{M}_4$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$\mathbf{M}_1 = [180\mathbf{j}]\ \text{lb}\cdot\text{ft}$

$\mathbf{M}_2 = [-90\mathbf{i}]\ \text{lb}\cdot\text{ft}$

$$\mathbf{M}_3 = M_3\mathbf{u} = 120\left[\frac{(2-0)\mathbf{i}+(-2-0)\mathbf{j}+(1+0)\mathbf{k}}{\sqrt{(2-0)^2+(-2-0)^2+(1-0)^2}}\right] = [80\mathbf{i}-80\mathbf{j}+40\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_4 = 150[\cos 45^\circ \sin 45^\circ\mathbf{i} - \cos 45^\circ \cos 45^\circ\mathbf{j} - \sin 45^\circ\mathbf{k}] = [75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$
$$= 180\mathbf{j} - 90\mathbf{i} + (80\mathbf{i} - 80\mathbf{j} + 40\mathbf{k}) + (75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k})$$
$$= [65\mathbf{i} + 25\mathbf{j} - 66.07\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

The magnitude of $(\mathbf{M}_c)_R$ is

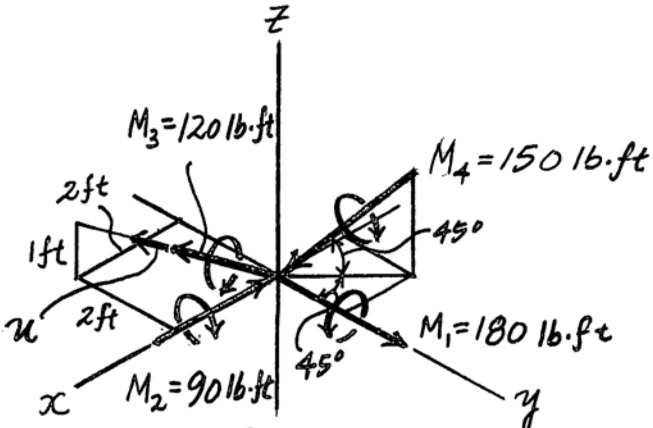
$$(M_c)_R = \sqrt{[(M_c)_R]_x^{\,2} + [(M_c)_R]_y^{\,2} + [(M_c)_R]_z^{\,2}}$$
$$= \sqrt{(65)^2 + (25)^2 + (-66.07)^2}$$
$$= 95.99\ \text{lb}\cdot\text{ft} = 96.0\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}_c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{65}{95.99}\right) = 47.4^\circ \qquad \textbf{Ans.}$$

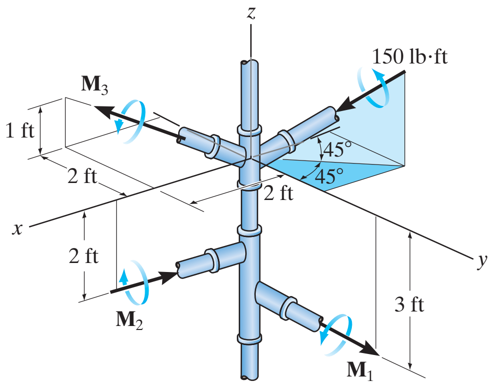
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{25}{95.99}\right) = 74.9^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-66.07}{95.99}\right) = 133^\circ \qquad \textbf{Ans.}$$



(a)

•4–101. Determine the magnitudes of couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ so that the resultant couple moment is zero.



Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, and $\mathbf{M}_4$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$$\mathbf{M}_1 = M_1\mathbf{j}$$

$$\mathbf{M}_2 = -M_2\mathbf{i}$$

$$\mathbf{M}_3 = M_3\mathbf{u} = M_3\left[\frac{(2-0)\mathbf{i}+(-2-0)\mathbf{j}+(1+0)\mathbf{k}}{\sqrt{(2-0)^2+(-2-0)^2+(1-0)^2}}\right] = \frac{2}{3}M_3\mathbf{i} - \frac{2}{3}M_3\mathbf{j} + \frac{1}{3}M_3\mathbf{k}$$

$$\mathbf{M}_4 = 150[\cos 45^\circ \sin 45^\circ \mathbf{i} - \cos 45^\circ \cos 45^\circ \mathbf{j} - \sin 45^\circ \mathbf{k}] = [75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k}]\ \text{lb}\cdot\text{ft}$$

The resultant couple moment is required to be zero. Thus,

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$

$$0 = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

$$0 = M_1\mathbf{j} + (-M_2\mathbf{i}) + \left(\frac{2}{3}M_3\mathbf{i} - \frac{2}{3}M_3\mathbf{j} + \frac{1}{3}M_3\mathbf{k}\right) + (75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k})$$

$$0 = \left(-M_2 + \frac{2}{3}M_3 + 75\right)\mathbf{i} + \left(M_1 - \frac{2}{3}M_3 - 75\right)\mathbf{j} + \left(\frac{1}{3}M_3 - 106.07\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -M_2 + \frac{2}{3}M_3 + 75 \qquad (1)$$

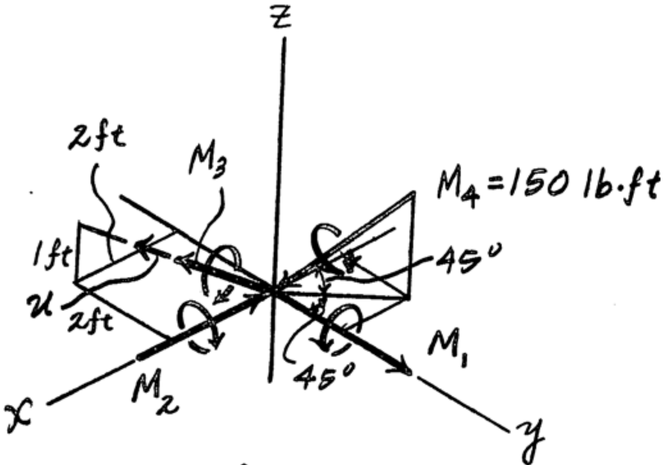
$$0 = M_1 - \frac{2}{3}M_3 - 75 \qquad (2)$$

$$0 = \frac{1}{3}M_3 - 106.07 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

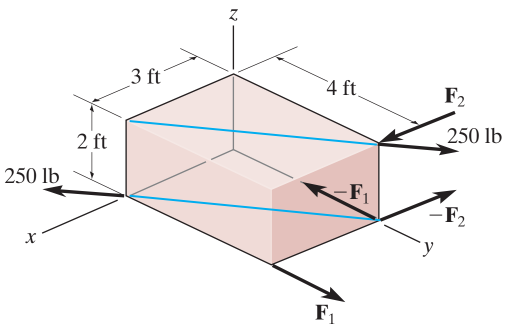
$M_3 = 318\ \text{lb}\cdot\text{ft}$ **Ans.**

$M_1 = M_2 = 287\ \text{lb}\cdot\text{ft}$ **Ans.**



(a)

4–102. If $F_1 = 100$ lb and $F_2 = 200$ lb, determine the magnitude and coordinate direction angles of the resultant couple moment.



Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = [-2\mathbf{k}]$ ft $\qquad \mathbf{r}_2 = [2\mathbf{k}]$ ft $\qquad \mathbf{r}_3 = [2\mathbf{k}]$ ft

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = [100\mathbf{j}]$ lb $\qquad \mathbf{F}_2 = [200\mathbf{i}]$ lb

$$\mathbf{F}_3 = F_3\mathbf{u} = 250\left[\frac{(0-3)\mathbf{i}+(4-0)\mathbf{j}+(2-2)\mathbf{k}}{\sqrt{(0-3)^2+(4-0)^2+(2-2)^2}}\right] = [-150\mathbf{i}+200\mathbf{j}]\text{ lb}$$

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (-2\mathbf{k})\times(100\mathbf{j}) = [200\mathbf{i}]\text{ lb}\cdot\text{ft}$$

$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (2\mathbf{k})\times(200\mathbf{i}) = [400\mathbf{j}]\text{ lb}\cdot\text{ft}$$

$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (2\mathbf{k})\times(-150\mathbf{i}+200\mathbf{j}) = [-400\mathbf{i}-300\mathbf{j}]\text{ lb}\cdot\text{ft}$$

Resultant Moment: The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1+\mathbf{M}_2+\mathbf{M}_3$$

$$= (200\mathbf{i})+(400\mathbf{j})+(-400\mathbf{i}-300\mathbf{j})$$

$$= [-200\mathbf{i}+100\mathbf{j}]\text{ lb}\cdot\text{ft}$$

The magnitude of the couple moment is

$$(M_c)_R = \sqrt{[(M_c)_R]_x^{\,2}+[(M_c)_R]_y^{\,2}+[(M_c)_R]_z^{\,2}}$$

$$= \sqrt{(-200)^2+(100)^2+(0)^2}$$

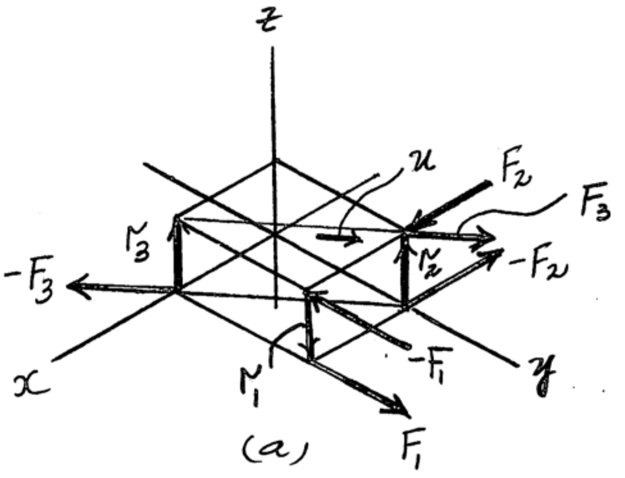
$$= 223.61\text{ N}\cdot\text{m} = 224\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{-200}{223.61}\right) = 153^\circ \qquad \textbf{Ans.}$$

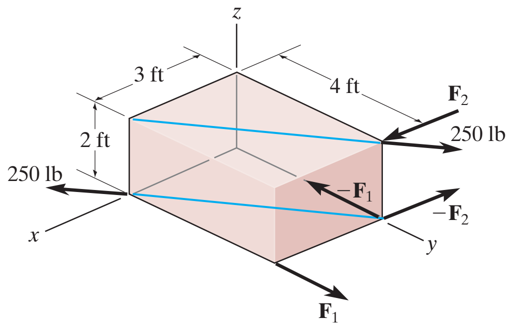
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{100}{223.61}\right) = 63.4^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{0}{223.61}\right) = 90^\circ \qquad \textbf{Ans.}$$



(a)

4–103. Determine the magnitude of couple forces $\mathbf{F}_1$ and $\mathbf{F}_2$ so that the resultant couple moment acting on the block is zero.



Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, Fig. a, must be determined first.

$\mathbf{r}_1 = [-2\mathbf{k}]\,\text{ft}$ $\quad$ $\mathbf{r}_2 = [2\mathbf{k}]\,\text{ft}$ $\quad$ $\mathbf{r}_3 = [2\mathbf{k}]\,\text{ft}$

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = F_1\mathbf{j}$ $\quad$ $\mathbf{F}_2 = F_2\mathbf{i}$

$$\mathbf{F}_3 = F_3\mathbf{u} = 250\left[\frac{(0-3)\mathbf{i}+(4-0)\mathbf{j}+(2-2)\mathbf{k}}{\sqrt{(0-3)^2+(4-0)^2+(2-2)^2}}\right] = [-150\mathbf{i}+200\mathbf{j}]\,\text{lb}$$

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (-2\mathbf{k}) \times (F_1\mathbf{j}) = 2F_1\mathbf{i}$$

$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (2\mathbf{k}) \times (F_2\mathbf{i}) = 2F_2\mathbf{j}$$

$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (2\mathbf{k}) \times (-150\mathbf{i} + 200\mathbf{j}) = [-400\mathbf{i} - 300\mathbf{j}]\,\text{lb}\cdot\text{ft}$$

Resultant Moment: Since the resultant couple moment is required to be equal to zero,

$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$ $\quad$ $\mathbf{0} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

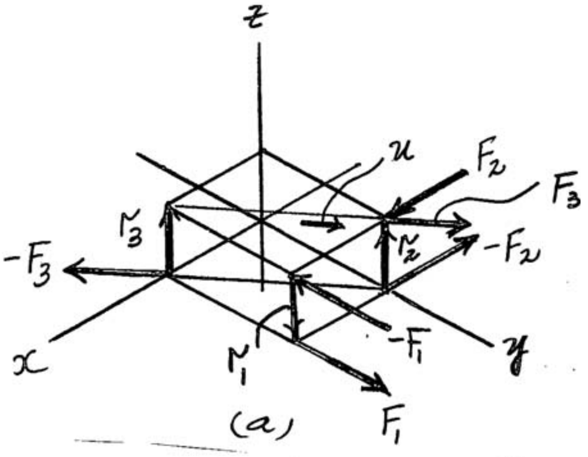
$$\mathbf{0} = (2F_1\mathbf{i}) + (2F_2\mathbf{j}) + (-400\mathbf{i} - 300\mathbf{j})$$

$$\mathbf{0} = (2F_1 - 400)\mathbf{i} + (2F_2 - 300)\mathbf{j}$$

Equating the **i**, **j**, and **k** components yields

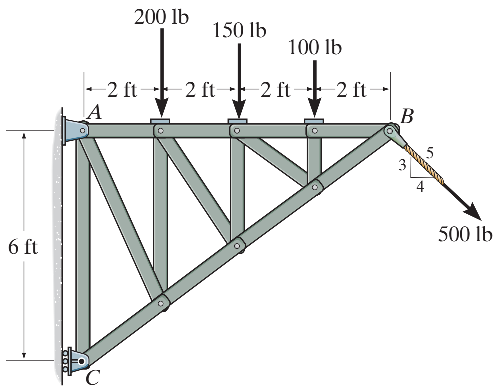
$0 = 2F_1 - 400$ $\quad$ $F_1 = 200\text{ lb}$ $\quad$ **Ans.**

$0 = 2F_2 - 300$ $\quad$ $F_2 = 150\text{ lb}$ $\quad$ **Ans.**



(a)

*__4–104.__ Replace the force system acting on the truss by a resultant force and couple moment at point *C*.



Equivalent Resultant Force: The 500-lb force is resolved into its *x* and *y* components, Fig. *a*. Summing these force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 500\left(\frac{4}{5}\right) = 400 \text{ lb} \rightarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \qquad (F_R)_y = -200 - 150 - 100 - 500\left(\frac{3}{5}\right) = -750 \text{ lb} = 750 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{400^2 + 750^2} = 850 \text{ lb} \qquad \textbf{Ans.}$$

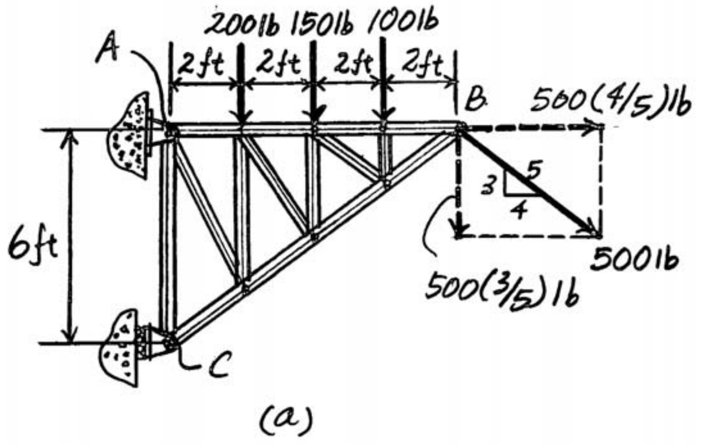
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{750}{400}\right] = 61.93° = 61.9° \qquad \textbf{Ans.}$$

Equivalent Couple Moment: Summing the moment of the forces and force components, Fig. *a*, algebraically about point *C*,

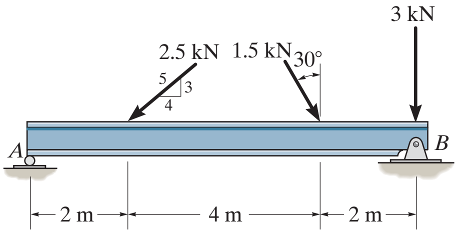
$$\circlearrowleft +(M_R)_C = \Sigma M_C; \qquad (M_R)_C = -200(2) - 150(4) - 100(6) - 500\left(\frac{3}{5}\right)(8) - 500\left(\frac{4}{5}\right)(6)$$

$$= -6400 \text{ lb}\cdot\text{ft} = 6.40 \text{ kip}\cdot\text{ft (clockwise)} \qquad \textbf{Ans.}$$



(a)

•4–105. Replace the force system acting on the beam by an equivalent force and couple moment at point *A*.

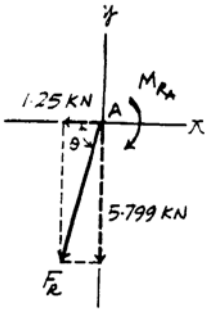


$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = 1.5\sin 30° - 2.5\left(\frac{4}{5}\right)$

$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = -1.5\cos 30° - 2.5\left(\frac{3}{5}\right) - 3$

$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$



Thus,

$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$ **Ans**

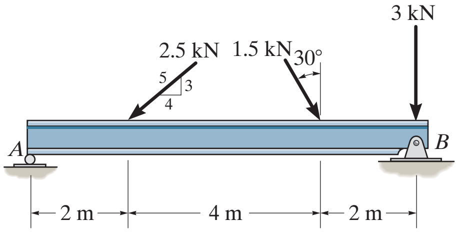
and

$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8°$ **Ans**

$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5\cos 30°(6) - 3(8)$

$= -34.8 \text{ kN}\cdot\text{m} = 34.8 \text{ kN}\cdot\text{m}$ ***(Clockwise)*** **Ans**

4–106. Replace the force system acting on the beam by an equivalent force and couple moment at point *B*.

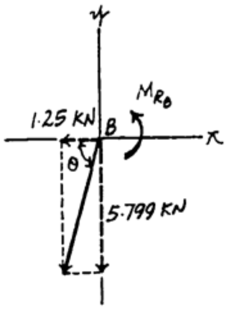


$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = 1.5\sin 30° - 2.5\left(\frac{4}{5}\right)$

$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = -1.5\cos 30° - 2.5\left(\frac{3}{5}\right) - 3$

$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$



Thus,

$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$ **Ans**

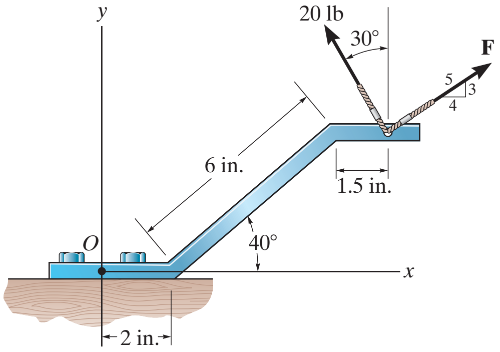
and

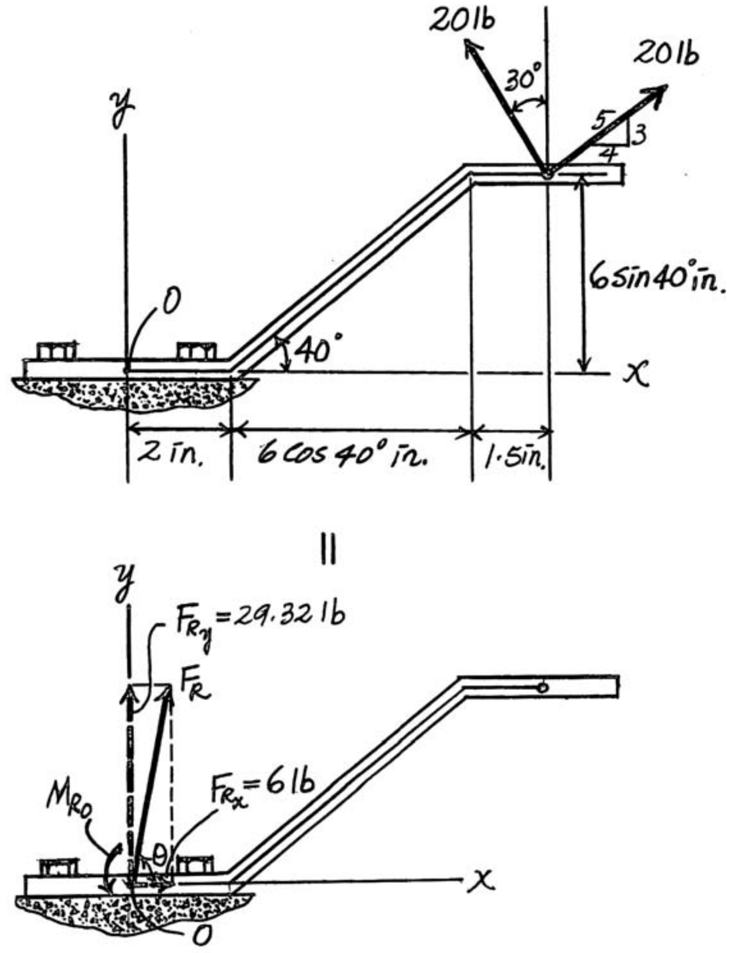
$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8°$ **Ans**

$\circlearrowleft + M_{R_B} = \Sigma M_B;\quad M_{R_B} = 1.5\cos 30°(2) + 2.5\left(\frac{3}{5}\right)(6)$

$= 11.6 \text{ kN}\cdot\text{m}$ ***(Counterclockwise)*** **Ans**

4–107. Replace the two forces by an equivalent resultant force and couple moment at point *O*. Set $F = 20$ lb.





$\xrightarrow{+} F_{R_x} = \Sigma F_x;\qquad F_{R_x} = \frac{4}{5}(20) - 20 \sin 30^\circ = 6 \text{ lb}$

$+\uparrow F_{R_y} = \Sigma F_y;\qquad F_{R_y} = 20 \cos 30^\circ + \frac{3}{5}(20) = 29.32 \text{ lb}$

$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{6^2 + (29.32)^2} = 29.9 \text{ lb}$ Ans

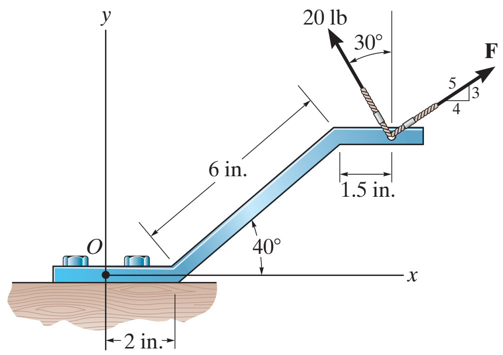
$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{29.32}{6}\right) = 78.4^\circ$ ∡ Ans

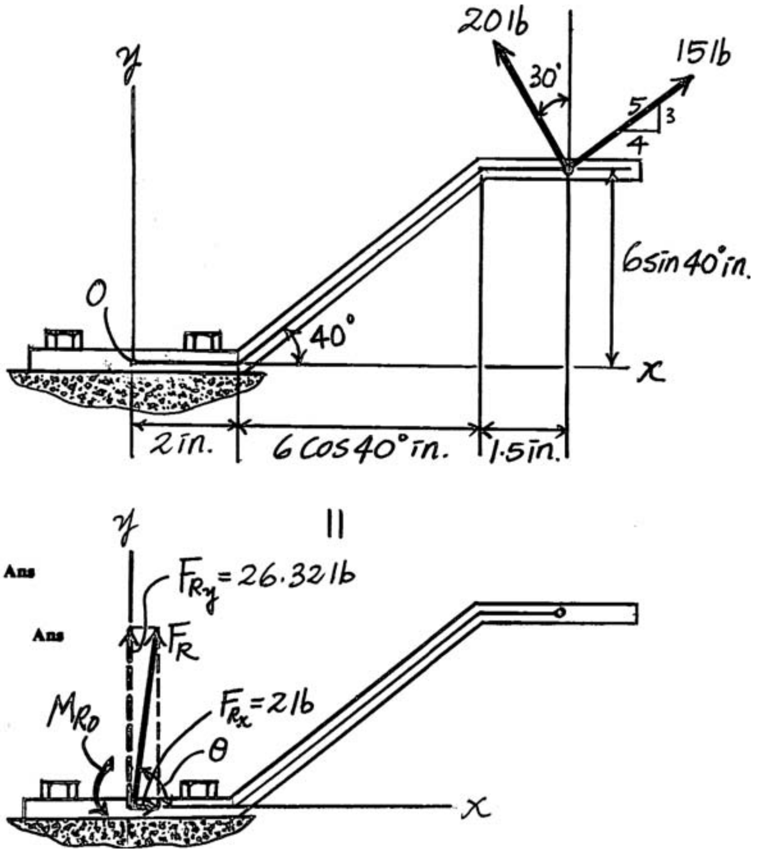
$\circlearrowleft + M_{R_O} = \Sigma M_O;\qquad M_{R_O} = 20 \sin 30^\circ(6 \sin 40^\circ) + 20 \cos 30^\circ(3.5 + 6 \cos 40^\circ)$

$- \frac{4}{5}(20)(6 \sin 40^\circ) + \frac{3}{5}(20)(3.5 + 6 \cos 40^\circ)$

$= 214 \text{ lb} \cdot \text{in.} \circlearrowleft$ Ans

***4–108.** Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 15$ lb.





$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = \frac{4}{5}(15) - 20\sin 30^\circ = 2\text{ lb}$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = 20\cos 30^\circ + \frac{3}{5}(15) = 26.32\text{ lb}$

$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{2^2 + 26.32^2} = 26.4\text{ lb}$ **Ans**

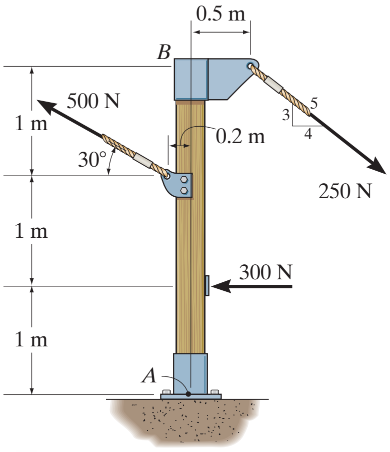
$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{26.32}{2}\right) = 85.7^\circ$ ∠ **Ans**

$\circlearrowleft + M_{R_O} = \Sigma M_O;\quad M_{R_O} = 20\sin 30^\circ(6\sin 40^\circ) + 20\cos 30^\circ(3.5 + 6\cos 40^\circ)$

$-\frac{4}{5}(15)(6\sin 40^\circ) + \frac{3}{5}(15)(3.5 + 6\cos 40^\circ)$

$= 205\text{ lb}\cdot\text{in.} \circlearrowright$ **Ans**

•4–109. Replace the force system acting on the post by a resultant force and couple moment at point *A*.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their *x* and *y* components, Fig. *a*. Summing these force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30^\circ - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500\sin 30^\circ - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

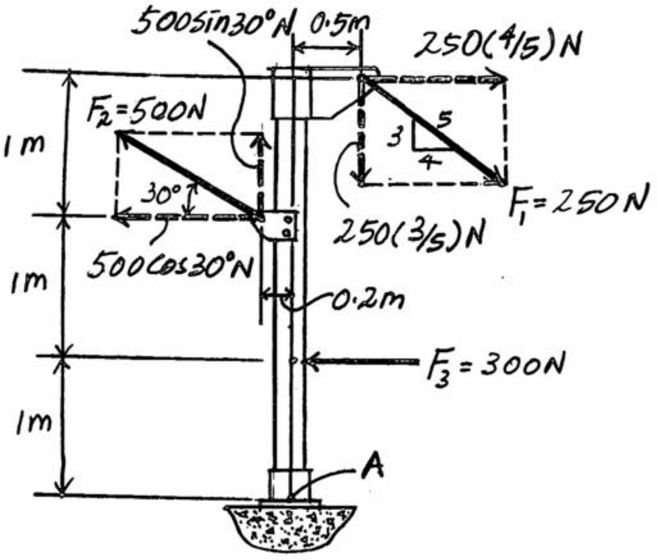
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63^\circ = 10.6^\circ \qquad \textbf{Ans.}$$

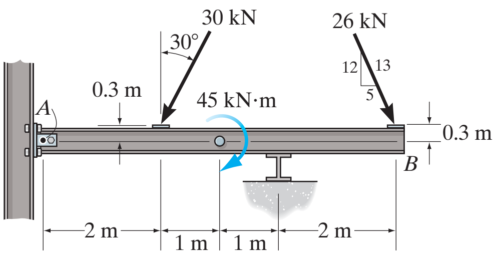
Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. *a*, and summing the moments of the force components algebraically about point *A*,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad (M_R)_A = 500\cos 30^\circ(2) - 500\sin 30^\circ(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$
$$= 441.02\text{ N}\cdot\text{m} = 441\text{ N}\cdot\text{m (counterclockwise)} \qquad \textbf{Ans.}$$



(a)

4–110. Replace the force and couple moment system acting on the overhang beam by a resultant force and couple moment at point *A*.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their *x* and *y* components, Fig. *a*. Summing these force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 26\left(\frac{5}{13}\right) - 30\sin 30° = -5\text{ kN} = 5\text{ kN} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -26\left(\frac{12}{13}\right) - 30\cos 30° = -49.98\text{ kN} = 49.98\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{5^2 + 49.98^2} = 50.23\text{ kN} = 50.2\text{ kN}$$ **Ans.**

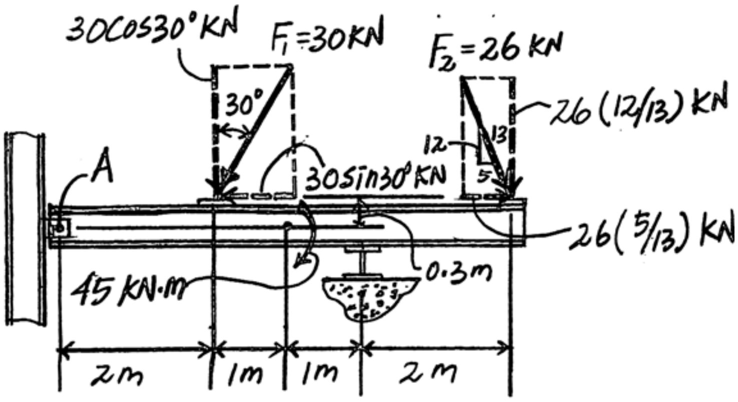
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{49.98}{5}\right] = 84.29° = 84.3° \measuredangle$$ **Ans.**

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point *A*,

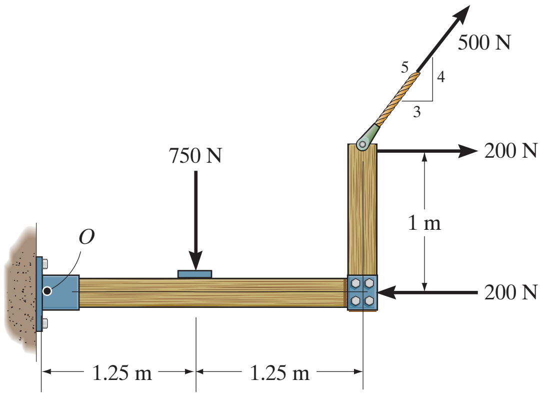
$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_A = 30\sin 30°(0.3) - 30\cos 30°(2) - 26\left(\frac{5}{13}\right)(0.3) - 26\left(\frac{12}{13}\right)(6) - 45$$

$$= -239.46\text{ kN}\cdot\text{m} = 239\text{ kN}\cdot\text{m (clockwise)}$$ **Ans.**



(a)

4–111. Replace the force system by a resultant force and couple moment at point *O*.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. *a*. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 200 - 200 + 500\left(\frac{3}{5}\right) = 300\text{ N} \rightarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -750 + 500\left(\frac{4}{5}\right) = -350\text{ N} = 350\text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{300^2 + 350^2} = 461.0\text{ N} = 461\text{ N} \qquad \textbf{Ans.}$$

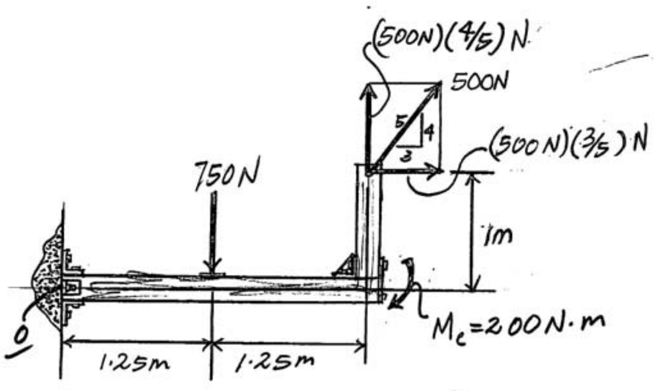
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{350}{300}\right] = 49.4° \searrow \qquad \textbf{Ans.}$$

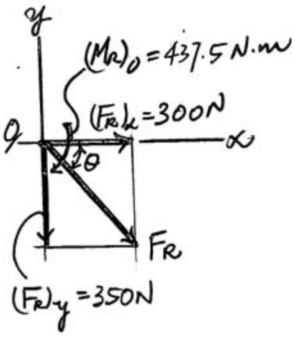
Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point *O*,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_O = -750(1.25) - 200(1) + 500\left(\frac{4}{5}\right)(2.50) - 500\left(\frac{3}{5}\right)(1)$$

$$= -438\text{ N}\cdot\text{m} = 438\text{ N}\cdot\text{m (clockwise)} \qquad \textbf{Ans.}$$

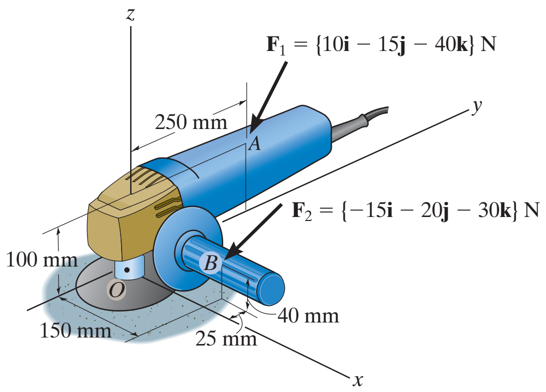


(b)



(c)

***4–112.** Replace the two forces acting on the grinder by a resultant force and couple moment at point *O*. Express the results in Cartesian vector form.


z
$\mathbf{F}_1 = \{10\mathbf{i} - 15\mathbf{j} - 40\mathbf{k}\}$ N
250 mm
y
A
$\mathbf{F}_2 = \{-15\mathbf{i} - 20\mathbf{j} - 30\mathbf{k}\}$ N
100 mm
B
O
40 mm
150 mm
25 mm
x

Equivalent Resultant Force: The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (10\mathbf{i} - 15\mathbf{j} - 40\mathbf{k}) + (-15\mathbf{i} - 20\mathbf{j} - 30\mathbf{k})$$
$$= [-5\mathbf{i} - 35\mathbf{j} - 70\mathbf{k}]\text{N} \qquad \textbf{Ans.}$$

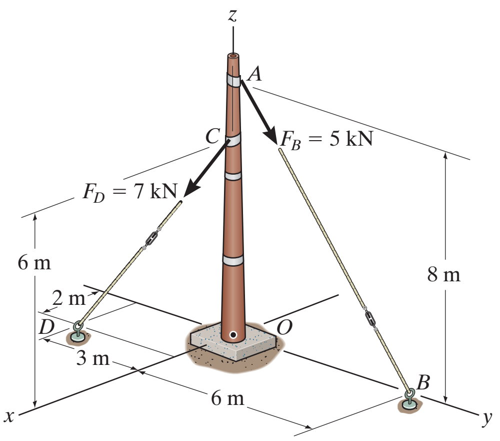
Equivalent Couple Moment: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$ are

$$\mathbf{r}_{OA} = (0 - 0)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0.1 - 0)\mathbf{k} = [0.25\mathbf{j} + 0.1\mathbf{k}]\text{ m}$$
$$\mathbf{r}_{OB} = (0.15 - 0)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0.04 - 0)\mathbf{k} = [0.15\mathbf{i} + 0.025\mathbf{j} + 0.04\mathbf{k}]\text{ m}$$

Thus, the resultant couple moment about point *O* is given by

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OB} \times \mathbf{F}_2$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.25 & 0.1 \\ 10 & -15 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.025 & 0.04 \\ -15 & -20 & -30 \end{vmatrix}$$
$$= [-8.45\mathbf{i} + 4.90\mathbf{j} - 5.125\mathbf{k}]\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

•4–113. Replace the two forces acting on the post by a resultant force and couple moment at point *O*. Express the results in Cartesian vector form.



Equivalent Resultant Force: The forces $\mathbf{F}_B$ and $\mathbf{F}_D$, Fig. *a*, expressed in Cartesian vector form can be written as

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 5\left[\frac{(0-0)\mathbf{i}+(6-0)\mathbf{j}+(0-8)\mathbf{k}}{(0-0)^2+(6-0)^2+(0-8)^2}\right] = [3\mathbf{j}-4\mathbf{k}]\text{kN}$$

$$\mathbf{F}_D = F_D\mathbf{u}_{CD} = 7\left[\frac{(2-0)\mathbf{i}+(-3-0)\mathbf{j}+(0-6)\mathbf{k}}{(2-0)^2+(-3-0)^2+(0-6)^2}\right] = [2\mathbf{i}-3\mathbf{j}-6\mathbf{k}]\text{kN}$$

The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_D$$
$$= (3\mathbf{j}-4\mathbf{k})+(2\mathbf{i}-3\mathbf{j}-6\mathbf{k})$$
$$= [2\mathbf{i}-10\mathbf{k}]\text{kN} \qquad \textbf{Ans.}$$

Equivalent Resultant Force: The position vectors $\mathbf{r}_{OB}$ and $\mathbf{r}_{OC}$ are

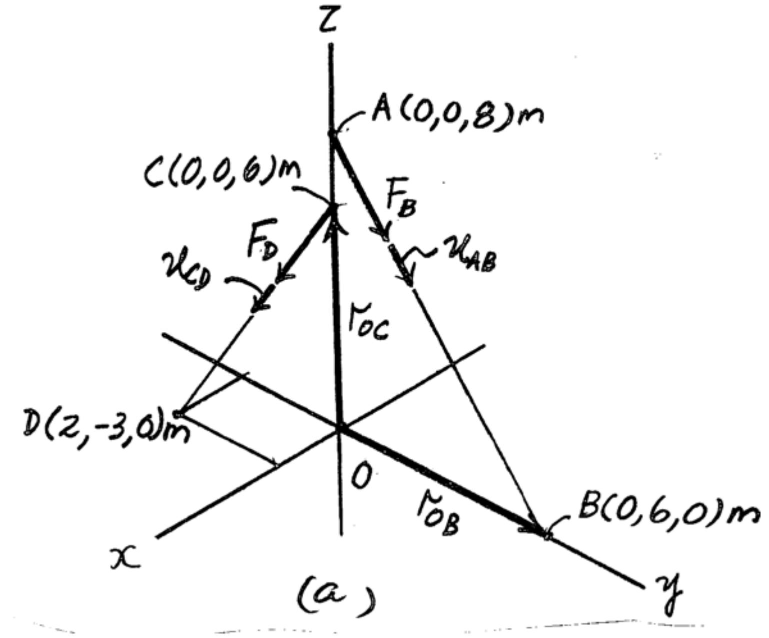
$\mathbf{r}_{OB} = \{6\mathbf{j}\}$ m $\qquad \mathbf{r}_{OC} = [6\mathbf{k}]$ m

Thus, the resultant couple moment about point *O* is given by

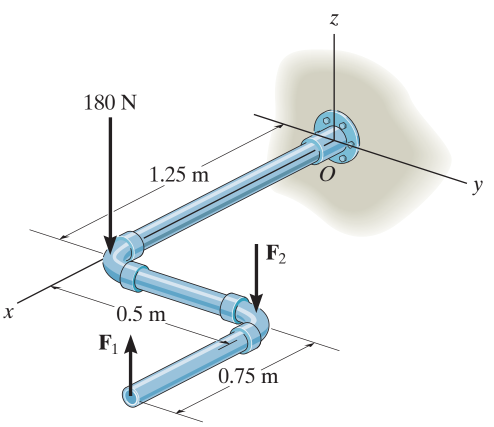
$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OB}\times\mathbf{F}_B + \mathbf{r}_{OC}\times\mathbf{F}_D$$
$$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 6 & 0\\ 0 & 3 & -4\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 6\\ 2 & -3 & -6\end{vmatrix}$$
$$= [-6\mathbf{i}+12\mathbf{j}]\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$M_{R_A} = \Sigma M_A;\quad 10750d = -3500(3) - 5500(17) - 1750(25)$$

$$d = 13.7\text{ ft} \qquad \textbf{Ans.}$$



4–114. The three forces act on the pipe assembly. If $F_1 = 50$ N and $F_2 = 80$ N, replace this force system by an equivalent resultant force and couple moment acting at O. Express the results in Cartesian vector form.



$$\mathbf{F}_R = \Sigma \mathbf{F}_z = \{-180\mathbf{k} + 50\mathbf{k} - 80\mathbf{k}\}\text{ N} = \{-210\mathbf{k}\}\text{ N} \qquad \textbf{Ans}$$

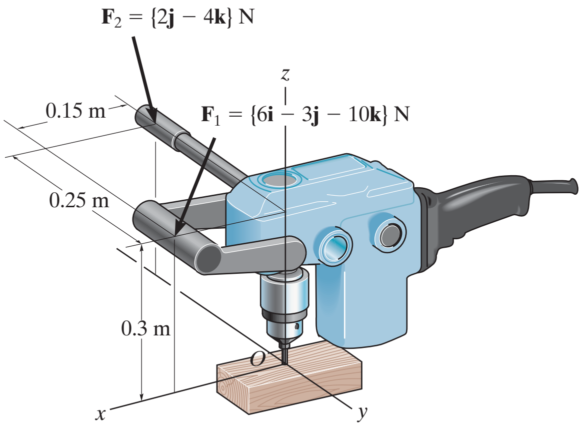
$$\mathbf{M}_{RO} = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.25 & 0 & 0 \\ 0 & 0 & -180 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.25 & 0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0.5 & 0 \\ 0 & 0 & 50 \end{vmatrix}$$

$$= (225\mathbf{j}) + (-40\mathbf{i} + 100\mathbf{j}) + (25\mathbf{i} - 100\mathbf{j})$$

$$= \{-15\mathbf{i} + 225\mathbf{j}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

4–115. Handle forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to the electric drill. Replace this force system by an equivalent resultant force and couple moment acting at point O. Express the results in Cartesian vector form.



$$\mathbf{F}_R = \Sigma\mathbf{F}; \quad \mathbf{F}_R = 6\,\mathbf{i} - 3\,\mathbf{j} - 10\,\mathbf{k} + 2\,\mathbf{j} - 4\,\mathbf{k}$$

$$= \{6\,\mathbf{i} - 1\,\mathbf{j} - 14\,\mathbf{k}\}\text{ N} \qquad \textbf{Ans}$$

$$\mathbf{M}_{RO} = \Sigma\mathbf{M}_O;$$

$$\mathbf{M}_{RO} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix}$$

$$= 0.9\,\mathbf{i} + 3.30\,\mathbf{j} - 0.450\,\mathbf{k} + 0.4\,\mathbf{i}$$

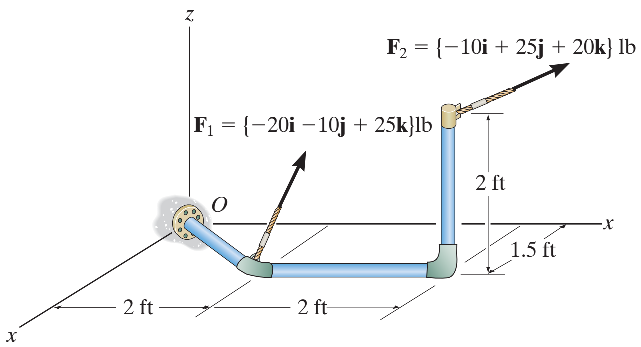
$$= \{1.30\,\mathbf{i} + 3.30\,\mathbf{j} - 0.450\,\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

Note that $F_{Rz} = -14$ N pushes the drill bit down into the stock.

$(M_{RO})_x = 1.30$ N · m and $(M_{RO})_y = 3.30$ N · m cause the drill bit to bend.

$(M_{RO})_z = -0.450$ N · m causes the drill case and the spinning drill bit to rotate about the z-axis.

***4–116.** Replace the force system acting on the pipe assembly by a resultant force and couple moment at point O. Express the results in Cartesian vector form.


z
$\mathbf{F}_2 = \{-10\mathbf{i} + 25\mathbf{j} + 20\mathbf{k}\}$ lb
$\mathbf{F}_1 = \{-20\mathbf{i} - 10\mathbf{j} + 25\mathbf{k}\}$lb
2 ft
O
x
1.5 ft
2 ft
2 ft
x

Equivalent Resultant Force: The resultant force $\mathbf{F}_R$ can be determined from

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (-20\mathbf{i} - 10\mathbf{j} + 25\mathbf{k}) + (-10\mathbf{i} + 25\mathbf{j} + 20\mathbf{k})$$

$$= [-30\mathbf{i} + 15\mathbf{j} + 45\mathbf{k}]\text{ lb} \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$, ~~Fig. a~~, are

$$\mathbf{r}_{OA} = (1.5 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [1.5\mathbf{i} + 2\mathbf{j}]\text{ ft}$$

$$\mathbf{r}_{OB} = (1.5 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = [1.5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}]\text{ ft}$$

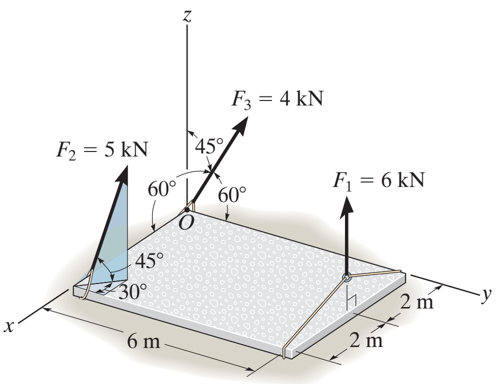
Thus, the resultant couple moment about point O is

$$\mathbf{M}_W = \Sigma\mathbf{M}_O;\qquad (\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OB} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix}$$

$$= [80\mathbf{i} - 87.5\mathbf{j} + 102.5\mathbf{k}]\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

•4–117. The slab is to be hoisted using the three slings shown. Replace the system of forces acting on slings by an equivalent force and couple moment at point O. The force $\mathbf{F}_1$ is vertical.



Force Vectors :

$$\mathbf{F}_1 = \{6.00\mathbf{k}\}\ \text{kN}$$

$$\begin{aligned}\mathbf{F}_2 &= 5(-\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k})\\ &= \{-1.768\mathbf{i} + 3.062\mathbf{j} + 3.536\mathbf{k}\}\ \text{kN}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_3 &= 4(\cos 60°\mathbf{i} + \cos 60°\mathbf{j} + \cos 45°\mathbf{k})\\ &= \{2.00\mathbf{i} + 2.00\mathbf{j} + 2.828\mathbf{k}\}\ \text{kN}\end{aligned}$$

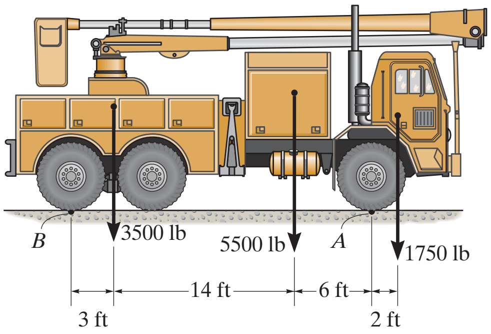
Equivalent Force and Couple Moment At Point O :

$$\begin{aligned}\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= (-1.768 + 2.00)\,\mathbf{i} + (3.062 + 2.00)\,\mathbf{j}\\ &\qquad + (6.00 + 3.536 + 2.828)\,\mathbf{k}\\ &= \{0.232\mathbf{i} + 5.06\mathbf{j} + 12.4\mathbf{k}\}\ \text{kN} \qquad \textbf{Ans}\end{aligned}$$

The position vectors are $\mathbf{r}_1 = \{2\mathbf{i} + 6\mathbf{j}\}$ m and $\mathbf{r}_2 = \{4\mathbf{i}\}$ m.

$$\begin{aligned}\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O;\quad \mathbf{M}_{R_O} &= \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 6 & 0\\ 0 & 0 & 6.00\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 4 & 0 & 0\\ -1.768 & 3.062 & 3.536\end{vmatrix}\\ &= \{36.0\mathbf{i} - 26.1\mathbf{j} + 12.2\mathbf{k}\}\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}\end{aligned}$$

4–118. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from B.

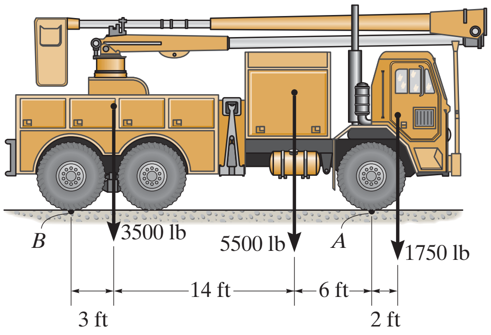


$$\begin{aligned}+\uparrow F_R = \Sigma F_y;\quad F_R &= -1750 - 5500 - 3500\\ &= -10750\ \text{lb} = 10.75\ \text{kip}\downarrow \qquad \textbf{Ans}\end{aligned}$$

$$\curvearrowleft + M_{R_A} = \Sigma M_A;\quad 10750d = -3500(3) - 5500(17) - 1750(25)$$

$$d = 13.7\ \text{ft} \qquad \textbf{Ans.}$$

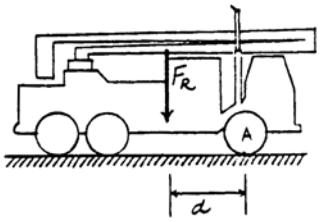
4–119. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.



Equivalent Force :

$$+\uparrow F_R = \Sigma F_y; \quad F_R = -1750 - 5500 - 3500$$
$$= -10750 \text{ lb} = 10.75 \text{ kip} \downarrow \qquad \textbf{Ans}$$

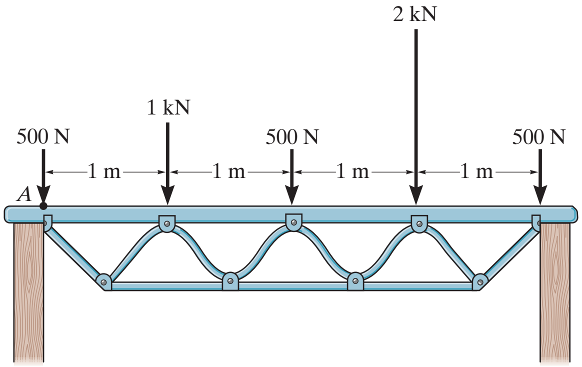
Location of Resultant Force From Point A :

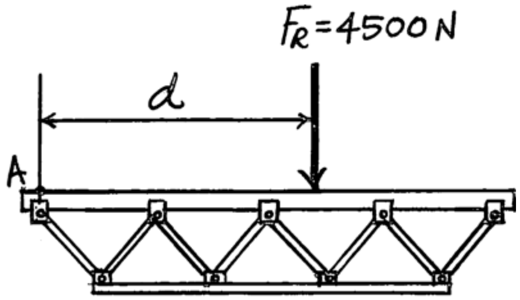


$$\curvearrowleft + M_{R_A} = \Sigma M_A; \quad 10750(d) = 3500(20) + 5500(6) - 1750(2)$$

$$d = 9.26 \text{ ft} \qquad \textbf{Ans}$$

***4–120.** The system of parallel forces acts on the top of the *Warren truss.* Determine the equivalent resultant force of the system and specify its location measured from point A.





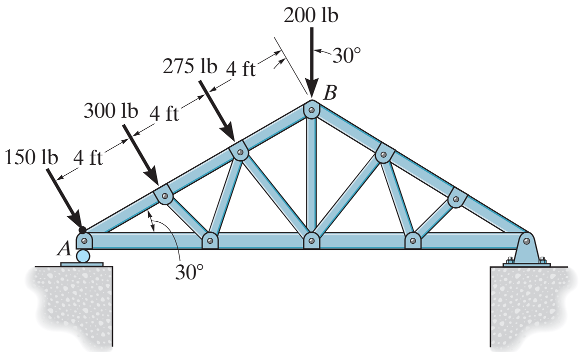
$$+\downarrow F_R = \Sigma F; \quad F_R = 500 + 1000 + 500 + 2000 + 500$$

$$F_R = 4500 \text{ N} = 4.50 \text{ kN} \qquad \textbf{Ans}$$

$$\curvearrowright + M_R = \Sigma M_A; \quad 4500(d) = 1000(1) + 500(2) + 2000(3) + 500(4)$$

$$d = 2.22 \text{ m} \qquad \textbf{Ans}$$

•4–121. The system of four forces acts on the roof truss. Determine the equivalent resultant force and specify its location along AB, measured from point A.



$\nearrow + F_{Rx} = \Sigma F_x;\quad F_{Rx} = 200 \sin 30° = 100 \text{ lb}$

$\searrow + F_{Ry} = \Sigma F_y;\quad F_{Ry} = 150 + 300 + 275 + 200 \cos 30° = 898.2 \text{ lb}$

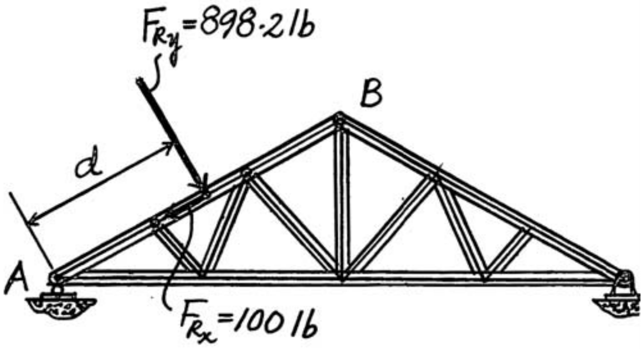
$F_R = \sqrt{(100)^2 + (898.2)^2} = 904 \text{ lb}$ **Ans**

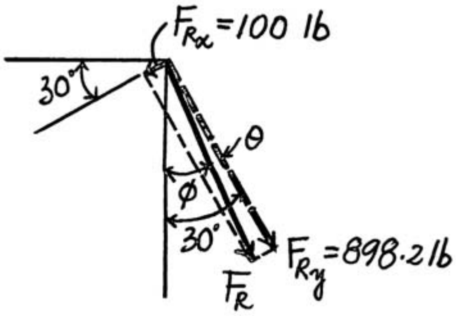
$\theta = \tan^{-1}\left(\dfrac{100}{898.2}\right) = 6.35°$

$\phi = 30° - 6.35° = 23.6°$ **Ans**

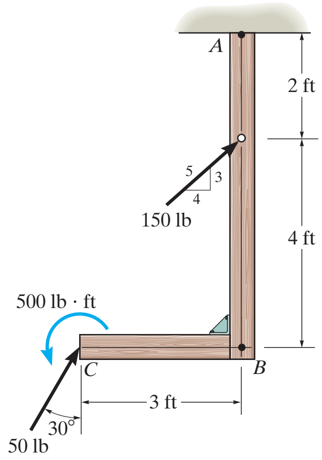
$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 898.2\,(d) = 4\,(300) + 8\,(275) + 12 \cos 30°\,(200)$

$d = 6.10 \text{ ft}$ **Ans**





4–122. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x; \quad F_{Rx} = 150\left(\frac{4}{5}\right) + 50 \sin 30° = 145 \text{ lb}$

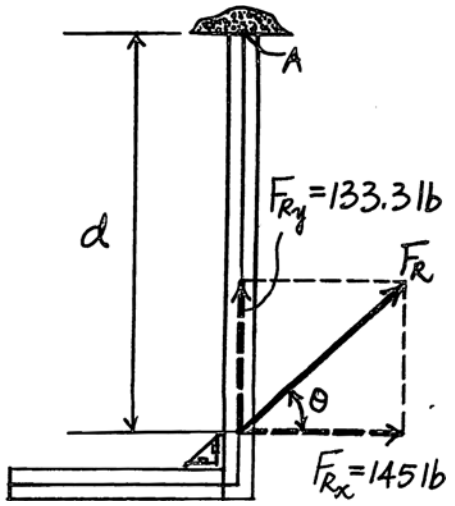
$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 50 \cos 30° + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$

$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb}$ **Ans**

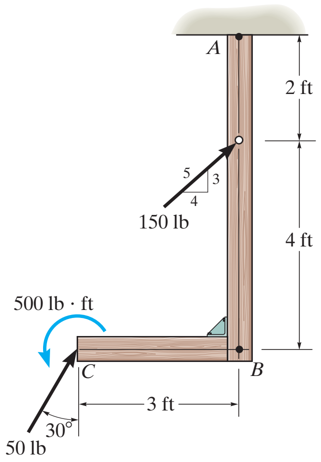
$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6° \measuredangle$ **Ans**

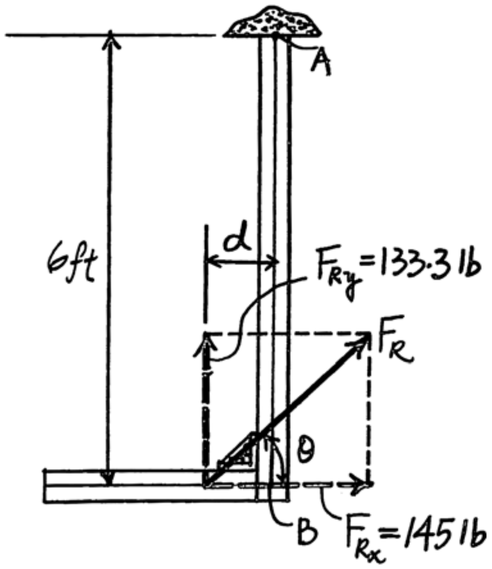
$\circlearrowleft + M_{RA} = \Sigma M_A; \quad 145(d) = 150\left(\frac{4}{5}\right)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500$

$d = 5.24 \text{ ft}$ **Ans**



4–123. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from *B*.





$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 150\left(\frac{4}{5}\right) + 50\sin 30^\circ = 145\text{ lb}$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 50\cos 30^\circ + 150\left(\frac{3}{5}\right) = 133.3\text{ lb}$

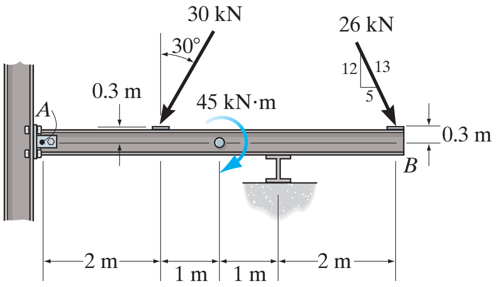
$F_R = \sqrt{(145)^2 + (133.3)^2} = 197\text{ lb}$ **Ans**

$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6^\circ$ ∠ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 145(6) - 133.3(d) = 150\left(\frac{4}{5}\right)(2) - 50\cos 30^\circ(3) + 50\sin 30^\circ(6) + 500$

$d = 0.824\text{ ft}$ **Ans**

***4–124.** Replace the force and couple moment system acting on the overhang beam by a resultant force, and specify its location along *AB* measured from point *A*.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. *a*. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 26\left(\frac{5}{13}\right) - 30\sin 30° = -5\text{ kN} = 5\text{ kN} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -26\left(\frac{12}{13}\right) - 30\cos 30° = -49.98\text{ kN} = 49.98\text{ kN} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{5^2 + 49.98^2} = 50.23\text{ kN} = 50.2\text{ kN} \quad \textbf{Ans.}$$

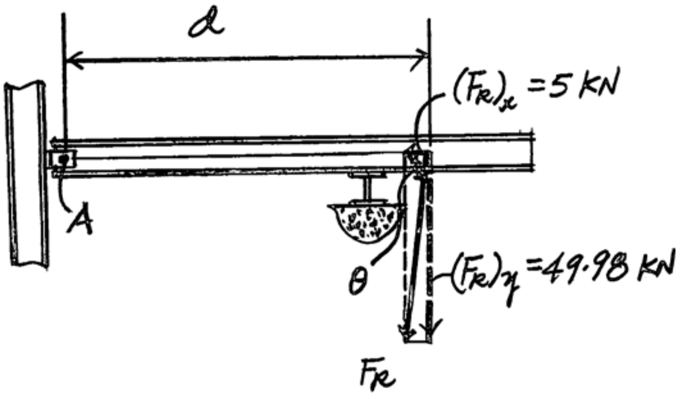
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{49.98}{5}\right] = 84.29° = 84.3° \qquad \textbf{Ans.}$$

Location of Resultant Force: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point A,

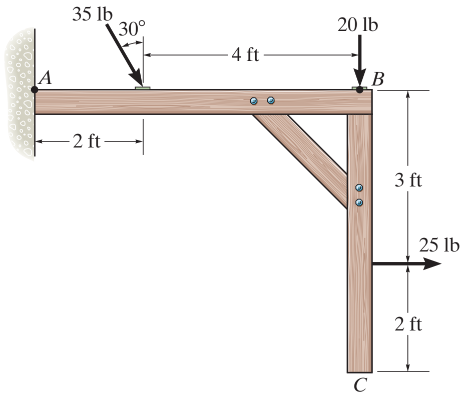
$$\circlearrowleft +(M_R)_A = \Sigma M_A; -49.98(d) = 30\sin 30°(0.3) - 30\cos 30°(2) - 26\left(\frac{5}{13}\right)(0.3) - 26\left(\frac{12}{13}\right)(6) - 45$$

$$d = 4.79\text{ m} \qquad \textbf{Ans.}$$



(b)

•4–125. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from point A.



$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = 35\sin 30° + 25 = 42.5\text{ lb}$

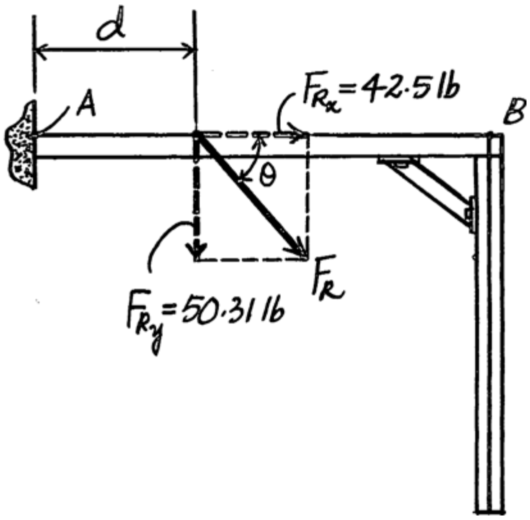
$+\downarrow F_{R_y} = \Sigma F_y;$ $\quad F_{R_y} = 35\cos 30° + 20 = 50.31\text{ lb}$

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9\text{ lb}$ **Ans**

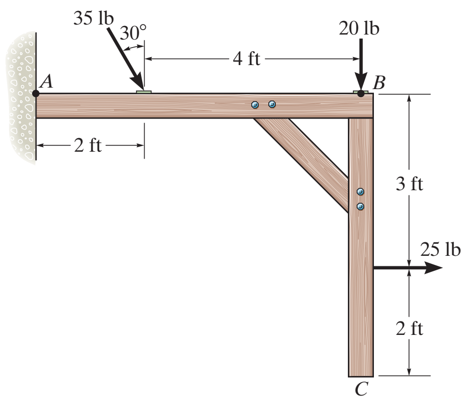
$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8° \searrow$ **Ans**

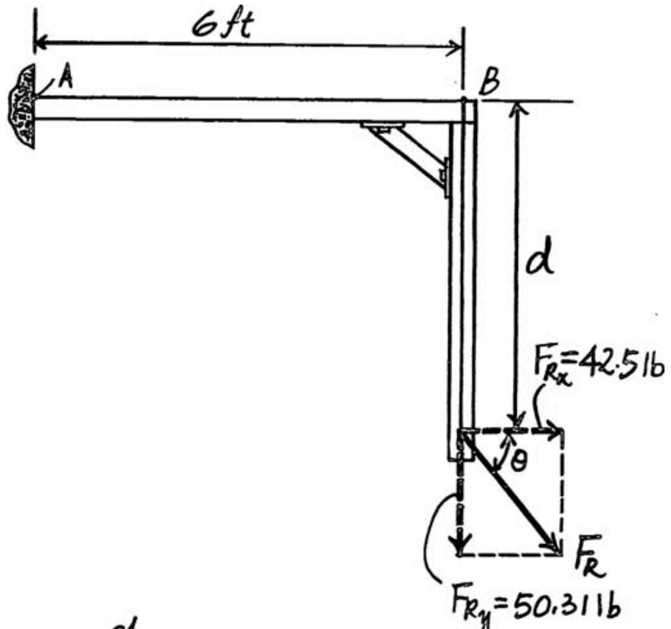
$\curvearrowleft + M_{R_A} = \Sigma M_A;$ $\quad 50.31\,(d) = 35\cos 30°\,(2) + 20\,(6) - 25\,(3)$

$d = 2.10\text{ ft}$ **Ans**



4–126. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from point *B*.





$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = 35\sin 30^\circ + 25 = 42.5\text{ lb}$

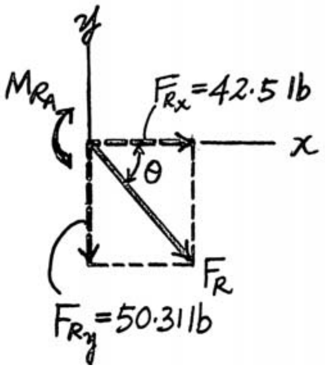
$+\downarrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = 35\cos 30^\circ + 20 = 50.31\text{ lb}$

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9\text{ lb}$ **Ans**

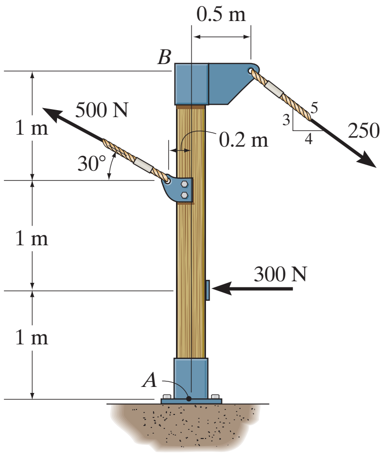
$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8^\circ$ ⦩ **Ans**

$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad 50.31(6) - 42.5(d) = 35\cos 30^\circ(2) + 20(6) - 25(3)$

$d = 4.62\text{ ft}$ **Ans**



4–127. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *A*.



Equivalent Resultant Force: Forces F_1 and F_2 are resolved into their *x* and *y* components, Fig. *a*. Summing these force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = 500\sin 30° - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

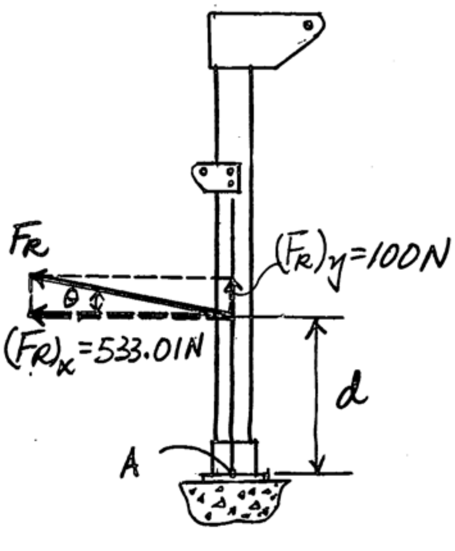
Location of the Resultant Force: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point *A*,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad 533.01(d) = 500\cos 30°(2) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$d = 0.8274\text{ m} = 827\text{ mm} \qquad \textbf{Ans.}$$

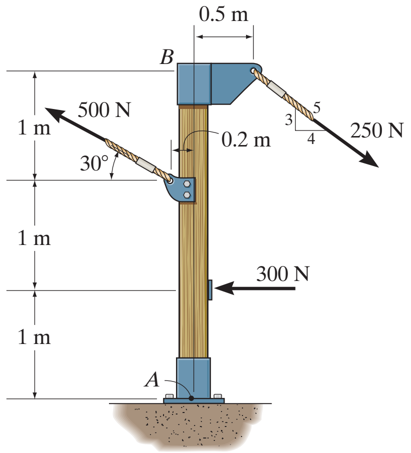
use Fig. (a)
of
Prob. 4-109

(a)



(b)

***4–128.** Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *B*.



Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their *x* and *y* components, Fig. *a*. Summing these force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30^\circ - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = 500\sin 30^\circ - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

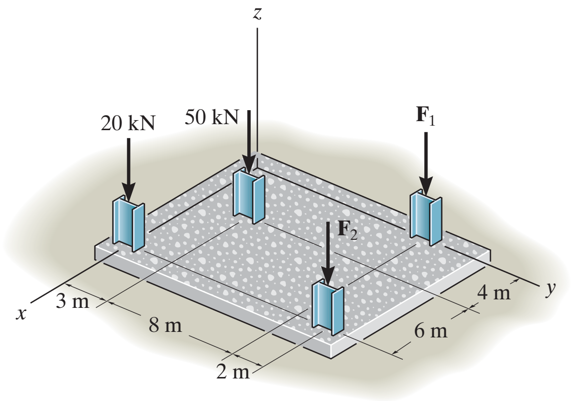
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63^\circ = 10.6^\circ \qquad \textbf{Ans.}$$

Location of the Resultant Force: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point *A*,

$$\circlearrowleft +(M_R)_b = \Sigma M_b; \quad -533.01(d) = -500\cos 30^\circ(1) - 500\sin 30^\circ(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 300(2)$$

$$d = 2.17\text{ m} \qquad \textbf{Ans.}$$

•4–129. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (*x, y*) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.



$$+\uparrow F_R = \Sigma F_z; \quad F_R = -20 - 50 - 30 - 40 = -140\text{ kN} = 140\text{ kN}\downarrow \qquad \textbf{Ans}$$

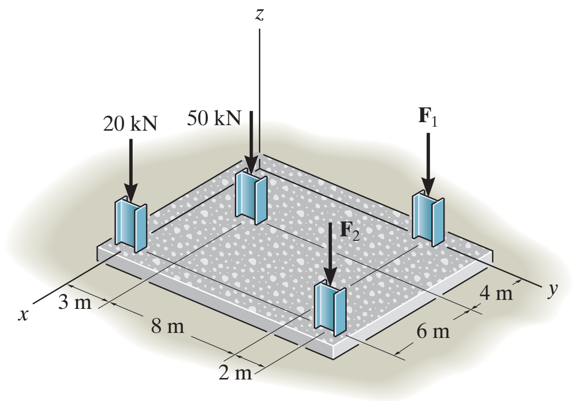
$$(M_R)_x = \Sigma M_x; \quad -140y = -50(3) - 30(11) - 40(13)$$

$$y = 7.14\text{ m} \qquad \textbf{Ans}$$

$$(M_R)_y = \Sigma M_y; \quad 140x = 50(4) + 20(10) + 40(10)$$

$$x = 5.71\text{ m} \qquad \textbf{Ans}$$

4–130. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



$+\downarrow F_R = \Sigma F_z;$ $F_R = 20 + 50 + 20 + 50 = 140$ kN **Ans**

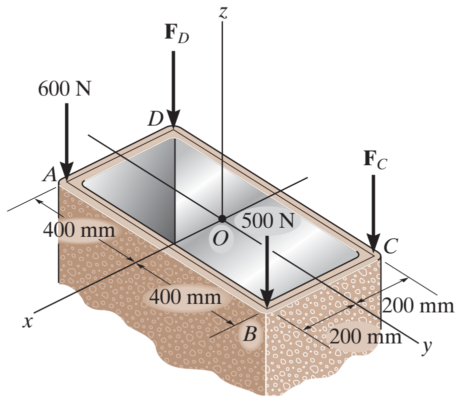
$M_{R\,y} = \Sigma M_y;$ $140(x) = (50)(4) + 20(10) + 50(10)$

$x = 6.43$ m **Ans**

$M_{R\,x} = \Sigma M_x;$ $-140(y) = -(50)(3) - 20(11) - 50(13)$

$y = 7.29$ m **Ans**

4–131. The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



Since the resultant force passes through point O, the resultant moment components about x and y axes are both zero.

$\Sigma M_x = 0;$ $F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0$

$F_C - F_D = 100$ (1)

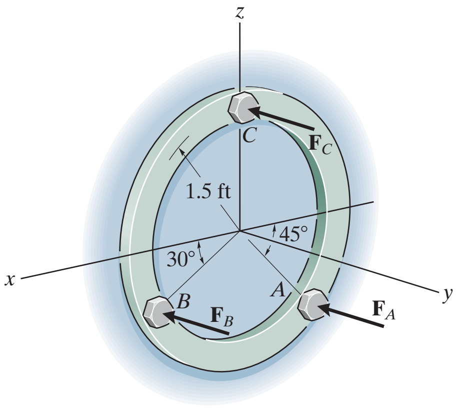
$\Sigma M_y = 0;$ $500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0$

$F_C + F_D = 1100$ (2)

Solving Eqs. (1) and (2) yields:

$F_C = 600$ N $F_D = 500$ N **Ans**

*__4–132.__ Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location (x, z) on the plate. $F_A = 200$ lb, $F_B = 100$ lb, and $F_C = 400$ lb.



Equivalent Force :

$$F_R = \Sigma F_y; \qquad -F_R = -400 - 200 - 100$$

$$F_R = 700 \text{ lb} \qquad \textbf{Ans}$$

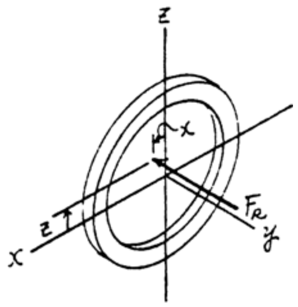
Location of Resultant Force :

$$M_{R_x} = \Sigma M_x; \qquad 700(z) = 400(1.5) - 200(1.5\sin 45°) - 100(1.5\sin 30°)$$

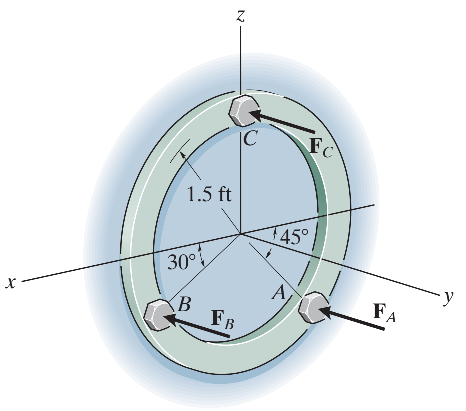
$$z = 0.447 \text{ ft} \qquad \textbf{Ans}$$

$$M_{R_z} = \Sigma M_z; \qquad -700(x) = 200(1.5\cos 45°) - 100(1.5\cos 30°)$$

$$x = -0.117 \text{ ft} \qquad \textbf{Ans}$$



•4–133. The three parallel bolting forces act on the circular plate. If the force at A has a magnitude of $F_A = 200$ lb, determine the magnitudes of $\mathbf{F}_B$ and $\mathbf{F}_C$ so that the resultant force $\mathbf{F}_R$ of the system has a line of action that coincides with the y axis. *Hint:* This requires $\Sigma M_x = 0$ and $\Sigma M_z = 0$.



Since $\mathbf{F}_R$ coincides with y axis, $M_{R_x} = M_{R_z} = 0$.

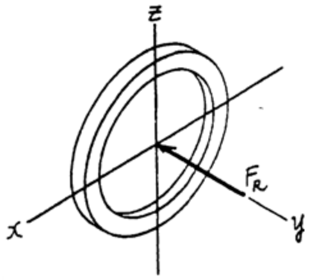
$M_{R_z} = \Sigma M_z$; $\quad 0 = 200(1.5\cos 45°) - F_B(1.5\cos 30°)$

$F_B = 163.30 \text{ lb} = 163 \text{ lb}$ **Ans**

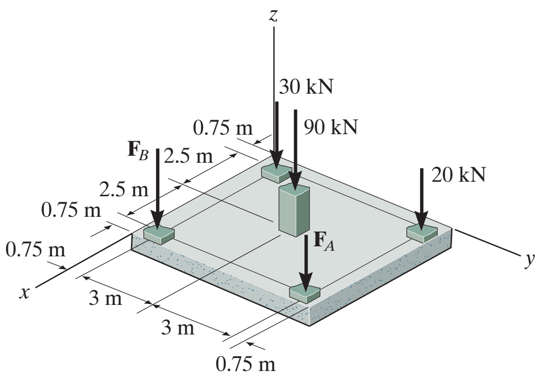
Using the result $F_B = 163.30$ lb,

$M_{R_x} = \Sigma M_x$; $\quad 0 = F_C(1.5) - 200(1.5\sin 45°)$
$- 163.30(1.5\sin 30°)$

$F_C = 223 \text{ lb}$ **Ans**



4–134. If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.



Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$, F[illegible] *b*,

$+\uparrow F_R = \Sigma F_z$; $\quad -F_R = -30 - 20 - 90 - 35 - 40$

$F_R = 215 \text{ kN}$ **Ans.**

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

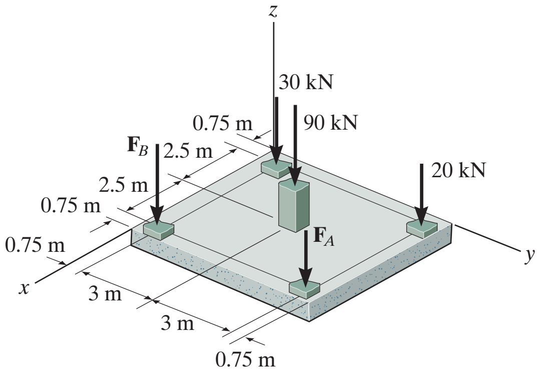
$(M_R)_x = \Sigma M_x$; $\quad -215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75)$

$y = 3.68 \text{ m}$ **Ans.**

$(M_R)_y = \Sigma M_y$; $\quad 215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)$

$x = 3.54 \text{ m}$ **Ans.**

4–135. If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$,

$+\uparrow F_R = \Sigma F_z;$ $\quad -F_R = -30 - 20 - 90 - F_A - F_B$

$$F_R = 140 + F_A + F_B \qquad (1)$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$(M_R)_x = \Sigma M_x;$ $\quad -F_R(3.75) = -F_B(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - F_A(6.75)$

$$F_R = 0.2F_B + 1.8F_A + 132 \qquad (2)$$

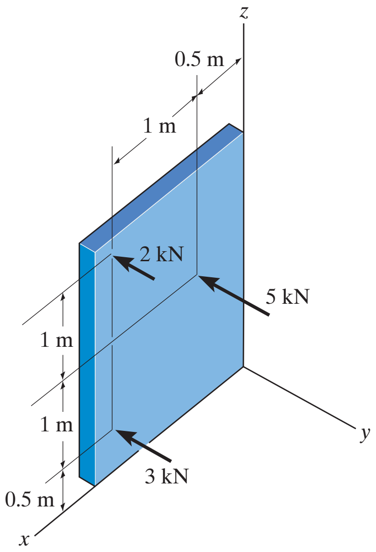
$(M_R)_y = \Sigma M_y;$ $\quad F_R(3.25) = 30(0.75) + 20(0.75) + 90(3.25) + F_A(5.75) + F_B(5.75)$

$$F_R = 1.769F_A + 1.769F_B + 101.54 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_A = 30\text{ kN}$ $\qquad F_B = 20\text{ kN}$ $\qquad F_R = 190\text{ kN}$ **Ans.**

***4–136.** Replace the parallel force system acting on the plate by a resultant force and specify its location on the x–z plane.



Resultant Force: Summing the forces acting on the plate,

$(F_R)_y = \Sigma F_y;$ $\quad F_R = -5\text{ kN} - 2\text{ kN} - 3\text{ kN}$

$$= -10\text{ kN}$$ **Ans.**

The negative sign indicates that $\mathbf{F}_R$ acts along the negative y axis.

Resultant Moment: Using the right - hand rule, and equating the moment of $\mathbf{F}_R$ to the sum of the moments of the force system about the x and z axes,

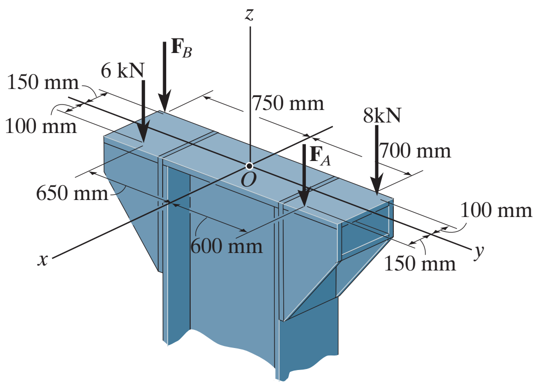
$(M_R)_x = \Sigma M_x;$ $\quad (10\text{ kN})(z) = (3\text{ kN})(0.5\text{ m}) + (5\text{ kN})(1.5\text{ m}) + 2\text{ kN}(2.5\text{ m})$

$$z = 1.40\text{ m}$$ **Ans.**

$(M_R)_z = \Sigma M_z;$ $\quad -(10\text{ kN})(x) = -(5\text{ kN})(0.5\text{ m}) - (2\text{ kN})(1.5\text{ m}) - (3\text{ kN})(1.5\text{ m})$

$$x = 1.00\text{ m}$$ **Ans.**

•4–137. If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system acting on the corbels by a resultant force, and specify its location on the *x*–*y* plane.



Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the *z* axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$+\uparrow F_R = \Sigma F_z;$ $\quad -F_R = -6-5-7-8$

$F_R = 26$ kN **Ans.**

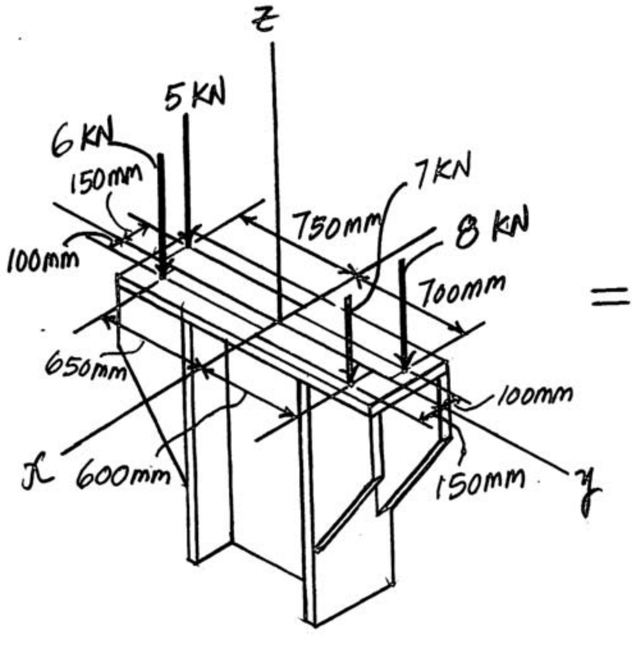
Point of Application: By equating the moment of the forces shown in Fig. *a* and $\mathbf{F}_R$, Fig. *b*, about the *x* and *y* axes,

$(M_R)_x = \Sigma M_x;$ $\quad -26(y) = 6(650)+5(750)-7(600)-8(700)$

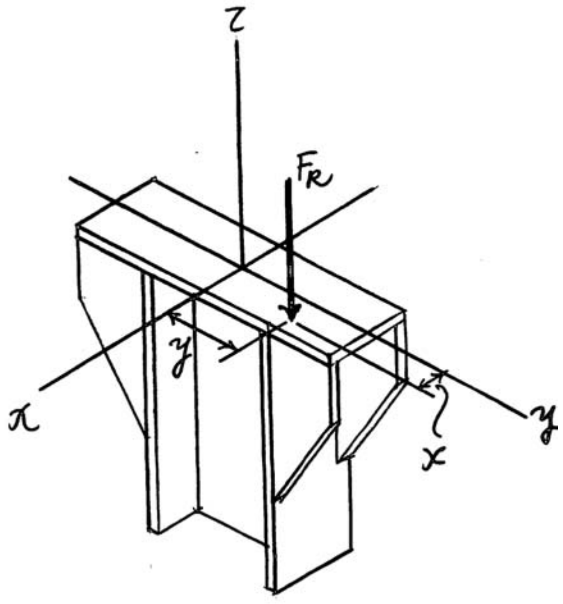
$y = 82.7$ mm **Ans.**

$(M_R)_y = \Sigma M_y;$ $\quad 26(x) = 6(100)+7(150)-5(150)-8(100)$

$x = 3.85$ mm **Ans.**

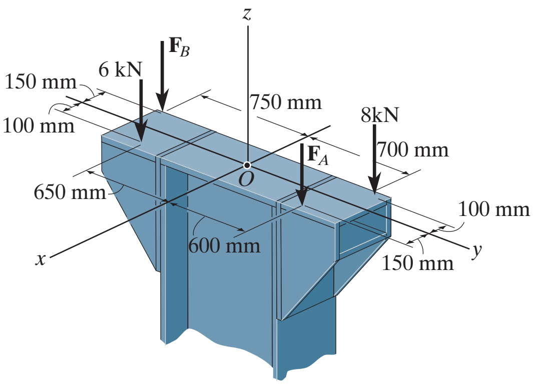


(a)



(b)

4–138. Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O of the column.



Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the *z* axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -F_A - F_B - 8 - 6$$

$$F_R = F_A + F_B + 14 \qquad (1)$$

Point of Application: Since $\mathbf{F}_R$ is required to pass through point O, the moment of $\mathbf{F}_R$ about the x and y axes are equal to zero. Thus,

$$(M_R)_x = \Sigma M_x; \qquad 0 = F_B(750) + 6(650) - F_A(600) - 8(700)$$

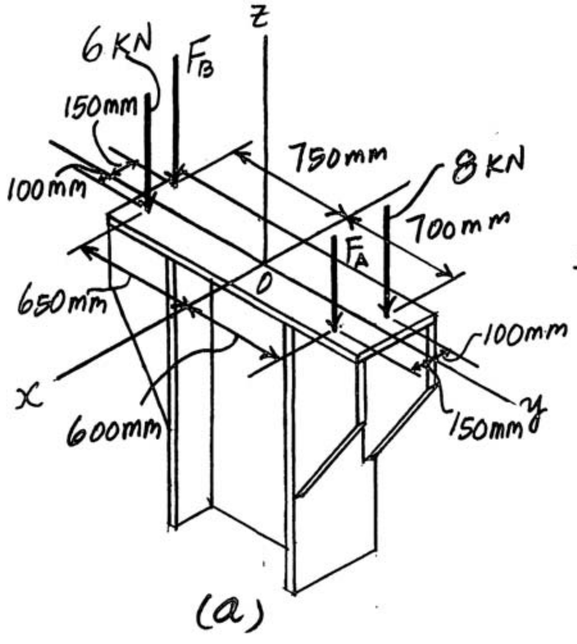
$$750F_B - 600F_A - 1700 = 0 \qquad (2)$$

$$(M_R)_y = \Sigma M_y; \qquad 0 = F_A(150) + 6(100) - F_B(150) - 8(100)$$

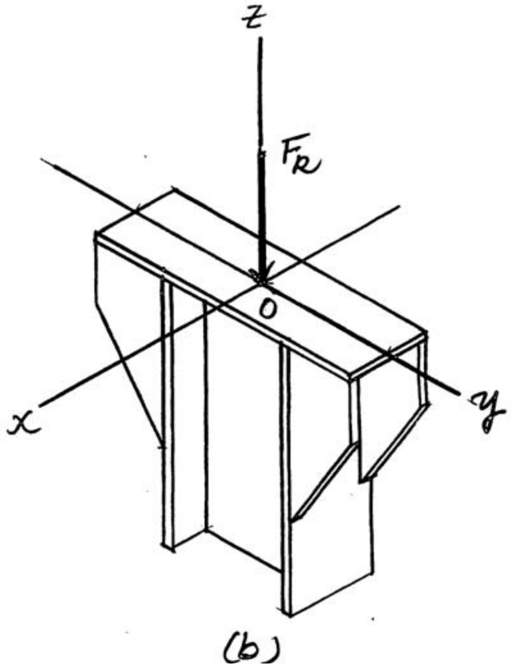
$$150F_A - 150F_B + 200 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_A = 18.0\text{ kN}$ $\qquad F_B = 16.7\text{ kN}$ $\qquad F_R = 48.7\text{ kN}$ **Ans.**

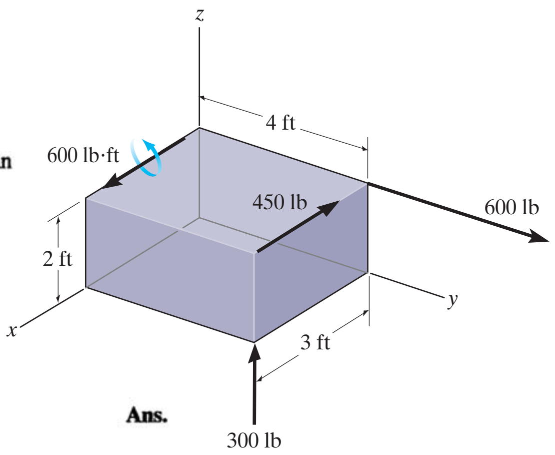


(a)



(b)

4–139. Replace the force and couple moment system acting on the rectangular block by a wrench. Specify the magnitude of the force and couple moment of the wrench and where its line of action intersects the *x*–*y* plane.



Equivalent Resultant Force: The resultant forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ expressed in Cartesian vector form can be written as $\mathbf{F}_1 = [600\mathbf{j}]$ lb, $\mathbf{F}_2 = [-450\mathbf{i}]$ lb, and $\mathbf{F}_3 = [300\mathbf{k}]$ lb. The force of the wrench can be determined from

$$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= 600\mathbf{j} - 450\mathbf{i} + 300\mathbf{k} = [-450\mathbf{i} + 600\mathbf{j} + 300\mathbf{k}]\text{ lb}$$

Thus, the magnitude of the wrench force is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2} = \sqrt{(-450)^2 + 600^2 + 300^2} = 807.77\text{ lb} = 808\text{ lb}$$ **Ans.**

Equivalent Couple Moment: Here, we will assume that the axis of the wrench passes through point *P*, Figs. *a* and *b*. Since $\mathbf{M}_W$ is collinear with $\mathbf{F}_R$,

$$\mathbf{M}_W = M_W\mathbf{u}_{F_R} = M_W\left[\frac{-450\mathbf{i} + 600\mathbf{j} + 300\mathbf{k}}{\sqrt{(-450)^2 + 600^2 + 300^2}}\right]$$

$$= -0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k}$$

The position vectors $\mathbf{r}_{PA}$, $\mathbf{r}_{PB}$, and $\mathbf{r}_{PC}$ are

$$\mathbf{r}_{PA} = (0 - x)\mathbf{i} + (4 - y)\mathbf{j} + (2 - 0)\mathbf{k} = -x\mathbf{i} + (4 - y)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{PB} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + (0 - 0)\mathbf{k} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j}$$

$$\mathbf{r}_{PC} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + (2 - 0)\mathbf{k} = (3 - x)\mathbf{i} + (4 - y)\mathbf{j} + 2\mathbf{k}$$

The couple moment $\mathbf{M}$ expressed in Cartesian vector form is written as $\mathbf{M} = [600\mathbf{i}]$ lb·ft. Summing the moments of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ about point *P* and including $\mathbf{M}$,

$$\mathbf{M}_W = \Sigma\mathbf{M}_P;\qquad \mathbf{M}_W = \mathbf{r}_{PA}\times\mathbf{F}_1 + \mathbf{r}_{PC}\times\mathbf{F}_2 + \mathbf{r}_{PB}\times\mathbf{F}_3 + \mathbf{M}$$

$$-0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -x & (4-y) & 2\\ 0 & 600 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ (3-x) & (4-y) & 2\\ -450 & 0 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ (3-x) & (4-y) & 0\\ 0 & 0 & 300\end{vmatrix} + 600\mathbf{i}$$

$$-0.5571M_w\mathbf{i} + 0.7428M_w\mathbf{j} + 0.3714M_w\mathbf{k} = (600 - 300y)\mathbf{i} + (300x - 1800)\mathbf{j} + (1800 - 600x - 450y)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$-0.5571M_w = 600 - 300y \qquad (1)$$

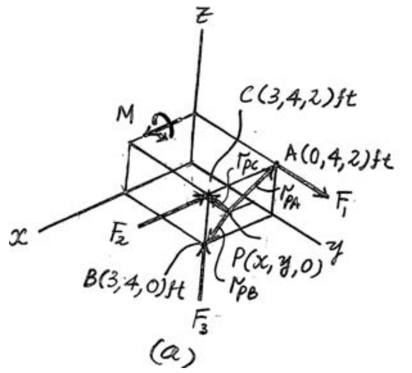
$$0.7428M_w = 300x - 1800 \qquad (2)$$

$$0.3714M_w = 1800 - 600x - 450y \qquad (3)$$

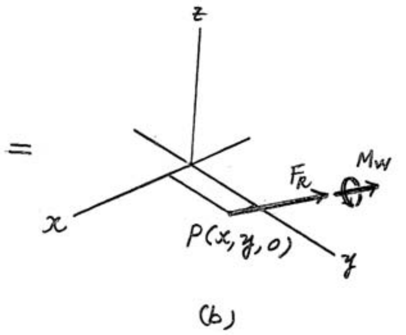
Solving Eqs. (1), (2), and (3) yields

$x = 3.52$ ft $\qquad y = 0.138$ ft $\qquad M_W = -1003$ lb·ft **Ans.**

The negative sign indicates that $\mathbf{M}_W$ acts in the opposite sense to that of $\mathbf{F}_R$.

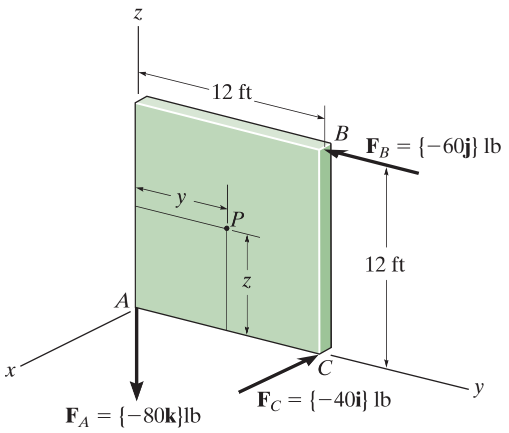


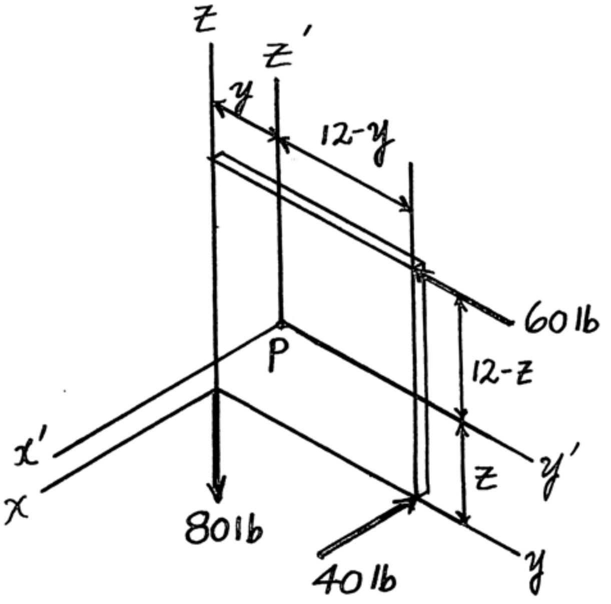
(a)



(b)

***4–140.** Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.


z
12 ft
B
$\mathbf{F}_B = \{-60\mathbf{j}\}$ lb
y
P
z
12 ft
A
x
C
y
$\mathbf{F}_C = \{-40\mathbf{i}\}$ lb
$\mathbf{F}_A = \{-80\mathbf{k}\}$lb



Resultant Force Vector :

$$\mathbf{F}_R = \{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(-40)^2 + (-60)^2 + (-80)^2} = 107.70\text{ lb} = 108\text{ lb} \qquad \textbf{Ans}$$

$$\mathbf{u}_{F_R} = \frac{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}}{107.70}$$
$$= -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$$

Resultant Moment: The line of action of $\mathbf{M}_R$ of the wrench is parallel to the line of action of $\mathbf{F}_R$. Assume that both $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Therefore, $\mathbf{u}_{M_R} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$.

$$(M_R)_{x'} = \Sigma M_{x'}; \qquad -0.3714M_R = 60(12 - z) + 80y \qquad [1]$$

$$(M_R)_{y'} = \Sigma M_{y'}; \qquad -0.5571M_R = 40z \qquad [2]$$

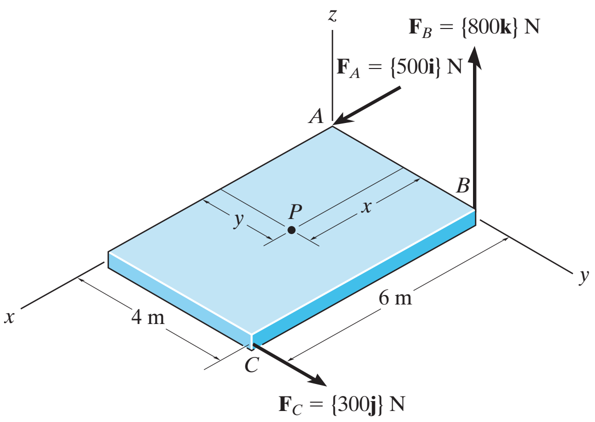
$$(M_R)_{z'} = \Sigma M_{z'}; \qquad -0.7428M_R = 40(12 - y) \qquad [3]$$

Solving Eqs.[1], [2], and [3] yields :

$$M_R = -624\text{ lb}\cdot\text{ft} \qquad z = 8.69\text{ ft} \qquad y = 0.414\text{ ft} \qquad \textbf{Ans}$$

The negative sign indicates that the line of action for $\mathbf{M}_R$ is directed in the opposite sense to that of $\mathbf{F}_R$.

•4–141. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.


z
$\mathbf{F}_B = \{800\mathbf{k}\}$ N
$\mathbf{F}_A = \{500\mathbf{i}\}$ N
A
B
P
y
x
x
4 m
6 m
y
C
$\mathbf{F}_C = \{300\mathbf{j}\}$ N

$$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}\ \text{N}$$

$$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{u}_{FR} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$$

$$M_{R_{x'}} = \Sigma M_{x'}; \qquad M_{R_{x'}} = 800(4-y)$$

$$M_{R_{y'}} = \Sigma M_{y'}; \qquad M_{R_{y'}} = 800x$$

$$M_{R_{z'}} = \Sigma M_{z'}; \qquad M_{R_{z'}} = 500y + 300(6-x)$$

Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{FR}$,

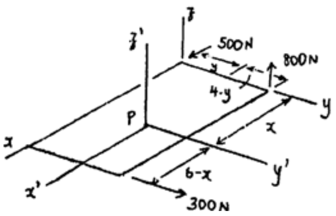
$$M_R(0.5051) = 800(4-y)$$

$$M_R(0.3030) = 800x$$

$$M_R(0.8081) = 500y + 300(6-x)$$

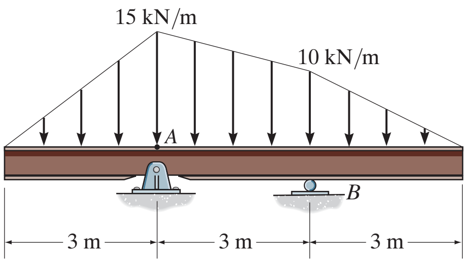
$$M_R = 3.07\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$x = 1.16\ \text{m} \qquad \textbf{Ans}$$



$$y = 2.06\ \text{m} \qquad \textbf{Ans}$$

4–142. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point *A*.



Loading: The distributed loading can be divided into four parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also indicated in Fig. *a*.

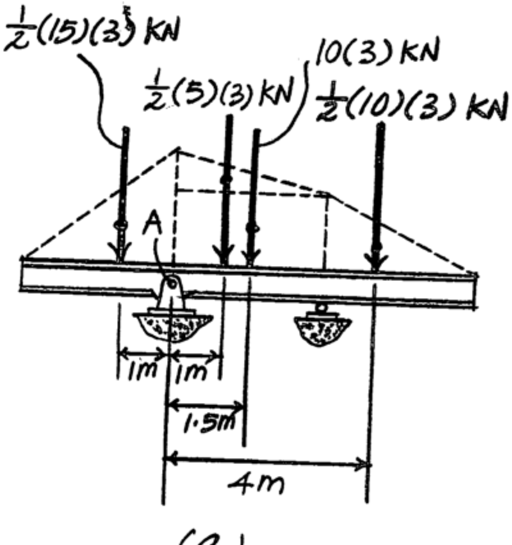
Resultants: Equating the sum of the forces along the *y* axis of Figs. *a* and *b*,

$$+\downarrow F_R = \Sigma F_y; \qquad F_R = \frac{1}{2}(15)(3) + \frac{1}{2}(5)(3) + 10(3) + \frac{1}{2}(10)(3) = 75\text{ kN} \downarrow \qquad \textbf{Ans.}$$

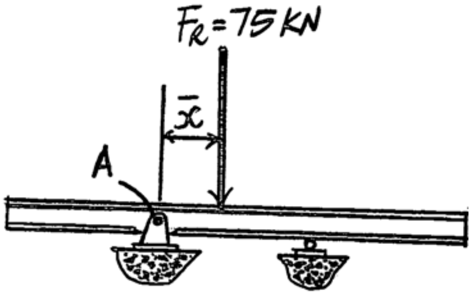
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point *A*,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -75(\bar{x}) = \frac{1}{2}(15)(3)(1) - \frac{1}{2}(5)(3)(1) - 10(3)(1.5) - \frac{1}{2}(10)(3)(4)$$

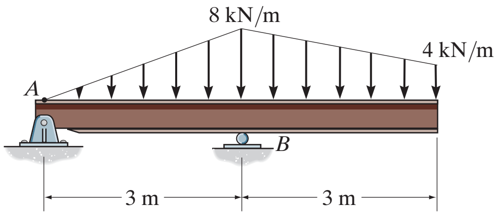
$$\bar{x} = 1.20\text{ m} \qquad \textbf{Ans.}$$



(a)



4–143. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Loading: The distributed loading can be divided into three parts as shown in Fig. *a*.

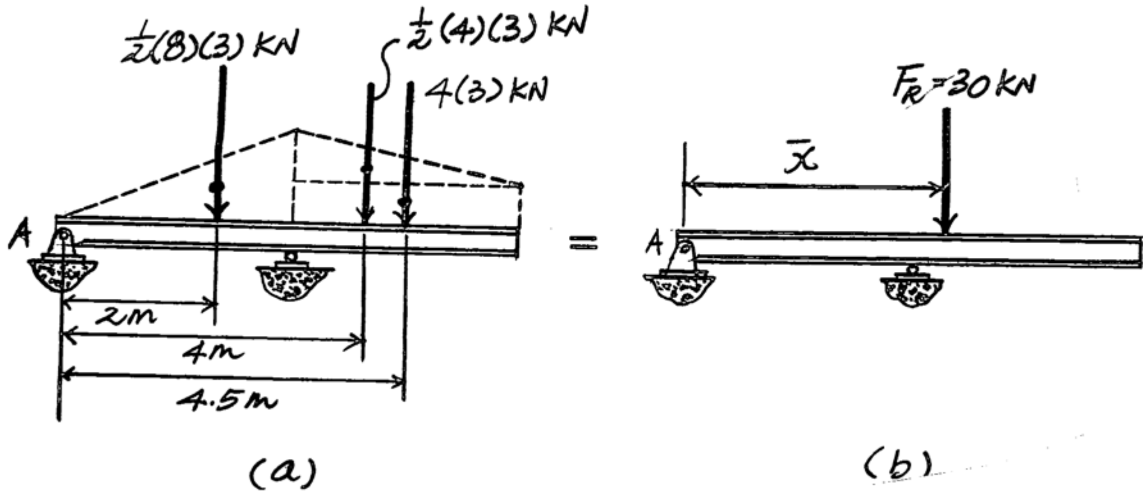
Resultants: Equating the sum of the forces along the y axis of Figs. *a* and *b*,

$$+\downarrow F_R = \Sigma F_y; \qquad F_R = \frac{1}{2}(8)(3) + \frac{1}{2}(4)(3) + 4(3) = 30 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

If we equate the moments of F_R, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point A,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -30(\bar{x}) = -\frac{1}{2}(8)(3)(2) - \frac{1}{2}(4)(3)(4) - 4(3)(4.5)$$

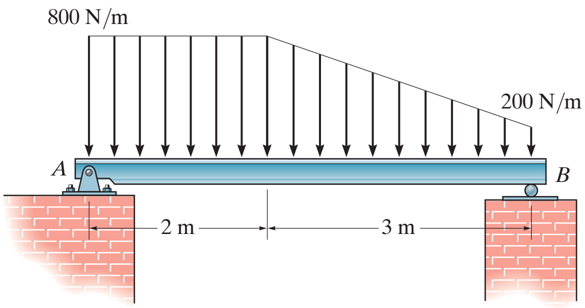
$$(\bar{x}) = 3.4 \text{ m} \qquad \textbf{Ans.}$$



(a)

(b)

***4–144.** Replace the distributed loading by an equivalent resultant force and specify its location, measured from point *A*.

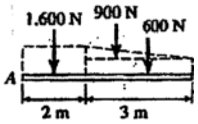


$+\downarrow F_R = \Sigma F;\quad F_R = 1600 + 900 + 600 = 3100\text{ N}$

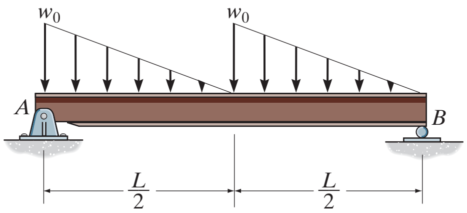
$F_R = 3.10\text{ kN}\downarrow$ **Ans**

$\curvearrowleft + M_{RA} = \Sigma M_A;\quad x(3100) = 1600(1) + 900(3) + 600(3.5)$

$x = 2.06\text{ m}$ **Ans**



•4–145. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point *A*.



Loading: The distributed loading can be divided into two parts as shown in Fig. *a*. The magnitude and location of the resultant force of each part acting on the beam are also shown in Fig. *a*.

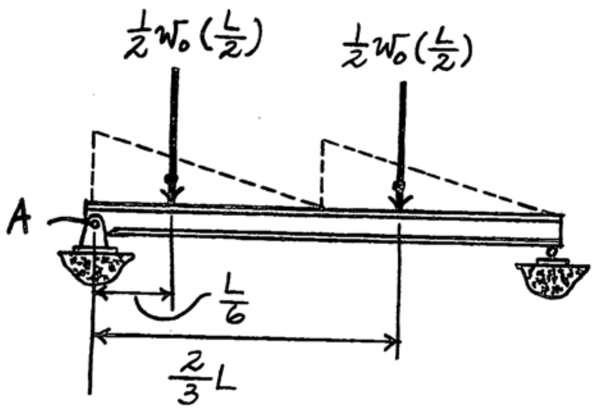
Resultants: Equating the sum of the forces along the *y* axis of Figs. *a* and *b*,

$+\downarrow F_R = \Sigma F;\qquad F_R = \frac{1}{2}w_0\left(\frac{L}{2}\right) + \frac{1}{2}w_0\left(\frac{L}{2}\right) = \frac{1}{2}w_0 L\downarrow$ **Ans.**

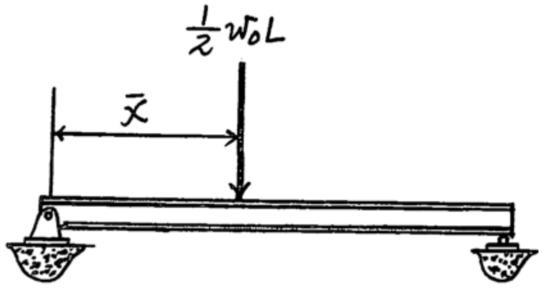
If we equate the moments of $\mathbf{F}_R$, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point *A*,

$\curvearrowleft + (M_R)_A = \Sigma M_A;\quad -\frac{1}{2}w_0 L(\bar{x}) = -\frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{2}{3}L\right)$

$\bar{x} = \frac{5}{12}L$ **Ans.**

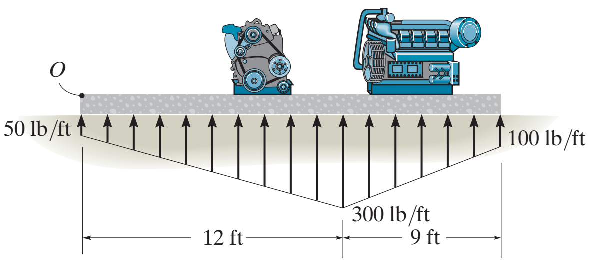


(a)



(b)

4–146. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.

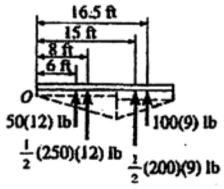


$$+\uparrow F_R = \Sigma F_y;\quad F_R = 50(12) + \tfrac{1}{2}(250)(12) + \tfrac{1}{2}(200)(9) + 100(9)$$

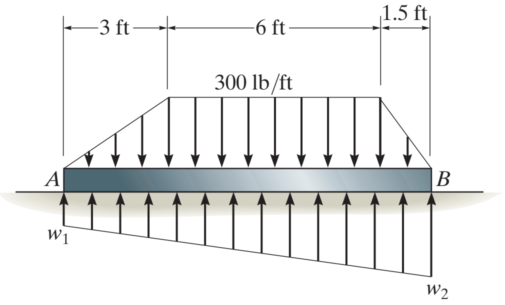
$$= 3900 \text{ lb} = 3.90 \text{ kip}\uparrow \qquad \textbf{Ans}$$

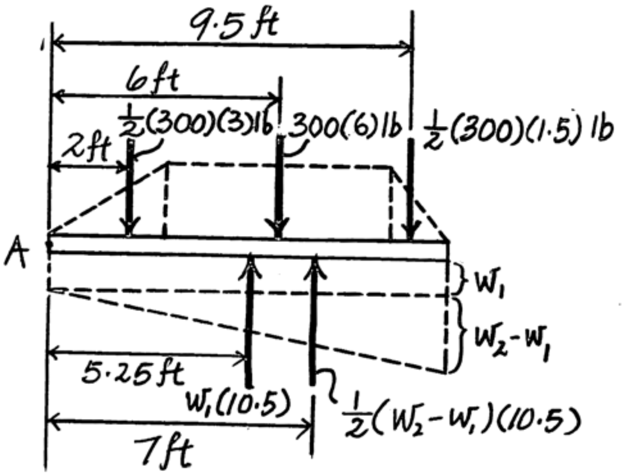
$$\curvearrowleft + M_{R_O} = \Sigma M_O;\quad 3900(d) = 50(12)(6) + \tfrac{1}{2}(250)(12)(8) + \tfrac{1}{2}(200)(9)(15) + 100(9)(16.5)$$

$$d = 11.3 \text{ ft} \qquad \textbf{Ans}$$



4–147. Determine the intensities w_1 and w_2 of the distributed loading acting on the bottom of the slab so that this loading has an equivalent resultant force that is equal but opposite to the resultant of the distributed loading acting on the top of the plate.





$$\uparrow + F_R = \Sigma F;\quad 0 = w_1(10.5) + \frac{1}{2}(w_2 - w_1)(10.5) - \frac{1}{2}(300)(3) - 300(6) - \frac{1}{2}(300)(1.5)$$

$$w_1 + w_2 = 471.429 \qquad (1)$$

$$\curvearrowleft + M_{RA} = \Sigma M_A;\quad 0 = w_1(10.5)(5.25) + \frac{1}{2}(w_2 - w_1)(10.5)(7) - \frac{1}{2}(300)(3)(2)$$

$$- 300(6)(6) - \frac{1}{2}(300)(1.5)(9.5)$$

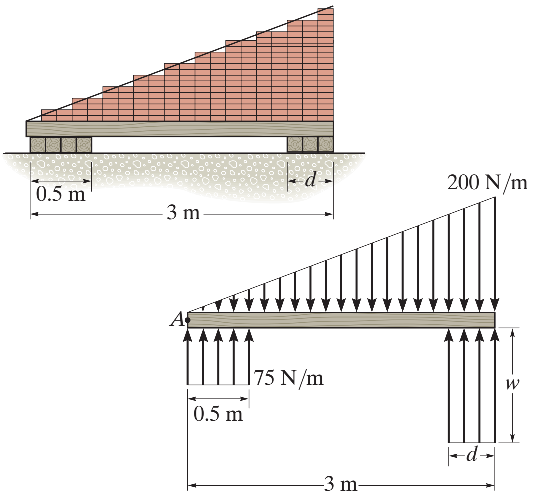
$$w_1 + 2w_2 = 753.061 \qquad (2)$$

Solving Eqs. (1) and (2),

$w_1 = 190$ lb/ft **Ans**

$w_2 = 282$ lb/ft **Ans**

***4–148.** The bricks on top of the beam and the supports at the bottom create the distributed loading shown in the second figure. Determine the required intensity *w* and dimension *d* of the right support so that the resultant force and couple moment about point *A* of the system are both zero.

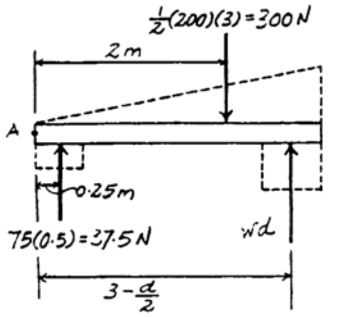


Require $F_R = 0$.

$$+\uparrow F_R = \Sigma F_y; \qquad 0 = wd + 37.5 - 300$$
$$wd = 262.5 \qquad [1]$$

Require $M_{R_A} = 0$.

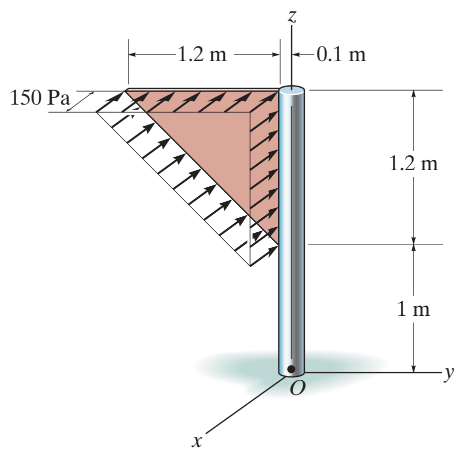
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad 0 = 37.5(0.25) + wd\left(3 - \frac{d}{2}\right) - 300(2)$$
$$3wd - \frac{wd^2}{2} = 590.625 \qquad [2]$$

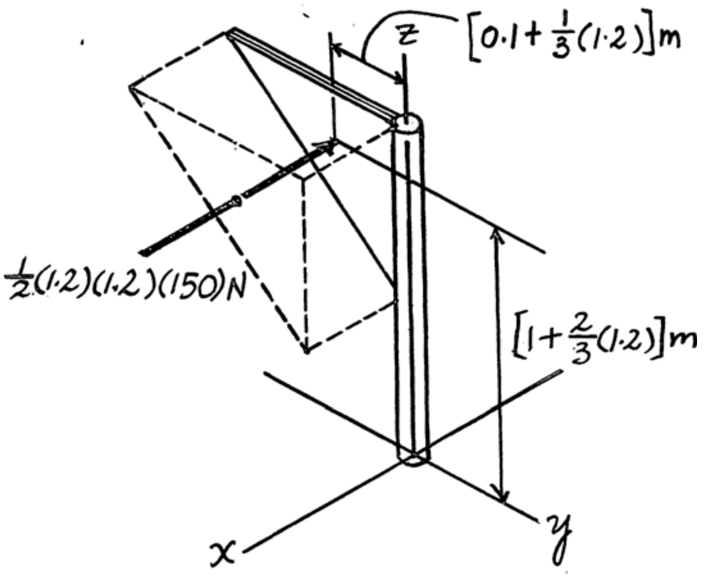


Solving Eqs. [1] and [2] yields

$d = 1.50$ m $\qquad w = 175$ N/m $\qquad$ **Ans**

•4–149. The wind pressure acting on a triangular sign is uniform. Replace this loading by an equivalent resultant force and couple moment at point *O*.





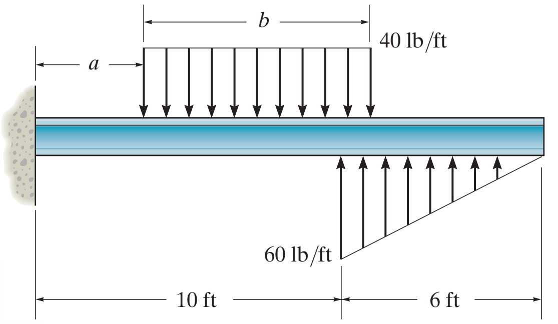
$$F_R = \frac{1}{2}(1.2)(1.2)(150)$$

$$\mathbf{F}_R = \{-108\,\mathbf{i}\}\text{ N} \qquad \text{Ans}$$

$$\mathbf{M}_{RO} = -\left(1+\frac{2}{3}(1.2)\right)(108)\,\mathbf{j} - \left(0.1+\frac{1}{3}(1.2)\right)(108)\,\mathbf{k}$$

$$\mathbf{M}_{RO} = \{-194\,\mathbf{j} - 54\,\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \text{Ans}$$

4–150. The beam is subjected to the distributed loading. Determine the length *b* of the uniform load and its position *a* on the beam such that the resultant force and couple moment acting on the beam are zero.



Require $F_R = 0$.

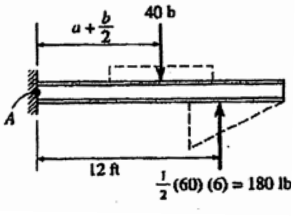
$+\uparrow F_R = \Sigma F_y;\quad 0 = 180 - 40b$

$b = 4.50$ ft **Ans**

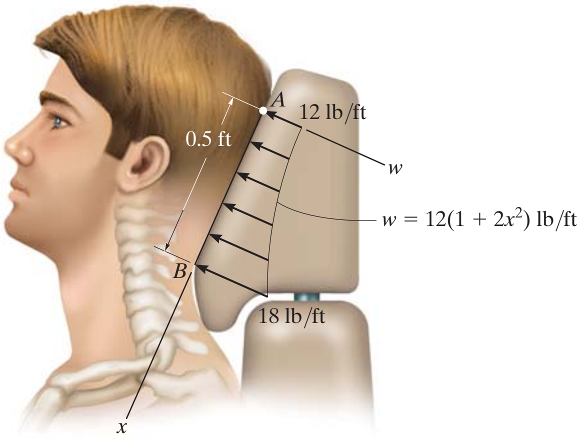
Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad 0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$

$a = 9.75$ ft **Ans**



4–151. Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point *A*.

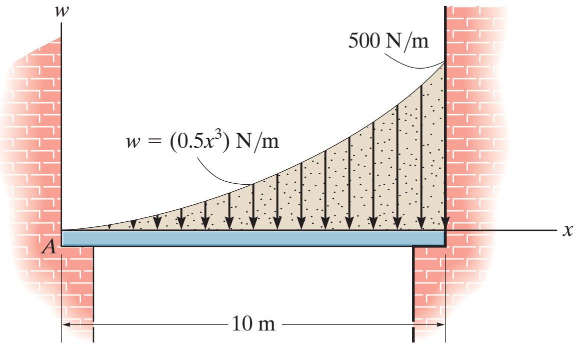


$$F_R = \int w(x)\,dx = \int_0^{0.5} 12\left(1 + 2x^2\right)dx = 12\left[x + \frac{2}{3}x^3\right]_0^{0.5} = 7 \text{ lb} \quad \textbf{Ans}$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^{0.5} x(12)\left(1 + 2x^2\right)\,dx}{7} = \frac{12\left[\frac{x^2}{2} + (2)\frac{x^4}{4}\right]_0^{0.5}}{7}$$

$\bar{x} = 0.268$ ft **Ans**

***4–152.** Wind has blown sand over a platform such that the intensity of the load can be approximated by the function $w = (0.5x^3)$ N/m. Simplify this distributed loading to an equivalent resultant force and specify its magnitude and location measured from A.



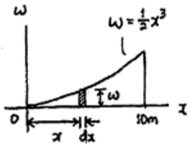
$dA = w\,dx$

$F_R = \int dA = \int_0^{10} \frac{1}{2}x^3\,dx$

$= \left[\frac{1}{8}x^4\right]_0^{10}$

$= 1250$ N

$F_R = 1.25$ kN **Ans**



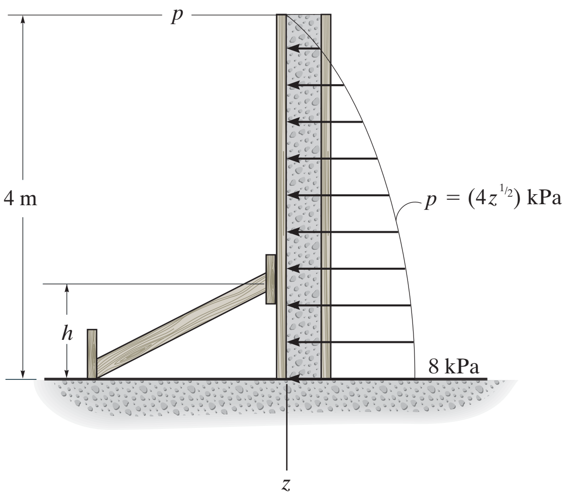
$\int \tilde{x}\,dA = \int_0^{10} \frac{1}{2}x^4\,dx$

$= \left[\frac{1}{10}x^5\right]_0^{10}$

$= 10\,000$ N·m

$\bar{x} = \frac{10\,000}{1250} = 8.00$ m **Ans**

•4–153. Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.



Equivalent Resultant Force :

$\xrightarrow{+} F_R = \Sigma F_x; \quad -F_R = -\int_A dA = -\int_0^z w\,dz$

$F_R = \int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)\left(10^3\right) dz$

$= 106.67\left(10^3\right)$ N $= 107$ kN ← **Ans**

Location of Equivalent Resultant Force :

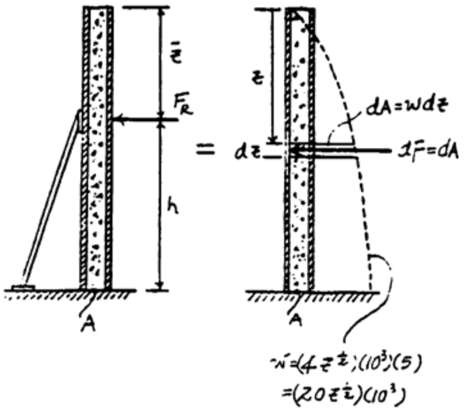
$\bar{z} = \frac{\int_A z\,dA}{\int_A dA} = \frac{\int_0^z zw\,dz}{\int_0^z w\,dz}$

$= \frac{\int_0^{4\text{m}} z\left[\left(20z^{\frac{1}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)(10^3)\,dz}$

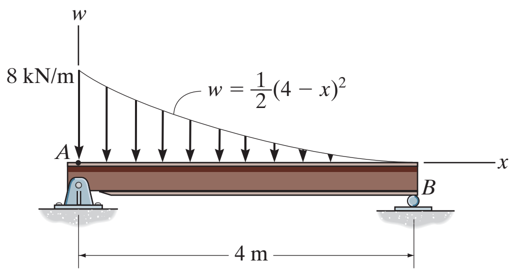
$= \frac{\int_0^{4\text{m}} \left[\left(20z^{\frac{3}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)(10^3)\,dz}$

$= 2.40$ m

Thus, $h = 4 - \bar{z} = 4 - 2.40 = 1.60$ m **Ans**


$\bar{z}$
F_R
h
A
=
z
dA = w dz
dz
dF = dA
A
$w = (4z^{\frac{1}{2}})(10^3)(5)$
$= (20z^{\frac{1}{2}})(10^3)$

4–154. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Resultant: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. a. Thus,

$$dF_R = w\,dx = \frac{1}{2}(4-x)^2\,dx = \left(\frac{x^2}{2} - 4x + 8\right)dx$$

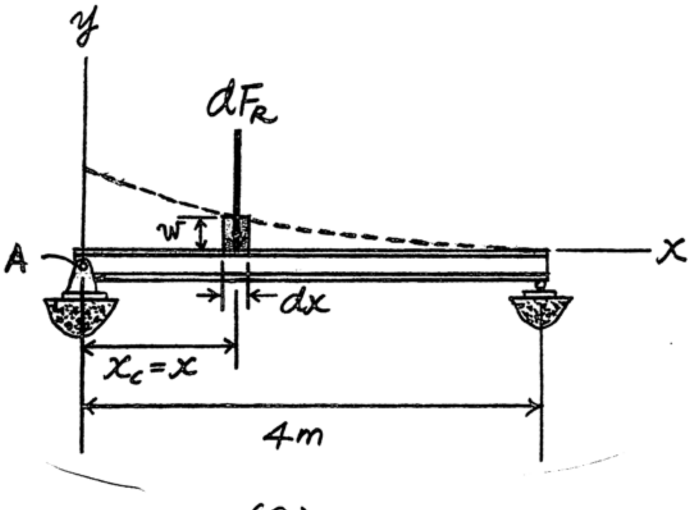
Integrating $d\mathbf{F}_R$ over the entire length of the beam gives the resultant force $\mathbf{F}_R$.

$$+\downarrow \qquad F_R = \int_L dF_R = \int_0^{4\text{ m}}\left(\frac{x^2}{2} - 4x + 8\right)dx = \left(\frac{x^3}{6} - 2x^2 + 8x\right)\Bigg|_0^{4\text{ m}}$$

$$= 10.667\text{ kN} = 10.7\text{ kN}\downarrow \qquad \textbf{Ans.}$$

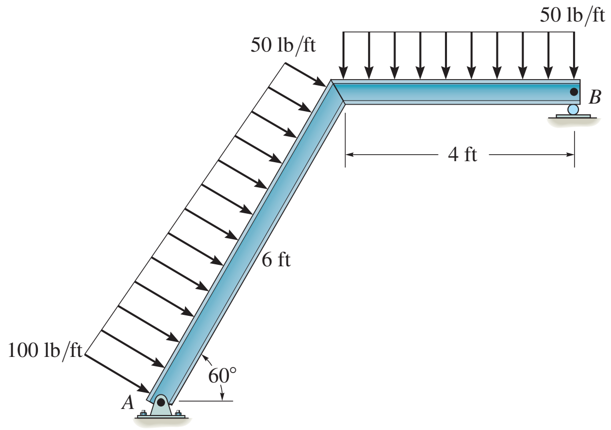
Location. The location of $d\mathbf{F}_R$ on the beam is $x_c = x$, measured from point A. Thus, the location $\bar{x}$ of $\mathbf{F}_R$ measured from point A is

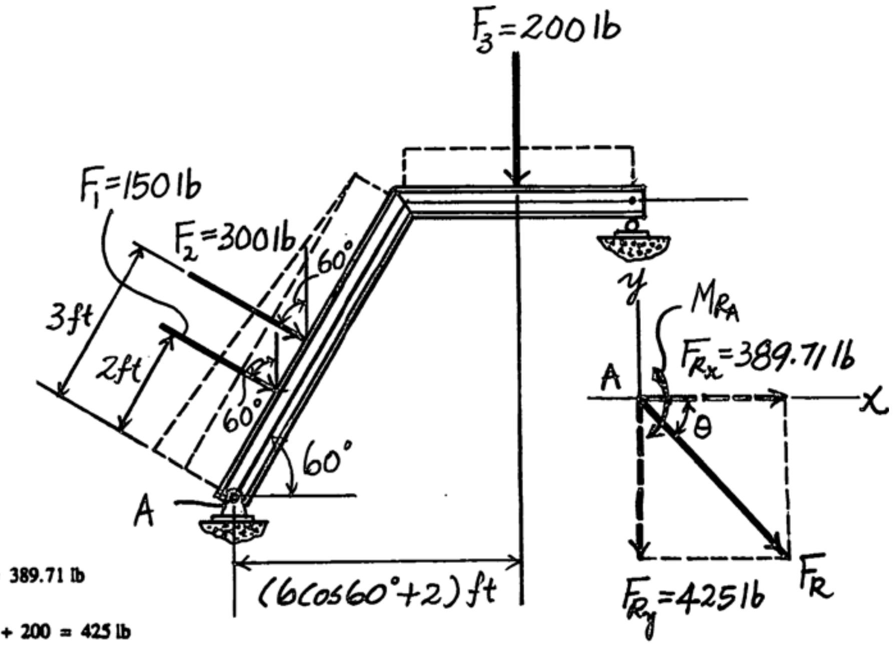
$$\bar{x} = \frac{\int_L x_c\,dF_R}{\int_L dF_R} = \frac{\int_0^{4\text{ m}} x\left(\frac{x^2}{2} - 4x + 8\right)dx}{10.667} = \frac{\left(\frac{x^4}{8} - \frac{4x^3}{3} + 4x^2\right)\Big|_0^{4\text{ m}}}{10.667} = 1\text{ m} \qquad \textbf{Ans.}$$



(a)

4–155. Replace the loading by an equivalent resultant force and couple moment at point *A*.





$$F_1 = \frac{1}{2}(6)(50) = 150\text{ lb}$$

$$F_2 = (6)(50) = 300\text{ lb}$$

$$F_3 = (4)(50) = 200\text{ lb}$$

$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x\,; \quad F_{Rx} = 150\sin 60^\circ + 300\sin 60^\circ = 389.71\text{ lb}$$

$$+\downarrow F_{Ry} = \Sigma F_y\,; \quad F_{Ry} = 150\cos 60^\circ + 300\cos 60^\circ + 200 = 425\text{ lb}$$

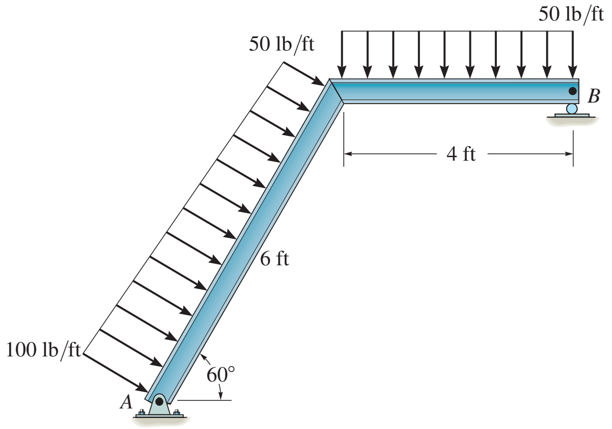
$$F_R = \sqrt{(389.71)^2 + (425)^2} = 577\text{ lb} \qquad \textbf{Ans}$$

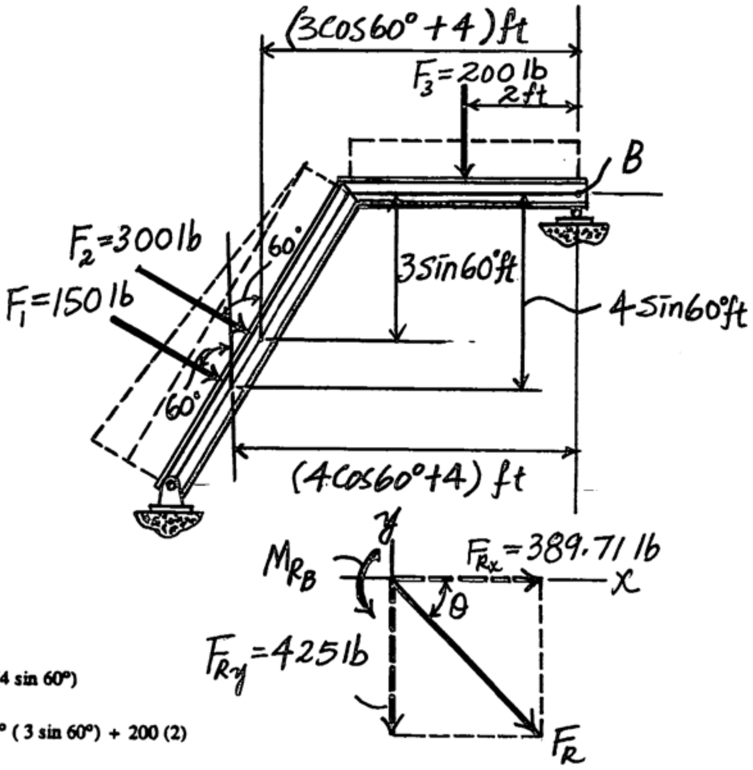
$$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5^\circ \measuredangle \qquad \textbf{Ans}$$

$$\circlearrowleft + M_{RA} = \Sigma M_A\,; \quad M_{RA} = 150(2) + 300(3) + 200(6\cos 60^\circ + 2)$$

$$= 2200\text{ lb}\cdot\text{ft} = 2.20\text{ kip}\cdot\text{ft} \circlearrowright \qquad \textbf{Ans}$$

***4–156.** Replace the loading by an equivalent resultant force and couple moment acting at point *B*.





$F_1 = \frac{1}{2}(6)(50) = 150 \text{ lb}$

$F_2 = (6)(50) = 300 \text{ lb}$

$F_2 = (4)(50) = 200 \text{ lb}$

$\xrightarrow{+} F_{Rx} = \Sigma F_x ; \quad F_{Rx} = 150 \sin 60^\circ + 300 \sin 60^\circ = 389.71 \text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y ; \quad F_{Ry} = 150 \cos 60^\circ + 300 \cos 60^\circ + 200 = 425 \text{ lb}$

$F_R = \sqrt{(389.71)^2 + (425)^2} = 577 \text{ lb}$ **Ans**

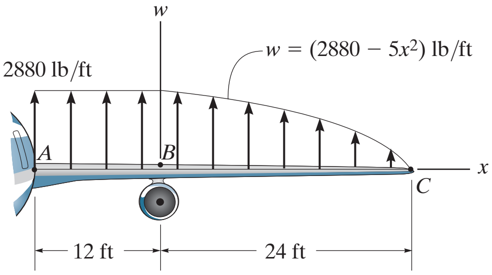
$\theta = \tan^{-1}\left(\frac{425}{389.71}\right) = 47.5^\circ$ ⦪ **Ans**

$\circlearrowleft + M_{RB} = \Sigma M_B ; \quad M_{RB} = 150 \cos 60^\circ (4 \cos 60^\circ + 4) + 150 \sin 60^\circ (4 \sin 60^\circ)$

$+ 300 \cos 60^\circ (3 \cos 60^\circ + 4) + 300 \sin 60^\circ (3 \sin 60^\circ) + 200(2)$

$M_{RB} = 2800 \text{ lb} \cdot \text{ft} = 2.80 \text{ kip} \cdot \text{ft} \circlearrowright$ **Ans**

•4–157. The lifting force along the wing of a jet aircraft consists of a uniform distribution along *AB*, and a semiparabolic distribution along *BC* with origin at *B*. Replace this loading by a single resultant force and specify its location measured from point *A*.



Equivalent Resultant Force :

$$+\uparrow F_R = \Sigma F_y; \quad F_R = 34560 + \int_0^x w\,dx$$

$$F_R = 34560 + \int_0^{24\text{ft}} \left(2880 - 5x^2\right) dx$$

$$= 80640 \text{ lb} = 80.6 \text{ kip} \uparrow \qquad \textbf{Ans}$$

Location of Equivalent Resultant Force :

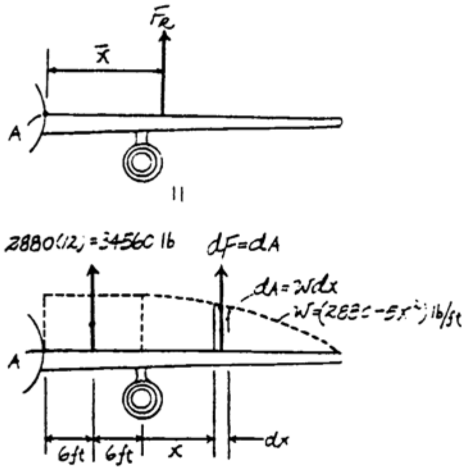
$$\circlearrowleft + M_{R_A} = \Sigma M_A;$$

$$80640\bar{x} = 34560(6) + \int_0^x (x+12)\,w\,dx$$

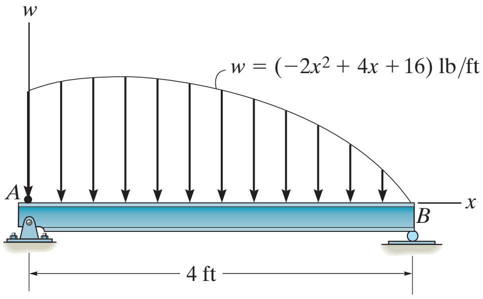
$$80640\bar{x} = 207360 + \int_0^{24\text{ft}} (x+12)\left(2880 - 5x^2\right) dx$$

$$80640\bar{x} = 207360 + \int_0^{24\text{ft}} \left(-5x^3 - 60x^2 + 2880x + 34560\right) dx$$

$$\bar{x} = 14.6 \text{ ft} \qquad \textbf{Ans}$$

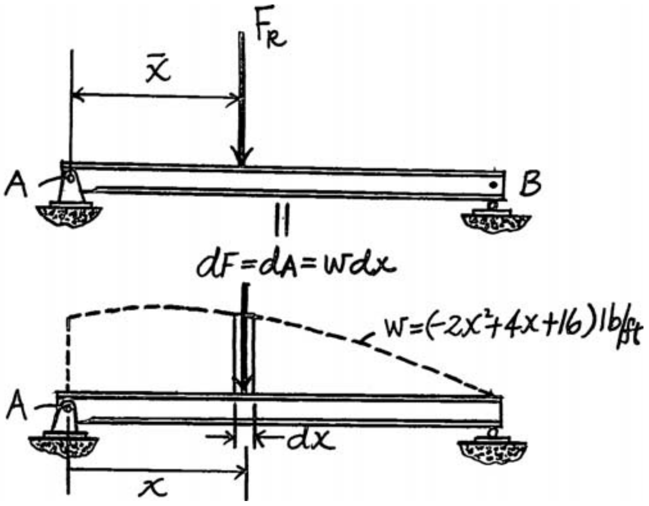


4–158. The distributed load acts on the beam as shown. Determine the magnitude of the equivalent resultant force and specify where it acts, measured from point *A*.

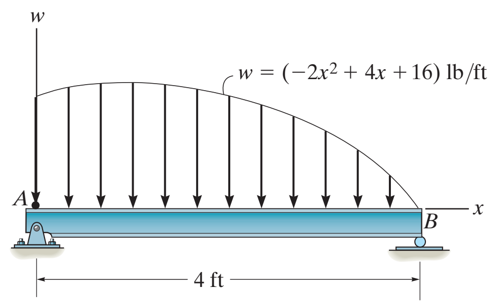


$$F_R = \int w(x)\,dx = \int_0^4 \left(-2x^2 + 4x + 16\right) dx = 53.333 = 53.3 \text{ lb} \qquad \textbf{Ans}$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^4 x\left(-2x^2 + 4x + 16\right) dx}{53.333} = 1.60 \text{ ft} \qquad \textbf{Ans}$$



4–159. The distributed load acts on the beam as shown. Determine the maximum intensity $w_{\max}$. What is the magnitude of the equivalent resultant force? Specify where it acts, measured from point B.



$$\frac{dw}{dx} = -4x + 4 = 0$$

$$x = 1$$

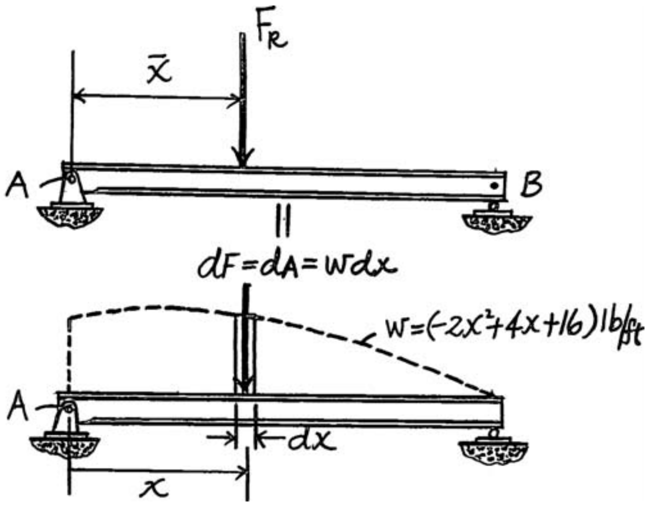
$$w_{max} = -2(1)^2 + 4(1) + 16 = 18 \text{ lb/ft} \qquad \textbf{Ans}$$

$$F_R = \int w(x)\,dx = \int_0^4 (-2x^2 + 4x + 16)\,dx = 53.333 = 53.3 \text{ lb} \qquad \textbf{Ans}$$

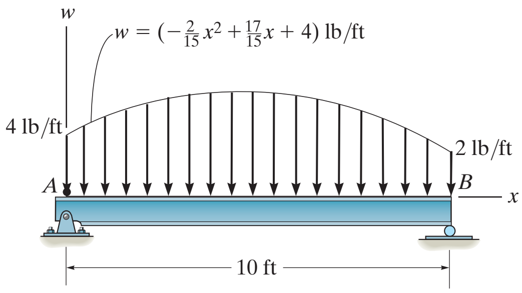
$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^4 x(-2x^2 + 4x + 16)\,dx}{53.333} = 1.60 \text{ ft}$$

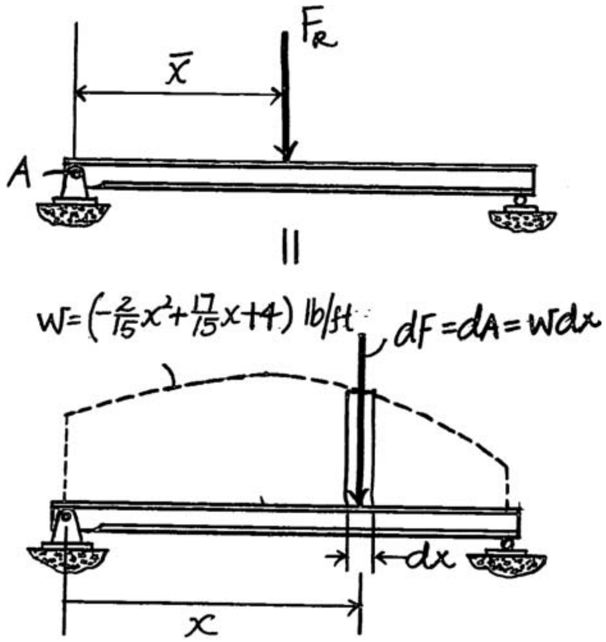
So that from B,

$$x' = 4 - 1.60 = 2.40 \text{ ft} \qquad \textbf{Ans}$$



***4–160.** The distributed load acts on the beam as shown. Determine the magnitude of the equivalent resultant force and specify its location, measured from point *A*.



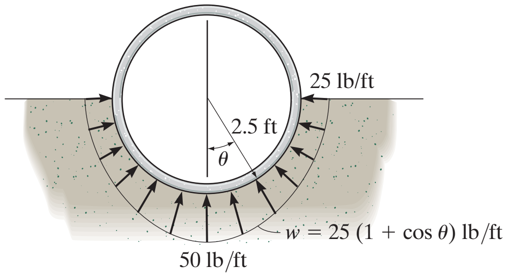


$$F_R = \int w(x)\,dx = \int_0^{10}\left(-\frac{2}{15}x^2 + \frac{17}{15}x + 4\right)dx = 52.22 = 52.2\text{ lb} \quad \textbf{Ans}$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx} = \frac{\int_0^{10} x\left(-\frac{2}{15}x^2 + \frac{17}{15}x + 4\right)dx}{52.22} = \frac{244.44}{52.22}$$

$$\bar{x} = 4.68\text{ ft} \quad \textbf{Ans}$$

•4–161. If the distribution of the ground reaction on the pipe per foot of length can be approximated as shown, determine the magnitude of the resultant force due to this loading.



Resultant Components: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. *a*.

$$d\mathbf{F}_R = wr\ d\theta = 25(1+\cos\theta)(2.5\ d\theta) = 62.5(1+\cos\theta)\ d\theta$$

The horizontal and vertical components of $d\mathbf{F}_R$ are given by

$$\overset{+}{\leftarrow} \qquad (dF_R)_x = dF_R \sin\theta = 62.5(1+\cos\theta)\sin\theta\ d\theta = 62.5\left(\sin\theta + \frac{\sin 2\theta}{2}\right)d\theta$$

$$+\uparrow \qquad (dF_R)_y = dF_R \cos\theta = 62.5(1+\cos\theta)\cos\theta\ d\theta = 62.5\left(\cos\theta + \frac{\cos 2\theta + 1}{2}\right)d\theta$$

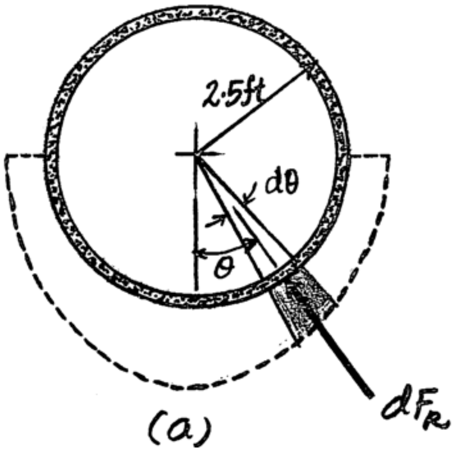
Integrating $(dF_R)_x$ and $(dF_R)_y$ from $\theta = -\frac{\pi}{2}$ rad to $\theta = \frac{-\pi}{2}$ rad gives the horizontal and vertical components of the resultant for $\mathbf{F}_R$.

$$\overset{+}{\leftarrow}(F_R)_x = \int_{-\pi/2}^{\pi/2} 62.5\left(\sin\theta + \frac{\sin 2\theta}{2}\right)d\theta = 62.5\left(-\cos\theta - \frac{\cos 2\theta}{4}\right)\Bigg|_{-\pi/2}^{\pi/2} = 0$$

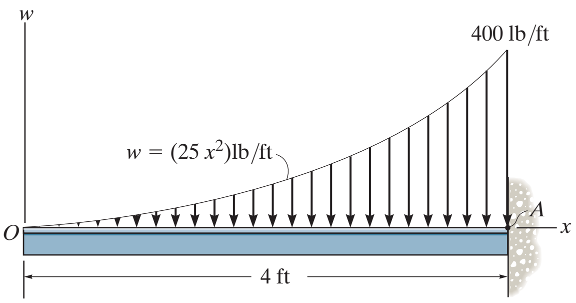
$$+\uparrow(F_R)_y = \int_{-\pi/2}^{\pi/2} 62.5\left(\cos\theta + \frac{\cos 2\theta + 1}{2}\right)d\theta = 62.5\left(\sin\theta + \frac{\sin 2\theta}{4} + \frac{1}{2}\theta\right)\Bigg|_{-\pi/2}^{\pi/2} = 62.5\left(2 + \frac{\pi}{2}\right) = 223.17\text{ lb}\ \uparrow$$

Thus,

$$F_R = (F_R)_y = 223.17\text{ lb} = 223\text{ lb}\ \uparrow \qquad \textbf{Ans.}$$



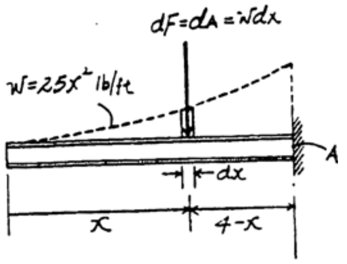
4–162. The beam is subjected to the parabolic loading. Determine an equivalent force and couple system at point A.



$+\uparrow F_R = \Sigma F_y;\quad F_R = -\int_A dA = -\int_0^x w\,dx$

$F_R = -\int_0^{4\text{ ft}} (25x^2)\,dx$

$= -533.33 \text{ lb} = 533 \text{ lb} \downarrow$ **Ans**



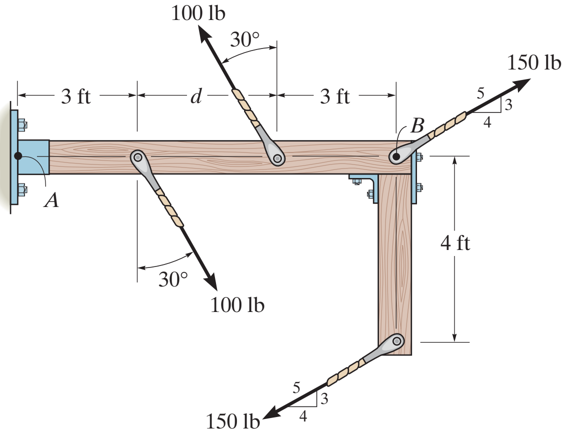
$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad M_{R_A} = \int_0^x (4-x)\,w\,dx$

$= \int_0^{4\text{ ft}} (4-x)(25x^2)\,dx$

$= \int_0^{4\text{ ft}} (-25x^3 + 100x^2)\,dx$

$= 533 \text{ lb}\cdot\text{ft}$ (*Counterclockwise*) **Ans**

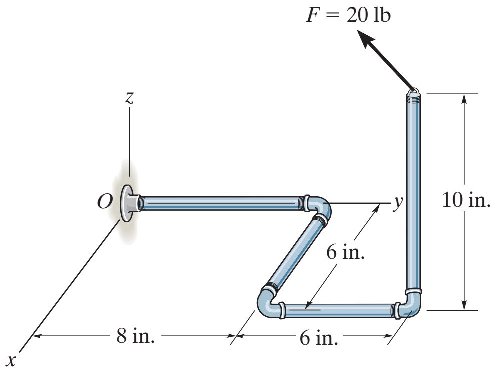
4–163. Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the 100-lb couple forces.



$\circlearrowleft + M_R = 0 = \Sigma M;\quad 0 = 100\cos 30°(d) - \frac{4}{5}(150)(4)$

$d = 5.54 \text{ ft}$ **Ans**

***4–164.** Determine the coordinate direction angles α, β, γ of **F**, which is applied to the end of the pipe assembly, so that the moment of **F** about O is zero.



Require $\mathbf{M}_O = \mathbf{0}$. This happens when force $\mathbf{F}$ is directed along line OA either from point O to A or from point A to O. The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_{AO}$ are

$$\mathbf{u}_{OA} = \frac{(6-0)\mathbf{i}+(14-0)\mathbf{j}+(10-0)\mathbf{k}}{\sqrt{(6-0)^2+(14-0)^2+(10-0)^2}}$$
$$= 0.3293\mathbf{i}+0.7683\mathbf{j}+0.5488\mathbf{k}$$

Thus,

$\alpha = \cos^{-1}0.3293 = 70.8^\circ$ **Ans**

$\beta = \cos^{-1}0.7683 = 39.8^\circ$ **Ans**

$\gamma = \cos^{-1}0.5488 = 56.7^\circ$ **Ans**

$$\mathbf{u}_{AO} = \frac{(0-6)\mathbf{i}+(0-14)\mathbf{j}+(0-10)\mathbf{k}}{\sqrt{(0-6)^2+(0-14)^2+(0-10)^2}}$$
$$= -0.3293\mathbf{i}-0.7683\mathbf{j}-0.5488\mathbf{k}$$

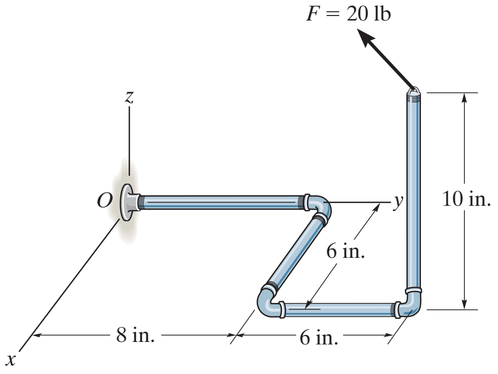
Thus,

$\alpha = \cos^{-1}(-0.3293) = 109^\circ$ **Ans**

$\beta = \cos^{-1}(-0.7683) = 140^\circ$ **Ans**

$\gamma = \cos^{-1}(-0.5488) = 123^\circ$ **Ans**

•4–165. Determine the moment of the force **F** about point O. The force has coordinate direction angles of $\alpha = 60^\circ$, $\beta = 120^\circ$, $\gamma = 45^\circ$. Express the result as a Cartesian vector.



Position Vector And Force Vectors :

$$\mathbf{r}_{OA} = \{(6-0)\mathbf{i}+(14-0)\mathbf{j}+(10-0)\mathbf{k}\} \text{ in.}$$
$$= \{6\mathbf{i}+14\mathbf{j}+10\mathbf{k}\} \text{ in.}$$

$$\mathbf{F} = 20(\cos 60^\circ\mathbf{i}+\cos 120^\circ\mathbf{j}+\cos 45^\circ\mathbf{k}) \text{ lb}$$
$$= \{10.0\mathbf{i}-10.0\mathbf{j}+14.142\mathbf{k}\} \text{ lb}$$

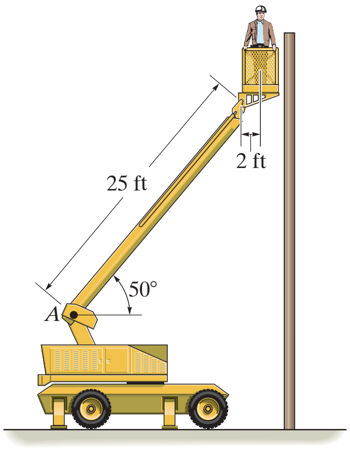
Moment of Force* F *About Point O : Applying Eq. 4 – 7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA}\times\mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 14 & 10 \\ 10.0 & -10.0 & 14.142 \end{vmatrix}$$

$= \{298\mathbf{i}+15.1\mathbf{j}-200\mathbf{k}\}$ lb · in **Ans**

4–166. The snorkel boom lift is extended into the position shown. If the worker weighs 160 lb, determine the moment of this force about the connection at A.

$$M_A = 160(2 + 25\cos 50^\circ) = 2891\ \text{lb}\cdot\text{ft} = 2.89\ \text{kip}\cdot\text{ft} \curvearrowright \qquad \textbf{Ans}$$



4–167. Determine the moment of the force $\mathbf{F}_C$ about the door hinge at A. Express the result as a Cartesian vector.

Position Vector And Force Vector :

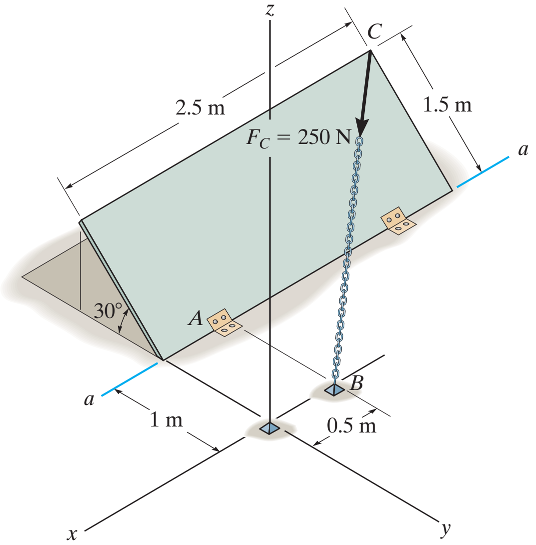
$$\mathbf{r}_{AB} = \{[-0.5-(-0.5)]\mathbf{i}+[0-(-1)]\mathbf{j}+(0-0)\mathbf{k}\}\ \text{m} = \{1\mathbf{j}\}\ \text{m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i}+\{0-[-(1+1.5\cos 30^\circ)]\}\mathbf{j}+(0-1.5\sin 30^\circ)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2+\{0-[-(1+1.5\cos 30^\circ)]\}^2+(0-1.5\sin 30^\circ)^2}}\right)\text{N}$$
$$= \{159.33\mathbf{i}+183.15\mathbf{j}-59.75\mathbf{k}\}\ \text{N}$$

Moment of Force $\mathbf{F}_C$ About Point A : Applying Eq. 4–7, we have

$$\mathbf{M}_A = \mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

$$= \{-59.7\mathbf{i} - 159\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$



***4–168.** Determine the magnitude of the moment of the force $\mathbf{F}_C$ about the hinged axis aa of the door.

$$\mathbf{r}_{AB} = \{[-0.5-(-0.5)]\mathbf{i}+[0-(-1)]\mathbf{j}+(0-0)\mathbf{k}\}\ \text{m} = \{1\mathbf{j}\}\ \text{m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i}+\{0-[-(1+1.5\cos 30^\circ)]\}\mathbf{j}+(0-1.5\sin 30^\circ)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2+\{0-[-(1+1.5\cos 30^\circ)]\}^2+(0-1.5\sin 30^\circ)^2}}\right)\text{N}$$
$$= \{159.33\mathbf{i}+183.15\mathbf{j}-59.75\mathbf{k}\}\ \text{N}$$

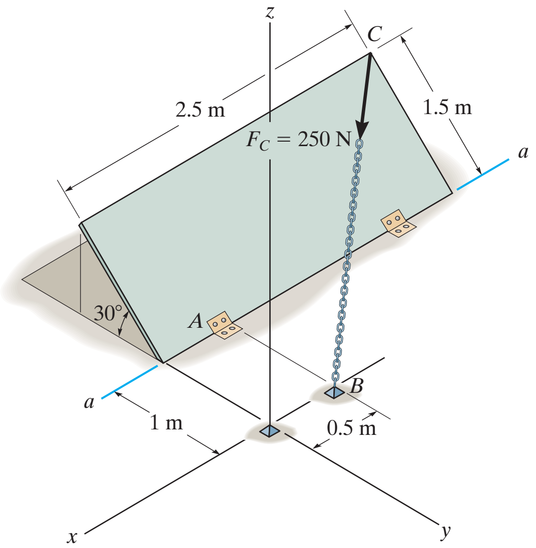
Moment of Force $\mathbf{F}_C$ About a-aAxis : The unit vector along the a–a axis is $\mathbf{i}$. Applying Eq. 4–11, we have

$$M_{a-a} = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F}_C) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

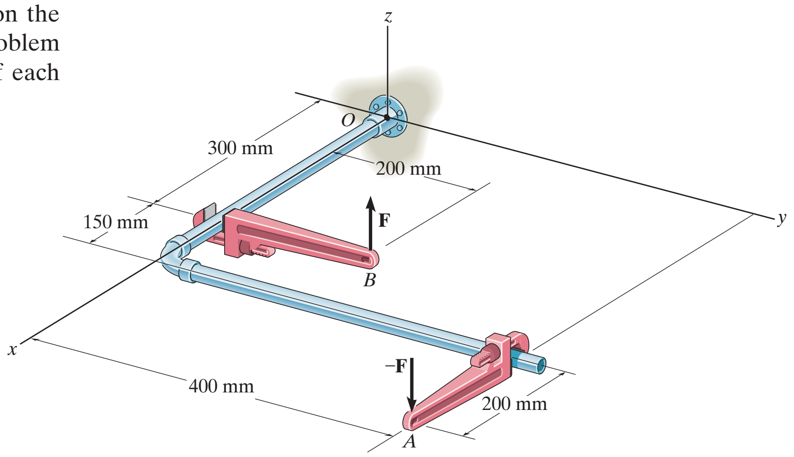
$$= 1[1(-59.75)-(183.15)(0)] - 0 + 0$$
$$= -59.7\ \text{N}\cdot\text{m}$$

The negative sign indicates that $\mathbf{M}_{a-a}$ is directed toward negative x axis.

$$\mathbf{M}_{a-a} = 59.7\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$



•4–169. Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem (a) using Eq. 4–13 and (b) summing the moment of each force about point *O*. Take $\mathbf{F} = \{25\mathbf{k}\}$ N.



a) $\mathbf{M}_C = \mathbf{r}_{AB} \times (25\,\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}$$

$\mathbf{M}_C = \{-5\,\mathbf{i} + 8.75\,\mathbf{j}\}$ N·m Ans

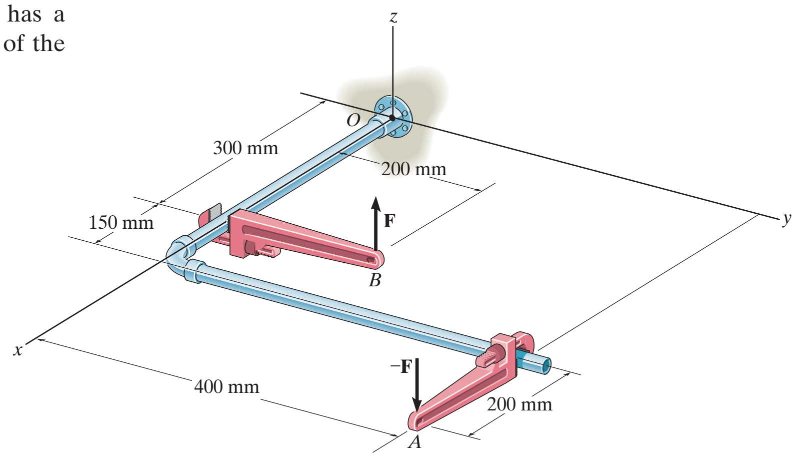
(b) $\mathbf{M}_C = \mathbf{r}_{OB} \times (25\,\mathbf{k}) + \mathbf{r}_{OA} \times (-25\,\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.4 & 0 \\ 0 & 0 & -25 \end{vmatrix}$$

$\mathbf{M}_C = (5 - 10)\,\mathbf{i} + (-7.5 + 16.25)\,\mathbf{j}$

$\mathbf{M}_C = \{-5\,\mathbf{i} + 8.75\,\mathbf{j}\}$ N·m Ans

4–170. If the couple moment acting on the pipe has a magnitude of 400 N · m, determine the magnitude *F* of the vertical force applied to each wrench.



$\mathbf{M}_C = \mathbf{r}_{AB} \times (F\,\mathbf{k})$

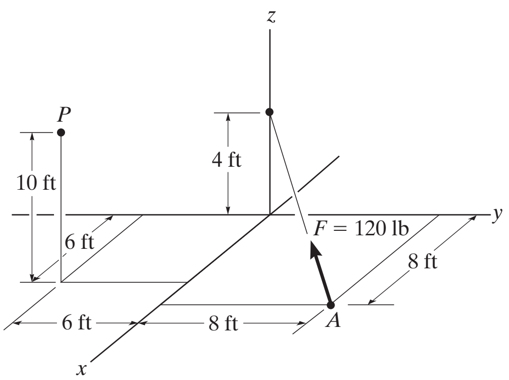
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$\mathbf{M}_C = \{-0.2F\,\mathbf{i} + 0.35F\,\mathbf{j}\}$ N·m

$M_C = \sqrt{(-0.2F)^2 + (0.35F)^2} = 400$

$$F = \frac{400}{\sqrt{(-0.2)^2 + (0.35)^2}} = 992 \text{ N}$$

4–171. Replace the force at *A* by an equivalent resultant force and couple moment at point *P*. Express the results in Cartesian vector form.

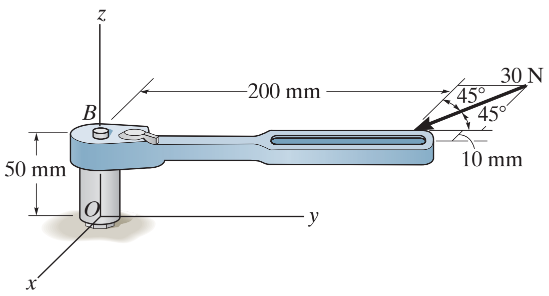


$$\mathbf{F}_R = 120\left(\frac{-8\,\mathbf{i} - 8\,\mathbf{j} + 4\,\mathbf{k}}{\sqrt{(-8)^2 + (-8)^2 + 4^2}}\right) = \{-80\,\mathbf{i} - 80\,\mathbf{j} + 40\,\mathbf{k}\}\text{ lb} \qquad \text{Ans}$$

$$\mathbf{M}_{RP} = \Sigma\mathbf{M}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 14 & -10 \\ -80 & -80 & 40 \end{vmatrix}$$

$$= \{-240\,\mathbf{i} + 720\,\mathbf{j} + 960\,\mathbf{k}\}\text{ lb}\cdot\text{ft} \qquad \text{Ans}$$

***4–172.** The horizontal 30-N force acts on the handle of the wrench. Determine the moment of this force about point *O*. Specify the coordinate direction angles α, β, γ of the moment axis.



Position Vector And Force Vectors :

$$\mathbf{r}_{OA} = \{(-0.01-0)\mathbf{i} + (0.2-0)\mathbf{j} + (0.05-0)\mathbf{k}\}\text{ m}$$
$$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 30(\sin 45°\,\mathbf{i} - \cos 45°\,\mathbf{j})\text{ N}$$
$$= \{21.213\mathbf{i} - 21.213\mathbf{j}\}\text{ N}$$

Moment of Force* F *About Point O : Applying Eq. 4 – 7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$$= \{1.061\mathbf{i} + 1.061\mathbf{j} - 4.031\mathbf{k}\}\text{ N}\cdot\text{m}$$
$$= \{1.06\mathbf{i} + 1.06\mathbf{j} - 4.03\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \text{Ans}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \sqrt{1.061^2 + 1.061^2 + (-4.031)^2} = 4.301\text{ N}\cdot\text{m}$$

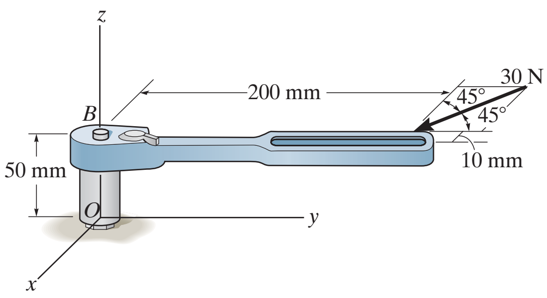
The coordinate direction angles for $\mathbf{M}_O$ are

$$\alpha = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7° \qquad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7° \qquad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-4.031}{4.301}\right) = 160° \qquad \text{Ans}$$

•4–173. The horizontal 30-N force acts on the handle of the wrench. What is the magnitude of the moment of this force about the z axis?



Position Vector And Force Vectors :

$$\mathbf{r}_{BA} = \{-0.01\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}$$

$$\begin{aligned}\mathbf{r}_{OA} &= \{(-0.01-0)\mathbf{i} + (0.2-0)\mathbf{j} + (0.05-0)\mathbf{k}\}\ \text{m}\\ &= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\ \text{m}\end{aligned}$$

$$\begin{aligned}\mathbf{F} &= 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})\ \text{N}\\ &= \{21.213\mathbf{i} - 21.213\mathbf{j}\}\ \text{N}\end{aligned}$$

Moment of Force* F *About z Axis : The unit vector along the z axis is **k**. Applying Eq. 4 – 11, we have

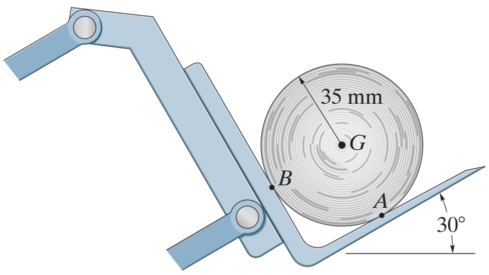
$$\begin{aligned}M_z &= \mathbf{k}\cdot(\mathbf{r}_{BA} \times \mathbf{F})\\ &= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0 \\ 21.213 & -21.213 & 0 \end{vmatrix}\\ &= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]\\ &= -4.03\ \text{N}\cdot\text{m} \qquad \textbf{Ans}\end{aligned}$$

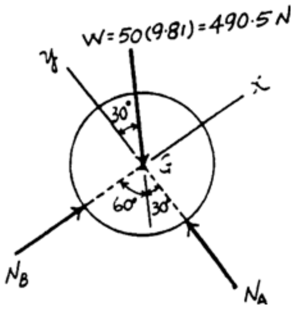
Or

$$\begin{aligned}M_z &= \mathbf{k}\cdot(\mathbf{r}_{BA} \times \mathbf{F})\\ &= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}\\ &= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]\\ &= -4.03\ \text{N}\cdot\text{m} \qquad \textbf{Ans}\end{aligned}$$

The negative sign indicates that $\mathbf{M}_z$ is directed along the negative z axis.

•5–1. Draw the free-body diagram of the 50-kg paper roll which has a center of mass at *G* and rests on the smooth blade of the paper hauler. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



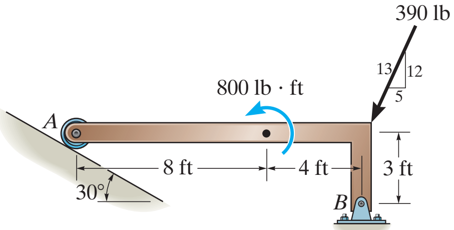


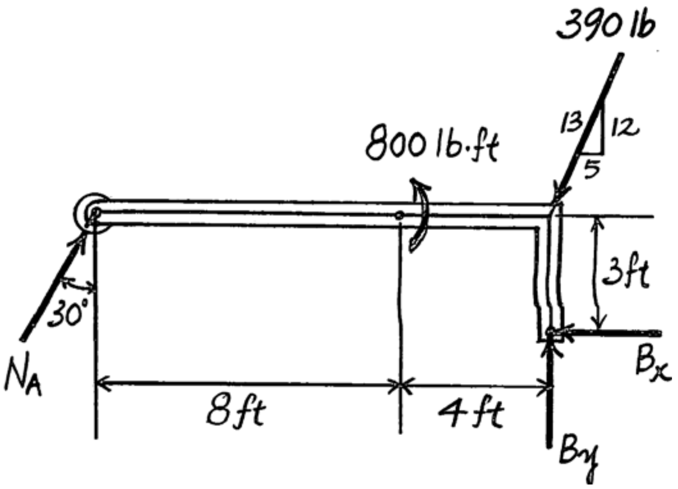
The Significance of Each Force :

W is the effect of gravity (weight) on the paper roll.

N_A and N_B are the smooth blade reactions on the paper roll.

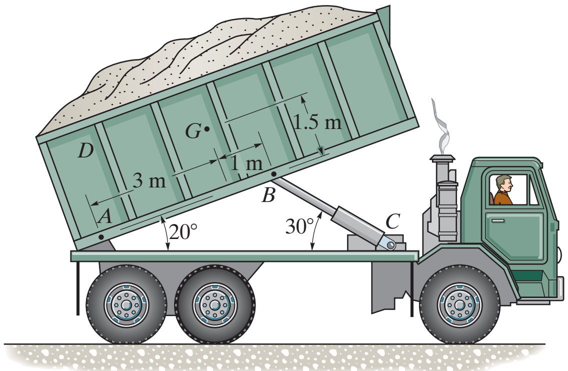
5–2. Draw the free-body diagram of member *AB*, which is supported by a roller at *A* and a pin at *B*. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)

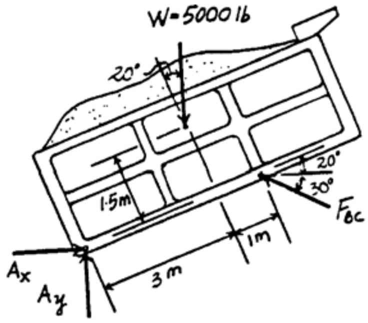




N_A force of plane on roller.
B_x, B_y force of pin on member.

5–3. Draw the free-body diagram of the dumpster D of the truck, which has a weight of 5000 lb and a center of gravity at G. It is supported by a pin at A and a pin-connected hydraulic cylinder BC (short link). Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)





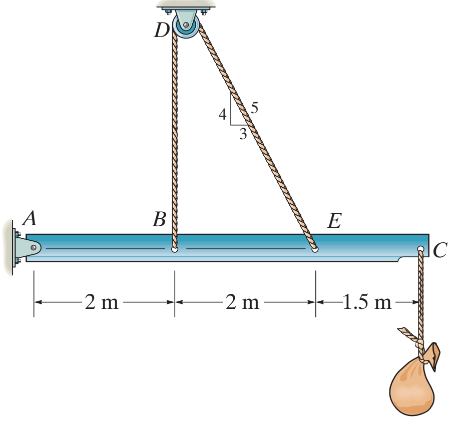
The Significance of Each Force :

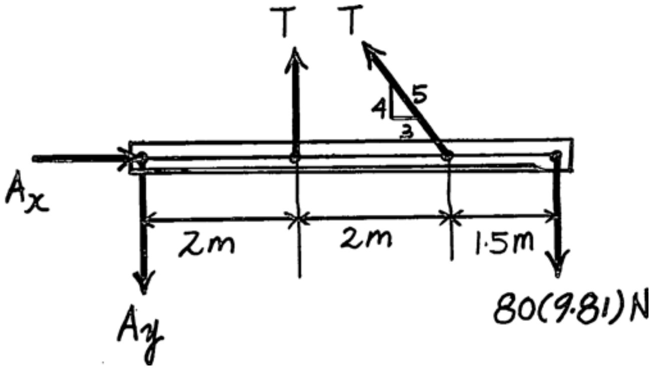
W is the effect of gravity (weight) on the dumpster.

A_y and A_x are the pin A reactions on the dumpster.

F_{BC} is the hydraulic cylinder BC reaction on the dumpster.

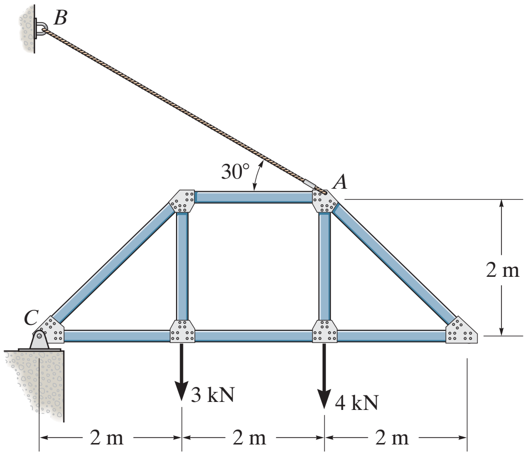
***5–4.** Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at A and a cable which wraps around the pulley at D. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)





T force of cable on beam.
A_x, A_y force of pin on beam.
80(9.81)N force of cable on beam.

•5–5. Draw the free-body diagram of the truss that is supported by the cable *AB* and pin *C*. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)

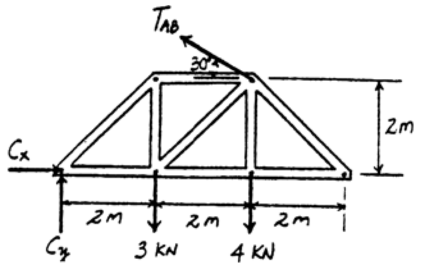


The Significance of Each Force :

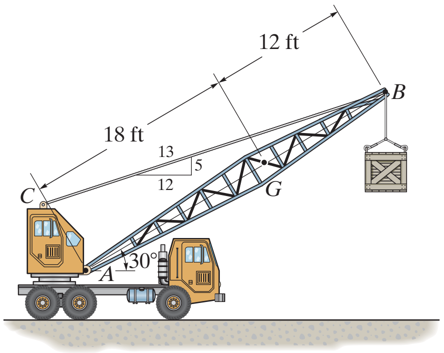
C_y and C_x are the pin C reactions on the truss.

T_{AB} is the cable AB tension on the truss.

3 kN and 4 kN force are the effect of external applied forces on the truss.



5–6. Draw the free-body diagram of the crane boom *AB* which has a weight of 650 lb and center of gravity at *G*. The boom is supported by a pin at *A* and cable *BC*. The load of 1250 lb is suspended from a cable attached at *B*. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



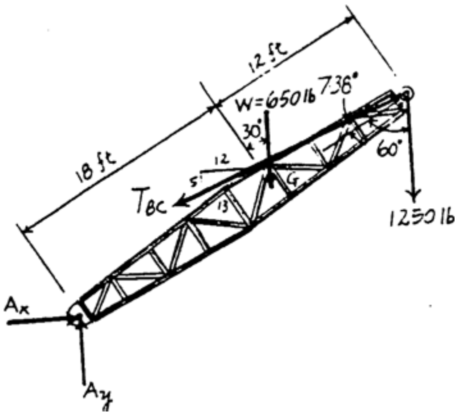
The Significance of Each Force :

W is the effect of gravity (weight) on the boom.

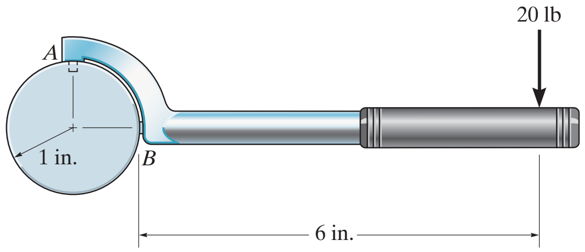
A_y and A_x are the pin A reactions on the boom.

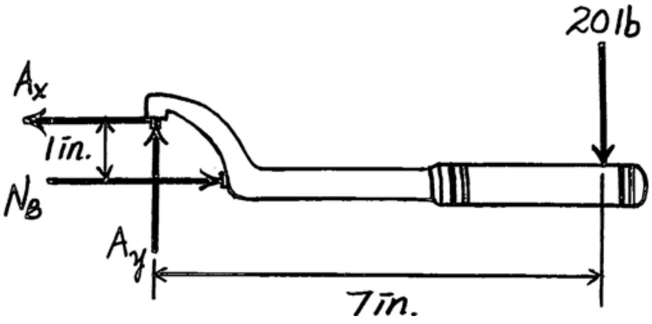
T_{BC} is the cable BC force reactions on the boom.

1250 lb force is the suspended load reaction on the boom.



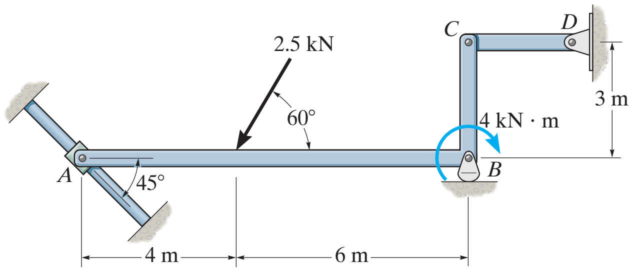
5–7. Draw the free-body diagram of the "spanner wrench" subjected to the 20-lb force. The support at *A* can be considered a pin, and the surface of contact at *B* is smooth. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)





A_x, A_y, N_B force of cylinder on wrench.

***5–8.** Draw the free-body diagram of member *ABC* which is supported by a smooth collar at *A*, roller at *B*, and short link *CD*. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



The Significance of Each Force :

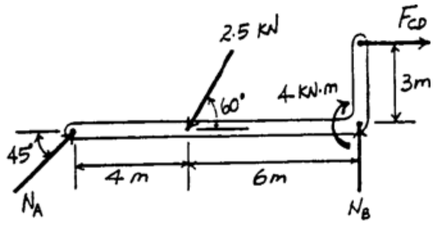
N_A is the smooth collar reaction on member *ABC*.

N_B is the roller support *B* reaction on member *ABC*.

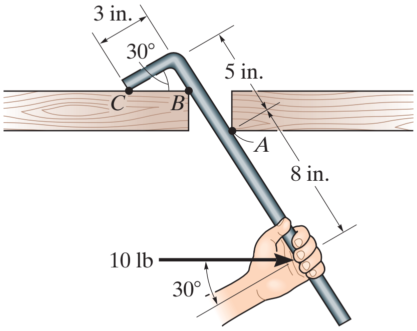
F_{CD} is the short link reaction on member *ABC*.

2.5 kN is the effect of external applied force on member *ABC*.

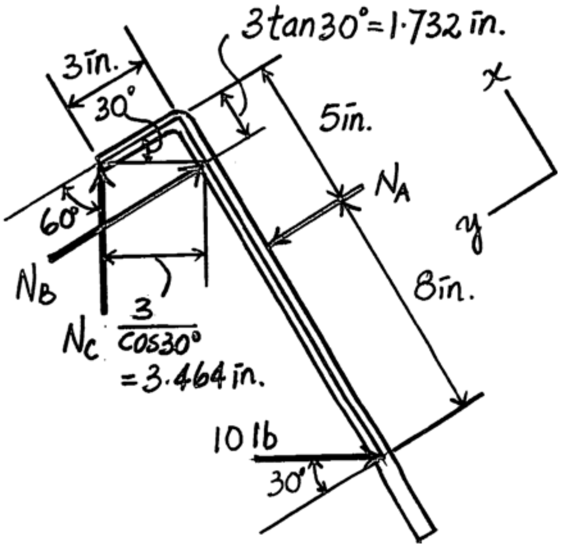
4 kN · m is the effect of external applied couple moment on member *ABC*.



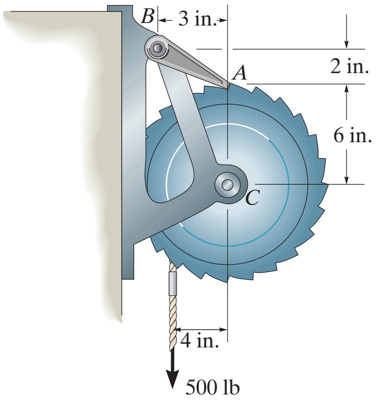
•5–9. Draw the free-body diagram of the bar, which has a negligible thickness and smooth points of contact at A, B, and C. Explain the significance of each force on the diagram. (See Fig. 5–7b.)

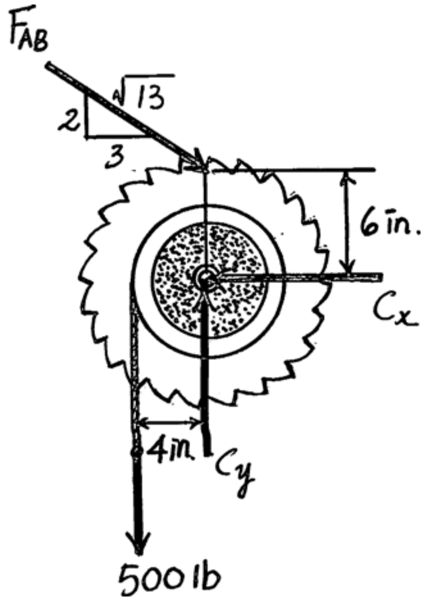


N_A, N_B, N_C force of wood on bar.
10 lb force of hand on bar.



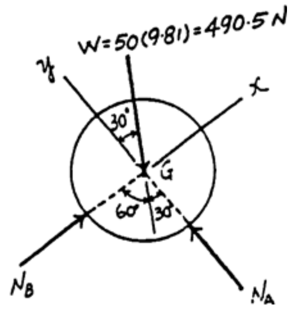
5–10. Draw the free-body diagram of the winch, which consists of a drum of radius 4 in. It is pin-connected at its center *C*, and at its outer rim is a ratchet gear having a mean radius of 6 in. The pawl *AB* serves as a two-force member (short link) and prevents the drum from rotating. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)





C_x, C_y force of pin on drum.
F_{AB} force of pawl on drum gear.
500 lb force of cable on drum.

5–11. Determine the normal reactions at *A* and *B* in Prob. 5–1.



Equations of Equilibrium : By setting up the *x* and *y* axes in the manner shown, one can obtain the direct solution for N_A and N_B.

$\nearrow + \Sigma F_x = 0;$ $N_B - 490.5\sin 30° = 0$ $N_B = 245 \text{ N}$ **Ans**

$\nwarrow + \Sigma F_y = 0;$ $N_A - 490.5\cos 30° = 0$ $N_A = 425 \text{ N}$ **Ans**

***5–12.** Determine the tension in the cord and the horizontal and vertical components of reaction at support *A* of the beam in Prob. 5–4.

$\circlearrowleft + \Sigma M_A = 0; \quad T(2) + T\left(\frac{4}{5}\right)(4) - 80(9.81)(5.5) = 0$

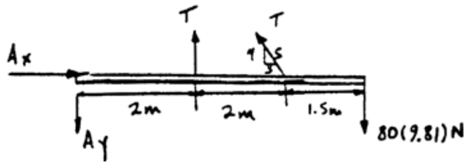
$T = 830.1\text{ N} = 830\text{ N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 830.1\left(\frac{3}{5}\right) = 0$

$A_x = 498\text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -A_y + 830.1 + 830.1\left(\frac{4}{5}\right) - 80(9.81) = 0$

$A_y = 709\text{ N}$ **Ans**



•5–13. Determine the horizontal and vertical components of reaction at *C* and the tension in the cable *AB* for the truss in Prob. 5–5.

Equations of Equilibrium : The tension in the cable can be obtained directly by summing moments about point *C*.

$\circlearrowleft + \Sigma M_C = 0; \quad T_{AB}\cos 30°(2) + T_{AB}\sin 30°(4) - 3(2) - 4(4) = 0$

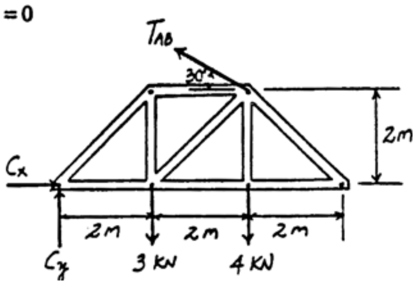
$T_{AB} = 5.89\text{ kN}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 5.89\cos 30° = 0$

$C_x = 5.11\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad C_y + 5.89\sin 30° - 3 - 4 = 0$

$C_y = 4.05\text{ kN}$ **Ans**



5–14. Determine the horizontal and vertical components of reaction at *A* and the tension in cable *BC* on the boom in Prob. 5–6.

Equations of Equilibrium : The force in cable *BC* can be obtained directly by summing moments about point *A*.

$$\circlearrowleft + \Sigma M_A = 0; \quad T_{BC} \sin 7.380°(30) - 650 \cos 30°(18) - 1250 \sin 60°(30) = 0$$

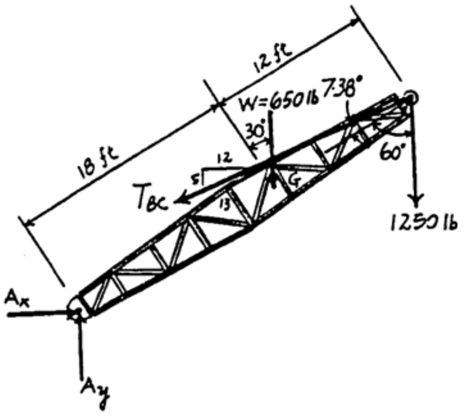
$$T_{BC} = 11056.9 \text{ lb} = 11.1 \text{ kip} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 11056.9\left(\frac{12}{13}\right) = 0$$

$$A_x = 10206.4 \text{ lb} = 10.2 \text{ kip} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 650 - 1250 - 11056.9\left(\frac{5}{13}\right) = 0$$

$$A_y = 6152.7 \text{ lb} = 6.15 \text{ kip} \qquad \textbf{Ans}$$



5–15. Determine the horizontal and vertical components of reaction at *A* and the normal reaction at *B* on the spanner wrench in Prob. 5–7.

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(1) - 20(7) = 0$$

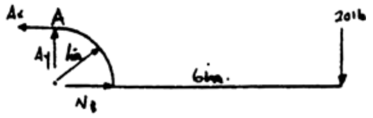
$$N_B = 140 \text{ lb} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 140 = 0$$

$$A_x = 140 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 20 = 0$$

$$A_y = 20 \text{ lb} \qquad \textbf{Ans}$$



***5–16.** Determine the normal reactions at *A* and *B* and the force in link *CD* acting on the member in Prob. 5–8.

Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point *C*.

$$\circlearrowleft + \Sigma M_C = 0; \quad 2.5 \sin 60°(6) - 2.5 \cos 60°(3) - 4 + N_A \cos 45°(3) - N_A \sin 45°(10) = 0$$

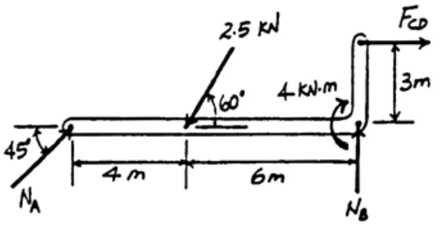
$$N_A = 1.059 \text{ kN} = 1.06 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 1.059 \cos 45° - 2.5 \cos 60° + F_{CD} = 0$$

$$F_{CD} = 0.501 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad N_B + 1.059 \sin 45° - 2.5 \sin 60° = 0$$

$$N_B = 1.42 \text{ kN} \qquad \textbf{Ans}$$



•5–17. Determine the normal reactions at the points of contact at *A*, *B*, and *C* of the bar in Prob. 5–9.

$+\nwarrow \Sigma F_x = 0;$ $N_C \sin 60° - 10 \sin 30° = 0$

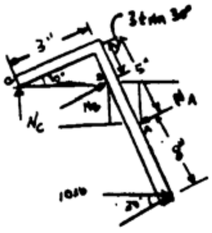
$N_C = 5.77 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_B = 0;$ $10 \cos 30°(13 - 1.732) - N_A(5 - 1.732) - 5.77(3.464) = 0$

$N_A = 23.7 \text{ lb}$ **Ans**

$+\nearrow \Sigma F_y = 0;$ $N_B + 5.77 \cos 60° + 10 \cos 30° - 23.7 = 0$

$N_B = 12.2 \text{ lb}$ **Ans**



5–18. Determine the horizontal and vertical components of reaction at pin *C* and the force in the pawl of the winch in Prob. 5–10.

$\circlearrowleft + \Sigma M_C = 0;$ $F_{AB}\left(\frac{3}{\sqrt{13}}\right)6 - 500(4) = 0$

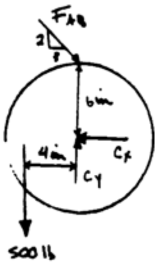
$F_{AB} = 400.6 \text{ lb} = 401 \text{ lb}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;$ $-C_x + 400.6\left(\frac{3}{\sqrt{13}}\right) = 0$

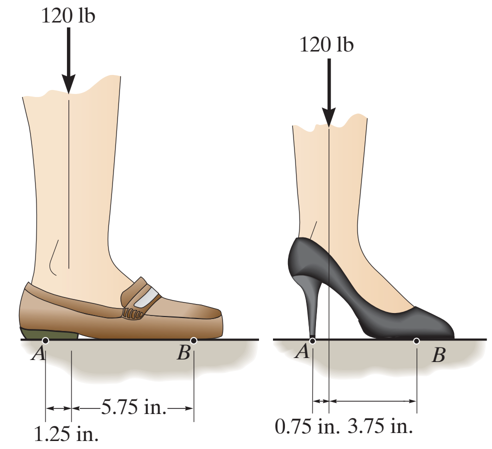
$C_x = 333 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $-500 + C_y - 400.6\left(\frac{2}{\sqrt{13}}\right) = 0$

$C_y = 722 \text{ lb}$ **Ans**



5–19. Compare the force exerted on the toe and heel of a 120-lb woman when she is wearing regular shoes and stiletto heels. Assume all her weight is placed on one foot and the reactions occur at points *A* and *B* as shown.



$\circlearrowleft + \Sigma M_B = 0; \quad 120(5.75) - (N_A)_r(7) = 0$

$(N_A)_r = 98.6 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad (N_B)_r + 98.6 - 120 = 0$

$(N_B)_r = 21.4 \text{ lb}$ **Ans.**

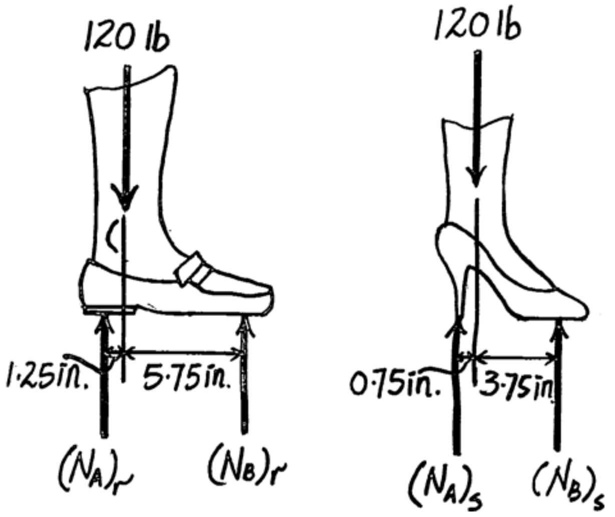
Stiletto heal shoe,

$\circlearrowleft + \Sigma M_B = 0; \quad 120(3.75) - (N_A)_s(4.5) = 0$

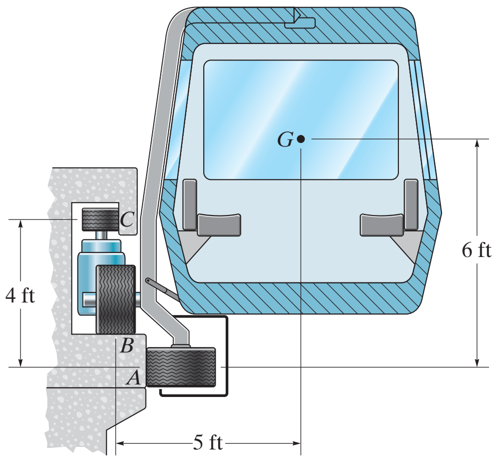
$(N_A)_s = 100 \text{ lb}$ **Ans**

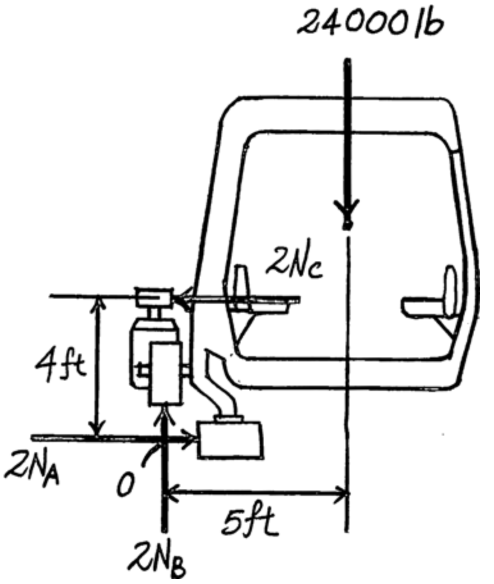
$+\uparrow \Sigma F_y = 0; \quad (N_B)_s + 100 - 120 = 0$

$(N_B)_s = 20 \text{ lb}$ **Ans.**



***5–20.** The train car has a weight of 24 000 lb and a center of gravity at *G*. It is suspended from its front and rear on the track by six tires located at *A*, *B*, and *C*. Determine the normal reactions on these tires if the track is assumed to be a smooth surface and an equal portion of the load is supported at both the front and rear tires.





$\circlearrowleft + \Sigma M_O = 0; \quad (2N_C)(4) - 24\,000(5) = 0$

$N_C = 15\,000 \text{ lb} = 15 \text{ kip}$ **Ans**

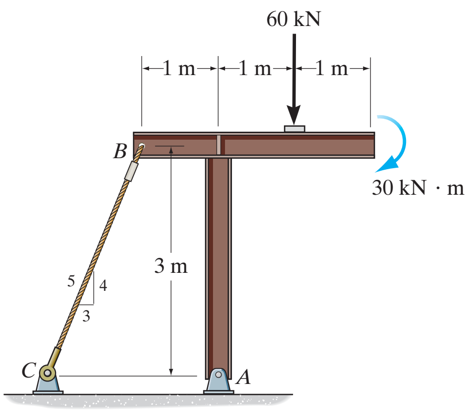
$\xrightarrow{+} \Sigma F_x = 0; \quad 2N_A - 2(15) = 0$

$N_A = 15 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 2N_B - 24\,000 = 0$

$N_B = 12 \text{ kip}$ **Ans**

•5–21. Determine the horizontal and vertical components of reaction at the pin A and the tension developed in cable BC used to support the steel frame.



Equations of Equilibrium: From the free - body diagram of the frame, Fig. *a*, the tension T of cable BC can be obtained by writing the moment equation of equilibrium about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T\left(\frac{3}{5}\right)(3) + T\left(\frac{4}{5}\right)(1) - 60(1) - 30 = 0$$

$$T = 34.62\text{ kN} = 34.62\text{ kN} \qquad \textbf{Ans.}$$

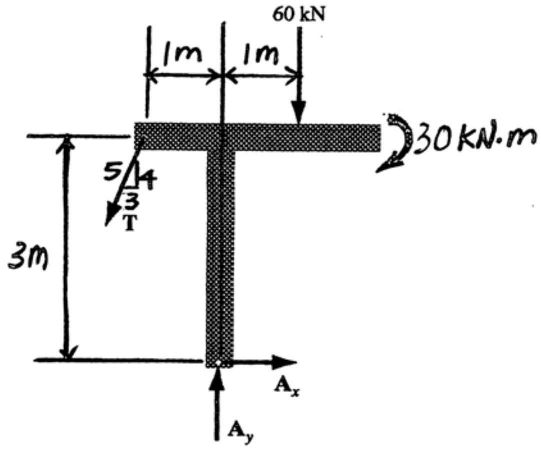
Using this result and writing the force equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - 34.62\left(\frac{3}{5}\right) = 0$$

$$A_x = 20.77\text{ kN} = 20.8\text{ kN} \qquad \textbf{Ans.}$$

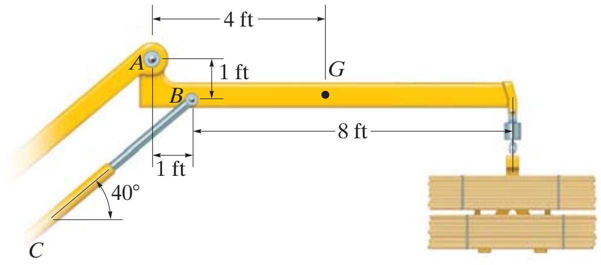
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 60 - 34.62\left(\frac{4}{5}\right) = 0$$

$$A_y = 87.69\text{ kN} = 87.7\text{ kN} \qquad \textbf{Ans.}$$



(a)

5–22. The articulated crane boom has a weight of 125 lb and center of gravity at *G*. If it supports a load of 600 lb, determine the force acting at the pin *A* and the force in the hydraulic cylinder *BC* when the boom is in the position shown.



$\curvearrowleft +\Sigma M_A = 0;\quad F_B \cos 40^\circ (1) + F_B \sin 40^\circ (1) - 125(4) - 600(9) = 0$

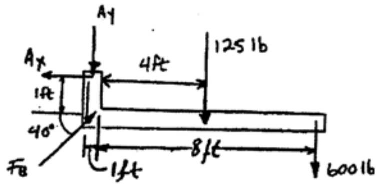
$F_B = 4188 \text{ lb} = 4.19 \text{ kip}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + 4188 \cos 40^\circ = 0$

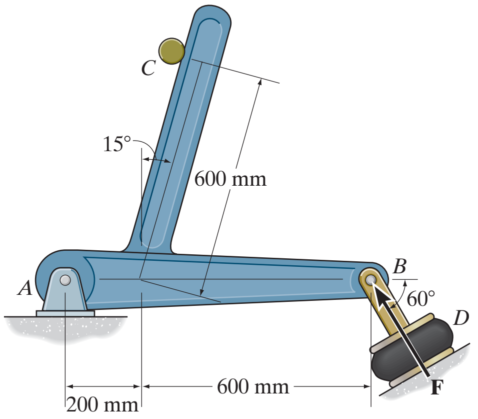
$A_x = 3208 \text{ lb} = 3.21 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad -A_y + 4188 \sin 40^\circ - 600 - 125 = 0$

$A_y = 1.97 \text{ kip}$ **Ans**



5–23. The airstroke actuator at D is used to apply a force of $F = 200$ N on the member at B. Determine the horizontal and vertical components of reaction at the pin A and the force of the smooth shaft at C on the member.



Equations of Equilibrium: From the free - body diagram of member ABC, Fig. a, N_C can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 200 \sin 60° (800) - N_C (600 + 200 \sin 15°) = 0$

$N_C = 212.60 \text{ N} = 213 \text{ N}$ **Ans.**

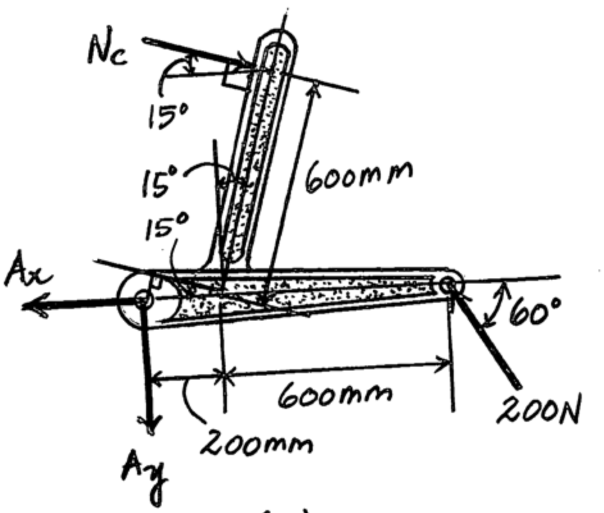
Using this result and writing the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -A_x + 212.60 \cos 15° - 200 \cos 60° = 0$

$A_x = 105 \text{ N}$ **Ans.**

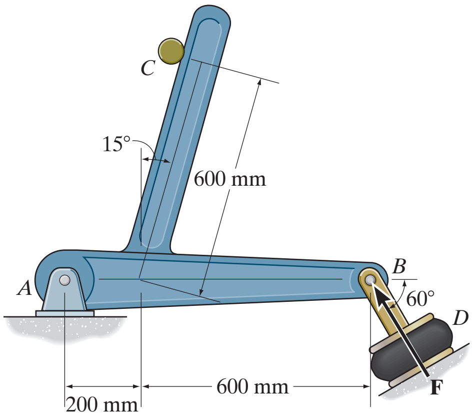
$+\uparrow \Sigma F_y = 0;$ $\quad -A_y - 212.60 \sin 15° + 200 \sin 60° = 0$

$A_y = 118 \text{ N}$ **Ans.**



(a)

***5–24.** The airstroke actuator at *D* is used to apply a force of **F** on the member at *B*. The normal reaction of the smooth shaft at *C* on the member is 300 N. Determine the magnitude of **F** and the horizontal and vertical components of reaction at pin *A*.



Equations of Equilibrium: From the free - body diagram of member *ABC*, Fig. *a*, force *F* can be obtained by writing the moment equation of equilibrium about point *A*.

$\curvearrowleft + \Sigma M_A = 0;$ $\quad F\sin 60°(800) - 300(600 + 200\sin 15°) = 0$

$F = 282.22\text{ N} = 282\text{ N}$ **Ans.**

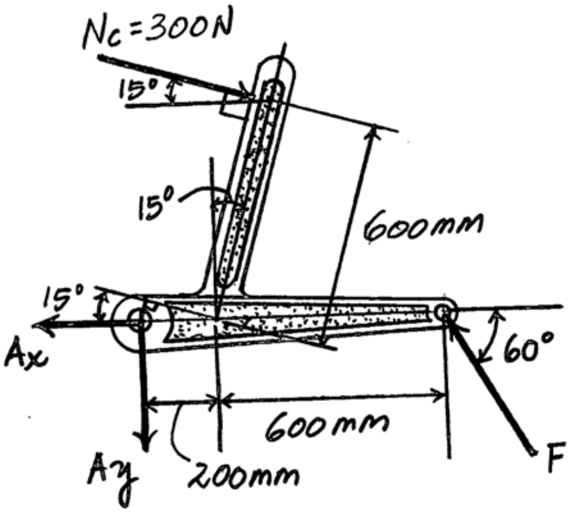
Using this result and writing the force equations of equilibrium along the *x* and *y* axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -A_x + 300\cos 15° - 282.22\cos 60° = 0$

$A_x = 149\text{ N}$ **Ans.**

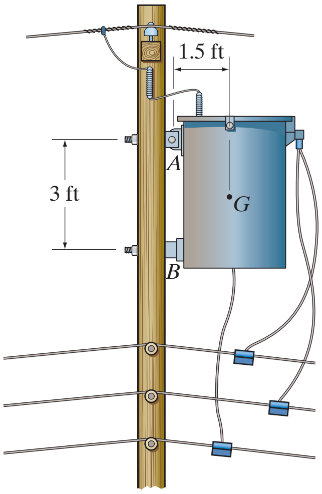
$+\uparrow \Sigma F_y = 0;$ $\quad -A_y - 300\sin 15° + 282.22\sin 60° = 0$

$A_y = 167\text{ N}$ **Ans.**



(a)

•5–25. The 300-lb electrical transformer with center of gravity at G is supported by a pin at A and a smooth pad at B. Determine the horizontal and vertical components of reaction at the pin A and the reaction of the pad B on the transformer.



Equations of Equilibrium: From the free - body diagram of the transformer, Fig. *a*, N_B and A_y can be obtained by writing the moment equation of equilibrium about point A and the force equation of equilibrium along the y axis.

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(3) - 300(1.5) = 0$$

$$N_B = 150 \text{ lb} \qquad \textbf{Ans.}$$

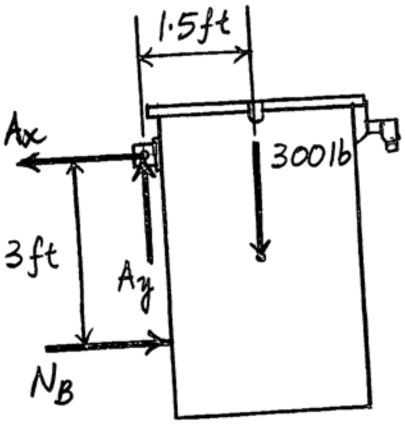
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 300 = 0$$

$$A_y = 300 \text{ lb} \qquad \textbf{Ans.}$$

Using the result $N_B = 150$ lb and writing the force equation of equilibrium along the x axis,

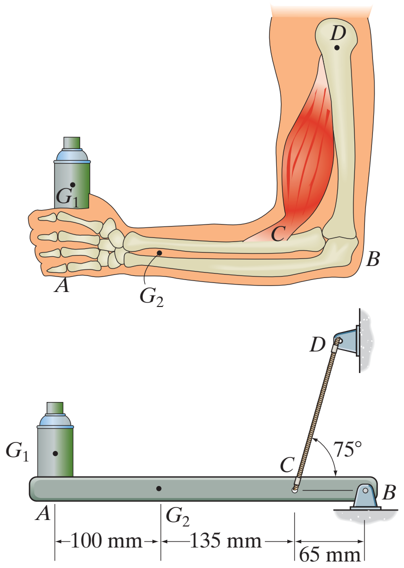
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 150 - A_x = 0$$

$$A_x = 150 \text{ lb} \qquad \textbf{Ans.}$$



(a)

5–26. A skeletal diagram of a hand holding a load is shown in the upper figure. If the load and the forearm have masses of 2 kg and 1.2 kg, respectively, and their centers of mass are located at G_1 and G_2, determine the force developed in the biceps *CD* and the horizontal and vertical components of reaction at the elbow joint *B*. The forearm supporting system can be modeled as the structural system shown in the lower figure.



Equations of Equilibrium: From the free - body diagram of the structural system, Fig. *a*, F_{CD} can be obtained by writing the moment equation of equilibrium about point *B*.

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 2(9.81)(100 + 135 + 65) + 1.2(9.81)(135 + 65) - F_{CD} \sin 75°(65) = 0$

$F_{CD} = 131.25 \text{ N} = 131 \text{ N}$ **Ans.**

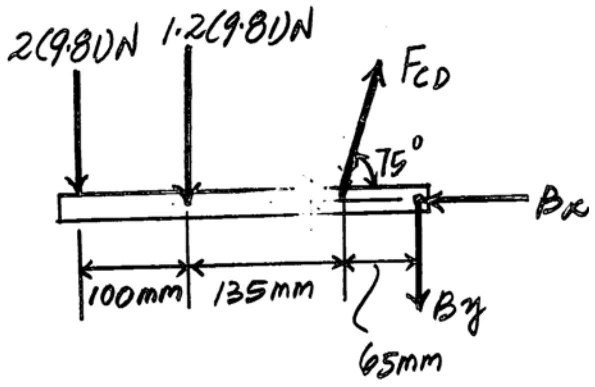
Using the above result and writing the force equations of equilibrium along the *x* and *y* axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 131.25 \cos 75° - B_x = 0$

$B_x = 33.97 \text{ N} = 34.0 \text{ N}$ **Ans.**

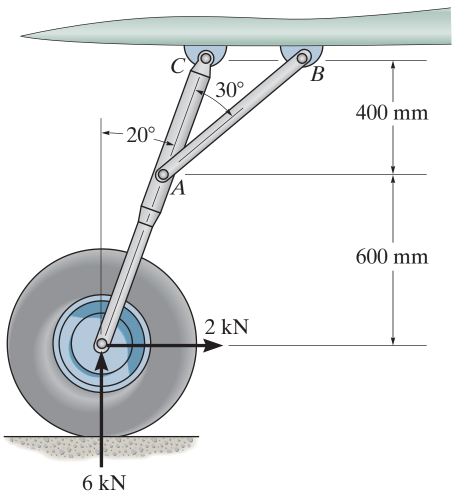
$+\uparrow \Sigma F_y = 0;$ $\quad 131.25 \sin 75° - 2(9.81) - 1.2(9.81) - B_y = 0$

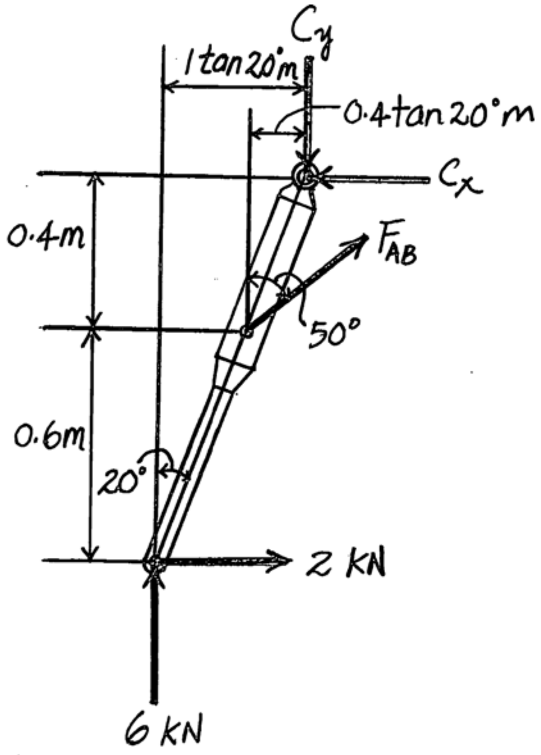
$B_y = 95.38 \text{ N} = 95.4 \text{ N}$ **Ans.**



(a)

5–27. As an airplane's brakes are applied, the nose wheel exerts two forces on the end of the landing gear as shown. Determine the horizontal and vertical components of reaction at the pin C and the force in strut AB.





Equations of Equilibrium : The force in strut AB can be obtained directly by summing moments about point C.

$$\circlearrowleft + \Sigma M_C = 0; \quad 2(1) - 6(1\tan 20°) + F_{AB}\sin 50°(0.4) - F_{AB}\cos 50°(0.4\tan 20°) = 0$$

$$F_{AB} = 0.8637\text{ kN} = 0.864\text{ kN} \qquad \textbf{Ans}$$

Using the result $F_{AB} = 0.8637$ kN and sum forces along x and y axes, we have,

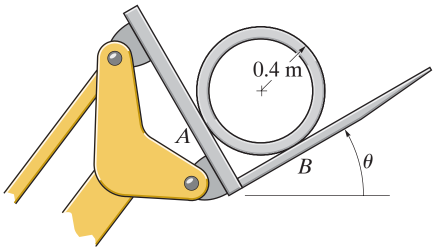
$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.8637\sin 50° + 2 - C_x = 0$$

$$C_x = 2.66\text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 6 + 0.8637\cos 50° - C_y = 0$$

$$C_y = 6.56\text{ kN} \qquad \textbf{Ans}$$

***5–28.** The 1.4-Mg drainpipe is held in the tines of the fork lift. Determine the normal forces at *A* and *B* as functions of the blade angle θ and plot the results of force (vertical axis) versus θ (horizontal axis) for $0 \leq \theta \leq 90°$.

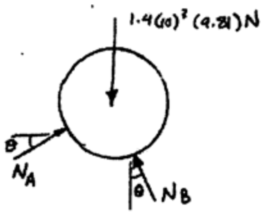


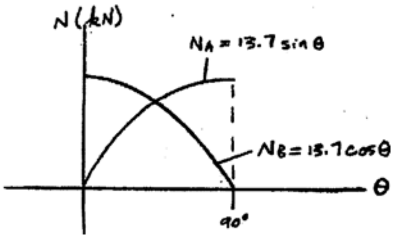
$+\nearrow\Sigma F_x = 0;\quad N_A - 1.4(10)^3(9.81)\sin\theta = 0$

$N_A = 13.7\sin\theta \text{ kN}$ **Ans**

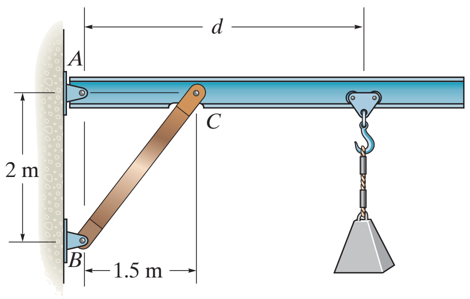
$+\nwarrow\Sigma F_y = 0;\quad N_B - 1.4(10)^3(9.81)\cos\theta = 0$

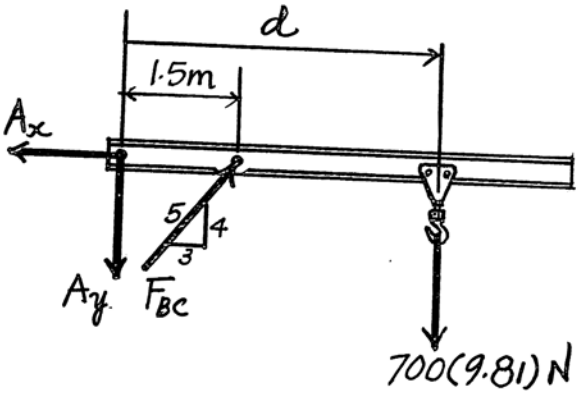
$N_B = 13.7\cos\theta \text{ kN}$ **Ans**





•5–29. The mass of 700 kg is suspended from a trolley which moves along the crane rail from $d = 1.7$ m to $d = 3.5$ m. Determine the force along the pin-connected knee strut BC (short link) and the magnitude of force at pin A as a function of position d. Plot these results of F_{BC} and F_A (vertical axis) versus d (horizontal axis).





$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC}\left(\frac{4}{5}\right)(1.5) - 700(9.81)(d) = 0$

$F_{BC} = 5722.5d$ **Ans**

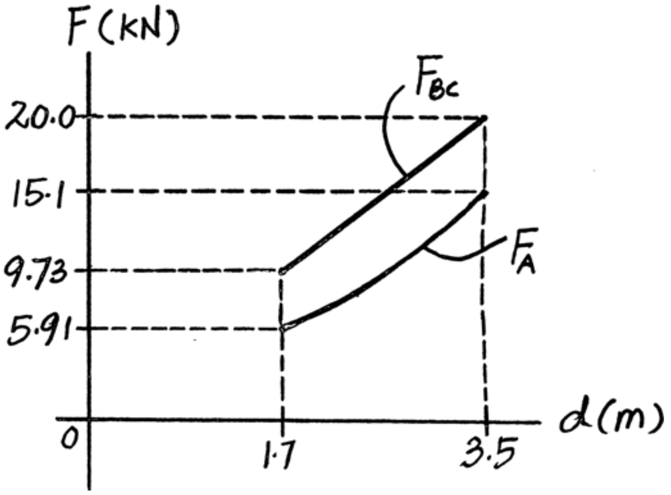
$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + (5722.5d)\left(\frac{3}{5}\right) = 0$

$A_x = 3433.5d$

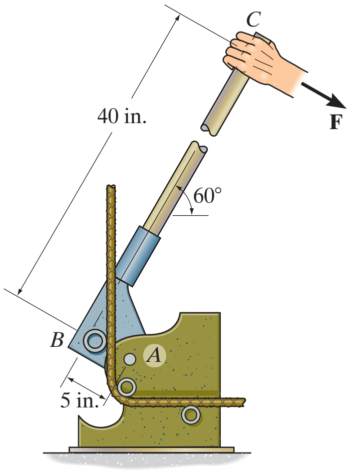
$+\uparrow \Sigma F_y = 0;\quad -A_y + (5722.5d)\left(\frac{4}{5}\right) - 700(9.81) = 0$

$A_y = 4578d - 6867$

$F_A = \sqrt{(3433.5d)^2 + (4578d - 6867)^2}$ **Ans**



5–30. If the force of $F = 100$ lb is applied to the handle of the bar bender, determine the horizontal and vertical components of reaction at pin A and the reaction of the roller B on the smooth bar.



Equations of Equilibrium: From the free - body diagram of the handle of the bar bender, Fig. *a*, A_y and N_B can be obtained by writing the force equation of equilibrium along the *y* axis and the moment equation of equilibrium about point *A*, respectively.

$+\uparrow \Sigma F_y = 0; \qquad A_y - 100\sin 30° = 0$

$A_y = 50 \text{ lb}$ **Ans.**

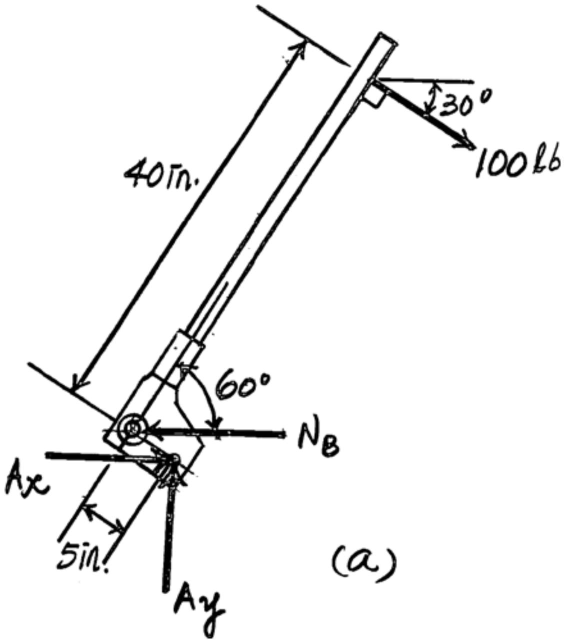
$\circlearrowleft + \Sigma M_A = 0; \qquad N_B \cos 60°(5) - 100(40) = 0$

$N_B = 1600 \text{ N} = 1.60 \text{ kip}$ **Ans.**

Using the result $N_B = 1600$ N and writing the force equation of equilibrium along the x axis,

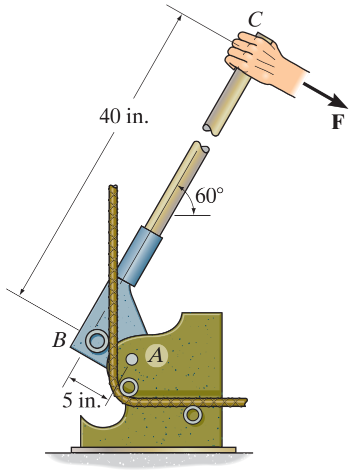
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 1600 + 100\cos 30° = 0$

$A_x = 1513.40 \text{ N} = 1.51 \text{ kip}$ **Ans.**



(a)

5–31. If the force of the smooth roller at *B* on the bar bender is required to be 1.5 kip, determine the horizontal and vertical components of reaction at pin *A* and the required magnitude of force **F** applied to the handle.



Equations of Equilibrium: From the free - body diagram of the handle of the bar bender, Fig. *a*, force *F* can be obtained by writing the moment equation of equilibrium about point *A*.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 1500\cos 60°(5) - F(40) = 0$

$F = 93.75 \text{ lb}$ **Ans.**

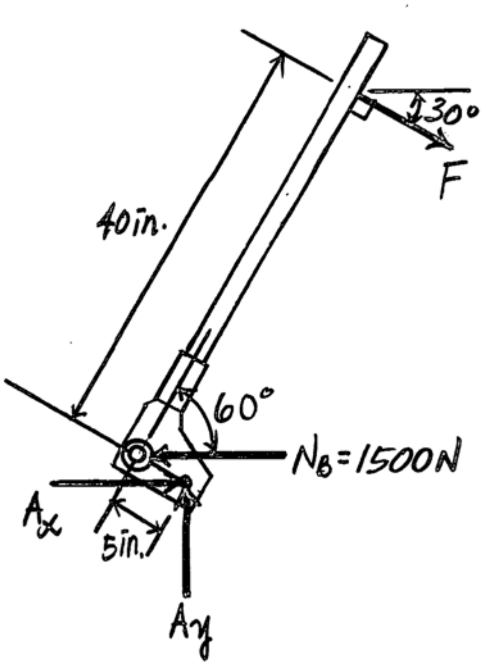
Using the above result and writing the force equation of equilibrium along the *x* and *y* axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x + 93.75\cos 30° - 1500 = 0$

$A_x = 1418.81 \text{ lb} = 1.42 \text{ kip}$ **Ans.**

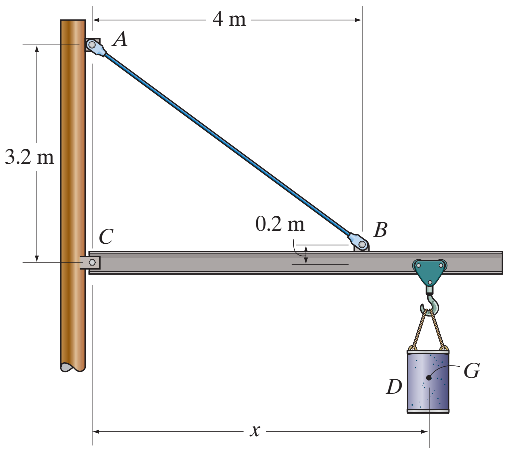
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 93.75\sin 30° = 0$

$A_y = 46.875 \text{ lb} = 46.9 \text{ lb}$ **Ans.**



(a)

***5–32.** The jib crane is supported by a pin at C and rod AB. If the load has a mass of 2 Mg with its center of mass located at G, determine the horizontal and vertical components of reaction at the pin C and the force developed in rod AB on the crane when $x = 5$ m.



Equations of Equilibrium: Realizing that rod AB is a two - force member, it will exert a force $\mathbf{F}_{AB}$ directed along its axis on the beam, as shown on the free - body diagram in Fig. *a*. From the free - body diagram, F_{AB} can be obtained by writing the moment equation of equilibrium about point C.

$$\circlearrowleft +\Sigma M_C = 0; \qquad F_{AB}\left(\frac{3}{5}\right)(4) + F_{AB}\left(\frac{4}{5}\right)(0.2) - 2000(9.81)(5) = 0$$

$$F_{AB} = 38\,320.31 \text{ N} = 38.3 \text{ kN} \qquad \textbf{Ans.}$$

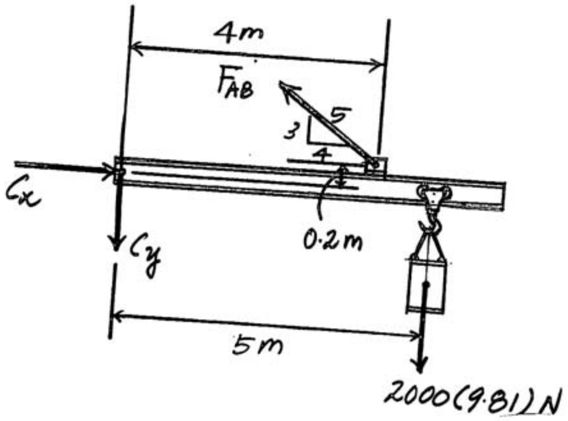
Using the above result and writing the force equations of equilibrium along the x and y axes.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad C_x - 38\,320.31\left(\frac{4}{5}\right) = 0$$

$$C_x = 30\,656.25 \text{ N} = 30.7 \text{ kN} \qquad \textbf{Ans.}$$

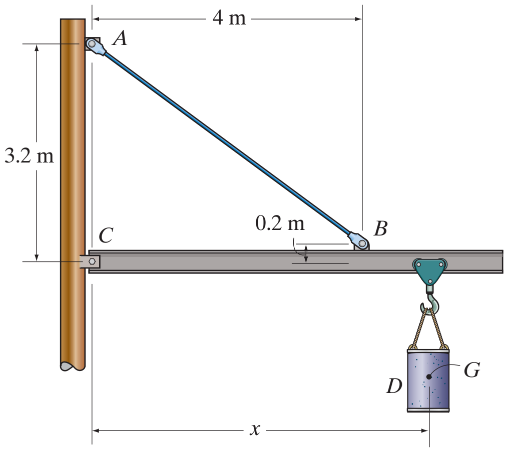
$$+\uparrow \Sigma F_y = 0; \qquad 38\,320.31\left(\frac{3}{5}\right) - 2000(9.81) - C_y = 0$$

$$C_y = 3372.19 \text{ N} = 3.37 \text{ kN} \qquad \textbf{Ans.}$$



(a)

•5–33. The jib crane is supported by a pin at C and rod AB. The rod can withstand a maximum tension of 40 kN. If the load has a mass of 2 Mg, with its center of mass located at G, determine its maximum allowable distance x and the corresponding horizontal and vertical components of reaction at C.



Equations of Equilibrium: Realizing that rod AB is a two - force member, it will exert a force $\mathbf{F}_{AB}$ directed along its axis on the beam, as shown on the free - body diagram in Fig. *a*. From the free - body diagram, the distance x can be obtained by writing the moment equation of equilibrium about point C.

$$\circlearrowleft + \Sigma M_C = 0; \qquad 40\,000\left(\frac{3}{5}\right)(4) + 40\,000\left(\frac{4}{5}\right)(0.2) - 2000(9.81)(x) = 0$$

$$x = 5.22 \text{ m} \qquad \textbf{Ans.}$$

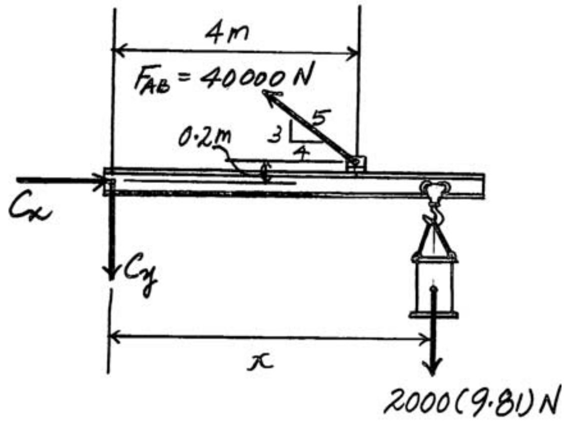
Writing the force equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 40\,000\left(\frac{4}{5}\right) = 0$$

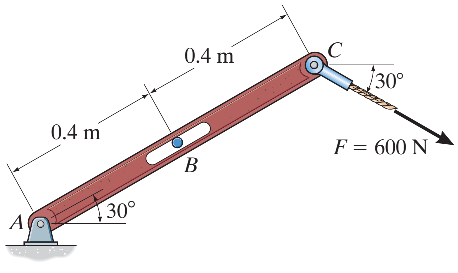
$$C_x = 32\,000 \text{ N} = 32 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 40\,000\left(\frac{3}{5}\right) - 2000(9.81) - C_y = 0$$

$$C_y = 4380 \text{ N} = 4.38 \text{ kN} \qquad \textbf{Ans.}$$



5–34. Determine the horizontal and vertical components of reaction at the pin *A* and the normal force at the smooth peg *B* on the member.



Equations of Equilibrium: From the free - body diagram of the member, Fig. *a*, N_B can be obtained by writing the moment equation of equilibrium about point *A*.

$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(0.4) - 600\cos 30°(0.8) = 0$

$N_B = 1039.23\text{ N} = 1.04\text{ kN}$ **Ans.**

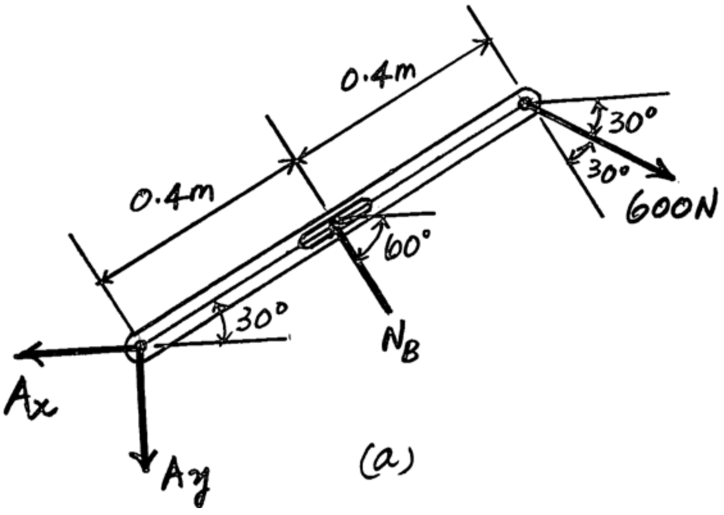
Using this result and writing the force equations of equilibrium along the *x* and *y* axes,

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 600\cos 30° - 1039.23\cos 60° - A_x = 0$

$A_x = 0$ **Ans.**

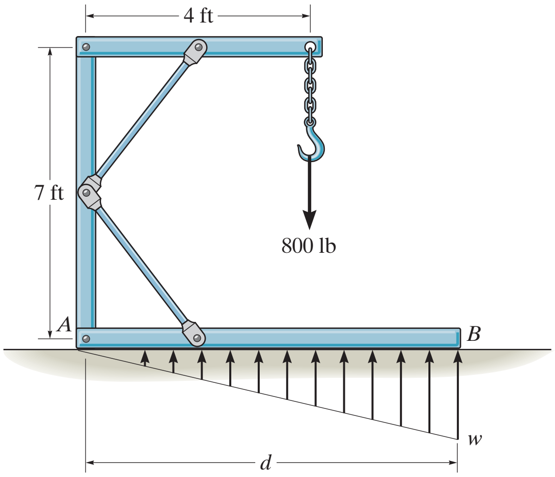
$+\uparrow \Sigma F_y = 0; \qquad -A_y + 1039.23\sin 60° - 600\sin 30° = 0$

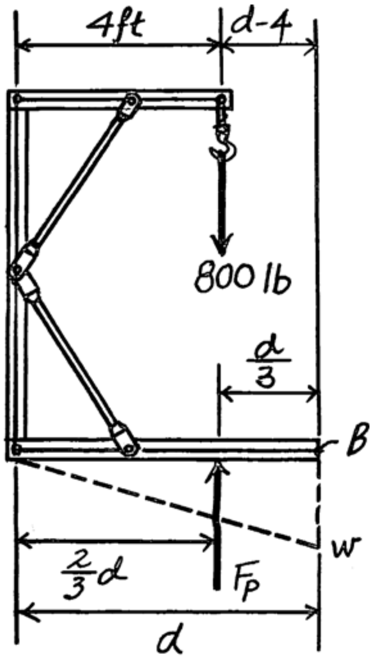
$A_y = 600\text{ N}$ **Ans.**



(a)

5–35. The framework is supported by the member *AB* which rests on the smooth floor. When loaded, the pressure distribution on *AB* is linear as shown. Determine the length *d* of member *AB* and the intensity *w* for this case.





$+\uparrow \Sigma F_y = 0; \quad F_P - 800 = 0$

$F_P = 800 \text{ lb}$

When tipping;

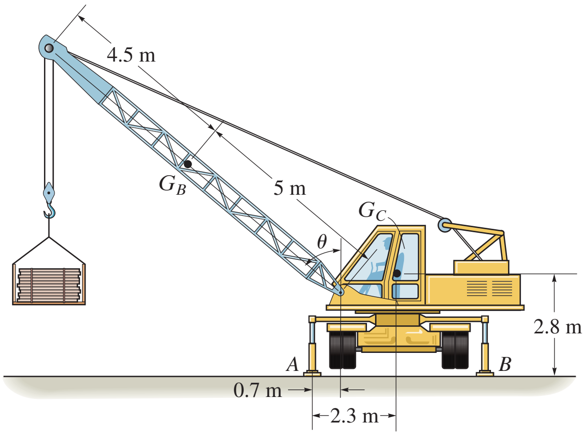
$\circlearrowleft + \Sigma M_B = 0; \quad -800\left(\frac{d}{3}\right) + 800(d-4) = 0$

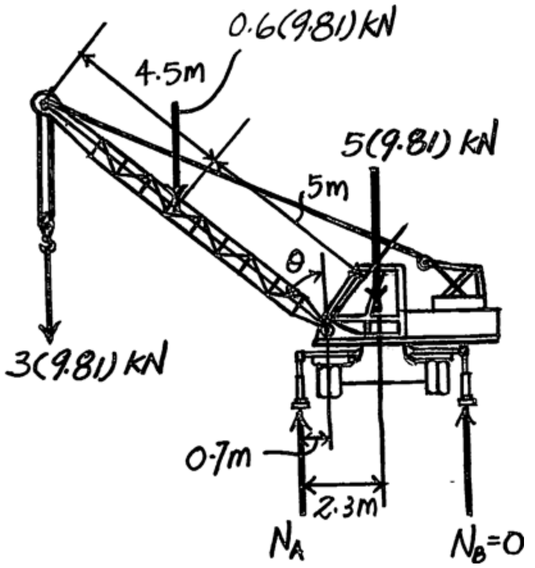
$d = 6 \text{ ft}$ **Ans**

$F_P = \frac{1}{2} wd = \frac{1}{2}(w)(6) = 800$

$w = 267 \text{ lb/ft}$ **Ans**

***5–36.** Outriggers *A* and *B* are used to stabilize the crane from overturning when lifting large loads. If the load to be lifted is 3 Mg, determine the *maximum* boom angle θ so that the crane does not overturn. The crane has a mass of 5 Mg and center of mass at G_C, whereas the boom has a mass of 0.6 Mg and center of mass at G_B.



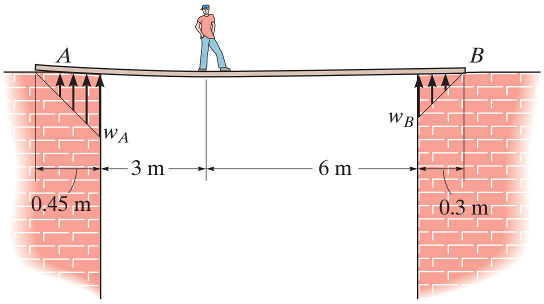


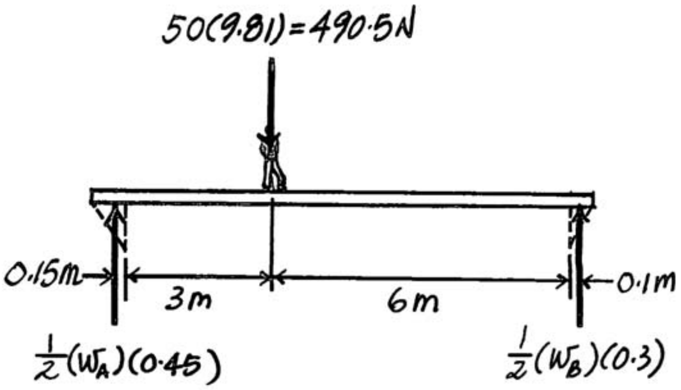
$$\circlearrowleft + \Sigma M_A = 0; \quad -5(9.81)(2.3) + 3(9.81)(9.5\sin\theta - 0.7)$$

$$+ 0.6(9.81)(5\sin\theta - 0.7) = 0$$

$$\theta = 26.4^\circ \qquad \text{Ans}$$

•5–37. The wooden plank resting between the buildings deflects slightly when it supports the 50-kg boy. This deflection causes a triangular distribution of load at its ends, having maximum intensities of w_A and w_B. Determine w_A and w_B, each measured in N/m, when the boy is standing 3 m from one end as shown. Neglect the mass of the plank.





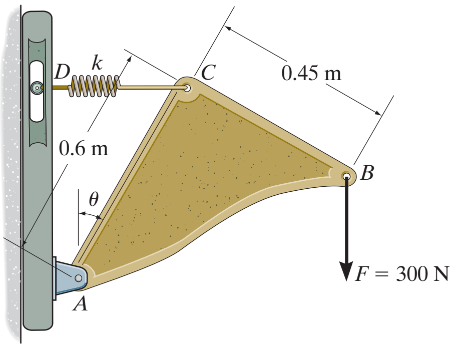
$\circlearrowleft + \Sigma M_A = 0; \quad -490.5\,(3.15) + \frac{1}{2}\,w_B\,(0.3)\,(9.25) = 0$

$w_B = 1113.6\ \text{N/m} = 1.11\ \text{kN/m}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}\,w_A\,(0.45) + \frac{1}{2}\,(1113.6)\,(0.3) - 490.5 = 0$

$w_A = 1437.6\ \text{N/m} = 1.44\ \text{kN/m}$ **Ans**

5–38. Spring *CD* remains in the horizontal position at all times due to the roller at *D*. If the spring is unstretched when $\theta = 0°$ and the bracket achieves its equilibrium position when $\theta = 30°$, determine the stiffness *k* of the spring and the horizontal and vertical components of reaction at pin *A*.



Spring Force Formula: At the equilibrium position, the spring elongates $x = 0.6 \sin 30°$ m. Using the spring force formula, the force in spring *CD* is found to be $F_{sp} = kx = 0.3k$.

Equations of Equilibrium: From the free - body diagram of the bracket, Fig. *a*, the stiffness *k* of spring *CD* and A_y can be obtained by writing the moment equation of equilibrium about point *A* and the force equation of equilibrium along the *x* axis, respectively.

$$\circlearrowleft + \Sigma M_A = 0; \qquad 0.3k \cos 30°(0.6) - 300 \cos 30°(0.45) - 300 \sin 30°(0.6) = 0$$

$$k = 1327.35 \text{ N/m} = 1.33 \text{ kN/m} \qquad \textbf{Ans.}$$

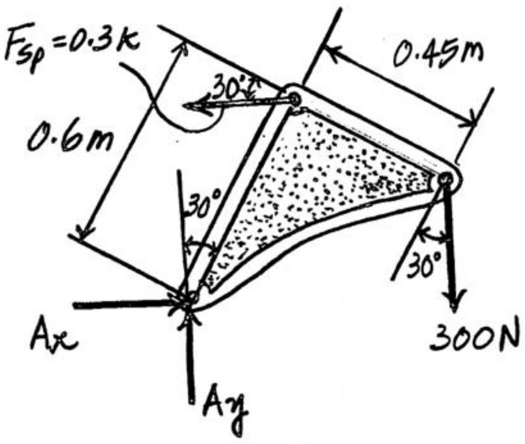
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 300 = 0$$

$$A_y = 300 \text{ N} \qquad \textbf{Ans.}$$

Using the result $k = 1327.35$ N / m and writing the force equation of equilibrium along the *x* axis,

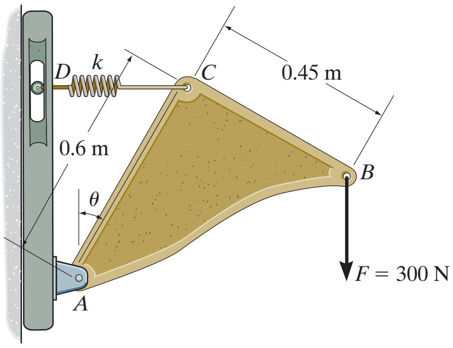
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 0.3(1327.35) = 0$$

$$A_x = 398.21 \text{ N} = 398 \text{ N} \qquad \textbf{Ans.}$$



(a)

5–39. Spring *CD* remains in the horizontal position at all times due to the roller at *D*. If the spring is unstretched when $\theta = 0°$ and the stiffness is $k = 1.5$ kN/m, determine the smallest angle θ for equilibrium and the horizontal and vertical components of reaction at pin *A*.



Spring Force Formula: At the equilibrium position, the spring elongates $x = 0.6\sin\theta$. Using the spring force formula, the force in spring *CD* is found to be $F_{sp} = kx = 1500(0.6\sin\theta) = 900\sin\theta$.

Equations of Equilibrium: From the free - body diagram of the bracket, Fig. *a*, the equilibrium position θ and A_y can be obtained by writing the moment equation of equilibrium about point *A* and the force equation of equilibrium along the *y* axis, respectively.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 900\sin\theta\cos\theta(0.6) - 300\sin\theta(0.6) - 300\cos\theta(0.45) = 0$

$540\sin\theta\cos\theta - 180\sin\theta - 135\cos\theta = 0$

Solving by trial and error yields

$\theta = 23.083° = 23.1°$ **Ans.**

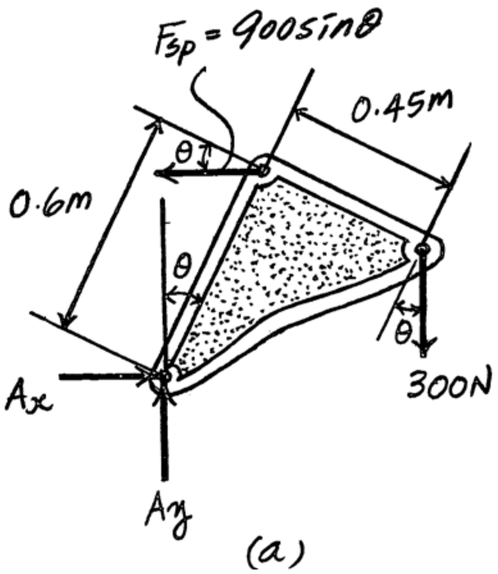
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 300 = 0$

$A_y = 300\,\text{N}$ **Ans.**

Using the result $\theta = 23.083°$ and writing the force equation of equilibrium along the *x* axis,

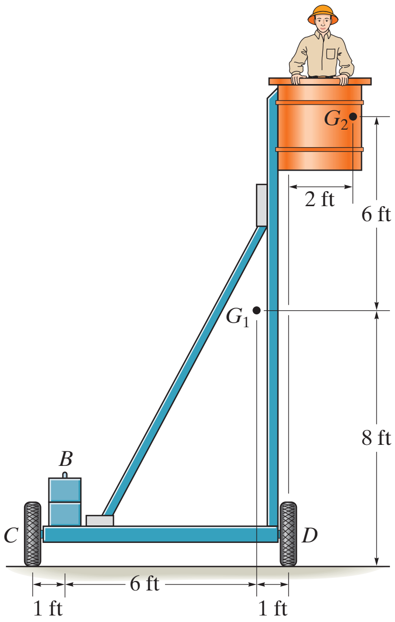
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 900\sin 23.083° = 0$

$A_x = 352.86\,\text{N} = 353\,\text{N}$ **Ans.**



(a)

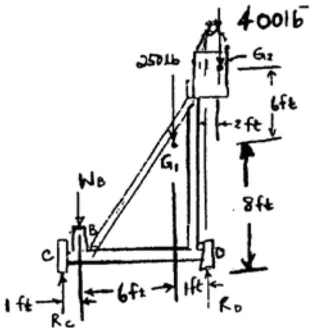
***5–40.** The platform assembly has a weight of 250 lb and center of gravity at G_1. If it is intended to support a maximum load of 400 lb placed at point G_2, determine the smallest counterweight W that should be placed at B in order to prevent the platform from tipping over.



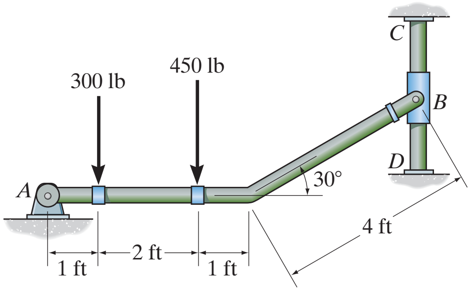
When tipping occurs, $R_C = 0$

$\circlearrowleft +\Sigma M_D = 0;\qquad -400(2) + 250(1) + W_B(7) = 0$

$W_B = 78.6 \text{ lb}$ **Ans**



•5–41. Determine the horizontal and vertical components of reaction at the pin *A* and the reaction of the smooth collar *B* on the rod.



Equations of Equilibrium: From the free - body diagram, A_y and N_B can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about point A.

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 300 - 450 = 0$

$A_y = 750 \text{ lb}$ **Ans.**

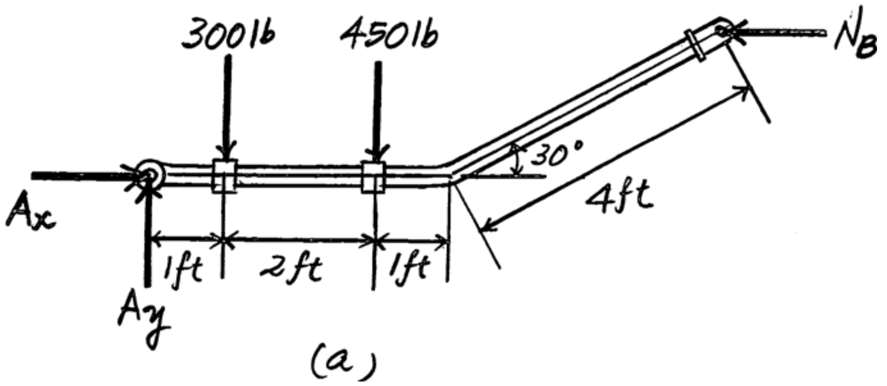
$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B(4\sin 30°) - 300(1) - 450(3) = 0$

$N_B = 825 \text{ lb}$ **Ans.**

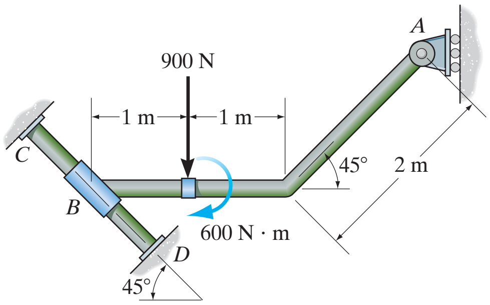
Using the result $N_B = 825$ lb and writing the force equation of equilibrium along the x axis,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 825 = 0$

$A_x = 825 \text{ lb}$ **Ans.**



5–42. Determine the support reactions of roller *A* and the smooth collar *B* on the rod. The collar is fixed to the rod *AB*, but is allowed to slide along rod *CD*.



Equations of Equilibrium: From the free - body diagram of the rod, Fig. *a*, N_B can be obtained by writing the force equation of equilibrium along the *y* axis.

$+\uparrow \Sigma F_y = 0;$ $N_B \sin 45° - 900 = 0$

$N_B = 1272.79\text{ N} = 1.27\text{ kN}$ **Ans.**

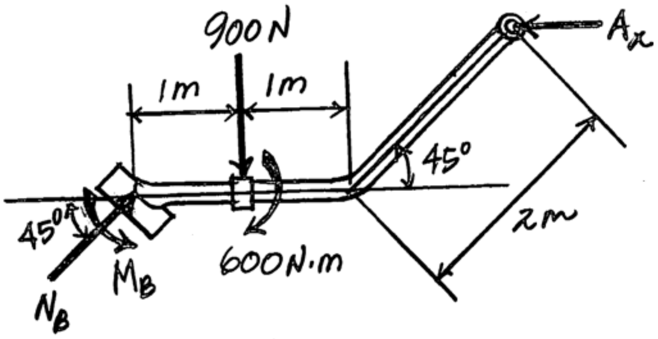
Using the above result and writing the force equation of equilibrium and the moment equation of equilibrium about point *B*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $1272.79\cos 45° - A_x = 0$

$A_x = 900\text{ N}$ **Ans.**

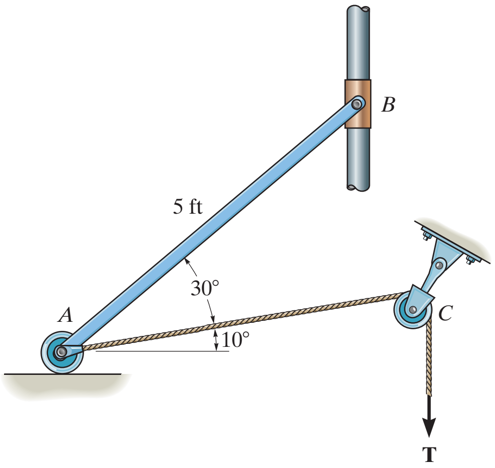
$\circlearrowleft + \Sigma M_B = 0;$ $-900(1) + 900(2)\sin 45° - 600 + M_B = 0$

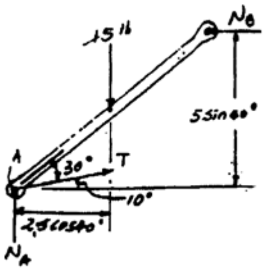
$M_B = 227\text{ N}\cdot\text{m}$ **Ans.**



(a)

5–43. The uniform rod *AB* has a weight of 15 lb. Determine the force in the cable when the rod is in the position shown.

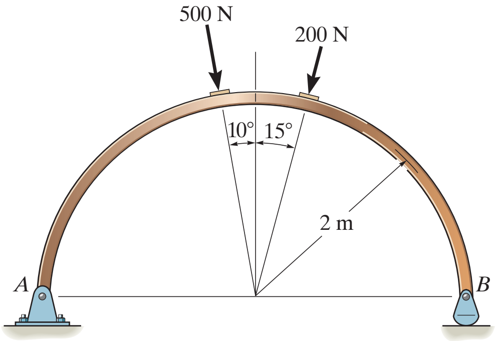


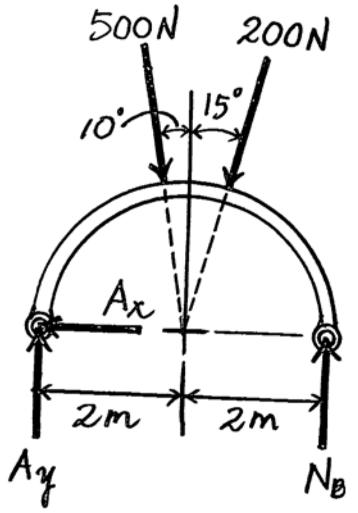


$\circlearrowleft + \Sigma M_A = 0;\quad N_B(5 \sin 40°) - 15(2.5 \cos 40°) = 0 \quad N_B = 8.938 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad T \cos 10° - 8.938 = 0 \quad T = 9.08 \text{ lb}$ **Ans**

***5–44.** Determine the horizontal and vertical components of force at the pin *A* and the reaction at the rocker *B* of the curved beam.





$\circlearrowleft + \Sigma M_A = 0;\quad N_B(4) - 200 \cos 15° (2) - 500 \cos 10° (2) = 0$

$N_B = 342.79 = 343 \text{ N}$ **Ans**

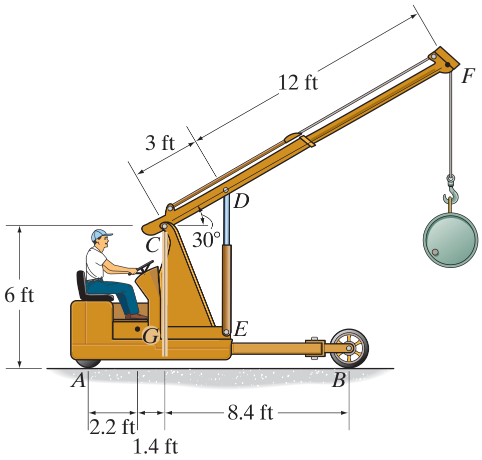
$+\uparrow \Sigma F_y = 0;\quad A_y - 500 \cos 10° - 200 \cos 15° + 342.79 = 0$

$A_y = 342.8 = 343 \text{ N}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -A_x + 500 \sin 10° - 200 \sin 15° = 0$

$A_x = 35.1 \text{ N}$ **Ans**

•5–45. The floor crane and the driver have a total weight of 2500 lb with a center of gravity at *G*. If the crane is required to lift the 500-lb drum, determine the normal reaction on *both* the wheels at *A* and *both* the wheels at *B* when the boom is in the position shown.



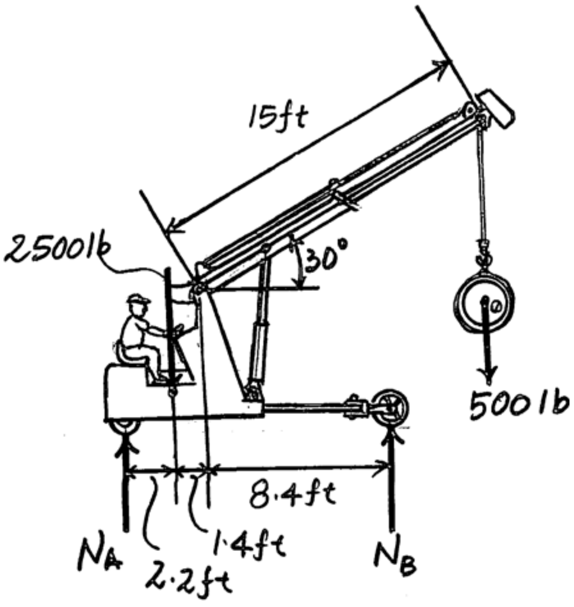
Equations of Equilibrium: From the free - body diagram of the floor crane, Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;$ $2500(1.4 + 8.4) - 500(15\cos 30° - 8.4) - N_A(2.2 + 1.4 + 8.4) = 0$

$N_A = 1850.40 \text{ lb} = 1.85 \text{ kip}$ **Ans.**

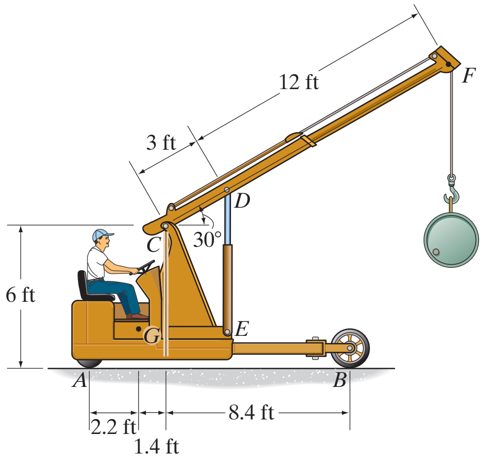
$+\uparrow \Sigma F_y = 0;$ $1850.40 - 2500 - 500 + N_B = 0$

$N_B = 1149.60 \text{ lb} = 1.15 \text{ kip}$ **Ans.**



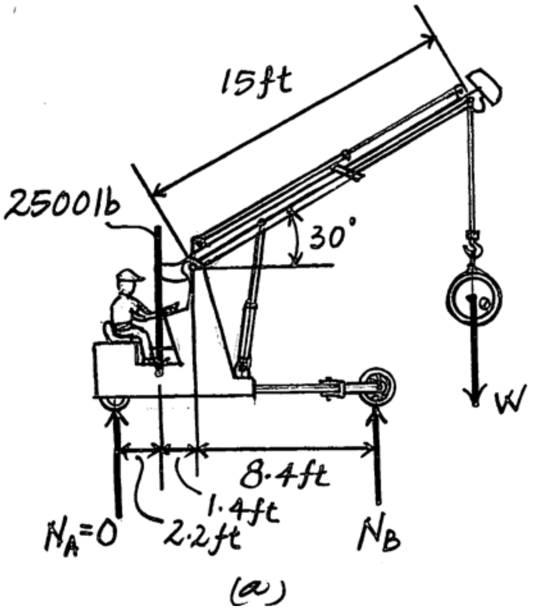
(a)

5–46. The floor crane and the driver have a total weight of 2500 lb with a center of gravity at *G*. Determine the largest weight of the drum that can be lifted without causing the crane to overturn when its boom is in the position shown.

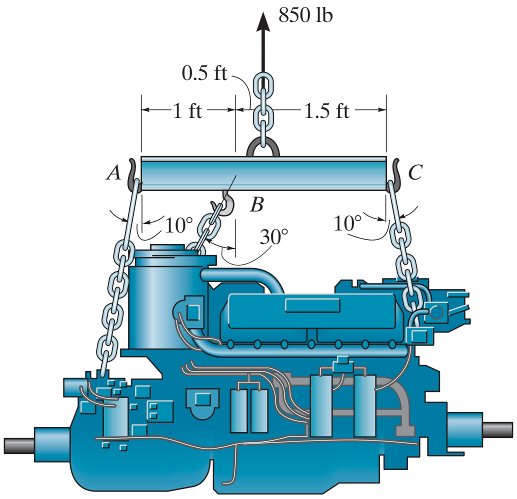


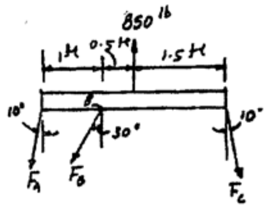
Equations of Equilibrium: Since the floor crane tends to overturn about point *B*, the wheel at *A* will leave the ground and $N_A = 0$. From the free - body diagram of the floor crane, Fig. *a*, *W* can be obtained by writing the moment equation of equilibrium about point *B*.

$\circlearrowleft + \Sigma M_B = 0;$ $$2500(1.4 + 8.4) - W(15\cos 30° - 8.4) = 0$$
$$W = 5337.25 \text{ lb} = 5.34 \text{ kip}$$ **Ans.**



5–47. The motor has a weight of 850 lb. Determine the force that each of the chains exerts on the supporting hooks at *A*, *B*, and *C*. Neglect the size of the hooks and the thickness of the beam.





$\curvearrowleft + \Sigma M_B = 0;\quad F_A \cos 10°(1) + 850(0.5) - F_C \cos 10°(2) = 0 \qquad (1)$

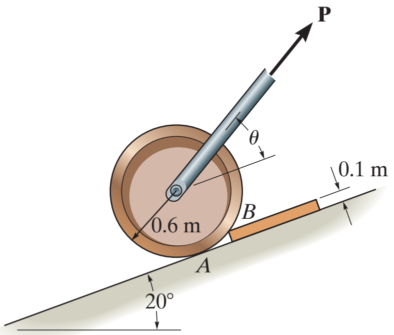
$\xrightarrow{+} \Sigma F_x = 0;\quad F_C \sin 10° - F_B \sin 30° - F_A \sin 10° = 0 \qquad (2)$

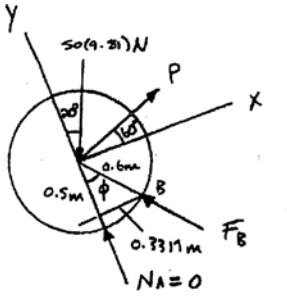
$+\uparrow \Sigma F_y = 0;\quad 850 - F_A \cos 10° - F_B \cos 30° - F_C \cos 10° = 0 \qquad (3)$

Solving Eqs.(1), (2) and (3) yields:

$F_A = 432 \text{ lb} \qquad F_B = 0 \qquad F_C = 432 \text{ lb}$ **Ans**

*__5–48.__ Determine the force *P* needed to pull the 50-kg roller over the smooth step. Take $\theta = 60°$.





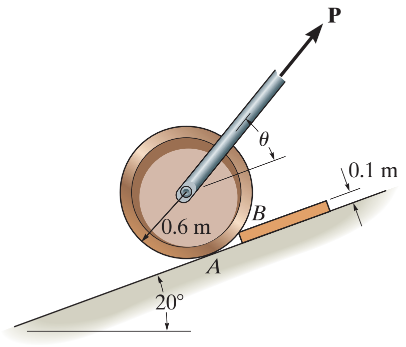
$\phi = \cos^{-1}\left(\frac{0.5}{0.6}\right) = 33.56°$

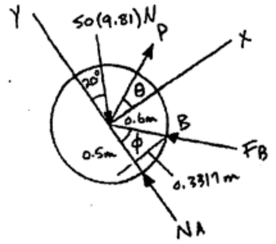
$\curvearrowleft + \Sigma M_B = 0;\quad 50(9.81) \sin 20° (0.5) + 50(9.81) \cos 20° (0.3317) - P \cos 60° (0.5)$

$- P \sin 60° (0.3317) = 0$

$P = 441 \text{ N}.$ **Ans**

•5–49. Determine the magnitude and direction θ of the minimum force P needed to pull the 50-kg roller over the smooth step.





For P_{min}, $N_A \to 0$, $\phi = \cos^{-1}\left(\frac{0.5}{0.6}\right) = 33.56°$

$$\circlearrowleft + \Sigma M_B = 0; \quad 50(9.81)\sin 20°(0.5) + 50(9.81)\cos 20°(0.3317) - P\cos\theta(0.5)$$

$$- P\sin\theta(0.3317) = 0$$

$$236.75 - P\cos\theta(0.5) - P\sin\theta(0.3317) = 0$$

$$P = \frac{236.75}{(0.5\cos\theta + 0.3317\sin\theta)}$$

For P_{min};

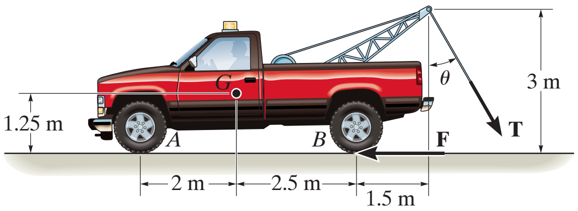
$$\frac{dP}{d\theta} = \frac{-236.75(-0.5\sin\theta + 0.3317\cos\theta)}{(0.5\cos\theta + 0.3317\sin\theta)^2} = 0$$

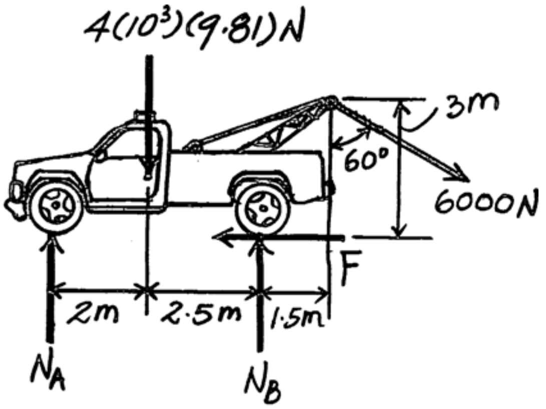
$$\tan\theta = \frac{0.3317}{0.5}$$

$\theta = 33.6°$ **Ans**

$P_{min} = 395\text{ N}$ **Ans**

5–50. The winch cable on a tow truck is subjected to a force of $T = 6$ kN when the cable is directed at $\theta = 60°$. Determine the magnitudes of the total brake frictional force **F** for the rear set of wheels B and the total normal forces at *both* front wheels A and both rear wheels B for equilibrium. The truck has a total mass of 4 Mg and mass center at G.





$\xrightarrow{+} \Sigma F_x = 0; \quad 6000 \sin 60° - F = 0$

$F = 5196 \text{ N} = 5.20 \text{ kN}$ Ans

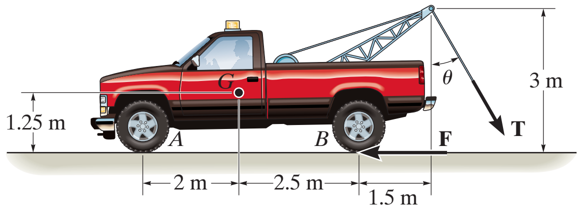
$\circlearrowleft + \Sigma M_B = 0; \quad -N_A(4.5) + 4(10^3)(9.81)(2.5) - 6000 \sin 60° (3) - 6000 \cos 60° (1.5) = 0$

$N_A = 17\,336 \text{ N} = 17.3 \text{ kN}$ Ans

$+\uparrow \Sigma F_y = 0; \quad 17\,336 - 4(10^3)(9.81) - 6000 \cos 60° + N_B = 0$

$N_B = 24\,904 \text{ N} = 24.9 \text{ kN}$ Ans

5–51. Determine the minimum cable force T and critical angle θ which will cause the tow truck to start tipping, i.e., for the normal reaction at A to be zero. Assume that the truck is braked and will not slip at B. The truck has a total mass of 4 Mg and mass center at G.x



$$\circlearrowleft + \Sigma M_B = 0; \quad 4(10^3)(9.81)(2.5) - T\sin\theta\,(3) - T\cos\theta\,(1.5) = 0$$

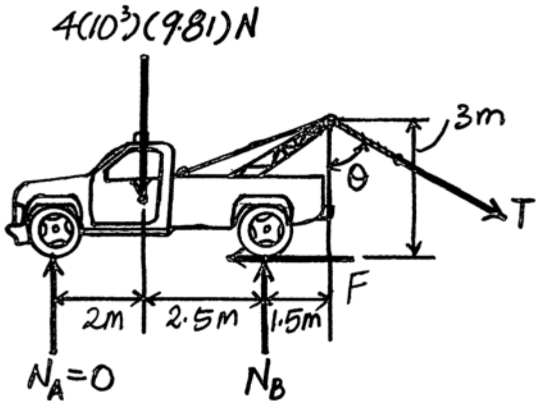
$$T = \frac{65\,400}{(\cos\theta + 2\sin\theta)}$$

$$\frac{dT}{d\theta} = \frac{-65\,400(-\sin\theta + 2\cos\theta)}{(\cos\theta + 2\sin\theta)^2} = 0$$

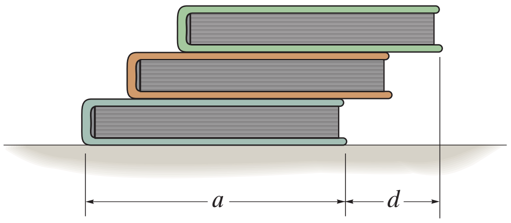
$$-\sin\theta + 2\cos\theta = 0$$

$$\theta = \tan^{-1}2 = 63.43^\circ = 63.4^\circ \qquad \text{Ans}$$

$$T = \frac{65\,400}{(\cos 63.43^\circ + 2\sin 63.43^\circ)} = 29.2\text{ kN} \qquad \text{Ans}$$



***5–52.** Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.

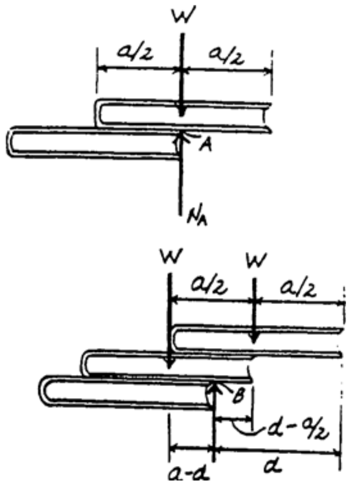


Equilibrium : For top two books, the upper book will topple when the center of gravity of this book is to the right of point A. Therefore, the maximum distance from the right edge of this book to point A is $a/2$.

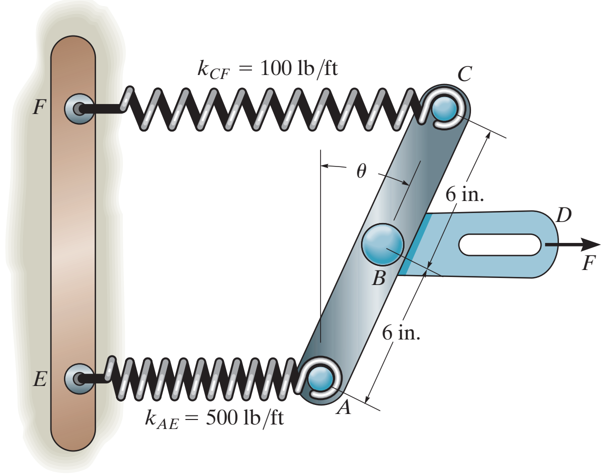
Equation of Equilibrium : For the entire three books, the top two books will topple about point B.

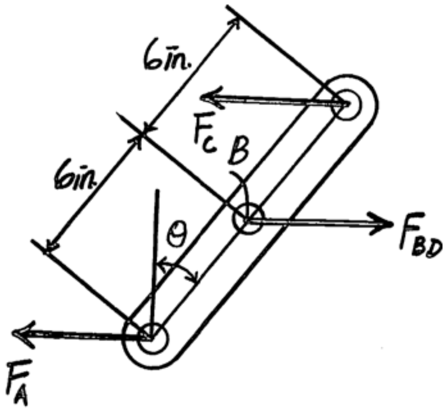
$$\circlearrowleft + \Sigma M_B = 0; \quad W(a-d) - W\left(d - \frac{a}{2}\right) = 0$$

$$d = \frac{3a}{4} \qquad \text{Ans}$$



•5–53. Determine the angle θ at which the link ABC is held in equilibrium if member BD moves 2 in. to the right. The springs are originally unstretched when $\theta = 0°$. Each spring has the stiffness shown. The springs remain horizontal since they are attached to roller guides.





$$\curvearrowleft +\Sigma M_B = 0; \quad F_C(6\cos\theta) - F_A(6\cos\theta) = 0 \quad F_C = F_A = F$$

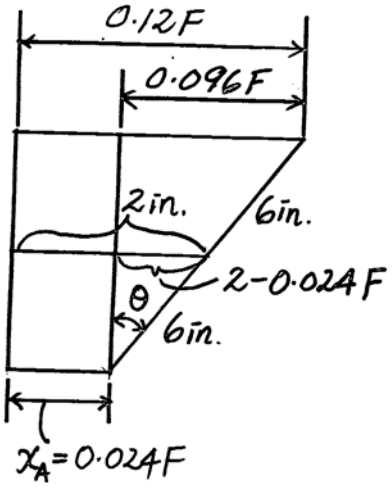
$$x_C = \frac{F}{\left(\frac{100}{12}\right)} = 0.12F \text{ and } x_A = \frac{F}{\left(\frac{500}{12}\right)} = 0.024F$$

Using similar triangles

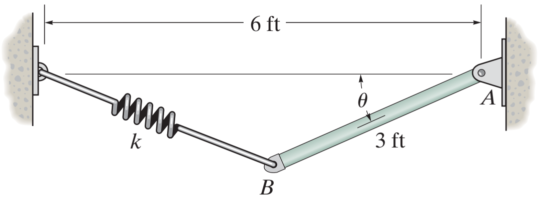
$$\frac{0.096F}{12} = \frac{2 - 0.024F}{6} \qquad F = 27.77 \text{ lb}$$

$$\sin\theta = \frac{0.096(27.77)}{12} = 0.2222$$

$$\theta = 12.8° \qquad \text{Ans}$$



5–54. The uniform rod AB has a weight of 15 lb and the spring is unstretched when $\theta = 0°$. If $\theta = 30°$, determine the stiffness k of the spring.

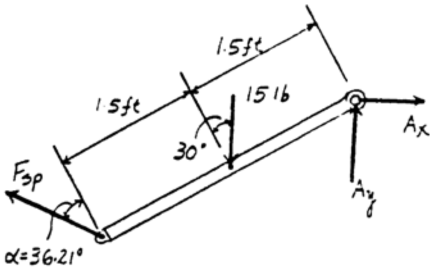


Geometry : From triangle CDB, the cosine law gives

$$l=\sqrt{2.536^2+1.732^2-2(2.536)(1.732)\cos 120°}=3.718\text{ ft}$$

Using the sine law,

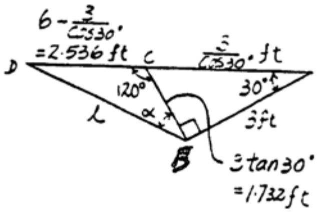
$$\frac{\sin\alpha}{2.536}=\frac{\sin 120°}{3.718}\qquad \alpha=36.21°$$



Equations of Equilibrium : The force in the spring can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A=0;\qquad 15\cos 30°(1.5)-F_{sp}\cos 36.21°(3)=0$$

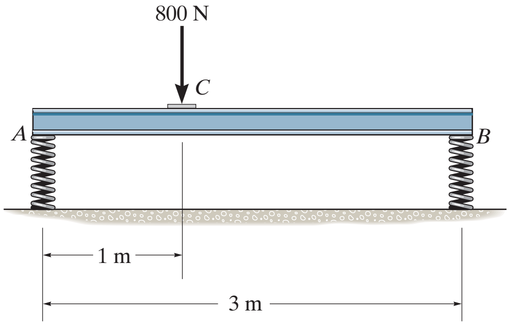
$$F_{sp}=8.050\text{ lb}$$



Spring Force Formula : The spring stretches $x=3.718-3=0.718$ ft.

$$k=\frac{F_{sp}}{x}=\frac{8.050}{0.718}=11.2\text{ lb/ft}\qquad \textbf{Ans}$$

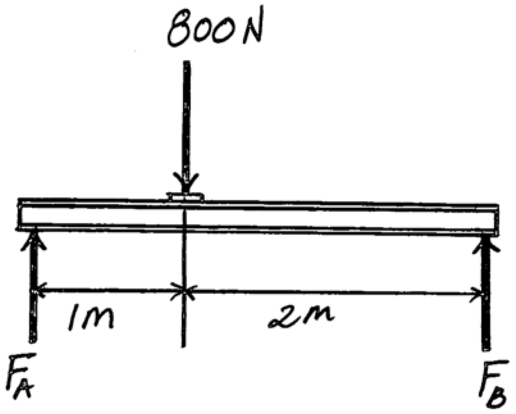
5–55. The horizontal beam is supported by springs at its ends. Each spring has a stiffness of $k = 5$ kN/m and is originally unstretched so that the beam is in the horizontal position. Determine the angle of tilt of the beam if a load of 800 N is applied at point C as shown.



Equations of Equilibrium : The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\circlearrowleft + \Sigma M_B=0;\qquad 800(2)-F_A(3)=0\qquad F_A=533.33\text{ N}$$

$$\circlearrowleft + \Sigma M_A=0;\qquad F_B(3)-800(1)=0\qquad F_B=266.67\text{ N}$$



Spring Formula : Applying $\Delta=\frac{F}{k}$, we have

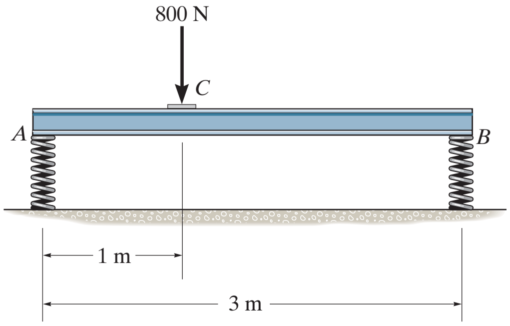
$$\Delta_A=\frac{533.33}{5(10^3)}=0.1067\text{ m}$$

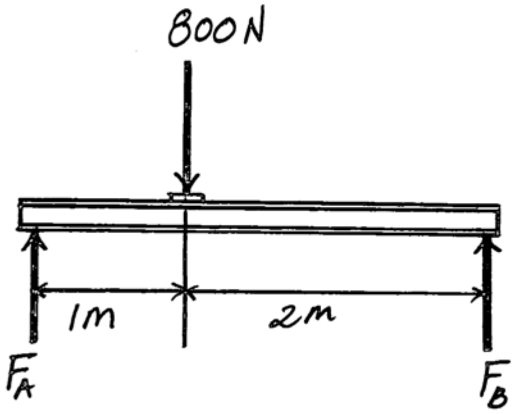
$$\Delta_B=\frac{266.67}{5(10^3)}=0.05333\text{ m}$$

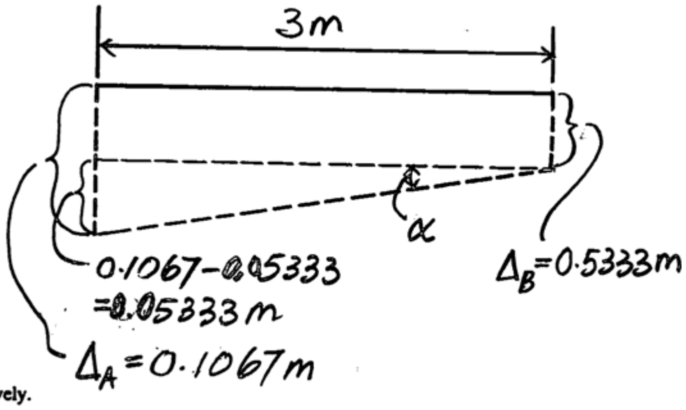
Geometry : The angle of tilt α is

$$\alpha=\tan^{-1}\left(\frac{0.05333}{3}\right)=1.02°\qquad \textbf{Ans}$$

***5–56.** The horizontal beam is supported by springs at its ends. If the stiffness of the spring at A is $k_A = 5$ kN/m, determine the required stiffness of the spring at B so that if the beam is loaded with the 800 N it remains in the horizontal position. The springs are originally constructed so that the beam is in the horizontal position when it is unloaded.







Equations of Equilibrium : The spring forces at A and B can be obtained directly by summing moments about points B and A respectively.

$$\curvearrowleft + \Sigma M_B = 0; \quad 800(2) - F_A(3) = 0 \quad F_A = 533.33 \text{ N}$$

$$\curvearrowleft + \Sigma M_A = 0; \quad F_B(3) - 800(1) = 0 \quad F_B = 266.67 \text{ N}$$

Spring Formula : Applying $\Delta = \dfrac{F}{k}$, we have

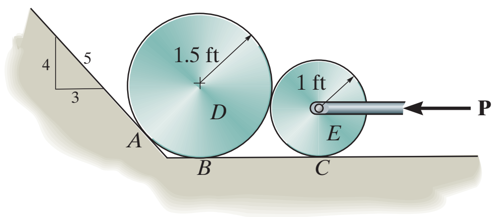
$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067 \text{ m} \qquad \Delta_B = \frac{266.67}{k_B}$$

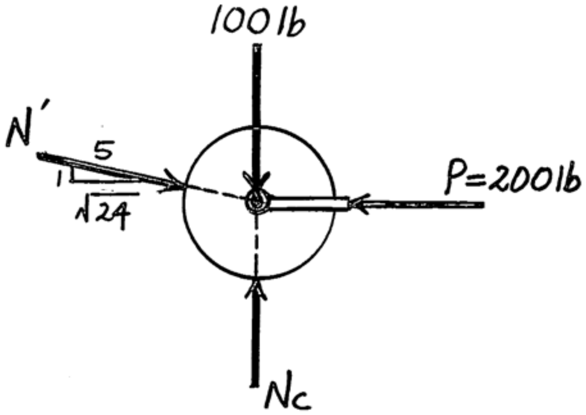
Geometry : Requires, $\Delta_B = \Delta_A$. Then

$$\frac{266.67}{k_B} = 0.1067$$

$$k_B = 2500 \text{ N/m} = 2.50 \text{ kN/m} \qquad \textbf{Ans}$$

•5–57. The smooth disks D and E have a weight of 200 lb and 100 lb, respectively. If a horizontal force of $P = 200$ lb is applied to the center of disk E, determine the normal reactions at the points of contact with the ground at A, B, and C.





For disk E:

$\xrightarrow{+} \Sigma F_x = 0;\quad -P + N'\left(\frac{\sqrt{24}}{5}\right) = 0$

$+\uparrow \Sigma F_y = 0;\quad N_C - 100 - N'\left(\frac{1}{5}\right) = 0$

For disk D:

$\xrightarrow{+} \Sigma F_x = 0;\quad N_A\left(\frac{4}{5}\right) - N'\left(\frac{\sqrt{24}}{5}\right) = 0$

$+\uparrow \Sigma F_y = 0;\quad N_A\left(\frac{3}{5}\right) + N_B - 200 + N'\left(\frac{1}{5}\right) = 0$

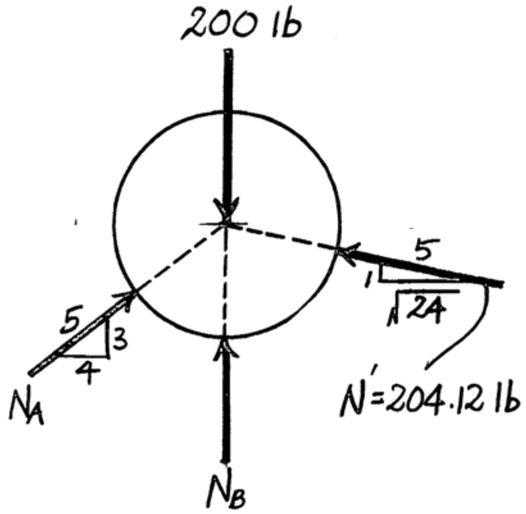
Set $P = 200$ lb and solve:

$N' = 204.12$ lb

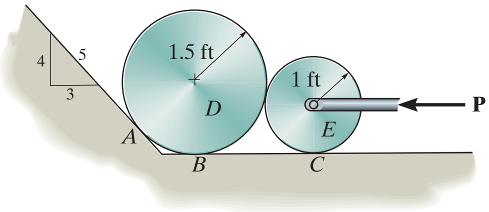
$N_A = 250$ lb **Ans**

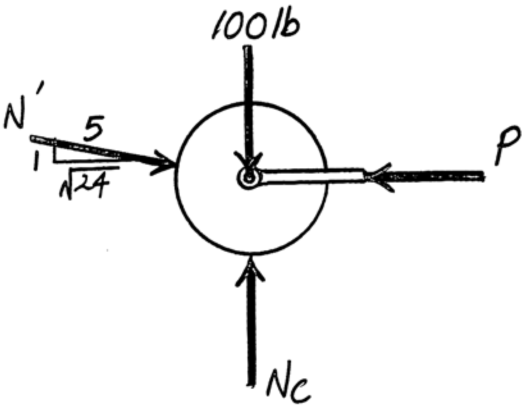
$N_B = 9.18$ lb **Ans**

$N_C = 141$ lb **Ans**



5–58. The smooth disks D and E have a weight of 200 lb and 100 lb, respectively. Determine the largest horizontal force P that can be applied to the center of disk E without causing the disk D to move up the incline.





For disk E:

$$\xrightarrow{+} \Sigma F_x = 0; \quad -P + N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad N_C - 100 - N'\left(\frac{1}{5}\right) = 0$$

For disk D:

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_A\left(\frac{4}{5}\right) - N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

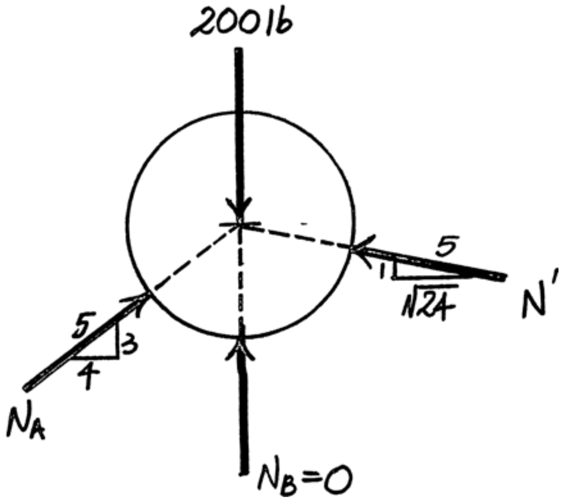
$$+\uparrow \Sigma F_y = 0; \quad N_A\left(\frac{3}{5}\right) + N_B - 200 + N'\left(\frac{1}{5}\right) = 0$$

Require $N_B = 0$ for P_{max}. Solving,

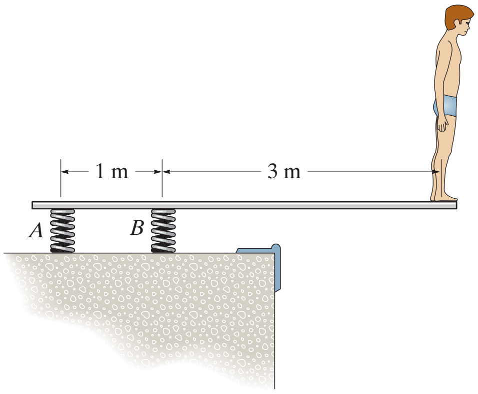
$P_{max} = 210$ lb **Ans**

$N_A = 262$ lb **Ans**

$N_C = 143$ lb **Ans**



5–59. A man stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15$ kN/m. In the position shown the board is horizontal. If the man has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.



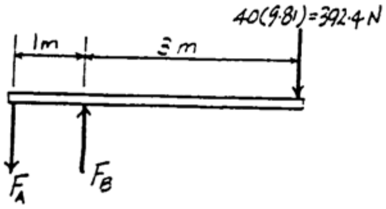
Equations of Equilibrium : The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\curvearrowleft + \Sigma M_B = 0; \qquad F_A(1) - 392.4(3) = 0 \qquad F_A = 1177.2\text{ N}$$

$$\curvearrowleft + \Sigma M_A = 0; \qquad F_B(1) - 392.4(4) = 0 \qquad F_B = 1569.6\text{ N}$$

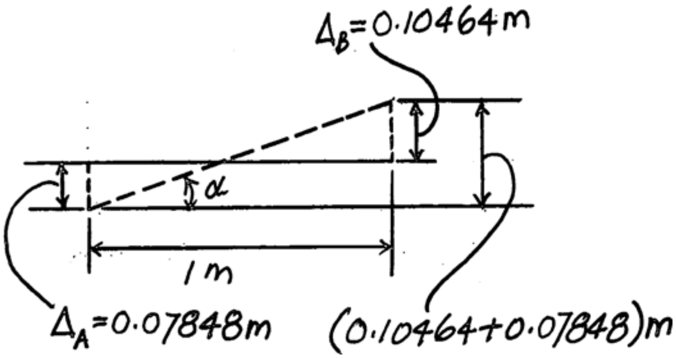
Spring Formula : Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848\text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464\text{ m}$$

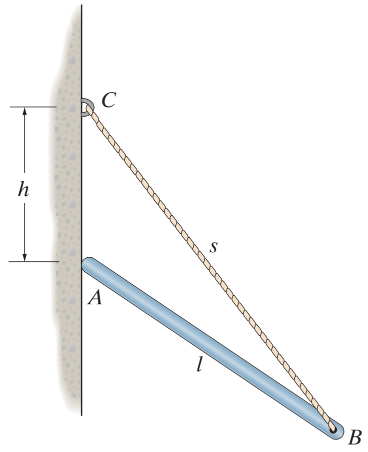


Geometry : The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4° \qquad \textbf{Ans}$$



***5–60.** The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Show that for equilibrium it is required that $h = [(s^2 - l^2)/3]^{1/2}$.



Equations of Equilibrium : The tension in the cable can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T\sin\phi\,(l) - W\sin\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\sin\theta}{2\sin\phi}$$

Using the result $T = \dfrac{W\sin\theta}{2\sin\phi}$,

$$+\uparrow \Sigma F_y = 0; \qquad \frac{W\sin\theta}{2\sin\phi}\cos(\theta-\phi) - W = 0$$

$$\sin\theta\cos(\theta-\phi) - 2\sin\phi = 0 \qquad [1]$$

Geometry : Applying the sine law with $\sin(180° - \theta) = \sin\theta$, we have

$$\frac{\sin\phi}{h} = \frac{\sin\theta}{s} \qquad \sin\phi = \frac{h}{s}\sin\theta \qquad [2]$$

Substituting Eq.[2] into [1] yields

$$\cos(\theta-\phi) = \frac{2h}{s} \qquad [3]$$

Using the cosine law,

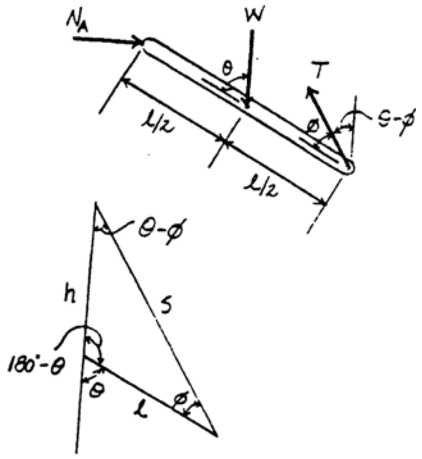
$$l^2 = h^2 + s^2 - 2hs\cos(\theta-\phi)$$

$$\cos(\theta-\phi) = \frac{h^2+s^2-l^2}{2hs} \qquad [4]$$

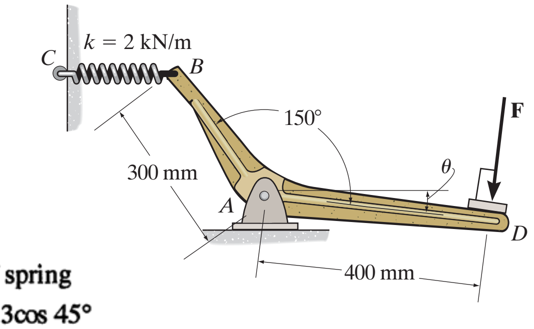
Equating Eqs.[3] and [4] yields

$$\frac{2h}{s} = \frac{h^2+s^2-l^2}{2hs}$$

$$h = \sqrt{\frac{s^2-l^2}{3}} \qquad (Q.E.D)$$



•5–61. If spring BC is unstretched with $\theta = 0°$ and the bell crank achieves its equilibrium position when $\theta = 15°$, determine the force $\mathbf{F}$ applied perpendicular to segment AD and the horizontal and vertical components of reaction at pin A. Spring BC remains in the horizontal postion at all times due to the roller at C.



Spring Force Formula: From the geometry shown in Fig. *a*, the stretch of spring BC when the bell crank rotates $\theta = 15°$ about point A is $x = 0.3\cos 30° - 0.3\cos 45° = 0.04768$ m. Thus, the force developed in spring BC is given by

$$F_{sp} = kx = 2000(0.04768) = 95.35\text{ N}$$

Equations of Equilibrium: From the free - body diagram of the bell crank, Fig. *b*, F can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;\qquad 95.35\sin 45°(300) - F(400) = 0$

$F = 50.57\text{ N} = 50.6\text{ N}$ **Ans.**

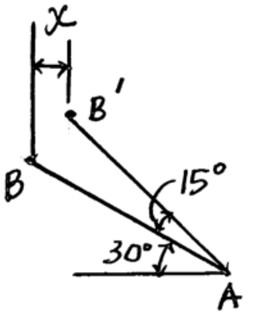
Using the above result and writing the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad A_x - 50.57\sin 15° - 95.35 = 0$

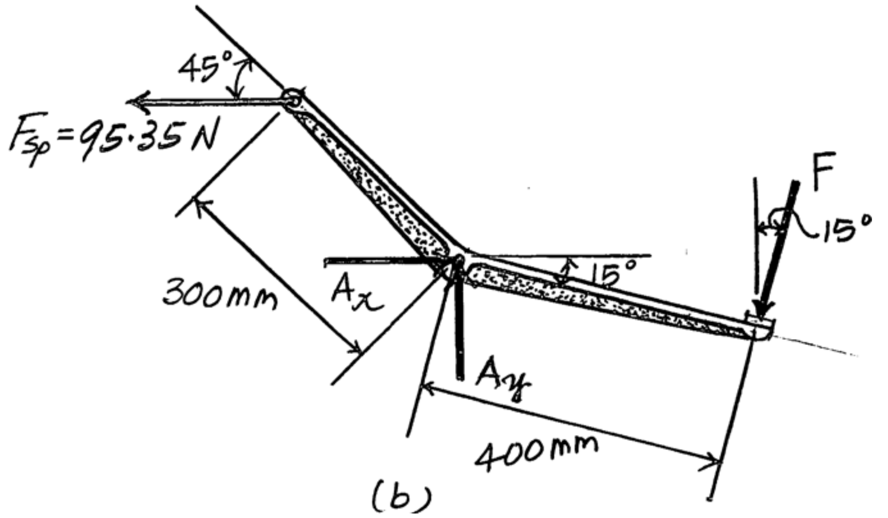
$A_x = 108.44\text{ N} = 108\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y - 50.57\cos 15° = 0$

$A_y = 48.84\text{ N} = 48.8\text{ N}$ **Ans.**

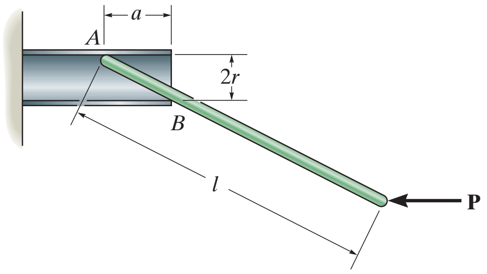


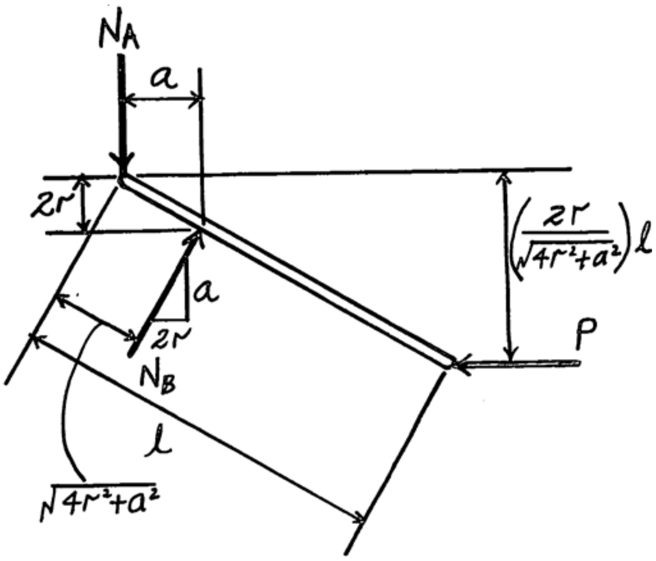
(a)



(b)

5–62. The thin rod of length *l* is supported by the smooth tube. Determine the distance *a* needed for equilibrium if the applied load is **P**.





$\xrightarrow{+}\Sigma F_x = 0;\quad \frac{2r}{\sqrt{4r^2+a^2}}N_B - P = 0$

$\circlearrowleft+\Sigma M_A = 0;\quad -P\left(\frac{2r}{\sqrt{4r^2+a^2}}\right)l + N_B\sqrt{4r^2+a^2} = 0$

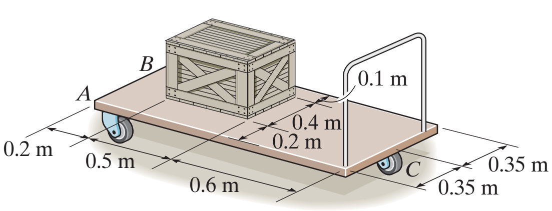
$$\frac{4r^2 l}{4r^2+a^2} - \sqrt{4r^2+a^2} = 0$$

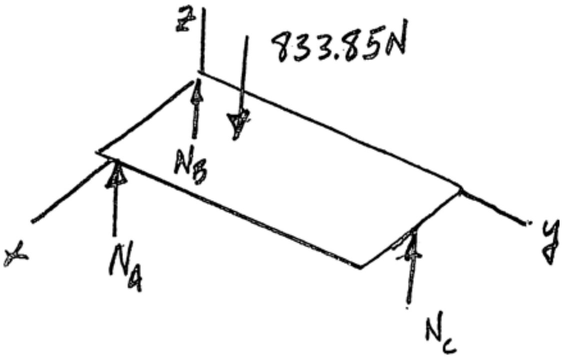
$$4r^2 l = (4r^2+a^2)^{\frac{3}{2}}$$

$$(4r^2 l)^{\frac{2}{3}} = 4r^2 + a^2$$

$$a = \sqrt{(4r^2 l)^{\frac{2}{3}} - 4r^2}$$ **Ans**

5–63. The cart supports the uniform crate having a mass of 85 kg. Determine the vertical reactions on the three casters at *A*, *B*, and *C*. The caster at *B* is not shown. Neglect the mass of the cart.





Equations of Equilibrium : The normal reaction N_C can be obtained directly by summing moments about x axis.

$\Sigma M_x = 0;$ $N_C(1.3) - 833.85(0.45) = 0$

$N_C = 288.64\ \text{N} = 289\ \text{N}$ **Ans**

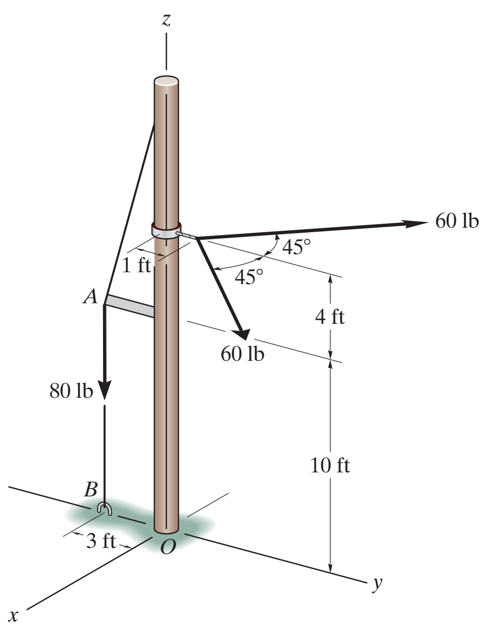
$\Sigma M_y = 0;$ $833.85(0.3) - 288.64(0.35) - N_A(0.7) = 0$

$N_A = 213.04\ \text{N} = 213\ \text{N}$ **Ans**

$\Sigma F_z = 0;$ $N_B + 288.64 + 213.04 - 833.85 = 0$

$N_B = 332\ \text{N}$ **Ans**

***5–64.** The pole for a power line is subjected to the two cable forces of 60 lb, each force lying in a plane parallel to the $x-y$ plane. If the tension in the guy wire AB is 80 lb, determine the x, y, z components of reaction at the fixed base of the pole, O.



Equations of Equilibrium :

$\Sigma F_x = 0;\quad O_x + 60\overrightarrow{\sin} 45° - 60\sin 45° = 0$

$O_x = 0$ **Ans**

$\Sigma F_y = 0;\quad O_y + 60\cos 45° + 60\cos 45° = 0$

$O_y = -84.9 \text{ lb}$ **Ans**

$\Sigma F_z = 0;\quad O_z - 80 = 0 \qquad O_z = 80.0 \text{ lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_O)_x + 80(3) - 2[60\cos 45°(14)] = 0$

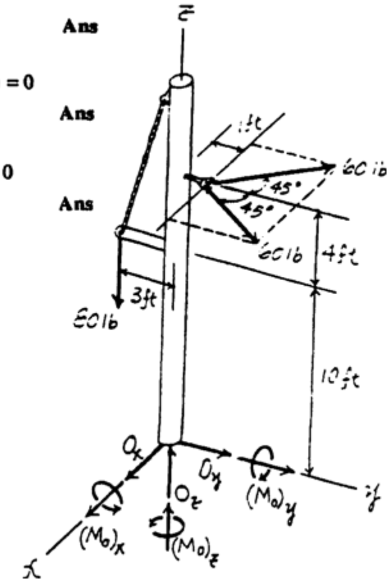
$(M_O)_x = 948 \text{ lb}\cdot\text{ft}$ **Ans**

$\Sigma M_y = 0;\quad (M_O)_y + 60\sin 45°(14) - 60\sin 45°(14) = 0$

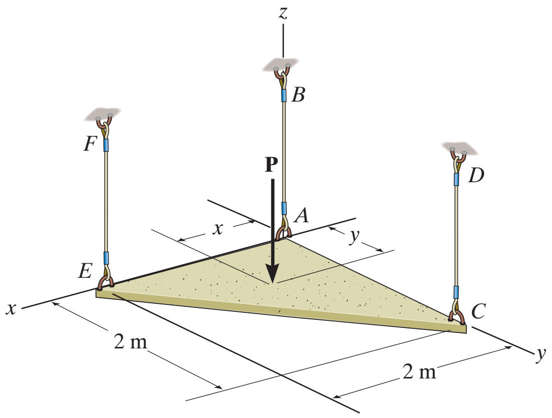
$(M_O)_y = 0$ **Ans**

$\Sigma M_z = 0;\quad (M_O)_y + 60\sin 45°(1) - 60\sin 45°(1) = 0$

$(M_O)_z = 0$ **Ans**



•5–65. If $P = 6$ kN, $x = 0.75$ m and $y = 1$ m, determine the tension developed in cables *AB, CD,* and *EF*. Neglect the weight of the plate.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, T_{CD} and T_{EF} can be obtained by writing the moment equation of equilibrium about the *x* and *y* axes, respectively.

$\Sigma M_x = 0;\ T_{CD}(2) - 6(1) = 0$

$T_{CD} = 3\text{ kN}$ **Ans.**

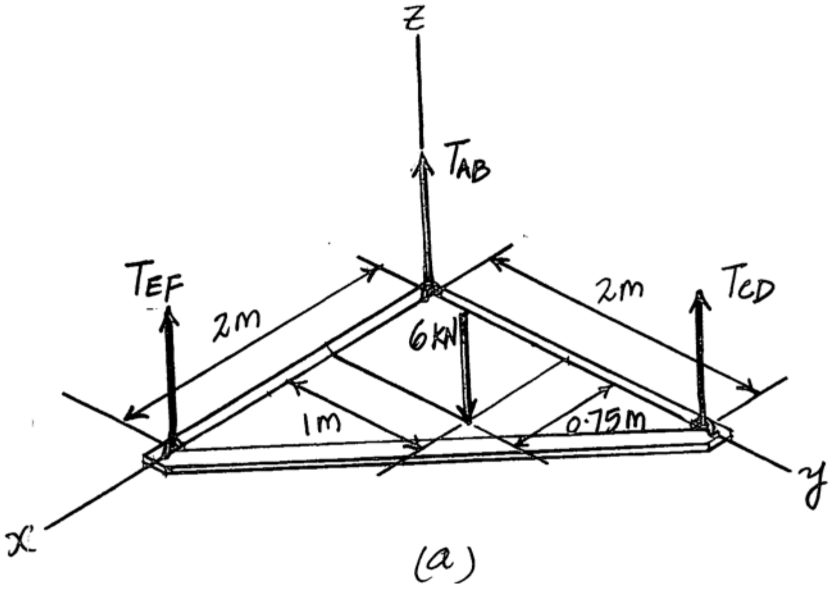
$\Sigma M_y = 0;\ T_{EF}(2) - 6(0.75) = 0$

$T_{EF} = 2.25\text{ kN}$ **Ans.**

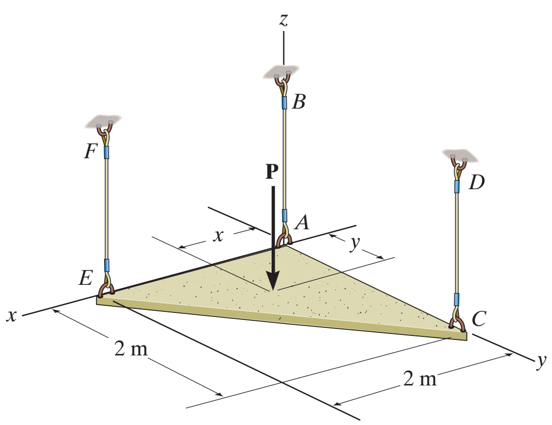
Using the above results and writing the force equation of equilibrium along the *z* axis,

$\Sigma F_z = 0;\quad T_{AB} + 3 + 2.25 - 6 = 0$

$T_{AB} = 0.75\text{ kN}$ **Ans.**



5–66. Determine the location x and y of the point of application of force **P** so that the tension developed in cables AB, CD, and EF is the same. Neglect the weight of the plate.



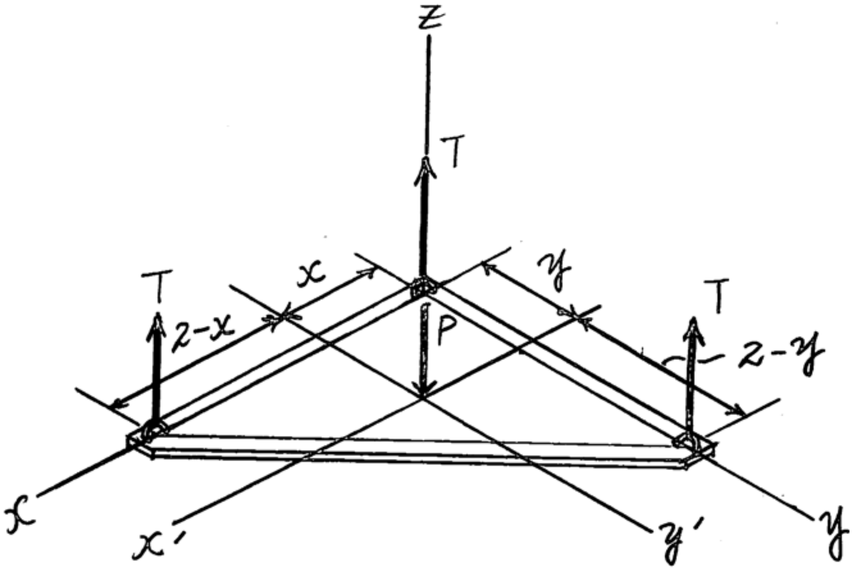
Equations of Equilibrium: From the free - body diagram of the plate, Fig. a, and writing the moment equations of equilibrium about the x' and y' axes,

$\Sigma M_{x'} = 0;$ $\quad T(2-y) - 2T(y) = 0$

$y = 0.667 \text{ m}$ **Ans.**

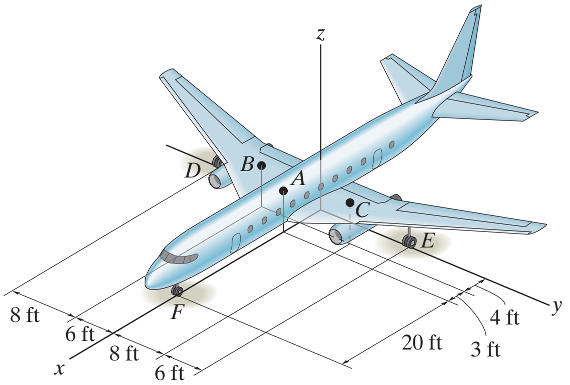
$\Sigma M_{y'} = 0;$ $\quad 2T(x) - T(2-x) = 0$

$x = 0.667 \text{ m}$ **Ans.**



(a)

5–67. Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage *A* and wings *B* and *C* are located as shown. If these components have weights $W_A = 45\,000$ lb, $W_B = 8000$ lb, and $W_C = 6000$ lb, determine the normal reactions of the wheels *D*, *E*, and *F* on the ground.



$\Sigma M_x = 0;\quad 8000\,(6) - R_D\,(14) - 6000\,(8) + R_E\,(14) = 0$

$\Sigma M_y = 0;\quad 8000\,(4) + 45\,000\,(7) + 6000\,(4) - R_F\,(27) = 0$

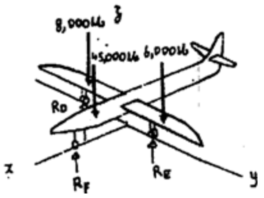
$\Sigma F_z = 0;\quad R_D + R_E + R_F - 8000 - 6000 - 45\,000 = 0$

Solving,

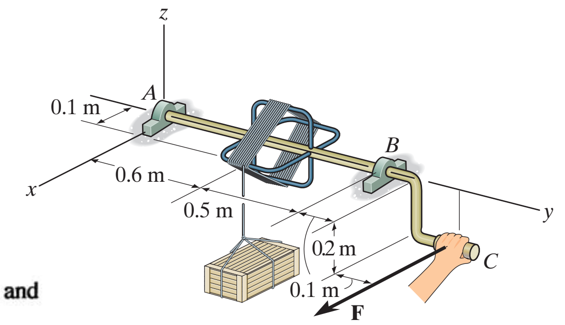
$R_D = 22.6$ kip **Ans**

$R_E = 22.6$ kip **Ans**

$R_F = 13.7$ kip **Ans**



***5–68.** Determine the magnitude of force **F** that must be exerted on the handle at *C* to hold the 75-kg crate in the position shown. Also, determine the components of reaction at the thrust bearing *A* and smooth journal bearing *B*.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, F, B_z, A_z, and A_y can be obtained by writing the moment equation of equilibrium about the y, x, and x' axes and the force equation of equilibrium along the y axis.

$\Sigma M_y = 0;\quad -F(0.2) + 75(9.81)(0.1) = 0$

$F = 367.875 \text{ N} = 368 \text{ N}$ **Ans.**

$\Sigma M_x = 0;\quad B_z(0.5 + 0.6) - 75(9.81)(0.6) = 0$

$B_z = 401.32 \text{ N} = 401 \text{ N}$ **Ans.**

$\Sigma M_{x'} = 0;\quad -A_z(0.6 + 0.5) + 75(9.81)(0.5) = 0$

$A_z = 334.43 \text{ N} = 334 \text{ N}$ **Ans.**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans.**

Using the result $F = 367.875$ N and writing the moment equations of equilibrium about the z and z' axes,

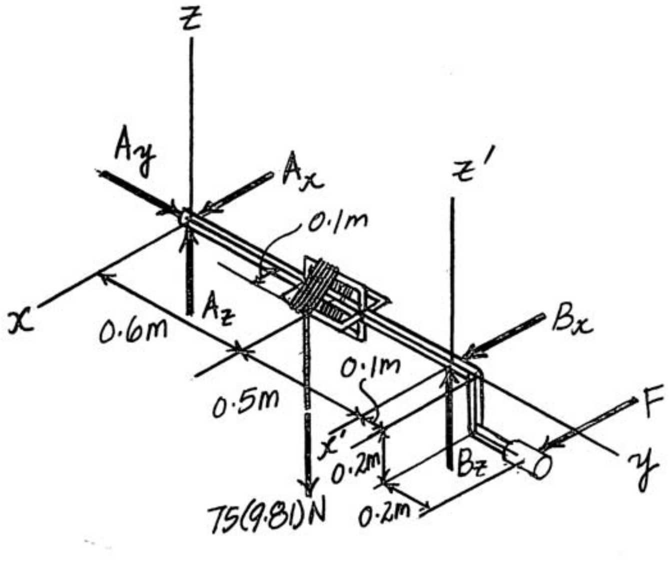
$\Sigma M_z = 0;\quad -B_x(0.5 + 0.6) - 367.875(0.2 + 0.1 + 0.5 + 0.6) = 0$

$B_x = -468.20 \text{ N} = -468 \text{ N}$ **Ans.**

$\Sigma M_{z'} = 0;\quad A_x(0.6 + 0.5) - 367.875(0.2 + 0.1) = 0$

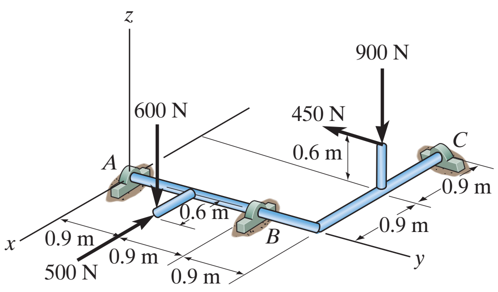
$A_x = 100.33 \text{ N} = 100 \text{ N}$ **Ans.**

The negative signs indicate that $\mathbf{B}_x$ act in the opposite sense to that shown on the free - body diagram.



(a)

•5–69. The shaft is supported by three smooth journal bearings at *A*, *B*, and *C*. Determine the components of reaction at these bearings.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, C_y and C_z can be obtained by writing the force equation of equilibrium along the *y* axis and the moment equation of equilibrium about the *y* axis.

$\Sigma F_y = 0;\quad C_y - 450 = 0$

$C_y = 450\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad C_z(0.9 + 0.9) - 900(0.9) + 600(0.6) = 0$

$C_z = 250\text{ N}$ **Ans.**

Using the above results

$\Sigma M_x = 0;\quad B_z(0.9 + 0.9) + 250(0.9 + 0.9 + 0.9) + 450(0.6) - 900(0.9 + 0.9 + 0.9) - 600(0.9) = 0$

$B_z = 1125\text{ N} = 1.125\text{ kN}$ **Ans.**

$\Sigma M_{x'} = 0;\quad 600(0.9) + 450(0.6) - 900(0.9) + 250(0.9) - A_z(0.9 + 0.9) = 0$

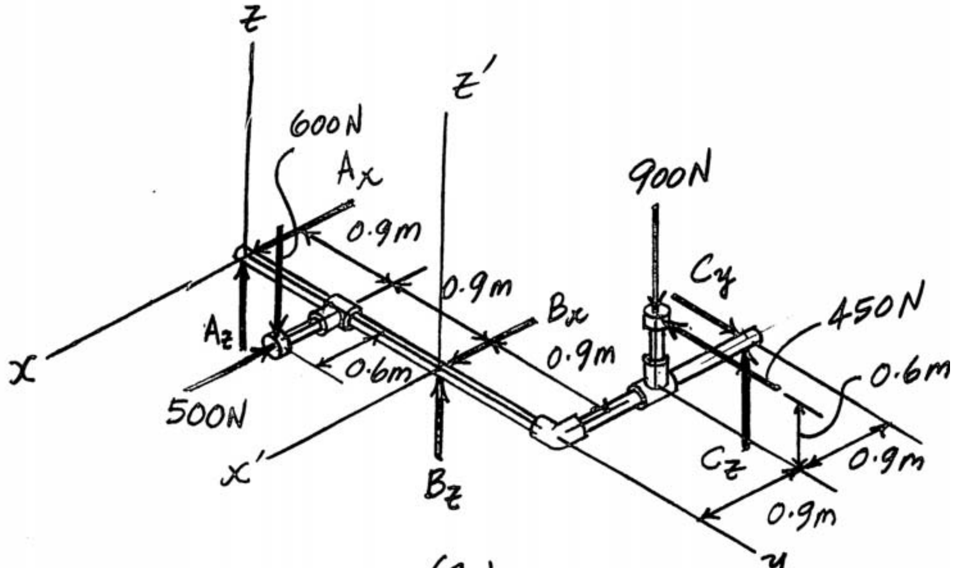
$A_z = 125\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad -B_x(0.9 + 0.9) + 500(0.9) + 450(0.9) - 450(0.9 + 0.9) = 0$

$B_x = 25\text{ N}$ **Ans.**

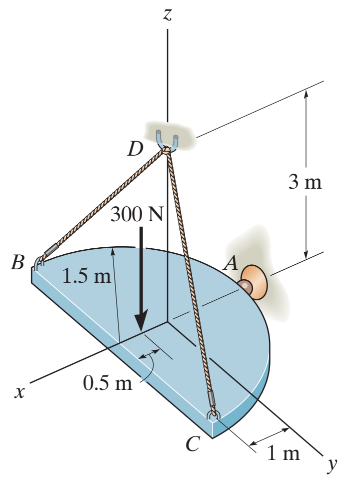
$\Sigma F_x = 0;\quad A_x + 25 - 500 = 0$

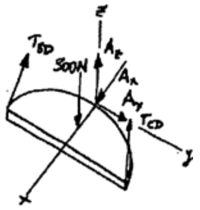
$A_x = 475\text{ N}$ **Ans.**



(a)

5–70. Determine the tension in cables *BD* and *CD* and the *x, y, z* components of reaction at the ball-and-socket joint at *A*.





$\mathbf{r}_{BD} = \{-1\,\mathbf{i} + 1.5\,\mathbf{j} + 3\,\mathbf{k}\}$ m; $\quad r_{BD} = 3.50$ m

$$\mathbf{T}_{BD} = T_{BD}\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = -0.2857\,T_{BD}\,\mathbf{i} + 0.4286\,T_{BD}\,\mathbf{j} + 0.8571\,T_{BD}\,\mathbf{k}$$

In a similar manner,

$$\mathbf{T}_{CD} = T_{CD}\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = -0.2857\,T_{CD}\,\mathbf{i} - 0.4286\,T_{CD}\,\mathbf{j} + 0.8571\,T_{CD}\,\mathbf{k}$$

Thus, using the components of $\mathbf{T}_{BD}$ and $\mathbf{T}_{CD}$, the scalar equations of equilibrium become :

$\Sigma F_x = 0;\quad A_x - 0.2857\,T_{BD} - 0.2857\,T_{CD} = 0$

$\Sigma F_y = 0;\quad A_y + 0.4286\,T_{BD} - 0.4286\,T_{CD} = 0$

$\Sigma F_z = 0;\quad A_z + 0.8571\,T_{BD} + 0.8571\,T_{CD} - 300 = 0$

$\Sigma M_{Ax} = 0;\quad -(0.8571\,T_{BD})(1.5) + (0.8571\,T_{CD})(1.5) = 0$

$\Sigma M_{Ay} = 0;\quad 300\,(1) - (0.8571\,T_{BD})(1.5) - (0.8571\,T_{CD})(1.5) = 0$

Solving

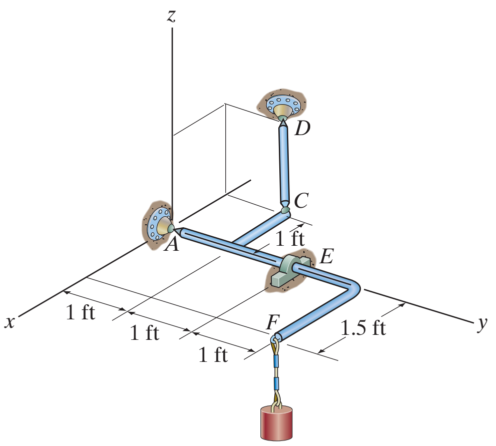
$T_{BD} = T_{CD} = 117$ N **Ans**

$A_x = 66.7$ N **Ans**

$A_y = 0$ **Ans**

$A_z = 100$ N **Ans**

5–71. The rod assembly is used to support the 250-lb cylinder. Determine the components of reaction at the ball-and-socket joint *A*, the smooth journal bearing *E*, and the force developed along rod *CD*. The connections at *C* and *D* are ball-and-socket joints.



Equations of Equilibrium: Since rod *CD* is a two-force member, it exerts a force $\mathbf{F}_{DC}$ directed along its axis as defined by $\mathbf{u}_{DC}$ on rod *BC*, Fig. *a*. Expressing each of the forces indicated on the free-body diagram in Cartesian vector form,

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$
$$\mathbf{F}_E = E_x\mathbf{i} + E_z\mathbf{k}$$
$$\mathbf{W} = [-250\mathbf{k}]\text{ lb}$$
$$\mathbf{F}_{DC} = -F_{DC}\,\mathbf{k}$$

Applying the force equation of equilibrium

$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_A + \mathbf{F}_E + \mathbf{F}_{DC} + \mathbf{W} = \mathbf{0}$

$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + (E_x\mathbf{i} + E_z\mathbf{k}) + (-F_{DC}\,\mathbf{k}) + (-250\mathbf{k}) = \mathbf{0}$$

$(A_x + E_x)\mathbf{i} + (A_y)\mathbf{j} + (A_z + E_z - F_{DC})\mathbf{k} = \mathbf{0}$

Equating **i**, **j**, and **k** components,

$A_x + E_x = 0 \qquad (1)$

$A_y = 0 \qquad (2)$

$A_z + E_z - F_{DC} - 250 = 0 \qquad (3)$

In order to apply the moment equation of equilibrium about point *A*, the position vectors $\mathbf{r}_{AC}$, $\mathbf{r}_{AE}$, and $\mathbf{r}_{AF}$, Fig. *a*, must be determined first.

$\mathbf{r}_{AC} = [-1\mathbf{i} + 1\mathbf{j}]\text{ft}$

$\mathbf{r}_{AE} = [2\mathbf{j}]\text{ft}$

$\mathbf{r}_{AF} = [1.5\mathbf{i} + 3\mathbf{j}]\text{ft}$

Thus,

$\Sigma\mathbf{M}_A = \mathbf{0};\ (\mathbf{r}_{AC} \times \mathbf{F}_{DC}) + (\mathbf{r}_{AE} \times \mathbf{F}_E) + (\mathbf{r}_{AF} \times \mathbf{W}) = 0$

$(-1\mathbf{i} + 1\mathbf{j}) \times (-F_{DC}\,\mathbf{k}) + (2\mathbf{j}) \times (E_x\mathbf{i} + E_z\mathbf{k}) + (1.5\mathbf{i} + 3\mathbf{j}) \times (-250\mathbf{k}) = 0$

$(-F_{DC} + 2E_z - 750)\mathbf{i} + (375 - F_{DC})\mathbf{j} + (-2E_x)\mathbf{k} = \mathbf{0}$

Equating **i**, **j**, and **k** components,

$-F_{DC} + 2E_z - 750 = 0$ (4)

$375 - F_{DC} = 0$ (5)

$-2E_x = 0$ (6)

Solving Eqs. (1) through (6), yields

$F_{DC} = 375\text{ lb}$ **Ans.**

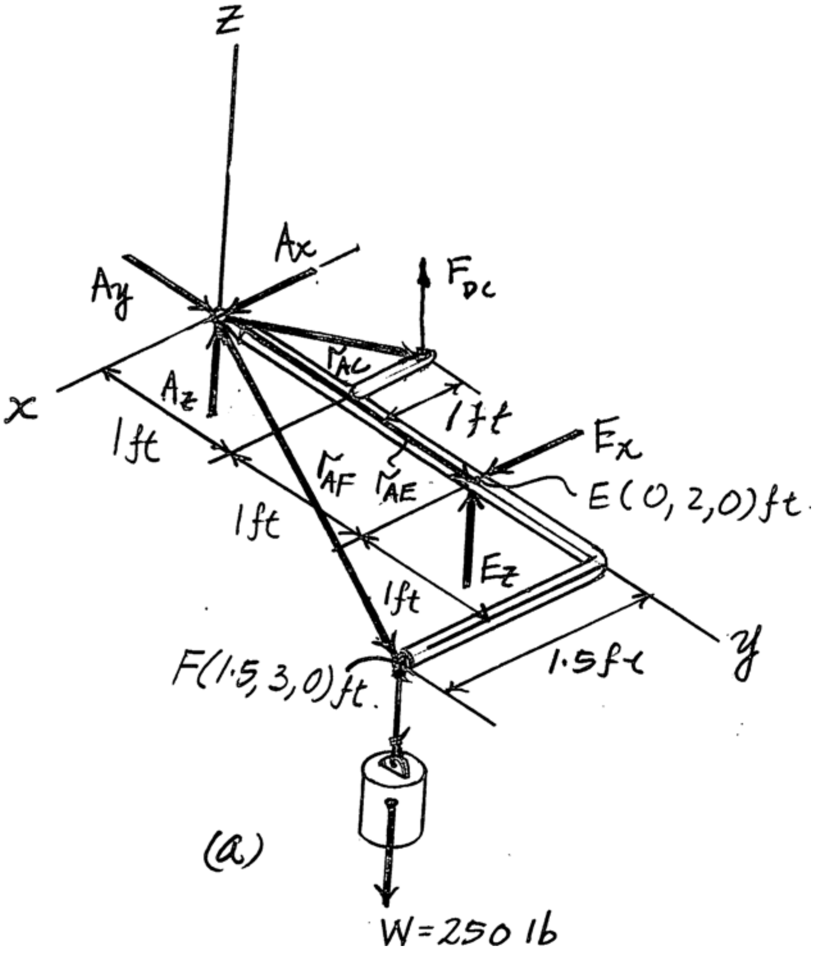
$E_x = 0$ **Ans.**

$E_z = 562.5\text{ lb}$ **Ans.**

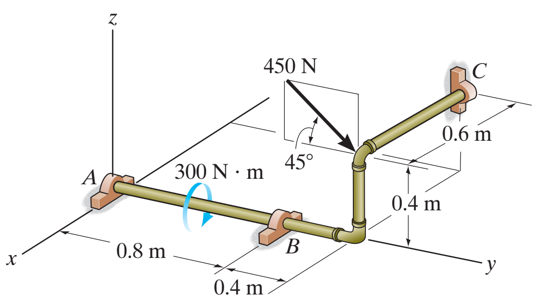
$A_x = 0$ **Ans.**

$A_y = 0$ **Ans.**

$A_z = 62.5\text{ lb}$ **Ans.**



***5–72.** Determine the components of reaction acting at the smooth journal bearings *A*, *B*, and *C*.



Equations of Equilibrium: From the free - body diagram of the shaft, Fig. *a*, C_y and C_z can be obtained by writing the force equation of equilibrium along the *y* axis and the moment equation of equilibrium about the *y* axis.

$\Sigma F_y = 0;\quad 450\cos 45° + C_y = 0$

$C_y = -318.20\text{ N} = -318\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad C_z(0.6) - 300 = 0$

$C_z = 500\text{ N}$ **Ans.**

Using the above results and writing the moment equations of equilibrium about the *x* and *z* axes,

$\Sigma M_x = 0;\quad B_z(0.8) - 450\cos 45°(0.4) - 450\sin 45°(0.8 + 0.4) + (318.20)(0.4) + 500(0.8 + 0.4) = 0$

$B_z = -272.70\text{ N} = -273\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad -B_x(0.8) - (-318.20)(0.6) = 0$

$B_x = 238.65\text{ N} = 239\text{ N}$ **Ans.**

Finally, using the above results and writing the force equations of equilibrium along the *x* and *y* axes,

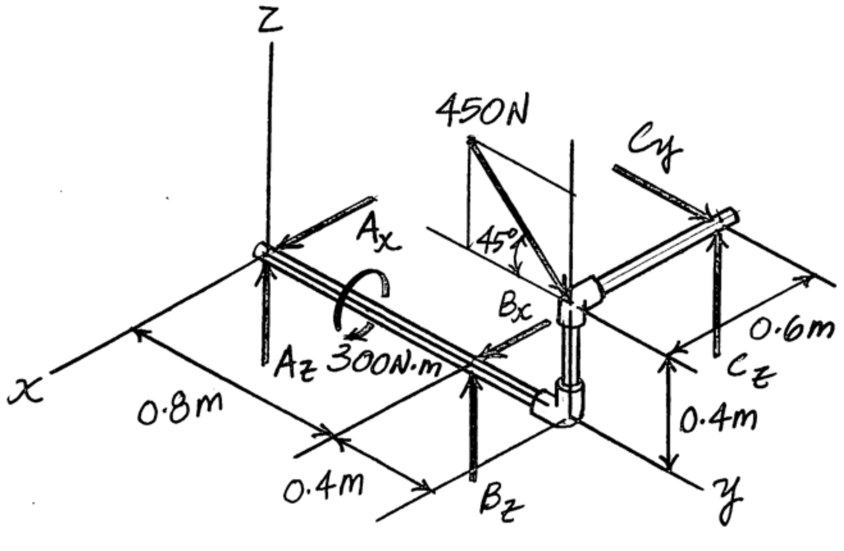
$\Sigma F_x = 0;\quad A_x + 238.5 = 0$

$A_x = -238.65\text{ N} = -239\text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad A_z - (-272.70) + 500 - 450\sin 45° = 0$

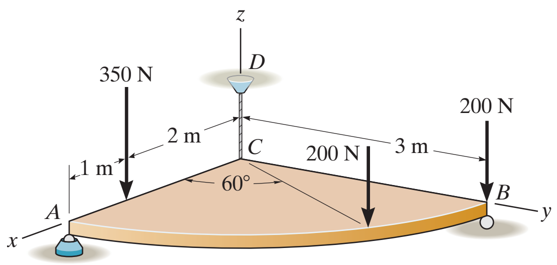
$A_z = 90.90\text{ N} = 90.9\text{ N}$ **Ans.**

The negative signs indicate that $\mathbf{C}_y$, $\mathbf{B}_z$ and $\mathbf{A}_x$ act in the opposite sense of that shown on the free - body diagram.



(a)

•5–73. Determine the force components acting on the ball-and-socket at *A*, the reaction at the roller *B* and the tension on the cord *CD* needed for equilibrium of the quarter circular plate.



Equations of Equilibrium : **The normal reaction N_B and A_z can be obtained directly by summing moments about the *x* and *y* axes respectively.**

$\Sigma M_x = 0;\quad N_B(3) - 200(3) - 200(3\sin 60°) = 0$

$N_B = 373.21\text{ N} = 373\text{ N}$ **Ans**

$\Sigma M_y = 0;\quad 350(2) + 200(3\cos 60°) - A_z(3) = 0$

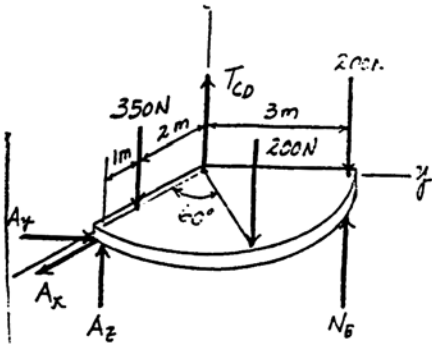
$A_z = 333.33\text{ N} = 333\text{ N}$ **Ans**

$\Sigma F_z = 0;\quad T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

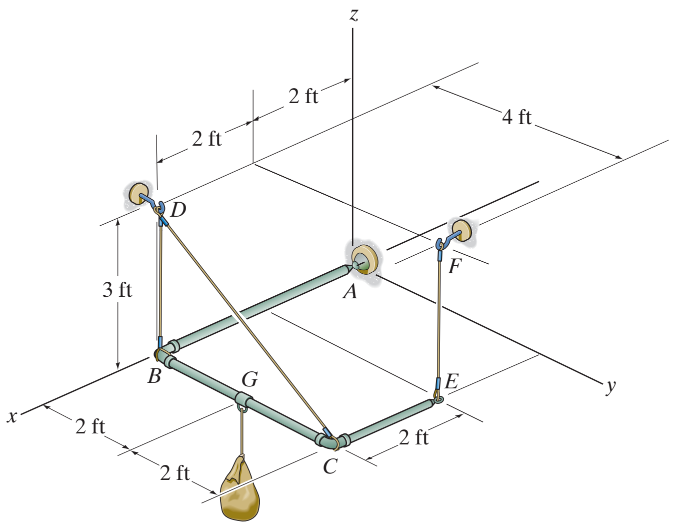
$T_{CD} = 43.5\text{ N}$ **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans**



5–74. If the load has a weight of 200 lb, determine the *x, y, z* components of reaction at the ball-and-socket joint *A* and the tension in each of the wires.



Equations of Equilibrium: Expressing the forces indicated on the free - body diagram, Fig. *a*, in Cartesian vector form,

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$
$$\mathbf{W} = [-200\mathbf{k}]\ \text{lb}$$
$$\mathbf{F}_{BD} = F_{BD}\mathbf{k}$$
$$\mathbf{F}_{CD} = F_{CD}\mathbf{u}_{CD} = F_{CD}\left[\frac{(4-4)\mathbf{i}+(0-4)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(4-4)^2+(0-4)^2+(3-0)^2}}\right] = \left(-\frac{4}{5}F_{CD}\mathbf{j}+\frac{3}{5}F_{CD}\mathbf{k}\right)$$
$$\mathbf{F}_{EF} = F_{EF}\mathbf{k}$$

Applying the force equation of equilibrium,

$$\Sigma\mathbf{F} = 0; \quad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{CD} + \mathbf{F}_{EF} + \mathbf{W} = \mathbf{0}$$
$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + F_{BD}\mathbf{k} + \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right) + F_{EF}\mathbf{k} + (-200\mathbf{k}) = \mathbf{0}$$
$$(A_x)\mathbf{i} + \left(A_y - \frac{4}{5}F_{CD}\right)\mathbf{j} + \left(A_z + F_{BD} + \frac{3}{5}F_{CD} + F_{EF} - 200\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$$A_x = 0 \qquad (1)$$
$$A_y - \frac{4}{5}F_{CD} = 0 \qquad (2)$$
$$A_z + F_{BD} + \frac{3}{5}F_{CD} + F_{EF} - 200 = 0 \qquad (3)$$

In order to write the moment equation of equilibrium about point *A*, the position vectors $\mathbf{r}_{AB}$, $\mathbf{r}_{AG}$, $\mathbf{r}_{AC}$, and $\mathbf{r}_{AE}$ must be determined first.

$$\mathbf{r}_{AB} = [4\mathbf{i}]\text{ft}$$
$$\mathbf{r}_{AG} = [4\mathbf{i} + 2\mathbf{j}]\text{ft}$$
$$\mathbf{r}_{AC} = [4\mathbf{i} + 4\mathbf{j}]\text{ft}$$
$$\mathbf{r}_{AE} = [2\mathbf{i} + 4\mathbf{j}]\text{ft}$$

Thus,

$$\Sigma\mathbf{M}_A = \mathbf{0}; \ (\mathbf{r}_{AB} \times \mathbf{F}_{BD}) + (\mathbf{r}_{AC} \times \mathbf{F}_{CD}) + (\mathbf{r}_{AE} \times \mathbf{F}_{EF}) + (\mathbf{r}_{AG} \times \mathbf{W}) = \mathbf{0}$$
$$(4\mathbf{i}) \times (F_{BD}\mathbf{k}) + (4\mathbf{i} + 4\mathbf{j}) \times \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right) + (2\mathbf{i} + 4\mathbf{j}) \times (F_{EF}\mathbf{k}) + (4\mathbf{i} + 2\mathbf{j}) \times (-200\mathbf{k})$$
$$\left(\frac{12}{5}F_{CD} + 4F_{EF} - 400\right)\mathbf{i} + \left(-4F_{BD} - \frac{12}{5}F_{CD} - 2F_{EF} + 800\right)\mathbf{j} + \left(-\frac{16}{5}F_{CD}\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$$\frac{12}{5}F_{CD} + 4F_{EF} - 400 = 0 \qquad (4)$$

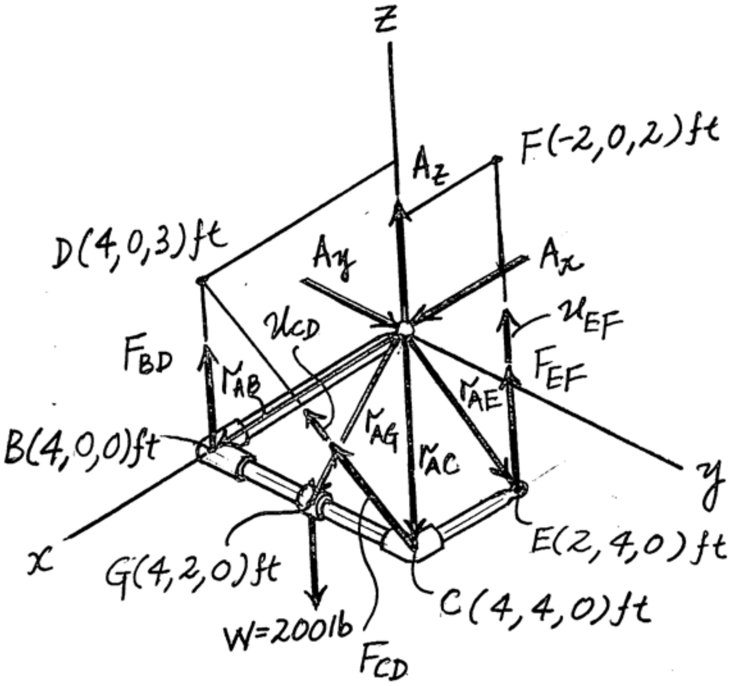
$$-4F_{BD} - \frac{12}{15}F_{CD} - 2F_{EF} + 800 = 0 \qquad (5)$$

$$-\frac{16}{5}F_{CD} = 0 \qquad (6)$$

Solving Eqs. (1) through (6),

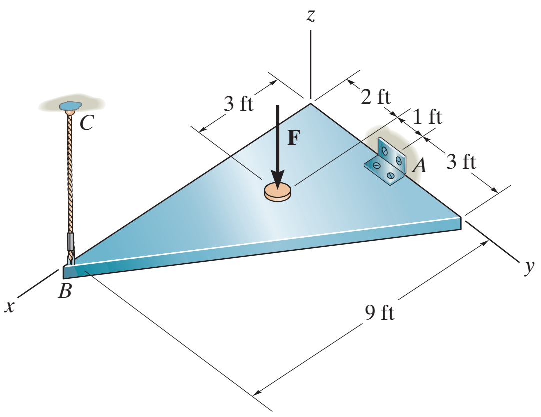
$F_{CD} = 0$ **Ans.**
$F_{EF} = 100 \text{ lb}$ **Ans.**
$F_{BD} = 150 \text{ lb}$ **Ans.**
$A_x = 0$ **Ans.**
$A_y = 0$ **Ans.**
$A_z = 100 \text{ lb}$ **Ans.**

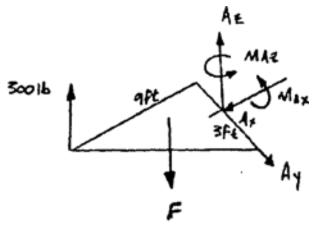
The negative signs indicate that $\mathbf{A}_z$ acts in the opposite sense to that on the free-body diagram.



(a)

5–75. If the cable can be subjected to a maximum tension of 300 lb, determine the maximum force *F* which may be applied to the plate. Compute the *x, y, z* components of reaction at the hinge *A* for this loading.





$\Sigma M_y = 0;\quad 3(F) - 300(9) = 0$

$F = 900 \text{ lb}$ **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**

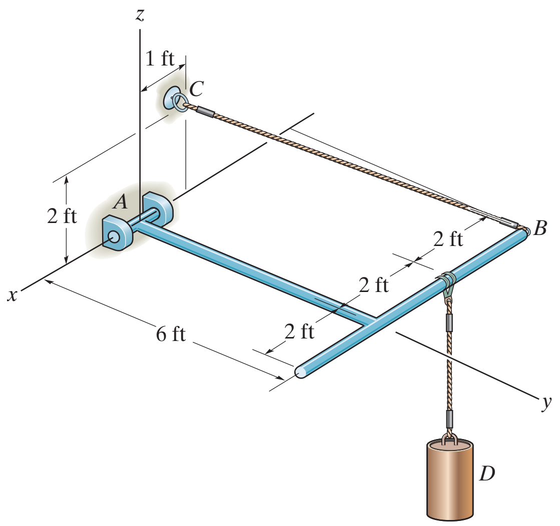
$\Sigma F_y = 0;\quad A_y = 0$ **Ans**

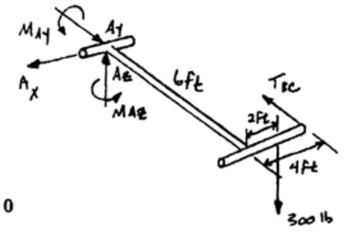
$\Sigma F_z = 0;\quad -900 + 300 + A_z = 0;\quad A_z = 600 \text{ lb}$ **Ans**

$\Sigma M_{Ax} = 0;\quad M_{Ax} + 900(1) - 3(300) = 0;\quad M_{Ax} = 0$ **Ans**

$\Sigma M_{Az} = 0;\quad M_{Az} = 0$ **Ans**

***5–76.** The member is supported by a pin at *A* and a cable *BC*. If the load at *D* is 300 lb, determine the *x, y, z* components of reaction at the pin *A* and the tension in cable *B C*.





$\mathbf{T}_{BC} = T_{BC}\left\{\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right\}$ ft

$\Sigma F_x = 0;\quad A_x + \left(\frac{3}{7}\right)T_{BC} = 0$

$\Sigma F_y = 0;\quad A_y - \left(\frac{6}{7}\right)T_{BC} = 0$

$\Sigma F_z = 0;\quad A_z - 300 + \left(\frac{2}{7}\right)T_{BC} = 0$

$\Sigma M_x = 0;\quad -300(6) + \left(\frac{2}{7}\right)T_{BC}\ (6) = 0$

$\Sigma M_y = 0;\quad M_{Ay} - 300(2) + \left(\frac{2}{7}\right)T_{BC}\ (4) = 0$

$\Sigma M_z = 0;\quad M_{Az} - \left(\frac{3}{7}\right)T_{BC}\ (6) + \left(\frac{6}{7}\right)T_{BC}\ (4) = 0$

Solving,

$T_{BC} = 1.05 \text{ kip}$ **Ans**

$A_x = -450 \text{ lb}$ **Ans**

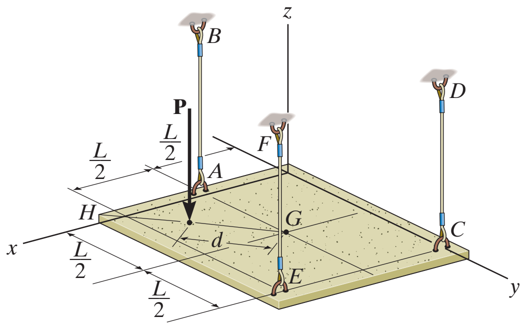
$A_y = 900 \text{ lb}$ **Ans**

$A_z = 0$ **Ans**

$M_{Ay} = -600 \text{ lb} \cdot \text{ft}$ **Ans**

$M_{Az} = -900 \text{ lb} \cdot \text{ft}$ **Ans**

•5–77. The plate has a weight of W with center of gravity at G. Determine the distance d along line GH where the vertical force $P = 0.75W$ will cause the tension in wire CD to become zero.



Equations of Equilibrium: From the free - body diagram, Fig. a,

$$\Sigma M_x = 0; \quad T_{EF}(L) - W\left(\frac{L}{2}\right) - 0.75W\left(\frac{L}{2} - d\cos 45°\right) = 0$$

$$T_{EF}L - 0.875WL + 0.5303Wd = 0 \qquad (1)$$

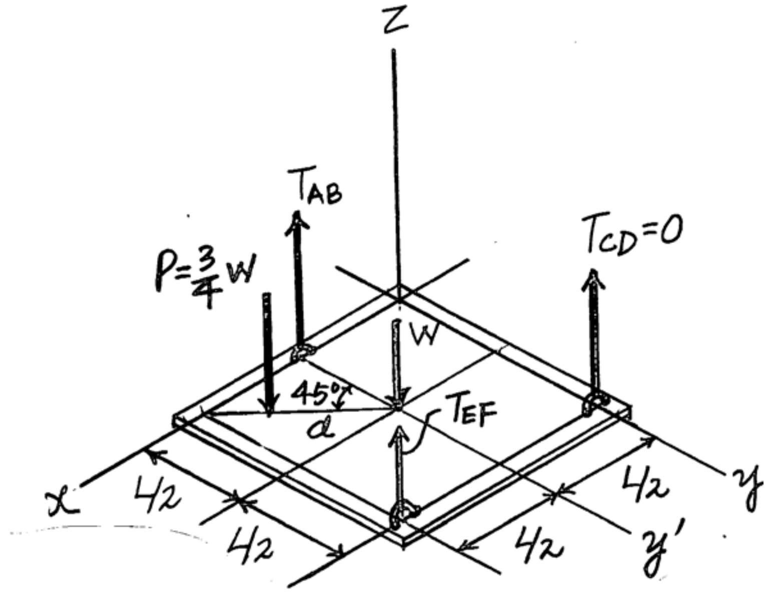
$$\Sigma M_{y'} = 0; \quad 0.75W(d\sin 45°) - T_{EF}\left(\frac{L}{2}\right) = 0$$

$$1.0607Wd - T_{EF}L = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields

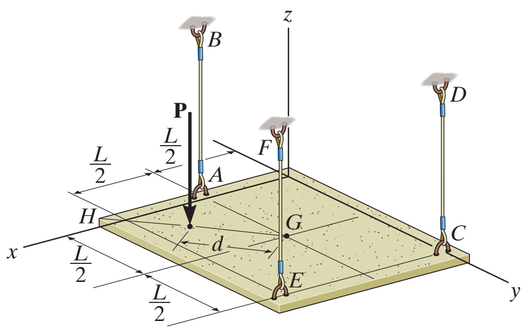
$$d = 0.550L \qquad \textbf{Ans.}$$

$$T_{EF} = 0.583W \qquad \textbf{Ans.}$$



(a)

5–78. The plate has a weight of W with center of gravity at G. Determine the tension developed in wires AB, CD, and EF if the force $P = 0.75W$ is applied at $d = L/2$.



Equations of Equilibrium: From the free - body diagram, Fig. a, T_{AB} can be obtained by writing the moment equation of equilibrium about the x' axis.

$\Sigma M_{x'} = 0;$ $$0.75W\left[\frac{L}{2}+\frac{L}{2}\cos 45°\right]+W\left(\frac{L}{2}\right)-T_{AB}(L)=0$$

$$T_{AB} = 1.1402\ W = 1.14\ W$$ **Ans.**

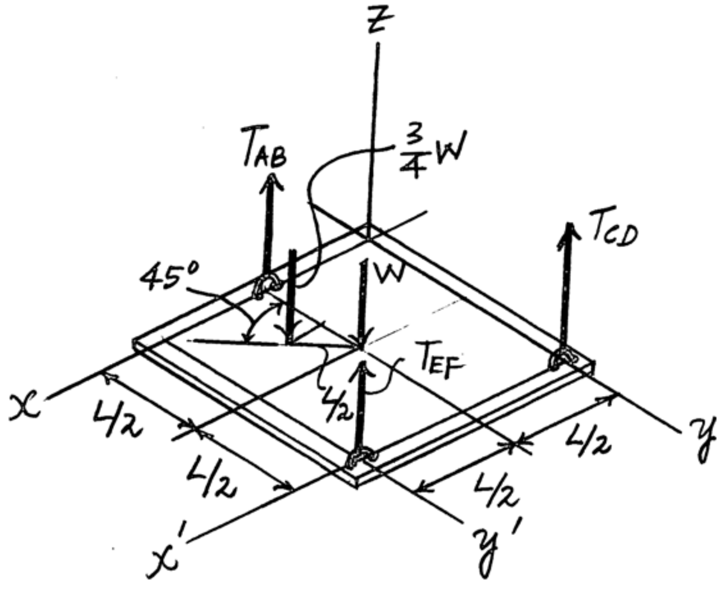
Using the above result and writing the moment equations of equilibrium about the y and y' axes,

$\Sigma M_y = 0;$ $$W\left(\frac{L}{2}\right)+0.75W\left[\frac{L}{2}+\frac{L}{2}\sin 45°\right]-1.1402W\left(\frac{L}{2}\right)-T_{EF}(L)=0$$

$$T_{EF} = 0.570\ W$$ **Ans.**

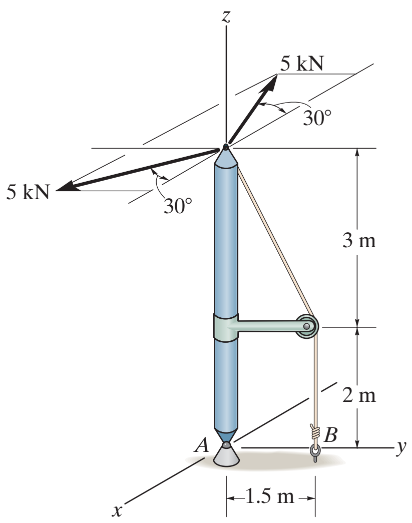
$\Sigma M_{y'} = 0;$ $$T_{CD}(L)+1.1402W\left(\frac{L}{2}\right)-W\left(\frac{L}{2}\right)-0.75W\left[\frac{L}{2}-\frac{L}{2}\sin 45°\right]=0$$

$$T_{CD} = 0.0398\ W$$ **Ans.**



(a)

5–79. The boom is supported by a ball-and-socket joint at A and a guy wire at B. If the 5-kN loads lie in a plane which is parallel to the x–y plane, determine the x, y, z components of reaction at A and the tension in the cable at B.



Equations of Equilibrium :

$\Sigma M_x = 0;$ $2[5\sin 30°(5)] - T_B(1.5) = 0$

$T_B = 16.67\ \text{kN} = 16.7\ \text{kN}$ **Ans**

$\Sigma M_y = 0;$ $5\cos 30°(5) - 5\cos 30°(5) = 0$ (***Statisfied!***)

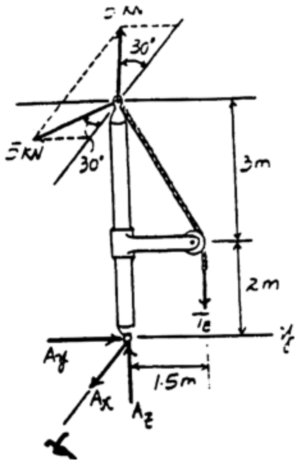
$\Sigma F_x = 0;$ $A_x + 5\cos 30° - 5\cos 30° = 0$

$A_x = 0$ **Ans**

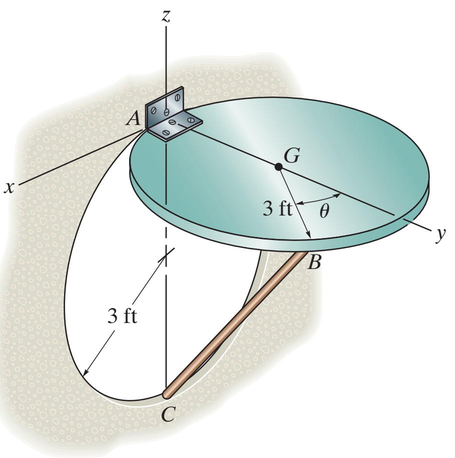
$\Sigma F_y = 0;$ $A_y - 2(5\sin 30°) = 0$

$A_y = 5.00\ \text{kN}$ **Ans**

$\Sigma F_z = 0;$ $A_z - 16.67 = 0$ $A_z = 16.7\ \text{kN}$ **Ans**



***5–80.** The circular door has a weight of 55 lb and a center of gravity at G. Determine the x, y, z components of reaction at the hinge A and the force acting along strut CB needed to hold the door in equilibrium. Set $\theta = 45°$.



$$\mathbf{r}_{CB} = 3\sin 45°\,\mathbf{i} + (3 + 3\cos 45°)\,\mathbf{j} + 6\,\mathbf{k}$$

$$= \{2.121\,\mathbf{i} + 5.121\,\mathbf{j} + 6\,\mathbf{k}\}\text{ ft}$$

$$r_{CB} = \sqrt{(2.121)^2 + (5.121)^2 + (6)^2} = 8.169$$

$$\Sigma F_x = 0; \quad A_x + \left(\frac{2.121}{8.169}\right)F_{CB} = 0$$

$$\Sigma F_y = 0; \quad A_y + \left(\frac{5.121}{8.169}\right)F_{CB} = 0$$

$$\Sigma F_z = 0; \quad A_z - 55 + \left(\frac{6}{8.169}\right)F_{CB} = 0$$

$$\Sigma M_x = 0; \quad -55(3) + \left(\frac{6}{8.169}\right)F_{CB}(3 + 3\cos 45°) = 0$$

$$F_{CB} = 43.9\text{ lb} \qquad \textbf{Ans}$$

$$A_x = -11.4\text{ lb} \qquad \textbf{Ans}$$

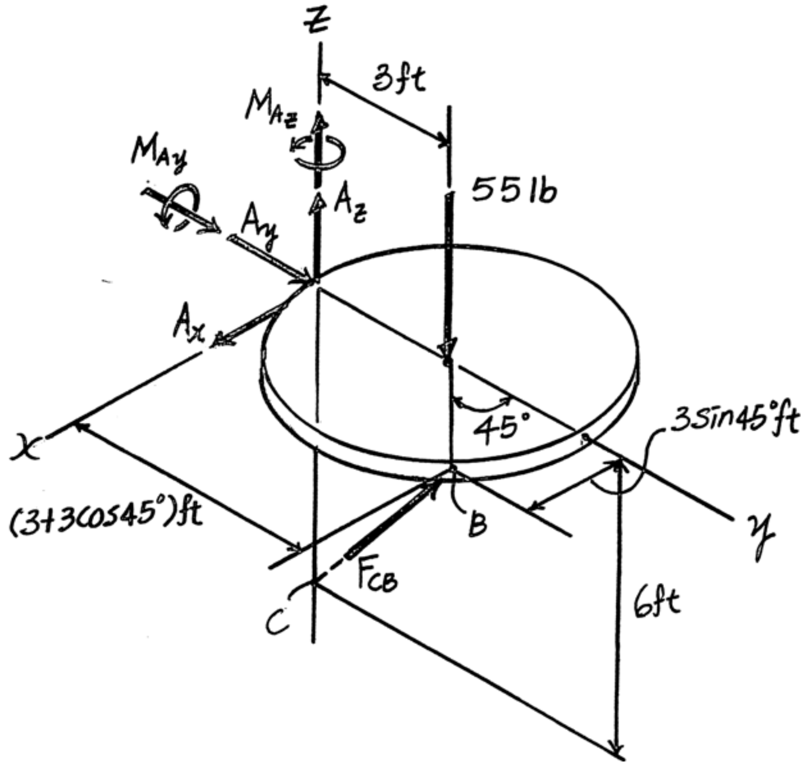
$$A_y = -27.5\text{ lb} \qquad \textbf{Ans}$$

$$A_z = 22.8\text{ lb} \qquad \textbf{Ans}$$

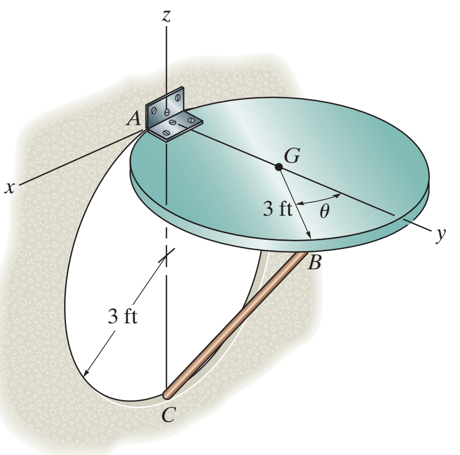
$$\Sigma M_y = 0; \quad M_{Ay} - \left(\frac{6}{8.169}\right)(43.9)(3\sin 45°) = 0; \quad M_{Ay} = 68.3\text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$\Sigma M_z = 0; \quad M_{Az} - \left(\frac{2.121}{8.169}\right)(43.9)(3 + 3\cos 45°) + \left(\frac{5.121}{8.169}\right)(43.9)(3\sin 45°) = 0$$

$$M_{Az} = 0 \qquad \textbf{Ans}$$



•5–81. The circular door has a weight of 55 lb and a center of gravity at *G*. Determine the *x*, *y*, *z* components of reaction at the hinge *A* and the force acting along strut *CB* needed to hold the door in equilibrium. Set $\theta = 90°$.



$\mathbf{r}_{CB} = \{3\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k}\}$ ft

$r_{CB} = \sqrt{(3)^2 + (3)^2 + (6)^2} = \sqrt{54}$

$\Sigma F_x = 0; \quad A_x + \left(\frac{3}{\sqrt{54}}\right)F_{CB} = 0$

$\Sigma F_y = 0; \quad A_y + \left(\frac{3}{\sqrt{54}}\right)F_{CB} = 0$

$\Sigma F_z = 0; \quad A_z - 55 + \left(\frac{6}{\sqrt{54}}\right)F_{CB} = 0$

$\Sigma M_x = 0; \quad -55\,(3) + \left(\frac{6}{\sqrt{54}}\right)F_{CB}\,(3) = 0$

$F_{CB} = 67.36 = 67.4$ lb **Ans**

Thus,

$A_x = -27.5$ lb **Ans**

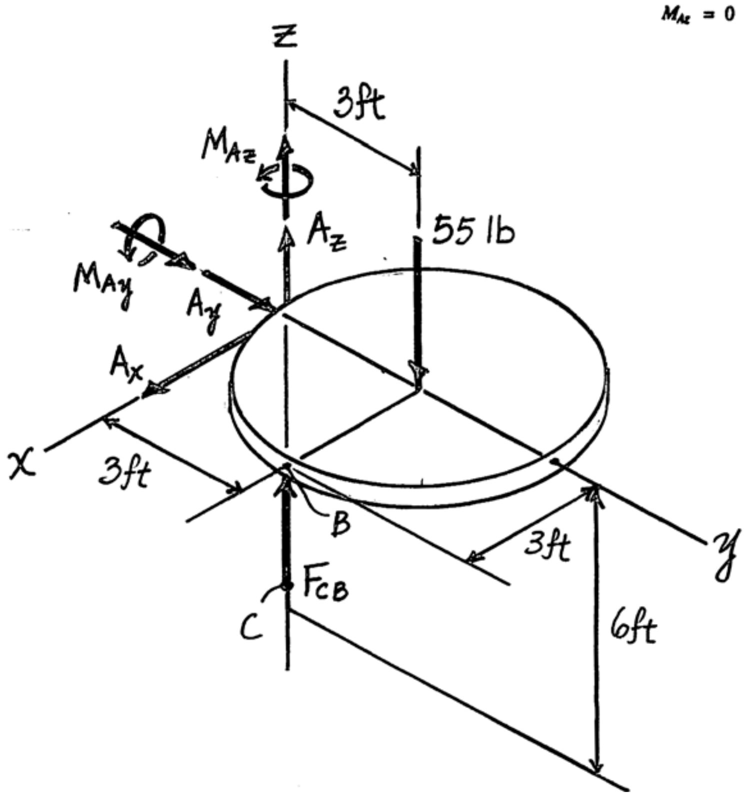
$A_y = -27.5$ lb **Ans**

$A_z = 0$ **Ans**

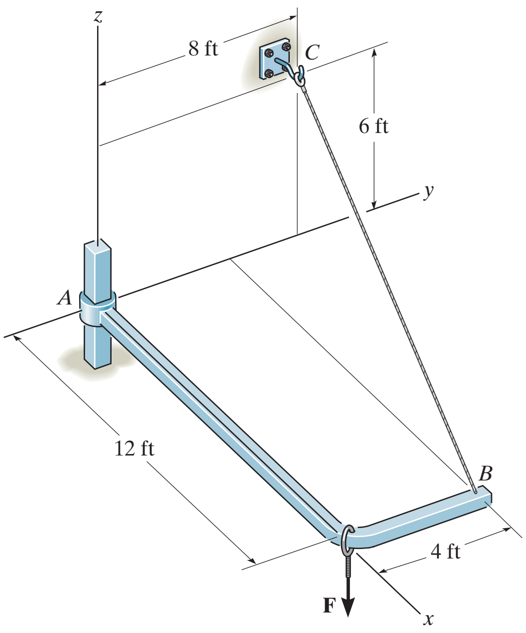
$\Sigma M_y = 0; \quad M_{Ay} - \left(\frac{6}{\sqrt{54}}\right)(67.36)\,(3) = 0; \quad M_{Ay} = 165$ lb·ft **Ans**

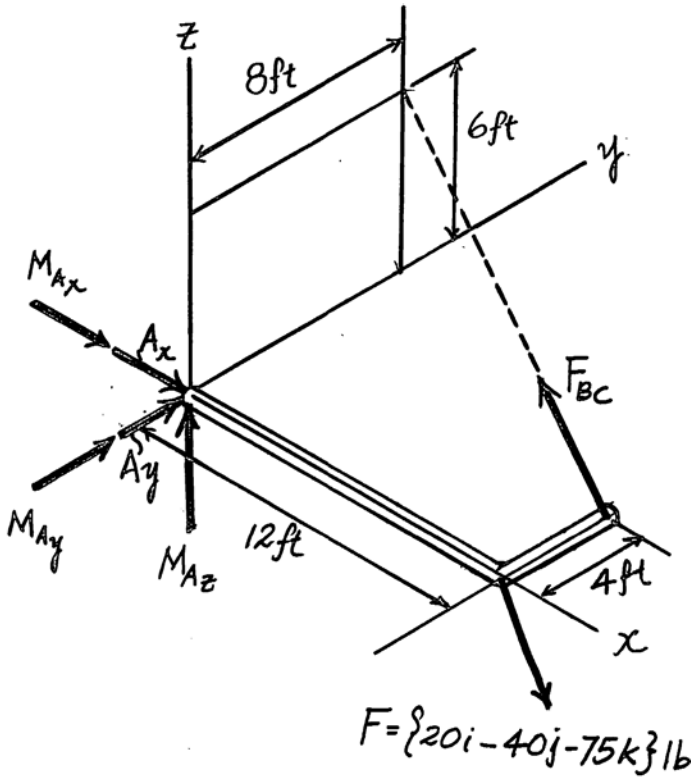
$\Sigma M_z = 0; \quad M_{Az} - \left(\frac{3}{\sqrt{54}}\right)(67.36)\,(3) + \left(\frac{3}{\sqrt{54}}\right)(67.36)\,(3) = 0$

$M_{Az} = 0$ **Ans**



5–82. Member AB is supported at B by a cable and at A by a smooth fixed *square* rod which fits loosely through the square hole of the collar. If $\mathbf{F} = \{20\mathbf{i} - 40\mathbf{j} - 75\mathbf{k}\}$ lb, determine the x, y, z components of reaction at A and the tension in the cable.





$\mathbf{F}_{BC} = -\frac{12}{14}F_{BC}\mathbf{i} + \frac{4}{14}F_{BC}\mathbf{j} + \frac{6}{14}F_{BC}\mathbf{k}$

$\Sigma F_x = 0;$ $\quad A_x + 20 - \frac{12}{14}F_{BC} = 0$

$\Sigma F_y = 0;$ $\quad A_y - 40 + \frac{4}{14}F_{BC} = 0$

$\Sigma F_z = 0;$ $\quad -75 + \frac{6}{14}F_{BC} = 0$

$F_{BC} = 175 \text{ lb}$ **Ans**

$A_x = 130 \text{ lb}$ **Ans**

$A_y = -10 \text{ lb}$ **Ans**

$\Sigma M_x = 0;$ $\quad \frac{6}{14}(175)(4) + M_{Ax} = 0$

$M_{Ax} = -300 \text{ lb}\cdot\text{ft}$ **Ans**

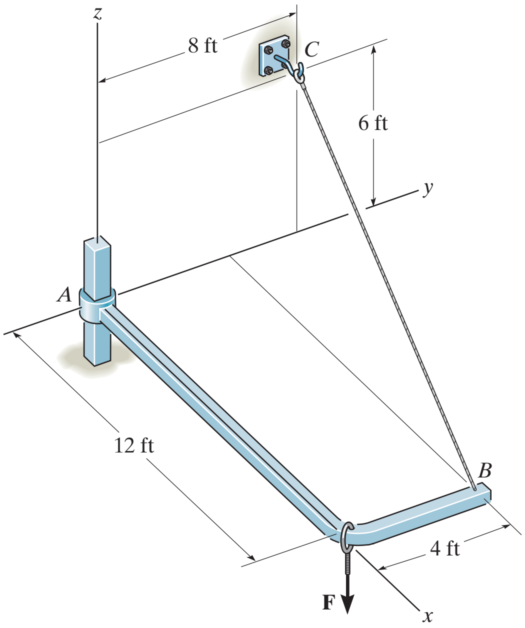
$\Sigma M_y = 0;$ $\quad 75(12) - \frac{6}{14}(175)(12) + M_{Ay} = 0$

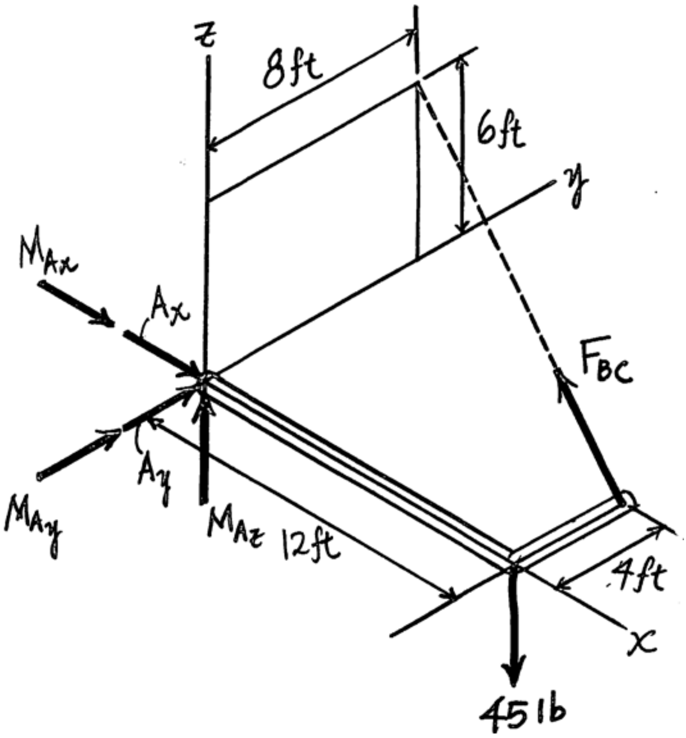
$M_{Ay} = 0$ **Ans**

$\Sigma M_z = 0;$ $\quad -40(12) + \frac{12}{14}(175)(4) + \frac{4}{14}(175)(12) + M_{Az} = 0$

$M_{Az} = -720 \text{ lb}\cdot\text{ft}$ **Ans**

5–83. Member *AB* is supported at *B* by a cable and at *A* by a smooth fixed *square* rod which fits loosely through the square hole of the collar. Determine the tension in cable *BC* if the force **F** = {−45**k**} lb.



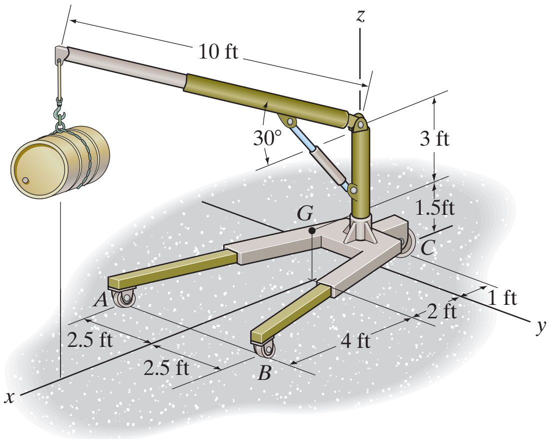


$$\mathbf{F}_{BC} = -\frac{12}{14}F_{BC}\mathbf{i} +$$

$\Sigma F_z = 0;$ $\quad \frac{6}{14}F_{BC} - 45 = 0$

$F_{BC} = 105\text{ lb}$ **Ans**

***5–84.** Determine the largest weight of the oil drum that the floor crane can support without overturning. Also, what are the vertical reactions at the smooth wheels *A, B,* and *C* for this case. The floor crane has a weight of 300 lb, with its center of gravity located at *G*.



Equations of Equilibrium: The floor crane has a tendency to overturn about the y' axis, as shown on the free - body diagram in Fig. *a*. When the crane is about to overturn, the wheel at C loses contact with the ground. Thus

$$N_C = 0 \qquad \textbf{Ans.}$$

Applying the moment equation of equilibrium about the y' axis,

$$\Sigma M_{y'} = 0; \qquad W(10\cos 30° - 2 - 4) - 300(4) = 0$$

$$W = 451.08 \text{ lb} = 451 \text{ lb} \qquad \textbf{Ans.}$$

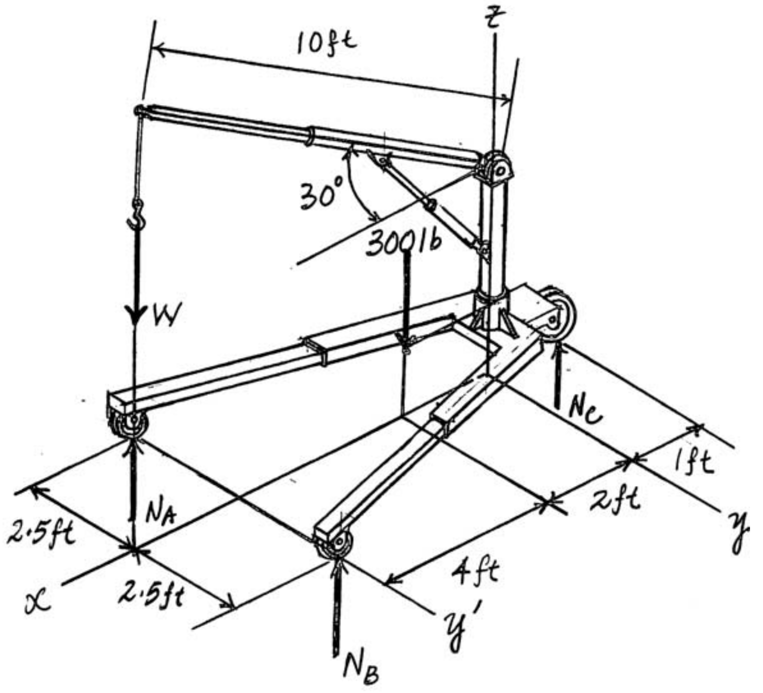
Using this result and writing the moment equation of equilibrium about the x axis and the force equation of equilibrium along the z axis,

$$\Sigma M_x = 0; \quad N_B(2.5) - N_A(2.5) = 0 \qquad (1)$$

$$\Sigma F_z = 0; \quad N_A + N_B - 300 - 451.08 = 0 \qquad (2)$$

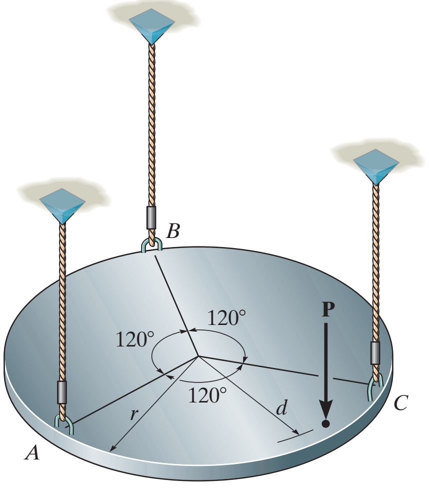
Solving Eqs. (1) and (2), yields

$$N_A = N_B = 375.54 \text{ lb} = 376 \text{ lb} \qquad \textbf{Ans.}$$



(a)

•5–85. The circular plate has a weight *W* and center of gravity at its center. If it is supported by three vertical cords tied to its edge, determine the largest distance *d* from the center to where any vertical force **P** can be applied so as not to cause the force in any one of the cables to become zero.



Assume $T_A = T_B = 0$

$\Sigma M_{a-a} = 0; \quad T_C(r + r\cos 60^\circ) - W(r\cos 60^\circ) - P(d + r\cos 60^\circ) = 0$

$\Sigma F_z = 0; \quad T_C - W - P = 0$

Eliminating T_C we get

$$Wr + Pr - Pd = 0$$

$$d = r\left(1 + \frac{W}{P}\right)$$

Assume $T_C = 0$

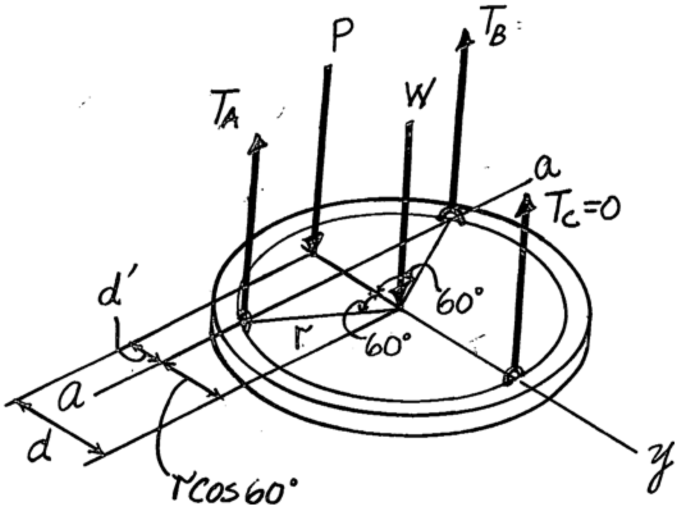
$\Sigma M_{a-a} = 0; \quad W(r\cos 60^\circ) - P(d') = 0$

$$d' = \frac{Wr}{2P}$$

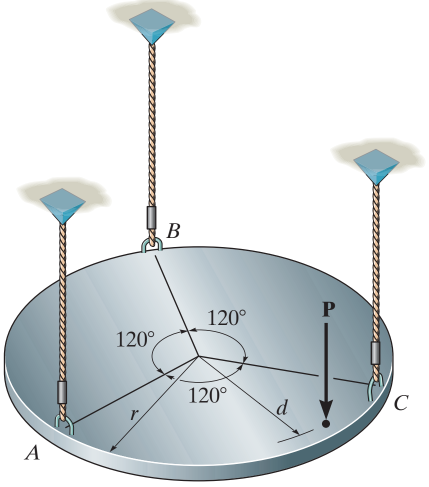
Thus

$$d = r\cos 60^\circ + \frac{Wr}{2P}$$

$$d = \frac{r}{2}\left(1 + \frac{W}{P}\right) \qquad \text{Ans}$$



5–86. Solve Prob. 5–85 if the plate's weight W is neglected.



Assume $T_A = T_B = 0$

$\Sigma M_{a-a} = 0;$ $T_C(r + r\cos 60°) - P(d + r\cos 60°) = 0$

$\Sigma F_z = 0;$ $T_C - P = 0$

$d = r$

Assume $T_C = 0$:

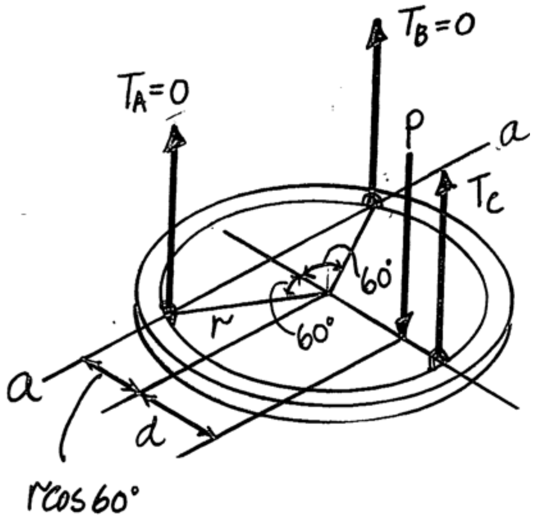
$\Sigma M_{a-a} = 0;$ $P(d') = 0$

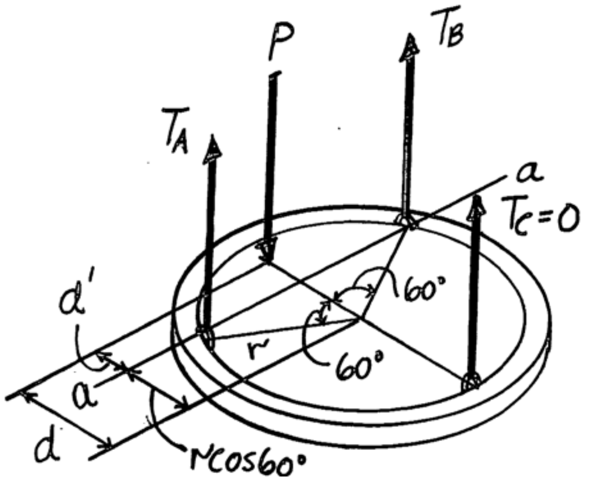
$d' = 0$

Thus,

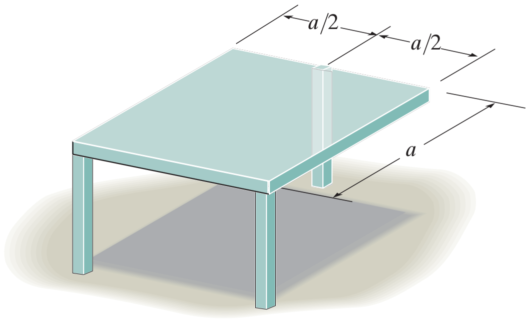
$d = r\cos 60° + 0$

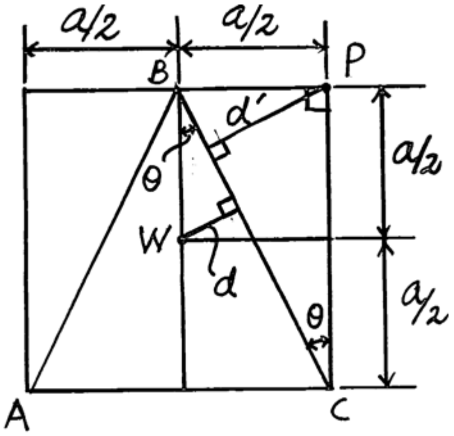
$d = \frac{r}{2}$ **Ans**





5–87. A uniform square table having a weight *W* and sides *a* is supported by three vertical legs. Determine the smallest vertical force **P** that can be applied to its top that will cause it to tip over.





$$\theta = \tan^{-1}\left(\frac{\frac{a}{2}}{a}\right) = 26.565^\circ$$

$$d = \left(\frac{a}{2}\right)\sin 26.565^\circ = 0.2236\,a < \frac{a}{2}$$

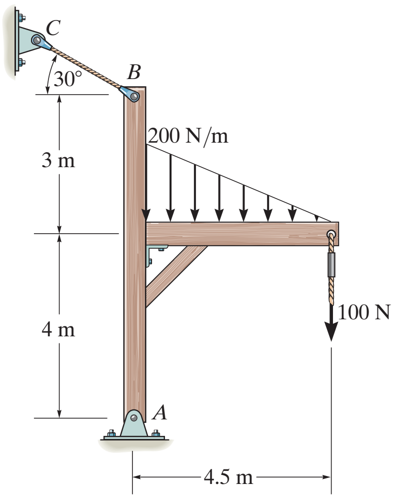
$$d' = a\sin 26.565^\circ = 0.4472\,a$$

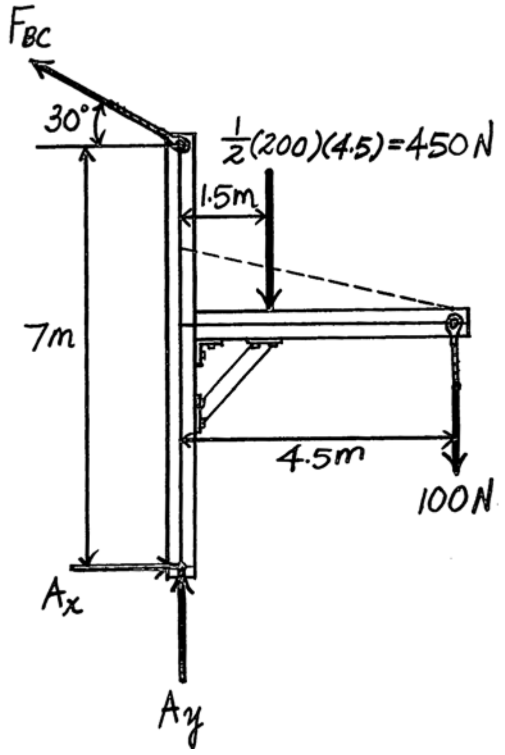
For P_{min}, put P at the corner as shown.

$$\Sigma M_{BC} = 0; \quad W(0.2236\,a) - P(0.4472\,a) = 0$$

$$P = 0.5\,W \qquad \text{Ans}$$

***5–88.** Determine the horizontal and vertical components of reaction at the pin *A* and the force in the cable *BC*. Neglect the thickness of the members.





$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC}\cos 30°(7) - 450(1.5) - 100(4.5) = 0$

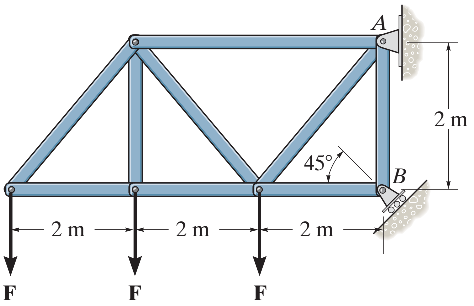
$F_{BC} = 185.58\ N = 186\ N$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 185.58\cos 30° = 0 \quad A_x = 161\ N$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + 185.58\sin 30° - 450 - 100 = 0$

$A_y = 457\ N$ **Ans**

•5–89. Determine the horizontal and vertical components of reaction at the pin *A* and the reaction at the roller *B* required to support the truss. Set $F = 600$ N.

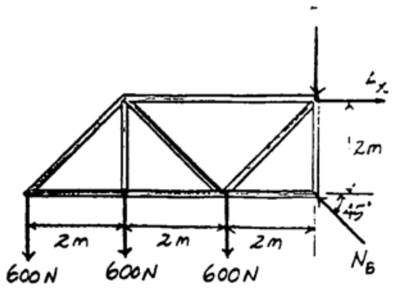


Equations of Equilibrium : The normal reaction N_B can be obtained directly by summing moments about point A.

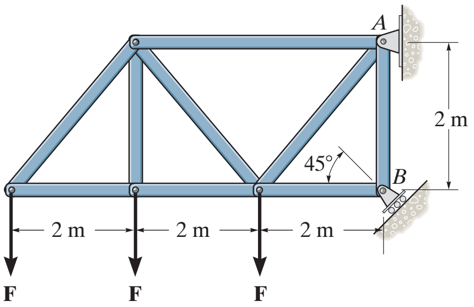
$$\curvearrowleft + \Sigma M_A = 0; \quad 600(6) + 600(4) + 600(2) - N_B \cos 45°(2) = 0$$
$$N_B = 5091.17 \text{ N} = 5.09 \text{ kN} \quad \textbf{Ans}$$

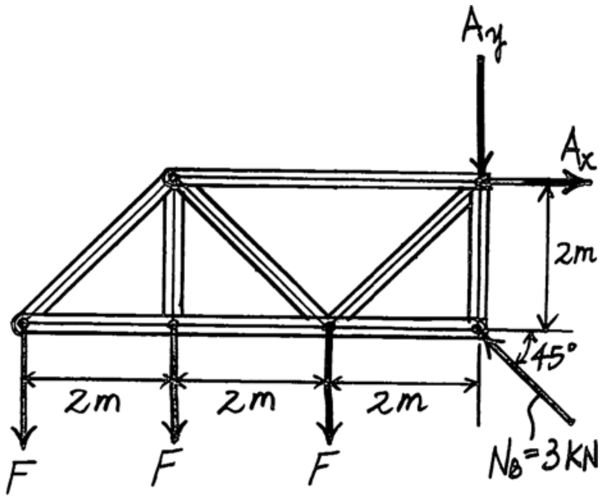
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 5091.17 \cos 45° = 0$$
$$A_x = 3600 \text{ N} = 3.60 \text{ kN} \quad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 5091.17 \sin 45° - 3(600) - A_y = 0$$
$$A_y = 1800 \text{ N} = 1.80 \text{ kN} \quad \textbf{Ans}$$



5–90. If the roller at *B* can sustain a maximum load of 3 kN, determine the largest magnitude of each of the three forces **F** that can be supported by the truss.

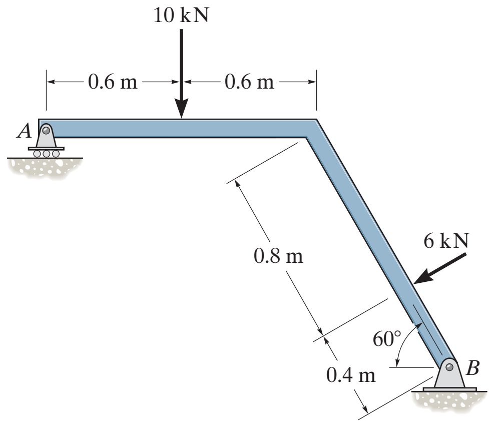




Equations of Equilibrium : The unknowns A_x and A_y can be eliminated by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \quad F(6) + F(4) + F(2) - 3\cos 45°(2) = 0$$
$$F = 0.3536 \text{ kN} = 354\text{N} \quad \textbf{Ans}$$

5–91. Determine the normal reaction at the roller *A* and horizontal and vertical components at pin *B* for equilibrium of the member.



Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point B.

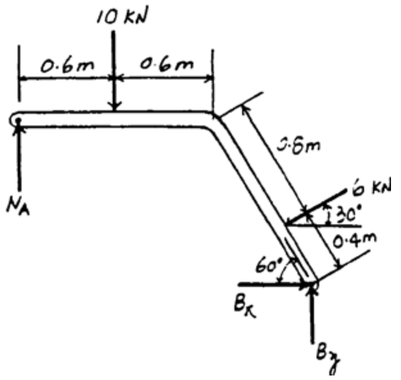
$$\curvearrowleft + \Sigma M_A = 0; \quad 10(0.6 + 1.2\cos 60°) + 6(0.4) - N_A(1.2 + 1.2\cos 60°) = 0$$

$$N_A = 8.00 \text{ kN} \qquad \textbf{Ans}$$

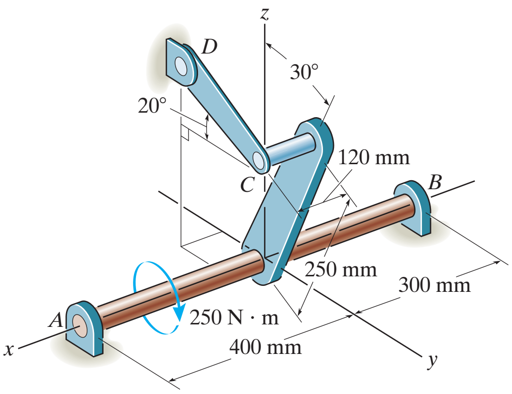
$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 6\cos 30° = 0 \quad B_x = 5.20 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad B_y + 8.00 - 6\sin 30° - 10 = 0$$

$$B_y = 5.00 \text{ kN} \qquad \textbf{Ans}$$



***5–92.** The shaft assembly is supported by two smooth journal bearings *A* and *B* and a short link *DC*. If a couple moment is applied to the shaft as shown, determine the components of force reaction at the journal bearings and the force in the link. The link lies in a plane parallel to the *y*–*z* plane and the bearings are properly aligned on the shaft.



$$\Sigma M_x = 0; \quad -250 + F_{CD}\cos 20°(0.25\cos 30°) + F_{CD}\sin 20°(0.25\sin 30°) = 0$$

$$F_{CD} = 1015.43\text{ N} = 1.02\text{ kN} \qquad \text{Ans}$$

$$\Sigma(M_B)_y = 0; \quad -A_z(0.7) - 1015.43\sin 20°(0.42) = 0$$

$$A_z = -208.38 = -208\text{ N} \qquad \text{Ans}$$

$$\Sigma F_z = 0; \quad -208.38 + 1015.43\sin 20° + B_z = 0$$

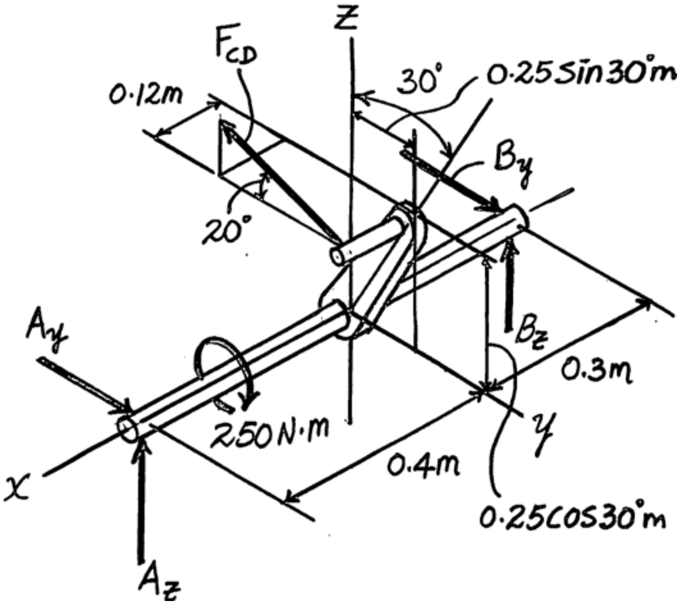
$$B_z = -139\text{ N} \qquad \text{Ans}$$

$$\Sigma(M_B)_z = 0; \quad A_y(0.7) - 1015.43\cos 20°(0.42) = 0$$

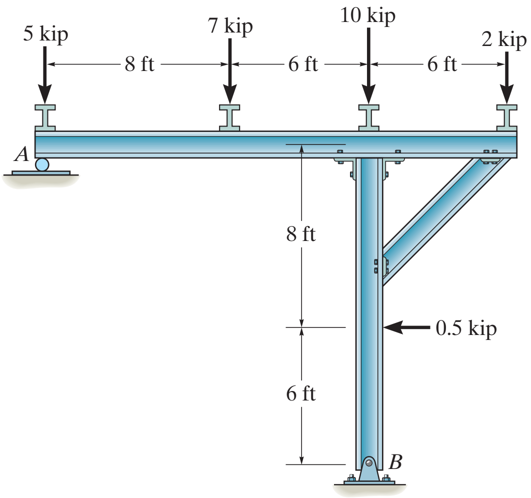
$$A_y = 572.51 = 573\text{ N} \qquad \text{Ans}$$

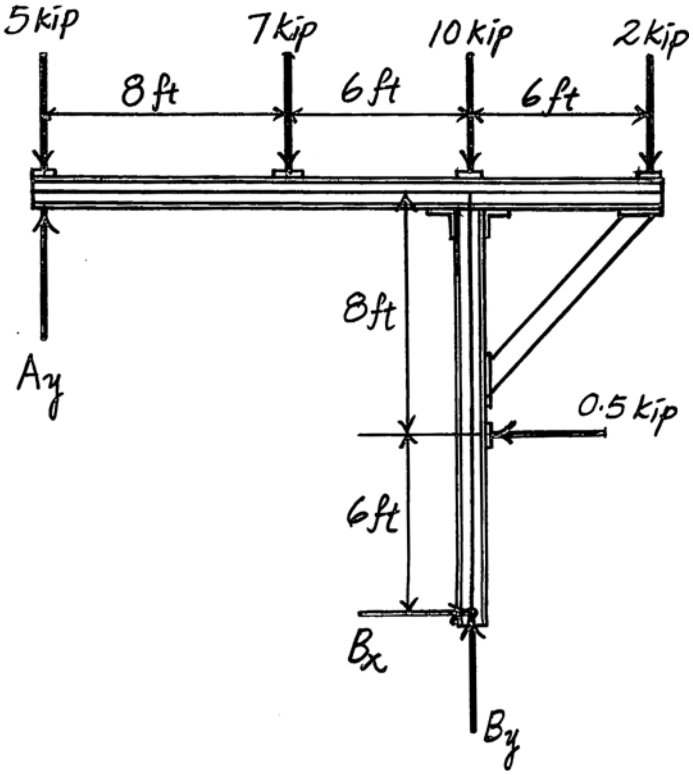
$$\Sigma F_y = 0; \quad 572.51 - 1015.43\cos 20° + B_y = 0$$

$$B_y = 382\text{ N} \qquad \text{Ans}$$



•5–93. Determine the reactions at the supports *A* and *B* of the frame.





$\curvearrowleft + \Sigma M_B = 0;$ $5(14) + 7(6) + 0.5(6) - 2(6) - A_y(14) = 0$

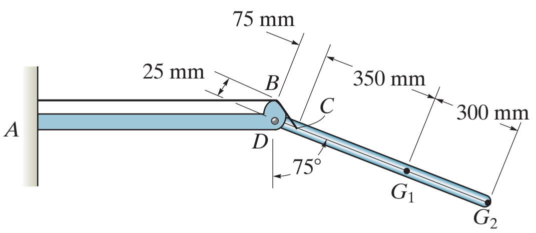
$A_y = 7.357$ kip $= 7.36$ kip **Ans**

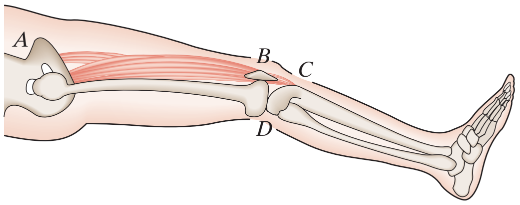
$\xrightarrow{+} \Sigma F_x = 0;$ $B_x - 0.5 = 0$ $B_x = 0.5$ kip **Ans**

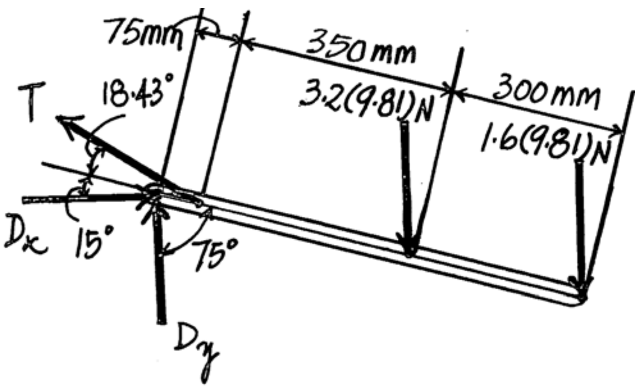
$+\uparrow \Sigma F_y = 0;$ $B_y + 7.357 - 5 - 7 - 10 - 2 = 0$

$B_y = 16.6$ kip **Ans**

5–94. A skeletal diagram of the lower leg is shown in the lower figure. Here it can be noted that this portion of the leg is lifted by the quadriceps muscle attached to the hip at *A* and to the patella bone at *B*. This bone slides freely over cartilage at the knee joint. The quadriceps is further extended and attached to the tibia at *C*. Using the mechanical system shown in the upper figure to model the lower leg, determine the tension in the quadriceps at *C* and the magnitude of the resultant force at the femur (pin), *D*, in order to hold the lower leg in the position shown. The lower leg has a mass of 3.2 kg and a mass center at G_1; the foot has a mass of 1.6 kg and a mass center at G_2.







$\circlearrowleft + \Sigma M_D = 0;\quad T\sin 18.43°(75) - 3.2(9.81)(425\sin 75°)$

$- 1.6(9.81)(725\sin 75°) = 0$

$T = 1006.82\text{ N} = 1.01\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad D_y + 1006.82\sin 33.43° - 3.2(9.81) - 1.6(9.81) = 0$

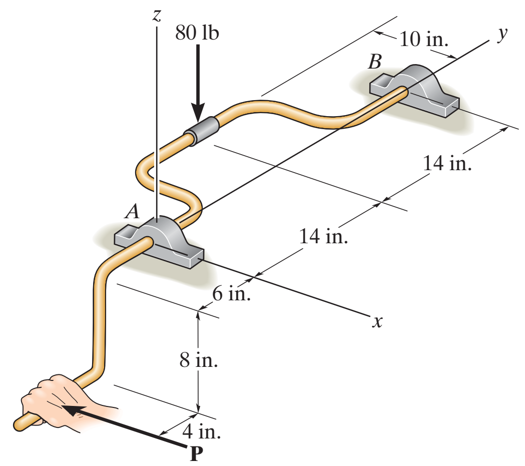
$D_y = -507.66\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0;\quad D_x - 1006.82\cos 33.43° = 0$

$D_x = 840.20\text{ N}$

$F_D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-507.66)^2 + 840.20^2} = 982\text{ N}$ **Ans**

5–95. A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force *P* that must be applied to the handle and the *x, y, z* components of force at the smooth journal bearing *A* and the thrust bearing *B*. The bearings are properly aligned and exert only force reactions on the shaft.



$\Sigma M_y = 0;\quad P(8) - 80(10) = 0 \qquad P = 100\text{ lb}$ **Ans**

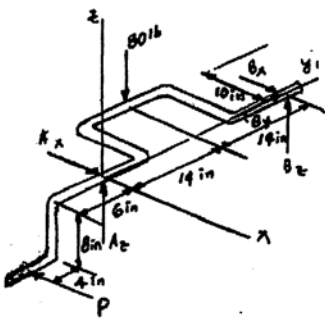
$\Sigma M_x = 0;\quad B_z(28) - 80(14) = 0 \qquad B_z = 40\text{ lb}$ **Ans**

$\Sigma M_z = 0;\quad -B_x(28) - 100(10) = 0 \qquad B_x = -35.7\text{ lb}$ **Ans**

$\Sigma F_x = 0;\quad A_x + (-35.7) - 100 = 0 \qquad A_x = 136\text{ lb}$ **Ans**

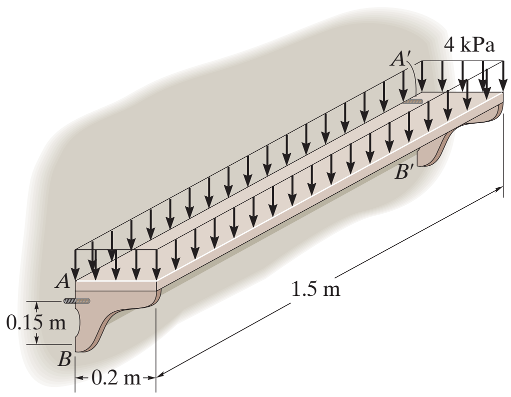
$\Sigma F_y = 0;\quad B_y = 0$ **Ans**

$\Sigma F_z = 0;\quad A_z + 40 - 80 = 0 \qquad A_z = 40\text{ lb}$ **Ans**



Negative sign indicates that B_x acts in the opposite sense to that shown on the FBD.

***5–96.** The symmetrical shelf is subjected to a uniform load of 4 kPa. Support is provided by a bolt (or pin) located at each end A and A' and by the symmetrical brace arms, which bear against the smooth wall on both sides at B and B'. Determine the force resisted by each bolt at the wall and the normal force at B for equilibrium.



Equations of Equilibrium : Each shelf's post at its end supports half of the applied load, ie, $4000(0.2)(0.75) = 600$ N. The normal reaction N_B can be obtained directly by summing moments about point A.

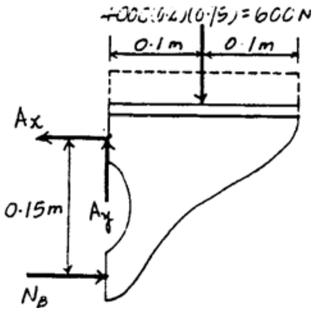
$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(0.15) - 600(0.1) = 0 \quad N_B = 400 \text{ N} \quad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 400 - A_x = 0 \quad A_x = 400\text{N}$$

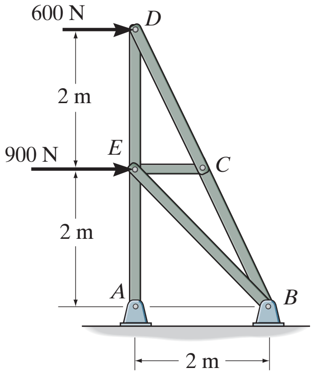
$$+\uparrow \Sigma F_y = 0; \quad A_y - 600 = 0 \quad A_y = 600 \text{ N}$$

The force resisted by the bolt at A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{400^2 + 600^2} = 721 \text{ N} \quad \textbf{Ans}$$



•6–1. Determine the force in each member of the truss, and state if the members are in tension or compression.



Method of Joints: We will begin by analyzing the equilibrium of joint D, and then proceed to analyze joints C and D.

Joint D: From geometry, $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$. Thus, from the free - body diagram in Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $600 - F_{DC}\ \sin 26.57° = 0$

$F_{DC} = 1341.64\text{ N} = 1.34\text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $1341.64 \cos 26.57° - F_{DE} = 0$

$F_{DE} = 1200\text{N} = 1.20\text{kN (T)}$ **Ans.**

Joint C: From the free - body diagram in Fig. b,

$+\nearrow \Sigma F_{x'} = 0;$ $-F_{CE}\ \cos 26.57° = 0$

$F_{CE} = 0$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0;$ $F_{CB} - 1341.64 = 0$

$F_{CB} = 1341.64\text{ N} = 1.34\text{ kN (C)}$ **Ans.**

Joint E: From the free - body diagram in Fig. c,

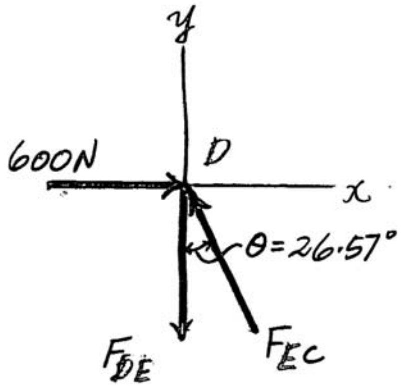
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $900 - F_{EB}\ \sin 45° = 0$

$F_{EB} = 1272.79\text{N} = 1.27\text{ kN (C)}$ **Ans.**

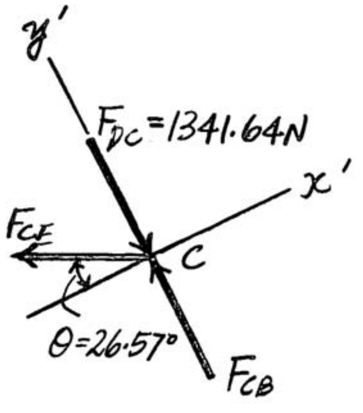
$+\uparrow \Sigma F_y = 0;$ $1200 + 1272.79 \cos 45° - F_{EA} = 0$

$F_{EA} = 2100\text{N} = 2.10\text{kN (T)}$ **Ans.**

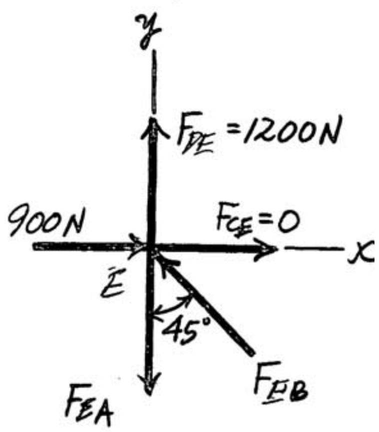
Note: The equilibrium analysis of joint A can be used to determine the components of support reaction at A.



(a)

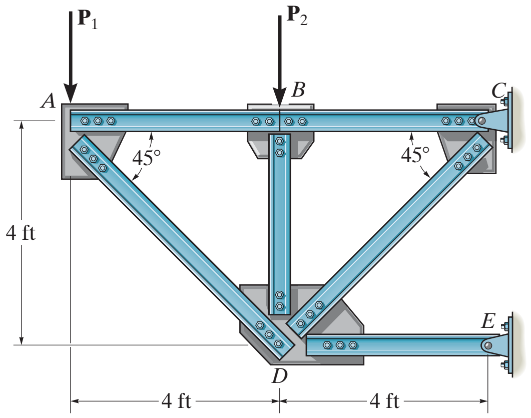


(b)



(c)

6–2. The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression. Set $P_1 = 600$ lb, $P_2 = 400$ lb.



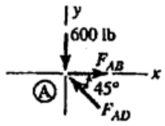
Joint *A* :

$+\uparrow \Sigma F_y = 0;$ $F_{AD}\sin 45° - 600 = 0$

$F_{AD} = 848.528 = 849 \text{ lb (C)}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $F_{AB} - 848.528\cos 45° = 0$

$F_{AB} = 600 \text{ lb (T)}$ **Ans**



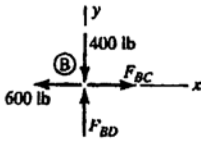
Joint *B* :

$+\uparrow \Sigma F_y = 0;$ $F_{BD} - 400 = 0$

$F_{BD} = 400 \text{ lb (C)}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $F_{BC} - 600 = 0$

$F_{BC} = 600 \text{ lb (T)}$ **Ans**



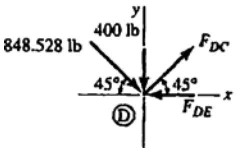
Joint *D*:

$+\uparrow \Sigma F_y = 0;$ $F_{DC}\sin 45° - 400 - 848.528\sin 45° = 0$

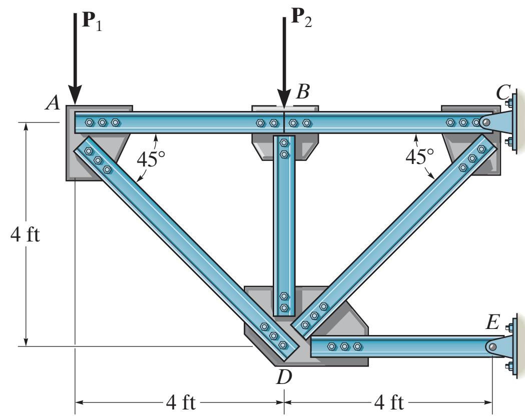
$F_{DC} = 1414.214 \text{ lb} = 1.41 \text{ kip (T)}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $848.528\cos 45° + 1414.214\cos 45° - F_{DE} = 0$

$F_{DE} = 1600 \text{ lb} = 1.60 \text{ kip (C)}$ **Ans**



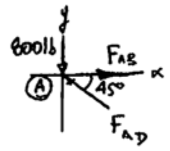
6–3. The truss, used to support a balcony, is subjected to the loading shown. Approximate each joint as a pin and determine the force in each member. State whether the members are in tension or compression. Set $P_1 = 800$ lb, $P_2 = 0$.



Joint ***A*** **:**

$+\uparrow\Sigma F_y = 0;$ $\quad F_{AD}\sin 45° - 800 = 0$

$F_{AD} = 1131.4 \text{ lb} = 1.13 \text{ kip (C)}$ **Ans**



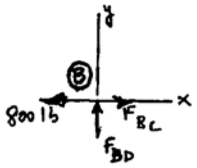
$\xrightarrow{+}\Sigma F_x = 0;$ $\quad F_{AB} - 1131.4\cos 45° = 0$

$F_{AB} = 800 \text{ lb (T)}$ **Ans**

Joint ***B*** **:**

$+\uparrow\Sigma F_y = 0;$ $\quad F_{BD} - 0 = 0$

$F_{BD} = 0$ **Ans**



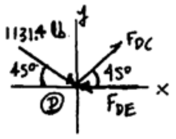
$\xrightarrow{+}\Sigma F_x = 0;$ $\quad F_{BC} - 800 = 0$

$F_{BC} = 800 \text{ lb (T)}$ **Ans**

Joint ***D*** **:**

$+\uparrow\Sigma F_y = 0;$ $\quad F_{DC}\sin 45° - 0 - 1131.4\sin 45° = 0$

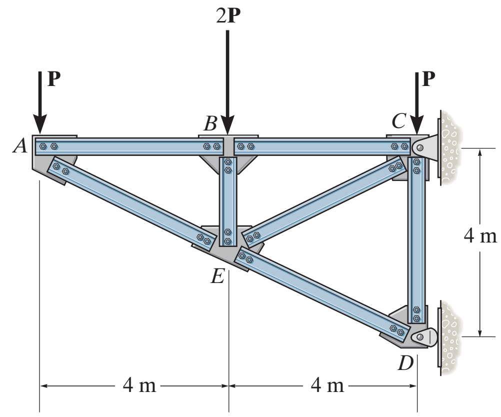
$F_{DC} = 1131.4 \text{ lb} = 1.13 \text{ kip (T)}$ **Ans**



$\xrightarrow{+}\Sigma F_x = 0;$ $\quad 1131.4\cos 45° + 1131.4\cos 45° - F_{DE} = 0$

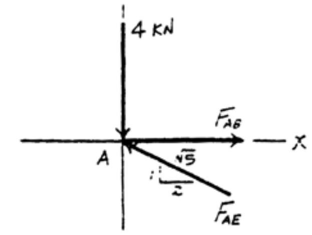
$F_{DE} = 1600 \text{ lb} = 1.60 \text{ kip (C)}$ **Ans**

***6–4.** Determine the force in each member of the truss and state if the members are in tension or compression. Assume each joint as a pin. Set $P = 4$ kN.



Method of Joints : In this case, the support reactions are not required for determining the member forces.

Joint *A*



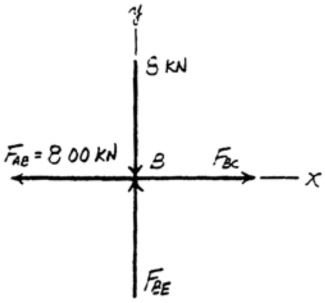
$+\uparrow \Sigma F_y = 0;\quad F_{AE}\left(\frac{1}{\sqrt{5}}\right) - 4 = 0$

$F_{AE} = 8.944$ kN (C) $= 8.94$ kN (C) **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{AB} - 8.944\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{AB} = 8.00$ kN (T) **Ans**

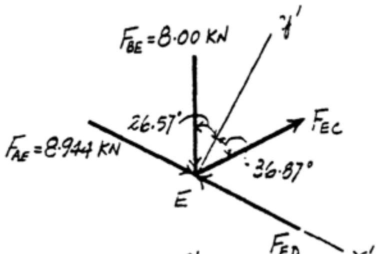
Joint *B*



$\xrightarrow{+} \Sigma F_x = 0;\quad F_{BC} - 8.00 = 0 \quad F_{BC} = 8.00$ kN (T) **Ans**

$+\uparrow \Sigma F_y = 0;\quad F_{BE} - 8 = 0 \quad F_{BE} = 8.00$ kN (C) **Ans**

Joint *E*



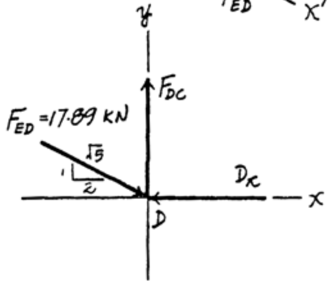
$+\nearrow \Sigma F_{y'} = 0;\quad F_{EC}\cos 36.87° - 8.00\cos 26.57° = 0$

$F_{EC} = 8.944$ kN (T) $= 8.94$ kN (T) **Ans**

$\nwarrow + \Sigma F_{x'} = 0;\quad 8.944 + 8.00\sin 26.57° + 8.944\sin 36.87° - F_{ED} = 0$

$F_{ED} = 17.89$ kN (C) $= 17.9$ kN (C) **Ans**

Joint *D*

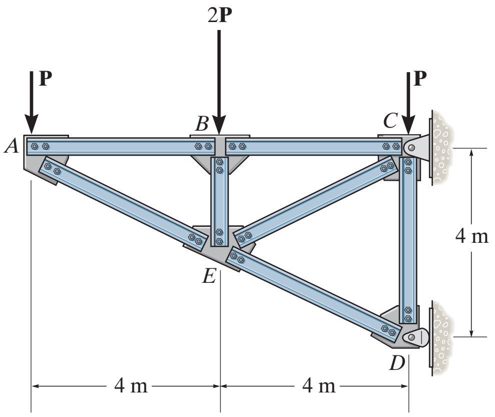


$+\uparrow \Sigma F_y = 0;\quad F_{DC} - 17.89\left(\frac{1}{\sqrt{5}}\right) = 0 \quad F_{DC} = 8.00$ kN (T) **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad D_x - 17.89\left(\frac{2}{\sqrt{5}}\right) = 0 \quad D_x = 16.0$ kN

Note : The support reactions C_x and C_y can be determined by analysing Joint C using the results obtained above.

•6–5. Assume that each member of the truss is made of steel having a mass per length of 4 kg/m. Set $P = 0$, determine the force in each member, and indicate if the members are in tension or compression. Neglect the weight of the gusset plates and assume each joint is a pin. Solve the problem by assuming the weight of each member can be represented as a vertical force, half of which is applied at the end of each member.



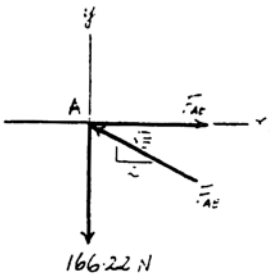
Joint Forces :

$$F_A = 4(9.81)\left(2+\frac{\sqrt{20}}{2}\right) = 166.22\ \text{N}$$

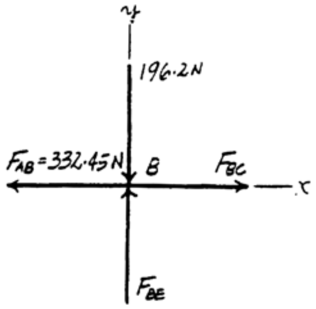
$$F_B = 4(9.81)(2+2+1) = 196.2\ \text{N}$$

$$F_E = 4(9.81)\left[1+3\left(\frac{\sqrt{20}}{2}\right)\right] = 302.47\ \text{N}$$

$$F_D = 4(9.81)\left(2+\frac{\sqrt{20}}{2}\right) = 166.22\ \text{N}$$



Method of Joints : In this case, the support reactions are not required for determining the member forces.



Joint *A*

$$+\uparrow \Sigma F_y = 0; \qquad F_{AE}\left(\frac{1}{\sqrt{5}}\right) - 166.22 = 0$$

$$F_{AE} = 371.69\ \text{N (C)} = 372\ \text{N (C)} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} - 371.69\left(\frac{2}{\sqrt{5}}\right) = 0$$

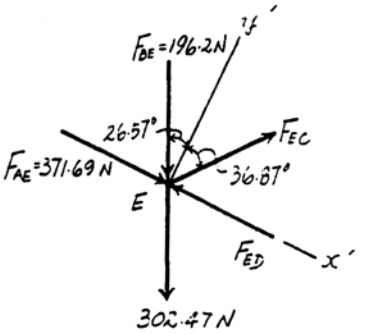
$$F_{AB} = 332.45\ \text{N (T)} = 332\ \text{N (T)} \qquad \textbf{Ans}$$

Joint *B*

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} - 332.45 = 0 \qquad F_{BC} = 332\ \text{N (T)} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BE} - 196.2 = 0$$

$$F_{BE} = 196.2\ \text{N (C)} = 196\ \text{N (C)} \qquad \textbf{Ans}$$



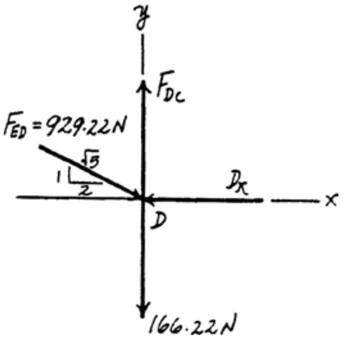
Joint *E*

$$\nearrow + \Sigma F_{y'} = 0; \qquad F_{EC}\cos 36.87° - (196.2 + 302.47)\cos 26.57° = 0$$

$$F_{EC} = 557.53\ \text{N (T)} = 558\ \text{N (T)} \qquad \textbf{Ans}$$

$$\searrow + \Sigma F_{x'} = 0; \qquad 371.69 + (196.2 + 302.47)\sin 26.57° + 557.53\sin 36.87° - F_{ED} = 0$$

$$F_{ED} = 929.22\ \text{N (C)} = 929\ \text{N (C)} \qquad \textbf{Ans}$$

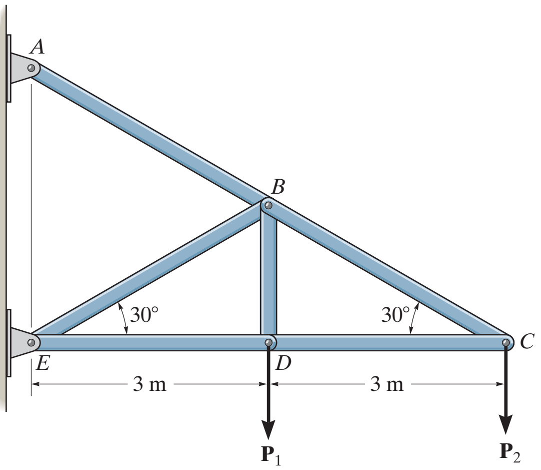


Joint *D*:

$$+\uparrow \Sigma F_y = 0; \qquad -166.22 - 929.22\frac{1}{\sqrt{5}} + F_{DC} = 0$$

$$F_{DC} = 582\text{N (T)} \qquad \textbf{Ans.}$$

6–6. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 2$ kN and $P_2 = 1.5$ kN.



Method of Joints : In this case, the support reactions are not required for determining the member forces.

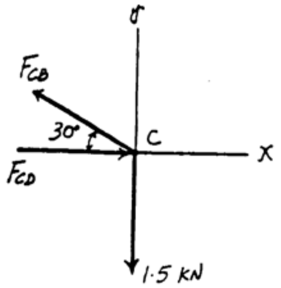
Joint *C*

$+\uparrow \Sigma F_y = 0;$ $F_{CB} \sin 30° - 1.5 = 0$

$F_{CB} = 3.00 \text{ kN (T)}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{CD} - 3.00 \cos 30° = 0$

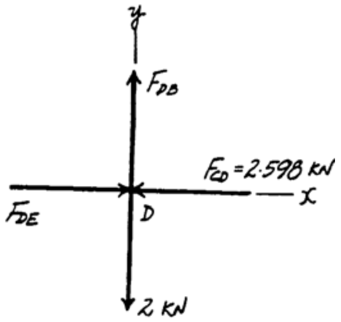
$F_{CD} = 2.598 \text{ kN (C)} = 2.60 \text{ kN (C)}$ **Ans**



Joint *D*

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{DE} - 2.598 = 0$ $F_{DE} = 2.60 \text{ kN (C)}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $F_{DB} - 2 = 0$ $F_{DB} = 2.00 \text{ kN (T)}$ **Ans**



Joint *B*

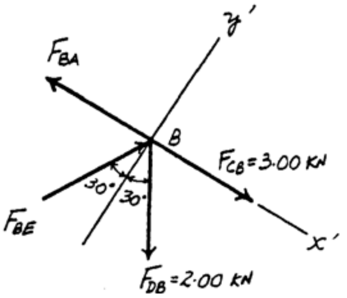
$+\nearrow \Sigma F_{y'} = 0;$ $F_{BE} \cos 30° - 2.00 \cos 30° = 0$

$F_{BE} = 2.00 \text{ kN (C)}$ **Ans**

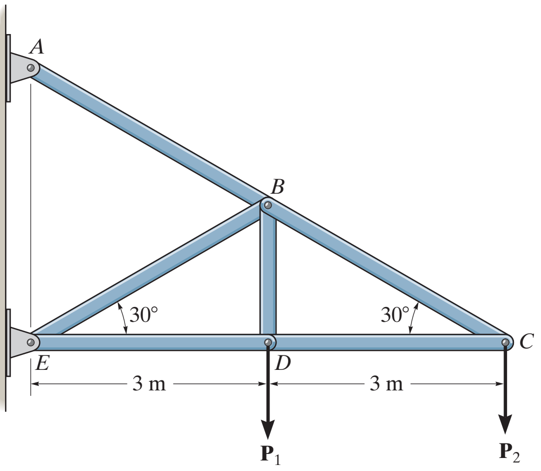
$\nwarrow + \Sigma F_{x'} = 0;$ $(2.00 + 2.00) \sin 30° + 3.00 - F_{BA} = 0$

$F_{BA} = 5.00 \text{ kN (T)}$ **Ans**

Note : The support reactions at support *A* and *E* can be determined by analyzing Joints *A* and *E* respectively using the results obtained above.



6–7. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = P_2 = 4$ kN.



Method of Joints : In this case, the support reactions are not required for determining the member forces.

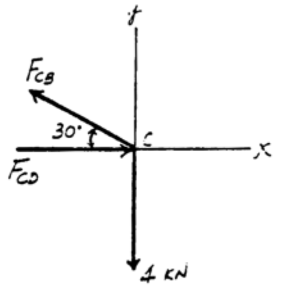
Joint *C*

$+\uparrow \Sigma F_y = 0;$ $F_{CB} \sin 30° - 4 = 0$

$F_{CB} = 8.00$ kN (T) **Ans**

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{CD} - 8.00 \cos 30° = 0$

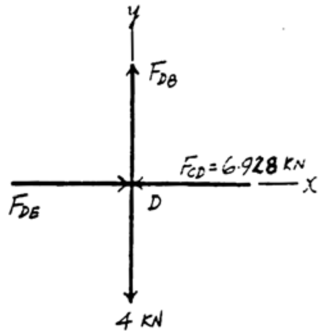
$F_{CD} = 6.928$ kN (C) $= 6.93$ kN (C) **Ans**



Joint *D*

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{DE} - 6.928 = 0$ $F_{DE} = 6.93$ kN (C) **Ans**

$+\uparrow \Sigma F_y = 0;$ $F_{DB} - 4 = 0$ $F_{DB} = 4.00$ kN (T) **Ans**



Joint *B*

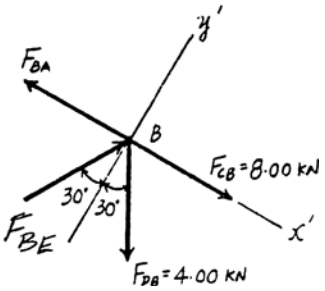
$\nearrow + \Sigma F_{y'} = 0;$ $F_{BE} \cos 30° - 4.00 \cos 30° = 0$

$F_{BE} = 4.00$ kN (C) **Ans**

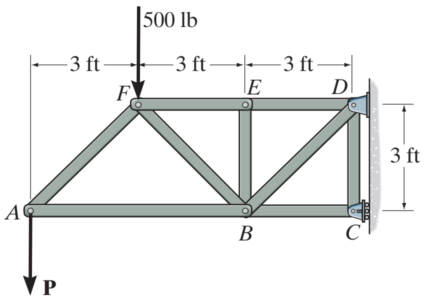
$\searrow + \Sigma F_{x'} = 0;$ $(4.00 + 4.00) \sin 30° + 8.00 - F_{BA} = 0$

$F_{BA} = 12.0$ kN (T) **Ans**

Note : The support reactions at support *A* and *E* can be determined by analyzing Joints *A* and *E* respectively using the results obtained above.



***6–8.** Determine the force in each member of the truss, and state if the members are in tension or compression. Set $P = 800$ lb.



Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $A \rightarrow F \Rightarrow E \rightarrow B \Rightarrow C$.

Joint A**:** From the free - body diagram in Fig. a,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{AF} \sin 45° - 800 = 0$

$F_{AF} = 1131.37 \text{ lb} = 1131 \text{ lb (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1131.37 \cos 45° - F_{AB} = 0$

$F_{AB} = 800 \text{ lb (C)}$ **Ans.**

Joint F**:** From the free - body diagram in Fig. b,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{FB} \cos 45° - 1131.37 \cos 45° - 500 = 0$

$F_{FB} = 1838.48 \text{ lb} = 1838 \text{ lb (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{FE} - 1838.48 \sin 45° - 1131.37 \sin 45° = 0$

$F_{FE} = 2100 \text{ lb (T)}$ **Ans.**

Joint E**:** From the free - body diagram in Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{ED} - 2100 = 0$

$F_{ED} = 2100 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{EB} = 0$ **Ans.**

Joint B**:** From the free - body diagram in Fig. d,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD} \sin 45° - 1838.48 \sin 45° = 0$

$F_{BD} = 1838.48 \text{ lb} = 1838 \text{ lb (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 800 + 1838.48 \cos 45° + 1838.48 \cos 45° - F_{BC} = 0$

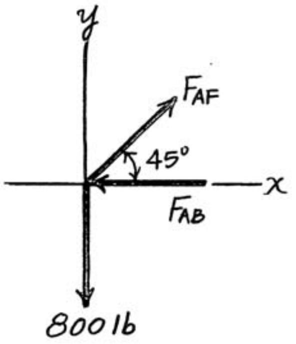
$F_{BC} = 3400 \text{ lb (C)}$ **Ans.**

Joint C**:** From the free - body diagram in Fig. e,

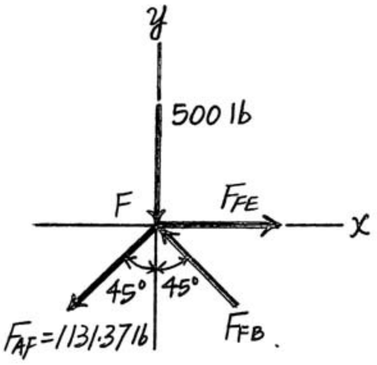
$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD} = 0$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 3400 - N_C = 0$

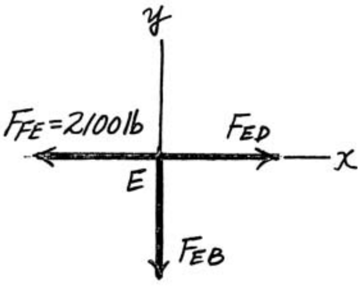
$N_C = 3400 \text{ lb}$



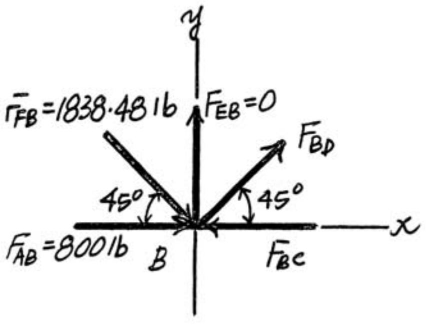
(a)



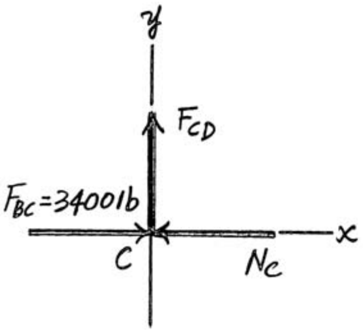
(b)



(C)

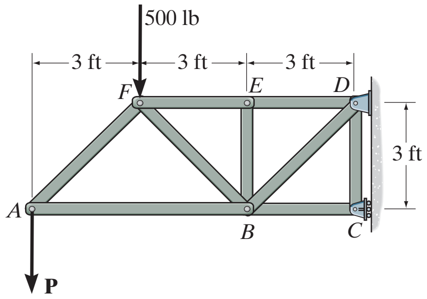


(d)



(e)

•6–9. Remove the 500-lb force and then determine the greatest force *P* that can be applied to the truss so that none of the members are subjected to a force exceeding either 800 lb in tension or 600 lb in compression.



Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $A \rightarrow F \rightarrow E \rightarrow B \rightarrow C$.

Joint *A*: From the free - body diagram in Fig. *a*,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{AF} \sin 45° - P = 0$

$F_{AF} = 1.4142P \text{ (T)}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1.4142P \cos 45° - F_{AB} = 0$

$F_{AB} = P \text{ (C)}$

Joint *F*: From the free - body diagram in Fig. *b*,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{FB} \cos 45° - 1.4142P \cos 45° = 0$

$F_{FB} = 1.4142P \text{ (C)}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{FE} - 1.4142P \sin 45° - 1.4142P \sin 45° = 0$

$F_{FE} = 2P \text{ (T)}$

Joint *E*: From the free - body diagram in Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{ED} - 2P = 0$

$F_{ED} = 2P \text{ (T)}$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{EB} = 0$

Joint *B*: From the free - body diagram in Fig. *d*,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD} \sin 45° - 1.4142P \sin 45° = 0$

$F_{BD} = 1.4142P \text{ (T)}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad P + 1.4142P \cos 45° + 1.4142P \cos 45° - F_{BC} = 0$

$F_{BC} = 3P \text{ (C)}$

Joint *C*: From the free - body diagram in Fig. *e*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 3P - N_C = 0$

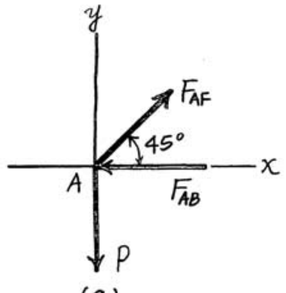
$N_C = 3P$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD} = 0$

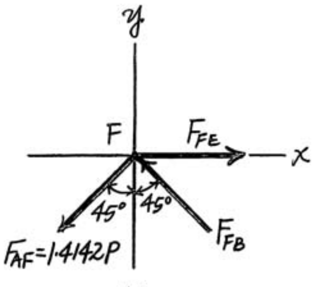
From the above results, the greatest compressive and tensile forces developed in the member are $3P$ and $2P$, respectively.

$2P = 800 \text{ lb}$ $\qquad P = 400 \text{ lb}$

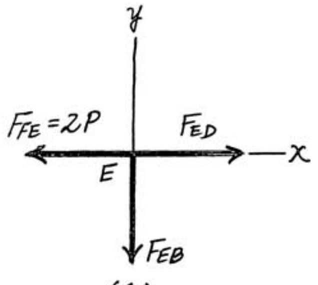
$3P = 600 \text{ lb}$ $\qquad P = 200 \text{ lb}$ $\qquad$ (controls) **Ans.**



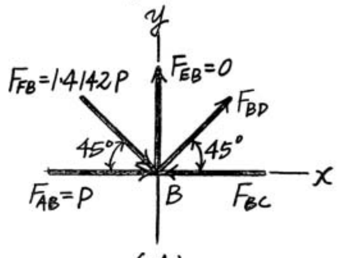
(a)



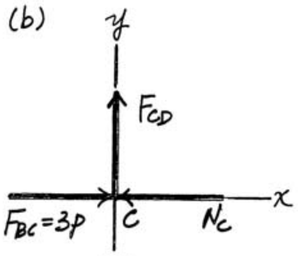
(b)



(c)

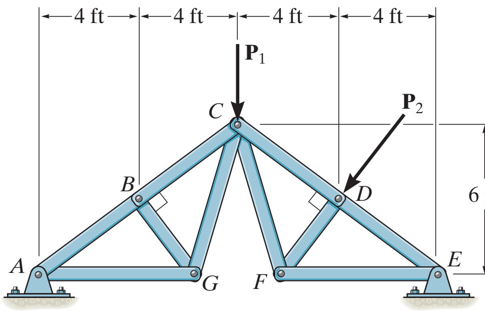


(d)



(e)

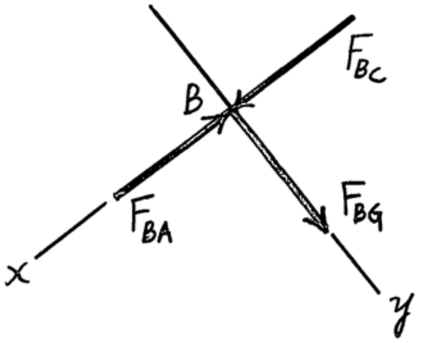
6–10. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 800$ lb, $P_2 = 0$.



Joint B:

$\searrow + \Sigma F_y = 0;\quad F_{BG} = 0$

$+\nearrow \Sigma F_x = 0;\quad F_{BA} = F_{BC}$

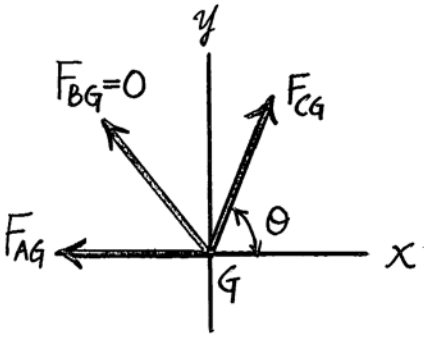


Joint G:

$+\uparrow \Sigma F_y = 0;\quad F_{CG}\sin\theta = 0$

$F_{CG} = 0$

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{AG} = 0$

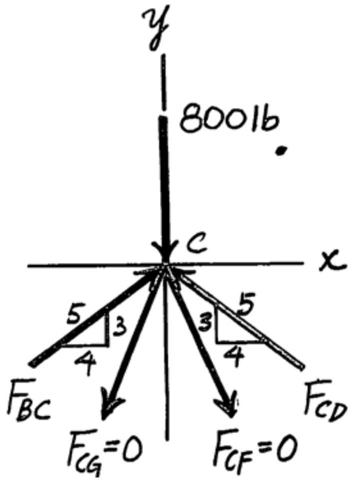


Joint C:

$\xrightarrow{+} \Sigma F_x = 0;\quad \frac{4}{5}F_{BC} - \frac{4}{5}F_{CD} = 0$

$+\uparrow \Sigma F_y = 0;\quad \frac{3}{5}(F_{BC}) + \frac{3}{5}(F_{CD}) - 800 = 0$

$F_{BC} = F_{CD} = 667$ lb (C)



Due to symmetry:

$F_{DF} = F_{BG} = 0$ Ans

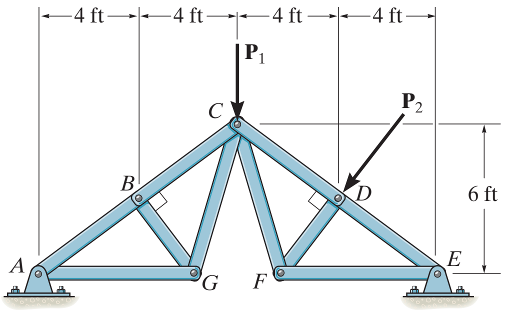
$F_{CF} = F_{CG} = 0$ Ans

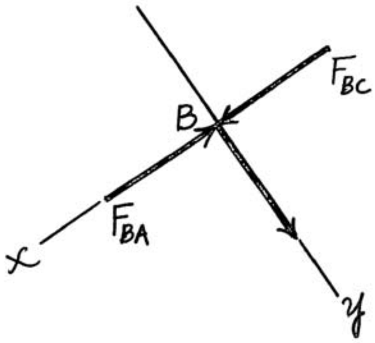
$F_{EF} = F_{AG} = 0$ Ans

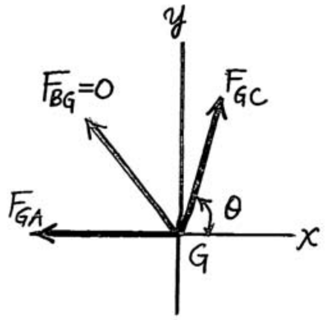
$F_{AB} = F_{DE} = 667$ lb (C) Ans

$F_{BC} = F_{CD} = 667$ lb (C) Ans

6–11. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 600$ lb, $P_2 = 400$ lb.







Joint *B* :

$+\searrow\Sigma F_y = 0;\quad F_{BG} = 0$ **Ans**

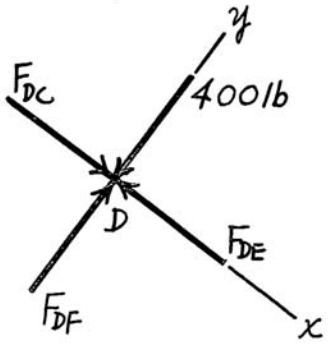
$+\nearrow\Sigma F_x = 0;\quad F_{BC} = F_{BA}$

Joint *G* :

$+\uparrow\Sigma F_y = 0;\quad F_{GC}\sin\theta = 0$

$F_{GC} = 0$ **Ans**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{GA} = 0$ **Ans**



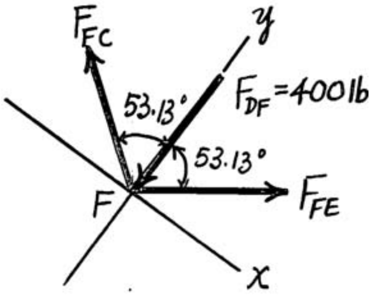
Joint *D* :

$+\nwarrow\Sigma F_x = 0;\quad F_{DE} - F_{DC} = 0$

$F_{DE} = F_{DC}$

$+\nearrow\Sigma F_y = 0;\quad F_{DF} - 400 = 0$

$F_{DF} = 400$ lb (C) **Ans**



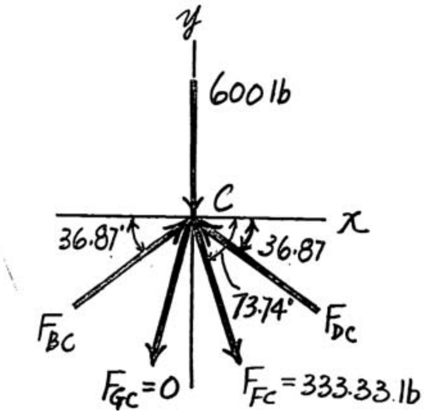
Joint *F* :

$+\searrow\Sigma F_x = 0;\quad F_{FE}\sin 53.13° - F_{FC}\sin 53.13° = 0$

$F_{FE} = F_{FC}$

$+\nearrow\Sigma F_y = 0;\quad 2F\cos 53.13° - 400 = 0$

$F_{FC} = F_{FE} = 333.33 = 333$ lb (T) **Ans**



Joint *C* :

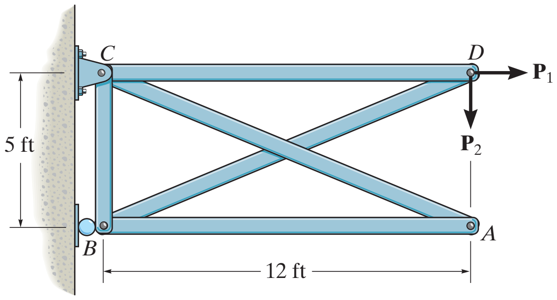
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{BC}\cos 36.87° - F_{DC}\cos 36.87° + 333.33\cos 73.74° = 0$

$+\uparrow\Sigma F_y = 0;\quad F_{BC}\sin 36.87° + F_{DC}\sin 36.87° - 600 - 333.33\sin 73.74° = 0$

$F_{BC} = F_{BA} = 708$ lb (C) **Ans**

$F_{DC} = F_{DE} = 825$ lb (C) **Ans**

***6–12.** Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 240$ lb, $P_2 = 100$ lb.



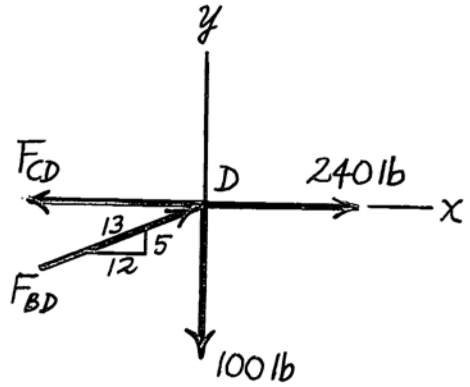
Joint *D*:

$$+\uparrow \Sigma F_y = 0; \quad F_{BD}\left(\frac{5}{13}\right) - 100 = 0$$

$$F_{BD} = 260 \text{ lb (C)} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 240 - F_{CD} + 260\left(\frac{12}{13}\right) = 0$$

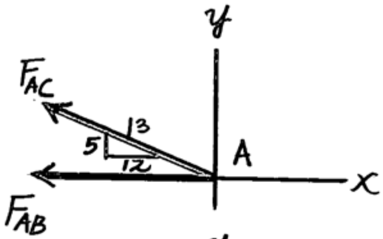
$$F_{CD} = 480 \text{ lb (T)} \qquad \textbf{Ans}$$



Joint *A*:

$$+\uparrow \Sigma F_y = 0; \quad F_{AC} = 0 \qquad \textbf{Ans}$$

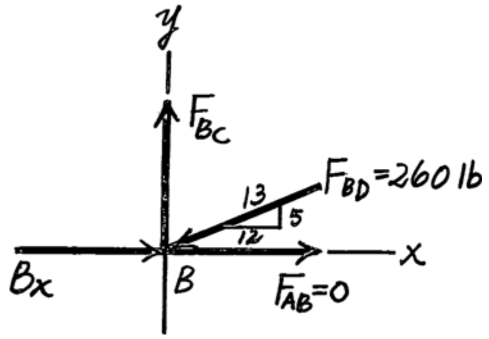
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AB} = 0 \qquad \textbf{Ans}$$



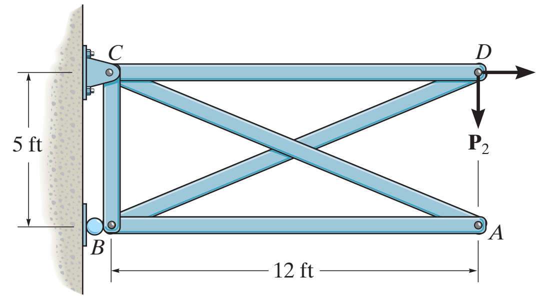
Joint *B*:

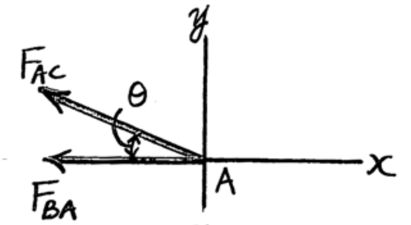
$$+\uparrow \Sigma F_y = 0; \quad F_{BC} - 260\left(\frac{5}{13}\right) = 0$$

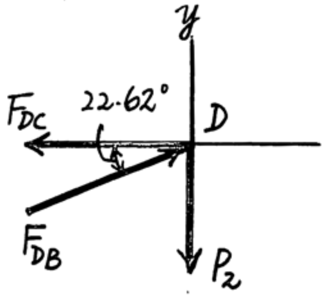
$$F_{BC} = 100 \text{ lb (T)} \qquad \textbf{Ans}$$

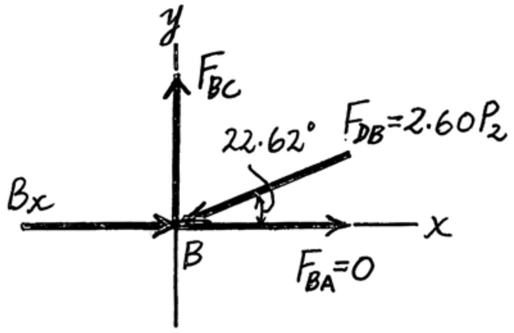


•6–13. Determine the largest load P_2 that can be applied to the truss so that the force in any member does not exceed 500 lb (T) or 350 lb (C). Take $P_1 = 0$.









Joint *A*:

$+\uparrow\Sigma F_y = 0;\quad F_{AC}\sin\theta = 0$

$F_{AC} = 0$

$\xrightarrow{+}\Sigma F_x = 0;\quad F_{AB} = 0$

Joint *D*:

$+\uparrow\Sigma F_y = 0;\quad -P_2 + F_{DB}\sin 22.62° = 0$

$F_{DB} = 2.60\,P_2$ (C)

$\xrightarrow{+}\Sigma F_x = 0;\quad 2.60\,P_2\cos 22.62° - F_{DC} = 0$

$F_{DC} = 2.40\,P_2$ (T)

Joint *B*:

$+\uparrow\Sigma F_y = 0;\quad F_{BC} - 2.60\,P_2\sin 22.62° = 0$

$F_{BC} = P_2$ (T)

Maximum tension member is *DC*:

$500 = 2.40\,P_2$

$P_2 = 208$ lb

Maximum compression member is *DB*:

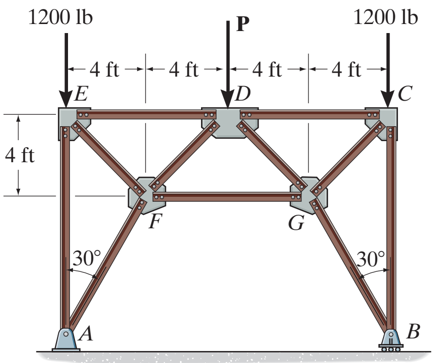
$350 = 2.60\,P_2$

$P_2 = 135$ lb

Thus member *DB* reaches the critical value first.

$P_2 = 135$ lb **Ans**

6–14. Determine the force in each member of the truss, and state if the members are in tension or compression. Set $P = 2500$ lb.



Support Reactions: Applying the moment equation of equilibrium about point A to the free-body diagram of the truss, Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(8+8) - 1200(8+8) - 2500(8) = 0$$
$$N_B = 2450 \text{ lb}$$

Method of Joints: We will begin by analyzing the equilibrium of joint B, and then that of joints C and G.

Joint B: From the free-body diagram in Fig. b,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BG} = 0 \qquad \textbf{Ans.}$$
$$+\uparrow \Sigma F_y = 0; \qquad 2450 - F_{BC} = 0$$
$$F_{BC} = 2450 \text{ lb (C)} \qquad \textbf{Ans.}$$

Joint C: From the free-body diagram in Fig. c,

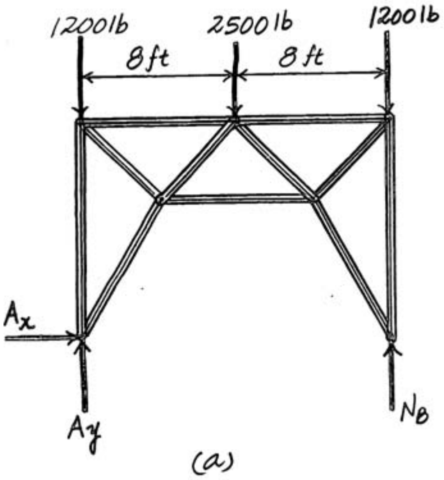
$$+\uparrow \Sigma F_y = 0; \qquad 2450 - 1200 - F_{CG}\sin 45° = 0$$
$$F_{CG} = 1767.77 \text{ lb} = 1768 \text{ lb (T)} \qquad \textbf{Ans.}$$
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CD} - 1767.77\cos 45° = 0$$
$$F_{CD} = 1250 \text{ lb (C)} \qquad \textbf{Ans.}$$

Joint G: From the free-body diagram in Fig. d,

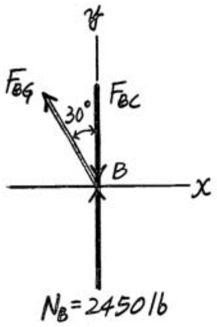
$$+\uparrow \Sigma F_y = 0; \qquad 1767.77\cos 45° - F_{GD}\cos 45° = 0$$
$$F_{GD} = 1767.77 \text{ lb} = 1768 \text{ lb (C)} \qquad \textbf{Ans.}$$
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 1767.77\sin 45° + 1767.77\sin 45° - F_{GF} = 0$$
$$F_{GF} = 2500 \text{ lb (T)} \qquad \textbf{Ans.}$$

Due to the symmetry of the system and the loading,

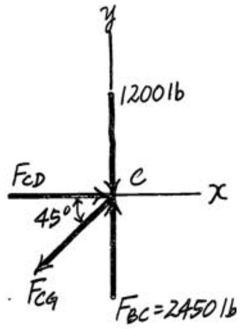
$$F_{AE} = F_{BC} = 2450 \text{ lb (C)} \qquad \textbf{Ans.}$$
$$F_{AF} = F_{BG} = 0 \qquad \textbf{Ans.}$$
$$F_{ED} = F_{CD} = 1250 \text{ lb (C)} \qquad \textbf{Ans.}$$
$$F_{EF} = F_{CG} = 1767.77 \text{ lb} = 1768 \text{ lb (T)} \qquad \textbf{Ans.}$$
$$F_{FD} = F_{GD} = 1767.77 \text{ lb} = 1768 \text{ lb (C)} \qquad \textbf{Ans.}$$



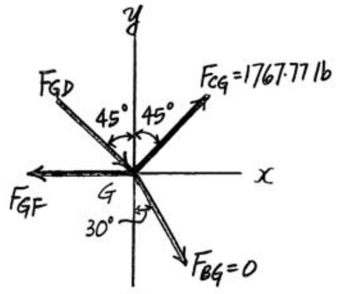
(a)



(b)

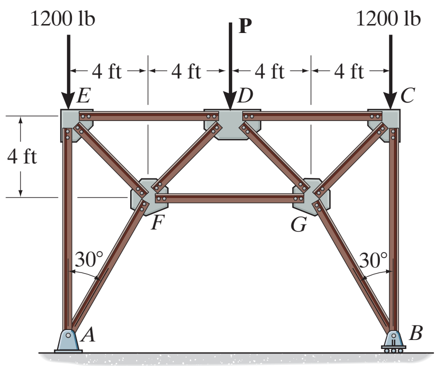


(c)



(d)

6–15. Remove the 1200-lb forces and determine the greatest force P that can be applied to the truss so that none of the members are subjected to a force exceeding either 2000 lb in tension or 1500 lb in compression.



Support Reactions: Applying the moment equation of equilibrium about point A to the free-body diagram of the truss, Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(8+8) - P(8) = 0$$

$$N_B = 0.5P$$

Method of Joints: We will begin by analyzing the equilibrium of joint B, and then that of joints C and G.

Joint B: From the free-body diagram in Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BG} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 0.5P - F_{BC} = 0$$

$$F_{BC} = 0.5P \text{ (C)}$$

Joint C: From the free-body diagram in Fig. c,

$$+\uparrow \Sigma F_y = 0; \qquad 0.5P - F_{CG}\sin 45° = 0$$

$$F_{CG} = 0.7071P \text{ (T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CD} - 0.7071P\cos 45° = 0$$

$$F_{CD} = 0.5P \text{ (C)}$$

Joint G: From the free-body diagram in Fig. d,

$$+\uparrow \Sigma F_y = 0; \qquad 0.7071P\cos 45° - F_{GD}\cos 45° = 0$$

$$F_{GD} = 0.7071P \text{(C)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.7071P\sin 45° + 0.7071P\sin 45° - F_{GF} = 0$$

$$F_{GF} = P \text{ (T)}$$

Due to the symmetry of the system and the loading,

$F_{AE} = F_{BC} = 0.5P$ (C)

$F_{AF} = F_{BG} = 0$

$F_{ED} = F_{CD} = 0.5P$(C)

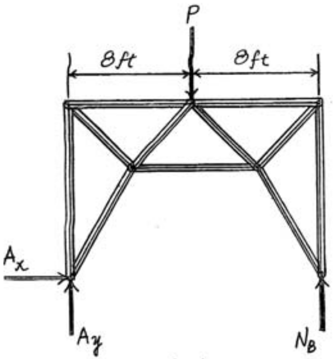
$F_{EF} = F_{CG} = 0.7071P$(T)

$F_{FD} = F_{GD} = 0.7071P$ (C)

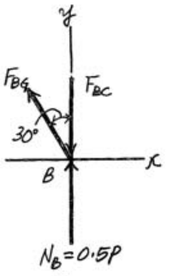
From the above results, the greatest tensile and compressive forces developed in the member of the truss are P and $0.7071P$, respectively. Thus,

$0.7071P = 1500$ $\qquad P = 2121.33$ lb

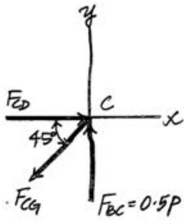
$P = 2000$ lb (controls) **Ans.**



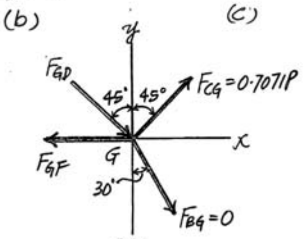
(a)



(b)

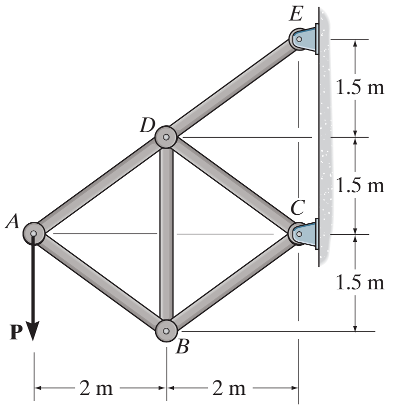


(c)



(d)

***6–16.** Determine the force in each member of the truss, and state if the members are in tension or compression. Set $P = 5$ kN.



Method of Joints: We will begin by analyzing the equilibrium of joint A, and then proceed to analyzing that of joints B and D.

Joint A: From the free - body diagram in Fig. a,

$\Sigma F_{y'} = 0;\quad F_{AD}\sin 73.74° - 5\sin 53.13° = 0$

$F_{AD} = 4.167\text{ kN} = 4.17\text{ kN (T)}$ **Ans.**

$\Sigma F_{x'} = 0;\quad 4.167\cos 73.74° + 5\cos 53.13° - F_{AB} = 0$

$F_{AB} = 4.167\text{ kN} = 4.17\text{ kN (C)}$ **Ans.**

Joint B: From the free - body diagram in Fig. b,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 4.167\left(\frac{4}{5}\right) - F_{BC}\left(\frac{4}{5}\right) = 0$

$F_{BC} = 4.167\text{ kN} = 4.17\text{ kN (C)}$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad F_{BD} - 4.167\left(\frac{3}{5}\right) - 4.167\left(\frac{3}{5}\right) = 0$

$F_{BD} = 5\text{ kN (T)}$ **Ans.**

Joint D: From the free - body diagram in Fig. c,

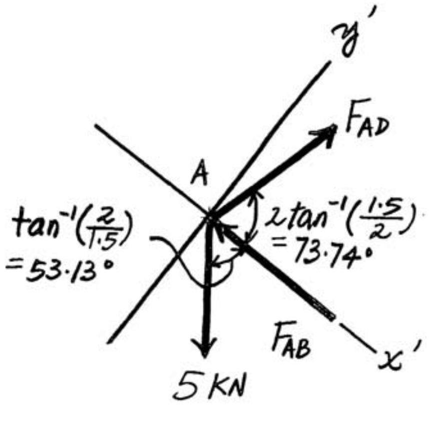
$+\nwarrow\Sigma F_{y'} = 0;\quad F_{DC}\sin 73.74° - 5\sin 53.13° = 0$

$F_{DC} = 4.167\text{ kN} = 4.17\text{ kN (C)}$ **Ans.**

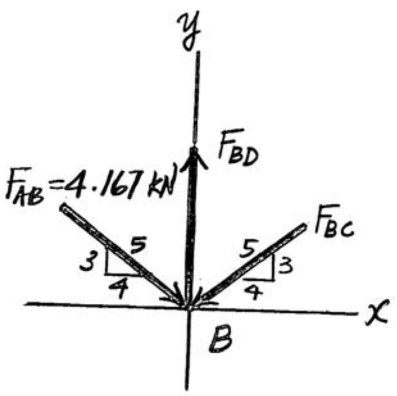
$\overset{+}{\nearrow}\Sigma F_{x'} = 0;\quad F_{DE} - 4.167 - 5\cos 53.13° - 4.167\cos 73.74° = 0$

$F_{DE} = 8.333\text{ kN} = 8.33\text{ kN (T)}$ **Ans.**

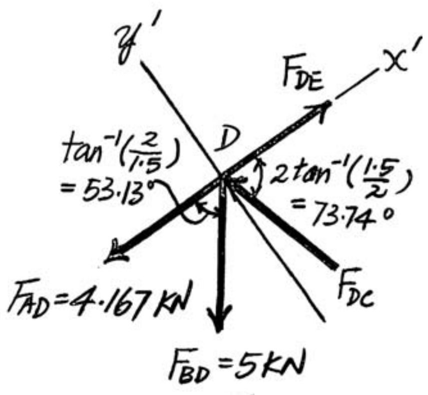
Note: The equilibrium analysis of joints E and C can be used to determine the components of the support reaction at supports E and C, respectively.



(a)

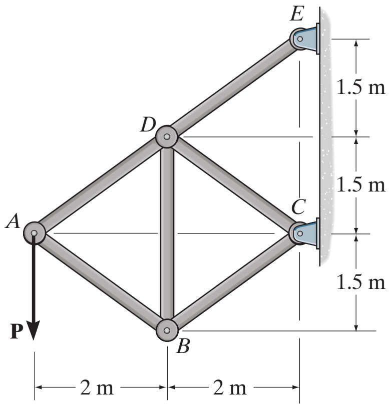


(b)



(c)

•6–17. Determine the greatest force *P* that can be applied to the truss so that none of the members are subjected to a force exceeding either 2.5 kN in tension or 2 kN in compression.



Method of Joints: We will begin by analyzing the equilibrium of joint *A*, and then proceed to analyzing that of joints *B* and *D*.

Joint *A***:** From the free - body diagram in Fig. *a*,

$\overset{+}{\nearrow}\Sigma F_{y'} = 0;\quad F_{AD}\sin 73.74° - P\sin 53.13° = 0$

$F_{AD} = 0.8333P\ (\mathrm{T})$

$\overset{+}{\searrow}\Sigma F_{x'} = 0;\quad 0.8333P\cos 73.74° + P\cos 53.13° - F_{AB} = 0$

$F_{AB} = 0.8333P\ (\mathrm{C})$

Joint *B***:** From the free - body diagram in Fig. *b*,

$\xrightarrow{+}\Sigma F_x = 0;\quad 0.8333P\left(\frac{4}{5}\right) - F_{BC}\left(\frac{4}{5}\right) = 0$

$F_{BC} = 0.8333P\ (\mathrm{C})$

$+\uparrow\Sigma F_y = 0;\quad F_{BD} - 0.8333P\left(\frac{3}{5}\right) - 0.8333P\left(\frac{3}{5}\right) = 0$

$F_{BD} = P\ (\mathrm{T})$

Joint *D***:** From the free - body diagram in Fig. *c*,

$\overset{+}{\nwarrow}\Sigma F_{y'} = 0;\quad F_{DC}\sin 73.74° - P\sin 53.13° = 0$

$F_{DC} = 0.8333P(\mathrm{C})$

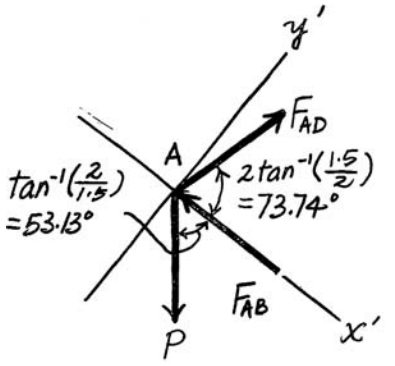
$\overset{+}{\nearrow}\Sigma F_{x'} = 0;\quad F_{DE} - 0.8333P - P\cos 53.13° - 0.8333P\cos 73.74° = 0$

$F_{DE} = 1.6667P\ (\mathrm{T})$

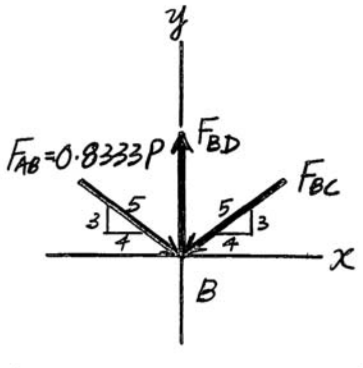
From the above results, the greatest compressive and tensile forces developed in the member are 0.8333*P* and 1.6667*P*, respectively. Thus,

$0.8333P = 2 \qquad P = 2.40\ \mathrm{kN}$

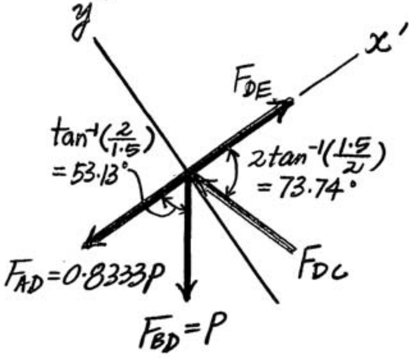
$1.6667P = 2.5 \qquad P = 1.50\ \mathrm{kN}$ (controls) **Ans.**



(a)

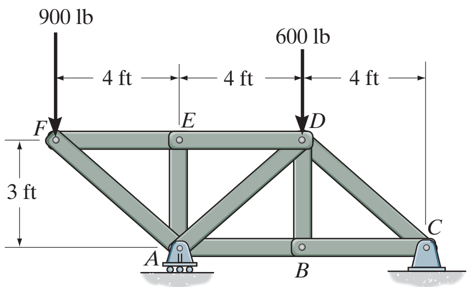


(b)



(C)

6–18. Determine the force in each member of the truss, and state if the members are in tension or compression.



Support Reactions: Applying the moment equation of equilibrium about point C to the free-body diagram of the truss, Fig. a,

$\curvearrowleft +\Sigma M_C = 0;$ $\quad 600(4) + 900(4+4+4) - N_A = 0$

$N_A = 1650 \text{ lb}$

Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $F \rightarrow E \rightarrow A \rightarrow B \rightarrow D$.

Joint F: From the free-body diagram in Fig. b,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{FA}\left(\frac{3}{5}\right) - 900 = 0$

$F_{FA} = 1500 \text{ lb (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{FE} - 1500\left(\frac{4}{5}\right) = 0$

$F_{FE} = 1200 \text{ lb (T)}$ **Ans.**

Joint E: From the free-body diagram in Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{ED} - 1200 = 0$

$F_{ED} = 1200 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{EA} = 0$ **Ans.**

Joint A: From the free-body diagram in Fig. d,

$+\uparrow \Sigma F_y = 0;$ $\quad 1650 - 1500\left(\frac{3}{5}\right) - F_{AD}\left(\frac{3}{5}\right) = 0$

$F_{AD} = 1250 \text{ lb (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1500\left(\frac{4}{5}\right) - 1250\left(\frac{4}{5}\right) - F_{AB} = 0$

$F_{AB} = 200 \text{ lb (C)}$ **Ans.**

Joint B: From the free-body diagram in Fig. e,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 200 - F_{BC} = 0$

$F_{BC} = 200 \text{ lb (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD} = 0$ **Ans.**

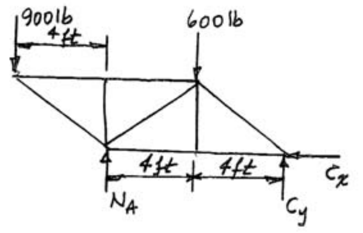
Joint D: From the free-body diagram in Fig. f,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{DC}\left(\frac{4}{5}\right) + 1250\left(\frac{4}{5}\right) - 1200 = 0$

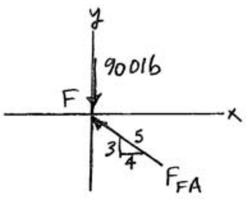
$F_{DC} = 250 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 1250\left(\frac{3}{5}\right) - 250\left(\frac{3}{5}\right) - 600 = 0$ (check)

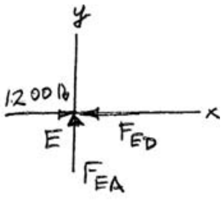
Note: The equilibrium analysis of joint C must be used to determine the components of the support reaction at C.



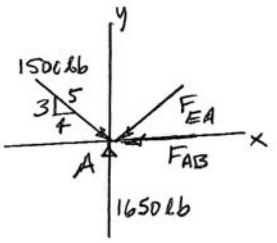
(a)



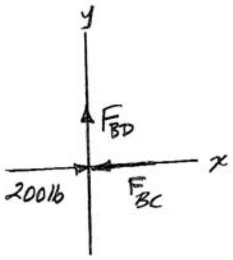
(b)



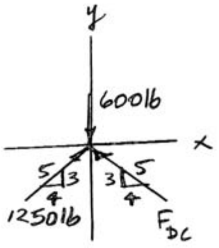
(c)



(d)

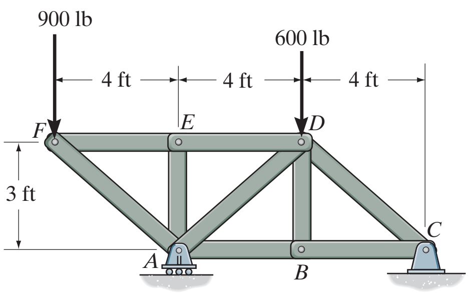


(e)



(f)

6–19. The truss is fabricated using members having a weight of 10 lb/ft. Remove the external forces from the truss, and determine the force in each member due to the weight of the members. State whether the members are in tension or compression. Assume that the total force acting on a joint is the sum of half of the weight of every member connected to the joint.



Joint Loadings:

$$F_C = F_F = 10\left(\frac{4+5}{2}\right) = 45 \text{ lb}$$

$$F_E = F_B = 10\left(\frac{4+4+3}{2}\right) = 55 \text{ lb}$$

$$F_A = F_D = 10\left(\frac{5+5+4+3}{2}\right) = 85 \text{ lb}$$

Support Reactions: Applying the moment equation of equilibrium about point C to the free-body diagram of the truss, Fig. a,

$$\circlearrowleft +\Sigma M_C = 0; \qquad 45(4+4+4)+55(4+4)+85(4+4)+85(4)+55(4)-N_A(4+4)=0$$

$$N_A = 277.5 \text{ lb}$$

Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $F \rightarrow E \rightarrow A \rightarrow B \rightarrow D$.

Joint F: From the free-body diagram in Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad F_{FA}\left(\frac{3}{5}\right) - 45 = 0$$

$$F_{FA} = 75 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{FE} - 75\left(\frac{4}{5}\right) = 0$$

$$F_{FE} = 60 \text{ lb (T)} \qquad \textbf{Ans.}$$

Joint E: From the free-body diagram in Fig. c,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{ED} - 60 = 0$$

$$F_{ED} = 60 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{EA} - 55 = 0$$

$$F_{EA} = 55 \text{ lb (C)} \qquad \textbf{Ans.}$$

Joint A: From the free-body diagram in Fig. d,

$$+\uparrow \Sigma F_y = 0; \qquad 277.5 - 55 - 85 - 75\left(\frac{3}{5}\right) - F_{AD}\left(\frac{3}{5}\right) = 0$$

$$F_{AD} = 154.17 \text{ lb} = 154 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} + 75\left(\frac{4}{5}\right) - 154.17\left(\frac{4}{5}\right) = 0$$

$$F_{AB} = 63.33 \text{ lb} = 63.3 \text{ lb (T)} \qquad \textbf{Ans.}$$

Joint B: From the free-body diagram in Fig. e,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} - 63.33 = 0$$

$$F_{BC} = 63.33 \text{ lb} = 63.3 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BD} - 55 = 0$$

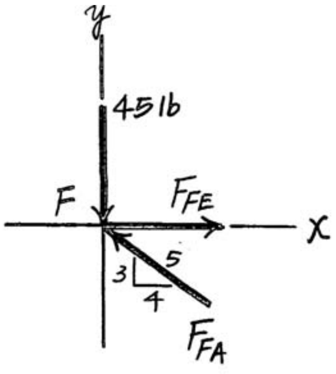
$$F_{BD} = 55 \text{ lb (T)} \qquad \textbf{Ans.}$$

Joint D: From the free-body diagram in Fig. f,

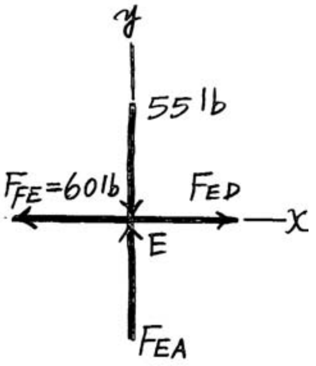
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 154.17\left(\frac{4}{5}\right) - 60 - F_{DC}\left(\frac{4}{5}\right) = 0$$

$$F_{DC} = 79.17 \text{ lb} = 79.2 \text{ lb (C)} \qquad \textbf{Ans.}$$

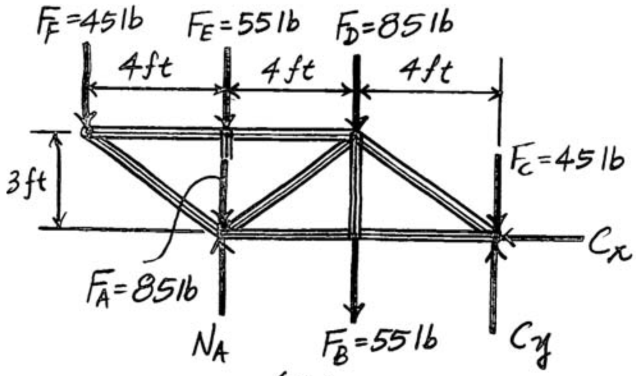
$$+\uparrow \Sigma F_y = 0; \qquad 154.17\left(\frac{3}{5}\right) + 79.17\left(\frac{3}{5}\right) - 85 - 55 = 0 \qquad \text{(check)}$$


y
45 lb
F
F_{FE}
x
3
4
5
F_{FA}

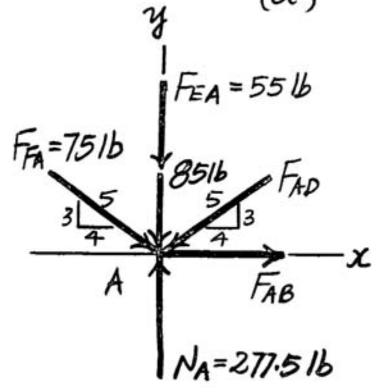
(b)


y
55 lb
F_{FE}=60 lb
F_{ED}
x
E
F_{EA}

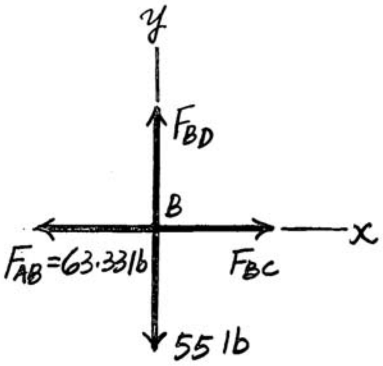
(c)


F_F=45 lb
F_E=55 lb
F_D=85 lb
4 ft
4 ft
4 ft
3 ft
F_C=45 lb
C_x
F_A=85 lb
N_A
F_B=55 lb
C_y

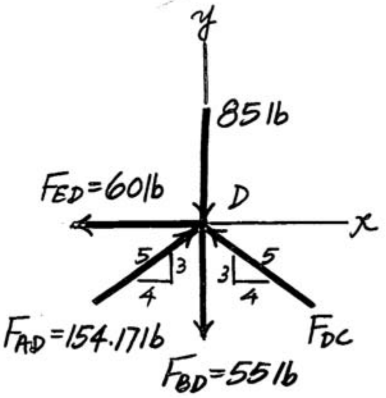
(a)


y
F_{EA} = 55 lb
F_{FA}=75 lb
85 lb
F_{AD}
3
4
5
5
3
4
x
A
F_{AB}
N_A=277.5 lb

(d)

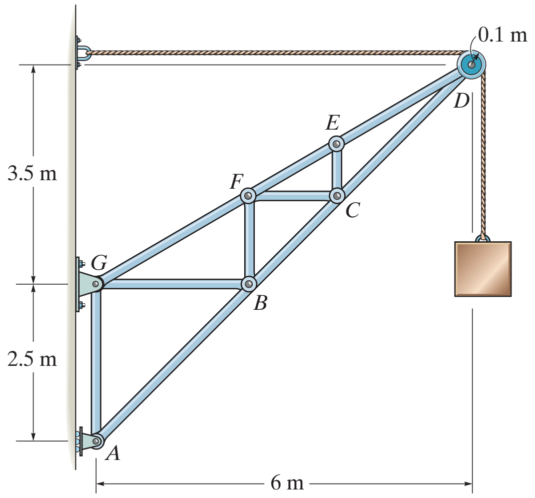

y
F_{BD}
B
x
F_{AB}=63.33 lb
F_{BC}
55 lb

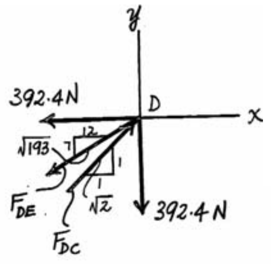
(e)


y
85 lb
F_{ED}=60 lb
D
x
5
3
4
3
5
4
F_{AD}=154.17 lb
F_{DC}
F_{BD}=55 lb

(f)

***6–20.** Determine the force in each member of the truss and state if the members are in tension or compression. The load has a mass of 40 kg.





Joint D :

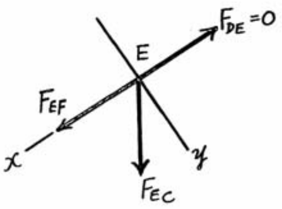
$\xrightarrow{+} \Sigma F_x = 0;\quad F_{DC}\left(\frac{1}{\sqrt{2}}\right) - 392.4 - F_{DE}\left(\frac{12}{\sqrt{193}}\right) = 0$

$+\uparrow \Sigma F_y = 0;\quad F_{DC}\left(\frac{1}{\sqrt{2}}\right) - F_{DE}\left(\frac{7}{\sqrt{193}}\right) - 392.4 = 0$

Solving,

$F_{DE} = 0$ Ans

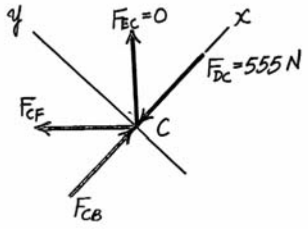
$F_{DC} = 555\text{ N (C)}$ Ans



Joint E :

$+\searrow \Sigma F_y = 0;\quad F_{EC} = 0$ Ans

$+\nearrow \Sigma F_x = 0;\quad F_{EF} = 0$ Ans

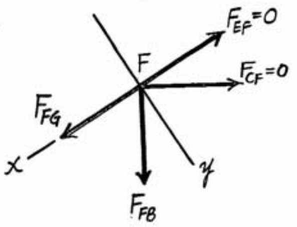


Joint C :

$+\nwarrow \Sigma F_y = 0;\quad F_{CF} = 0$ Ans

$+\nearrow \Sigma F_x = 0;\quad -555 + F_{CB} = 0$

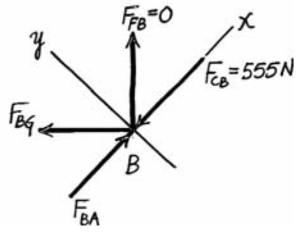
$F_{CB} = 555\text{ N (C)}$ Ans



Joint F :

$+\searrow \Sigma F_y = 0;\quad F_{FB} = 0$ Ans

$+\nearrow \Sigma F_x = 0;\quad F_{FG} = 0$ Ans



Joint B :

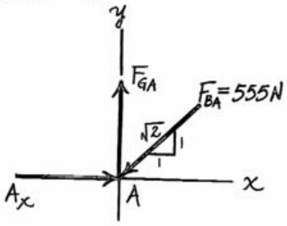
$+\searrow \Sigma F_y = 0;\quad F_{BG} = 0$ Ans

$+\nearrow \Sigma F_x = 0;\quad F_{BA} = 555\text{ N (C)}$ Ans

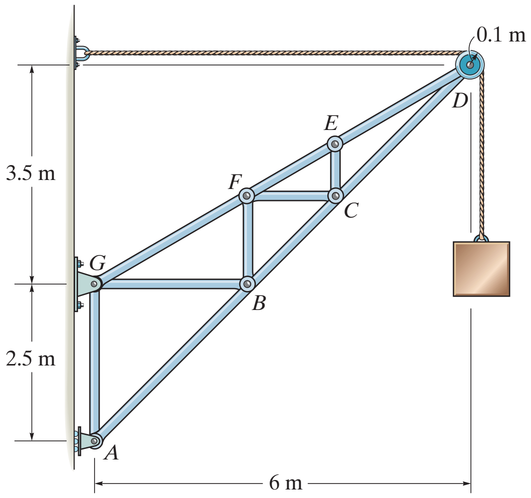
Joint A :

$+\uparrow \Sigma F_y = 0;\quad F_{GA} - 555\left(\frac{1}{\sqrt{2}}\right) = 0$

$F_{GA} = 392\text{ N (T)}$ Ans



•6–21. Determine the largest mass *m* of the suspended block so that the force in any member does not exceed 30 kN (T) or 25 kN (C).



Inspection of joints *E*, *C*, *F*, and *B* indicates that *EC*, *CF*, *FB*, and *BG* are all zero - force members.

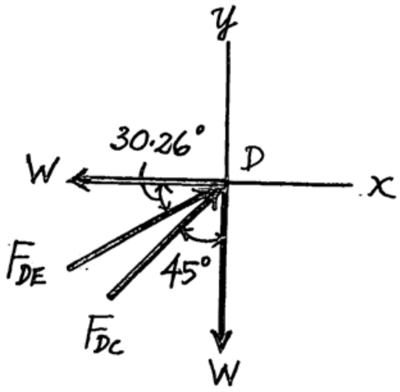
Joint *D* :

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{DC} \sin 45° + F_{DE} \cos 30.25° - W = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{DC} \cos 45° + F_{DE} \sin 30.25° - W = 0$

$F_{DC} = 1.414\,W\ (C)$

$F_{DE} = 0$



Joint *A* :

$+\uparrow \Sigma F_y = 0; \quad F_{AG} - 1.414\,W \sin 45° = 0$

$F_{AG} = W\ (T)$

For compression of members *DC*, *BC*, and *AB*,

$25\text{ kN} = 1.414\,W$

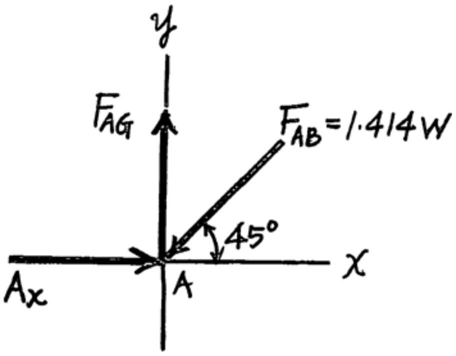
$W = 17.678\text{ kN}$

For tension of member *AG*,

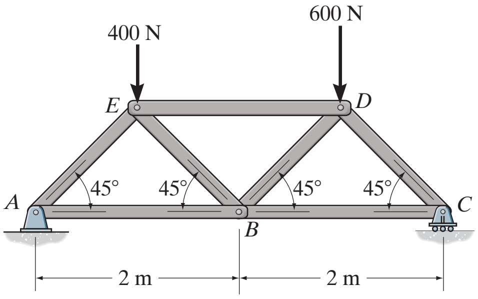
$W = 30\text{ kN}$

Thus the critical value is compression.

$$m = \frac{17.678\,(10^3)\text{ N}}{9.81} = 1.80\text{ Mg} \qquad \text{Ans}$$



6–22. Determine the force in each member of the truss, and state if the members are in tension or compression.



Support Reaction: Applying the moment equation of equilibrium about point A on the free-body diagram of the truss, Fig. a,

$+\Sigma M_A = 0;\qquad N_C(4) - 400(1) - 600(3) = 0$

$N_C = 550\text{ N}$

Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $C \rightarrow D \rightarrow E \rightarrow B$.

Joint C**:** From the free-body diagram in Fig. b,

$+\uparrow \Sigma F_y = 0;\qquad 550 - F_{CD}\sin 45° = 0$

$F_{CD} = 777.82\text{ N} = 778\text{ N (C)}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 777.82\cos 45° - F_{CB} = 0$

$F_{CB} = 550\text{ N (T)}$ **Ans.**

Joint D**:** From the free-body diagram in Fig. c,

$+\uparrow \Sigma F_y = 0;\qquad F_{DB}\sin 45° + 777.82\sin 45° - 600 = 0$

$F_{DB} = 70.71\text{ N} = 70.7\text{ N (C)}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad F_{DE} + 70.71\cos 45° - 777.82\cos 45° = 0$

$F_{DE} = 500\text{ N (C)}$ **Ans.**

Joint E**:** From the free-body diagram in Fig. d,

$\Sigma F_{x'} = 0;\quad F_{EA} - 400\sin 45° - 500\sin 45° = 0$

$F_{EA} = 636.40\text{ N} = 636\text{ N (C)}$ **Ans.**

$\Sigma F_{y'} = 0;\quad 500\cos 45° - 400\cos 45° - F_{EB} = 0$

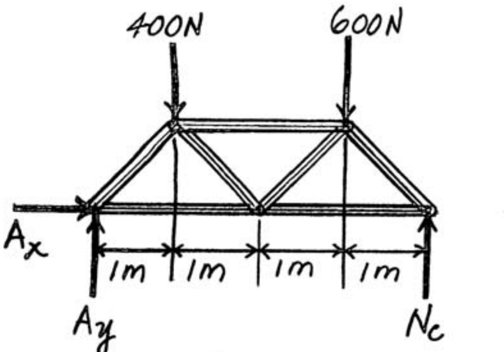
$F_{EB} = 70.71\text{N} = 70.7\text{ N (T)}$ **Ans.**

Joint B**:** From the free-body diagram in Fig. e,

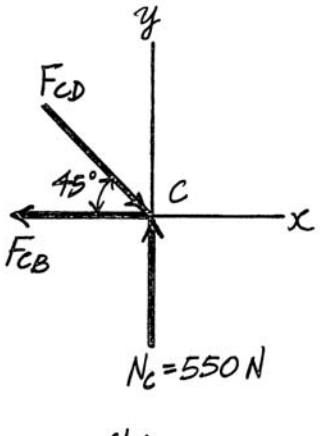
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 550 - 70.71\cos 45° - 70.71\cos 45° - F_{BA} = 0$

$F_{BA} = 450\text{ N (T)}$ **Ans.**

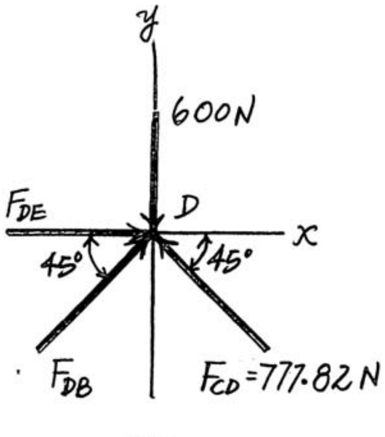
$+\uparrow \Sigma F_y = 0;\qquad 70.71\sin 45° - 70.71\sin 45° = 0$ (check)



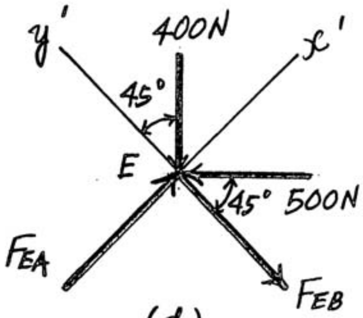
(a)



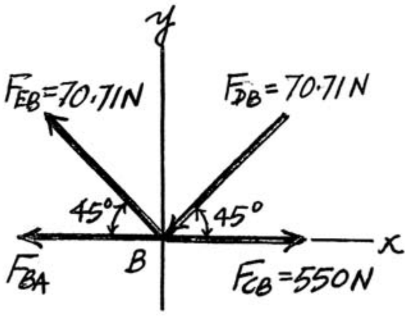
(b)



(c)

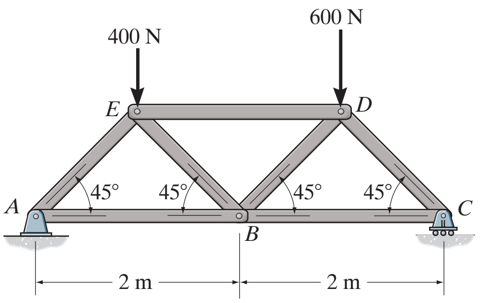


(d)



(e)

6–23. The truss is fabricated using uniform members having a mass of 5 kg/m. Remove the external forces from the truss, and determine the force in each member due to the weight of the truss. State whether the members are in tension or compression. Assume that the total force acting on a joint is the sum of half of the weight of every member connected to the joint.



Joint Loading:

$$F_C = F_A = 5(9.81)\left[\frac{2+\sqrt{2}}{2}\right] = 83.73\text{N}$$

$$F_B = 5(9.81)\left[\frac{2+2+\sqrt{2}+\sqrt{2}}{2}\right] = 167.47\text{N}$$

$$F_E = F_D = 5(9.81)\left[\frac{2+\sqrt{2}+\sqrt{2}}{2}\right] = 118.2\text{N}$$

Support Reactions: Applying the moment equation of equilibrium about point A to the free-body diagram of the truss, Fig. a,

$\curvearrowleft +\Sigma M_A = 0;$ $\quad N_C(4) - 83.73(4) - 118.42(3) - 167.47(2) - 118.42(1) = 0$

$N_C = 285.88\text{ N}$

Method of Joints: We will begin by analyzing the equilibrium of joint C, and then that of joint D.

Joint C: From the free-body diagram in Fig. b,

$+\uparrow \Sigma F_y = 0;$ $\quad 285.88 - 83.73 - F_{CD}\sin 45° = 0$

$F_{CD} = 285.88\text{ N} = 286\text{ N (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 285.88\cos 45° - F_{CB} = 0$

$F_{CB} = 202.15\text{ N} = 202\text{ N (T)}$ **Ans.**

Joint D: From the free-body diagram in Fig. c,

$+\uparrow \Sigma F_y = 0;$ $\quad 285.88\sin 45° - 118.42 - F_{DB}\sin 45° = 0$

$F_{DB} = 118.42\text{ N} = 118\text{ N (T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{DE} - 285.88\cos 45° - 118.42\cos 45° = 0$

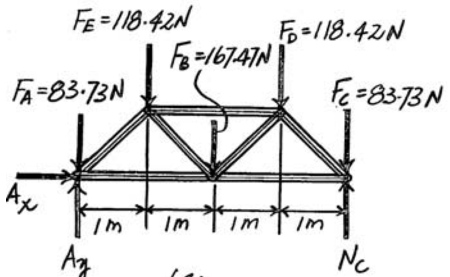
$F_{DE} = 285.88\text{ N} = 286\text{ N (C)}$ **Ans.**

Due to the symmetry of the system and the loading,

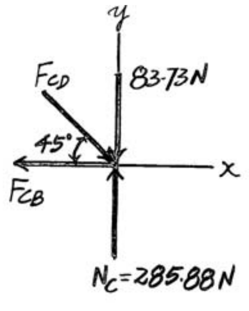
$F_{BE} = F_{DB} = 118.42\text{ N} = 118\text{ N (T)}$ **Ans.**

$F_{BA} = F_{CB} = 202.15\text{ N} = 202\text{ N (T)}$ **Ans.**

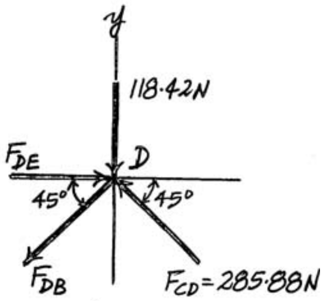
$F_{EA} = F_{CD} = 285.88\text{ N} = 286\text{ N (C)}$ **Ans.**



(a)

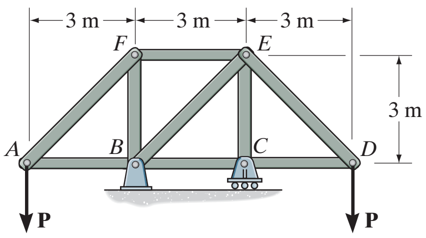


(b)



(c)

***6–24.** Determine the force in each member of the truss, and state if the members are in tension or compression. Set $P = 4$ kN.



Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $A \rightarrow D \rightarrow F \rightarrow E \rightarrow C$.

Joint *A*: From the free - body diagram in Fig. *a*,

$+\uparrow \Sigma F_y = 0;\quad F_{AF}\ \sin 45° - 4 = 0$

$F_{AF} = 5.657\text{ kN} = 5.66\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 5.657\cos 45° - F_{AB} = 0$

$F_{AB} = 4\text{ kN (C)}$ **Ans.**

Joint *D*: From the free - body diagram in Fig. *b*,

$+\uparrow \Sigma F_y = 0;\quad F_{DE}\ \sin 45° - 4 = 0$

$F_{DE} = 5.657\text{ kN} = 5.66\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{DC} - 5.657\cos 45° = 0$

$F_{DC} = 4\text{ kN (C)}$ **Ans.**

Joint *F*: From the free - body diagram in Fig. *c*,

$+\uparrow \Sigma F_y = 0;\quad F_{FB} - 5.657\cos 45° = 0$

$F_{FB} = 4\text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{FE} - 5.657\sin 45° = 0$

$F_{FE} = 4\text{ kN (T)}$ **Ans.**

Joint *E*: From the free - body diagram in Fig. *d*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 5.657\sin 45° - 4 - F_{EB}\ \sin 45° = 0$

$F_{EB} = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad F_{EC} - 5.657\cos 45° = 0$

$F_{EC} = 4\text{ kN (C)}$ **Ans.**

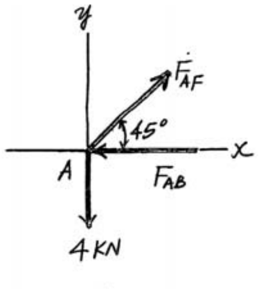
Joint *C*: From the free - body diagram in Fig. *e*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{CB} - 4 = 0$

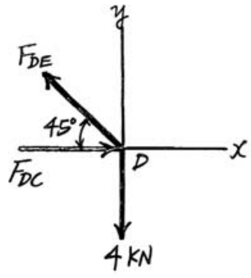
$F_{CB} = 4\text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad N_C - 4 = 0$

$N_C = 4\text{ kN}$



(a)



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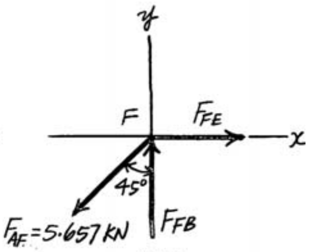
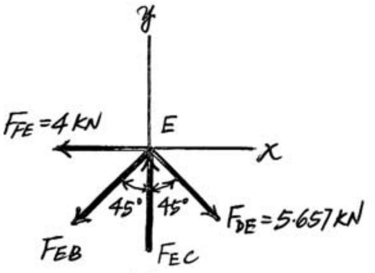
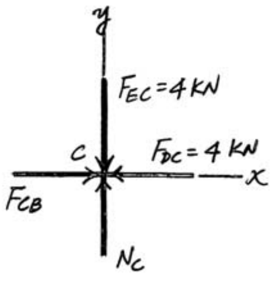


(C)

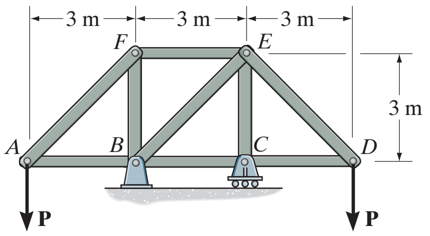


(d)



(e)

•6–25. Determine the greatest force P that can be applied to the truss so that none of the members are subjected to a force exceeding either 1.5 kN in tension or 1 kN in compression.



Method of Joints: We will analyze the equilibrium of the joints in the following sequence: $A \rightarrow D \rightarrow F \rightarrow E \rightarrow C$.

Joint A**:** From the free - body diagram in Fig. a,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{AF} \sin 45° - P = 0$

$F_{AF} = 1.4142P$ (T)

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1.4142P \cos 45° - F_{AB} = 0$

$F_{AB} = P$ (C)

Joint D**:** From the free - body diagram in Fig. b,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{DE} \sin 45° - P = 0$

$F_{DE} = 1.4142P$ (T)

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{DC} - 1.4142P \cos 45° = 0$

$F_{DC} = P$ (C)

Joint F**:** From the free - body diagram in Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{FE} - 1.4142P \sin 45° = 0$

$F_{FE} = P$ (T)

$+\uparrow \Sigma F_y = 0;$ $\quad F_{FB} - 1.4142P \cos 45° = 0$

$F_{FB} = P$ (C)

Joint E**:** From the free - body diagram in Fig. d,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1.4142P \sin 45° - P - F_{EB} \sin 45° = 0$

$F_{EB} = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{EC} - 1.4142P \cos 45° = 0$

$F_{EC} = P$ (C)

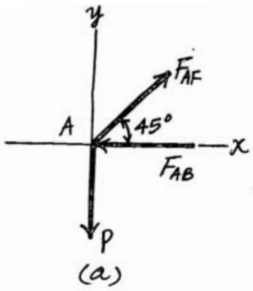
Joint C**:** From the free - body diagram in Fig. e,

$F_{CB} = P$ (C)

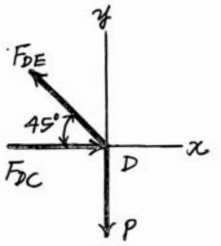
From the above results, the greatest compressive and tensile forces developed in the member are P and $1.4142P$, respectively. Thus,

$P = 1\text{kN}$ (controls) **Ans.**

$1.4142P = 1.5$ $\quad P = 1.06\text{kN}$



(a)



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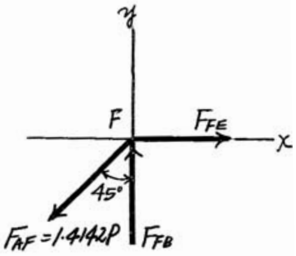

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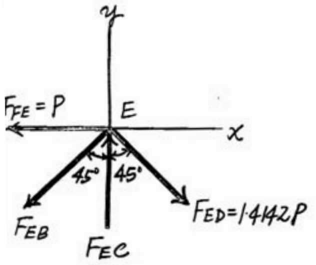

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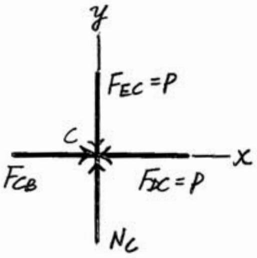
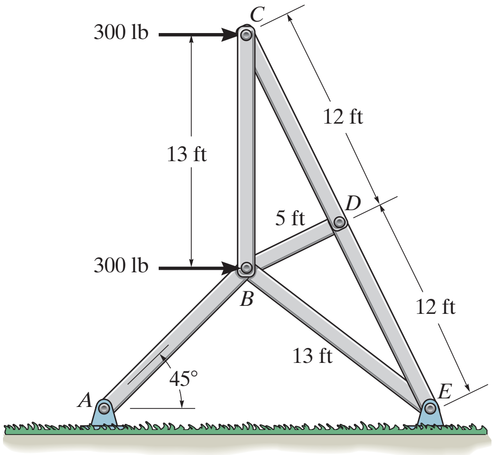
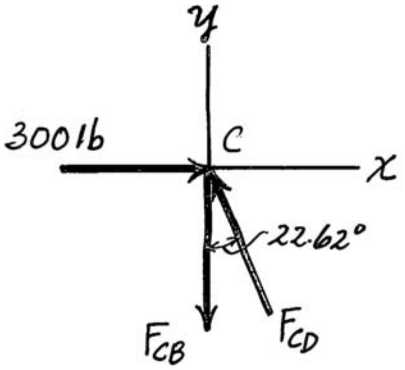


(e)

6–26. A sign is subjected to a wind loading that exerts horizontal forces of 300 lb on joints *B* and *C* of one of the side supporting trusses. Determine the force in each member of the truss and state if the members are in tension or compression.





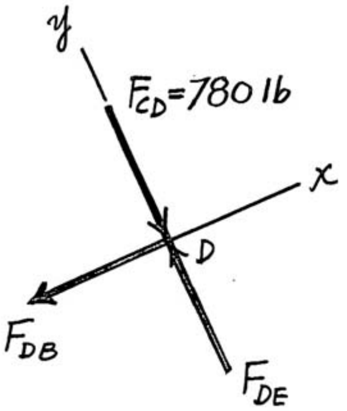
Joint *C*:

$\xrightarrow{+} \Sigma F_x = 0;$ $300 - F_{CD} \sin 22.62° = 0$

$F_{CD} = 780$ lb (C) Ans

$+\uparrow \Sigma F_y = 0;$ $-F_{CB} + 780 \cos 22.62° = 0$

$F_{CB} = 720$ lb (T) Ans

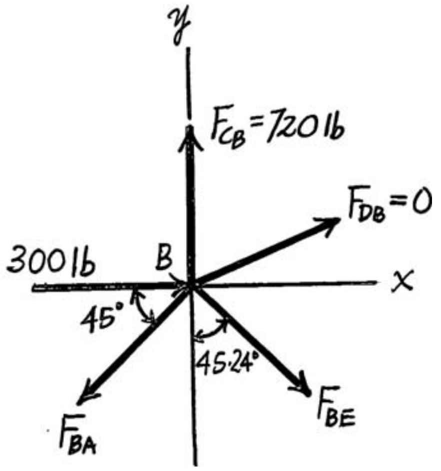


Joint *D*:

$\overset{+}{\nearrow} \Sigma F_x = 0;$ $F_{DB} = 0$ Ans

$\overset{+}{\searrow} \Sigma F_y = 0;$ $780 - F_{DE} = 0$

$F_{DE} = 780$ lb (C) Ans



Joint *B*:

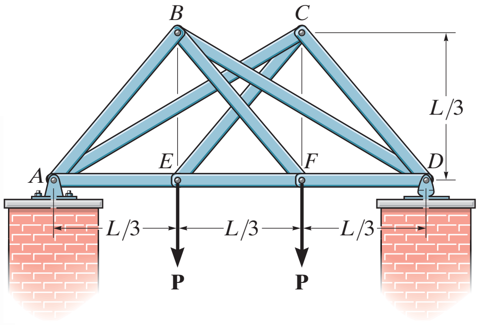
$\xrightarrow{+} \Sigma F_x = 0;$ $300 - F_{BA} \cos 45° + F_{BE} \sin 45.24° = 0$

$+\uparrow \Sigma F_y = 0;$ $720 - F_{BA} \sin 45° - F_{BE} \cos 45.24° = 0$

$F_{BE} = 297$ lb (T) Ans

$F_{BA} = 722$ lb (T) Ans

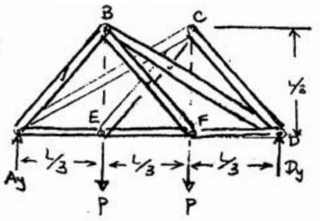
6–27. Determine the force in each member of the double scissors truss in terms of the load P and state if the members are in tension or compression.



$\circlearrowleft + \Sigma M_A = 0; \quad P(\frac{L}{3}) + P(\frac{2L}{3}) - (D_y)(L) = 0$

$D_y = P$

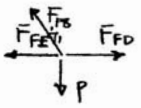
$+\uparrow \Sigma F_y = 0; \quad A_y = P$



Joint F:

$+\uparrow \Sigma F_y = 0; \quad F_{FB}(\frac{1}{\sqrt{2}}) - P = 0$

$F_{FB} = \sqrt{2}P = 1.41P\,(T)$



$\xrightarrow{+} \Sigma F_x = 0; \quad F_{FD} - F_{FE} - F_{FB}(\frac{1}{\sqrt{2}}) = 0$

$F_{FD} - F_{FE} = P \qquad (1)$

Cont'd

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CA}(\frac{2}{\sqrt{5}}) - \sqrt{2}P(\frac{1}{\sqrt{2}}) - F_{CD}(\frac{1}{\sqrt{2}}) = 0$

$$\frac{2}{\sqrt{5}}F_{CA} - \frac{1}{\sqrt{2}}F_{CD} = P$$

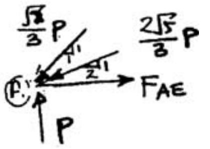
$$F_{CA} = \frac{2\sqrt{5}}{3}P = 1.4907P = 1.49P\,(C)$$

$$F_{CD} = \frac{\sqrt{2}}{3}P = 0.4714P = 0.471P\,(C)$$

Joint A:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AE} - \frac{\sqrt{2}}{3}P(\frac{1}{\sqrt{2}}) - \frac{2\sqrt{5}}{3}P(\frac{2}{\sqrt{5}}) = 0$

$$F_{AE} = \frac{5}{3}P = 1.67\,P\,(T)$$



From Eqs. (1) and (2):

$F_{FE} = 0.667\,P\,(T)$ **Ans**

$F_{FD} = 1.67\,P\,(T)$ **Ans**

$F_{AB} = 0.471\,P\,(C)$ **Ans**

$F_{AE} = 1.67\,P\,(T)$ **Ans**

$F_{AC} = 1.49P\,(C)$ **Ans**

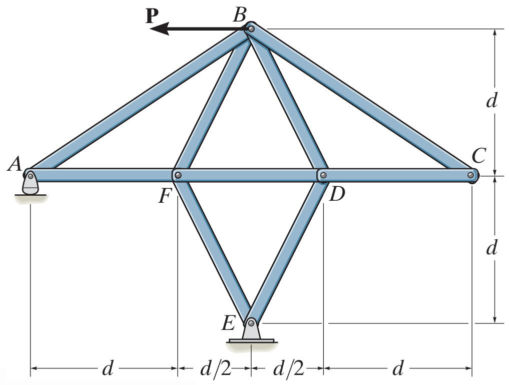
$F_{BF} = 1.41\,P\,(T)$ **Ans**

$F_{BD} = 1.49\,P\,(C)$ **Ans**

$F_{EC} = 1.41\,P\,(T)$ **Ans**

$F_{CD} = 0.471\,P\,(C)$ **Ans**

***6–28.** Determine the force in each member of the truss in terms of the load *P*, and indicate whether the members are in tension or compression.

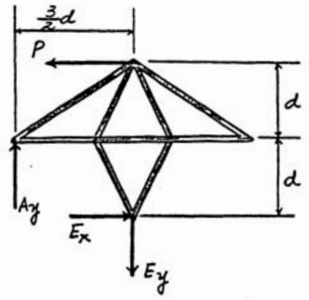


Support Reactions :

$$\circlearrowleft + \Sigma M_E = 0; \quad P(2d) - A_y\left(\frac{3}{2}d\right) = 0 \quad A_y = \frac{4}{3}P$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{4}{3}P - E_y = 0 \quad E_y = \frac{4}{3}P$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad E_x - P = 0 \quad E_x = P$$



Method of Joints : By inspection of joint *C*, members *CB* and *CD* are zero force member. Hence

$$F_{CB} = F_{CD} = 0 \qquad \textbf{Ans}$$

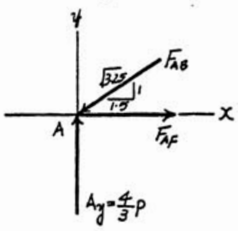
Joint *A*

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\left(\frac{1}{\sqrt{3.25}}\right) - \frac{4}{3}P = 0$$

$$F_{AB} = 2.404P\ (C) = 2.40P\ (C) \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AF} - 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) = 0$$

$$F_{AF} = 2.00P\ (T) \qquad \textbf{Ans}$$



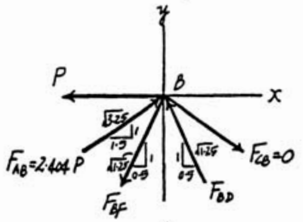
Joint *B*

$$\xrightarrow{+} \Sigma F_x = 0; \quad 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) - P - F_{BF}\left(\frac{0.5}{\sqrt{1.25}}\right) - F_{BD}\left(\frac{0.5}{\sqrt{1.25}}\right) = 0$$

$$1.00P - 0.4472F_{BF} - 0.4472F_{BD} = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad 2.404P\left(\frac{1}{\sqrt{3.25}}\right) + F_{BD}\left(\frac{1}{\sqrt{1.25}}\right) - F_{BF}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

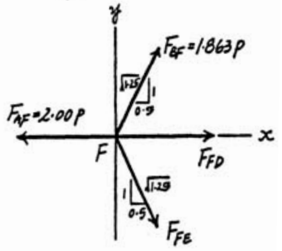
$$1.333P + 0.8944F_{BD} - 0.8944F_{BF} = 0 \qquad [2]$$



Solving Eqs. [1] and [2] yield,

$$F_{BF} = 1.863P(T) = 1.86P(T) \qquad \textbf{Ans}$$

$$F_{BD} = 0.3727P(C) = 0.373P(C) \qquad \textbf{Ans}$$



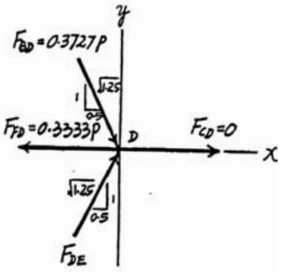
Joint *F*

$$+\uparrow \Sigma F_y = 0; \quad 1.863P\left(\frac{1}{\sqrt{1.25}}\right) - F_{FE}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{FE} = 1.863P(T) = 1.86P(T) \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{FD} + 2\left[1.863P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 2.00P = 0$$

$$F_{FD} = 0.3333P(T) = 0.333P\ (T) \qquad \textbf{Ans}$$



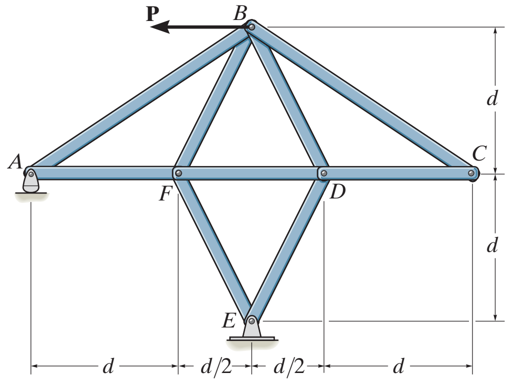
Joint *D*

$$+\uparrow \Sigma F_y = 0; \quad F_{DE}\left(\frac{1}{\sqrt{1.25}}\right) - 0.3727P\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{DE} = 0.3727P\ (C) = 0.373P\ (C) \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_y = 0; \quad 2\left[0.3727P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 0.3333P = 0\ (\textit{Check!})$$

•6–29. If the maximum force that any member can support is 4 kN in tension and 3 kN in compression, determine the maximum force *P* that can be applied at joint *B*. Take $d = 1$ m.



Support Reactions :

$$\circlearrowleft + \Sigma M_E = 0; \quad P(2d) - A_y\left(\frac{3}{2}d\right) = 0 \quad A_y = \frac{4}{3}P$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{4}{3}P - E_y = 0 \quad E_y = \frac{4}{3}P$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad E_x - P = 0 \quad E_x = P$$

Method of Joints : By inspection of joint *C*, members *CB* and *CD* are zero force members. Hence

$$F_{CB} = F_{CD} = 0$$

Joint *A*

$$+\uparrow \Sigma F_y = 0; \quad -F_{AB}\left(\frac{1}{\sqrt{3.25}}\right) + \frac{4}{3}P = 0 \quad F_{AB} = 2.404P \text{ (C)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AF} - 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) = 0 \quad F_{AF} = 2.00P \text{ (T)}$$

Joint *B*

$$\xrightarrow{+} \Sigma F_x = 0; \quad 2.404P\left(\frac{1.5}{\sqrt{3.25}}\right) - P - F_{BF}\left(\frac{0.5}{\sqrt{1.25}}\right) - F_{BD}\left(\frac{0.5}{\sqrt{1.25}}\right) = 0$$

$$1.00P - 0.4472F_{BF} - 0.4472F_{BD} = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad 2.404P\left(\frac{1}{\sqrt{3.25}}\right) + F_{BD}\left(\frac{1}{\sqrt{1.25}}\right) - F_{BF}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$1.333P + 0.8944F_{BD} - 0.8944F_{BF} = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yield,

$$F_{BF} = 1.863P\text{(T)} \qquad F_{BD} = 0.3727P\text{(C)}$$

Joint *F*

$$+\uparrow \Sigma F_y = 0; \quad 1.863P\left(\frac{1}{\sqrt{1.25}}\right) - F_{FE}\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

$$F_{FE} = 1.863P\text{(T)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{FD} + 2\left[1.863P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 2.00P = 0$$

$$F_{FD} = 0.3333P\text{(T)}$$

Joint *D*

$$+\uparrow \Sigma F_y = 0; \quad F_{DE}\left(\frac{1}{\sqrt{1.25}}\right) - 0.3727P\left(\frac{1}{\sqrt{1.25}}\right) = 0$$

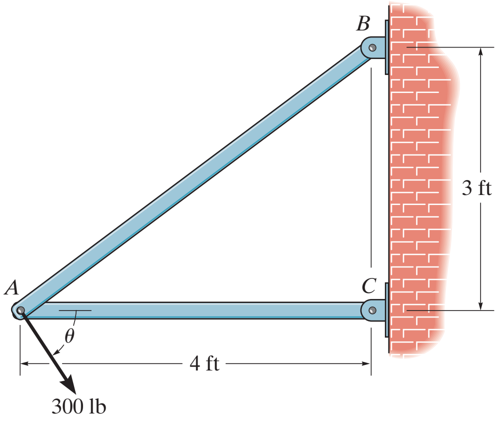
$$F_{DE} = 0.3727P \text{ (C)}$$

$$\xrightarrow{+} \Sigma F_y = 0; \quad 2\left[0.3727P\left(\frac{0.5}{\sqrt{1.25}}\right)\right] - 0.3333P = 0 \text{ (***Check!***)}$$

From the above analysis, the maximum compression and tension in the truss members are 2.404*P* and 2.00*P*, respectively. For this case, compression controls which requires

$$2.404P = 3$$
$$P = 1.25 \text{ kN}$$

6–30. The two-member truss is subjected to the force of 300 lb. Determine the range of θ for application of the load so that the force in either member does not exceed 400 lb (T) or 200 lb (C).



Joint *A* :

$$\xrightarrow{+} \Sigma F_x = 0; \quad 300\cos\theta + F_{AC} + F_{AB}\left(\frac{4}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad -300\sin\theta + F_{AB}\left(\frac{3}{5}\right) = 0$$

Thus,

$$F_{AB} = 500\sin\theta$$

$$F_{AC} = -300\cos\theta - 400\sin\theta$$

For *AB* require :

$$-200 \le 500\sin\theta \le 400$$

$$-2 \le 5\sin\theta \le 4 \qquad (1)$$

For *AC* require :

$$-200 \le -300\cos\theta - 400\sin\theta \le 400$$

$$-4 \le 3\cos\theta + 4\sin\theta \le 2 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$127^\circ \le \theta \le 196^\circ \qquad \text{Ans}$$

$$336^\circ \le \theta \le 347^\circ \qquad \text{Ans}$$

A possible hand solution :

$$\theta_2 = \theta_1 + \tan^{-1}\left(\frac{3}{4}\right) = \theta_1 + 36.870$$

Then

$$F_{AB} = 500\sin\theta_1$$

$$F_{AC} = -300\cos(\theta_2 - 36.870^\circ) - 400\sin(\theta_2 - 36.870^\circ)$$

$$= -300\,[\cos\theta_2\cos 36.870^\circ + \sin\theta_2\sin 36.870^\circ]$$

$$-400\,[\sin\theta_2\cos 36.870^\circ - \cos\theta_2\sin 36.870^\circ]$$

$$= -240\cos\theta_2 - 180\sin\theta_2 - 320\sin\theta_2 + 240\cos\theta_2$$

$$= -500\sin\theta_2$$

Thus, we require

$$-2 \le 5\sin\theta_1 \le 4 \quad \text{or} \quad -0.4 \le \sin\theta_1 \le 0.8 \qquad (1)$$

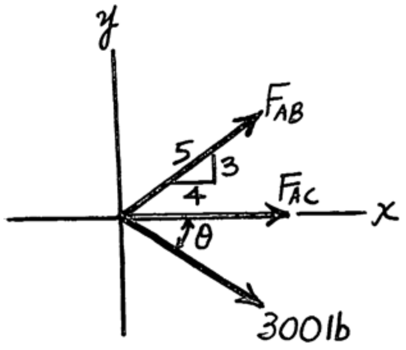
$$-4 \le 5\sin\theta_2 \le 2 \quad \text{or} \quad -0.8 \le \sin\theta_2 \le 0.4 \qquad (2)$$

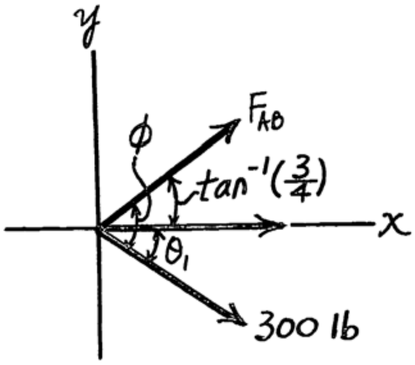
The range of values for Eqs. (1) and (2) are shown in the figures :

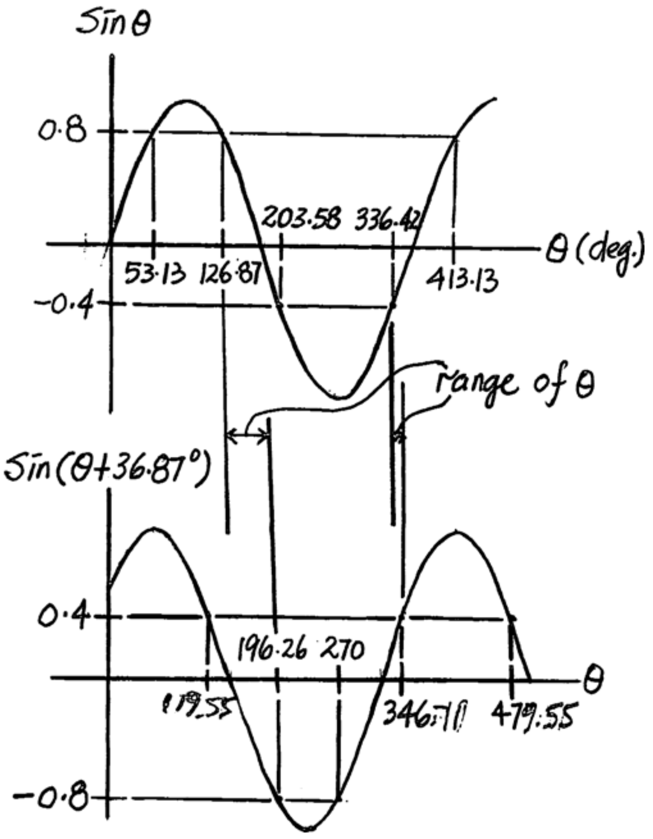
Since $\theta_1 = \theta_2 - 36.870^\circ$, the range of acceptable values for $\theta = \theta_1$ is

$$127^\circ \le \theta \le 196^\circ \qquad \text{Ans}$$

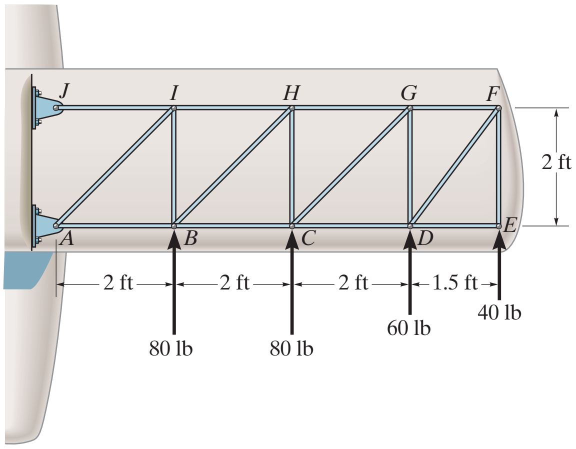
$$336^\circ \le \theta \le 347^\circ \qquad \text{Ans}$$

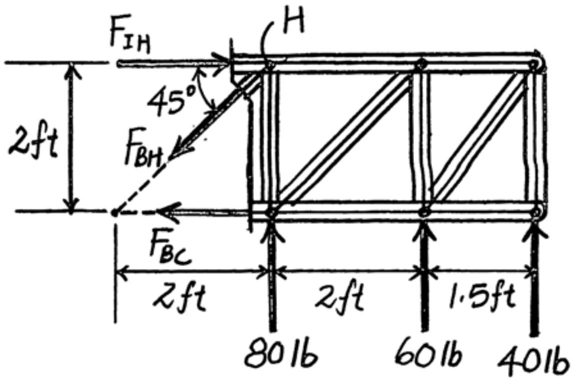

y
F_{AB}
5
3
4
F_{AC}
x
θ
300 lb


y
F_{AB}
φ
$\tan^{-1}(\frac{3}{4})$
x
θ_1
300 lb


Sin θ
0.8
-0.4
53.13
126.87
203.58
336.42
413.13
θ (deg.)
range of θ
Sin(θ+36.87°)
0.4
-0.8
119.55
196.26
270
346.71
479.55
θ

6–31. The internal drag truss for the wing of a light airplane is subjected to the forces shown. Determine the force in members *BC*, *BH*, and *HC*, and state if the members are in tension or compression.





$+\uparrow\Sigma F_y = 0;\quad 180 - F_{BH}\sin 45° = 0$

$F_{BH} = 255$ lb (T) Ans

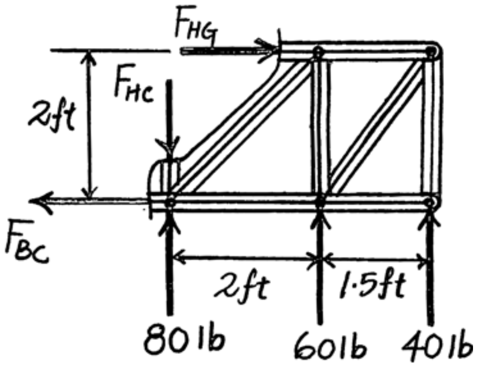
$\circlearrowleft+\Sigma M_H = 0;\quad -F_{BC}(2) + 60(2) + 40(3.5) = 0$

$F_{BC} = 130$ lb (T) Ans

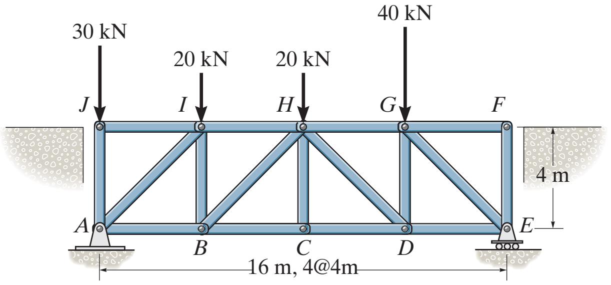
Section 2 :

$+\uparrow\Sigma F_y = 0;\quad 80 + 60 + 40 - F_{HC} = 0$

$F_{HC} = 180$ lb (C) Ans



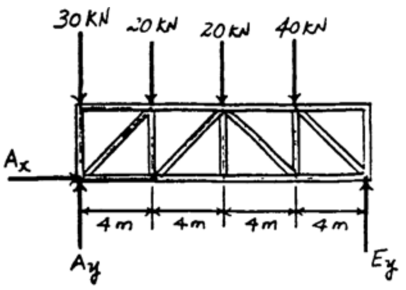
***6–32.** The *Howe bridge truss* is subjected to the loading shown. Determine the force in members *HD*, *CD*, and *GD*, and state if the members are in tension or compression.



Support Reactions :

$\circlearrowleft + \Sigma M_A = 0; \quad E_y(16) - 40(12) - 20(8) - 20(4) = 0$

$E_y = 45.0 \text{ kN}$



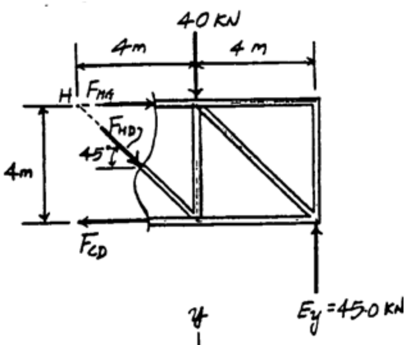
Method of Sections :

$\circlearrowleft + \Sigma M_H = 0; \quad 45.0(8) - 40(4) - F_{CD}(4) = 0$

$F_{CD} = 50.0 \text{ kN (T)}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 45.0 - 40 - F_{HD}\sin 45° = 0$

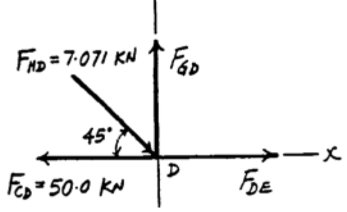
$F_{HD} = 7.071 \text{ kN (C)} = 7.07 \text{ kN (C)}$ **Ans**



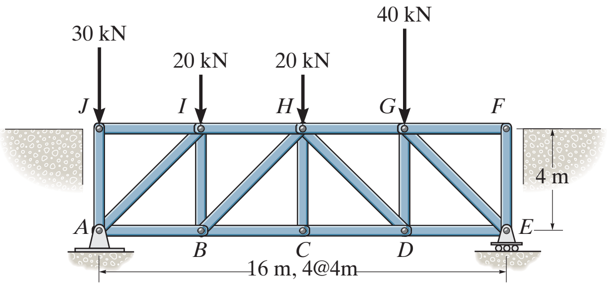
Method of Joints : Analysing joint *D*, we have

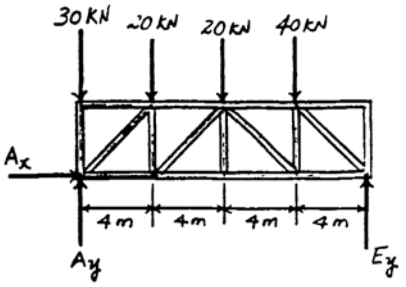
$+\uparrow \Sigma F_y = 0; \quad F_{GD} - 7.071\sin 45° = 0$

$F_{GD} = 5.00 \text{ kN (T)}$ **Ans**



•6–33. The *Howe bridge truss* is subjected to the loading shown. Determine the force in members *HI*, *HB*, and *BC*, and state if the members are in tension or compression.





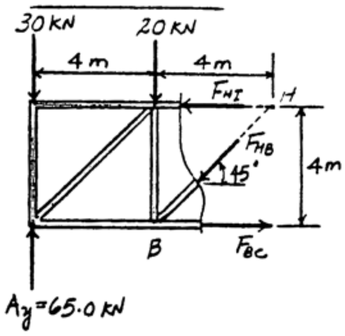
Support Reactions :

$\circlearrowleft + \Sigma M_E = 0;\quad 30(16) + 20(12) + 20(8) + 40(4) - A_y(16) = 0$

$A_y = 65.0 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$

Method of Sections :



$\circlearrowleft + \Sigma M_H = 0;\quad F_{BC}(4) + 20(4) + 30(8) - 65.0(8) = 0$

$F_{BC} = 50.0 \text{ kN (T)}$ **Ans**

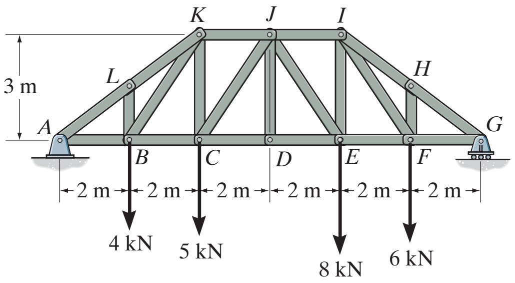
$\circlearrowleft + \Sigma M_B = 0;\quad F_{HI}(4) + 30(4) - 65.0(4) = 0$

$F_{HI} = 35.0 \text{ kN (C)}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 65.0 - 30 - 20 - F_{HB} \sin 45° = 0$

$F_{HB} = 21.2 \text{ kN (C)}$ **Ans**

6–34. Determine the force in members *JK*, *CJ*, and *CD* of the truss, and state if the members are in tension or compression.



Method of Joints: Applying the equations of equilibrium to the free-body diagram of the truss, Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad A_x = 0$

$\circlearrowleft + \Sigma M_G = 0$ $\quad 6(2) + 8(4) + 5(8) + 4(10) - A_y(12) = 0$

$A_y = 10.33 \text{ kN}$

Method of Sections: Using the left portion of the free-body diagram, Fig. *a*.

$\circlearrowleft + \Sigma M_C = 0;$ $\quad F_{JK}(3) + 4(2) - 10.33(4) = 0$

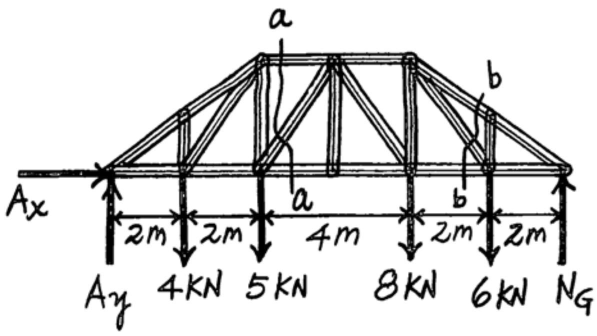
$F_{JK} = 11.111 \text{ kN} = 11.1 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_J = 0;$ $\quad F_{CD}(3) + 5(2) + 4(4) - 10.33(6) = 0$

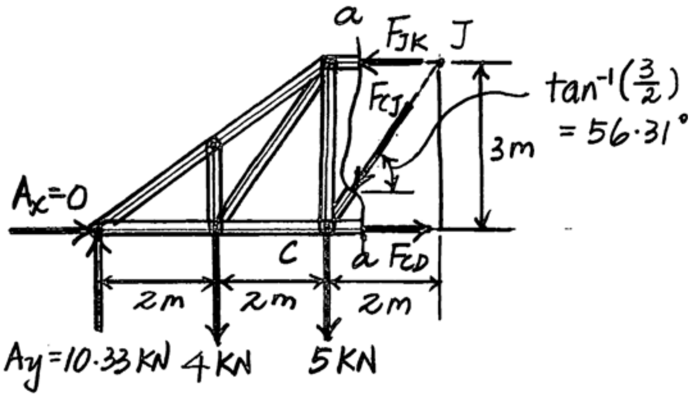
$F_{CD} = 12 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 10.33 - 4 - 5 - F_{CJ}\sin 56.31° = 0$

$F_{CJ} = 1.602 \text{ kN} = 1.60 \text{ kN (C)}$ **Ans.**

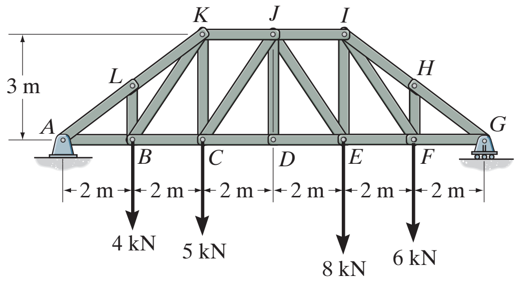


(a)



(b)

6–35. Determine the force in members *HI*, *FI*, and *EF* of the truss, and state if the members are in tension or compression.



Support Reactions: Applying the moment equation of equilibrium about point *A* to the free - body diagram of the truss, Fig. *a*,

$$\curvearrowleft +\Sigma M_A = 0; \qquad N_G(2) - 4(2) - 5(4) - 8(8) - 6(10) = 0$$

$$N_G = 12.67\ \text{kN}$$

Method of Sections: Using the right portion of the free - body diagram, Fig. *b*.

$$\curvearrowleft +\Sigma M_I = 0; \qquad 12.67(4) - 6(2) - F_{EF}(3) = 0$$

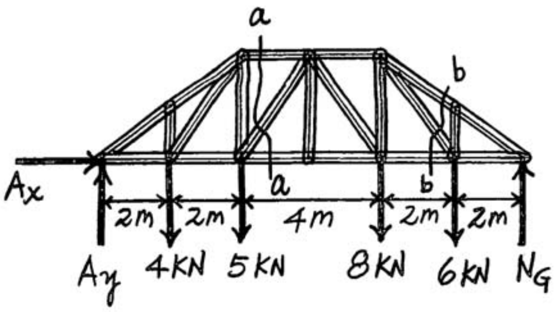
$$F_{EF} = 12.89\ \text{kN} = 12.9\ \text{kN (T)} \qquad \textbf{Ans.}$$

$$\curvearrowleft +\Sigma M_G = 0; \qquad -F_{FI}\sin 56.31°(2) + 6(2) = 0$$

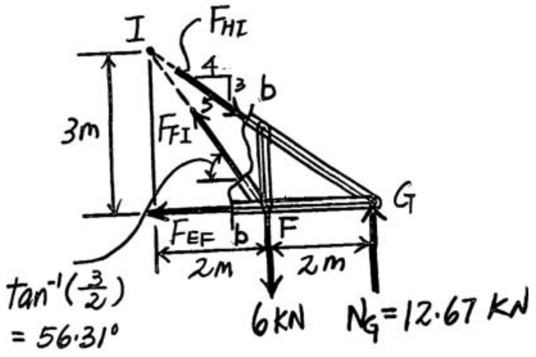
$$F_{FI} = 7.211\ \text{kN} = 7.21\ \text{kN (T)} \qquad \textbf{Ans.}$$

$$\curvearrowleft +\Sigma M_F = 0; \qquad 12.67(2) - F_{HI}\left(\frac{3}{5}\right)(2) = 0$$

$$F_{HI} = 21.11\ \text{kN} = 21.1\ \text{kN (C)} \qquad \textbf{Ans.}$$

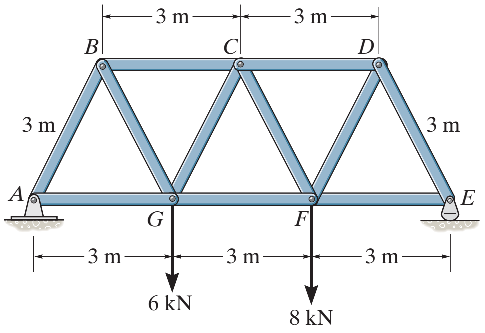


(a)



(b)

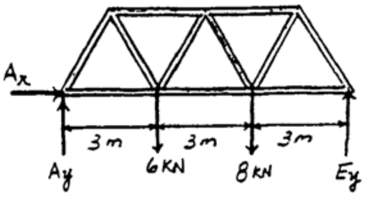
***6–36.** Determine the force in members *BC*, *CG*, and *GF* of the *Warren* truss. Indicate if the members are in tension or compression.



Support Reactions :

$\curvearrowleft + \Sigma M_E = 0; \quad 6(6) + 8(3) - A_y(9) = 0 \qquad A_y = 6.667 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$



Method of Sections :

$\curvearrowleft + \Sigma M_C = 0; \quad F_{GF}(3\sin 60°) + 6(1.5) - 6.667(4.5) = 0$

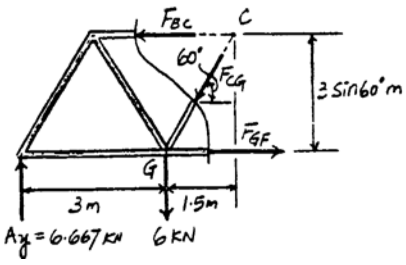
$F_{GF} = 8.08 \text{ kN (T)}$ **Ans**

$\curvearrowleft + \Sigma M_G = 0; \quad F_{BC}(3\sin 60°) - 6.667(3) = 0$

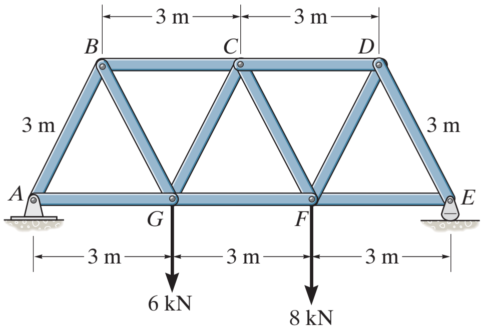
$F_{BC} = 7.70 \text{ kN (C)}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 6.667 - 6 - F_{CG}\sin 60° = 0$

$F_{CG} = 0.770 \text{ kN (C)}$ **Ans**



•6–37. Determine the force in members *CD*, *CF*, and *FG* of the *Warren truss*. Indicate if the members are in tension or compression.



Support Reactions :

$\curvearrowleft + \Sigma M_A = 0; \quad E_y(9) - 8(6) - 6(3) = 0 \qquad E_y = 7.333 \text{ kN}$

Method of Sections :

$\curvearrowleft + \Sigma M_C = 0; \quad 7.333(4.5) - 8(1.5) - F_{FG}(3\sin 60°) = 0$

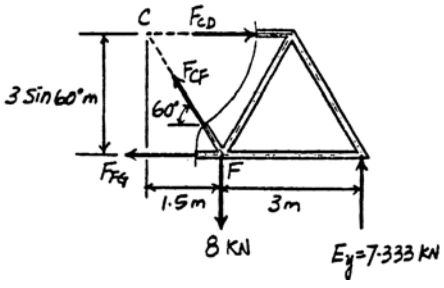
$F_{FG} = 8.08 \text{ kN (T)}$ **Ans**

$\curvearrowleft + \Sigma M_G = 0; \quad 7.333(3) - F_{CD}(3\sin 60°) = 0$

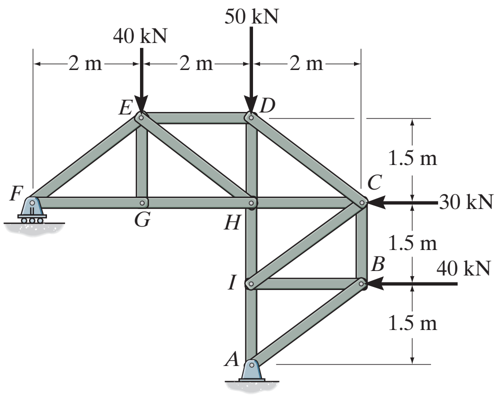
$F_{CD} = 8.47 \text{ kN (C)}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad F_{CF}\sin 60° + 7.333 - 8 = 0$

$F_{CF} = 0.770 \text{ kN (T)}$ **Ans**



6–38. Determine the force in members *DC*, *HC*, and *HI* of the truss, and state if the members are in tension or compression.



Support Reactions: Applying the moment equation of equilibrium about point A to the free - body diagram of the truss, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 40(1.5) + 30(3) + 40(2) - F_y(4) = 0$$

$$F_y = 57.5\text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - 30 - 40 = 0; \qquad A_x = 70\text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad 57.5 - 40 - 50 + A_y = 0; \qquad A_y = 32.5\text{ kN}$$

Method of Sections: Using the bottom portion of the free - body diagram, Fig. *b*.

$$\circlearrowleft + \Sigma M_C = 0; \qquad 70(3) - 32.5(2) - 40(1.5) - F_{HI}(2) = 0$$

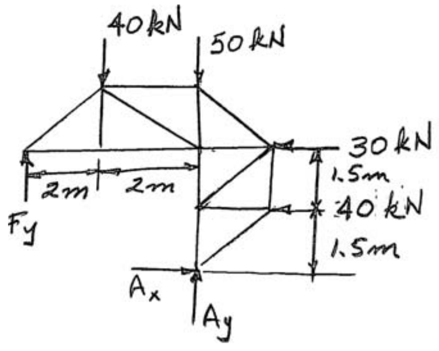
$$F_{HI} = 42.5\text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad 70(4.5) - 40(3) - 30(1.5) - F_{HC}(1.5) = 0$$

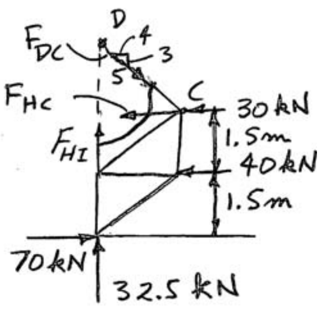
$$F_{HC} = 100\text{ kN (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 32.5 + 42.5 - F_{DC}\left(\frac{3}{5}\right) = 0$$

$$F_{DC} = 125\text{ kN (C)} \qquad \textbf{Ans.}$$

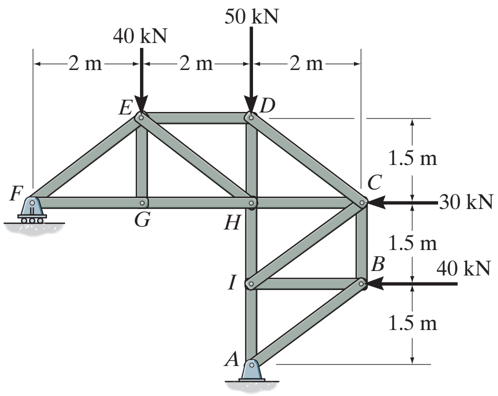


(a)



(b)

6–39. Determine the force in members *ED*, *EH*, and *GH* of the truss, and state if the members are in tension or compression.



Support Reactions: Applying the moment equation of equilibrium about point *A* to the free - body diagram of the truss, Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0;\quad 40(1.5) + 30(3) + 40(2) - F_y(4) = 0$

$F_y = 57.5\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 30 - 40 = 0;\quad A_x = 70\text{ kN}$

$+\uparrow \Sigma F_y = 0;\quad 57.5 - 40 - 50 + A_y = 0;\quad A_y = 32.5\text{ kN}$

Method of Sections: Using the left portion of the free - body diagram, Fig. *b*.

$\curvearrowleft + \Sigma M_E = 0;\quad -57.5(2) + F_{GH}(1.5) = 0$

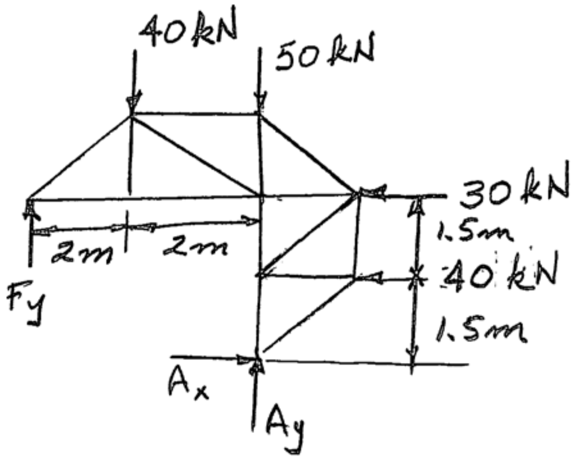
$F_{GH} = 76.7\text{ kN (T)}$ **Ans.**

$\curvearrowleft + \Sigma M_H = 0;\quad -57.5(4) + F_{ED}(1.5) + 40(2) = 0$

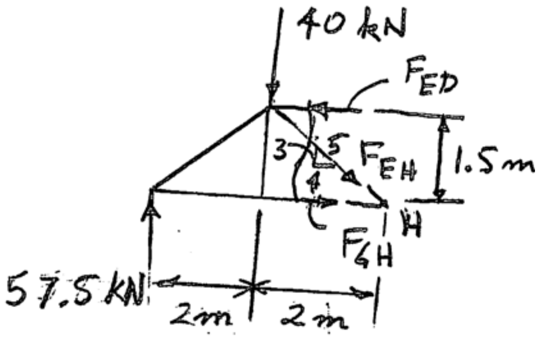
$F_{ED} = 100\text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 57.5 - F_{EH}\left(\frac{3}{5}\right) - 40 = 0$

$F_{EH} = 29.2\text{ kN (T)}$ **Ans.**

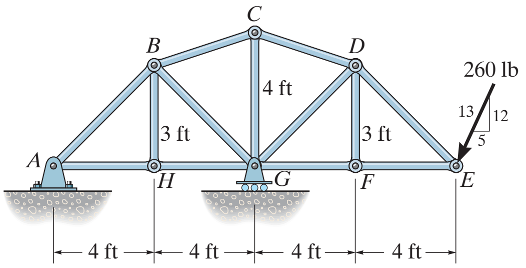


(a)



(b)

***6–40.** Determine the force in members *GF*, *GD*, and *CD* of the truss and state if the members are in tension or compression.



$\curvearrowleft + \Sigma M_O = 0;\quad \left(\frac{12}{13}\right)260\,(8) - F_{GD}\sin 36.87°\,(16) = 0$

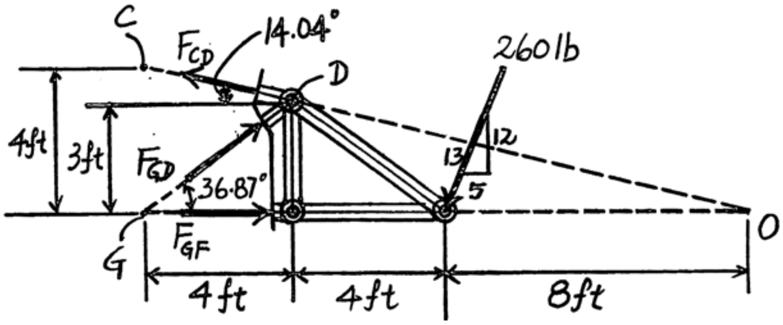
$F_{GD} = 200\text{ lb (C)}$ **Ans**

$\curvearrowleft + \Sigma M_D = 0;\quad F_{GF}\,(3) - \left(\frac{12}{13}\right)(260)\,(4) - \left(\frac{5}{13}\right)(260)\,(3) = 0$

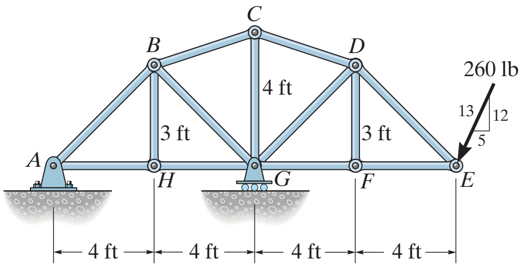
$F_{GF} = 420\text{ lb (C)}$ **Ans**

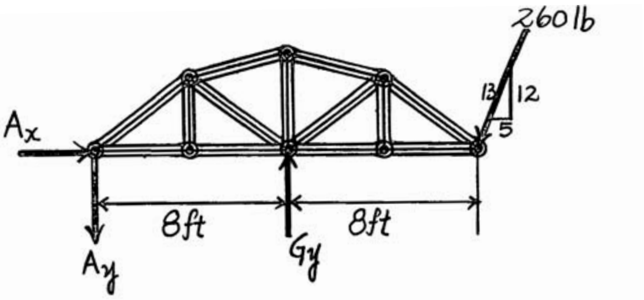
$\curvearrowleft + \Sigma M_G = 0;\quad F_{CD}\cos 14.04°\,(4) - \left(\frac{12}{13}\right)(260)\,(8) = 0$

$F_{CD} = 495\text{ lb (T)}$ **Ans**



•6–41. Determine the force in members *BG*, *BC*, and *HG* of the truss and state if the members are in tension or compression.





Entire truss :

$\curvearrowleft + \Sigma M_G = 0;\quad A_y\,(8) - \left(\frac{12}{13}\right)(260)\,(8) = 0$

$A_y = 240\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - \left(\frac{5}{13}\right)(260) = 0$

$A_x = 100\text{ lb}$

Section :

$\curvearrowleft + \Sigma M_G = 0;\quad 240\,(8) - F_{BC}\cos 14.04°\,(4) = 0$

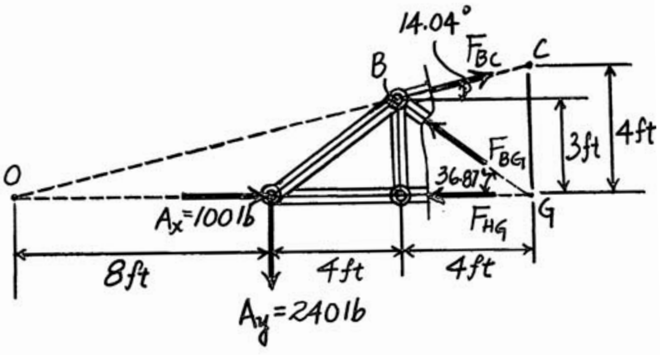
$F_{BC} = 495\text{ lb (T)}$ **Ans**

$\curvearrowleft + \Sigma M_B = 0;\quad 240\,(4) + 100\,(3) - F_{HG}\,(3) = 0$

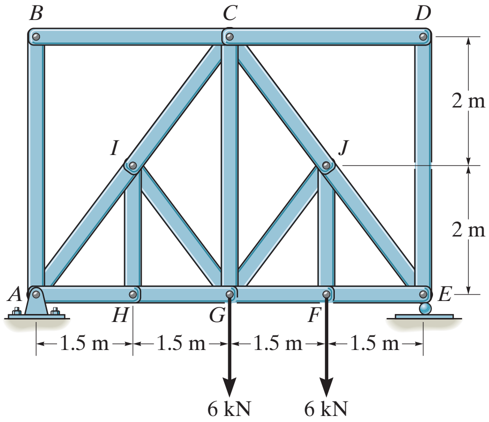
$F_{HG} = 420\text{ lb (C)}$ **Ans**

$\curvearrowleft + \Sigma M_O = 0;\quad -240\,(8) + F_{BG}\sin 36.87°\,(16) = 0$

$F_{BG} = 200\text{ lb (C)}$ **Ans**



6–42. Determine the force in members *IC* and *CG* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.

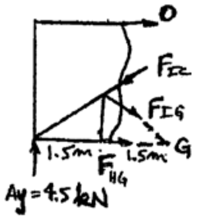


By inspection of joints *B*, *D*, *H* and *I*,

AB, *BC*, *CD*, *DE*, *HI*, and *GI* are all zero-force members. **Ans**

$\circlearrowleft +\Sigma M_G = 0; \quad -4.5(3) + F_{IC}(\frac{3}{5})(4) = 0$

$F_{IC} = 5.62 \text{ kN (C)}$ **Ans**

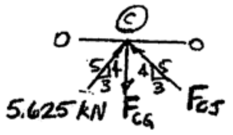


Joint *C*:

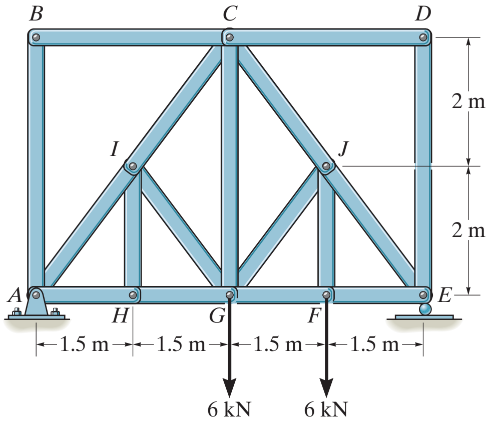
$\xrightarrow{+}\Sigma F_x = 0; \quad F_{CJ} = 5.625 \text{ kN}$

$+\uparrow\Sigma F_y = 0; \quad \frac{4}{5}(5.625) + \frac{4}{5}(5.625) - F_{CG} = 0$

$F_{CG} = 9.00 \text{ kN (T)}$ **Ans**



6–43. Determine the force in members *JE* and *GF* of the truss and state if these members are in tension or compression. Also, indicate all zero-force members.



By inspection of joints *B*, *D*, *H* and *I*,

AB, *BC*, *CD*, *DE*, *HI*, and *GI* are zero-force members. **Ans**

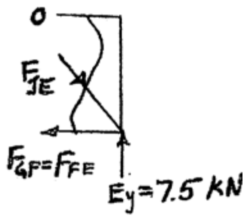
Joint *E*:

$+\uparrow\Sigma F_y = 0; \quad 7.5 - \frac{4}{5}F_{JE} = 0$

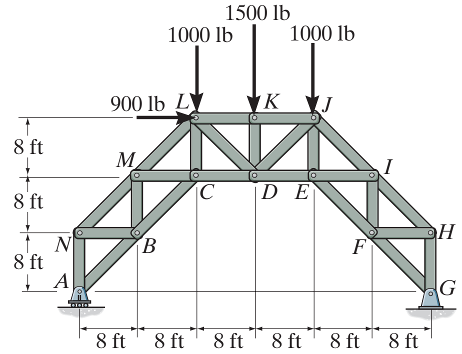
$F_{JE} = 9.375 = 9.38 \text{ kN (C)}$ **Ans**

$\xrightarrow{+}\Sigma F_x = 0; \quad \frac{3}{5}(9.375) - F_{GF} = 0$

$F_{GF} = 5.625 \text{ kN (T)}$ **Ans**



***6–44.** Determine the force in members *JI*, *EF*, *EI*, and *JE* of the truss, and state if the members are in tension or compression.



Support Reactions: Applying the equations of equilibrium to the free - body diagram of the truss, Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 900 - G_x = 0$

$G_x = 900 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 1000(16) + 1500(24) + 1000(32) + 900(24) - G_y(48) = 0$

$G_y = 2200 \text{ lb}$

Method of Sections: Using the right portion of the free - body diagram, Fig. *b*.

$\circlearrowleft + \Sigma M_E = 0;$ $\quad 2200(16) - 900(16) - F_{JI} \sin 45°(8) = 0$

$F_{JI} = 3676.96 \text{ lb} = 3677 \text{ lb (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_I = 0;$ $\quad 2200(8) - 900(16) - F_{EF} \cos 45°(8) = 0$

$F_{EF} = 565.69 \text{ ln} = 566 \text{ lb (T)}$ **Ans.**

Using the above results and writing the force equation of equilibrium along the *x* axis,

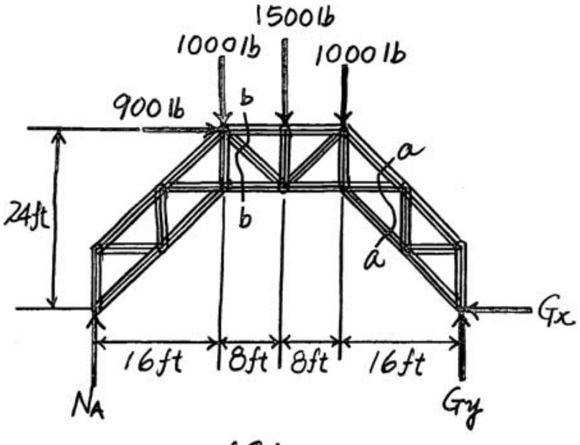
$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 3676.96 \cos 45° - 565.69 \cos 45° - 900 - F_{EI} = 0$

$F_{EI} = 1300 \text{ lb (T)}$ **Ans.**

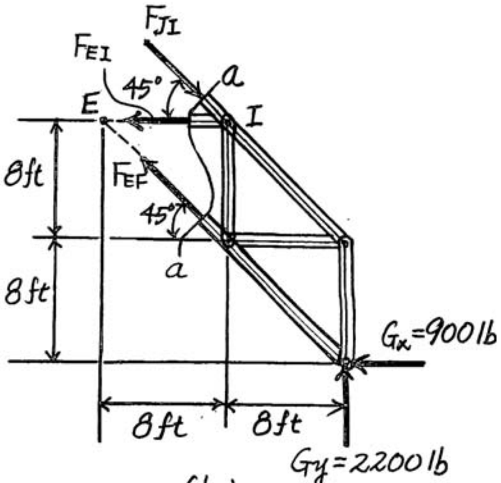
Method of Joints: From the free - body diagram of joint *E*, Fig. *c*,

$+\uparrow \Sigma F_y = 0;$ $\quad F_{JE} - 565.69 \sin 45° = 0$

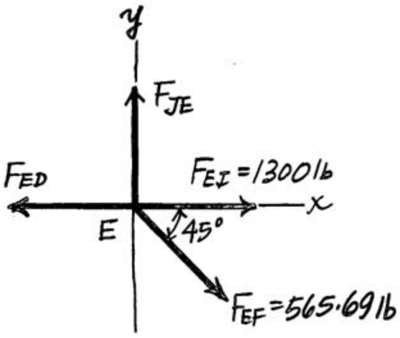
$F_{JE} = 400 \text{ lb (T)}$ **Ans.**



(a)

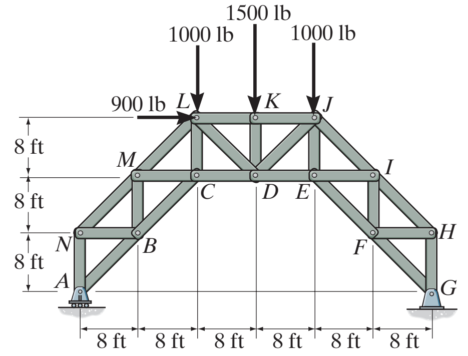


(b)



(c)

•6–45. Determine the force in members *CD*, *LD*, and *KL* of the truss, and state if the members are in tension or compression.



Support Reactions: Applying the equation of equilibrium about point *G* to the free - body diagram of the truss, Fig. *a*,

$$\circlearrowleft + \Sigma M_G = 0; \qquad 1000(16) + 1500(24) + 1000(32) - 900(24) - N_A(48) = 0$$

$$N_A = 1300 \text{ lb}$$

Method of Sections: Using the left portion of the free - body diagram, Fig. *b*.

$$\circlearrowleft + \Sigma M_D = 0; \qquad F_{KL}(8) + 1000(8) - 900(8) - 1300(24) = 0$$

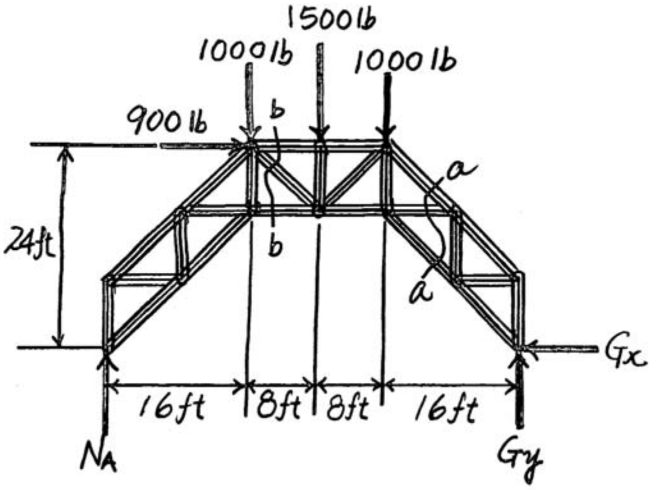
$$F_{KL} = 3800 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_L = 0; \qquad F_{CD}(8) - 1300(16) = 0$$

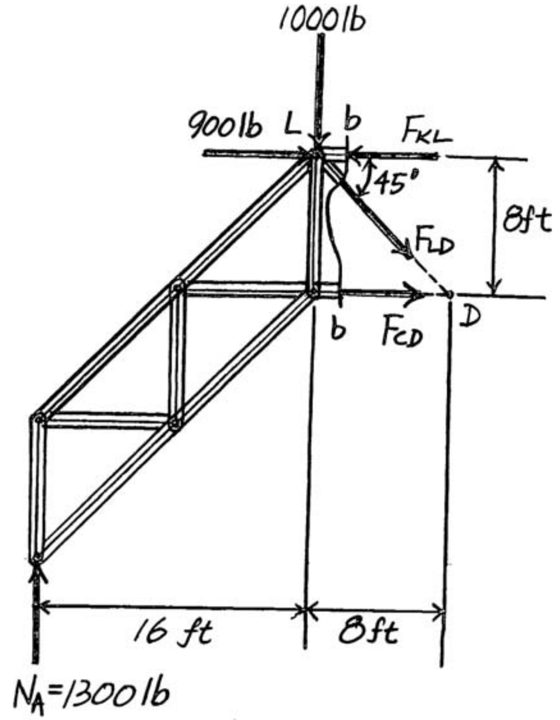
$$F_{CD} = 2600 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1300 - 1000 - F_{LD} \sin 45° = 0$$

$$F_{LD} = 424.26 \text{ lb} = 424 \text{ lb (T)} \qquad \textbf{Ans.}$$

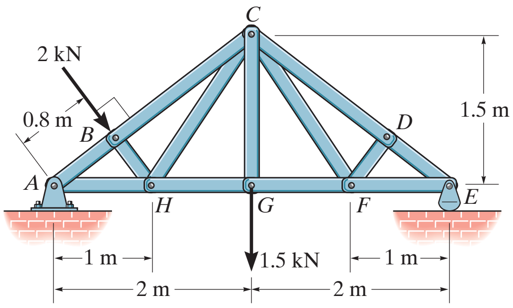


(a)



(b)

6–46. Determine the force developed in members *BC* and *CH* of the roof truss and state if the members are in tension or compression.



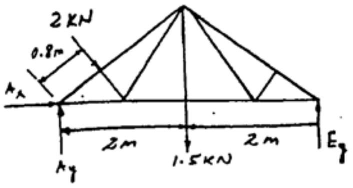
$\circlearrowleft + \Sigma M_A = 0;\quad E_y(4) - 2(0.8) - 1.5(2) = 0 \qquad E_y = 1.15\text{ kN}$

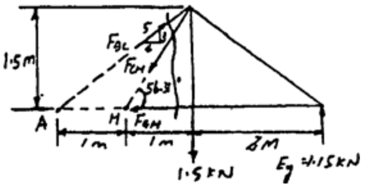
$\circlearrowleft + \Sigma M_H = 0;\quad 1.15(3) - 1.5(1) - \frac{3}{5}F_{BC}(1) = 0$

$F_{BC} = 3.25\text{ kN (C)}$ **Ans**

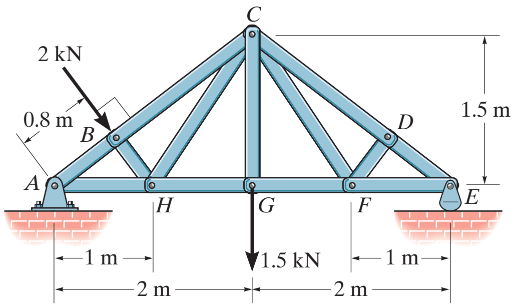
$\circlearrowleft + \Sigma M_A = 0;\quad 1.15(4) - 1.5(2) - F_{CH}\sin 56.31^\circ(1) = 0$

$F_{CH} = 1.92\text{ kN (T)}$ **An**





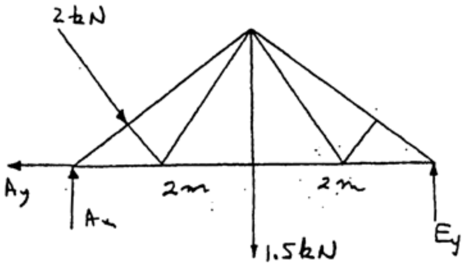
6–47. Determine the force in members *CD* and *GF* of the truss and state if the members are in tension or compression. Also indicate all zero-force members.



Entire truss :

$\circlearrowleft+\Sigma M_A = 0; \quad -2\,(0.8) - 1.5\,(2) + E_y\,(4) = 0$

$E_y = 1.15 \text{ kN}$



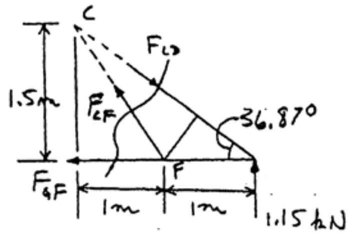
Section :

$\circlearrowleft+\Sigma M_F = 0; \quad 1.15\,(1) - F_{CD}\sin 36.87°\,(1) = 0$

$F_{CD} = 1.92 \text{ kN (C)}$ **Ans**

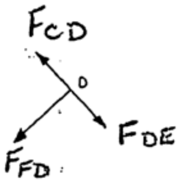
$\circlearrowleft+\Sigma M_C = 0; \quad -F_{GF}\,(1.5) + 1.15\,(2) = 0$

$F_{GF} = 1.53 \text{ kN (T)}$ **Ans**



Joint D :

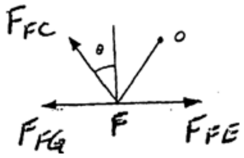
$+\nearrow\Sigma F_y = 0; \quad F_{FD} = 0$ **Ans**



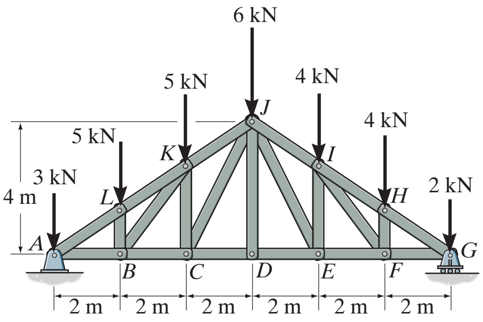
Joint F :

$+\uparrow\Sigma F_y = 0; \quad F_{FC}\cos\theta = 0$

$F_{FC} = 0$ **Ans**



***6–48.** Determine the force in members *IJ*, *EJ*, and *CD* of the *Howe* truss, and state if the members are in tension or compression.



Support Reactions: Applying the moment equation of equilibrium about point A to the free - body diagram of the truss, Fig. *a*,

$\circlearrowleft +\Sigma M_A = 0;$ $\quad N_G(12) - 2(12) - 4(10) - 4(8) - 6(6) - 5(4) - 5(2) = 0$

$N_G = 13.5\text{ kN}$

Method of Sections: By inspecting joint D, we find that member DJ is a zero - force member, thus $F_{DJ} = 0$. Using the right portion of the free - body diagram, Fig. *b*.

$\circlearrowleft +\Sigma M_J = 0;$ $\quad 13.5(6) - 4(2) - 4(4) - 2(6) - F_{CD}(4) = 0$

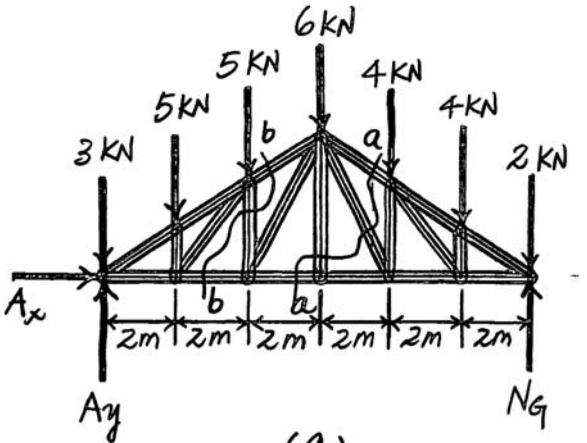
$F_{CD} = 11.25\text{ kN (T)}$ **Ans.**

$\circlearrowleft +\Sigma M_E = 0;$ $\quad 13.5(4) - 2(4) - 4(2) - F_{IJ}\sin 33.69°(4) = 0$

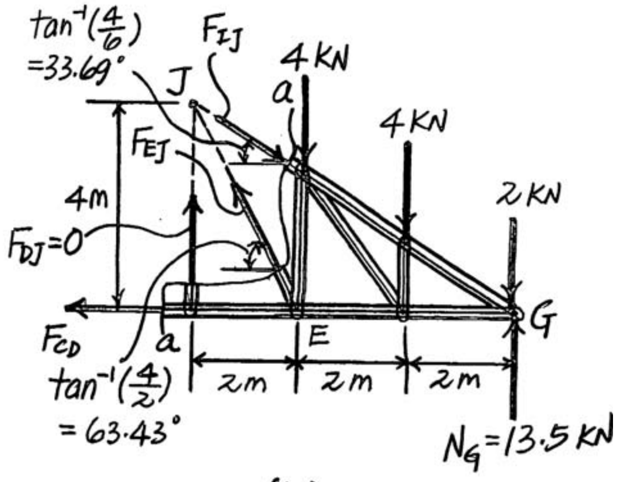
$F_{IJ} = 17.13\text{ kN} = 17.1\text{ kN (C)}$ **Ans.**

$\circlearrowleft +\Sigma M_G = 0;$ $\quad 4(2) + 4(4) - F_{EJ}\sin 63.43°(4) = 0$

$F_{EJ} = 6.708\text{ kN} = 6.71\text{ kN (T)}$ **Ans.**

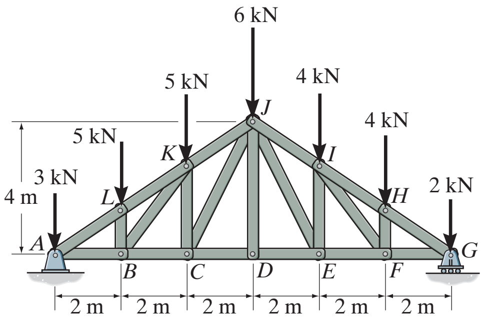


(a)



(b)

•6–49. Determine the force in members *KJ*, *KC*, and *BC* of the *Howe* truss, and state if the members are in tension or compression.



Support Reactions: Applying the equations of equilibrium to the free-body diagram of the truss, Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $A_x = 0$

$\curvearrowleft + \Sigma M_G = 0$ $3(12) + 5(10) + 5(8) + 6(6) + 4(4) + 4(2) - A_y(12) = 0$

$A_y = 15.5 \text{ kN}$

Method of Sections: Using the left portion of the free-body diagram, Fig. *a*.

$\curvearrowleft + \Sigma M_C = 0;$ $F_{KJ} \sin 33.69°(4) + 5(2) + 3(4) - 15.5(4) = 0$

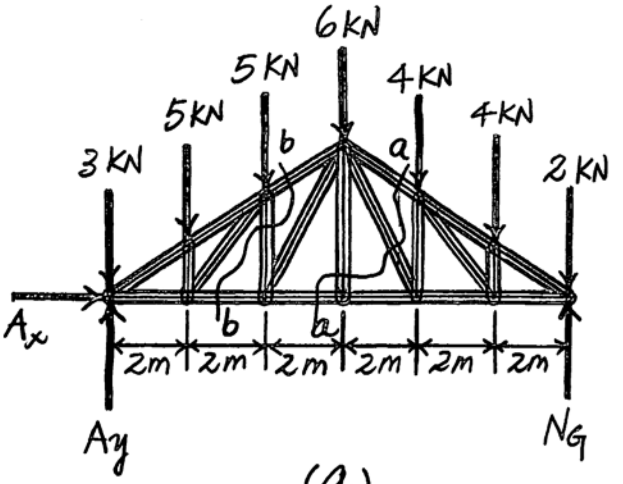
$F_{KJ} = 18.03 \text{ kN} = 18.0 \text{ kN (C)}$ **Ans.**

$\curvearrowleft + \Sigma M_A = 0;$ $F_{KC}(4) - 5(4) - 5(2) = 0$

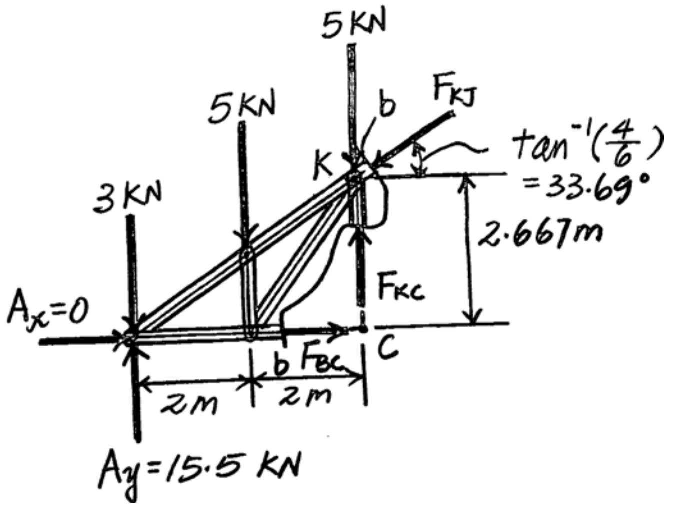
$F_{KC} = 7.50 \text{ kN (C)}$ **Ans.**

$\curvearrowleft + \Sigma M_K = 0;$ $F_{BC}(2.667) + 5(2) + 3(4) - 15.5(4) = 0$

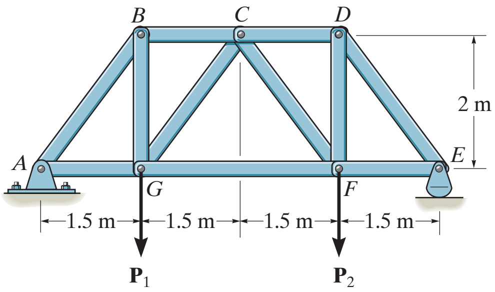
$F_{BC} = 15 \text{ kN (T)}$ **Ans.**



(a)



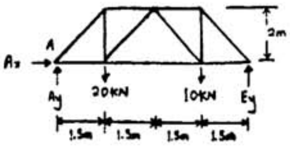
6–50. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 20$ kN, $P_2 = 10$ kN.



Entire truss :

$\circlearrowleft + \Sigma M_A = 0; \quad -20\,(1.5) - 10\,(4.5) + E_y\,(6) = 0$

$E_y = 12.5 \text{ kN}$



$+\uparrow \Sigma F_y = 0; \quad A_y - 20 - 10 + 12.5 = 0$

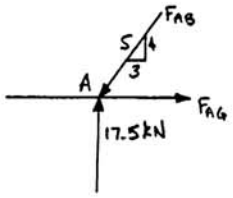
$A_y = 17.5 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$

Joint A :

$+\uparrow \Sigma F_y = 0; \quad 17.5 - \frac{4}{5} F_{AB} = 0$

$F_{AB} = 21.875 = 21.9 \text{ kN (C)}$ **Ans**



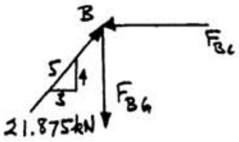
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AG} - \frac{3}{5}(21.875) = 0$

$F_{AG} = 13.125 = 13.1 \text{ kN (T)}$ **Ans**

Joint B :

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{3}{5}(21.875) - F_{BC} = 0$

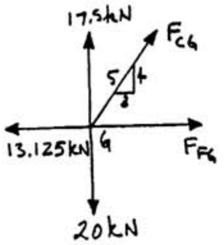
$F_{BC} = 13.125 = 13.1 \text{ kN (C)}$ **Ans**



$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}(21.875) - F_{BG} = 0$

$F_{BG} = 17.5 \text{ kN (T)}$ **Ans**

Joint G :



$+\uparrow \Sigma F_y = 0; \quad 17.5 - 20 + \frac{4}{5} F_{CG} = 0$

$F_{CG} = 3.125 = 3.12 \text{ kN (T)}$ **Ans**

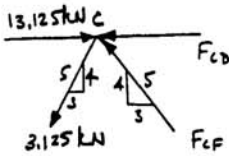
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{3}{5}(3.125) + F_{FG} - 13.125 = 0$

$F_{FG} = 11.25 = 11.2 \text{ kN (T)}$ **Ans**

Joint C :

$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5} F_{CF} - \frac{4}{5}(3.125) = 0$

$F_{CF} = 3.125 = 3.12 \text{ kN (C)}$ **Ans**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 13.125 - \frac{3}{5}(3.125) - \frac{3}{5}(3.125) - F_{CD} = 0$

$F_{CD} = 9.375 = 9.38 \text{ kN (C)}$ **Ans**

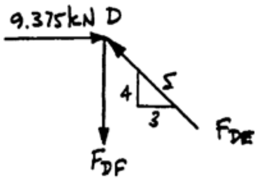
Joint *D* :

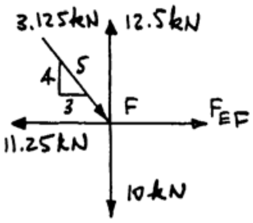
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 9.375 - \frac{3}{5} F_{DE} = 0$

$F_{DE} = 15.625 = 15.6 \text{ kN (C)}$ **Ans**

$+ \uparrow \Sigma F_y = 0; \quad \frac{4}{5}(15.625) - F_{DF} = 0$

$F_{DF} = 12.5 \text{ kN (T)}$ **Ans**





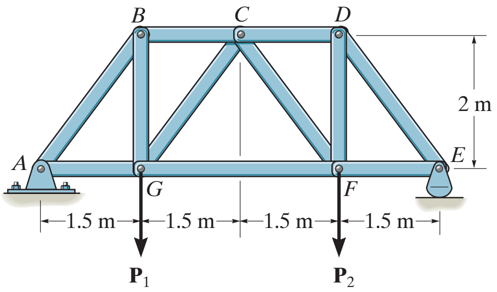
Joint *F* :

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{3}{5}(3.125) - 11.25 + F_{EF} = 0$

$F_{EF} = 9.38 \text{ kN (T)}$ **Ans**

$+ \uparrow \Sigma F_y = 0; \quad 12.5 - 10 - \frac{4}{5}(3.125) = 0$ **Check!**

6–51. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 40$ kN, $P_2 = 20$ kN.



Entire truss :

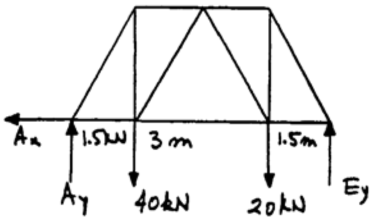
$\circlearrowleft + \Sigma M_A = 0; \quad -40\,(1.5) - 20\,(4.5) + E_y\ (6) = 0$

$E_y = 25$ kN

$+\uparrow \Sigma F_y = 0; \quad A_y - 40 - 20 + 25 = 0$

$A_y = 35$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$



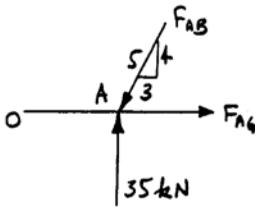
Joint A :

$+\uparrow \Sigma F_y = 0; \quad 35 - \frac{4}{5} F_{AB} = 0$

$F_{AB} = 43.75 = 43.8$ kN (C) **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AG} - \frac{3}{5}(43.75) = 0$

$F_{AG} = 26.25 = 26.2$ kN (T) **Ans**



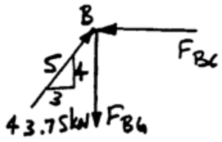
Joint B :

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{3}{5}(43.75) - F_{BC} = 0$

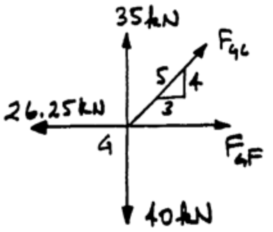
$F_{BC} = 26.25 = 26.2$ kN (C) **Ans**

$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}(43.75) - F_{BG} = 0$

$F_{BG} = 35.0$ kN (T) **Ans**



Joint G:



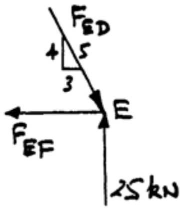
$+\uparrow\Sigma F_y = 0;\quad -40 + 35 + \frac{4}{5}F_{GC} = 0$

$F_{GC} = 6.25\text{ kN (T)}$ **Ans**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad \frac{3}{5}(6.25) + F_{GF} - 26.25 = 0$

$F_{GF} = 22.5\text{ kN (T)}$ **Ans**

Joint E:



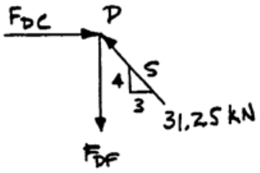
$+\uparrow\Sigma F_y = 0;\quad 25 - \frac{4}{5}F_{ED} = 0$

$F_{ED} = 31.25 = 31.2\text{ kN (C)}$ **Ans**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad -F_{EF} + \frac{3}{5}(31.25) = 0$

$F_{EF} = 18.75 = 18.8\text{ kN (T)}$ **Ans**

Joint D:



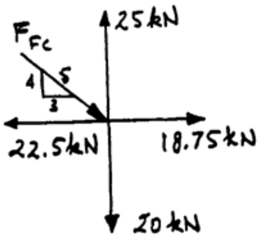
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{DC} - \frac{3}{5}(31.25) = 0$

$F_{DC} = 18.75 = 18.8\text{ kN (C)}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad \frac{4}{5}(31.25) - F_{DF} = 0$

$F_{DF} = 25.0\text{ kN (T)}$ **Ans**

Joint F:

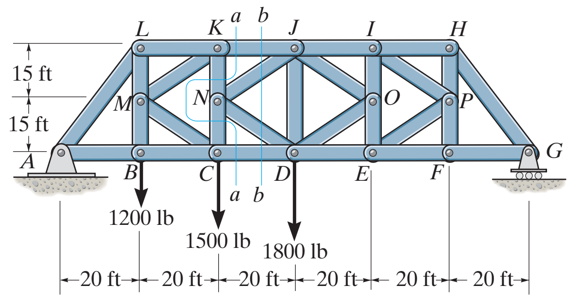


$+\uparrow\Sigma F_y = 0;\quad 25 - \frac{4}{5}(F_{FC}) - 20 = 0$

$F_{FC} = 6.25\text{ kN (C)}$ **Ans**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad -22.5 + 18.75 + \frac{3}{5}(6.25) = 0$ Check!

*__6–52.__ Determine the force in members *KJ*, *NJ*, *ND*, and *CD* of the *K truss*. Indicate if the members are in tension or compression. *Hint:* Use sections *aa* and *bb*.

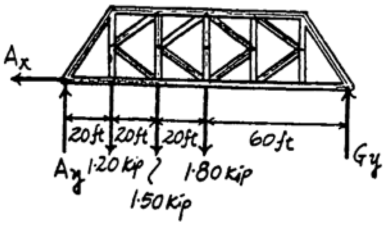


Support Reactions :

$$\curvearrowleft + \Sigma M_G = 0; \quad 1.20(100) + 1.50(80) + 1.80(60) - A_y(120) = 0$$

$$A_y = 2.90 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$$



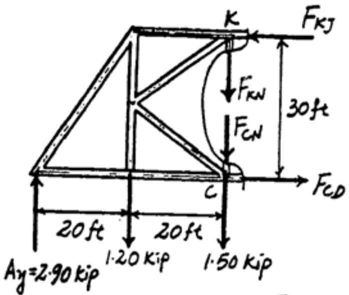
Method of Sections : From section $a-a$, F_{KJ} and F_{CD} can be obtained directly by summing moment about points C and K respectively.

$$\curvearrowleft + \Sigma M_C = 0; \quad F_{KJ}(30) + 1.20(20) - 2.90(40) = 0$$

$$F_{KJ} = 3.067 \text{ kip (C)} = 3.07 \text{ kip (C)} \qquad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M_K = 0; \quad F_{CD}(30) + 1.20(20) - 2.90(40) = 0$$

$$F_{CD} = 3.067 \text{ kip (T)} = 3.07 \text{ kip (T)} \qquad \textbf{Ans}$$



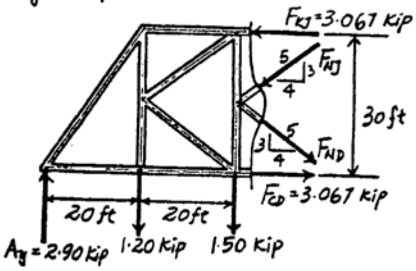
From sec $b-b$, summing forces along x and y axes yields

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{ND}\left(\frac{4}{5}\right) - F_{NJ}\left(\frac{4}{5}\right) + 3.067 - 3.067 = 0$$

$$F_{ND} = F_{NJ} \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad 2.90 - 1.20 - 1.50 - F_{ND}\left(\frac{3}{5}\right) - F_{NJ}\left(\frac{3}{5}\right) = 0$$

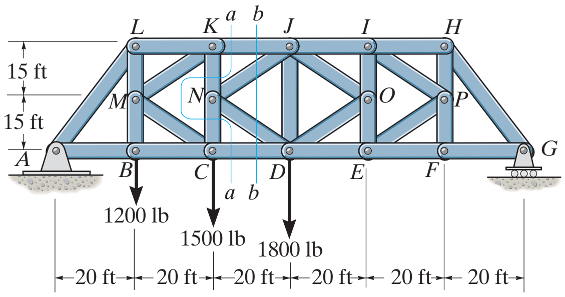
$$F_{ND} + F_{NJ} = 0.3333 \qquad [2]$$

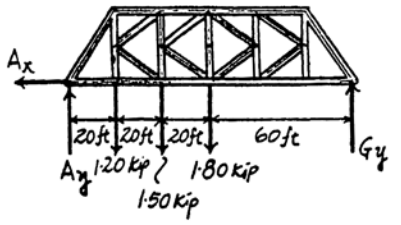


Solving Eqs. [1] and [2] yields

$$F_{ND} = 0.167 \text{ kip (T)} \qquad F_{NJ} = 0.167 \text{ kip (C)} \qquad \textbf{Ans}$$

•6–53. Determine the force in members *JI* and *DE* of the *K truss*. Indicate if the members are in tension or compression.





Support Reactions :

$\circlearrowleft + \Sigma M_A = 0;\quad G_y(120) - 1.80(60) - 1.50(40) - 1.20(20) = 0$

$G_y = 1.60 \text{ kip}$

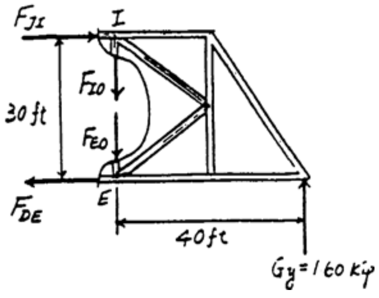
Method of Sections :

$\circlearrowleft + \Sigma M_E = 0;\quad 1.60(40) - F_{JI}(30) = 0$

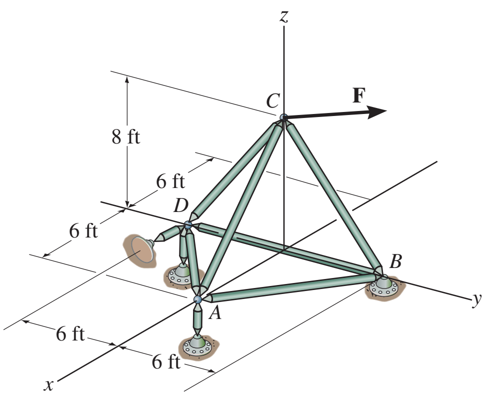
$F_{JI} = 2.13 \text{ kip (C)}$ **Ans**

$\circlearrowleft + \Sigma M_I = 0;\quad 1.60(40) - F_{DE}(30) = 0$

$F_{DE} = 2.13 \text{ kip (T)}$ **Ans**



6–54. The space truss supports a force $\mathbf{F} = \{-500\mathbf{i} + 600\mathbf{j} + 400\mathbf{k}\}$ lb. Determine the force in each member, and state if the members are in tension or compression.



Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint *C*, and then that of joints *A* and *D*.

Joint *C*: From the free - body diagram, Fig. *a*,

$\Sigma F_x = 0;\quad F_{CA}\left(\frac{3}{5}\right) - 500 = 0$

$F_{CA} = 833.33\text{ lb} = 833\text{ lb (T)}$ **Ans.**

$\Sigma F_y = 0;\quad F_{CB}\left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) + 600 = 0 \quad (1)$

$\Sigma F_z = 0;\quad 400 - 833.33\left(\frac{4}{5}\right) - F_{CD}\left(\frac{4}{5}\right) - F_{CB}\left(\frac{4}{5}\right) = 0 \quad (2)$

Solving Eqs. (1) and (2) yields

$F_{CB} = -666.67\text{ lb} = 667\text{ lb (C)}$ **Ans.**

$F_{CD} = 333.33\text{ lb} = 333\text{ lb (T)}$ **Ans.**

Joint *A*: From the free - body diagram, Fig. *b*,

$\Sigma F_x = 0;\quad F_{AD}\cos 45° - F_{AB}\cos 45° = 0$

$F_{AD} = F_{AB} = F$

$\Sigma F_y = 0;\quad F\sin 45° + F\sin 45° - 833.33\left(\frac{3}{5}\right) = 0$

$F = 353.55\text{ lb}$

Thus, $F_{AD} = F_{AB} = 353.55\text{ lb} = 354\text{ lb (C)}$ **Ans.**

$\Sigma F_z = 0;\quad 833.33\left(\frac{4}{5}\right) - A_z = 0$

$A_z = 666.67\text{ lb}$

Joint *D*: From the free - body diagram, Fig. *c*,

$\Sigma F_y = 0;\quad F_{DB} + 333.33\left(\frac{3}{5}\right) - 353.55\cos 45° = 0$

$F_{DB} = 50\text{ lb (T)}$ **Ans.**

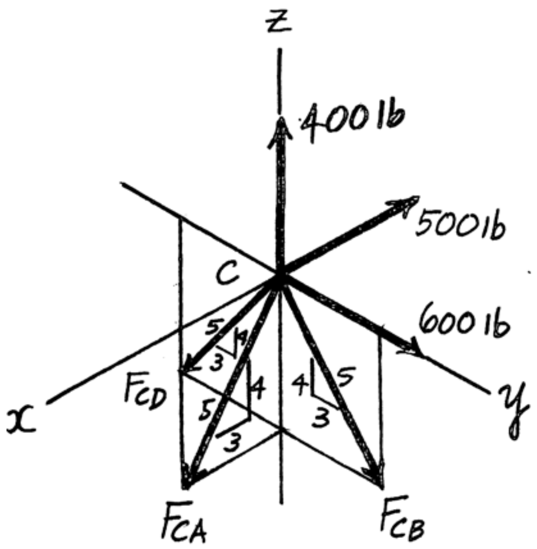
$\Sigma F_x = 0;\quad D_x - 353.55\sin 45° = 0$

$D_x = 250\text{ lb}$

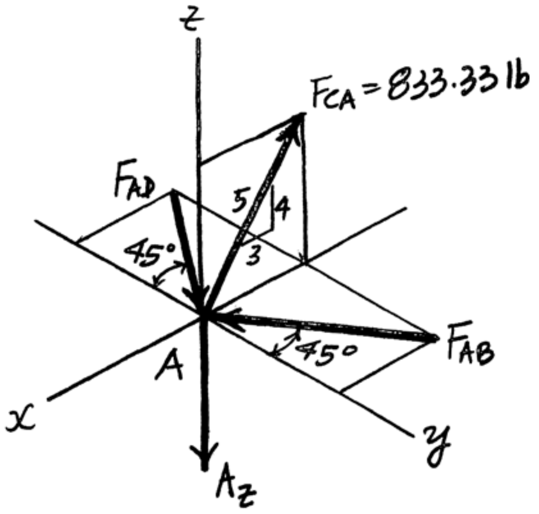
$\Sigma F_z = 0;\quad 333.33\left(\frac{4}{5}\right) - D_z = 0$

$D_z = 266.67\text{ lb}$

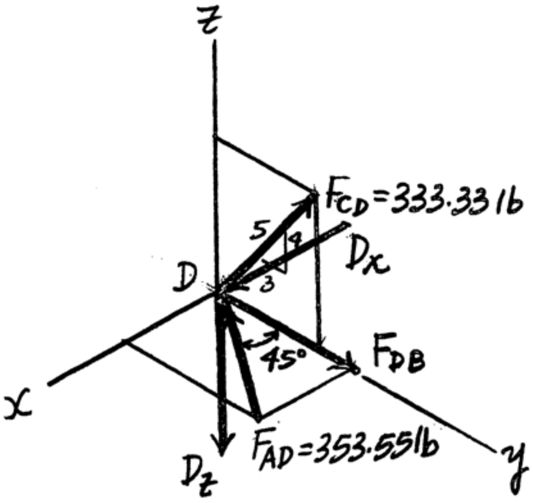
Note: The equilibrium analysis of joint *B* can be used to determine the components of support reaction of the ball and socket support at *B*.


z
400 lb
500 lb
600 lb
C
5
4
3
F_{CD}
5
4
3
4
5
3
x
y
F_{CA}
F_{CB}

(a)

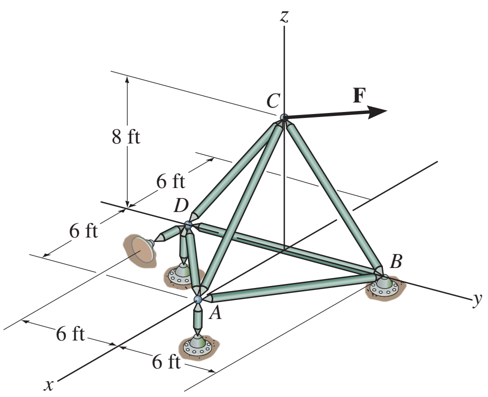

z
$F_{CA} = 833.33$ lb
F_{AD}
5
4
3
45°
A
F_{AB}
45°
x
y
A_z

(b)


z
$F_{CD} = 333.33$ lb
5
4
3
D
D_x
F_{DB}
45°
x
$F_{AD} = 353.55$ lb
D_z
y

(c)

6–55. The space truss supports a force $\mathbf{F} = \{600\mathbf{i} + 450\mathbf{j} - 750\mathbf{k}\}$ lb. Determine the force in each member, and state if the members are in tension or compression.



Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint C, and then that of joints A and D.

Joint C**:** From the free - body diagram, Fig. a,

$$\Sigma F_x = 0; \quad 600 + F_{CA}\left(\frac{3}{5}\right) = 0$$

$$F_{CA} = -1000 \text{ lb} = 1000 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \quad F_{CB}\left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) + 450 = 0 \qquad (1)$$

$$\Sigma F_z = 0; \quad -F_{CB}\left(\frac{4}{5}\right) - F_{CD}\left(\frac{4}{5}\right) - (-1000)\left(\frac{4}{5}\right) - 750 = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F_{CD} = 406.25 \text{ lb} = 406 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$F_{CB} = -343.75 \text{ lb} = 344 \text{ lb (C)} \qquad \textbf{Ans.}$$

Joint A**:** From the free - body diagram, Fig. b,

$$\Sigma F_y = 0; \quad F_{AB} \cos 45° - F_{AD} \cos 45° = 0$$

$$F_{AB} = F_{AD} = F$$

$$\Sigma F_x = 0; \quad 1000\left(\frac{3}{5}\right) - F \sin 45° - F \sin 45° = 0$$

$$F = 424.26 \text{ lb}$$

Thus, $F_{AB} = F_{AD} = 424.26 \text{ lb} = 424 \text{ lb (T)}$ **Ans.**

$$\Sigma F_z = 0; \quad A_z - 1000\left(\frac{4}{5}\right) = 0$$

$$A_z = 800 \text{ lb}$$

Joint D**:** From the free - body diagram, Fig. c,

$$\Sigma F_y = 0; \quad 406.25\left(\frac{3}{5}\right) + 406.25 \cos 45° - F_{DB} = 0$$

$$F_{DB} = 543.75 \text{ lb} = 544 \text{ lb (C)} \qquad \textbf{Ans.}$$

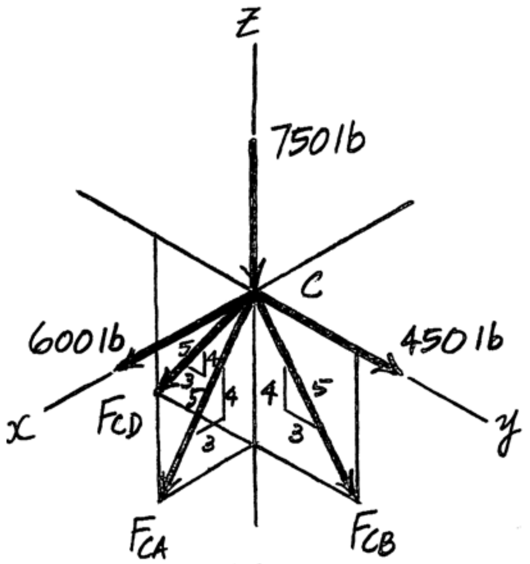
$$\Sigma F_x = 0; \quad 424.26 \sin 45° - D_x = 0$$

$$D_x = 300 \text{ lb}$$

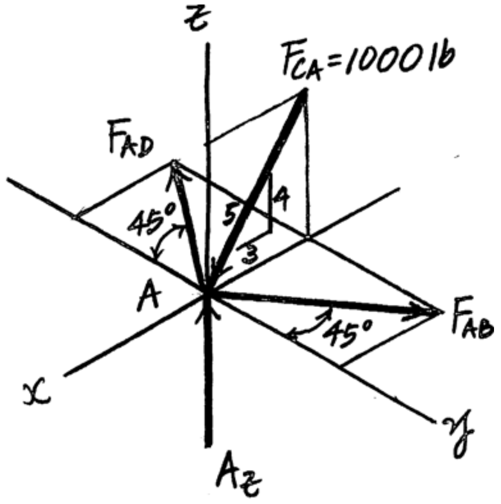
$$\Sigma F_z = 0; \quad 406.25\left(\frac{4}{5}\right) - D_z = 0$$

$$D_z = 325 \text{ lb}$$

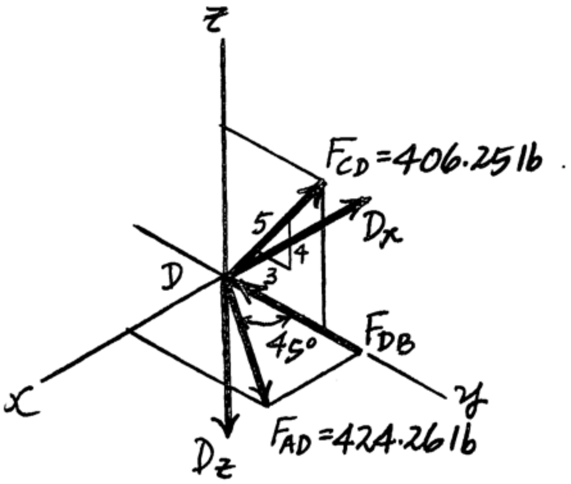
Note: The equilibrium analysis of joint B can be used to determine the components of support reaction of the ball and socket support at B.


z
750 lb
C
600 lb
450 lb
x
y
F_{CD}
F_{CA}
F_{CB}
5
3
4
3
4
5
3

(a)

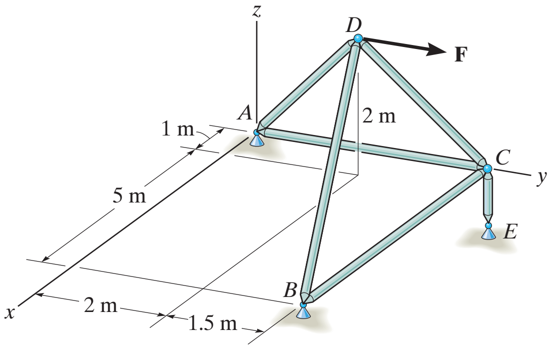

z
$F_{CA} = 1000$ lb
F_{AD}
45°
5
4
3
A
F_{AB}
45°
x
y
A_z

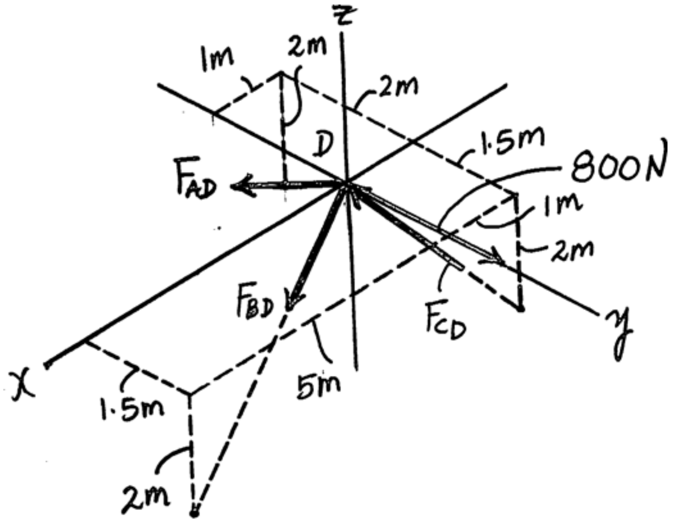
(b)


z
$F_{CD} = 406.25$ lb
5
4
3
D_x
D
45°
F_{DB}
x
y
D_z
$F_{AD} = 424.26$ lb

(c)

***6–56.** Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at A, B, and E. Set $\mathbf{F} = \{800\mathbf{j}\}$ N. *Hint:* The support reaction at E acts along member EC. Why?





Joint D :

$\Sigma F_x = 0;$ $\quad -\frac{1}{3}F_{AD} + \frac{5}{\sqrt{31.25}}F_{BD} + \frac{1}{\sqrt{7.25}}F_{CD} = 0$

$\Sigma F_y = 0;$ $\quad -\frac{2}{3}F_{AD} + \frac{1.5}{\sqrt{31.25}}F_{BD} - \frac{1.5}{\sqrt{7.25}}F_{CD} + 800 = 0$

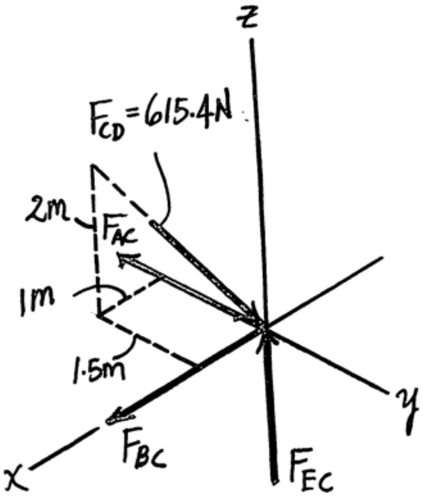
$\Sigma F_z = 0;$ $\quad -\frac{2}{3}F_{AD} - \frac{2}{\sqrt{31.25}}F_{BD} + \frac{2}{\sqrt{7.25}}F_{CD} = 0$

$F_{AD} = 686$ N (T) **Ans**

$F_{BD} = 0$ **Ans**

$F_{CD} = 615.4 = 615$ N (C) **Ans**

Joint C :



$\Sigma F_x = 0;$ $\quad F_{BC} - \frac{1}{\sqrt{7.25}}(615.4) = 0$

$F_{BC} = 229$ N (T) **Ans**

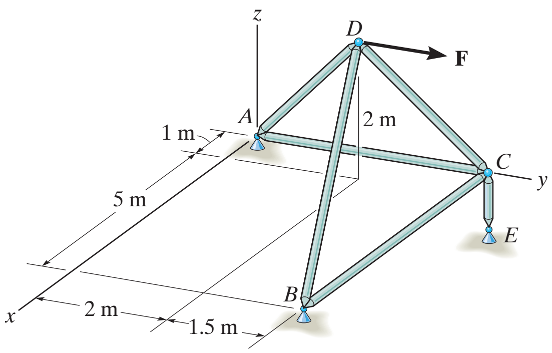
$\Sigma F_y = 0;$ $\quad \frac{1.5}{\sqrt{7.25}}(615.4) - F_{AC} = 0$

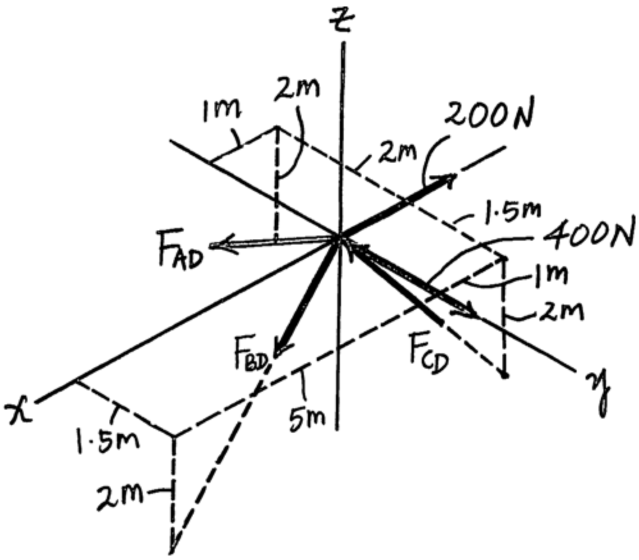
$F_{AC} = 343$ N (T) **Ans**

$\Sigma F_z = 0;$ $\quad F_{EC} - \frac{2}{\sqrt{7.25}}(615.4) = 0$

$F_{EC} = 457$ N (C) **Ans**

•6–57. Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at A, B, and E. Set $\mathbf{F} = \{-200\mathbf{i} + 400\mathbf{j}\}$ N. *Hint:* The support reaction at E acts along member EC. Why?





Joint D :

$\Sigma F_x = 0;\qquad -\frac{1}{3}F_{AD} + \frac{5}{\sqrt{31.25}}F_{BD} + \frac{1}{\sqrt{7.25}}F_{CD} - 200 = 0$

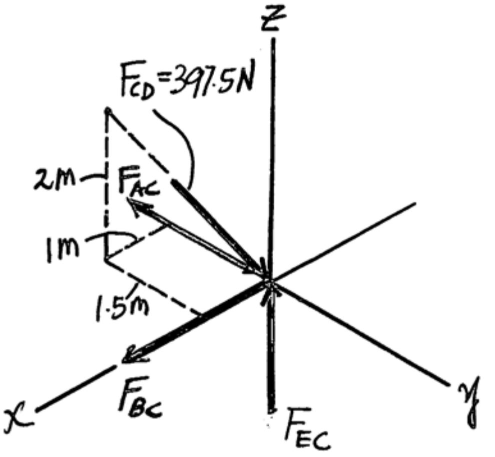
$\Sigma F_y = 0;\qquad -\frac{2}{3}F_{AD} + \frac{1.5}{\sqrt{31.25}}F_{BD} - \frac{1.5}{\sqrt{7.25}}F_{CD} + 400 = 0$

$\Sigma F_z = 0;\qquad -\frac{2}{3}F_{AD} - \frac{2}{\sqrt{31.25}}F_{BD} + \frac{2}{\sqrt{7.25}}F_{CD} = 0$

$F_{AD} = 343$ N (T) Ans

$F_{BD} = 186$ N (T) Ans

$F_{CD} = 397.5 = 397$ N (C) Ans



Joint C :

$\Sigma F_x = 0;\qquad F_{BC} - \frac{1}{\sqrt{7.25}}(397.5) = 0$

$F_{BC} = 148$ N (T) Ans

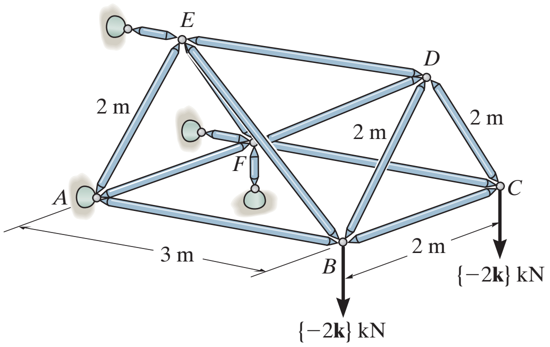
$\Sigma F_y = 0;\qquad \frac{1.5}{\sqrt{7.25}}(397.5) - F_{AC} = 0$

$F_{AC} = 221$ N (T) Ans

$\Sigma F_z = 0;\qquad F_{EC} - \frac{2}{\sqrt{7.25}}(397.5) = 0$

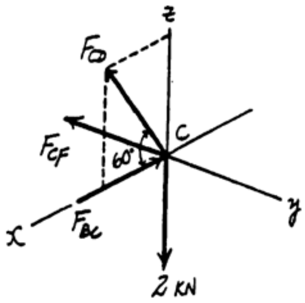
$F_{EC} = 295$ N (C) Ans

6–58. Determine the force in members *BE*, *DF*, and *BC* of the space truss and state if the members are in tension or compression.



Method of Joints : In this case, the support reactions are not required for determining the member forces.

Joint *C*

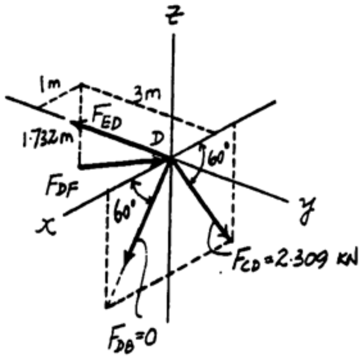


$\Sigma F_z = 0;\quad F_{CD}\sin 60^\circ - 2 = 0 \qquad F_{CD} = 2.309\text{ kN (T)}$

$\Sigma F_x = 0;\quad 2.309\cos 60^\circ - F_{BC} = 0$

$F_{BC} = 1.154\text{ kN (C)} = 1.15\text{ kN (C)}$ **Ans**

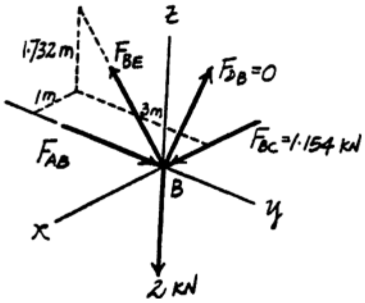
Joint *D* Since F_{CD}, F_{DF} and F_{DE} lie within the same plane and F_{DB} is out of this plane, then $F_{DB} = 0$.



$\Sigma F_x = 0;\quad F_{DF}\left(\frac{1}{\sqrt{13}}\right) - 2.309\cos 60^\circ = 0$

$F_{DF} = 4.16\text{ kN (C)}$ **Ans**

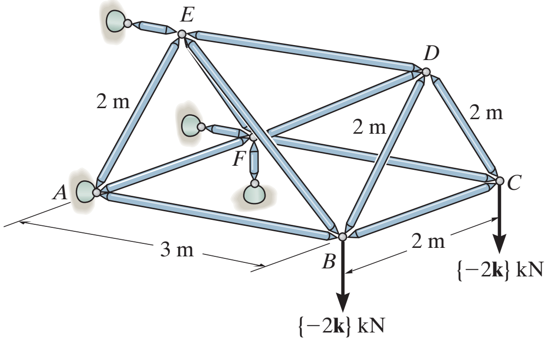
Joint *B*



$\Sigma F_z = 0;\quad F_{BE}\left(\frac{1.732}{\sqrt{13}}\right) - 2 = 0$

$F_{BE} = 4.16\text{ kN (T)}$ **Ans**

6–59. Determine the force in members *AB*, *CD*, *ED*, and *CF* of the space truss and state if the members are in tension or compression.



Method of Joints : In this case, the support reactions are not required for determining the member forces.

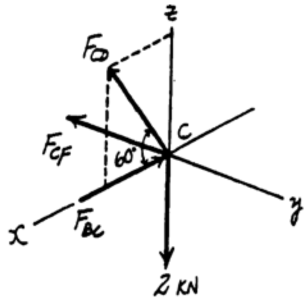
Joint *C* Since F_{CD}, F_{BC} and 2 kN force lie within the same plane and F_{CF} is out of this plane, then

$$F_{CF} = 0 \qquad \textbf{Ans}$$

$\Sigma F_z = 0;$ $\quad F_{CD}\sin 60° - 2 = 0$

$F_{CD} = 2.309\text{ kN (T)} = 2.31\text{ kN (T)}$ **Ans**

$\Sigma F_x = 0;$ $\quad 2.309\cos 60° - F_{BC} = 0 \qquad F_{BC} = 1.154\text{ kN (C)}$



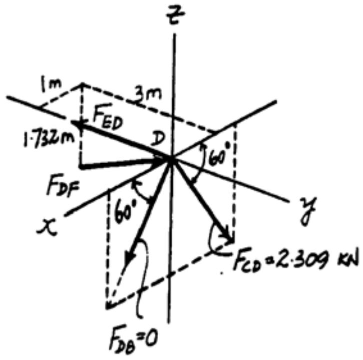
Joint *D* Since F_{CD}, F_{DE} and F_{DF} lie within the same plane and F_{DB} is out of this plane, then $F_{DB} = 0$.

$\Sigma F_x = 0;$ $\quad F_{DF}\left(\dfrac{1}{\sqrt{13}}\right) - 2.309\cos 60° = 0$

$F_{DF} = 4.163\text{ kN (C)}$

$\Sigma F_y = 0;$ $\quad 4.163\left(\dfrac{3}{\sqrt{13}}\right) - F_{ED} = 0$

$F_{ED} = 3.46\text{ kN (T)}$ **Ans**

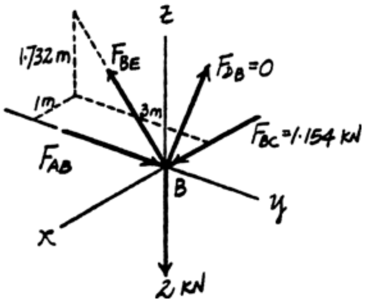


Joint *B*

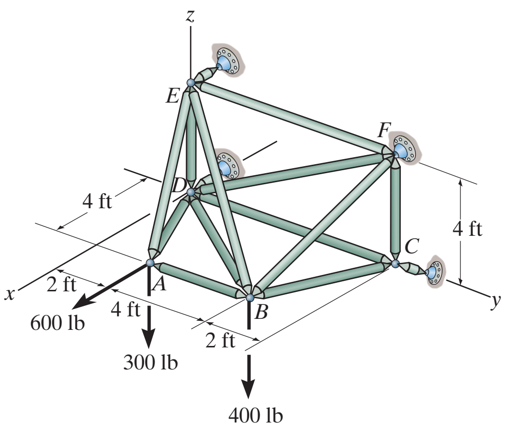
$\Sigma F_z = 0;$ $\quad F_{BE}\left(\dfrac{1.732}{\sqrt{13}}\right) - 2 = 0 \qquad F_{BE} = 4.163\text{ kN (T)}$

$\Sigma F_y = 0;$ $\quad F_{AB} - 4.163\left(\dfrac{3}{\sqrt{13}}\right) = 0$

$F_{AB} = 3.46\text{ kN (C)}$ **Ans**



***6–60.** Determine the force in the members AB, AE, BC, BF, BD, and BE of the space truss, and state if the members are in tension or compression.



Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint A, and then that of joints C and B.

Joint A**:** From the free - body diagram, Fig. a,

$$\Sigma F_z = 0; \qquad F_{AE}\left(\frac{4}{6}\right) - 300 = 0$$

$$F_{AE} = 450 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \quad 600 - 450\left(\frac{4}{6}\right) - F_{AD}\left(\frac{4}{\sqrt{20}}\right) = 0$$

$$F_{AD} = 335.41 \text{ lb (T)}$$

$$\Sigma F_y = 0; \quad F_{AB} - 335.41\left(\frac{2}{\sqrt{20}}\right) - 450\left(\frac{2}{6}\right) = 0$$

$$F_{AB} = 300 \text{ lb (T)} \qquad \textbf{Ans.}$$

Joint C**:** From the free - body diagram of the joint in Fig. b, notice that $\mathbf{F}_{CD}$, $\mathbf{F}_{CF}$, and $\mathbf{C}_y$ lie in the $y-z$ plane (shown shaded). Thus, if we write the force equation of equilibrium along the x axis, we have

$$\Sigma F_x = 0; \quad F_{BC}\left(\frac{4}{\sqrt{20}}\right) = 0$$

$$F_{BC} = 0 \qquad \textbf{Ans.}$$

Joint B**:** From the free - body diagram, Fig. c,

$$\Sigma F_x = 0; \quad -F_{BF}\left(\frac{4}{6}\right) - F_{BE}\left(\frac{4}{\sqrt{68}}\right) - F_{BD}\left(\frac{4}{\sqrt{52}}\right) = 0 \qquad (1)$$

$$\Sigma F_y = 0; \quad F_{BF}\left(\frac{2}{6}\right) - F_{BE}\left(\frac{6}{\sqrt{68}}\right) - F_{BD}\left(\frac{6}{\sqrt{52}}\right) - 300 = 0 \qquad (2)$$

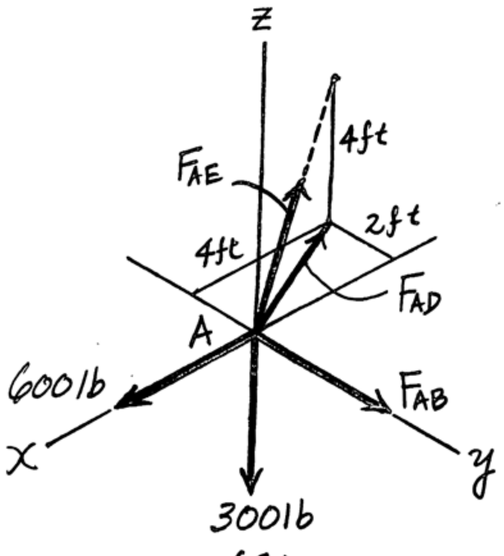
$$\Sigma F_z = 0; \qquad F_{BE}\left(\frac{4}{\sqrt{68}}\right) + F_{BF}\left(\frac{4}{6}\right) - 400 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

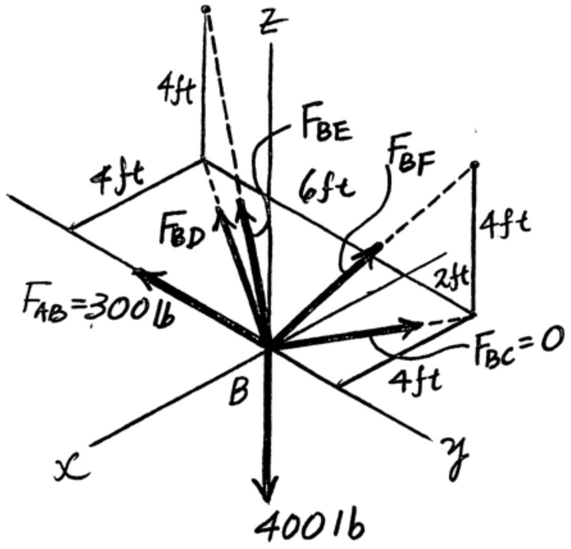
$$F_{BF} = 225 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$F_{BE} = 515.39 \text{ lb} = 515 \text{ lb (T)} \qquad \textbf{Ans.}$$

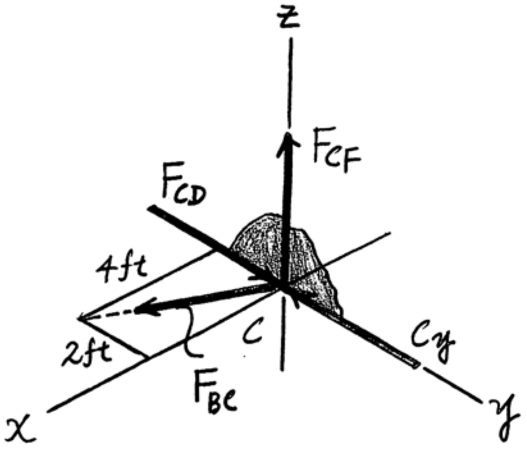
$$F_{BD} = -721.11 \text{ lb} = 721 \text{ lb (C)} \qquad \textbf{Ans.}$$


z
F_{AE}
4ft
2ft
4ft
F_{AD}
A
600lb
F_{AB}
x
y
300lb

(a)

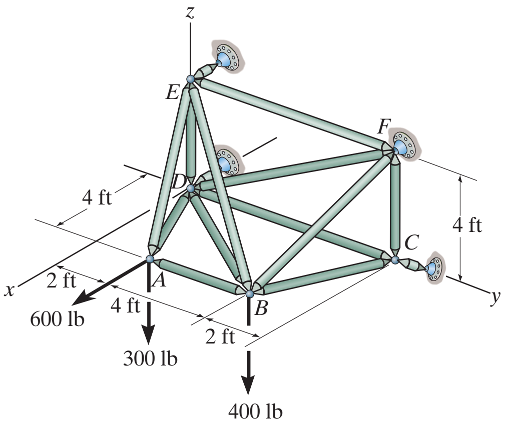

z
4ft
F_{BE}
F_{BF}
4ft
6ft
F_{BD}
4ft
2ft
$F_{AB}=300$ lb
$F_{BC}=0$
4ft
B
x
y
400lb

(c)


z
F_{CF}
F_{CD}
4ft
2ft
C
C_y
F_{BC}
x
y

(b)

•6–61. Determine the force in the members *EF*, *DF*, *CF*, and *CD* of the space truss, and state if the members are in tension or compression.



Support Reactions: In this case, it is easier to compute the support reactions first. From the free - body diagram of the truss, Fig. *a*, and writing the equations of equilibrium,

$\Sigma M_{y'} = 0;$ $\quad 400(4) + 300(4) - 600(4) - D_x(4) = 0$

$D_x = 100 \text{ lb}$

$\Sigma M_{x'} = 0;$ $\quad 400(2) + 300(6) - C_y(4) = 0$

$C_y = 650 \text{ lb}$

$\Sigma M_{z'} = 0;$ $\quad 600(6) + 100(8) - E_x(8) = 0$

$E_x = 550 \text{ lb}$

$\Sigma F_x = 0;$ $\quad -F_x + 600 + 100 - 550 = 0$

$F_x = 150 \text{ lb}$

$\Sigma F_y = 0;$ $\quad F_y - 650 = 0$

$F_y = 650 \text{ lb}$

$\Sigma F_z = 0;$ $\quad F_z - 300 - 400 = 0$

$F_z = 700 \text{ lb}$

Method of Joints: Using the above results, we will begin by analyzing the equilibrium of joint *C*, and then proceed to analyzing that of joint *F*.

Joint *C*: From the free - body diagram in Fig. *b*,

$\Sigma F_x = 0;$ $\quad F_{CB}\left(\frac{4}{\sqrt{20}}\right) = 0 \qquad F_{CB} = 0$

$\Sigma F_y = 0;$ $\quad F_{CD} - 650 = 0 \qquad F_{CD} = 650 \text{ lb (C)}$ **Ans.**

$\Sigma F_z = 0;$ $\quad F_{CF} = 0$ **Ans.**

Joint *F*: From the free - body diagram in Fig. *c*,

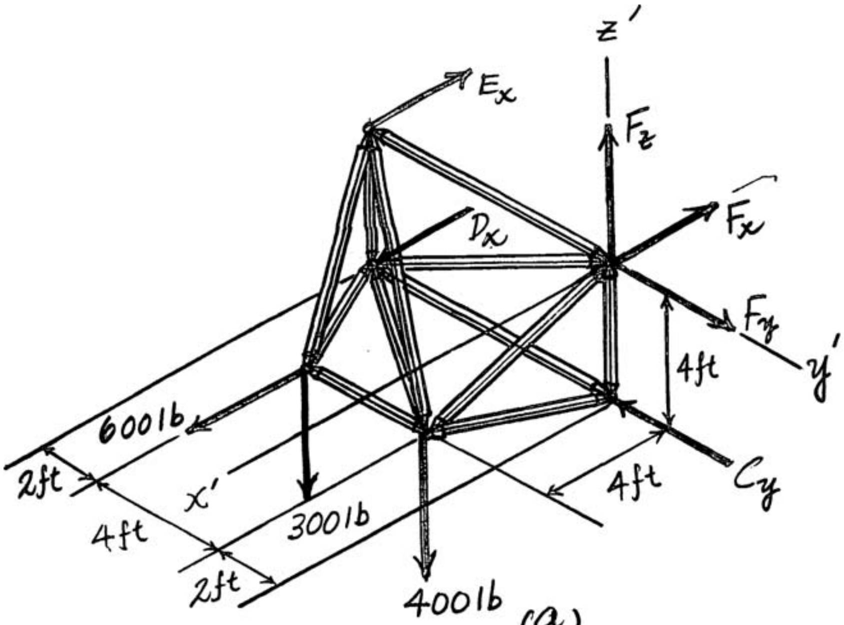
$\Sigma F_x = 0;$ $\quad F_{FB}\left(\frac{4}{6}\right) - 150 = 0 \qquad F_{BF} = 225 \text{ lb (T)}$

$\Sigma F_z = 0;$ $\quad 700 - 225\left(\frac{4}{6}\right) - F_{DF}\left(\frac{4}{\sqrt{80}}\right) = 0$

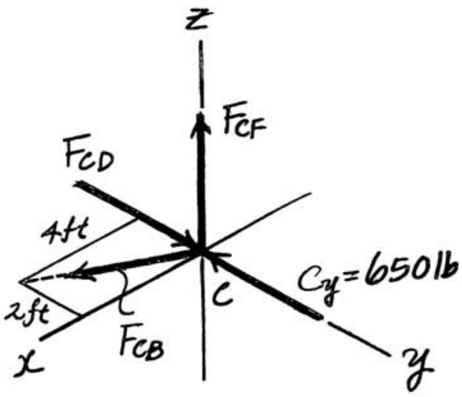
$F_{DF} = 1229.84 \text{ lb} = 1230 \text{ lb (T)}$ **Ans.**

$\Sigma F_y = 0;$ $\quad F_{EF} + 650 - 225\left(\frac{2}{6}\right) - 1229.84\left(\frac{8}{\sqrt{80}}\right) = 0$

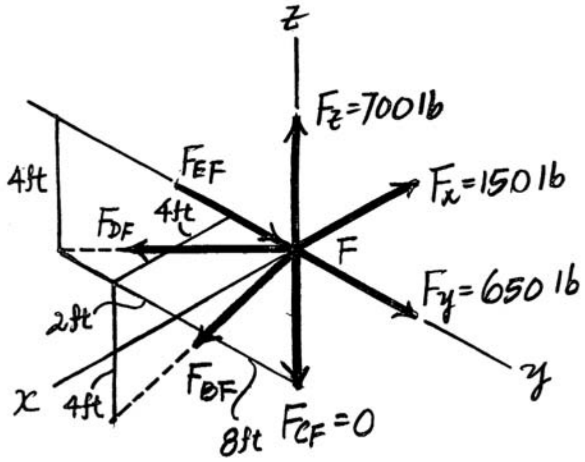
$F_{EF} = 525 \text{ lb (C)}$ **Ans.**


z'
E_x
F_z
F_x
D_x
F_y
y'
4ft
600 lb
2ft
x'
4ft
300 lb
2ft
400 lb
4ft
C_y

(a)

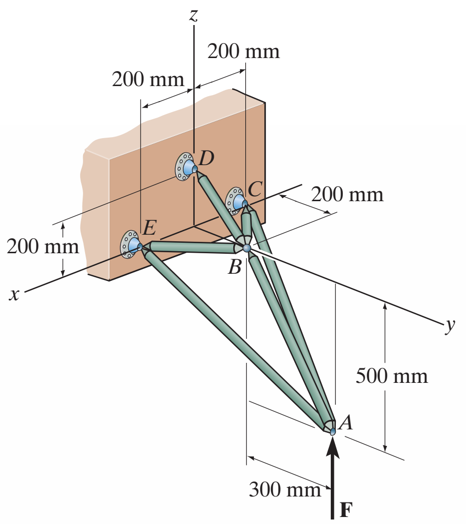

z
F_{CF}
F_{CD}
4ft
$C_y = 650$ lb
2ft
C
F_{CB}
x
y

(b)


z
$F_z = 700$ lb
4ft
F_{EF}
4ft
F_{DF}
$F_x = 150$ lb
F
$F_y = 650$ lb
2ft
x
4ft
F_{BF}
8ft
$F_{CF} = 0$
y

(c)

6–62. If the truss supports a force of $F = 200$ N, determine the force in each member and state if the members are in tension or compression.



Method of Joints: We will begin by analyzing the equilibrium of joint A, and then proceed to analyzing that of joint B.

Joint A: From the free - body diagram in Fig. b,

$$\Sigma F_x = 0;\quad F_{AE}\left(\frac{0.2}{\sqrt{0.54}}\right) - F_{AC}\left(\frac{0.2}{\sqrt{0.54}}\right) = 0 \qquad (1)$$

$$\Sigma F_y = 0;\quad F_{AB}\left(\frac{0.3}{\sqrt{0.34}}\right) - F_{AE}\left(\frac{0.5}{\sqrt{0.54}}\right) - F_{AC}\left(\frac{0.5}{\sqrt{0.54}}\right) = 0 \qquad (2)$$

$$\Sigma F_z = 0;\quad F_{AC}\left(\frac{0.5}{\sqrt{0.54}}\right) + F_{AE}\left(\frac{0.5}{\sqrt{0.54}}\right) - F_{AB}\left(\frac{0.5}{\sqrt{0.34}}\right) + 200 = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AE} = F_{AC} = 220.45\text{ N} = 220\text{ N (T)}$ **Ans.**

$F_{AB} = 583.10\text{ N} = 583\text{ N (C)}$ **Ans.**

Joint B: From the free - body diagram in Fig. b,

$$\Sigma F_z = 0;\quad 583.10\left(\frac{0.5}{\sqrt{0.34}}\right) - F_{BD}\sin 45° = 0$$

$F_{BD} = 707.11\text{ N} = 707\text{ N (C)}$ **Ans.**

$$\Sigma F_x = 0;\quad F_{BE}\cos 45° - F_{BC}\cos 45° = 0$$

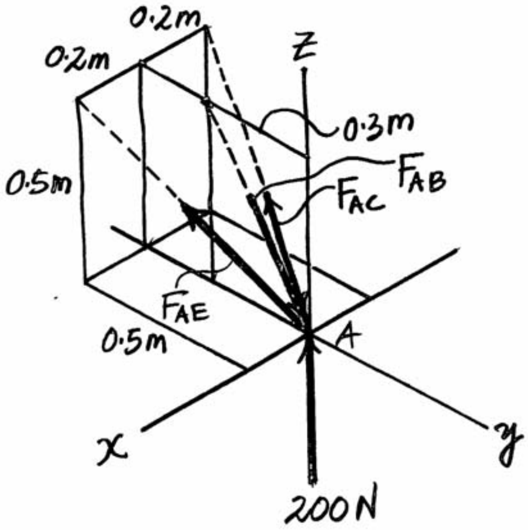
$$F_{BE} = F_{BC} = F$$

$$\Sigma F_y = 0;\quad 707.11\cos 45° - 583.10\left(\frac{0.3}{\sqrt{0.34}}\right) - 2F\sin 45° = 0$$

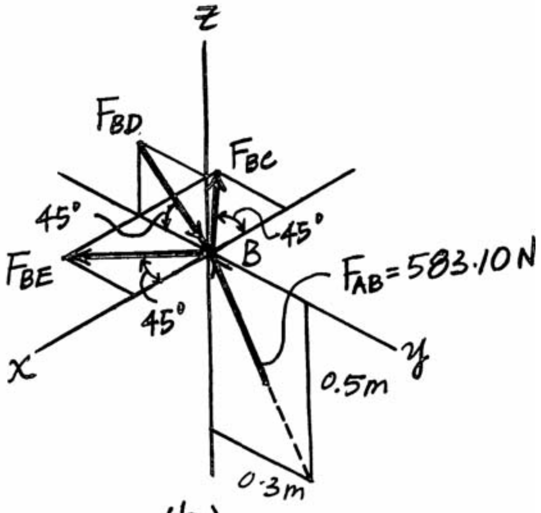
$F = 141.42\text{ N}$ **Ans.**

Thus,

$F_{BE} = F_{BC} = 141.42\text{ N} = 141\text{ N (T)}$ **Ans.**

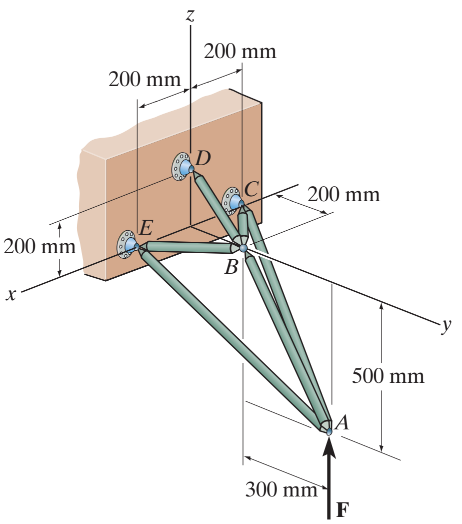


(a)



(b)

6–63. If each member of the space truss can support a maximum force of 600 N in compression and 800 N in tension, determine the greatest force *F* the truss can support.



Method of Joints: We will begin by analyzing the equilibrium of joint *A*, and then proceed to analyzing that of joint *B*.

Joint *A*: From the free - body diagram in Fig. *b*,

$$\Sigma F_x = 0; \quad F_{AE}\left(\frac{0.2}{\sqrt{0.54}}\right) - F_{AC}\left(\frac{0.2}{\sqrt{0.54}}\right) = 0 \qquad (1)$$

$$\Sigma F_y = 0; \quad F_{AB}\left(\frac{0.3}{\sqrt{0.34}}\right) - F_{AE}\left(\frac{0.5}{\sqrt{0.54}}\right) - F_{AC}\left(\frac{0.5}{\sqrt{0.54}}\right) = 0 \qquad (2)$$

$$\Sigma F_z = 0; \quad F_{AE}\left(\frac{0.5}{\sqrt{0.54}}\right) + F_{AC}\left(\frac{0.5}{\sqrt{0.54}}\right) - F_{AB}\left(\frac{0.5}{\sqrt{0.34}}\right) + F = 0 \qquad (3)$$

Solving Eqs. (1) through (3) yields

$F_{AB} = 2.9155F$ (C)

$F_{AC} = F_{AE} = 1.1023F$ (T)

Joint *B*: From the free - body diagram in Fig. *b*,

$$\Sigma F_z = 0; \quad 2.9155F\left(\frac{0.5}{\sqrt{0.34}}\right) - F_{BD}\sin 45° = 0$$

$$F_{BD} = 3.5355F \text{ (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \quad F_{BE}\cos 45° - F_{BC}\cos 45° = 0$$

$$F_{BE} = F_{BC} = F'$$

$$\Sigma F_y = 0; \quad 3.5355F\cos 45° - 2.9155F\left(\frac{0.3}{\sqrt{0.34}}\right) - 2F'\sin 45° = 0$$

$$F' = 0.7071F \qquad \textbf{Ans.}$$

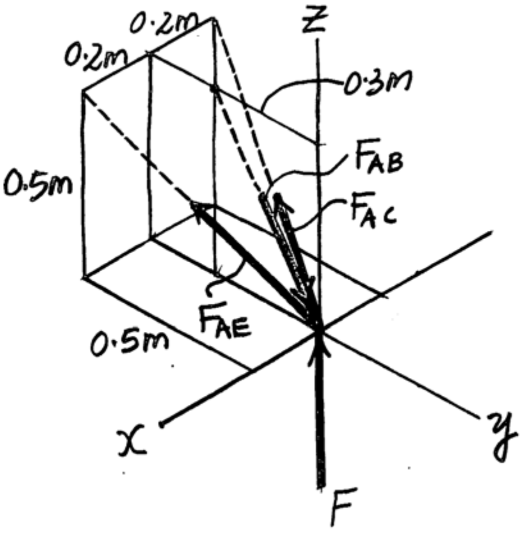
Thus,

$$F_{BE} = F_{BC} = 0.7071F \text{ (T)} \qquad \textbf{Ans.}$$

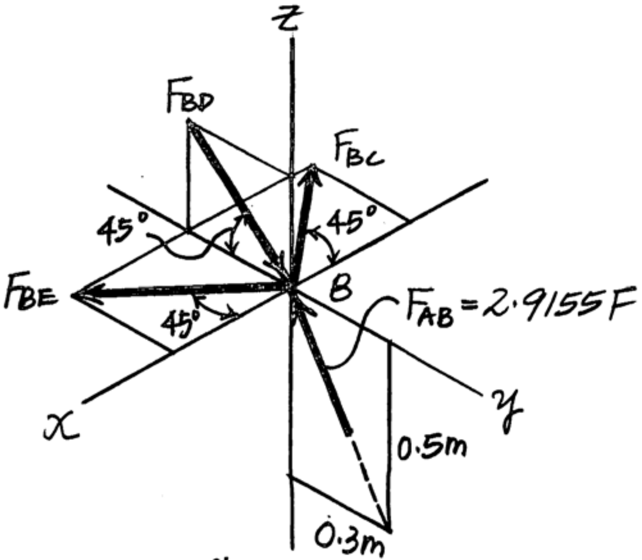
From the above results, the greatest tensile and compressive force developed in the members of the truss are $1.1023F$ and $3.5355F$, respectively. Thus,

$1.1023F = 800 \qquad F = 725.77\text{ N}$

$3.5355F = 600 \qquad F = 169.71\text{ N} = 170\text{ N (controls)}$ **Ans.**

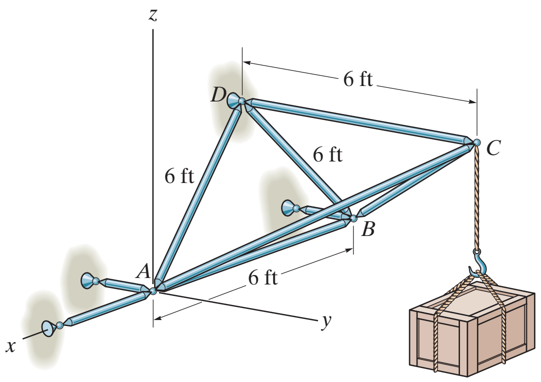


(a)



(b)

***6–64.** Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb.



$$\mathbf{F}_{CA} = F_{CA}\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\sin 60°\mathbf{k}}{\sqrt{8}}\right]$$

$$= -0.354\,F_{CA}\,\mathbf{i} + 0.707\,F_{CA}\,\mathbf{j} + 0.612\,F_{CA}\,\mathbf{k}$$

$$\mathbf{F}_{CB} = 0.354F_{CB}\,\mathbf{i} + 0.707F_{CB}\,\mathbf{j} + 0.612F_{CB}\,\mathbf{k}$$

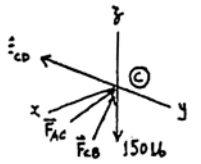
$$\mathbf{F}_{CD} = -\,F_{CD}\mathbf{j}$$

$$\mathbf{W} = -150\,\mathbf{k}$$

$\Sigma F_x = 0;$ $\quad -0.354F_{CA} + 0.354F_{CB} = 0$

$\Sigma F_y = 0;$ $\quad 0.707F_{CA} + 0.707F_{CB} - F_{CD} = 0$

$\Sigma F_z = 0;$ $\quad 0.612F_{CA} + 0.612F_{CB} - 150 = 0$



Solving :

$F_{CA} = F_{CB} = 122.5\text{ lb} = 122\text{ lb (C)}$ **Ans**

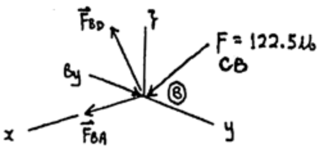
$F_{CD} = 173\text{ lb (T)}$ **Ans**

$$\mathbf{F}_{BA} = F_{BA}\,\mathbf{i}$$

$$\mathbf{F}_{BD} = F_{BD}\cos 60°\mathbf{i} + F_{BD}\sin 60°\,\mathbf{k}$$

$$\mathbf{F}_{CB} = 122.5\,(-0.354\mathbf{i} - 0.707\,\mathbf{j} - 0.612\mathbf{k})$$

$$= -43.3\mathbf{i} - 86.6\mathbf{j} - 75.0\mathbf{k}$$



$\Sigma F_x = 0;$ $\quad F_{BA} + F_{BD}\cos 60° - 43.3 = 0$

$\Sigma F_z = 0;$ $\quad F_{BD}\sin 60° - 75 = 0$

Solving :

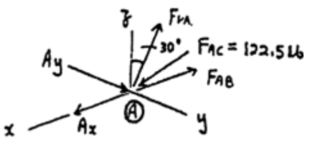
$F_{BD} = 86.6\text{ lb (T)}$ **Ans**

$F_{BA} = 0$ **Ans**

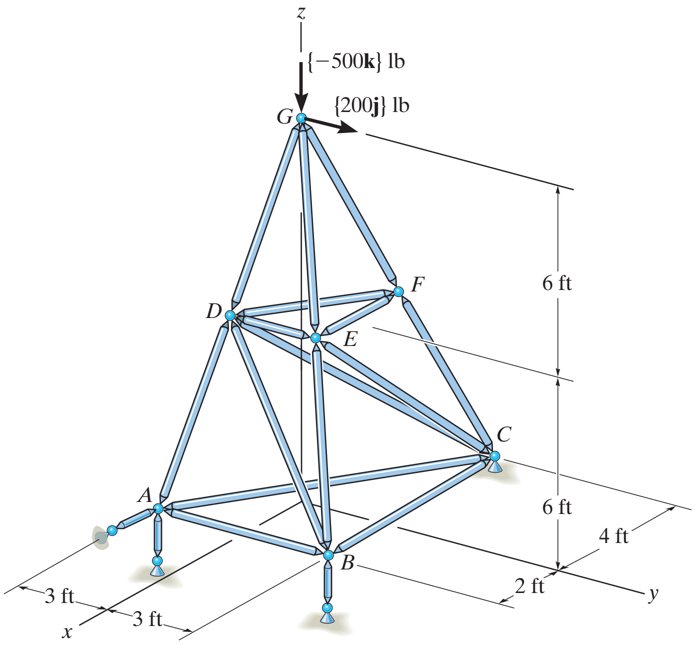
$$\mathbf{F}_{AC} = 122.5(0.354F_{AC}\mathbf{i} - 0.707F_{AC}\mathbf{j} - 0.612F_{AC}\mathbf{k}\,)$$

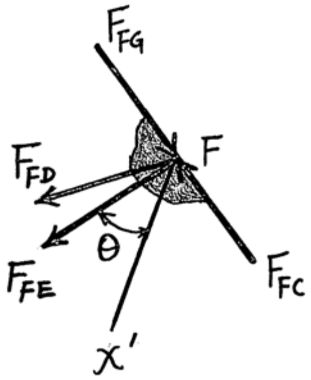
$\Sigma F_z = 0;$ $\quad F_{DA}\cos 30° - 0.612(122.5) = 0$

$F_{DA} = 86.6\text{ lb (T)}$ **Ans**



•6–65. Determine the force in members *FE* and *ED* of the space truss and state if the members are in tension or compression. The truss is supported by a ball-and-socket joint at *C* and short links at *A* and *B*.



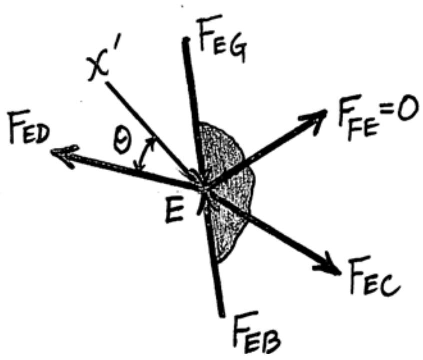


Joint F : F_{FG}, F_{FD}, and F_{FC} are lying in the same plane and x' axis is normal to that plane. Thus

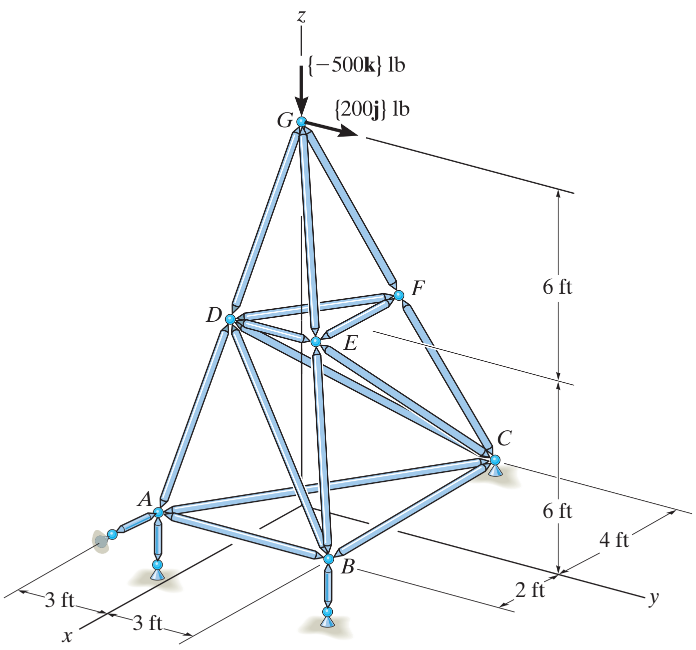
$$\Sigma F_{x'} = 0; \qquad F_{FE}\cos\theta = 0 \qquad F_{FE} = 0 \qquad \text{Ans}$$

Joint E : F_{EG}, F_{EC}, and F_{EB} are lying in the same plane and x' axis is normal to that plane. Thus

$$\Sigma F_{x'} = 0; \qquad F_{ED}\cos\theta = 0 \qquad F_{ED} = 0 \qquad \text{Ans}$$



6–66. Determine the force in members *GD*, *GE*, and *FD* of the space truss and state if the members are in tension or compression.



Joint *G*:

$$\mathbf{F}_{GD} = F_{GD}\left(-\frac{2}{12.53}\mathbf{i} + \frac{3}{12.53}\mathbf{j} + \frac{12}{12.53}\mathbf{k}\right)$$

$$\mathbf{F}_{GF} = F_{GF}\left(\frac{4}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} + \frac{12}{13}\mathbf{k}\right)$$

$$\mathbf{F}_{GE} = F_{GE}\left(-\frac{2}{12.53}\mathbf{i} - \frac{3}{12.53}\mathbf{j} + \frac{12}{12.53}\mathbf{k}\right)$$

$$\Sigma F_x = 0; \quad -F_{GD}\left(\frac{2}{12.53}\right) + F_{GF}\left(\frac{4}{13}\right) - F_{GE}\left(\frac{2}{12.53}\right) = 0$$

$$\Sigma F_y = 0; \quad F_{GD}\left(\frac{3}{12.53}\right) - F_{GF}\left(\frac{3}{13}\right) - F_{GE}\left(\frac{3}{12.53}\right) + 200 = 0$$

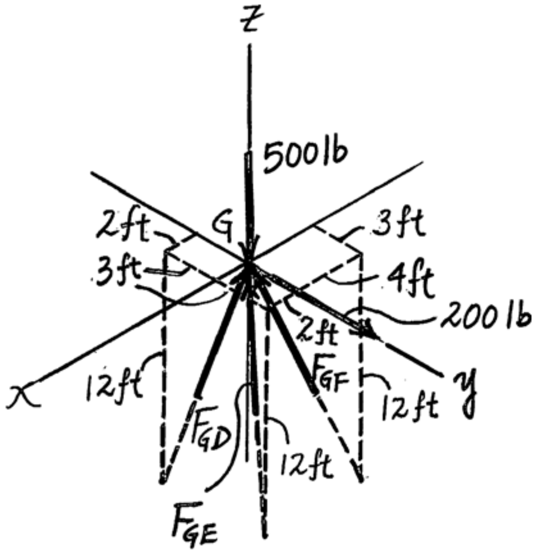
$$\Sigma F_z = 0; \quad F_{GD}\left(\frac{12}{12.53}\right) + F_{GF}\left(\frac{12}{13}\right) + F_{GE}\left(\frac{12}{12.53}\right) - 500 = 0$$

Solving,

$F_{GD} = -157 \text{ lb} = 157 \text{ lb (T)}$ **Ans**

$F_{GF} = 181 \text{ lb (C)}$

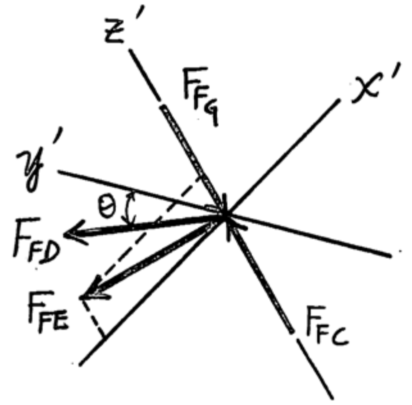
$F_{GE} = 505 \text{ lb (C)}$ **Ans**



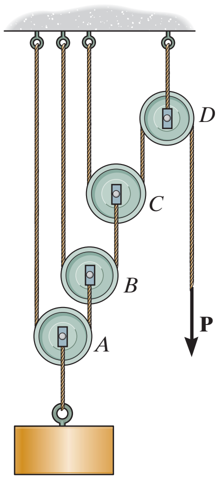
Joint *F*:

Orient the *x*', *y*', *z*' axes as shown.

$\Sigma F_{y'} = 0; \quad F_{FD} = 0$ **Ans**



6–67. Determine the force **P** required to hold the 100-lb weight in equilibrium.



Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of pulley A on the free-body diagram, Fig. a,

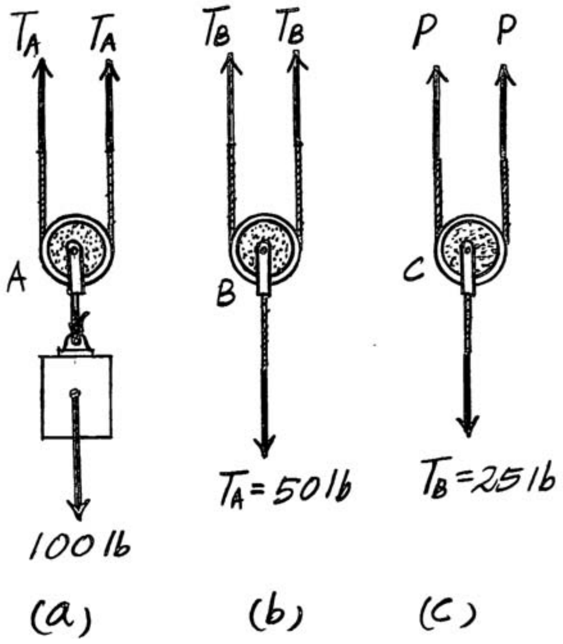
$+\uparrow \Sigma F_y = 0;$ $2T_A - 100 = 0$ $T_A = 50\text{ lb}$

Applying $\Sigma F_y = 0$ to the free-body diagram of pulley B, Fig. b,

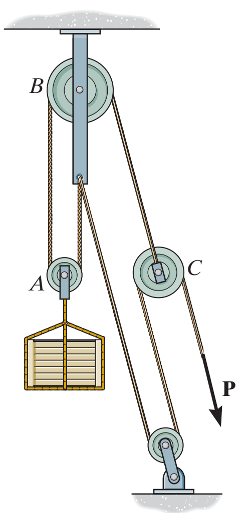
$+\uparrow \Sigma F_y = 0;$ $2T_B - 50 = 0$ $T_B = 25\text{ lb}$

From the free-body diagram of pulley C, Fig. c,

$+\uparrow \Sigma F_y = 0;$ $2P - 25 = 0$ $P = 12.5\text{ lb}$ **Ans.**



***6–68.** Determine the force **P** required to hold the 150-kg crate in equilibrium.

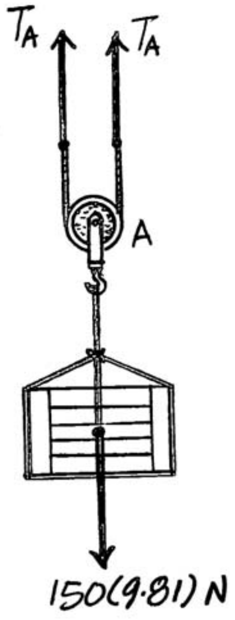


Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of pulley A on the free-body diagram, Fig. a,

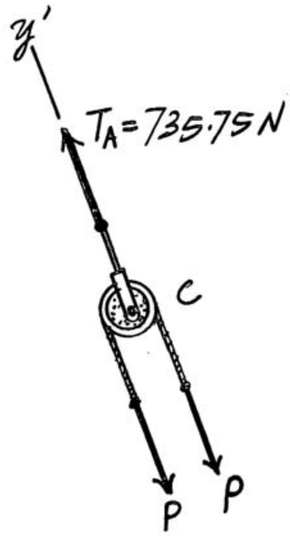
$$+\uparrow \Sigma F_y = 0; \qquad 2T_A - 150(9.81) = 0 \quad T_A = 735.75\text{ N}$$

Using the above result and writing the force equation of equilibrium along the y' axis of pulley C on the free-body diagram in Fig. b,

$$\Sigma F_{y'} = 0; \quad 735.75 - 2P = 0 \qquad P = 367.88\text{ N} = 368\text{ N}$$ **Ans.**

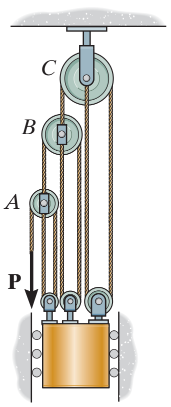


(a)



(b)

•6–69. Determine the force **P** required to hold the 50-kg mass in equilibrium.



Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of each pulley.

$+\uparrow \Sigma F_y = 0;$ $\quad R - 3P = 0;$ $\quad R = 3P$

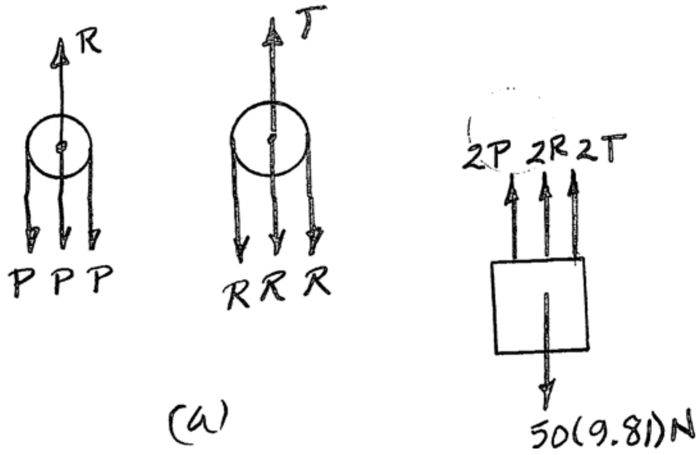
$+\uparrow \Sigma F_y = 0;$ $\quad T - 3R = 0;$ $\quad T = 3R = 9P$

$+\uparrow \Sigma F_y = 0;$ $\quad 2P + 2R + 2T - 50(9.81) = 0$ **Ans.**

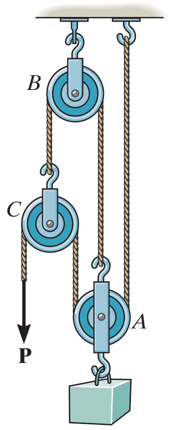
Substituting Eqs.(1) and (2) into Eq.(3) and solving for P,

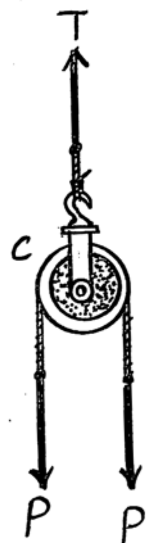
$$2P + 2(3P) + 2(9P) = 50(9.81)$$

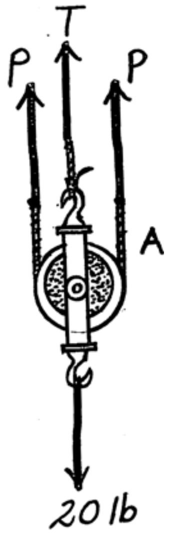
$$P = 18.9\text{ N}$$ **Ans.**



6–70. Determine the force **P** needed to hold the 20-lb block in equilibrium.







Pulley C:

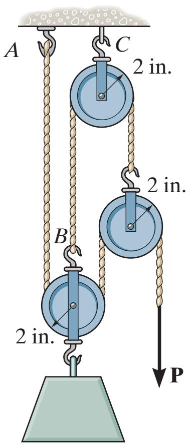
$+\uparrow\Sigma F_y = 0;\quad T - 2P = 0$

Pulley A:

$+\uparrow\Sigma F_y = 0;\quad 2P + T - 20 = 0$

$P = 5\text{ lb}$ Ans

6–71. Determine the force **P** needed to support the 100-lb weight. Each pulley has a weight of 10 lb. Also, what are the cord reactions at *A* and *B*?



Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad P' - 2P - 10 = 0 \qquad [1]$$

From FBD (b),

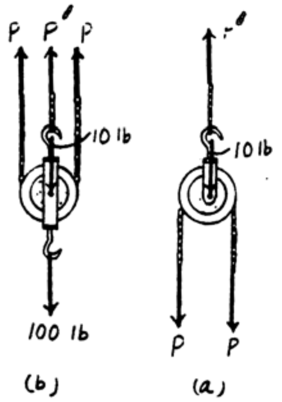
$$+\uparrow \Sigma F_y = 0; \quad 2P + P' - 100 - 10 = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields,

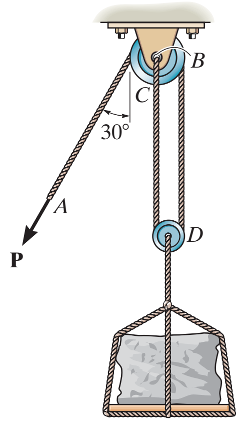
$$P = 25.0 \text{ lb} \qquad \textbf{Ans}$$
$$P' = 60.0 \text{ lb}$$

The cord reactions at *A* and *B* are

$$F_A = P = 25.0 \text{ lb} \quad F_B = P' = 60.0 \text{ lb} \qquad \textbf{Ans}$$



*__6–72.__ The cable and pulleys are used to lift the 600-lb stone. Determine the force that must be exerted on the cable at *A* and the corresponding magnitude of the resultant force the pulley at *C* exerts on pin *B* when the cables are in the position shown.



Pulley *D* :

$+\uparrow \Sigma F_y = 0; \quad 2T - 600 = 0$

$T = 300 \text{ lb}$ **Ans**

Pulley *B* :

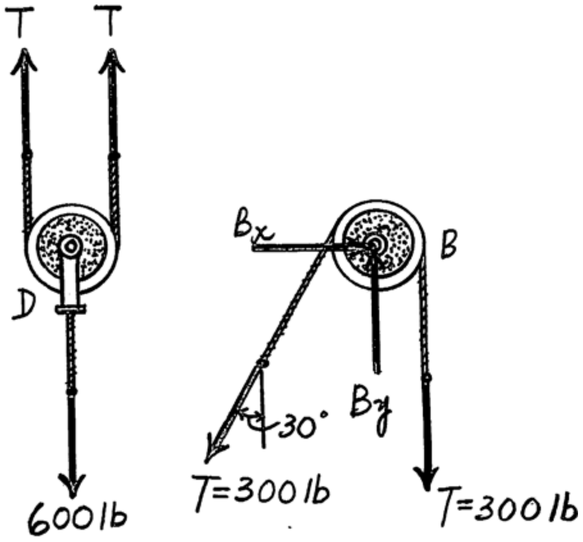
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 300 \sin 30^\circ = 0$

$B_x = 150 \text{ lb}$

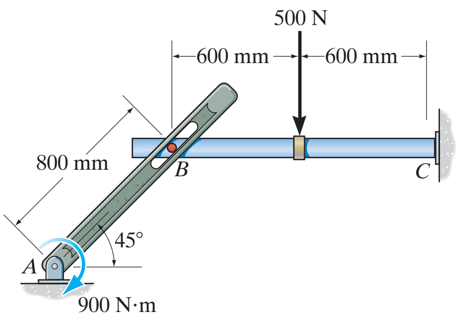
$+\uparrow \Sigma F_y = 0; \quad B_y - 300 - 300 \cos 30^\circ = 0$

$B_y = 559.8 \text{ lb}$

$F_B = \sqrt{(150)^2 + (559.8)^2} = 580 \text{ lb}$ **Ans**



•6–73. If the peg at *B* is smooth, determine the components of reaction at the pin *A* and fixed support *C*.



Equations of Equilibrium: From the free - body diagram of member *AB*, Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0;\quad N_B(0.8) - 900 = 0\qquad N_B = 1125\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 1125\cos 45° = 0\qquad A_x = 795.50\text{ N} = 795\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 1125\sin 45° - A_y = 0\qquad A_y = 795.50\text{ N} = 795\text{ N}$ **Ans.**

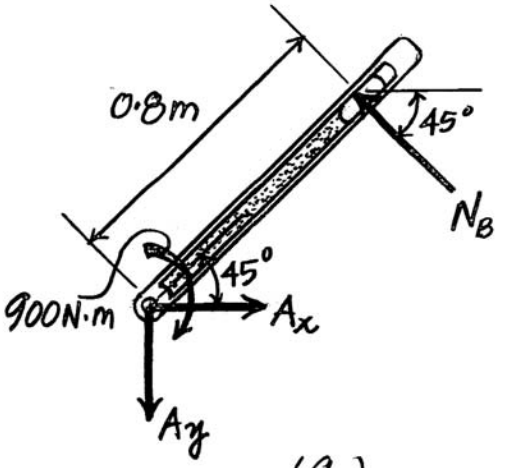
Applying the equations of equilibrium to the free - body diagram of member *BC*, Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 1125\cos 45° - C_x = 0\qquad C_x = 795.50\text{ N} = 795\text{ N}$ **Ans.**

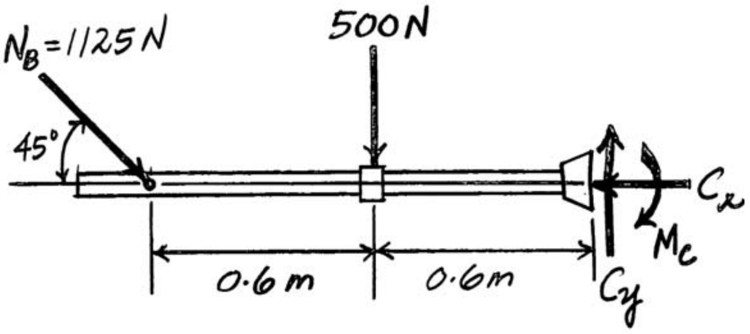
$+\uparrow \Sigma F_y = 0;\quad C_y - 1125\sin 45° - 500 = 0\qquad C_y = 1295.50\text{ N} = 1.30\text{ kN}$ **Ans.**

$\curvearrowleft + \Sigma M_C = 0;\ 1125\sin 45°(1.2) + 500(0.6) - M_C = 0$

$M_C = 1254.59\text{ N}\cdot\text{m} = 1.25\text{ kN}\cdot\text{m}$ **Ans.**

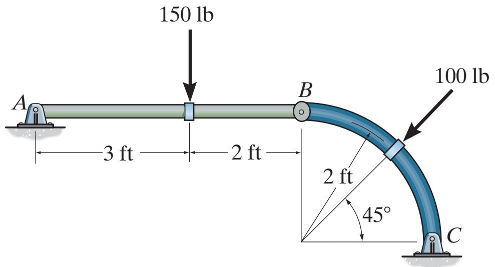


(a)



(b)

6–74. Determine the horizontal and vertical components of reaction at pins *A* and *C*.



Equations of Equilibrium: From the free-body diagram of member *AB* in Fig. *a*, we have

$\circlearrowleft + \Sigma M_A = 0;$ $B_y(5) - 150(3) = 0$ $B_y = 90 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;$ $150(2) - A_y(5) = 0$ $A_y = 60 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $A_x - B_x = 0$ (1)

From the free-body diagram of the member *BC* in Fig. *b* and using the result for B_y, we can write

$\circlearrowleft + \Sigma M_C = 0;$ $90(2) + 100 \sin 45°(2) - B_x(2) = 0$

$B_x = 160.71 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $C_y - 90 - 100 \sin 45° = 0$

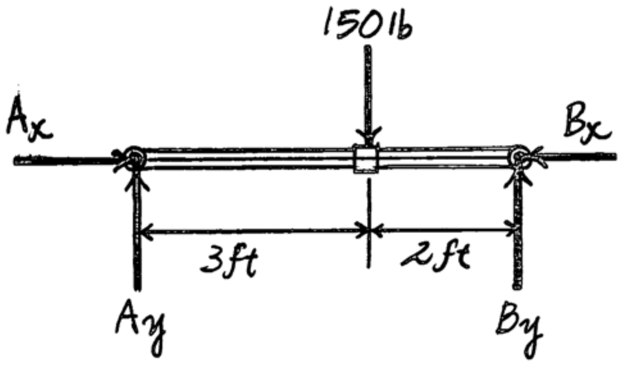
$C_y = 160.71 \text{ lb} = 161 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $160.71 - 100 \cos 45° - C_x = 0$

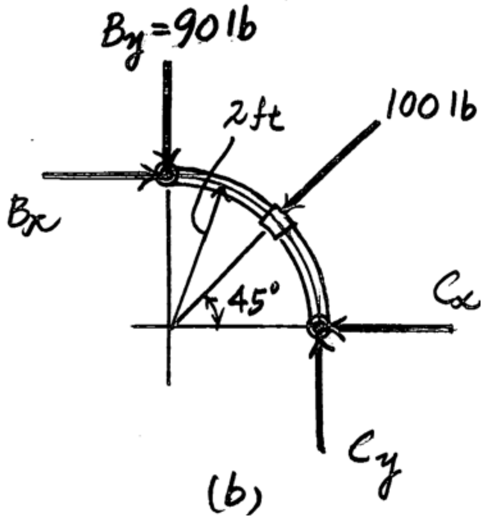
$C_x = 90 \text{ lb}$ **Ans.**

Substituting $B_x = 160.71$ lb into Eq. (1) yields

$A_x = 160.71 \text{ lb} = 161 \text{ lb}$ **Ans.**

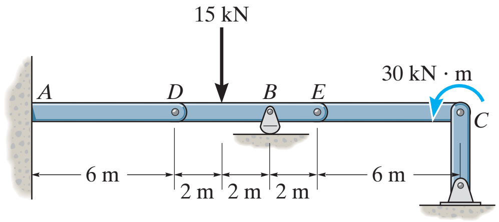


(a)



(b)

6–75. The compound beam is fixed at *A* and supported by rockers at *B* and *C*. There are hinges (pins) at *D* and *E*. Determine the components of reaction at the supports.



Equations of Equilibrium : From FBD(a),

$\circlearrowleft + \Sigma M_E = 0;\quad 30 - C_y(6) = 0 \qquad C_y = 5.00\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad E_y - 5.00 = 0 \qquad E_y = 5.00\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad E_x = 0$

From FBD(b),

$\circlearrowleft + \Sigma M_D = 0;\quad B_y(4) - 15(2) - 5.00(6) = 0$
$B_y = 15.0\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad D_y + 15.0 - 15 - 5.00 = 0$
$D_y = 5.00\text{ kN}$

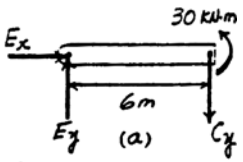
$\xrightarrow{+} \Sigma F_x = 0;\qquad D_x = 0$

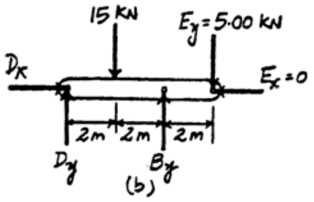
From FBD(c),

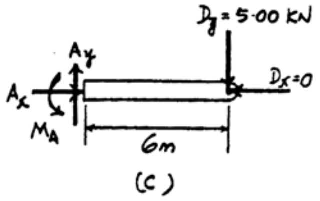
$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 5.00(6) = 0$
$M_A = 30.0\text{ kN}\cdot\text{m}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y - 5.00 = 0 \qquad A_y = 5.00\text{ kN}$ **Ans**

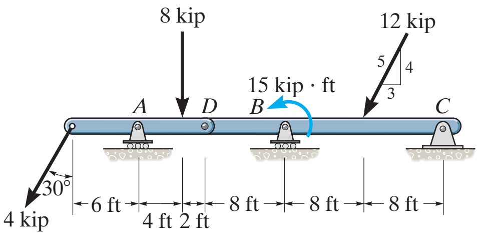
$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans**

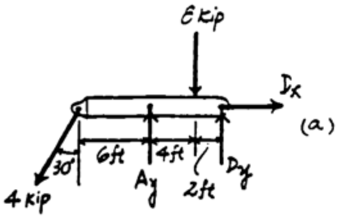


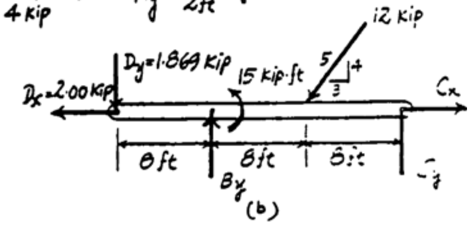




***6–76.** The compound beam is pin-supported at *C* and supported by rollers at *A* and *B*. There is a hinge (pin) at *D*. Determine the components of reaction at the supports. Neglect the thickness of the beam.







Equations of Equilibrium : **From FBD(a),**

$\curvearrowleft + \Sigma M_D = 0;\quad 4\cos 30°(12) + 8(2) - A_y(6) = 0$

$A_y = 9.595 \text{ kip} = 9.59 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad D_y + 9.595 - 4\cos 30° - 8 = 0$

$D_y = 1.869 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad D_x - 4\sin 30° = 0 \qquad D_x = 2.00 \text{ kip}$

From FBD(b),

$\curvearrowleft + \Sigma M_C = 0;\quad 1.869(24) + 15 + 12\left(\frac{4}{5}\right)(8) - B_y(16) = 0$

$B_y = 8.541 \text{ kip} = 8.54 \text{ kip}$ **Ans**

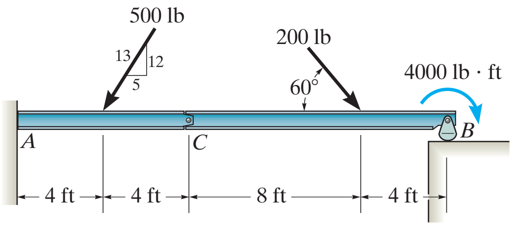
$+\uparrow \Sigma F_y = 0;\quad C_y + 8.541 - 1.869 - 12\left(\frac{4}{5}\right) = 0$

$C_y = 2.93 \text{ kip}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad C_x - 2.00 - 12\left(\frac{3}{5}\right) = 0$

$C_x = 9.20 \text{ kip}$ **Ans**

•6–77. The compound beam is supported by a rocker at *B* and is fixed to the wall at *A*. If it is hinged (pinned) together at *C*, determine the components of reaction at the supports. Neglect the thickness of the beam.



Member *CB*:

$\xrightarrow{+} \Sigma F_x = 0;$ — $-C_x + 200 \cos 60° = 0$

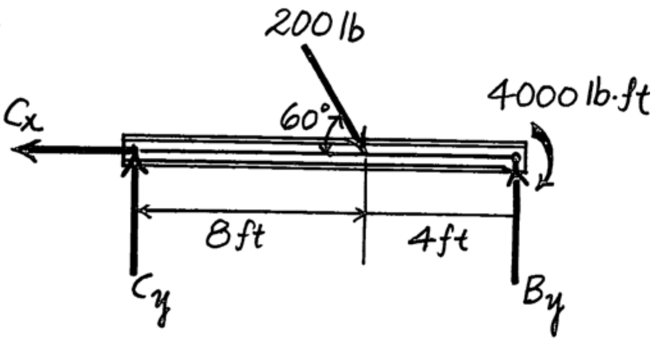
$C_x = 100 \text{ lb}$

$\circlearrowleft + \Sigma M_C = 0;$ $-200 \sin 60° (8) + B_y (12) - 4000 = 0$

$B_y = 448.8 \text{ lb} = 449 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $C_y - 200 \sin 60° + 448.8 = 0$

$C_y = -275.6 \text{ lb}$



Member *AC*:

$\xrightarrow{+} \Sigma F_x = 0;$ $A_x - 500\left(\frac{5}{13}\right) + 100 = 0$

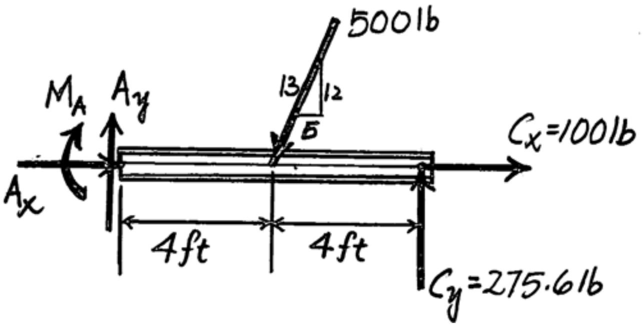
$A_x = 92.3 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $A_y - 500\left(\frac{12}{13}\right) + 275.6 = 0$

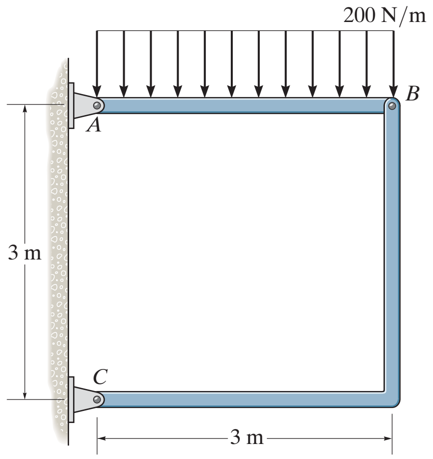
$A_y = 186 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_A = 0;$ $-M_A - 500\left(\frac{12}{13}\right)(4) + 275.6(8) = 0$

$M_A = 359 \text{ lb} \cdot \text{ft}$ **Ans**



6–78. Determine the horizontal and vertical components of reaction at pins A and C of the two-member frame.



Free Body Diagram : **The solution for this problem will be simplified if one realizes that member** ***BC*** **is a two force member.**

Equations of Equilibrium :

$\curvearrowleft + \Sigma M_A = 0; \quad F_{BC}\cos 45°(3) - 600(1.5) = 0$

$F_{BC} = 424.26 \text{ N}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 424.26\cos 45° - 600 = 0$

$A_y = 300 \text{ N}$ **Ans**

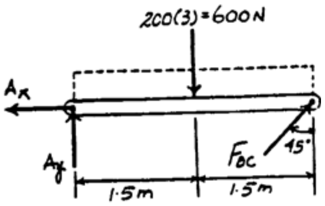
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 424.26\sin 45° - A_x = 0$

$A_x = 300 \text{ N}$ **Ans**

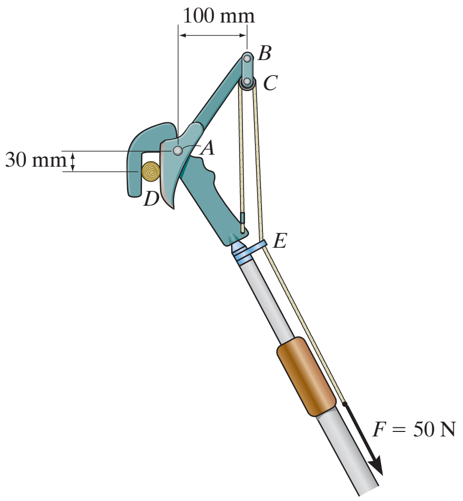
For pin C,

$C_x = F_{BC}\sin 45° = 424.26\sin 45° = 300 \text{ N}$ **Ans**

$C_y = F_{BC}\cos 45° = 424.26\cos 45° = 300 \text{ N}$ **Ans**



6–79. If a force of $F = 50$ N acts on the rope, determine the cutting force on the smooth tree limb at D and the horizontal and vertical components of force acting on pin A. The rope passes through a small pulley at C and a smooth ring at E.



Equations of Equilibrium: From the free - body diagram of pulley C in Fig. a,

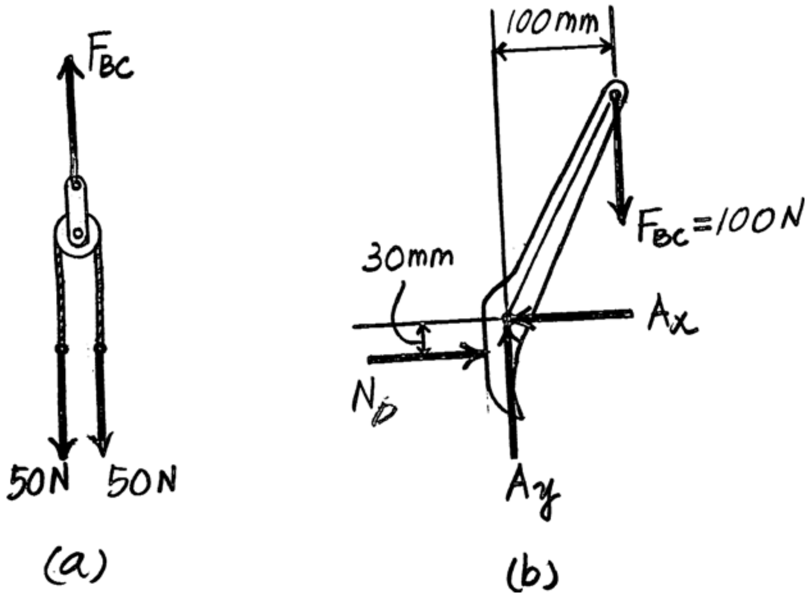
$+\uparrow \Sigma F_y = 0;$ $\quad F_{BC} - 50 - 50 = 0 \quad F_{BC} = 100\text{ N}$

From the free - body diagram of segment BAD in Fig. b and using the result $F_{BC} = 100\text{ N}$,

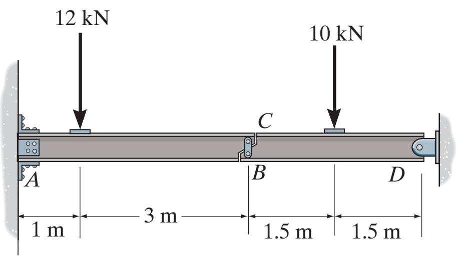
$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_D(30) - 100(100) = 0 \quad N_D = 333.33\text{ N} = 333\text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 333.33 - A_x = 0 \quad A_x = 333.33\text{ N} = 333\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 100 = 0 \quad A_y = 100\text{ N}$ **Ans.**



***6–80.** Two beams are connected together by the short link BC. Determine the components of reaction at the fixed support A and at pin D.



Equations of Equilibrium: First, we will consider the free - body diagram of member BD in Fig. a.

$\circlearrowleft + \Sigma M_D = 0;$ $10(1.5) - F_{BC}(3) = 0$

$F_{BC} = 5\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;$ $D_x = 0$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0;$ $D_y(3) - 10(1.5) = 0$

$D_y = 5\text{ kN}$ **Ans.**

Subsequently, the free - body diagram of member AC in Fig. b will be considered using the result $F_{BC} = 5\text{kN}$.

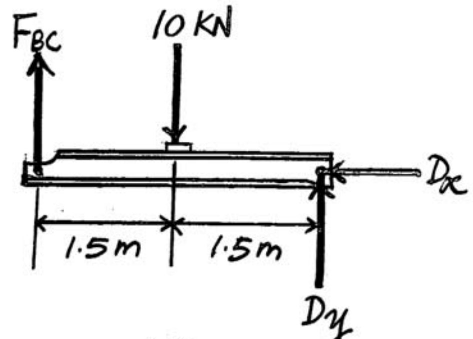
$\xrightarrow{+} \Sigma F_x = 0;$ $A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $A_y - 12 - 5 = 0$

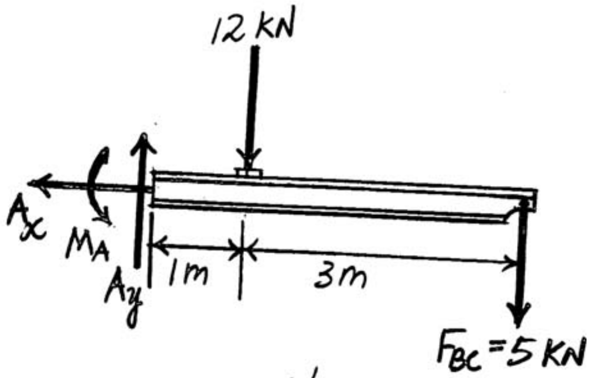
$A_y = 17\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;$ $M_A - 12(1) - 5(4) = 0$

$M_A = 32\text{ kN}\cdot\text{m}$ **Ans.**

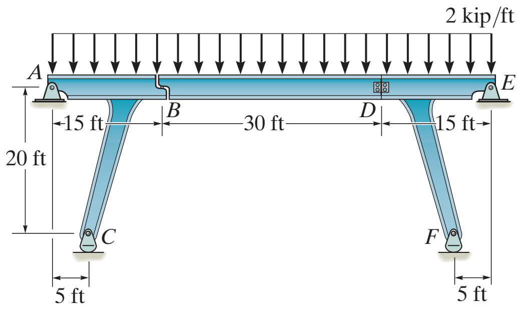


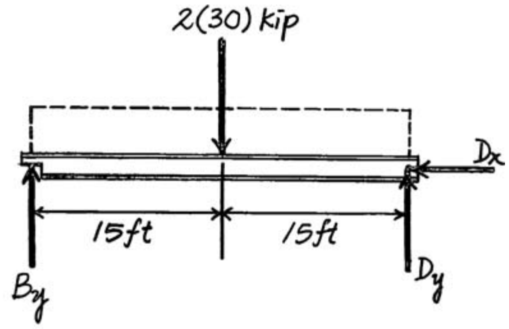
(a)

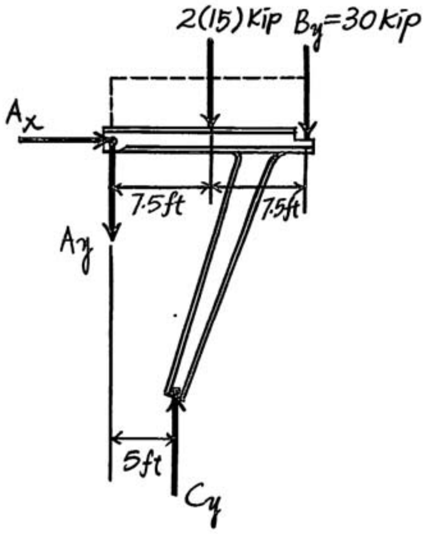


(b)

•6–81. The bridge frame consists of three segments which can be considered pinned at *A*, *D*, and *E*, rocker supported at *C* and *F*, and roller supported at *B*. Determine the horizontal and vertical components of reaction at all these supports due to the loading shown.







For segment *BD* :

$\curvearrowleft + \Sigma M_D = 0; \quad 2(30)(15) - B_y(30) = 0 \quad B_y = 30 \text{ kip}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad D_x = 0.$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad D_y + 30 - 2(30) = 0 \quad D_y = 30 \text{ kip}$ **Ans**

For segment *ABC* :

$\curvearrowleft + \Sigma M_A = 0; \quad C_y(5) - 2(15)(7.5) - 30(15) = 0 \quad C_y = 135 \text{ kip}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$ **Ans**

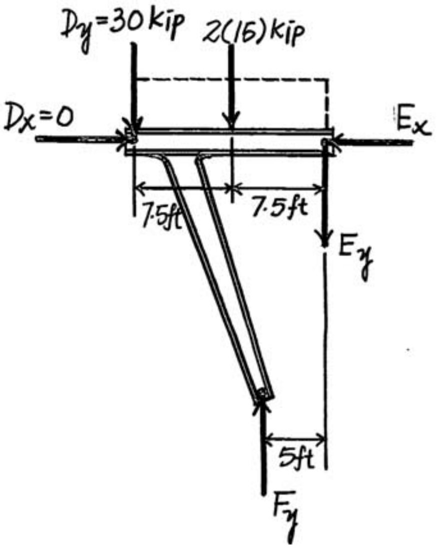
$+\uparrow \Sigma F_y = 0; \quad -A_y + 135 - 2(15) - 30 = 0 \quad A_y = 75 \text{ kip}$ **Ans**

For segment *DEF* :

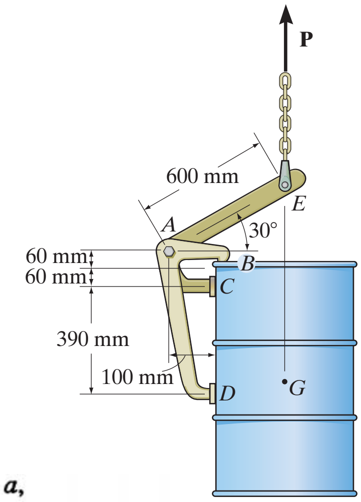
$\curvearrowleft + \Sigma M_E = 0; \quad -F_y(5) + 2(15)(7.5) + 30(15) = 0 \quad F_y = 135 \text{ kip}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad E_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -E_y + 135 - 2(15) - 30 = 0 \quad E_y = 75 \text{ kip}$ **Ans**



6–82. If the 300-kg drum has a center of mass at point *G*, determine the horizontal and vertical components of force acting at pin *A* and the reactions on the smooth pads *C* and *D*. The grip at *B* on member *DAB* resists both horizontal and vertical components of force at the rim of the drum.



Equations of Equilibrium: From the free - body diagram of segment *CAE* in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 300(9.81)(600\cos 30°) - N_C(120) = 0$

$N_C = 12\,743.56\text{ N} = 12.7\text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 12\,743.56 = 0$

$A_x = 12\,743.56\text{ N} = 12.7\text{ kN}$ **Ans.**

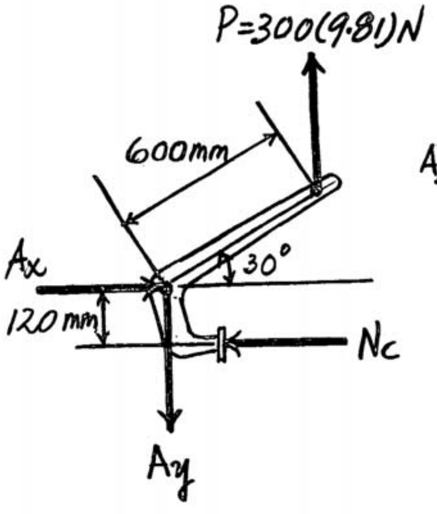
$+\uparrow \Sigma F_y = 0;$ $\quad 300(9.81) - A_y = 0$

$A_y = 2943\text{ N} = 2.94\text{ kN}$ **Ans.**

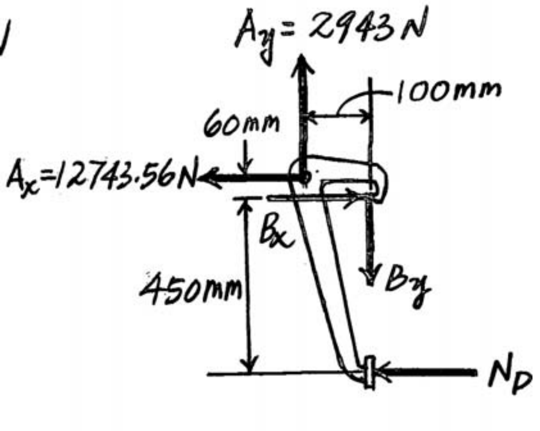
Using the results for A_x and A_y obtained above and applying the moment equation of equilibrium about point *B* on the free - body diagram of segment *BAD*, Fig. *b*,

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 12\,743.56(60) - 2943(100) - N_D(450) = 0$

$N_D = 1045.14\text{ N} = 1.05\text{ kN}$ **Ans.**

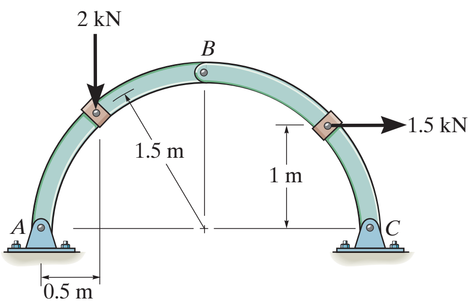


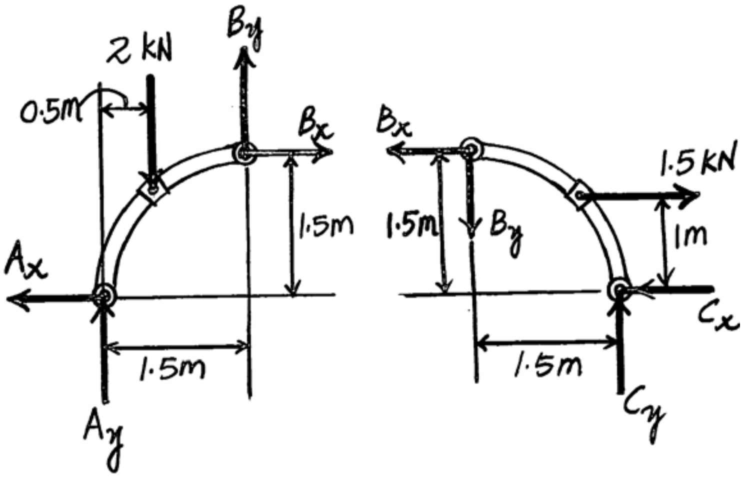
(a)



(b)

6–83. Determine the horizontal and vertical components of reaction that pins *A* and *C* exert on the two-member arch.





Member *AB*:

$\curvearrowleft + \Sigma M_A = 0; \quad -2(0.5) + B_y(1.5) - B_x(1.5) = 0$

Member *BC*:

$\curvearrowleft + \Sigma M_C = 0; \quad B_y(1.5) + B_x(1.5) - 1.5(1) = 0$

Solving:

$B_y = 0.8333 \text{ kN} = 833 \text{ N}$

$B_x = 0.1667 \text{ kN} = 167 \text{ N}$

Member *AB*:

$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 167 = 0$

$A_x = 167 \text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad A_y - 2000 + 833 = 0$

$A_y = 1.17 \text{ kN}$ **Ans**

Member *BC*:

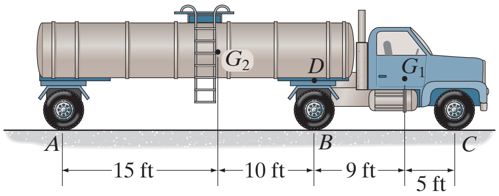
$\xrightarrow{+} \Sigma F_x = 0; \quad -C_x + 1500 - 167 = 0$

$C_x = 1.33 \text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad C_y - 833 = 0$

$C_y = 833 \text{ N}$ **Ans**

***6–84.** The truck and the tanker have weights of 8000 lb and 20 000 lb respectively. Their respective centers of gravity are located at points G_1 and G_2. If the truck is at rest, determine the reactions on both wheels at A, at B, and at C. The tanker is connected to the truck at the turntable D which acts as a pin.



Equations of Equilibrium: First, we will consider the free-body diagram of the tanker in Fig. *a*.

$\circlearrowleft + \Sigma M_D = 0;\quad 20\,000(10) - N_A(25) = 0$

$N_A = 8000 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad D_x = 0$

$+\uparrow \Sigma F_y = 0;\quad D_y + 8000 - 20\,000 = 0$

$D_y = 12\,000 \text{ lb}$

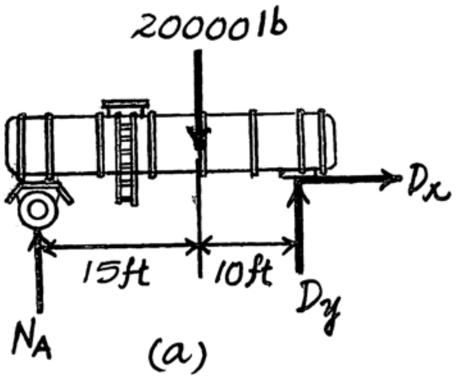
Using the results of D_x and D_y obtained above and considering the free-body diagram of the truck in Fig. *b*,

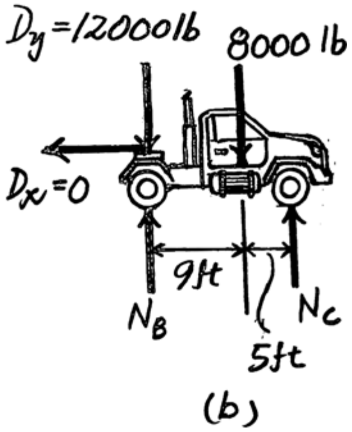
$\circlearrowleft + \Sigma M_D = 0;\quad N_C(14) - 8000(9) = 0$

$N_C = 5142.86 \text{ lb} = 5143 \text{ lb}$ **Ans.**

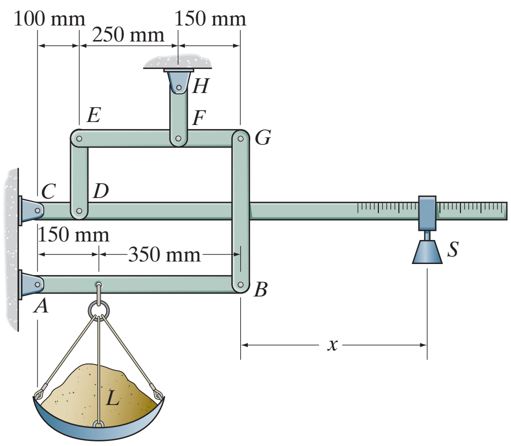
$+\uparrow \Sigma F_y = 0;\quad N_B + 5142.86 - 8000 - 12000 = 0$

$N_B = 14\,857.14 \text{ lb} = 14\,857 \text{ lb}$ **Ans.**





•6–85. The platform scale consists of a combination of third and first class levers so that the load on one lever becomes the effort that moves the next lever. Through this arrangement, a small weight can balance a massive object. If $x = 450$ mm, determine the required mass of the counterweight S required to balance a 90-kg load, L.



Equations of Equilibrium: Applying the moment equation of equilibrium about point A to the free - body diagram of member AB in Fig. a,

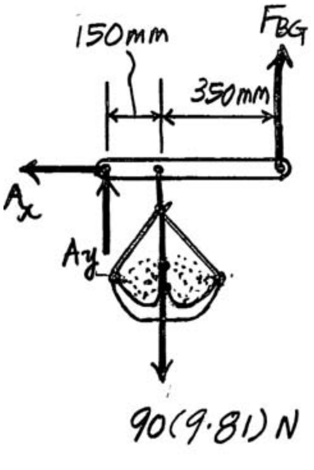
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BG}(500) - 90(9.81)(150) = 0$$
$$F_{BG} = 264.87 \text{ N}$$

Using the result of F_{BG} and writing the moment equation of equilibrium about point F on the free - body diagram of member EFG in Fig. b,

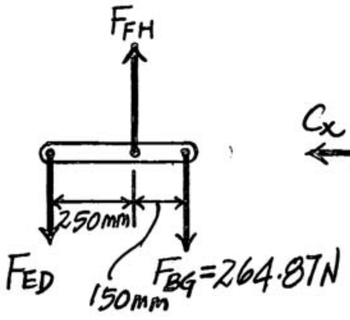
$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{ED}(250) - 264.87(150) = 0$$
$$F_{ED} = 158.922 \text{ N}$$

Using the result of F_{ED} and writing the moment equation of equilibrium about point C on the free - body diagram of member CDI in Fig. c,

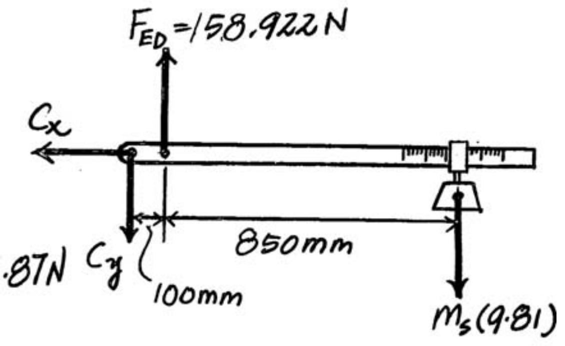
$$\circlearrowleft + \Sigma M_C = 0; \qquad 158.922(100) - m_s(9.81)(950) = 0$$
$$m_s = 1.705 \text{ kg} = 1.71 \text{ kg} \qquad \textbf{Ans.}$$



(a)

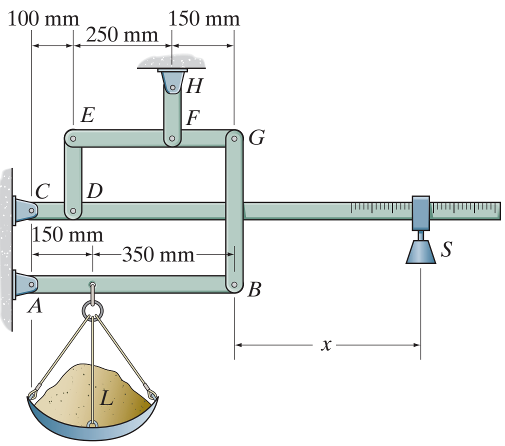


(b)



(c)

6–86. The platform scale consists of a combination of third and first class levers so that the load on one lever becomes the effort that moves the next lever. Through this arrangement, a small weight can balance a massive object. If $x = 450$ mm and, the mass of the counterweight S is 2 kg, determine the mass of the load L required to maintain the balance.



Equations of Equilibrium: Applying the moment equation of equilibrium about point A to the free - body diagram of member AB in Fig. a,

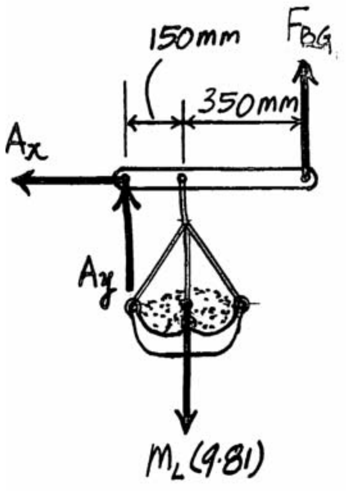
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BG}(500) - M_L(9.81)(150) = 0$$
$$F_{BG} = 2.943 \text{ lb}$$

Using the result of F_{BG} and writing the moment equation of equilibrium about point F on the free - body diagram of member EFG in Fig. b,

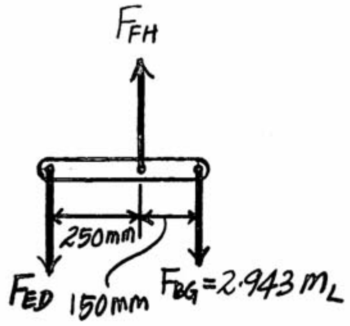
$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{ED}(250) - 2.943m_L(150) = 0$$
$$F_{ED} = 1.7658m_L$$

Using the result of F_{ED} and writing the moment equation of equilibrium about point C on the free - body diagram of member CDI in Fig. c,

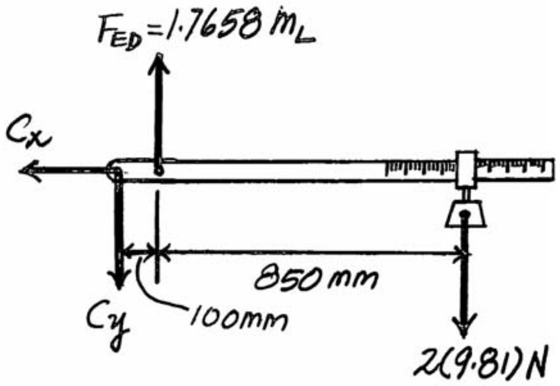
$$\circlearrowleft + \Sigma M_C = 0; \qquad 1.7658m_L(100) - 2(9.81)(950) = 0$$
$$m_L = 105.56 \text{ kg} = 106 \text{ kg} \qquad \textbf{Ans.}$$



(a)

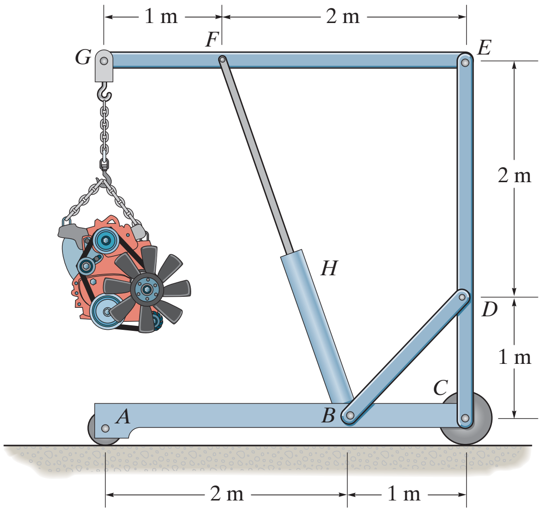


(b)



(C)

6–87. The hoist supports the 125-kg engine. Determine the force the load creates in member *DB* and in member *FB*, which contains the hydraulic cylinder *H*.



Free Body Diagram : **The solution for this problem will be simplified if one realizes that members *FB* and *DB* are two-force members.**

Equations of Equilibrium : **For FBD(a),**

$$\circlearrowleft + \Sigma M_E = 0; \quad 1226.25(3) - F_{FB}\left(\frac{3}{\sqrt{10}}\right)(2) = 0$$

$$F_{FB} = 1938.87\text{ N} = 1.94\text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 1938.87\left(\frac{3}{\sqrt{10}}\right) - 1226.25 - E_y = 0$$

$$E_y = 613.125\text{N}$$

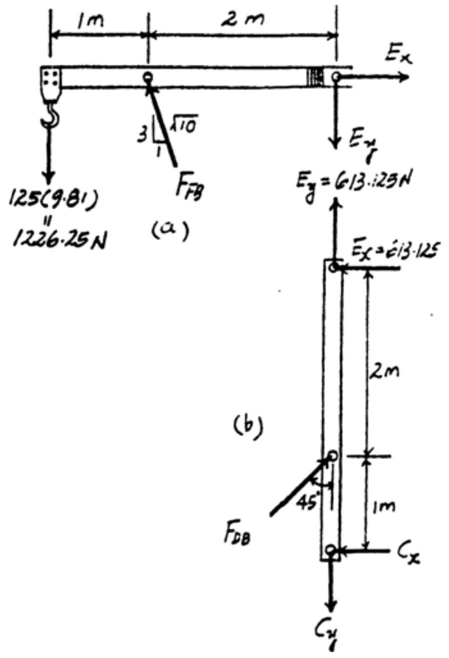
$$\xrightarrow{+} \Sigma F_x = 0; \quad E_x - 1938.87\left(\frac{1}{\sqrt{10}}\right) = 0$$

$$E_x = 613.125\text{ N}$$

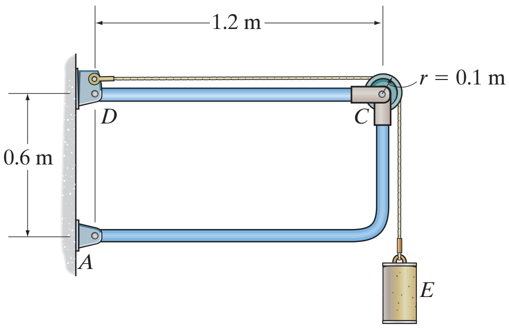
From FBD (b),

$$\circlearrowleft + \Sigma M_C = 0; \quad 613.125(3) - F_{BD}\sin 45°(1) = 0$$

$$F_{BD} = 2601.27\text{ N} = 2.60\text{ kN} \qquad \textbf{Ans}$$



***6–88.** The frame is used to support the 100-kg cylinder *E*. Determine the horizontal and vertical components of reaction at *A* and *D*.



Equations of Equilibrium: Member *DC* is a two - force member.

$D_y = 0$ **Ans.**

Consider the free - body diagram of member *AC* in Fig. *a*.

$\circlearrowleft +\Sigma M_A = 0;$ $D_x(0.6) - 981(1.2) + 981(0.6) = 0$

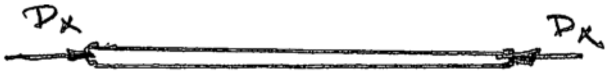
$D_x = 981 \text{ N}$ **Ans.**

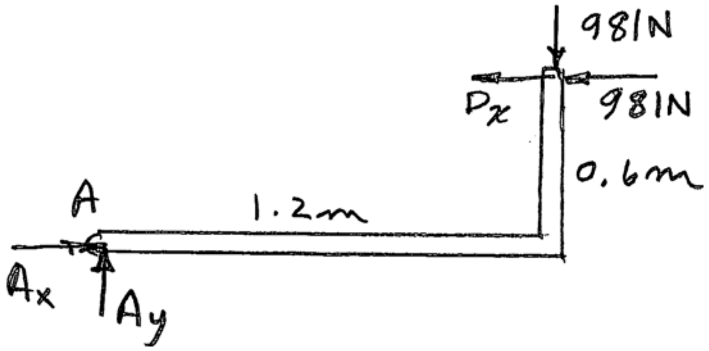
$\xrightarrow{+} \Sigma F_x = 0;$ $A_x - 981 - 981 = 0$

$A_x = 1962 \text{ N}$ **Ans.**

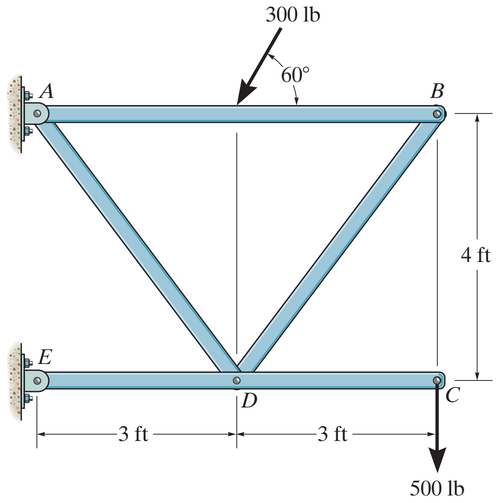
$+\uparrow \Sigma F_y = 0;$ $A_y - 981 = 0$

$A_y = 981 \text{ N}$ **Ans.**





•6–89. Determine the horizontal and vertical components of reaction which the pins exert on member *AB* of the frame.



Member *AB* :

$$\curvearrowleft + \Sigma M_A = 0; \qquad -300 \sin 60° (3) + \frac{4}{5} F_{BD} (6) = 0$$

$$F_{BD} = 162.4 \text{ lb}$$

Thus,

$$B_x = \frac{3}{5}(162.4) = 97.4 \text{ lb} \qquad \textbf{Ans}$$

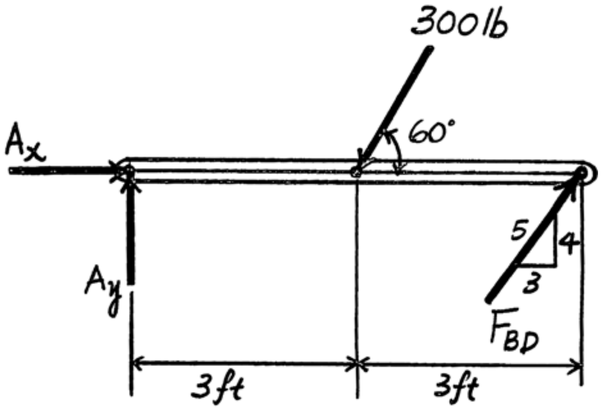
$$B_y = \frac{4}{5}(162.4) = 130 \text{ lb} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -300 \cos 60° + \frac{3}{5}(162.4) + A_x = 0$$

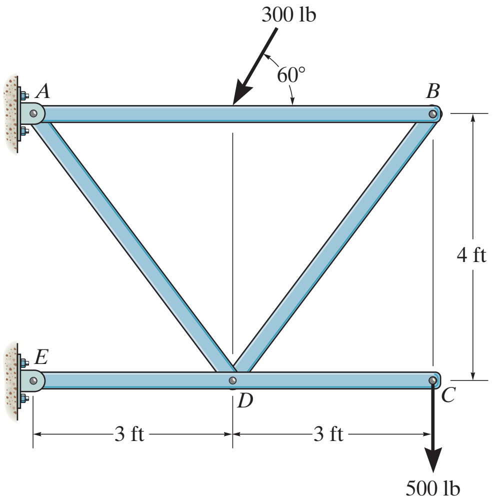
$$A_x = 52.6 \text{ lb} \qquad \textbf{Ans}$$

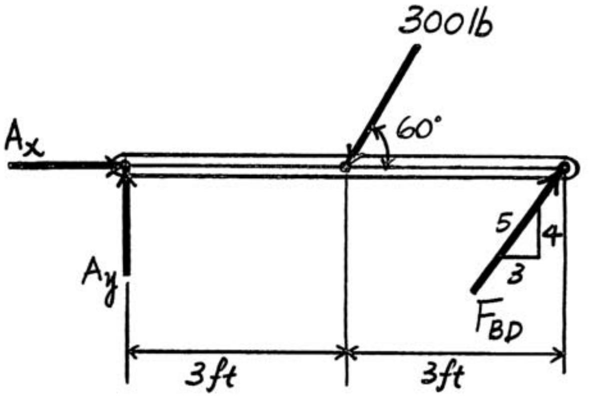
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 300 \sin 60° + \frac{4}{5}(162.4) = 0$$

$$A_y = 130 \text{ lb} \qquad \textbf{Ans}$$



6–90. Determine the horizontal and vertical components of reaction which the pins exert on member *EDC* of the frame.

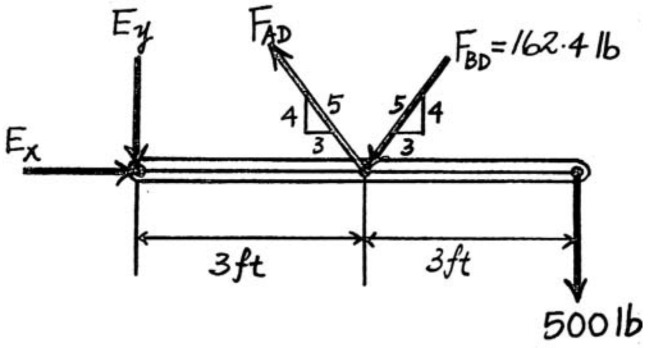




Member *AB*:

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -300 \sin 60°(3) + \frac{4}{5}F_{BD}(6) = 0$

$F_{BD} = 162.4$ lb



Member *EDC*:

$\circlearrowleft + \Sigma M_E = 0;$ $\quad -500(6) - \frac{4}{5}(162.4)(3) + \frac{4}{5}F_{AD}(3) = 0$

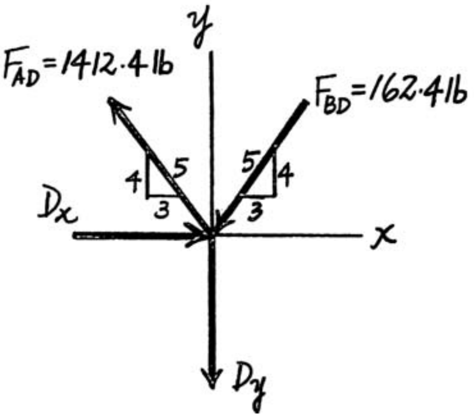
$F_{AD} = 1412.4$ lb

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad E_x - 162.4\left(\frac{3}{5}\right) - 1412.4\left(\frac{3}{5}\right) = 0$

$E_x = 945$ lb $\quad$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad -E_y + 1412.4\left(\frac{4}{5}\right) - 162.4\left(\frac{4}{5}\right) - 500 = 0$

$E_y = 500$ lb $\quad$ **Ans**



Pin *D*:

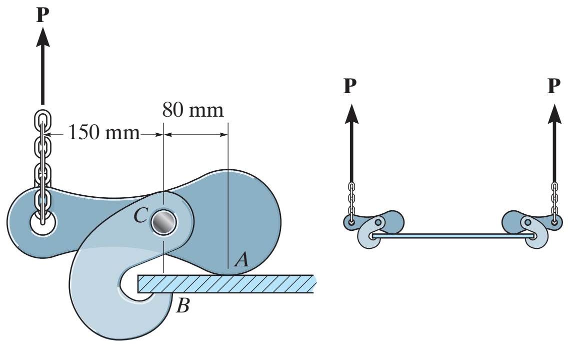
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad D_x - \frac{3}{5}(162.4) - \frac{3}{5}(1412.4) = 0$

$D_x = 945$ lb $\quad$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad -D_y - \frac{4}{5}(162.4) + \frac{4}{5}(1412.4) = 0$

$D_y = 1000$ lb $\quad$ **Ans**

6–91. The clamping hooks are used to lift the uniform smooth 500-kg plate. Determine the resultant compressive force that the hook exerts on the plate at *A* and *B*, and the pin reaction at *C*.



$\curvearrowleft + \Sigma M_C = 0;\quad N_A\,(80) - 2452.5\,(150) = 0$

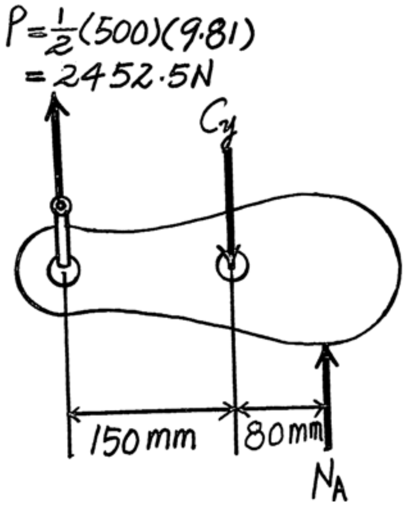
$N_A = 4598.4\text{ N} \doteq 4.60\text{ kN}$ **Ans**

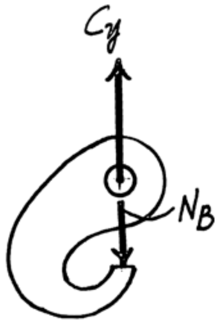
$+\uparrow \Sigma F_y = 0;\quad 2452.5 + 4598.4 - C_y = 0$

$C_y = 7050.9\text{ N} = 7.05\text{ kN}$ **Ans**

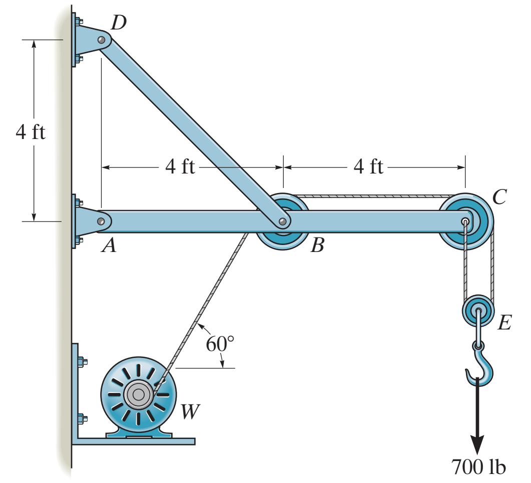
CB is a two-force member.

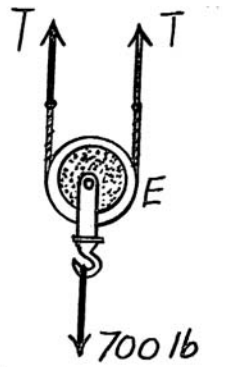
$N_B = C_y = 7.05\text{ kN}$ **Ans**





***6–92.** The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins *A* and *D*. Also, what is the force in the cable at the winch *W*?





Pulley *E*:

$+\uparrow \Sigma F_y = 0; \quad 2T - 700 = 0$

$T = 350 \text{ lb}$ Ans

Member *ABC*:

$\circlearrowleft + \Sigma M_A = 0; \quad T_{BD} \sin 45^\circ (4) - 350 \sin 60^\circ (4) - 700(8) = 0$

$T_{BD} = 2409 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -A_y + 2409 \sin 45^\circ - 350 \sin 60^\circ - 700 = 0$

$A_y = 700 \text{ lb}$ Ans

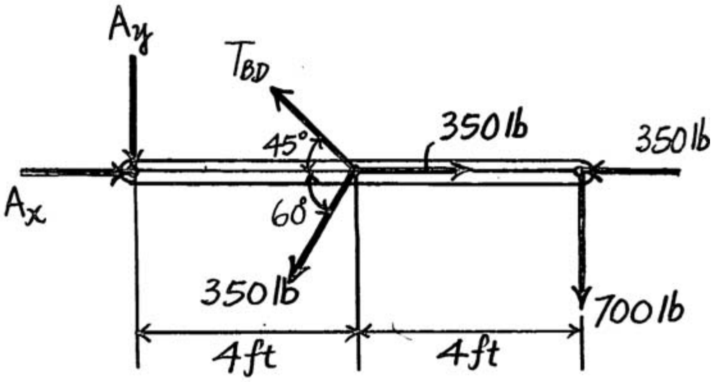
$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 2409 \cos 45^\circ - 350 \cos 60^\circ + 350 - 350 = 0$

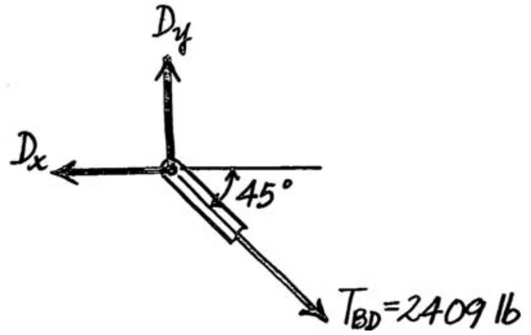
$A_x = 1.88 \text{ kip}$ Ans

At *D*:

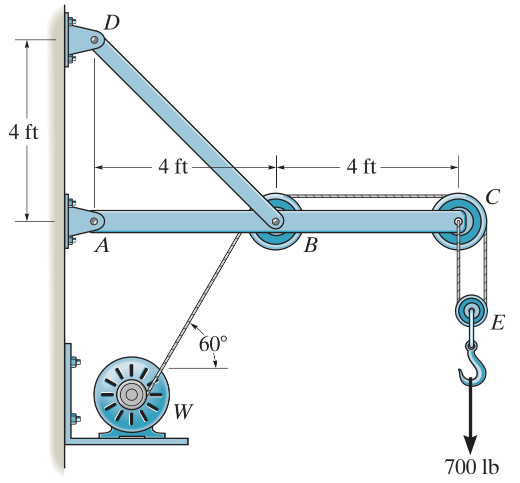
$D_x = 2409 \cos 45^\circ = 1703.1 \text{ lb} = 1.70 \text{ kip}$ Ans

$D_y = 2409 \sin 45^\circ = 1.70 \text{ kip}$ Ans

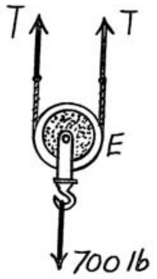




•6–93. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins *A* and *D*. Also, what is the force in the cable at the winch *W*? The jib *ABC* has a weight of 100 lb and member *BD* has a weight of 40 lb. Each member is uniform and has a center of gravity at its center.



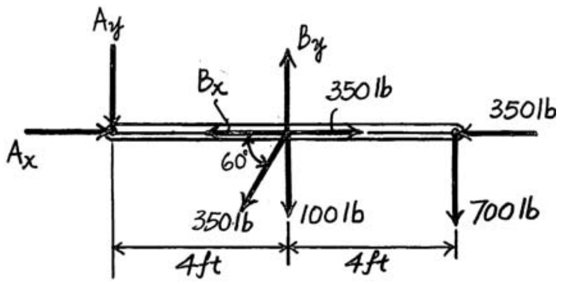
Pulley *E* :



$+\uparrow \Sigma F_y = 0;\quad 2T - 700 = 0$

$T = 350 \text{ lb}$ **Ans**

Member *ABC* :



$\curvearrowleft + \Sigma M_A = 0;\quad B_y(4) - 700(8) - 100(4) - 350 \sin 60^\circ (4) = 0$

$B_y = 1803.1 \text{ lb}$

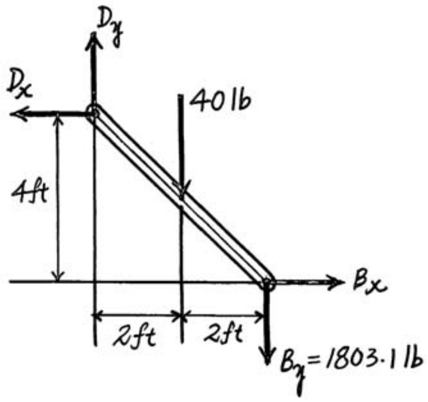
$+\uparrow \Sigma F_y = 0;\quad -A_y - 350 \sin 60^\circ - 100 - 700 + 1803.1 = 0$

$A_y = 700 \text{ lb}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 350 \cos 60^\circ - B_x + 350 - 350 = 0$

$A_x = B_x + 175 \quad (1)$

Member *DB* :



$\curvearrowleft + \Sigma M_D = 0;\quad -40(2) - 1803.1(4) + B_x(4) = 0$

$B_x = 1823.1 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad -D_x + 1823.1 = 0$

$D_x = 1.82 \text{ kip}$ **Ans**

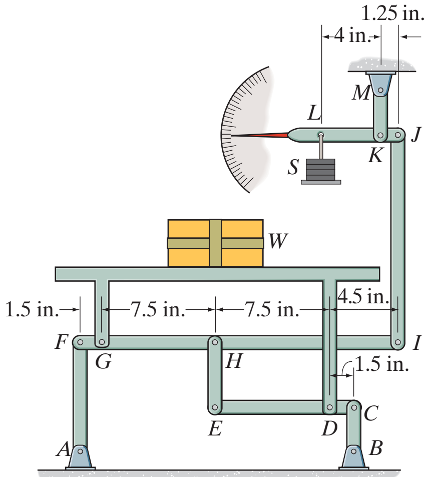
$+\uparrow \Sigma F_y = 0;\quad D_y - 40 - 1803.1 = 0$

$D_y = 1843.1 = 1.84 \text{ kip}$ **Ans**

From Eq. (1)

$A_x = 2.00 \text{ kip}$ **Ans**

6–94. The lever-actuated scale consists of a series of compound levers. If a load of weight $W = 150$ lb is placed on the platform, determine the required weight of the counterweight S to balance the load. Is it necessary to place the load symmetrically on the platform? Explain.



Equations of Equilibrium: First, we will consider the free-body diagram of the platform in Fig. *a*.

$\circlearrowleft + \Sigma M_D = 0; \quad 150(x) - G_y(15) = 0$

$G_y = 10x$

$+\uparrow \Sigma F_y = 0; \quad D_y + 10x - 150 = 0$

$D_y = 150 - 10x$

From the free-body diagram of member *CDE* in Fig. *b*,

$\circlearrowleft + \Sigma M_C = 0; \quad (150 - 10x)(1.5) - F_{EH}(9) = 0$

$F_{EH} = 25 - 1.6667x$

From the free-body diagram of member *FGHI* in Fig. C.

$\circlearrowleft + \Sigma M_F = 0; \quad F_{IJ}(21) - (25 - 1.6667x)(9) - 10x(1.5) = 0$

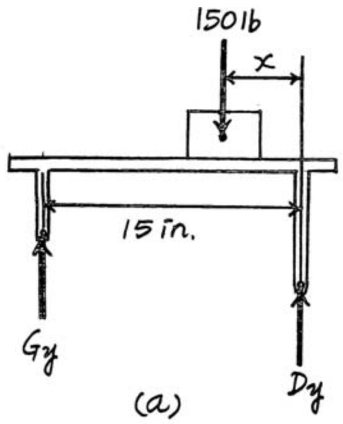
$F_{IJ} = 10.71 \text{ lb}$

Finally, from the free-body diagram of member *JKL* in Fig. *d*,

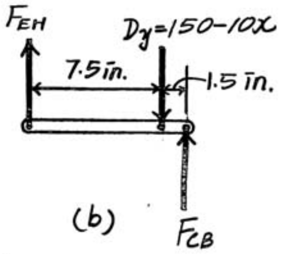
$\circlearrowleft + \Sigma M_K = 0; \quad W_s(4) - 10.71(1.25) = 0$

$W_s = 3.348 \text{ lb} = 3.35 \text{ lb}$ **Ans.**

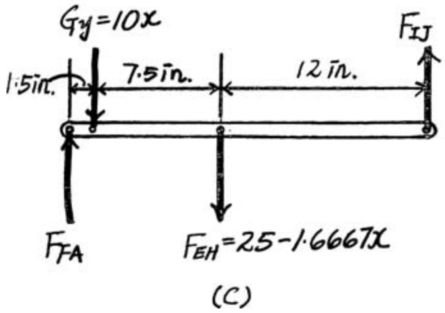
This result shows that W_s is independent of the position x of the load on the platform. Thus, the load can be placed at any position on the platform.



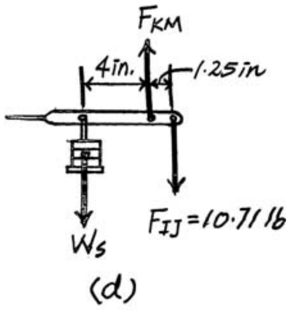
(a)



(b)

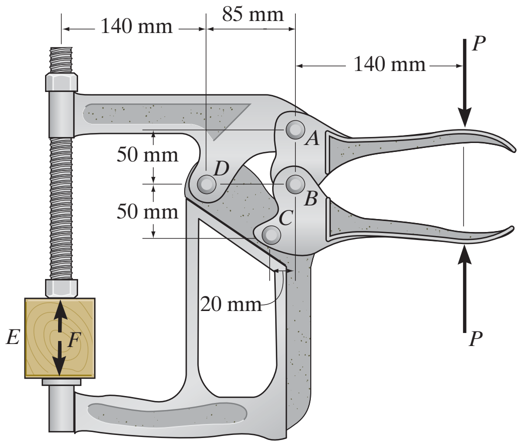


(C)



(d)

6–95. If $P = 75$ N, determine the force F that the toggle clamp exerts on the wooden block.



Equations of Equilibrium: First, we will consider the free-body diagram of the upper handle in Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_x(50) - 75(140) = 0$$
$$B_x = 210 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 210 - A_x = 0$$
$$A_x = 210 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - A_y - 75 = 0 \qquad (1)$$

Using the result for B_x and applying the moment equation of equilibrium about point C on the free-body diagram of the lower handle in Fig. *b*,

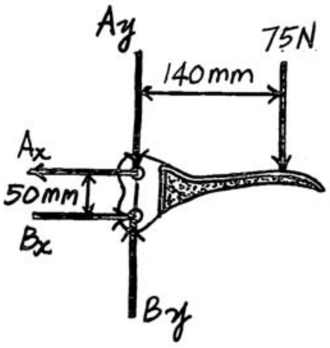
$$\circlearrowleft + \Sigma M_C = 0; \qquad 210(50) + 75(160) - B_y(20) = 0$$
$$B_y = 1125 \text{ N}$$

Substituting $B_y = 1125$ N into Eq. (1) yields

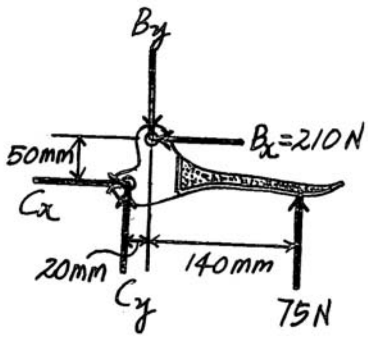
$$A_y = 1050 \text{ N}$$

Writing the moment equation of equilibrium about point D on the free-body diagram of the clamp shown in Fig. *c*,

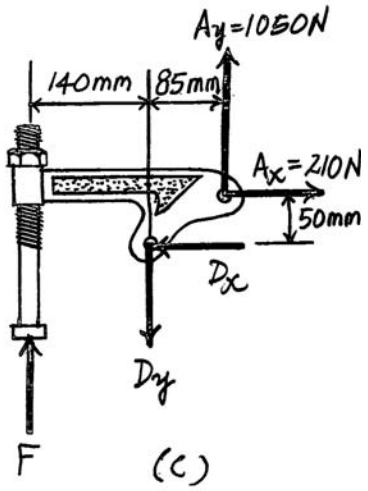
$$\circlearrowleft + \Sigma M_D = 0; \qquad 1050(85) - 210(50) - F(140) = 0$$
$$F = 562.5 \text{ N} \qquad \textbf{Ans.}$$



(a)

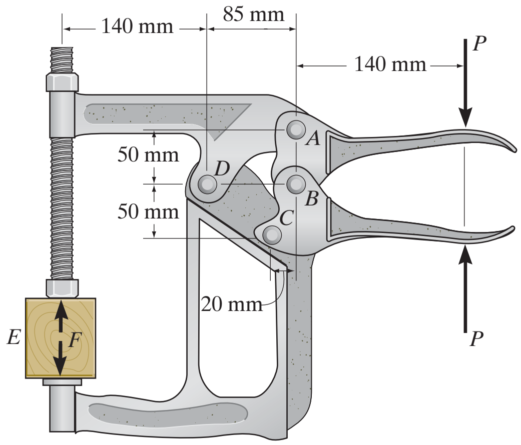


(b)



(c)

***6–96.** If the wooden block exerts a force of $F = 600$ N on the toggle clamp, determine the force P applied to the handle.



Equations of Equilibrium: First, we will consider the free-body diagram of the upper handle in Fig. *a*.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_x(50) - P(140) = 0$

$B_x = 2.8P$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 2.8P - A_x = 0$

$A_x = 2.8P$

$+\uparrow \Sigma F_y = 0;$ $\quad B_x - A_y - P = 0 \qquad (1)$

Using the result of B_x and applying the moment equation of equilibrium about point C on the free-body diagram of the lower handle in Fig. *b*,

$\circlearrowleft + \Sigma M_C = 0;$ $\quad P(160) + 2.8P(50) - B_y(20) = 0$

$B_y = 15P$

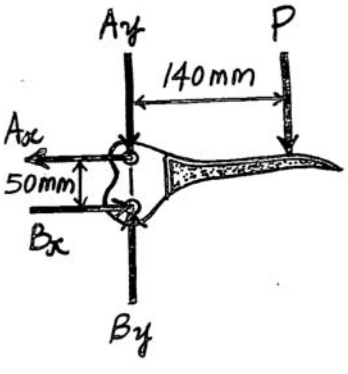
Substituting $B_y = 15P$ into Eq. (1) yields

$A_y = 14P$

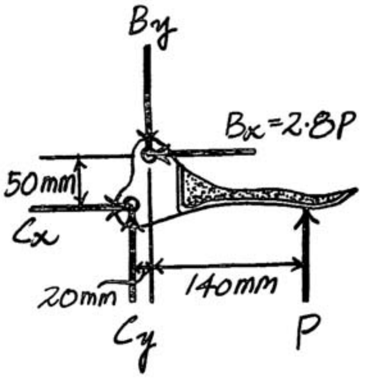
Writing the moment equation of equilibrium about point D on the free-body diagram of the clamp shown in Fig. *c*,

$\circlearrowleft + \Sigma M_D = 0;$ $\quad 14P(85) - 2.8P(50) - 600(140) = 0$

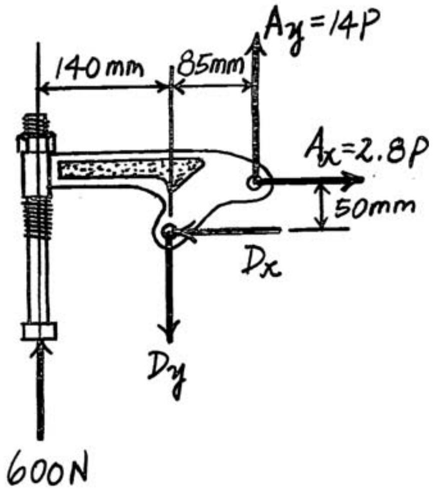
$P = 80\text{ N}$ **Ans.**



(a)

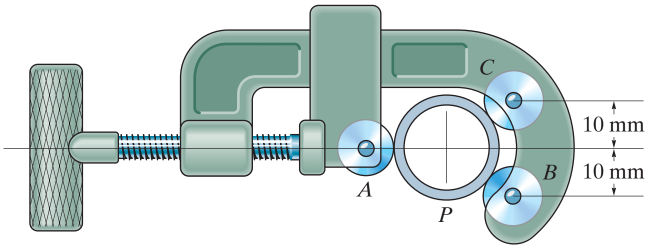


(b)



(c)

•6–97. The pipe cutter is clamped around the pipe *P*. If the wheel at *A* exerts a normal force of $F_A = 80$ N on the pipe, determine the normal forces of wheels *B* and *C* on the pipe. The three wheels each have a radius of 7 mm and the pipe has an outer radius of 10 mm.



$\theta = \sin^{-1}\left(\frac{10}{17}\right) = 36.03°$

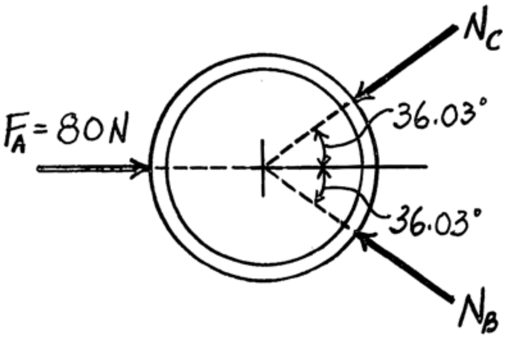
Equations of Equilibrium:

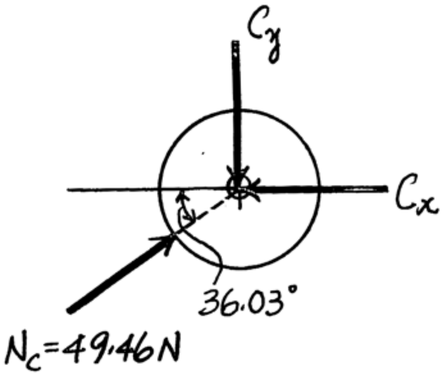
$+\uparrow \Sigma F_y = 0;\quad N_B \sin 36.03° - N_C \sin 36.03° = 0$

$N_B = N_C$

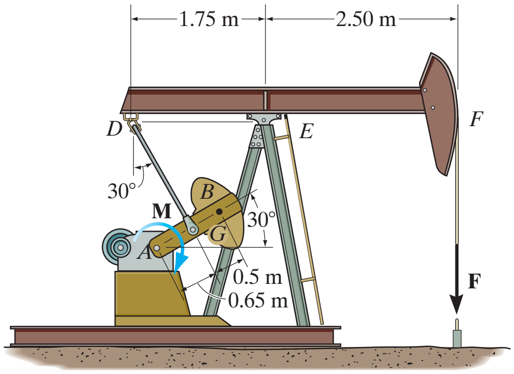
$\xrightarrow{+} \Sigma F_x = 0;\quad 80 - N_C \cos 36.03° - N_C \cos 36.03° = 0$

$N_B = N_C = 49.5 \text{ N}$ **Ans**





6–98. A 300-kg counterweight, with center of mass at G, is mounted on the pitman crank AB of the oil-pumping unit. If a force of $F = 5$ kN is to be developed in the fixed cable attached to the end of the walking beam DEF, determine the torque M that must be supplied by the motor.



Equations of Equilibrium: Applying the moment equation of equilibrium about point E to the free - body diagram of the walking beam in Fig. *a*,

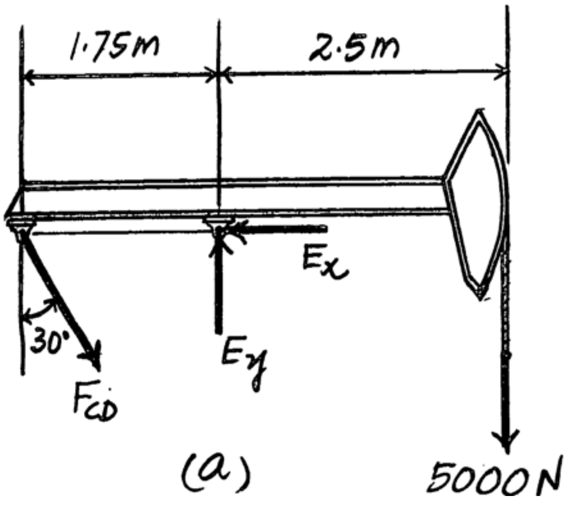
$$\circlearrowleft +\Sigma M_E = 0; \qquad F_{CD}\cos 30°(1.75) - 5000(2.5) = 0$$

$$F_{CD} = 8247.86\,\text{N}$$

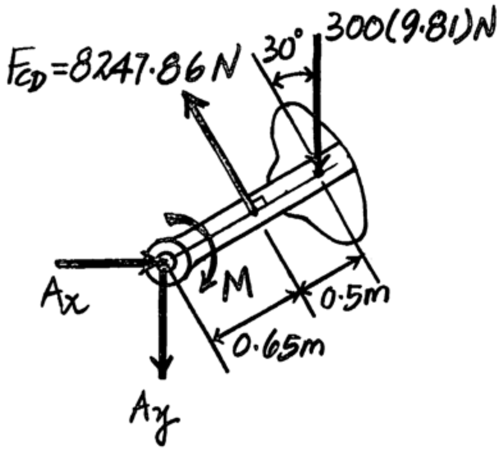
Using the result of F_{CD} and applying the moment equation of equilibrium about point A on the free - body diagram of the pitman crank in Fig. *b*,

$$\circlearrowleft +\Sigma M_A = 0; \qquad 8247.86(0.65) - 300(9.81)\cos 30°(1.15) - M = 0$$

$$M = 2430.09\,\text{N}\cdot\text{m} = 2.43\,\text{kN}\cdot\text{m}$$ **Ans.**

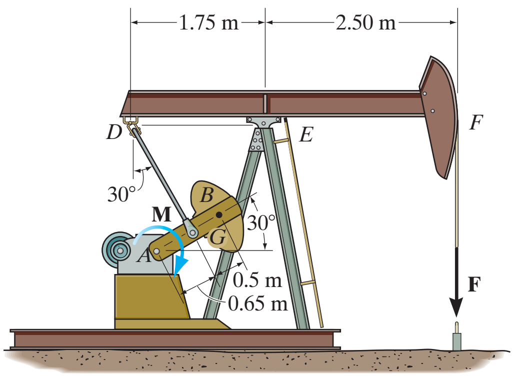


(a)



(b)

6–99. A 300-kg counterweight, with center of mass at G, is mounted on the pitman crank AB of the oil-pumping unit. If the motor supplies a torque of $M = 2500\ \text{N}\cdot\text{m}$, determine the force $\mathbf{F}$ developed in the fixed cable attached to the end of the walking beam DEF.

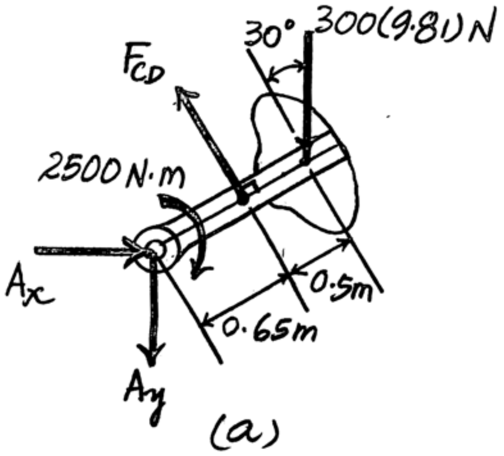


Equations of Equilibrium: Applying the moment equation of equilibrium about point A to the free-body diagram of the pitman crank in Fig. a,

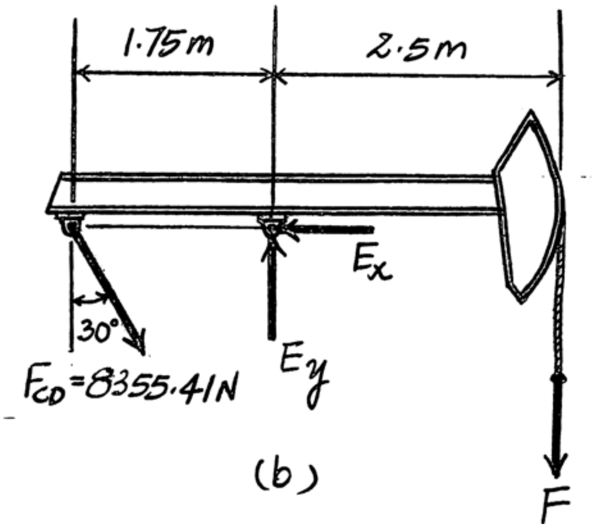
$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(0.65) - 300(9.81)\cos 30°(1.15) - 2500 = 0$$
$$F_{CD} = 8355.41\ \text{N}$$

Using the result of F_{CD} and applying the moment equation of equilibrium about point E on the free-body diagram of the walking beam in Fig. b,

$$\circlearrowleft +\Sigma M_E = 0; \qquad 8355.41\cos 30°(1.75) - F(2.5) = 0$$
$$F = 5065.20\ \text{N} = 5.07\ \text{kN} \qquad \textbf{Ans.}$$

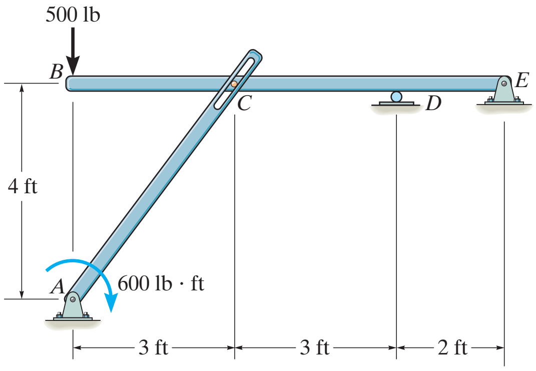


(a)



(b)

***6–100.** The two-member structure is connected at *C* by a pin, which is fixed to *BDE* and passes through the smooth slot in member *AC*. Determine the horizontal and vertical components of reaction at the supports.



Member *AC* :

$\circlearrowleft + \Sigma M_A = 0; \quad N_C\,(5) - 600 = 0$

$N_C = 120\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 120\left(\frac{4}{5}\right) = 0$

$A_x = 96\text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -A_y + 120\left(\frac{3}{5}\right) = 0$

$A_y = 72\text{ lb}$ **Ans**

Member *BDE* :

$\circlearrowleft + \Sigma M_E = 0; \quad 500\,(8) + 120\left(\frac{3}{5}\right)(5) - D_y\,(2) = 0$

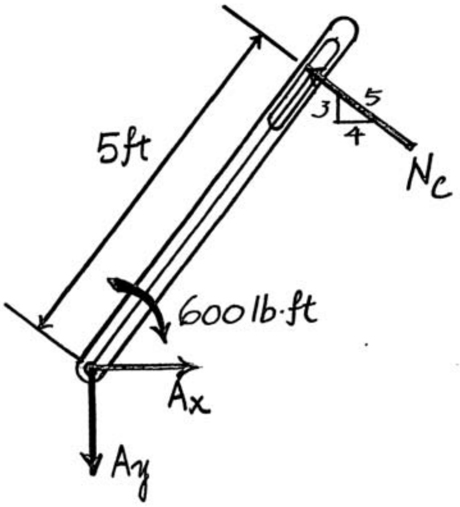
$D_y = 2180\text{ lb} = 2.18\text{ kip}$ **Ans**

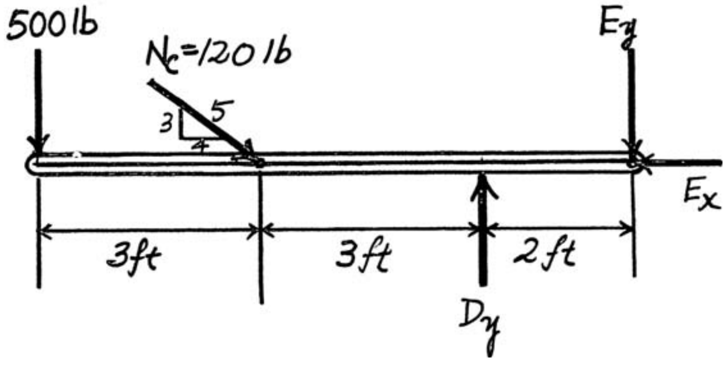
$\xrightarrow{+} \Sigma F_x = 0; \quad -E_x + 120\left(\frac{4}{5}\right) = 0$

$E_x = 96\text{ lb}$ **Ans**

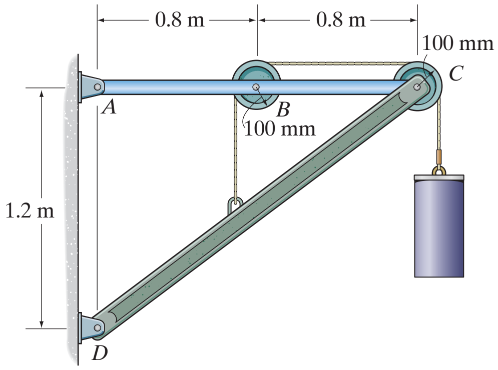
$+\uparrow \Sigma F_y = 0; \quad -500 - 120\left(\frac{3}{5}\right) + 2180 - E_y = 0$

$E_y = 1608\text{ lb} = 1.61\text{ kip}$ **Ans**





•6–101. The frame is used to support the 50-kg cylinder. Determine the horizontal and vertical components of reaction at *A* and *D*.



Equations of Equilibrium: First, we will consider member *ABC*.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad C_y(1.6) - 50(9.81)(0.7) - 50(9.81)(1.7) = 0$

$C_y = 735.75\text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 735.75 - 50(9.81) - 50(9.81) = 0$

$A_y = 245.25\text{ N} = 245\text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad C_x - A_x = 0$ (1)

Subsequently, we will consider member *CD*.

$\circlearrowleft + \Sigma M_D = 0;$ $\quad C_x(1.2) + 50(9.81)(0.7) - 735.75(1.6) = 0$

$C_x = 694.875\text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad D_y + 50(9.81) - 735.75 = 0$

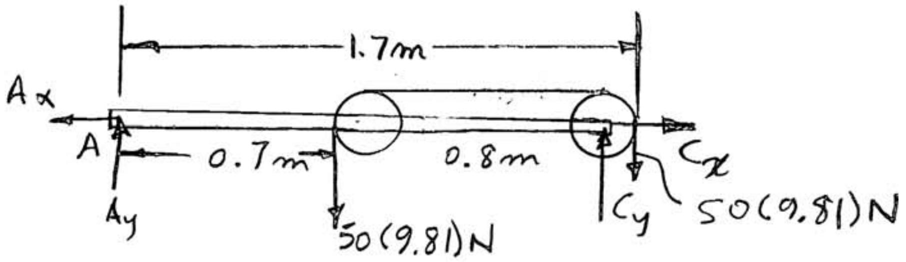
$D_y = 245.25\text{ N} = 245\text{ N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad D_x - 694.875 = 0$

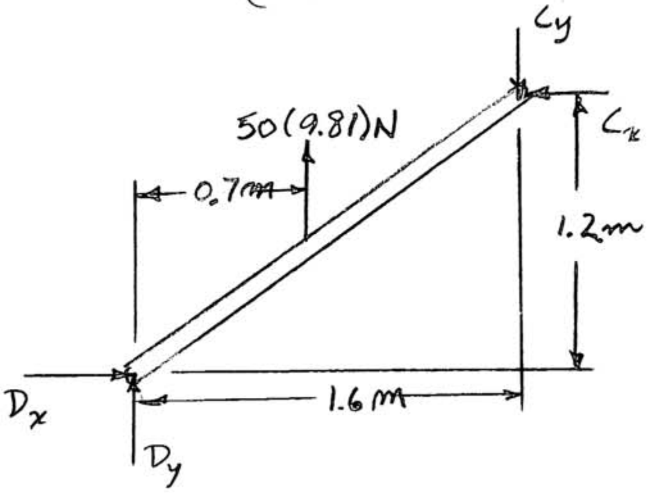
$D_x = 694.875\text{ N} = 695\text{ N}$ **Ans.**

Substituting $C_x = 694.875\text{ N}$ into Eq. (1) yields

$A_x = 694.875\text{ N} = 695\text{ N}$ **Ans.**

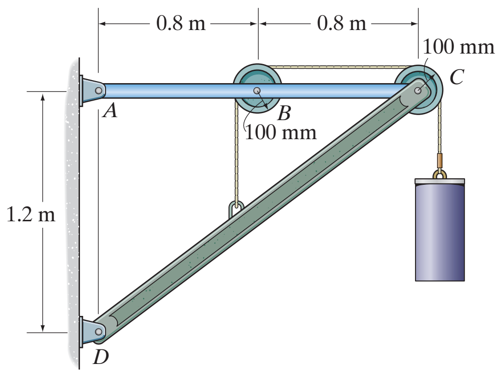


(a)



(b)

6–102. The frame is used to support the 50-kg cylinder. Determine the force of the pin at *C* on member *ABC* and on member *CD*.



Equations of Equilibrium: The horizontal and vertical components of force on members *CD* and *ABD* are denoted as C_x, C_y, C_x', and C_y', respectively. Writing the moment equation of equilibrium about point *A*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad C_y(1.6) - 50(9.81)(0.7) - 50(9.81)(1.7) = 0$$

$$C_y = 735.75\text{ N}$$

Using this and applying the moment equation of equilibrium about point *D* of member *CD*,

$$\circlearrowleft + \Sigma M_D = 0; \qquad C_x(1.2) + 50(9.81)(0.7) - 735.75(1.6) = 0$$

$$C_x = 694.875\text{ N}$$

Using the results of C_x and C_y and considering the free - body diagram of pulley *C*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 694.875 - 50(9.81) - C_{x'} = 0$$

$$C_{x'} = 204.375\text{ N}$$

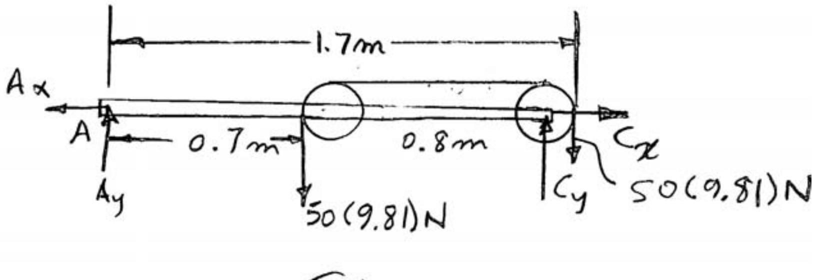
$$+\uparrow \Sigma F_y = 0; \qquad 735.75 - 50(9.81) - C_{y'} = 0$$

$$C_{y'} = 245.25\text{ N}$$

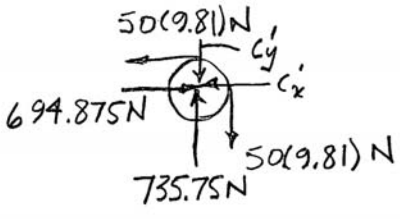
Thus, the force at pin *C* on members *CD* and *ABC* are given by

$$F_{CD} = \sqrt{C_x^2 + C_y^2} = \sqrt{694.875^2 + 735.75^2} = 1012.01\text{ N} = 1.01\text{ kN} \qquad \textbf{Ans.}$$

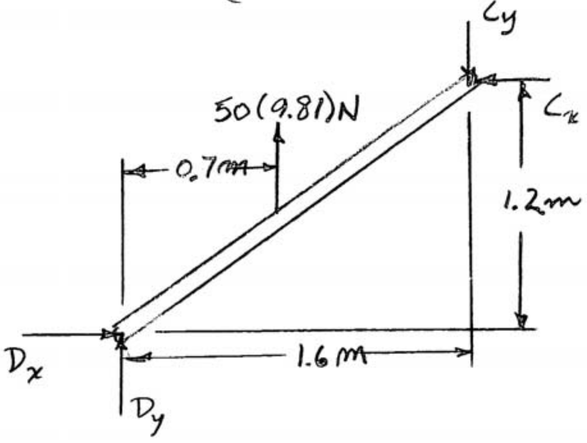
$$F_{ABC} = \sqrt{C_{x'}^2 + C_{y'}^2} = \sqrt{204.375^2 + 245.25^2} = 319.24\text{ N} = 319\text{ N} \qquad \textbf{Ans.}$$



(a)

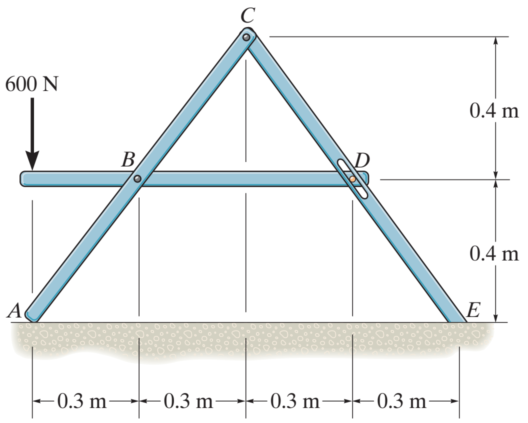


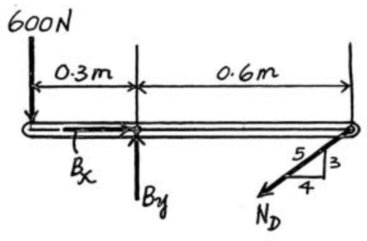
(c)



(b)

6–103. Determine the reactions at the fixed support *E* and the smooth support *A*. The pin, attached to member *BD*, passes through a smooth slot at *D*.





Member *BD* :

$\circlearrowleft \Sigma M_B = 0; \quad 600(0.3) - N_D\left(\frac{3}{5}\right)(0.6) = 0$

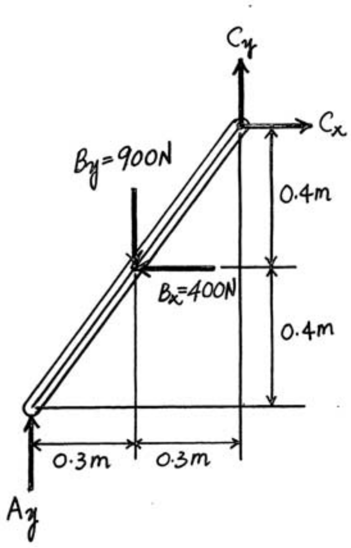
$N_D = 500\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - \frac{4}{5}(500) = 0$

$B_x = 400\text{ N}$

$+\uparrow \Sigma F_y = 0; \quad -600 + B_y - \frac{3}{5}(500) = 0$

$B_y = 900\text{ N}$



Member *ABC* :

$\circlearrowleft + \Sigma M_C = 0; \quad 900(0.3) - 400(0.4) - A_y(0.6) = 0$

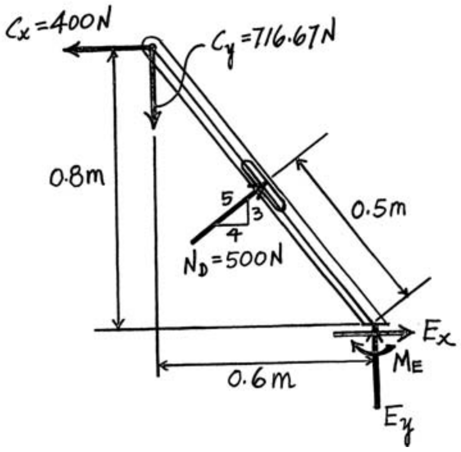
$A_y = 183.33 = 183\text{ N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad -400 + C_x = 0$

$C_x = 400\text{ N}$

$+\uparrow \Sigma F_y = 0; \quad 183.33 - 900 + C_y = 0$

$C_y = 716.67\text{ N}$



Member *CDE* :

$\xrightarrow{+} \Sigma F_x = 0; \quad -400 + 500\left(\frac{4}{5}\right) + E_x = 0$

$E_x = 0$ **Ans**

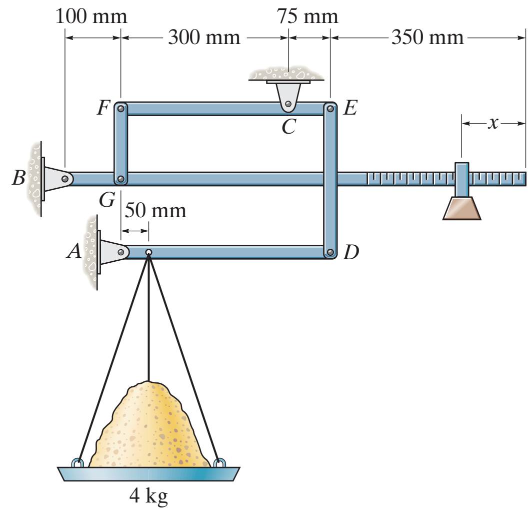
$+\uparrow \Sigma F_y = 0; \quad -716.67 + 500\left(\frac{3}{5}\right) + E_y = 0$

$E_y = 417\text{ N}$ **Ans**

$\circlearrowleft + \Sigma M_E = 0; \quad -M_E - 500(0.5) + 400(0.8) + 716.67(0.6) = 0$

$M_E = 500\text{ N}\cdot\text{m}$ **Ans**

***6–104.** The compound arrangement of the pan scale is shown. If the mass on the pan is 4 kg, determine the horizontal and vertical components at pins A, B, and C and the distance x of the 25-g mass to keep the scale in balance.



Free Body Diagram : The solution for this problem will be simplified if one realizes that members DE and FG are two-force members.

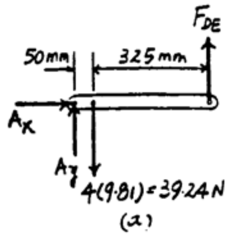
Equations of Equilibrium : From FBD (a).

$\circlearrowleft+\Sigma M_A = 0; \quad F_{DE}(375) - 39.24(50) = 0 \quad F_{DE} = 5.232\text{ N}$

$+\uparrow\Sigma F_y = 0; \quad A_y + 5.232 - 39.24 = 0$

$A_y = 34.0\text{ N}$ **Ans**

$\xrightarrow{+}\Sigma F_x = 0; \quad A_x = 0$ **Ans**



(a)

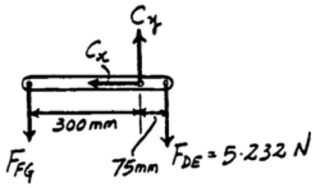
From (b).

$\circlearrowleft+\Sigma M_C = 0; \quad F_{FG}(300) - 5.232(75) = 0 \quad F_{FG} = 1.308\text{ N}$

$+\uparrow\Sigma F_y = 0; \quad C_y - 1.308 - 5.232 = 0$

$C_y = 6.54\text{ N}$ **Ans**

$\xrightarrow{+}\Sigma F_x = 0; \quad C_x = 0$ **Ans**



(b)

From (c).

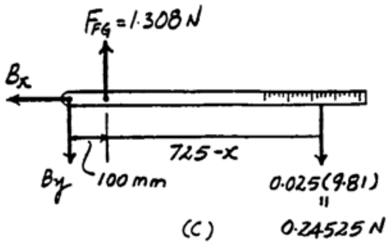
$\circlearrowleft+\Sigma M_B = 0; \quad 1.308(100) - 0.24525(825 - x) = 0$

$x = 292\text{ mm}$ **Ans**

$+\uparrow\Sigma F_y = 0; \quad 1.308 - 0.24525 - B_y = 0$

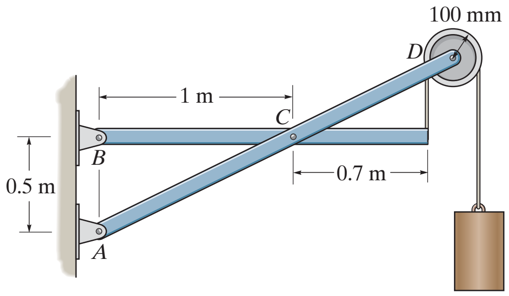
$B_y = 1.06\text{ N}$ **Ans**

$\xrightarrow{+}\Sigma F_x = 0; \quad B_x = 0$ **Ans**



(c)

•6–105. Determine the horizontal and vertical components of reaction that the pins at A, B, and C exert on the frame. The cylinder has a mass of 80 kg.



Equations of Equilibrium : From FBD (b),

$\curvearrowleft + \Sigma M_B = 0;\quad 784.8(1.7) - C_y(1) = 0$

$C_y = 1334.16\text{ N} = 1.33\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad B_y + 784.8 - 1334.16 = 0$

$B_y = 549\text{ N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad C_x - B_x = 0$ [1]

From FBD (a),

$\curvearrowleft + \Sigma M_A = 0;\quad C_x(0.5) + 1334.16(1) - 784.8(1.7) - 784.8(1.9) = 0$

$C_x = 2982.24\text{ N} = 2.98\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + 1334.16 - 784.8 - 784.8 = 0$

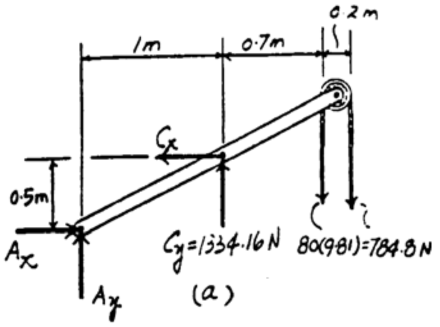
$A_y = 235\text{ N}$ **Ans**

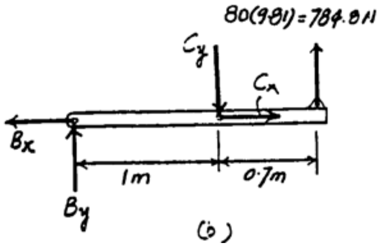
$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 2982.24 = 0$

$A_x = 2982.24\text{ N} = 2.98\text{ kN}$ **Ans**

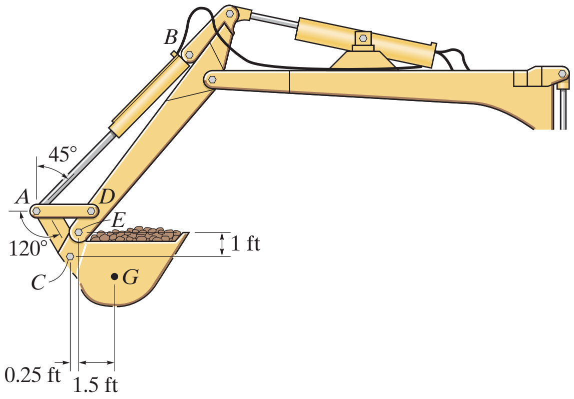
Substitute $C_x = 2982.24$ N into Eq.[1] yields,

$B_x = 2982.24\text{ N} = 2.98\text{ kN}$ **Ans**





6–106. The bucket of the backhoe and its contents have a weight of 1200 lb and a center of gravity at *G*. Determine the forces of the hydraulic cylinder *AB* and in links *AC* and *AD* in order to hold the load in the position shown. The bucket is pinned at *E*.

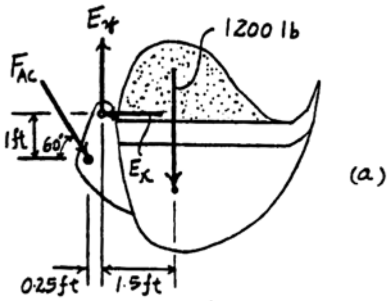


Free Body Diagram : The solution for this problem will be simplified if one realizes that the hydraulic cylinder *AB*, links *AD* and *AC* are two-force members.

Equations of Equilibrium : From FBD (a),

$$\curvearrowleft + \Sigma M_E = 0; \quad F_{AC}\cos 60°(1) + F_{AC}\sin 60°(0.25) - 1200(1.5) = 0$$

$$F_{AC} = 2512.19 \text{ lb} = 2.51 \text{ kip} \qquad \textbf{Ans}$$



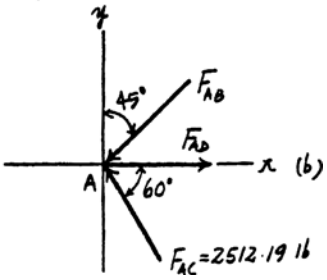
Using method of joint [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \quad 2512.19\sin 60° - F_{AB}\cos 45° = 0$$

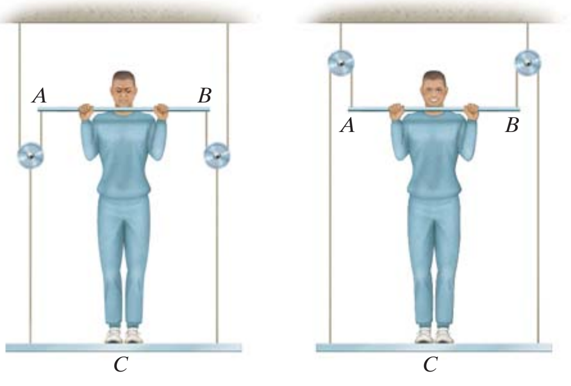
$$F_{AB} = 3076.79 \text{ lb} = 3.08 \text{ kip} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AD} - 3076.79\sin 45° - 2512.19\cos 60° = 0$$

$$F_{AD} = 3431.72 \text{ lb} = 3.43 \text{ kip} \qquad \textbf{Ans}$$



6–107. A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar *AB* in each case and the normal reaction he exerts on the platform at *C*. Neglect the weight of the platform.



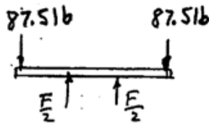
(a) (b)

(a)

Bar :

$$+\uparrow \Sigma F_y = 0; \quad 2(F/2) - 2(87.5) = 0$$

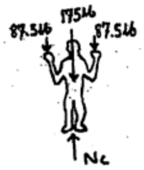
$$F = 175 \text{ lb} \qquad \textbf{Ans}$$



Man :

$$+\uparrow \Sigma F_y = 0; \quad N_C - 175 - 2(87.5) = 0$$

$$N_C = 350 \text{ lb} \qquad \textbf{Ans}$$

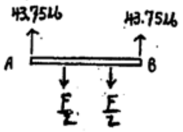


(b)

Bar :

$$+\uparrow \Sigma F_y = 0; \quad 2(43.75) - 2(F/2) = 0$$

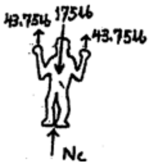
$$F = 87.5 \text{ lb} \qquad \textbf{Ans}$$



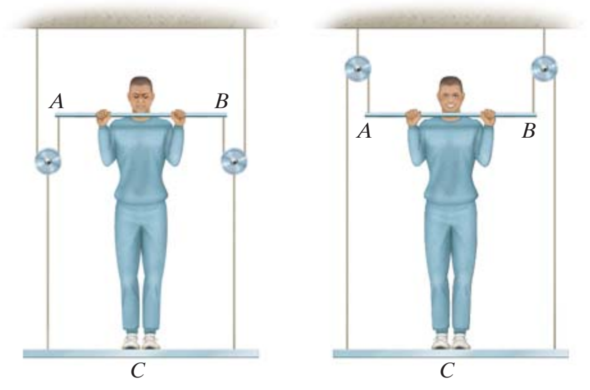
Man :

$$+\uparrow \Sigma F_y = 0; \quad N_C - 175 + 2(43.75) = 0$$

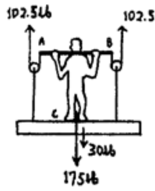
$$N_C = 87.5 \text{ lb} \qquad \textbf{Ans}$$

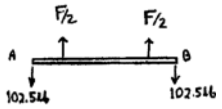


***6–108.** A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. The platform has a weight of 30 lb.



(a) (b)





(a)

Bar :

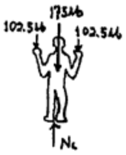
$+\uparrow\Sigma F_y = 0;\quad 2(F/2) - 102.5 - 102.5 = 0$

$F = 205\text{ lb}$ **Ans**

Man :

$+\uparrow\Sigma F_y = 0;\quad N_C - 175 - 102.5 - 102.5 = 0$

$N_C = 380\text{ lb}$ **Ans**



(b)

Bar :

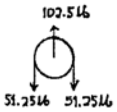
$+\uparrow\Sigma F_y = 0;\quad 2(F/2) - 51.25 - 51.25 = 0$

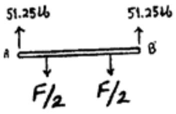
$F = 102\text{ lb}$ **Ans**

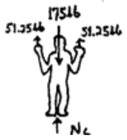
Man :

$+\uparrow\Sigma F_y = 0;\quad N_C - 175 + 51.25 + 51.25 = 0$

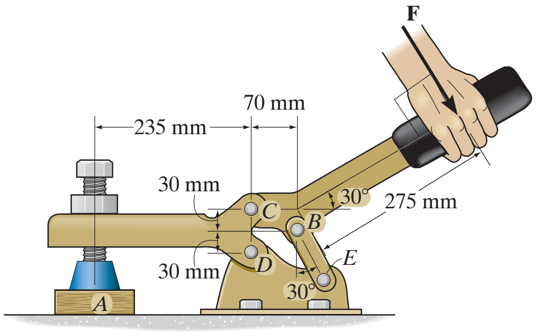
$N_C = 72.5\text{ lb}$ **Ans**







•6–109. If a clamping force of 300 N is required at *A*, determine the amount of force **F** that must be applied to the handle of the toggle clamp.



Equations of Equilibrium: First, we will consider the free-body diagram of the clamp in Fig. *a*. Writing the moment equation of equilibrium about point *D*,

$$\circlearrowleft + \Sigma M_D = 0; \qquad C_x(60) - 300(235) = 0$$
$$C_x = 1175 \text{ N}$$

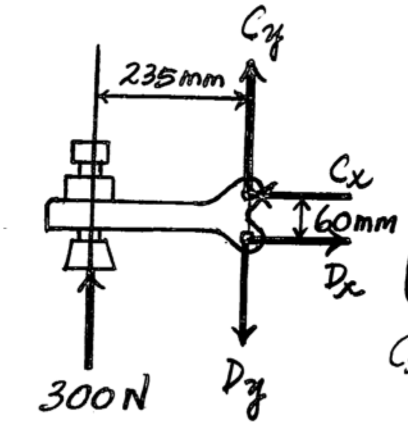
Subsequently, the free-body diagram of the handle in Fig. *b* will be considered.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{BE}\cos 30°(70) - F_{BE}\sin 30°(30) - F\cos 30°(275\cos 30° + 70)$$
$$-F\sin 30°(275\sin 30°) = 0$$
$$45.62F_{BE} - 335.62F = 0 \qquad (1)$$

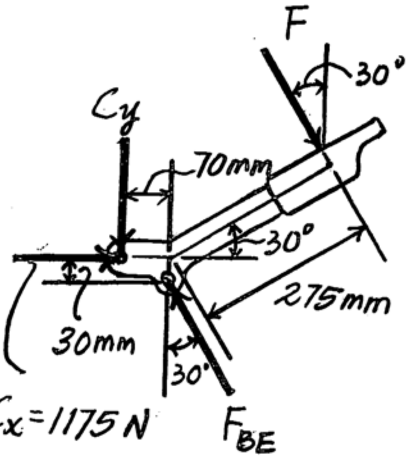
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 1175 + F\sin 30° - F_{BE}\sin 30° = 0$$
$$0.5F_{BE} - 0.5F = 1175 \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F = 369.69 \text{ N} = 370 \text{ N} \qquad \textbf{Ans.}$$
$$F_{BE} = 2719.69 \text{ N}$$

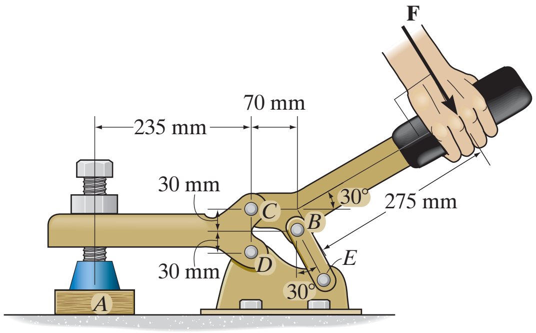


(a)



(b)

6–110. If a force of $F = 350$ N is applied to the handle of the toggle clamp, determine the resulting clamping force at A.



Equations of Equilibrium: First, we will consider the free-body diagram of the handle in Fig. a.

$$\circlearrowleft +\Sigma M_C = 0; \quad F_{BE}\cos 30°(70) - F_{BE}\sin 30°(30) - 350\cos 30°(275\cos 30° + 70) - 350\sin 30°(275\sin 30°) = 0$$

$$F_{BE} = 2574.81\text{ N}$$

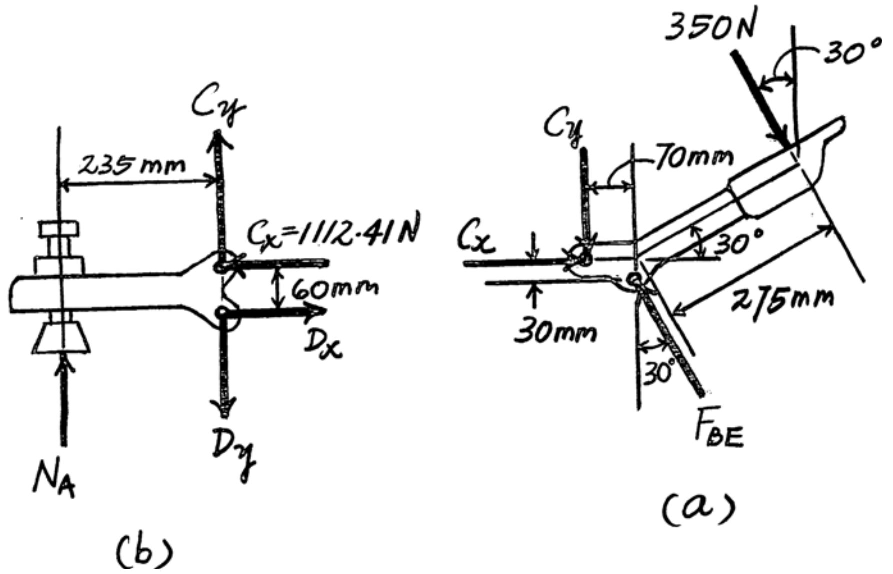
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad C_x - 2574.81\sin 30° + 350\sin 30° = 0$$

$$C_x = 1112.41\text{ N}$$

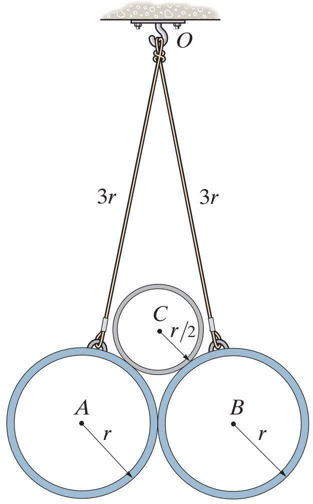
Subsequently, the free-body diagram of the clamp in Fig. b will be considered. Using the result of C_x and writing the moment equation of equilibrium about point D,

$$\circlearrowleft +\Sigma M_D = 0; \quad 1112.41(60) - N_A(235) = 0$$

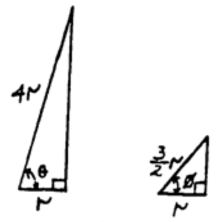
$$N_A = 284.01\text{ N} = 284\text{ N}$$ **Ans.**



6–111. Two smooth tubes *A* and *B*, each having the same weight, *W*, are suspended from a common point *O* by means of equal-length cords. A third tube, *C*, is placed between *A* and *B*. Determine the greatest weight of *C* without upsetting equilibrium.



Free Body Diagram : When the equilibrium is about to be upset, the reaction at *B* must be zero ($N_B = 0$). From the geometry, $\phi = \cos^{-1}\left(\frac{r}{\frac{3}{2}r}\right) = 48.19°$ and $\theta = \cos^{-1}\left(\frac{r}{4r}\right) = 75.52°$.



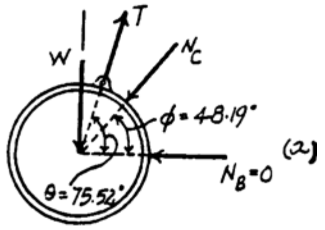
Equations of Equilibrium : From FBD (a),

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T\cos 75.52° - N_C \cos 48.19° = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad T\sin 75.52° - N_C \sin 48.19° - W = 0 \qquad [2]$$

Solving Eq. [1] and [2] yields,

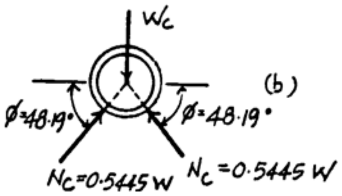
$$T = 1.452W \qquad N_C = 0.5445W$$



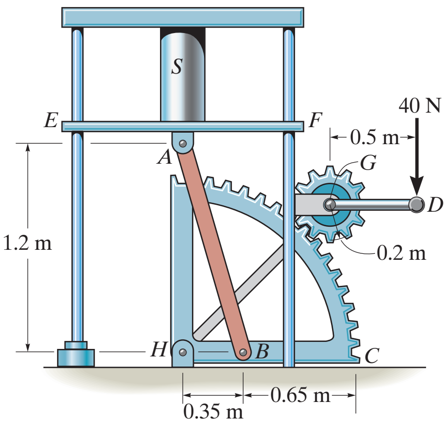
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad 2(0.5445W\sin 48.19°) - W_C = 0$$

$$W_C = 0.812W \qquad \textbf{Ans}$$



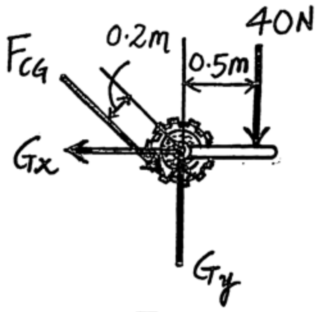
***6–112.** The handle of the sector press is fixed to gear *G*, which in turn is in mesh with the sector gear *C*. Note that *AB* is pinned at its ends to gear *C* and the underside of the table *EF*, which is allowed to move vertically due to the smooth guides at *E* and *F*. If the gears only exert tangential forces between them, determine the compressive force developed on the cylinder *S* when a vertical force of 40 N is applied to the handle of the press.



Member *GD*:

$\circlearrowleft + \Sigma M_G = 0; \quad -40(0.5) + F_{CG}(0.2) = 0$

$F_{CG} = 100 \text{ N}$



Sector gear:

$\circlearrowleft + \Sigma M_H = 0; \quad 100(1) - F_{AB}\left(\frac{1.2}{1.25}\right)(0.35) = 0$

$F_{AB} = 297.62 \text{ N}$

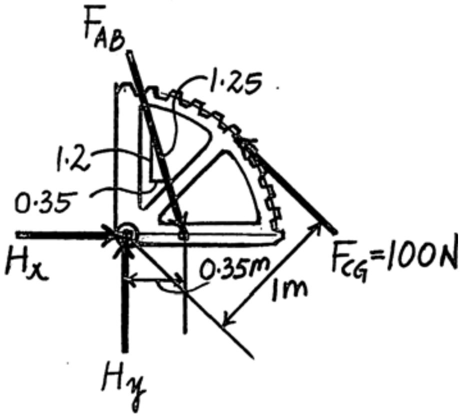
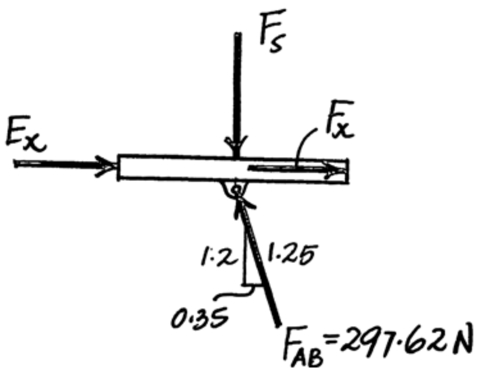


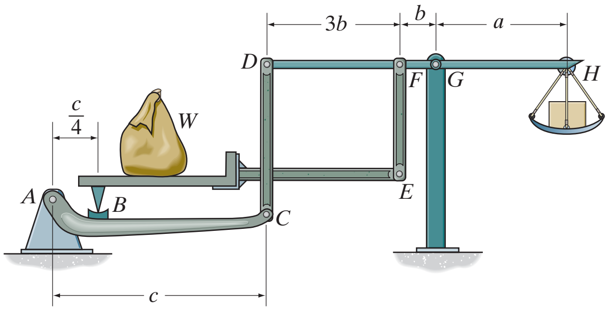
Table:

$+\uparrow \Sigma F_y = 0; \quad 297.62\left(\frac{1.2}{1.25}\right) - F_S = 0$

$F_S = 286 \text{ N}$ **Ans**



•6–113. Show that the weight W_1 of the counterweight at H required for equilibrium is $W_1 = (b/a)W$, and so it is independent of the placement of the load W on the platform.



Equations of Equilibrium: First, we will consider member *BE*.

$$\circlearrowleft+\Sigma M_E = 0; \qquad W(x) - N_B\left(3b + \frac{3}{4}c\right) = 0$$

$$N_B = \frac{Wx}{\left(3b + \frac{3}{4}c\right)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} + \frac{W_1 x}{\left(3b + \frac{3}{4}c\right)} - W = 0$$

$$F_{EF} = W\left(1 - \frac{x}{3b + \frac{3}{4}c}\right)$$

Using the result for N_B and applying the moment equation of equilibrium about point *A*,

$$\circlearrowleft+\Sigma M_A = 0; \qquad F_{CD}(c) - \frac{Wx}{\left(3b + \frac{3}{4}c\right)}\left(\frac{1}{4}c\right) = 0$$

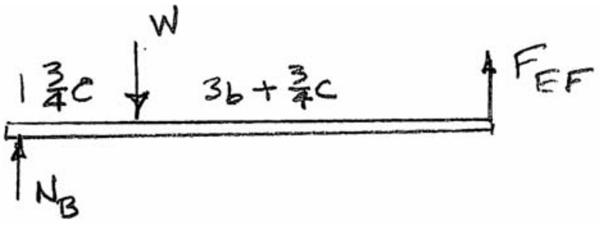
$$F_{CD} = \frac{Wx}{12b + 3c}$$

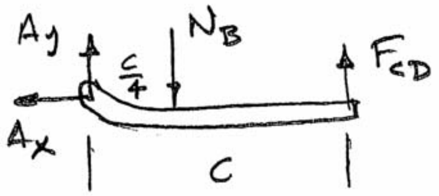
Writing the moment equation of equilibrium about point *G*,

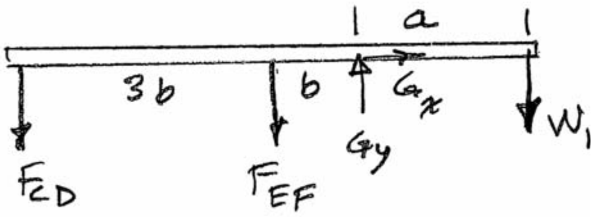
$$\circlearrowleft+\Sigma M_G = 0; \qquad \frac{Wx}{12b + 3c}(4b) + W\left(1 - \frac{x}{3b + \frac{3}{4}c}\right)(b) - W_1(a) = 0$$

$$W_1 = \frac{b}{a}W \qquad \textbf{Ans.}$$

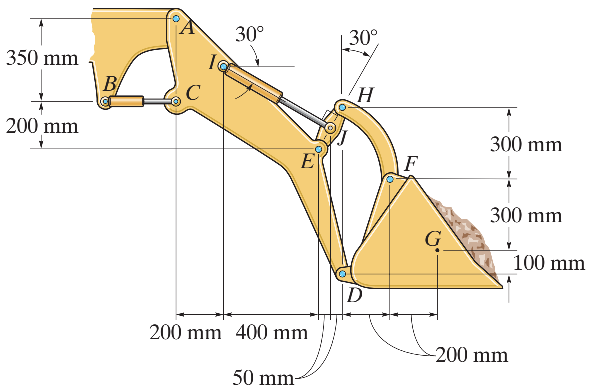
This result shows that the required weight W_1 of the counterweight is independent of the position x of the load on the platform.

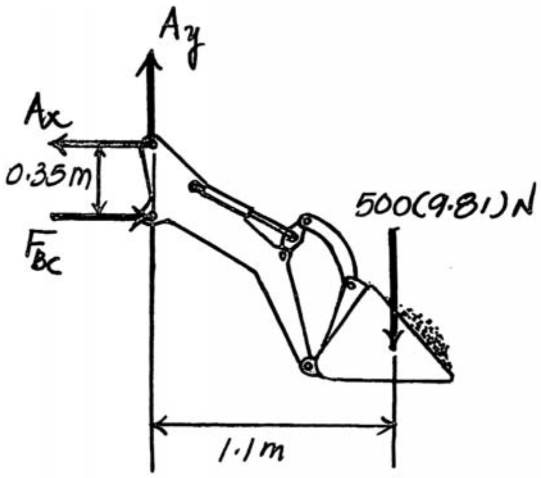






6–114. The tractor shovel carries a 500-kg load of soil, having a center of mass at G. Compute the forces developed in the hydraulic cylinders IJ and BC due to this loading.

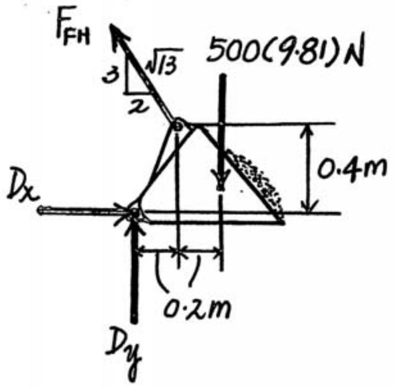




Shovel :

$$\circlearrowleft + \Sigma M_D = 0; \quad -500(9.81)(0.4) + F_{FH}\left(\frac{2}{\sqrt{13}}\right)(0.4) + F_{FH}\left(\frac{3}{\sqrt{13}}\right)(0.2) = 0$$

$$F_{FH} = 5052.92 \text{ N}$$



Member EH :

$$\circlearrowleft + \Sigma M_E = 0; \quad F_{IJ}\left(\frac{0.05}{\sin 30°}\right) - 5052.92\left(\frac{3}{\sqrt{13}}\right)(0.1)$$

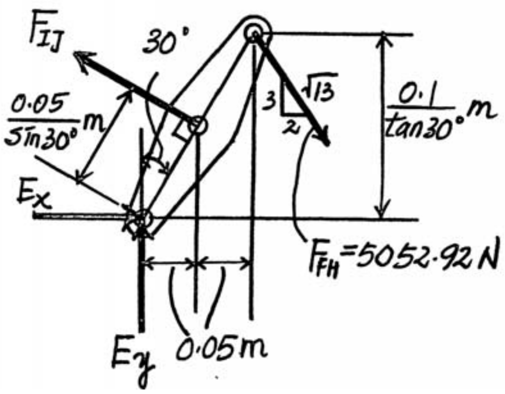
$$- 5052.92\left(\frac{2}{\sqrt{13}}\right)\left(\frac{0.1}{\tan 30°}\right) = 0$$

$$F_{IJ} = 9059 \text{ N} = 9.06 \text{ kN (T)} \qquad \text{Ans}$$

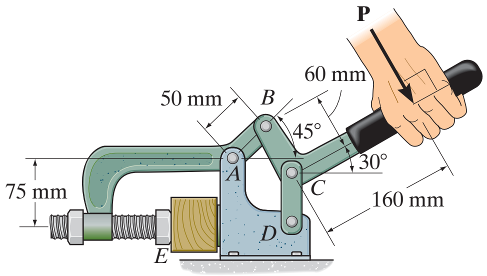
Assembly :

$$\circlearrowleft + \Sigma M_A = 0; \quad -500(9.81)(1.1) + F_{BC}(0.35) = 0$$

$$F_{BC} = 15\,415.7 \text{ N} = 15.4 \text{ kN (C)} \qquad \text{Ans}$$



6–115. If a force of $P = 100$ N is applied to the handle of the toggle clamp, determine the horizontal clamping force N_E that the clamp exerts on the smooth wooden block at E.



Equations of Equilibrium: First, we will consider the free-body diagram of the handle in Fig. *a*.

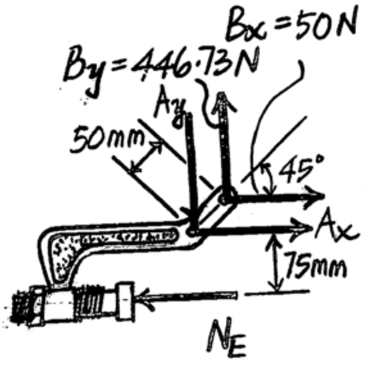
$$\circlearrowleft + \Sigma M_B = 0; \qquad F_{CD}\sin 30°(60) - 100(160) = 0$$
$$F_{CD} = 533.33\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 100\sin 30° - B_x = 0$$
$$B_x = 50\text{ N}$$

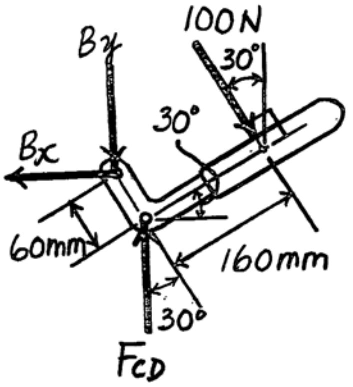
$$+\uparrow \Sigma F_y = 0; \qquad 533.33 - 100\cos 30° - B_y = 0$$
$$B_y = 446.73\text{ N}$$

Using the results of B_x and B_y obtained above and applying the moment equation of equilibrium about point A on the free-body diagram of the clamp in Fig. *b*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 446.73(50\cos 45°) - 50(50\sin 45°) - N_E(75) = 0$$
$$N_E = 187.02\text{ N} = 187\text{ N} \qquad \textbf{Ans.}$$

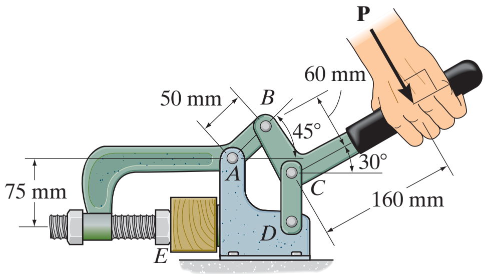


(b)



(a)

***6–116.** If the horizontal clamping force that the toggle clamp exerts on the smooth wooden block at E is $N_E = 200$ N, determine the force **P** applied to the handle of the clamp.



Equations of Equilibrium: First, we will consider the free-body diagram of the handle in Fig. *a*.

$\circlearrowleft + \Sigma M_B = 0;$ $\quad F_{CD}\sin 30°(60) - P(160) = 0$

$F_{CD} = 5.333P$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad P\sin 30° - B_x = 0$

$B_x = 0.5P$

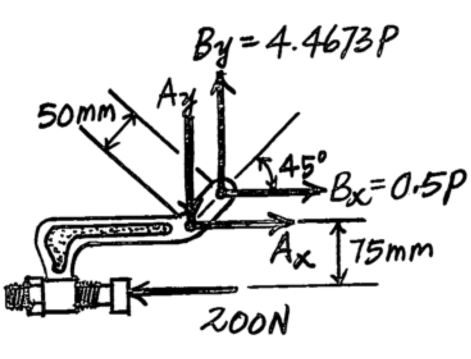
$+\uparrow \Sigma F_y = 0;$ $\quad 5.333P - P\cos 30° - B_y = 0$

$B_y = 4.4673P$

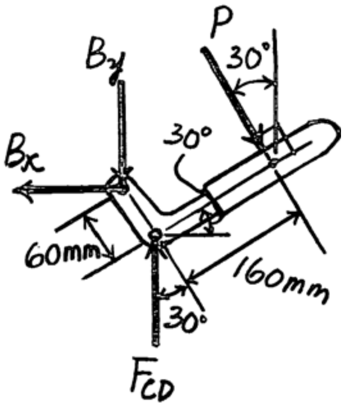
Using the results of B_x and B_y obtained above and applying the moment equation of equilibrium about point A on the free-body diagram of the clamp in Fig. *b*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 4.4673P(50\cos 45°) - 0.5P(50\sin 45°) - 200(75) = 0$

$P = 106.94\text{ N} = 107\text{ N}$ **Ans.**

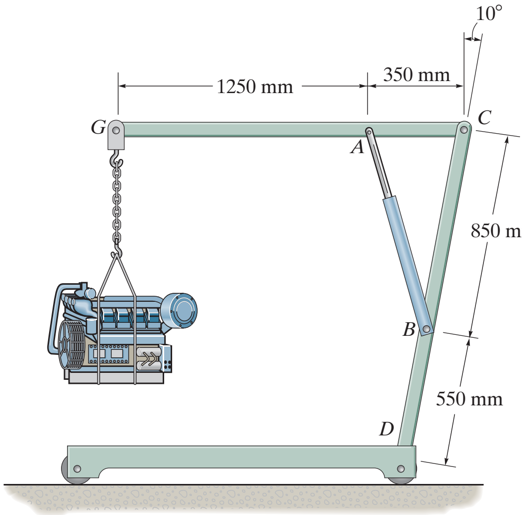


(b)



(a)

•6–117. The engine hoist is used to support the 200-kg engine. Determine the force acting in the hydraulic cylinder *AB*, the horizontal and vertical components of force at the pin *C*, and the reactions at the fixed support *D*.



Free Body Diagram : The solution for this problem will be simplified if one realizes that member *AB* is a two force member. From the geometry,

$$l_{AB} = \sqrt{350^2 + 850^2 - 2(350)(850)\cos 80°} = 861.21 \text{ mm}$$

$$\frac{\sin\theta}{850} = \frac{\sin 80°}{861.21} \qquad \theta = 76.41°$$

Equations of Equilibrium : From FBD (a),

$$\circlearrowleft + \Sigma M_C = 0; \quad 1962(1.60) - F_{AB}\sin 76.41°(0.35) = 0$$
$$F_{AB} = 9227.60 \text{ N} = 9.23 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 9227.60\cos 76.41° = 0$$
$$C_x = 2168.65 \text{ N} = 2.17 \text{ kN} \qquad \textbf{Ans}$$

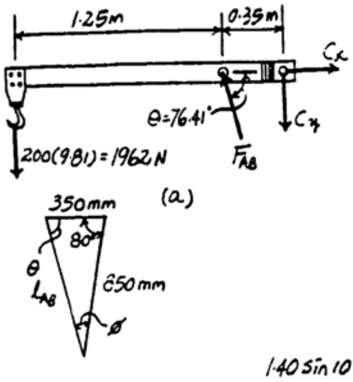
$$+\uparrow \Sigma F_y = 0; \quad 9227.60\sin 76.41° - 1962 - C_y = 0$$
$$C_y = 7007.14 \text{ N} = 7.01 \text{ kN} \qquad \textbf{Ans}$$

From FBD (b),

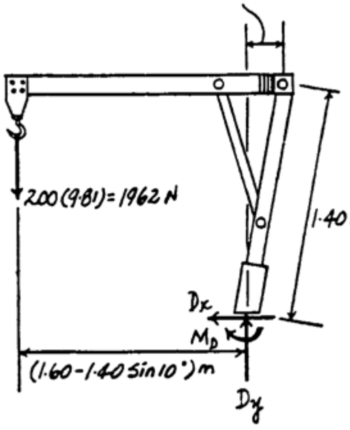
$$\xrightarrow{+} \Sigma F_x = 0; \quad D_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad D_y - 1962 = 0$$
$$D_y = 1962 \text{ N} = 1.96 \text{ kN} \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_D = 0; \quad 1962(1.60 - 1.40\sin 10°) - M_D = 0$$
$$M_D = 2662.22 \text{ N}\cdot\text{m} = 2.66 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

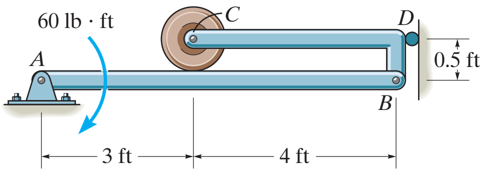


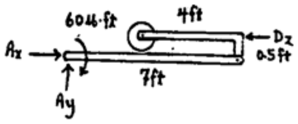
(a)



(b)

6–118. Determine the force that the smooth roller *C* exerts on member *AB*. Also, what are the horizontal and vertical components of reaction at pin *A*? Neglect the weight of the frame and roller.



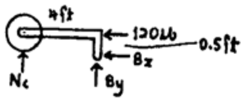


$\circlearrowleft + \Sigma M_A = 0;$ $\quad -60 + D_x(0.5) = 0$

$D_x = 120\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 120\text{ lb}$ **Ans**

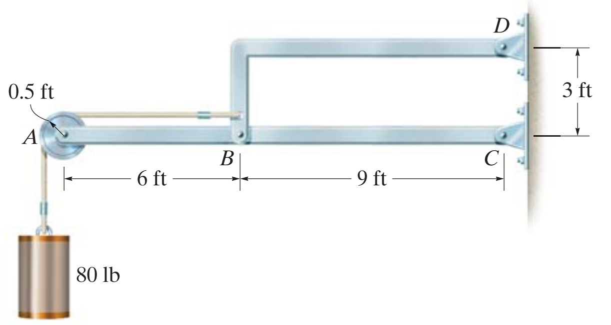
$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 0$ **Ans**

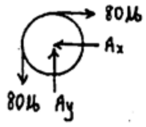


$\circlearrowleft + \Sigma M_B = 0;$ $\quad -N_C(4) + 120(0.5) = 0$

$N_C = 15.0\text{ lb}$ **Ans**

6–119. Determine the horizontal and vertical components of reaction which the pins exert on member *ABC*.



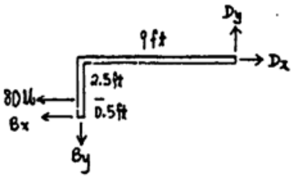


$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 80\text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 80\text{ lb}$ **Ans**

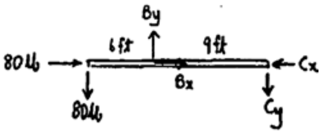
$\circlearrowleft + \Sigma M_C = 0;$ $\quad 80(15) - B_y(9) = 0$

$B_y = 133.3 = 133\text{ lb}$ **Ans**



$\circlearrowleft + \Sigma M_D = 0;$ $\quad -80(2.5) + 133.3(9) - B_x(3) = 0$

$B_x = 333\text{ lb}$ **Ans**



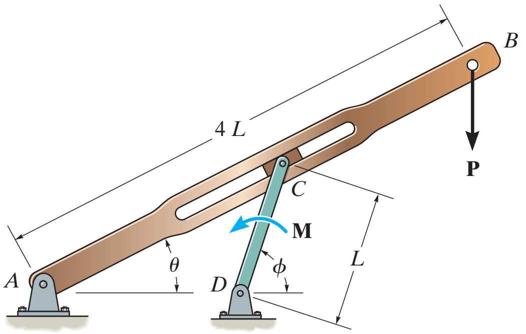
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 80 + 333 - C_x = 0$

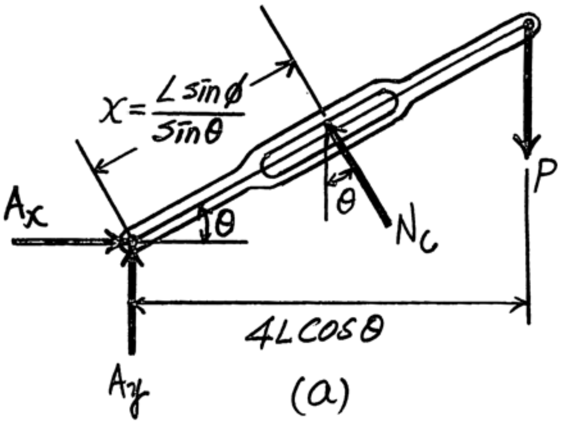
$C_x = 413\text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad -80 + 133.3 - C_y = 0$

$C_y = 53.3\text{ lb}$ **Ans**

***6–120.** Determine the couple moment **M** that must be applied to member *DC* for equilibrium of the quick-return mechanism. Express the result in terms of the angles ϕ and θ, dimension L, and the applied *vertical force* **P**. The block at C is confined to slide within the slot of member AB.





$$\frac{x}{4L} = \frac{L\sin\phi}{4L\sin\theta} \qquad x = \frac{L\sin\phi}{\sin\theta}$$

From FBD (a)

$$\curvearrowleft +\Sigma M_A = 0; \quad N_C\left(\frac{L\sin\phi}{\sin\theta}\right) - P(4L\cos\theta) = 0 \qquad N_C = \frac{4P\cos\theta\sin\theta}{\sin\phi}$$

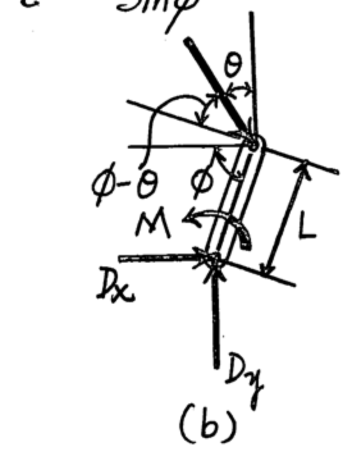
From FBD (b)

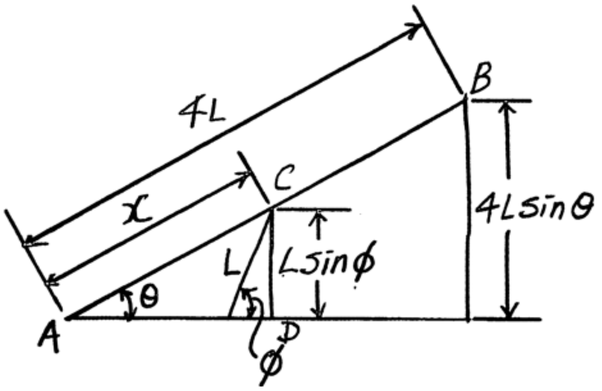
$$\curvearrowleft +\Sigma M_D = 0; \quad M - \frac{4P\cos\theta\sin\theta}{\sin\phi}[\cos(\phi-\theta)]L = 0$$

$$M = \frac{4PL\cos\theta\sin\theta}{\sin\phi}[\cos(\phi-\theta)]$$

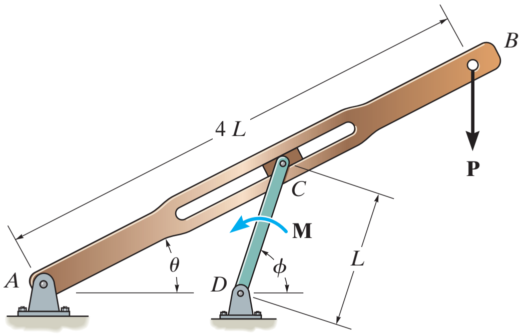
$$= \frac{2PL\sin 2\theta}{\sin\phi}[\cos(\phi-\theta)] \qquad \text{Ans}$$

$$N_C = \frac{4P\cos\theta\sin\theta}{\sin\phi}$$





•6–121. Determine the couple moment **M** that must be applied to member *DC* for equilibrium of the quick-return mechanism. Express the result in terms of the angles ϕ and θ, dimension L, and the applied force **P**, which should be changed in the figure and instead directed horizontally to the right. The block at C is confined to slide within the slot of member AB.



$$\frac{x}{4L} = \frac{L\sin\phi}{4L\sin\theta} \qquad x = \frac{L\sin\phi}{\sin\theta}$$

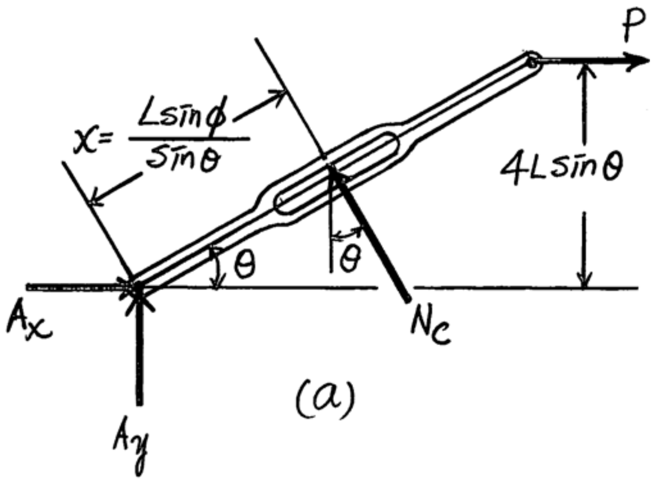
From FBD (a)

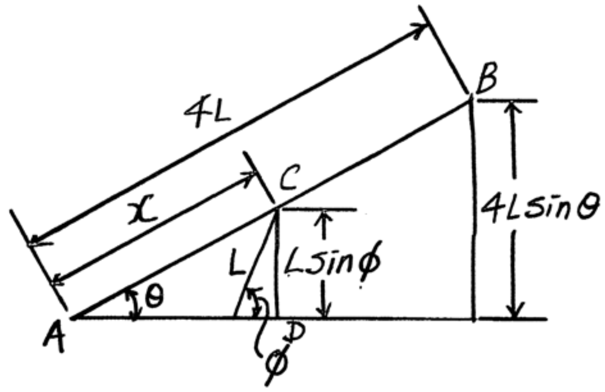
$$\circlearrowleft + \Sigma M_A = 0; \quad N_C\left(\frac{L\sin\phi}{\sin\theta}\right) - P(4L\sin\theta) = 0 \qquad N_C = \frac{4P\sin^2\theta}{\sin\phi}$$

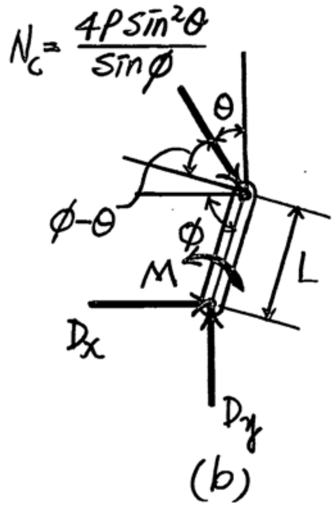
From FBD (b)

$$\circlearrowleft + \Sigma M_D = 0; \quad M - \frac{4P\sin^2\theta}{\sin\phi}[\cos(\phi-\theta)]L = 0$$

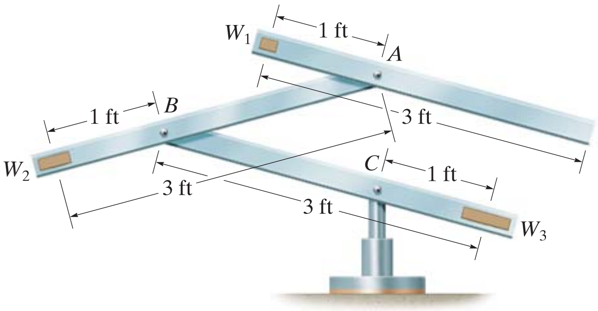
$$M = \frac{4PL\sin^2\theta}{\sin\phi}[\cos(\phi-\theta)] \qquad \text{Ans}$$





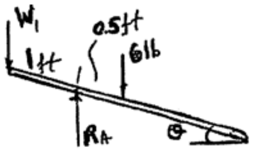


6–122. The kinetic sculpture requires that each of the three pinned beams be in perfect balance at all times during its slow motion. If each member has a uniform weight of 2 lb/ft and length of 3 ft, determine the necessary counterweights W_1, W_2, and W_3 which must be added to the ends of each member to keep the system in balance for any position. Neglect the size of the counterweights.



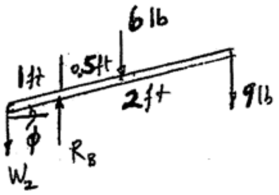
$$\circlearrowleft + \Sigma M_A = 0; \qquad W_1(1\cos\theta) - 6(0.5\cos\theta) = 0$$

$$W_1 = 3 \text{ lb} \qquad \textbf{Ans}$$



$$+\uparrow \Sigma F_y = 0; \qquad R_A - 3 - 6 = 0$$

$$R_A = 9 \text{ lb}$$

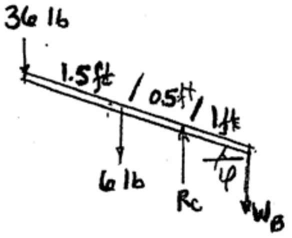


$$\circlearrowleft + \Sigma M_B = 0; \qquad W_2(1\cos\phi) - 6(0.5\cos\phi) - 9(2\cos\phi) = 0$$

$$W_2 = 21 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad R_B - 21 - 6 - 9 = 0$$

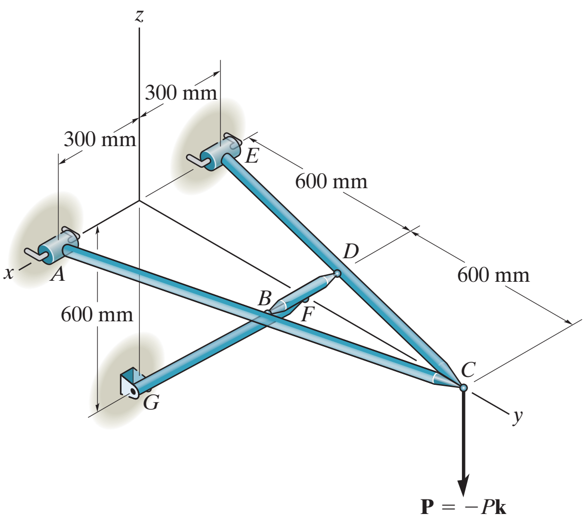
$$R_B = 36 \text{ lb}$$



$$\circlearrowleft + \Sigma M_C = 0; \qquad 36(2\cos\varphi) + 6(0.5\cos\varphi) - W_3(1\cos\varphi) = 0$$

$$W_3 = 75 \text{ lb} \qquad \textbf{Ans}$$

6–123. The four-member "A" frame is supported at *A* and *E* by smooth collars and at *G* by a pin. All the other joints are ball-and-sockets. If the pin at *G* will fail when the resultant force there is 800 N, determine the largest vertical force *P* that can be supported by the frame. Also, what are the *x, y, z* force components which member *BD* exerts on members *EDC* and *ABC*? The collars at *A* and *E* and the pin at *G* only exert force components on the frame.



GF is a two - force member, so the 800 - N force acts along the axis of *GF*. Using FBD (a),

$\Sigma M_x = 0;\quad -P(1.2) + 800 \sin 45° (0.6) = 0$

$P = 283 \text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad -A_y(0.3) + E_y(0.3) = 0$

$\Sigma F_y = 0;\quad -A_y - E_y + 800 \sin 45° = 0$

$A_y = E_y = 283 \text{ N}$

$\Sigma M_x = 0;\quad A_z(0.6) + E_z(0.6) - 283((0.6) = 0$

$\Sigma M_y = 0;\quad A_z(0.3) - E_z(0.3) = 0$

$A_z = E_z = 118 \text{N}$

Using FBD (b),

$\Sigma F_y = 0;\quad -B_y - D_y + 800 \sin 45° = 0$

$\Sigma M_z = 0;\quad D_y(0.3) - B_y(0.3) = 0$

$B_y = D_y = 283 \text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad -B_z - D_z + 800 \cos 45° = 0$

$\Sigma M_y = 0;\quad -D_z(0.3) + B_z(0.3) = 0$

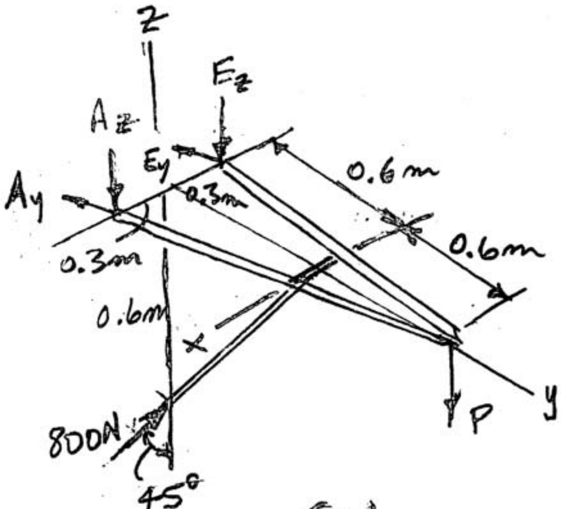
$B_z = D_z = 283 \text{ N}$ **Ans.**

$\Sigma F_x = 0;\quad -B_x + D_x = 0$

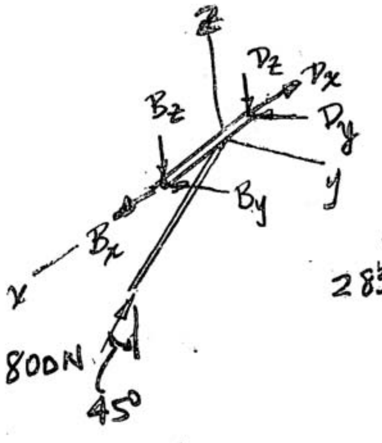
Using FBD (c),

$\Sigma M_z = 0;\quad -B_y(0.6) + 283(0.15) - 283(0.3) = 0$

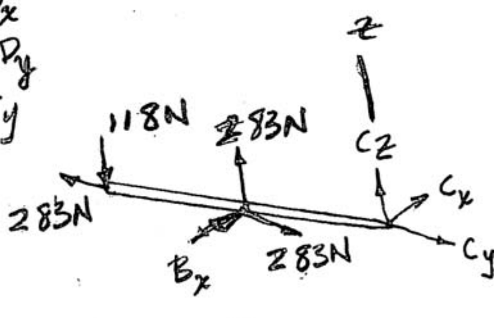
$B_x = D_x = 42.5 \text{ N}$ **Ans.**



(a)

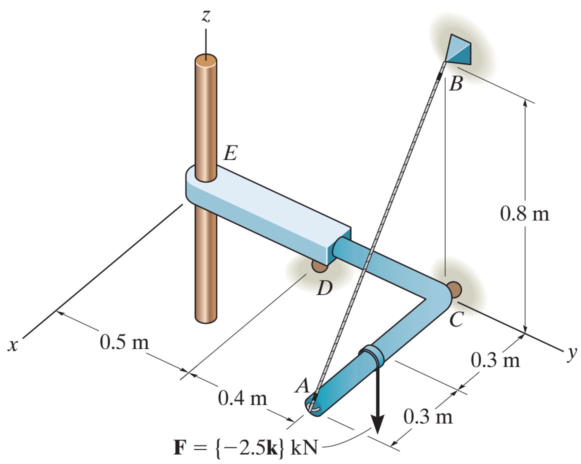


(b)



(c)

***6–124.** The structure is subjected to the loading shown. Member *AD* is supported by a cable *AB* and roller at *C* and fits through a smooth circular hole at *D*. Member *ED* is supported by a roller at *D* and a pole that fits in a smooth snug circular hole at *E*. Determine the *x, y, z* components of reaction at *E* and the tension in cable *AB*.



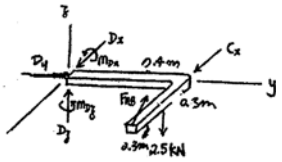
$\Sigma M_y = 0;$ $\quad -\frac{4}{5}F_{AB}(0.6) + 2.5(0.3) = 0$

$F_{AB} = 1.563 = 1.56\text{ kN}$ **Ans**

$\Sigma F_z = 0;$ $\quad \frac{4}{5}(1.563) - 2.5 + D_z = 0$

$D_z = 1.25\text{ kN}$

$\Sigma F_y = 0;$ $\quad D_y = 0$



$\Sigma F_x = 0;$ $\quad D_x + C_x - \frac{3}{5}(1.563) = 0 \qquad (1)$

$\Sigma M_x = 0;$ $\quad M_{Dx} + \frac{4}{5}(1.563)(0.4) - 2.5(0.4) = 0$

$M_{Dx} = 0.5\text{ kN}\cdot\text{m}$

$\Sigma M_z = 0;$ $\quad M_{Dz} + \frac{3}{5}(1.563)(0.4) - C_x(0.4) = 0 \qquad (2)$

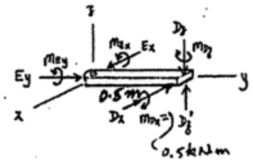
$\Sigma F_z = 0;$ $\quad D_z' = 1.25\text{ kN}$

$\Sigma M_x = 0;$ $\quad M_{Ex} = 0.5\text{ kN}\cdot\text{m}$ **Ans**

$\Sigma M_y = 0;$ $\quad M_{Ey} = 0$ **Ans**

$\Sigma F_y = 0;$ $\quad E_y = 0$ **Ans**

$\Sigma M_z = 0;$ $\quad D_x(0.5) - M_{Dz} = 0 \qquad (3)$



Solving Eqs. (1), (2) and (3):

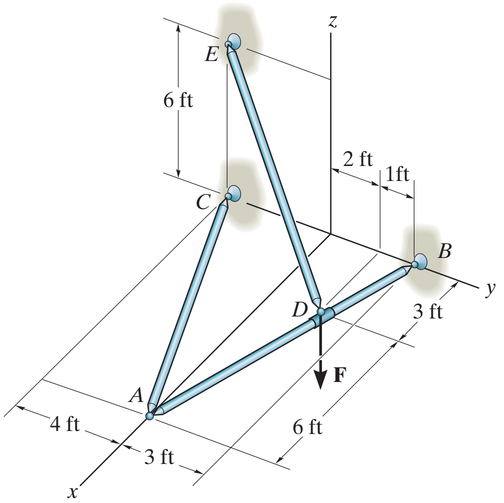
$C_x = 0.938\text{ kN}$

$M_{Dz} = 0$

$D_x = 0$

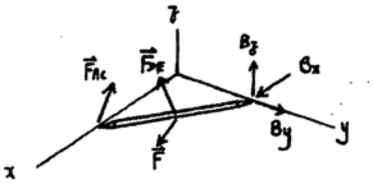
$\Sigma F_x = 0;$ $\quad E_x = 0$ **Ans**

•6–125. The three-member frame is connected at its ends using ball-and-socket joints. Determine the *x*, *y*, *z* components of reaction at *B* and the tension in member *ED*. The force acting at *D* is $\mathbf{F} = \{135\mathbf{i} + 200\mathbf{j} - 180\mathbf{k}\}$ lb.



***AC* is a two-force member.**

$$\mathbf{F} = \{135\mathbf{i} + 200\mathbf{j} - 180\mathbf{k}\}\text{ lb}$$



$\Sigma M_y = 0;$ $$-\frac{6}{9}F_{DE}(3) + 180(3) = 0$$

$$F_{DE} = 270\text{ lb} \qquad \textbf{Ans}$$

$\Sigma F_z = 0;$ $$B_z + \frac{6}{9}(270) - 180 = 0$$

$$B_z = 0 \qquad \textbf{Ans}$$

$\Sigma(M_B)_z = 0;$ $$-\frac{9}{\sqrt{97}}F_{AC}(3) - \frac{4}{\sqrt{97}}F_{AC}(9) + 135(1) + 200(3) - \frac{6}{9}(270)(3) - \frac{3}{9}(270)(1) = 0$$

$$F_{AC} = 16.41\text{ lb}$$

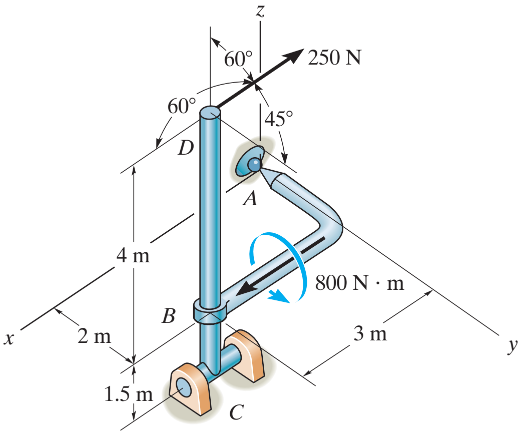
$\Sigma F_x = 0;$ $$135 - \frac{3}{9}(270) + B_x - \frac{9}{\sqrt{97}}(16.41) = 0$$

$$B_x = -30\text{ lb} \qquad \textbf{Ans}$$

$\Sigma F_y = 0;$ $$B_y - \frac{4}{\sqrt{97}}(16.41) + 200 - \frac{6}{9}(270) = 0$$

$$B_y = -13.3\text{ lb} \qquad \textbf{Ans}$$

6–126. The structure is subjected to the loadings shown. Member *AB* is supported by a ball-and-socket at *A* and smooth collar at *B*. Member *CD* is supported by a pin at *C*. Determine the *x, y, z* components of reaction at *A* and *C*.



From FBD (a)

$\Sigma M_y = 0;$ $M_{B\,y} = 0$

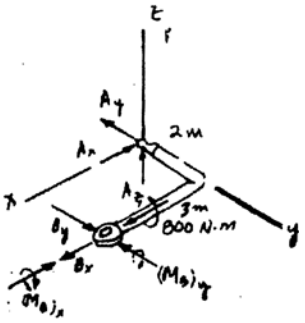
$\Sigma M_x = 0;$ $-M_{B\,x} + 800 = 0$ $M_{B\,x} = 800\text{ N}\cdot\text{m}$

$\Sigma M_z = 0;$ $B_y(3) - B_x(2) = 0$ (1)

$\Sigma F_z = 0;$ $A_z = 0$ Ans

$\Sigma F_x = 0;$ $-A_x + B_x = 0$ (2)

$\Sigma F_y = 0;$ $-A_y + B_y = 0$ (3)



(a)

From FBD (b)

$\Sigma M_x = 0;$ $B_y(1.5) + 800 - 250\cos 45°(5.5) = 0$ $B_y = 114.85\text{ N}$

From Eq. (1) $114.85(3) - B_x(2) = 0$ $B_x = 172.27\text{ N}$
From Eq. (2) $A_x = 172\text{ N}$ Ans
From Eq. (3) $A_y = 115\text{ N}$ Ans

$\Sigma F_x = 0;$ $C_x + 250\cos 60° - 172.27 = 0$ $C_x = 47.3\text{ N}$ Ans

$\Sigma F_y = 0;$ $250\cos 45° - 114.85 - C_y = 0$ $C_y = 61.9\text{ N}$ Ans

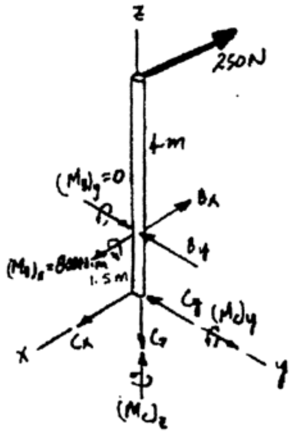
$\Sigma F_z = 0;$ $250\cos 60° - C_z = 0$ $C_z = 125\text{ N}$ Ans

$\Sigma M_y = 0;$ $M_{C\,y} - 172.27(1.5) + 250\cos 60°(5.5) = 0$

$M_{C\,y} = -429\text{ N}\cdot\text{m}$ Ans

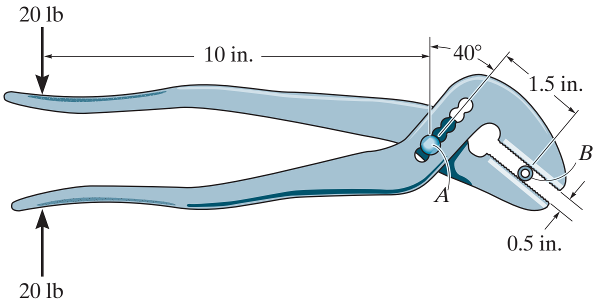
$\Sigma M_z = 0;$ $M_{C\,z} = 0$ Ans

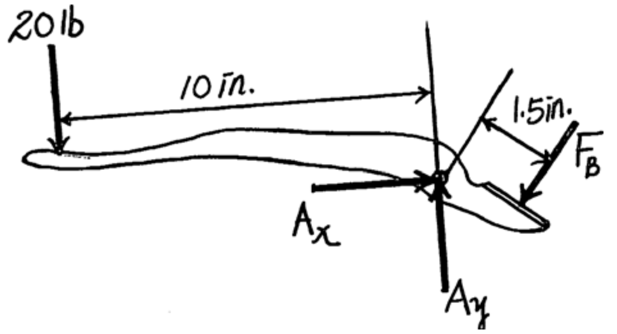
Negative sign indicates that $M_{C\,y}$ acts in the opposite sense to that shown on FBD.



(b)

6–127. Determine the clamping force exerted on the smooth pipe at *B* if a force of 20 lb is applied to the handles of the pliers. The pliers are pinned together at *A*.

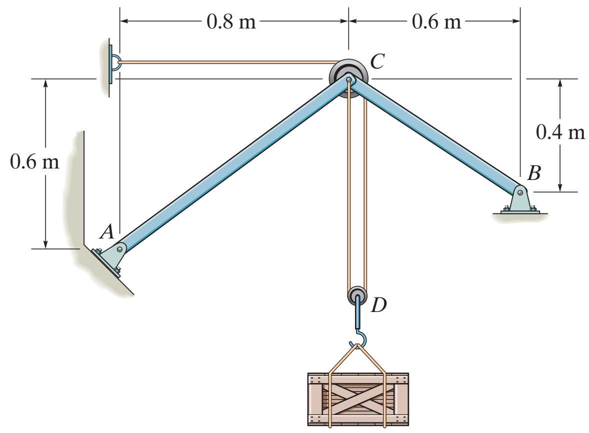




$\circlearrowleft + \Sigma M_A = 0; \quad 20(10) - 1.5(F_B) = 0$

$F_B = 133 \text{ lb}$ **Ans**

***6–128.** Determine the forces which the pins at *A* and *B* exert on the two-member frame which supports the 100-kg crate.



AC and BC are two - force members.

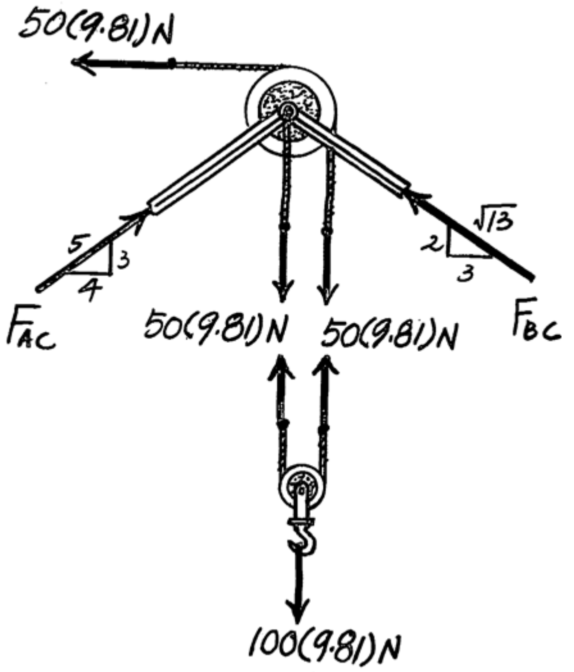
Pin C :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AC}\left(\frac{4}{5}\right) - F_{BC}\left(\frac{3}{\sqrt{13}}\right) - 50(9.81) = 0$$

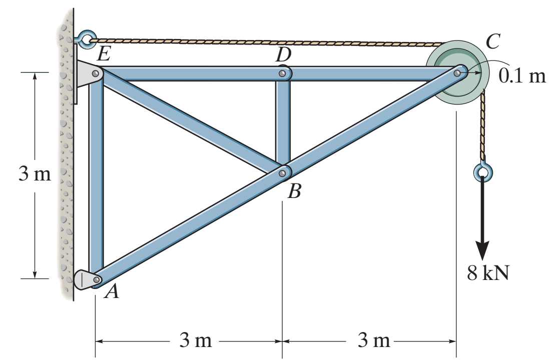
$$+\uparrow \Sigma F_y = 0; \quad F_{AC}\left(\frac{3}{5}\right) + F_{BC}\left(\frac{2}{\sqrt{13}}\right) - 100(9.81) = 0$$

$F_{AC} = 1154\text{ N} = 1.15\text{ kN}$ **Ans**

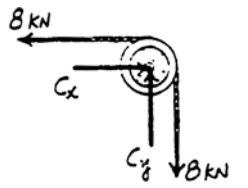
$F_{BC} = 520\text{ N}$ **Ans**



•6–129. Determine the force in each member of the truss and state if the members are in tension or compression.

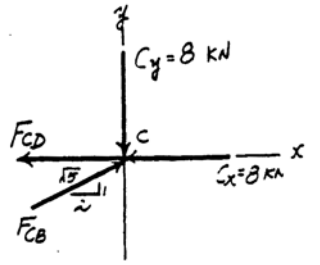


Method of Joint : In this case, support reactions are not required for determining the member forces. By inspection, members *DB* and *BE* are zero force members. Hence



$$F_{DB} = F_{BE} = 0 \qquad \textbf{Ans}$$

Joint *C*



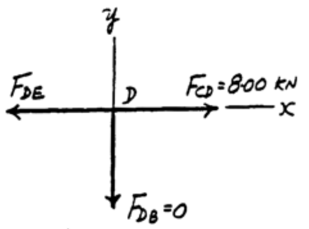
$$+\uparrow \Sigma F_y = 0; \qquad F_{CB}\left(\frac{1}{\sqrt{5}}\right) - 8 = 0$$

$$F_{CB} = 17.89 \text{ kN (C)} = 17.9 \text{ kN (C)} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 17.89\left(\frac{2}{\sqrt{5}}\right) - 8 - F_{CD} = 0$$

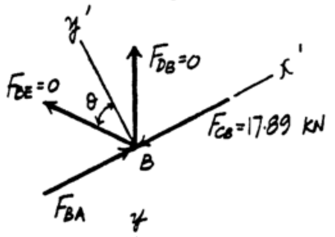
$$F_{CD} = 8.00 \text{ kN (T)} \qquad \textbf{Ans}$$

Joint *D*



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 8.00 - F_{DE} = 0 \qquad F_{DE} = 8.00 \text{ kN (T)} \qquad \textbf{Ans}$$

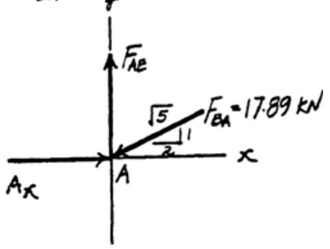
Joint *B*



$$+ \; \Sigma F_{x'} = 0; \qquad F_{BA} - 17.89 = 0$$

$$F_{BA} = 17.89 \text{ kN (C)} = 17.9 \text{ kN (C)} \qquad \textbf{Ans}$$

Joint *A*



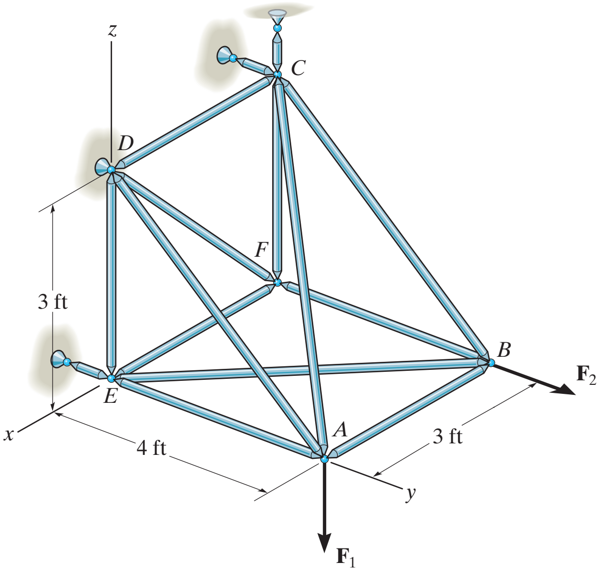
$$+\uparrow \Sigma F_y = 0; \qquad F_{AE} - 17.89\left(\frac{1}{\sqrt{5}}\right) = 0$$

$$F_{AE} = 8.00 \text{ kN (T)} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 17.89\left(\frac{2}{\sqrt{5}}\right) = 0 \qquad A_x = 16.0 \text{ kN}$$

Note : The support reactions E_x and E_y can be determined by analyzing Joint *E* using the results obtained above.

6–130. The space truss is supported by a ball-and-socket joint at *D* and short links at *C* and *E*. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{-500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



$\Sigma M_x = 0;\quad -C_y(3) - 400(3) = 0$

$C_y = -400\text{ lb}$

$\Sigma F_z = 0;\quad D_z = 0$

$\Sigma M_y = 0;\quad C_z = 0$

Joint *F*: $\Sigma F_y = 0;\quad F_{BF} = 0$ **Ans**

Joint *B*:

$\Sigma F_z = 0;\quad F_{BC} = 0$ **Ans**

$\Sigma F_y = 0;\quad 400 - \frac{4}{5}F_{BE} = 0$

$F_{BE} = 500\text{ lb (T)}$ **Ans**

$\Sigma F_x = 0;\quad F_{AB} - \frac{3}{5}(500) = 0$

$F_{AB} = 300\text{ lb (C)}$ **Ans**

Joint *A*:

$\Sigma F_x = 0;\quad 300 - \frac{3}{\sqrt{34}}F_{AC} = 0$

$F_{AC} = 583.1 = 583\text{ lb (T)}$ **Ans**

$\Sigma F_z = 0;\quad \frac{3}{\sqrt{34}}(583.1) - 500 + \frac{3}{5}F_{AD} = 0$

$F_{AD} = 333\text{ lb (T)}$ **Ans**

$\Sigma F_y = 0;\quad F_{AE} - \frac{4}{5}(333.3) - \frac{4}{\sqrt{34}}(583.1) = 0$

$F_{AE} = 667\text{ lb (C)}$ **Ans**

Joint *E*:

$\Sigma F_z = 0;\quad F_{DE} = 0$ **Ans**

$\Sigma F_x = 0;\quad F_{EF} - \frac{3}{5}(500) = 0$

$F_{EF} = 300\text{ lb (C)}$ **Ans**

Joint *C*:

$\Sigma F_x = 0;\quad \frac{3}{\sqrt{34}}(583.1) - F_{CD} = 0$

$F_{CD} = 300\text{ lb (C)}$ **Ans**

$\Sigma F_z = 0;\quad F_{CF} - \frac{3}{\sqrt{34}}(583.1) = 0$

$F_{CF} = 300\text{ lb (C)}$ **Ans**

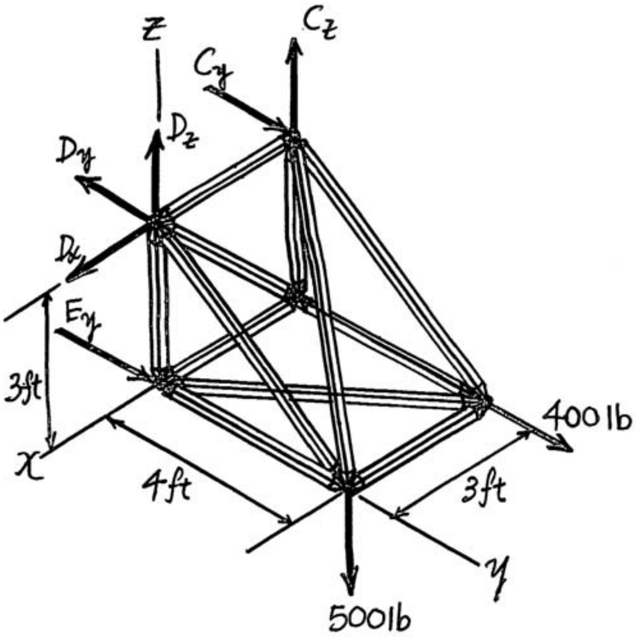
$\Sigma F_y = 0;\quad \frac{4}{\sqrt{34}}(583.1) - 400 = 0$ **Check!**

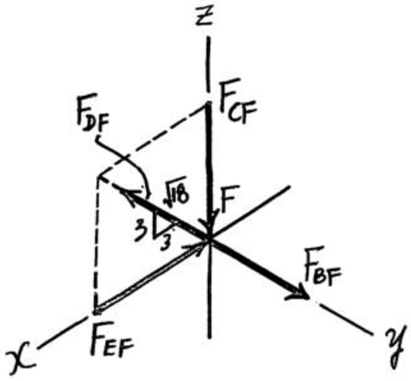
Joint *F*:

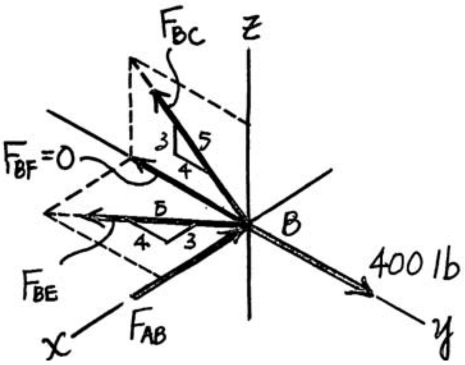
$\Sigma F_x = 0;\quad \frac{3}{\sqrt{18}}F_{DF} - 300 = 0$

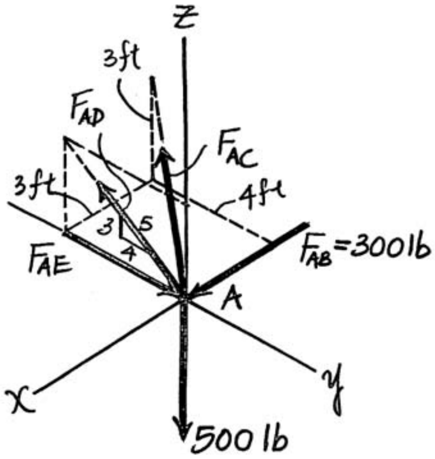
$F_{DF} = 424\text{ lb (T)}$ **Ans**

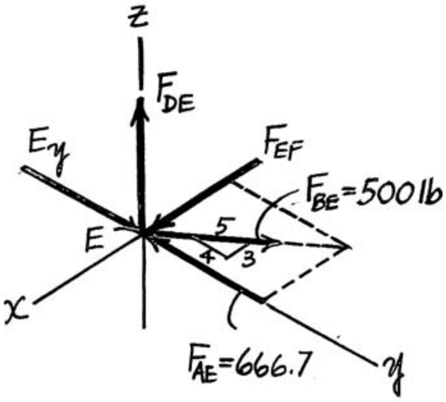
$\Sigma F_z = 0;\quad \frac{3}{\sqrt{18}}(424.3) - 300 = 0$ **Check!**

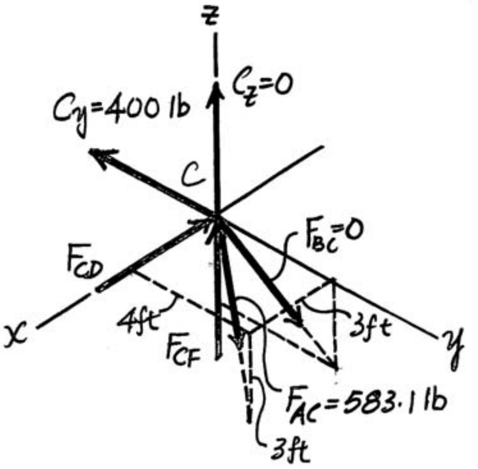

z
Cz
Cy
Dz
Dy
Dx
Ey
3ft
x
4ft
3ft
400 lb
y
500 lb


z
FCF
FDF
√18
F
3
3
FBF
x
FEF
y

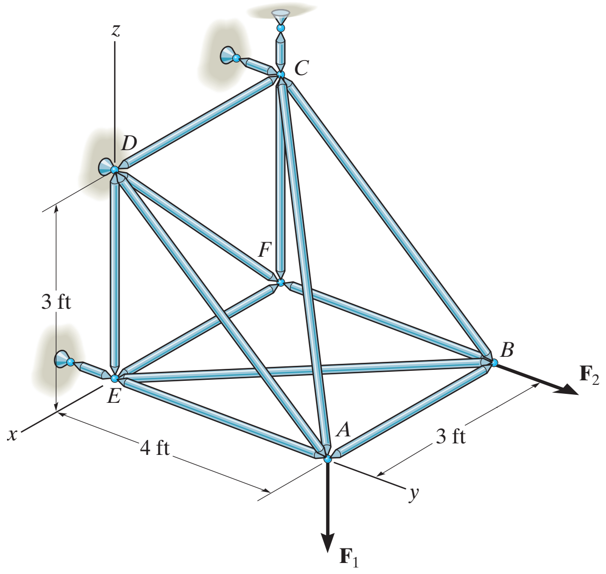

FBC
z
3
5
4
FBF=0
5
4
3
B
400 lb
FBE
x
FAB
y


z
3ft
FAD
FAC
3ft
4ft
3
5
4
FAE
FAB=300 lb
A
x
y
500 lb


z
FDE
Ey
FEF
FBE=500 lb
5
E
4
3
x
FAE=666.7
y


z
Cz=0
Cy=400 lb
C
FBC=0
FCD
4ft
3ft
x
FCF
y
FAC=583.1 lb
3ft

6–131. The space truss is supported by a ball-and-socket joint at *D* and short links at *C* and *E*. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{200\mathbf{i} + 300\mathbf{j} - 500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



$\Sigma F_x = 0;$ $D_x + 200 = 0$

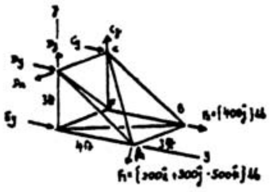
$D_x = -200$ lb

$\Sigma M_z = 0;$ $-C_y(3) - 400(3) - 200(4) = 0$

$C_y = -666.7$ lb

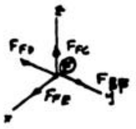
$\Sigma M_y = 0;$ $C_z(3) - 200(3) = 0$

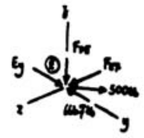
$C_z = 200$ lb



Joint *F* :

$\Sigma F_y = 0;$ $F_{BF} = 0$ Ans



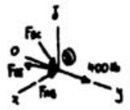


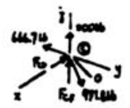
Joint *B* :

$\Sigma F_z = 0;$ $F_{BC} = 0$ Ans

$\Sigma F_y = 0;$ $400 - \frac{4}{5}F_{BE} = 0$

$F_{BE} = 500$ lb (T) Ans





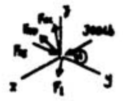
$\Sigma F_x = 0;$ $F_{AB} - \frac{3}{5}(500) = 0$

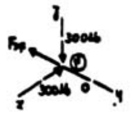
$F_{AB} = 300$ lb (C) Ans

Joint *A* :

$\Sigma F_x = 0;$ $300 + 200 - \frac{3}{\sqrt{34}}F_{AC} = 0$

$F_{AC} = 971.8 = 972$ lb (T) Ans





$\Sigma F_z = 0;$ $\frac{3}{\sqrt{34}}(971.8) - 500 + \frac{3}{5}F_{AD} = 0$

$F_{AD} = 0$ Ans

$\Sigma F_y = 0;$ $F_{AE} + 300 - \frac{4}{\sqrt{34}}(971.8) = 0$

$F_{AE} = 367$ lb (C) Ans

Joint *E* :

$\Sigma F_z = 0;$ $F_{DE} = 0$ Ans

$\Sigma F_x = 0;$ $F_{EF} - \frac{3}{5}(500) = 0$

$F_{EF} = 300$ lb (C) Ans

Joint *C* :

$\Sigma F_x = 0;$ $\frac{3}{\sqrt{34}}(971.8) - F_{CD} = 0$

$F_{CD} = 500$ lb (C) Ans

$\Sigma F_z = 0;$ $F_{CF} - \frac{3}{\sqrt{34}}(971.8) + 200 = 0$

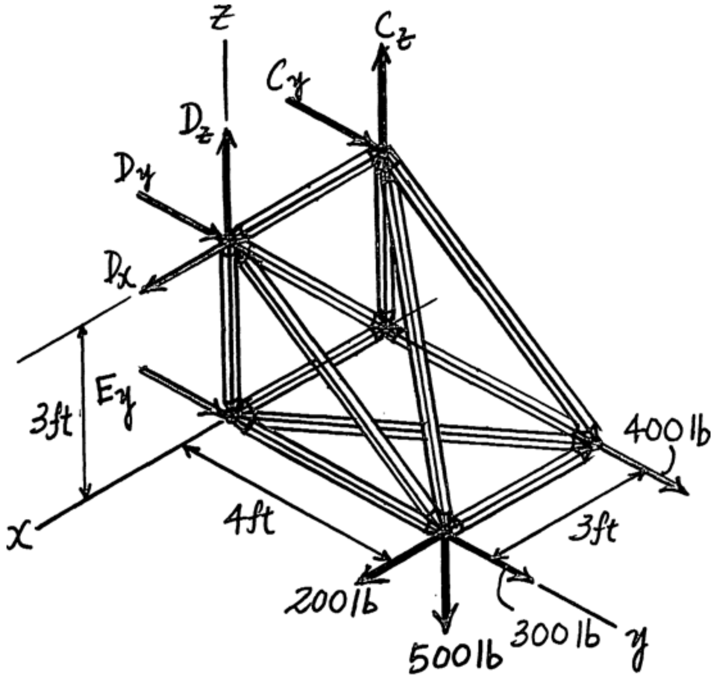
$F_{CF} = 300$ lb (C) Ans

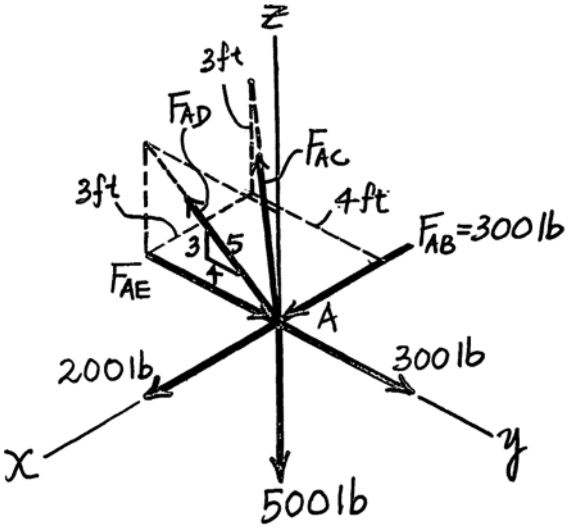
$\Sigma F_y = 0;$ $\frac{4}{\sqrt{34}}(971.8) - 666.7 = 0$ Check!

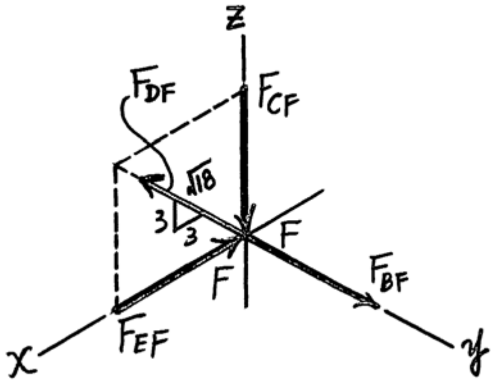
Joint *F* :

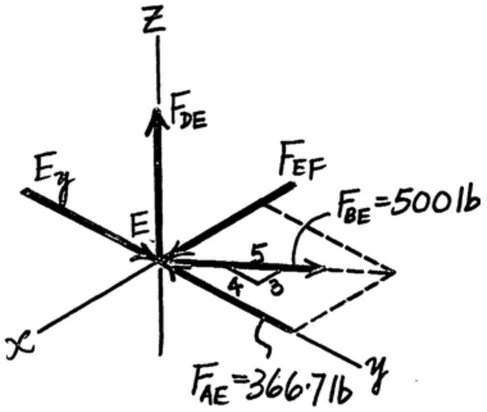
$\Sigma F_x = 0;$ $\frac{3}{\sqrt{18}}F_{DF} - 300 = 0$

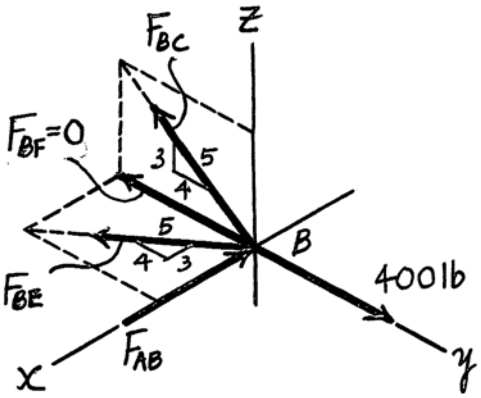
$F_{DF} = 424$ lb (T) Ans

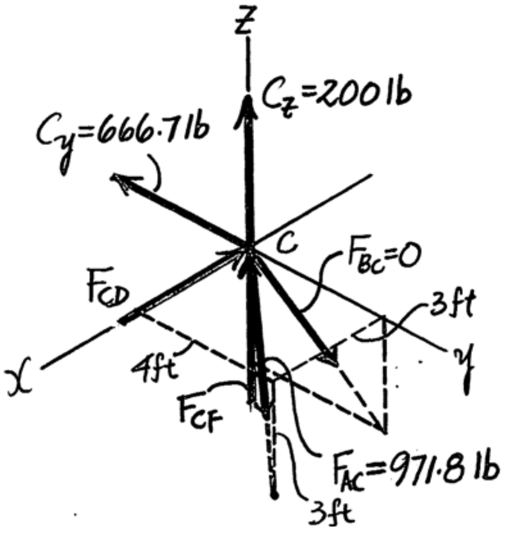

z
C_z
C_y
D_z
D_y
D_x
3ft
E_y
400 lb
x
4ft
3ft
200 lb
500 lb
300 lb
y


z
3ft
F_{AD}
F_{AC}
3ft
4ft
3
5
4
F_{AB}=300 lb
F_{AE}
A
200 lb
300 lb
x
y
500 lb

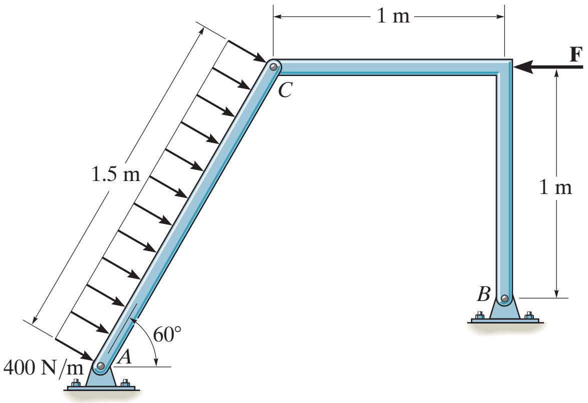

z
F_{DF}
F_{CF}
$\sqrt{18}$
3
3
F
F
F_{BF}
x
F_{EF}
y


z
F_{DE}
E_y
F_{EF}
F_{BE}=500 lb
E
5
4
3
x
y
F_{AE}=366.7 lb


F_{BC}
z
F_{BF}=0
3
5
4
5
4
3
B
F_{BE}
400 lb
F_{AB}
x
y


z
C_z=200 lb
C_y=666.7 lb
C
F_{BC}=0
F_{CD}
3ft
x
4ft
y
F_{CF}
F_{AC}=971.8 lb
3ft

***6–132.** Determine the horizontal and vertical components of reaction that the pins *A* and *B* exert on the two-member frame. Set $F = 0$.



CB is a two - force member.

Member *AC* :

$\circlearrowleft + \Sigma M_A = 0; \qquad -600\,(0.75) + 1.5\,(F_{CB} \sin 75°) = 0$

$F_{CB} = 310.6$

Thus,

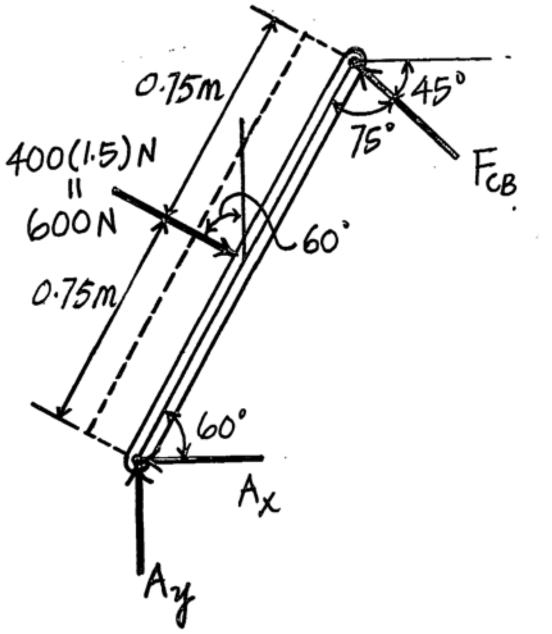
$B_x = B_y = 310.6\left(\frac{1}{\sqrt{2}}\right) = 220\text{ N} \qquad$ **Ans**

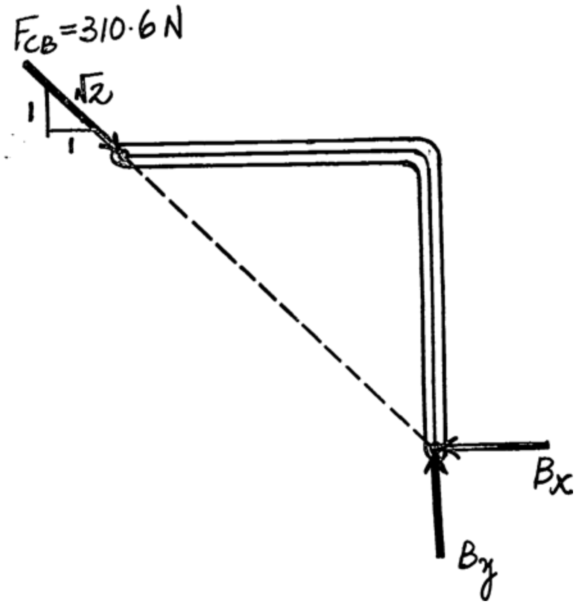
$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + 600 \sin 60° - 310.6 \cos 45° = 0$

$A_x = 300\text{ N} \qquad$ **Ans**

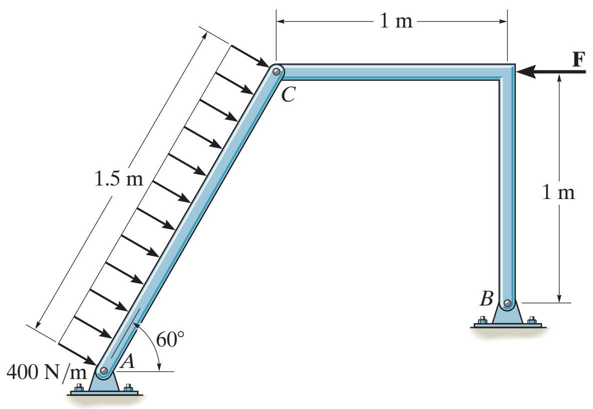
$+\uparrow \Sigma F_y = 0; \qquad A_y - 600 \cos 60° + 310.6 \sin 45° = 0$

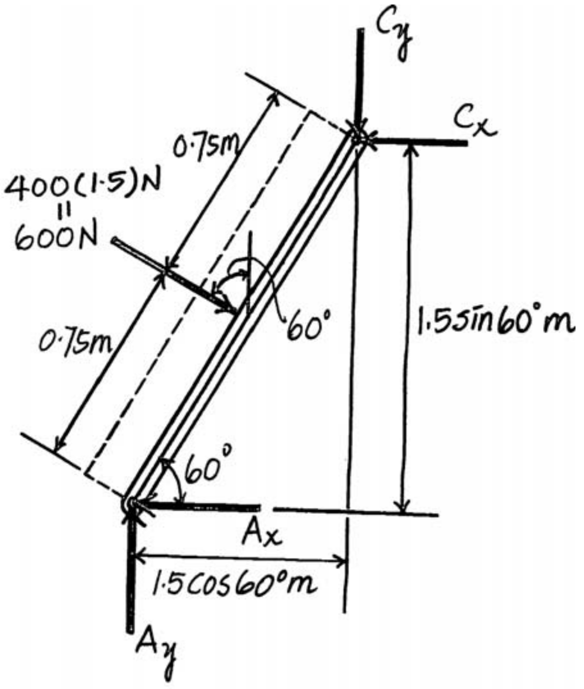
$A_y = 80.4\text{ N} \qquad$ **Ans**





•6–133. Determine the horizontal and vertical components of reaction that pins A and B exert on the two-member frame. Set $F = 500$ N.





Member AC:

$\circlearrowleft+\Sigma M_A = 0;\quad -600\,(0.75) - C_y\,(1.5\cos 60°) + C_x\,(1.5\sin 60°) = 0$

Member CB:

$\circlearrowleft+\Sigma M_B = 0;\quad -C_x\,(1) - C_y\,(1) + 500\,(1) = 0$

Solving,

$C_x = 402.6\text{ N}$

$C_y = 97.4\text{ N}$

Member AC:

$\xrightarrow{+}\Sigma F_x = 0;\quad -A_x + 600\sin 60° - 402.6 = 0$

$A_x = 117\text{ N}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad A_y - 600\cos 60° - 97.4 = 0$

$A_y = 397\text{ N}$ **Ans**

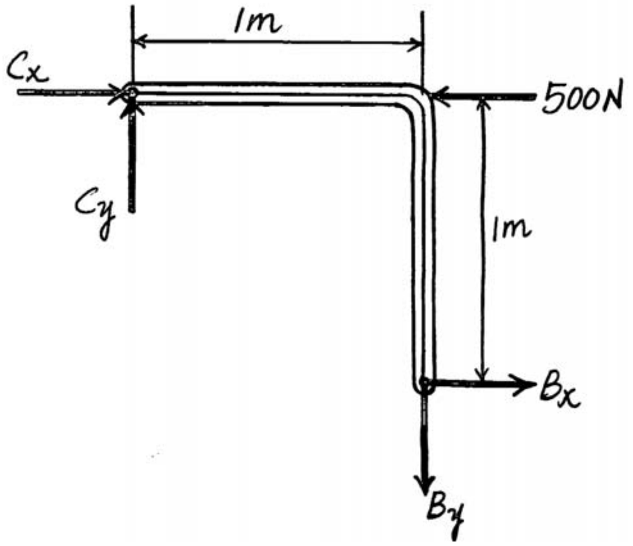
Member CB:

$\xrightarrow{+}\Sigma F_x = 0;\quad 402.6 - 500 + B_x = 0$

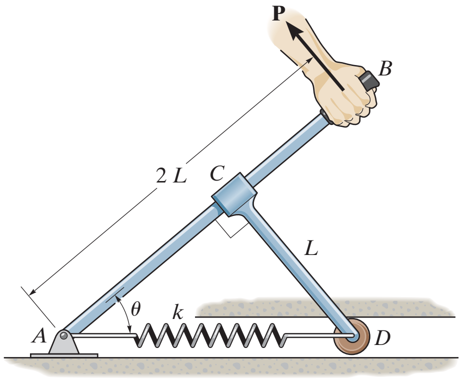
$B_x = 97.4\text{ N}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad -B_y + 97.4 = 0$

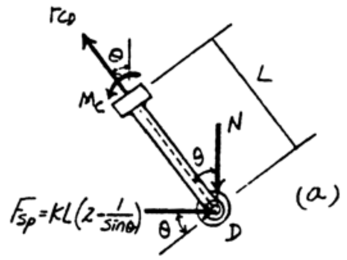
$B_y = 97.4\text{ N}$ **Ans**



6–134. The two-bar mechanism consists of a lever arm AB and smooth link CD, which has a fixed smooth collar at its end C and a roller at the other end D. Determine the force $\mathbf{P}$ needed to hold the lever in the position θ. The spring has a stiffness k and unstretched length $2L$. The roller contacts either the top or bottom portion of the horizontal guide.



Free Body Diagram : The spring compresses $x = 2L - \dfrac{L}{\sin\theta}$. Then, the spring force developed is $F_{sp} = kx = kL\left(2 - \dfrac{1}{\sin\theta}\right)$.



Equations of Equilibrium : From FBD (a),

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad kL\left(2 - \frac{1}{\sin\theta}\right) - F_{CD}\sin\theta = 0$$

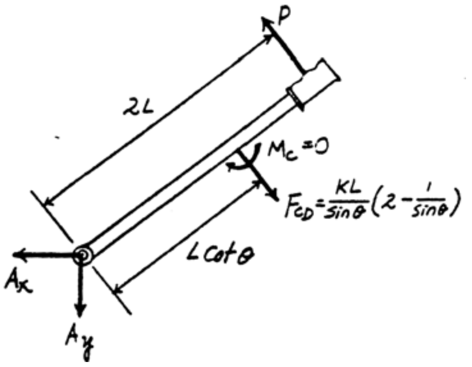
$$F_{CD} = \frac{kL}{\sin\theta}\left(2 - \frac{1}{\sin\theta}\right)$$

$$\circlearrowleft + \Sigma M_D = 0; \quad M_C = 0$$

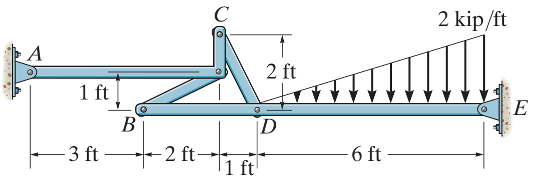
From FBD (b),

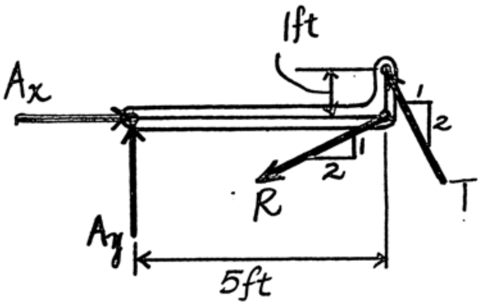
$$\circlearrowleft + \Sigma M_A = 0; \quad P(2L) - \frac{kL}{\sin\theta}\left(2 - \frac{1}{\sin\theta}\right)(L\cot\theta) = 0$$

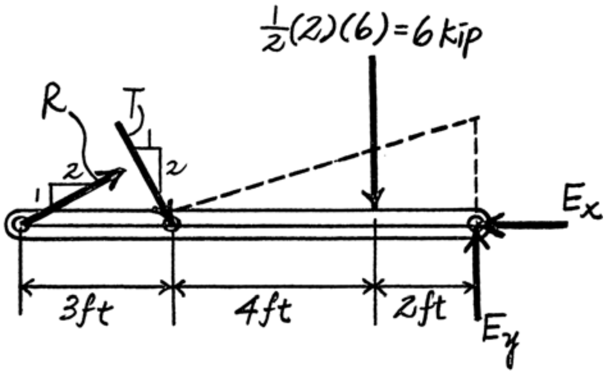
$$P = \frac{kL}{2\tan\theta\sin\theta}(2 - \csc\theta) \qquad \textbf{Ans}$$



6–135. Determine the horizontal and vertical components of reaction at the pin supports *A* and *E* of the compound beam assembly.







Member *BDE*:

$$\circlearrowleft + \Sigma M_E = 0; \qquad 6\,(2) + T\left(\frac{2}{\sqrt{5}}\right)(6) - R\left(\frac{1}{\sqrt{5}}\right)(9) = 0$$

Member *AC*:

$$\circlearrowleft + \Sigma M_A = 0; \qquad T\left(\frac{1}{\sqrt{5}}\right)(1) + T\left(\frac{2}{\sqrt{5}}\right)(5) - R\left(\frac{1}{\sqrt{5}}\right)(5) = 0$$

Solving,

$$T = 3.440 \text{ kip}, \qquad R = 7.568 \text{ kip}$$

Member *AC*:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 7.568\left(\frac{2}{\sqrt{5}}\right) - 3.440\left(\frac{1}{\sqrt{5}}\right) = 0$$

$$A_x = 8.31 \text{ kip} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 7.568\left(\frac{1}{\sqrt{5}}\right) + 3.440\left(\frac{2}{\sqrt{5}}\right) = 0$$

$$A_y = 0.308 \text{ kip} \qquad \textbf{Ans}$$

Member *BDE*:

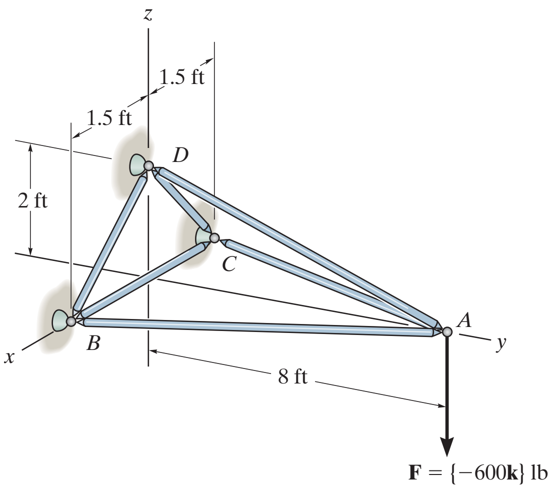
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 7.568\left(\frac{2}{\sqrt{5}}\right) + 3.440\left(\frac{1}{\sqrt{5}}\right) - E_x = 0$$

$$E_x = 8.31 \text{ kip} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad 7.568\left(\frac{1}{\sqrt{5}}\right) - 3.440\left(\frac{2}{\sqrt{5}}\right) - 6 + E_y = 0$$

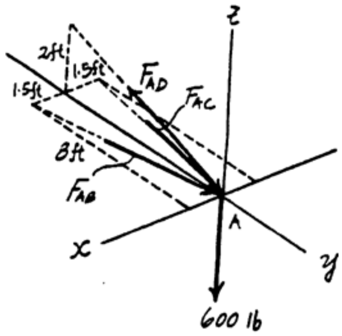
$$E_y = 5.69 \text{ kip} \qquad \textbf{Ans}$$

***6–136.** Determine the force in members *AB*, *AD*, and *AC* of the space truss and state if the members are in tension or compression.



Method of Joints : **In this case the support reactions are not required for determining the member forces.**

Joint *A*



$$\Sigma F_z = 0; \quad F_{AD}\left(\frac{2}{\sqrt{68}}\right) - 600 = 0$$

$$F_{AD} = 2473.86 \text{ lb (T)} = 2.47 \text{ kip (T)} \qquad \textbf{Ans}$$

$$\Sigma F_x = 0; \quad F_{AC}\left(\frac{1.5}{\sqrt{66.25}}\right) - F_{AB}\left(\frac{1.5}{\sqrt{66.25}}\right) = 0$$

$$F_{AC} = F_{AB} \qquad [1]$$

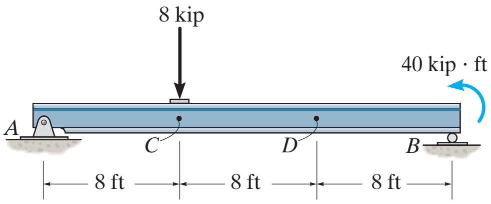
$$\Sigma F_y = 0; \quad F_{AC}\left(\frac{8}{\sqrt{66.25}}\right) + F_{AB}\left(\frac{8}{\sqrt{66.25}}\right) - 2473.86\left(\frac{8}{\sqrt{68}}\right) = 0$$

$$0.9829F_{AC} + 0.9829F_{AB} = 2400 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$F_{AC} = F_{AB} = 1220.91 \text{ lb (C)} = 1.22 \text{ kip (C)} \qquad \textbf{Ans}$$

•7–1. Determine the internal normal force and shear force, and the bending moment in the beam at points *C* and *D*. Assume the support at *B* is a roller. Point *C* is located just to the right of the 8-kip load.

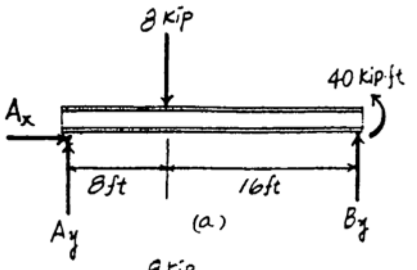


Support Reactions : **FBD (a).**

$\curvearrowleft + \Sigma M_A = 0;\quad B_y(24) + 40 - 8(8) = 0 \quad B_y = 1.00 \text{ kip}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 1.00 - 8 = 0 \quad A_y = 7.00 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x = 0$

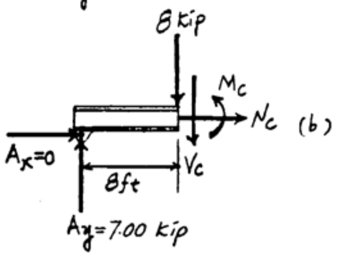


Internal Forces : Applying the equations of equilibrium to segment *AC* [FBD (b)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_C = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 7.00 - 8 - V_C = 0 \quad V_C = -1.00 \text{ kip}$ **Ans**

$\curvearrowleft + \Sigma M_C = 0;\quad M_C - 7.00(8) = 0 \quad M_C = 56.0 \text{ kip} \cdot \text{ft}$ **Ans**



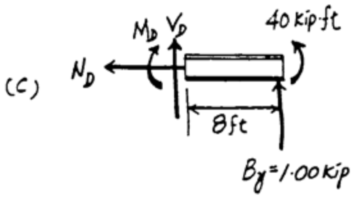
Applying the equations of equilibrium to segment *BD* [FBD (c)], we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans**

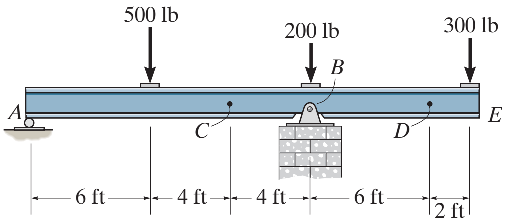
$+\uparrow \Sigma F_y = 0;\quad V_D + 1.00 = 0 \quad V_D = -1.00 \text{ kip}$ **Ans**

$\curvearrowleft + \Sigma M_D = 0;\quad 1.00(8) + 40 - M_D = 0$

$M_D = 48.0 \text{ kip} \cdot \text{ft}$ **Ans**



7–2. Determine the shear force and moment at points *C* and *D*.



Support Reactions : FBD (a).

$$\curvearrowleft + \Sigma M_B = 0; \quad 500(8) - 300(8) - A_y(14) = 0$$
$$A_y = 114.29 \text{ lb}$$

Internal Forces : Applying the equations of equilibrium to segment AC [FBD (b)], we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 114.29 - 500 - V_C = 0 \quad V_C = -386 \text{ lb} \qquad \textbf{Ans}$$

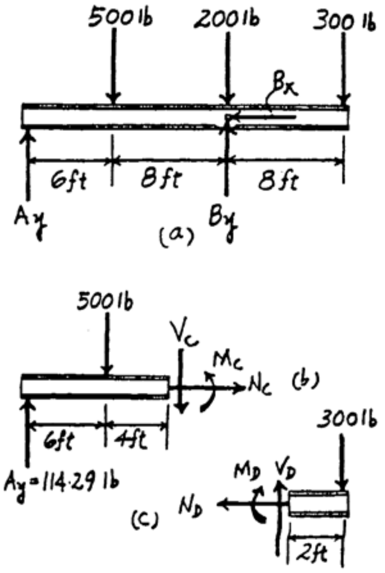
$$\curvearrowleft + \Sigma M_C = 0; \quad M_C + 500(4) - 114.29(10) = 0$$
$$M_C = -857 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

Applying the equations of equilibrium to segment ED [FBD (c)], we have

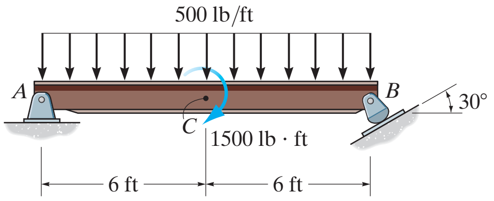
$$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_D - 300 = 0 \quad V_D = 300 \text{ lb} \qquad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M_D = 0; \quad -M_D - 300(2) = 0 \quad M_D = -600 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$



7–3. Determine the internal normal force, shear force, and moment at point C in the simply supported beam. Point C is located just to the right of the 1500-lb · ft couple moment.



Writing the moment equation of equilibrium about point A with reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $F_B \cos 30°(12) - 500(12)(6) - 1500 = 0$ $F_B = 3608.44 \text{ lb}$

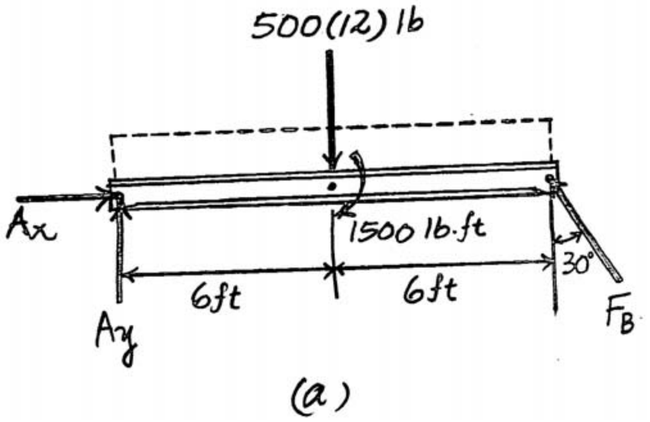
Using the result of $\mathbf{F}_B$ and referring to Fig. b,

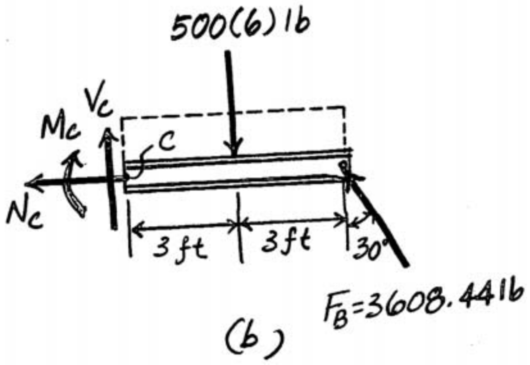
$\xrightarrow{+} \Sigma F_x = 0;$ $-N_C - 3608.44 \sin 30° = 0$ $N_C = -1804 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $V_C + 3608.44 \cos 30° - 500(6) = 0$ $V_C = -125 \text{ lb}$ **Ans.**

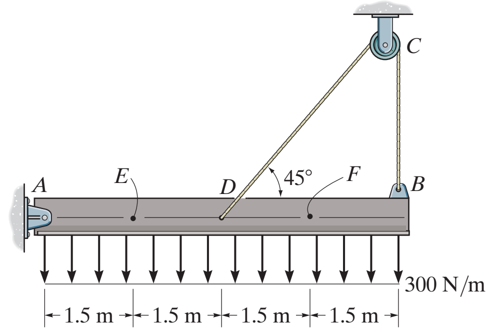
$\circlearrowleft + \Sigma M_C = 0;$ $3608.44 \cos 30°(6) - 500(6)(3) - M_C = 0$ $M_C = 9750 \text{ lb} \cdot \text{ft}$ **Ans.**

The negative sign indicates that $\mathbf{N}_C$ and $\mathbf{V}_C$ act in the opposite sense to that shown on the free-body diagram.





***7–4.** Determine the internal normal force, shear force, and moment at points *E* and *F* in the beam.



With reference to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad T(6) + T\sin 45°(3) - 300(6)(3) = 0$ $\quad T = 664.92\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 664.92\cos 45° - A_x = 0$ $\quad A_x = 470.17\text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 664.92\sin 45° + 664.92 - 300(6) = 0$ $\quad A_y = 664.92\text{ N}$

Using these results and referring to Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_E - 470.17 = 0$ $\quad N_E = 470\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 664.92 - 300(1.5) - V_E = 0$ $\quad V_E = 215\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad M_E + 300(1.5)(0.75) - 664.92(1.5) = 0$ $\quad M_E = 660\text{ N}\cdot\text{m}$ **Ans.**

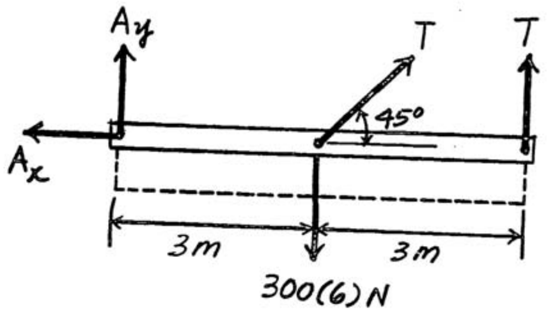
Also, by referring to Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_F = 0$ **Ans.**

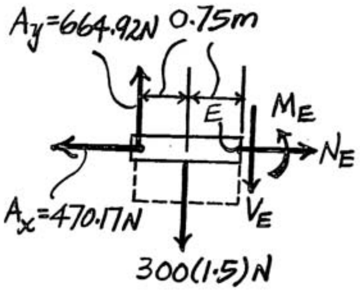
$+\uparrow \Sigma F_y = 0;$ $\quad V_F + 664.92 - 300(1.5) = 0$ $\quad V_F = -215\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0;$ $\quad 664.92(1.5) - 300(1.5)(0.75) - M_F = 0$ $\quad M_F = 660\text{ N}\cdot\text{m}$ **Ans.**

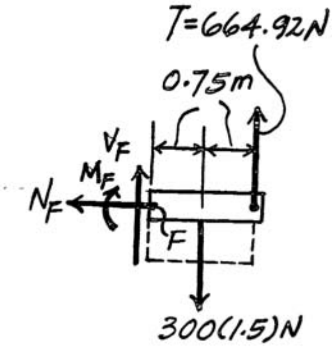
The negative sign indicates that $\mathbf{V}_F$ acts in the opposite sense to that shown on the free-body diagram.



(a)

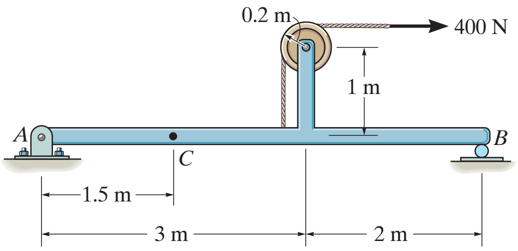


(b)



(c)

•7–5. Determine the internal normal force, shear force, and moment at point *C*.



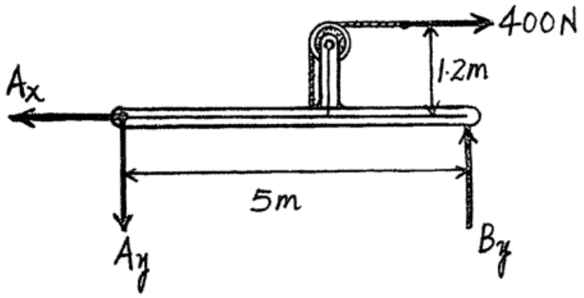
Beam :

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad -A_x + 400 = 0$

$A_x = 400\text{ N}$

$\circlearrowleft + \Sigma M_B = 0; \quad A_y(5) - 400(1.2) = 0$

$A_y = 96\text{ N}$



Segment ***AC*** **:**

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad N_C - 400 = 0$

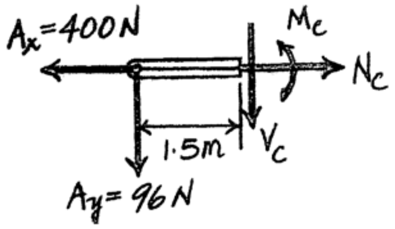
$N_C = 400\text{ N}$ **Ans**

$+\uparrow\Sigma F_y = 0; \quad -96 - V_C = 0$

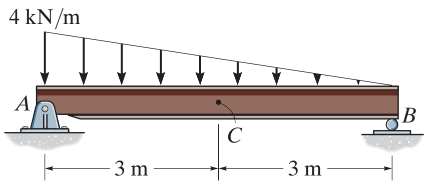
$V_C = -96\text{ N}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 96(1.5) = 0$

$M_C = -144\text{ N}\cdot\text{m}$ **Ans**



7–6. Determine the internal normal force, shear force, and moment at point *C* in the simply supported beam.



With reference to Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0;$ $\quad B_y(6) - \frac{1}{2}(4)(6)(2) = 0$ $\quad B_y = 4\,\text{kN}$

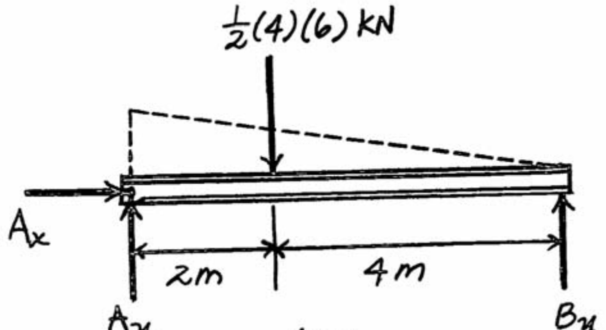
Using this result with reference to Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_C = 0$ **Ans.**

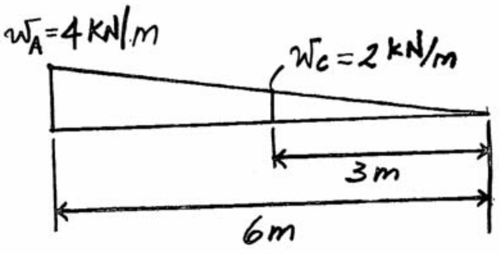
$+\uparrow \Sigma F_y = 0;$ $\quad 4 - \frac{1}{2}(2)(3) + V_C = 0$ $\quad V_C = -1\,\text{kN}$ **Ans.**

$\curvearrowleft + \Sigma M_C = 0;$ $\quad 4(3) - \frac{1}{2}(2)(3)(1) - M_C = 0$ $\quad M_C = 9\,\text{kN}\cdot\text{m}$ **Ans.**

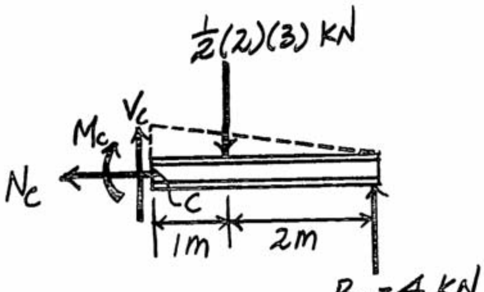
The negative sign indicates that $\mathbf{V}_C$ acts in the opposite sense to that shown on the free-body diagram.



(a)

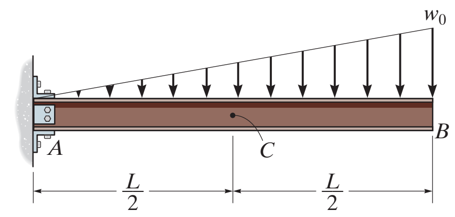


(b)



(c)

7–7. Determine the internal normal force, shear force, and moment at point C in the cantilever beam.



The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. a,

$$\frac{w_C}{L/2} = \frac{w_0}{L} \text{ or } w_C = w_0/2$$

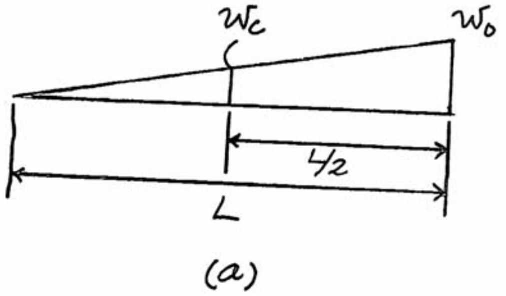
With reference to Fig. b,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_C = 0$ **Ans.**

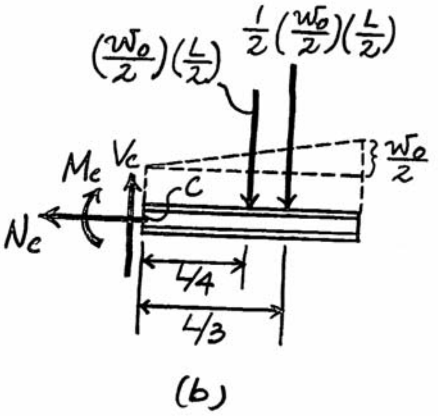
$+\uparrow \Sigma F_y = 0;$ $\quad V_C - \left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right) - \frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right) = 0 \quad V_C = \frac{3w_0L}{8}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -M_C - \left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) - \frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = 0 \quad M_C = -\frac{5}{48} w_0L^2$ **Ans.**

The negative sign indicates that $\mathbf{M}_C$ acts in the opposite sense to that shown on the free-body diagram.



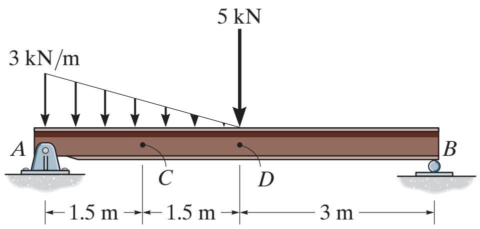
(a)



$\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)$
$\frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)$
M_C
V_C
C
N_C
$\frac{w_0}{2}$
L/4
L/3

(b)

***7–8.** Determine the internal normal force, shear force, and moment at points *C* and *D* in the simply supported beam. Point *D* is located just to the left of the 5-kN force.



The intensity of the triangular distributed loading at *C* can be computed using the similar triangles shown in Fig. *b*,

$$\frac{w_C}{1.5} = \frac{3}{3} \quad \text{or} \quad w_C = 1.5\ \text{kN/m}$$

With reference to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\quad B_y(6) - 5(3) - \frac{1}{2}(3)(3)(1) = 0 \qquad B_y = 3.25\ \text{kN}$

Using this result and referring to Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_C + 3.25 - \frac{1}{2}(1.5)(1.5) - 5 = 0 \qquad V_C = 2.875\ \text{kN}$ **Ans.**

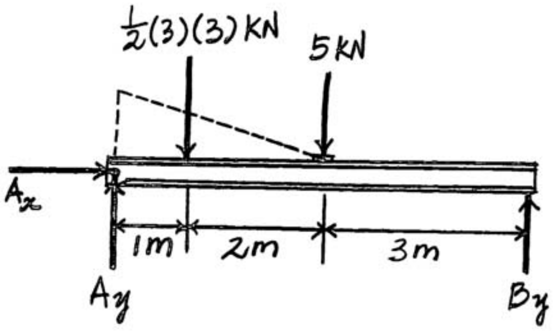
$\circlearrowleft + \Sigma M_C = 0;\quad 3.25(4.5) - \frac{1}{2}(1.5)(1.5)(0.5) - 5(1.5) - M_C = 0 \qquad M_C = 6.56\ \text{kN}\cdot\text{m}$ **Ans.**

Also, referring to Fig. *d*,

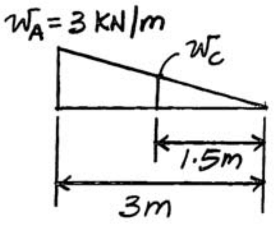
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_D + 3.25 - 5 = 0 \qquad V_D = 1.75\ \text{kN}$ **Ans.**

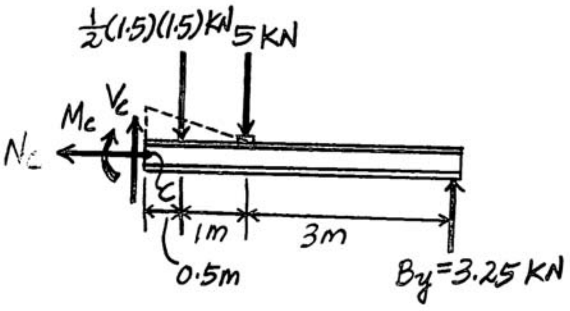
$\circlearrowleft + \Sigma M_D = 0;\quad 3.25(3) - M_D \qquad M_D = 9.75\ \text{kN}\cdot\text{m}$ **Ans.**



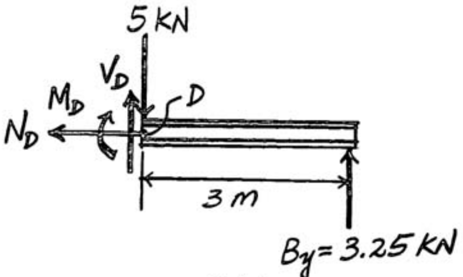
(a)



(b)

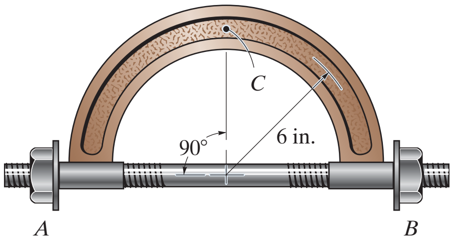


(c)



(d)

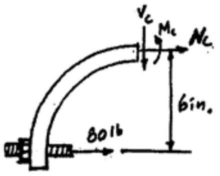
•7–9. The bolt shank is subjected to a tension of 80 lb. Determine the internal normal force, shear force, and moment at point *C*.



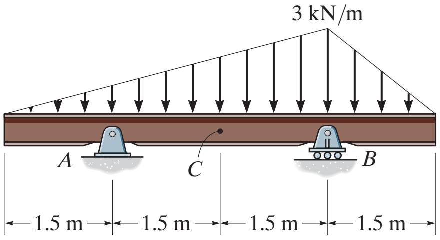
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_C + 80 = 0\quad N_C = -80\text{ lb}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad V_C = 0$ **Ans**

$\circlearrowleft+\Sigma M_C = 0;\quad M_C + 80(6) = 0\quad M_C = -480\text{ lb}\cdot\text{in.}$ **Ans**



7–10. Determine the internal normal force, shear force, and moment at point *C* in the double-overhang beam.



The intensity of the triangular distributed loading at *C* can be computed using the similar triangles shown in Fig. *b*,

$$\frac{w_C}{3} = \frac{3}{4.5} \text{ or } w_C = 2\text{ kN/m}$$

With reference to Fig. *a*,

$\curvearrowleft + \Sigma M_B = 0;$ $\quad \frac{1}{2}(3)(4.5)(1.5) - \frac{1}{2}(3)(1.5)(0.5) - A_y(3) = 0 \qquad A_y = 3\text{ kN}$

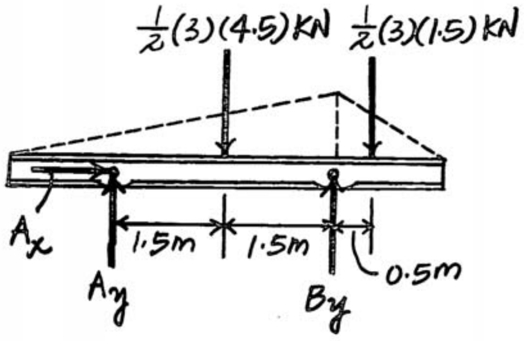
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 0$

Using the results of $\mathbf{A}_x$ and $\mathbf{A}_y$ and referring to Fig. *c*,

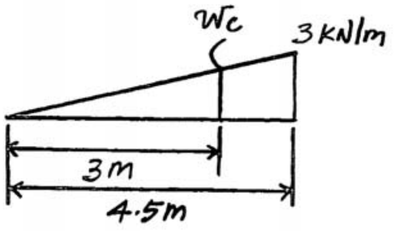
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 3 - \frac{1}{2}(2)(3) - V_C = 0 \qquad V_C = 0$ **Ans.**

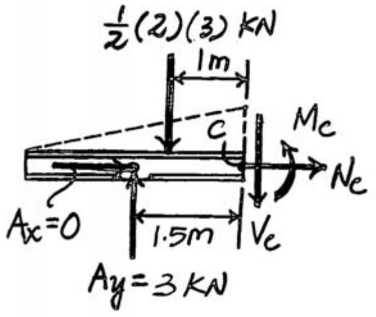
$\curvearrowleft + \Sigma M_C = 0;$ $\quad M_C + \frac{1}{2}(2)(3)(1) - 3(1.5) = 0 \qquad M_C = 1.5\text{ kN}\cdot\text{m}$ **Ans.**



(a)

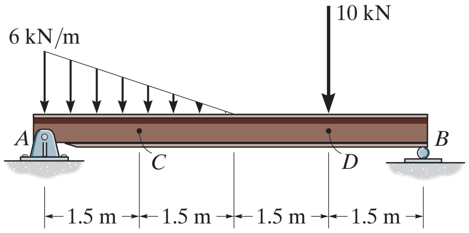


(b)



(c)

7–11. Determine the internal normal force, shear force, and moment at points *C* and *D* in the simply supported beam. Point *D* is located just to the left of the 10-kN concentrated load.



The intensity of the triangular distributed loading at *C* can be computed using the similar triangles shown in Fig. *b*,

$$\frac{w_C}{1.5} = \frac{6}{3} \text{ or } w_C = 3\text{kN / m}$$

With reference to Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0;$ $\quad B_y(6) - 10(4.5) - \frac{1}{2}(6)(3)(1) = 0$ $\quad B_y = 9\text{ kN}$

$\curvearrowleft + \Sigma M_B = 0;$ $\quad \frac{1}{2}(6)(3)(5) + 10(1.5) - A_y(6) = 0$ $\quad A_y = 10\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 0$

Using these results and refering to Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 10 - \frac{1}{2}(3)(1.5) - 3(1.5) - V_C = 0$ $\quad V_C = 3.25\text{ kN}$ **Ans.**

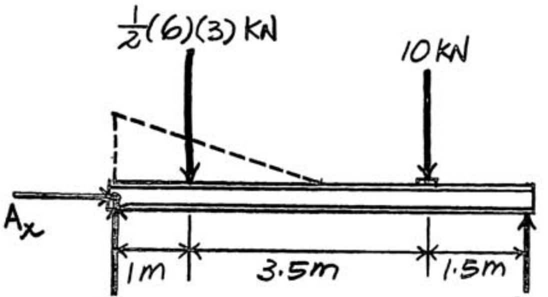
$\curvearrowleft + \Sigma M_C = 0;$ $\quad M_C + 3(1.5)(0.75) + \frac{1}{2}(3)(1.5)(1) - 10(1.5) = 0$ $\quad M_C = 9.375\text{ kN}\cdot\text{m}$ **Ans.**

Also, by referring to Fig. *d*,

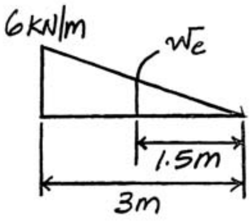
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_D + 9 - 10 = 0$ $\quad V_D = 1\text{ kN}$ **Ans.**

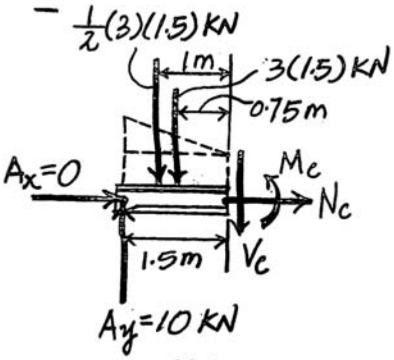
$\curvearrowleft + \Sigma M_D = 0;$ $\quad 9(1.5) - M_D = 0$ $\quad M_D = 13.5\text{ kN}\cdot\text{m}$ **Ans.**



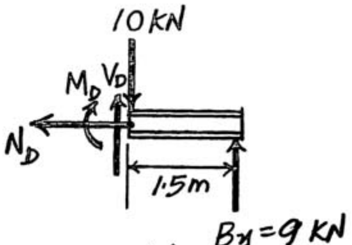
(a)



(b)

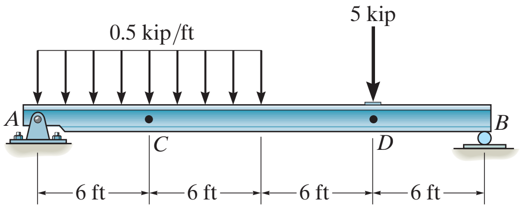


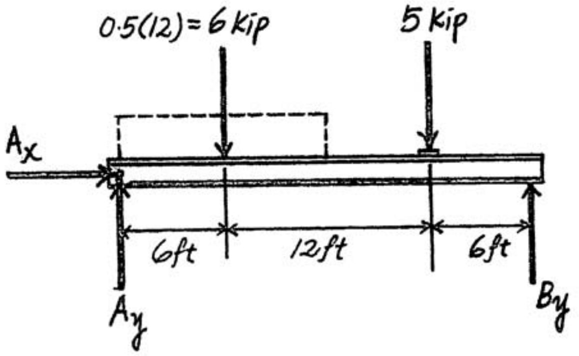
(C)



(d)

***7–12.** Determine the internal normal force, shear force, and moment in the beam at points *C* and *D*. Point *D* is just to the right of the 5-kip load.



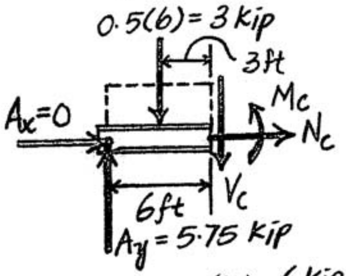


Entire beam :

$\circlearrowleft + \Sigma M_B = 0;$ $5(6) + 6(18) - A_y(24) = 0$

$A_y = 5.75$ kip

$\xrightarrow{+} \Sigma F_x = 0;$ $A_x = 0$



Segment *AC* :

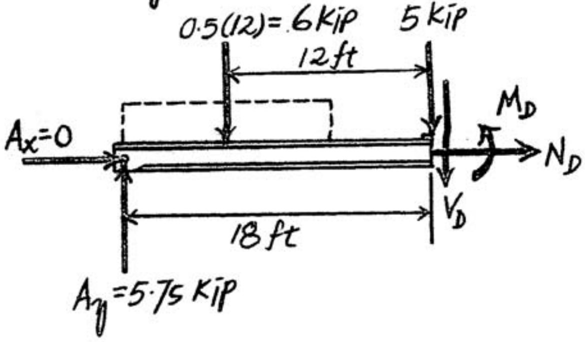
$\xrightarrow{+} \Sigma F_x = 0;$ $N_C = 0$ Ans

$+\uparrow \Sigma F_y = 0;$ $5.75 - 3 - V_C = 0$

$V_C = 2.75$ kip Ans

$\circlearrowleft + \Sigma M_C = 0;$ $M_C + 3(3) - 5.75(6) = 0$

$M_C = 25.5$ kip · ft Ans



Segment *AD* :

$\xrightarrow{+} \Sigma F_x = 0;$ $N_D = 0$ Ans

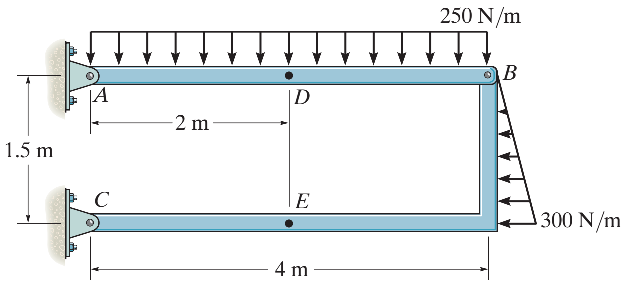
$+\uparrow \Sigma F_y = 0;$ $5.75 - 6 - 5 - V_D = 0$

$V_D = -5.25$ kip Ans

$\circlearrowleft + \Sigma M_D = 0;$ $M_D + 6(12) - 5.75(18) = 0$

$M_D = 31.5$ kip · ft Ans

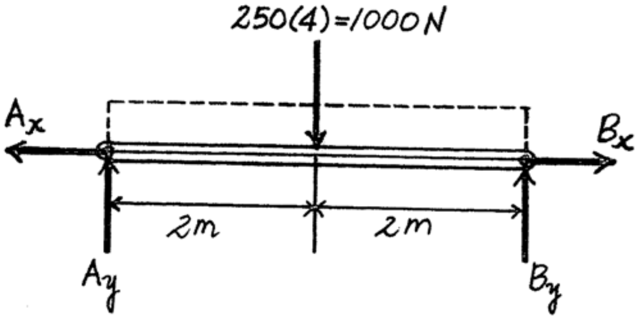
•7–13. Determine the internal normal force, shear force, and moment at point *D* of the two-member frame.



Member *AB* :

$\circlearrowleft + \Sigma M_A = 0;\qquad B_y\,(4) - 1000\,(2) = 0$

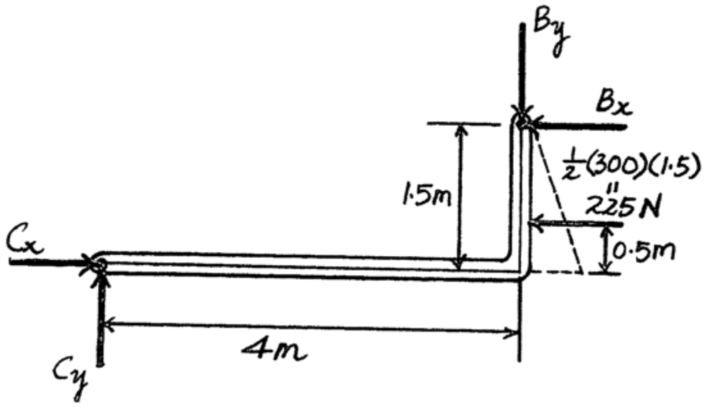
$B_y = 500\text{ N}$



Member *BC* :

$\circlearrowleft + \Sigma M_C = 0;\qquad -500\,(4) + 225\,(0.5) + B_x\,(1.5) = 0$

$B_x = 1258.33\text{ N}$



Segment *DB* :

$\xrightarrow{+} \Sigma F_x = 0;\qquad -N_D + 1258.33 = 0$

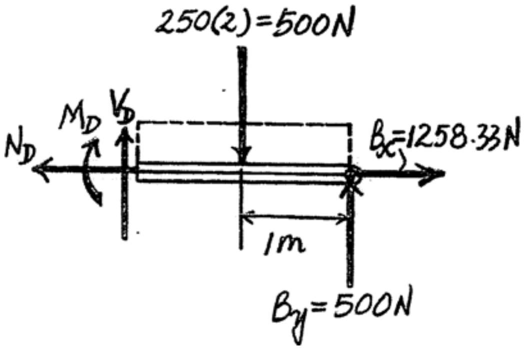
$N_D = 1.26\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\qquad V_D - 500 + 500 = 0$

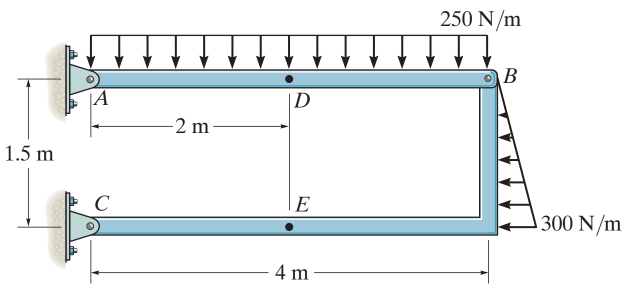
$V_D = 0$ **Ans**

$\circlearrowleft + \Sigma M_D = 0;\qquad -M_D + 500\,(1) = 0$

$M_D = 500\text{ N}\cdot\text{m}$ **Ans**



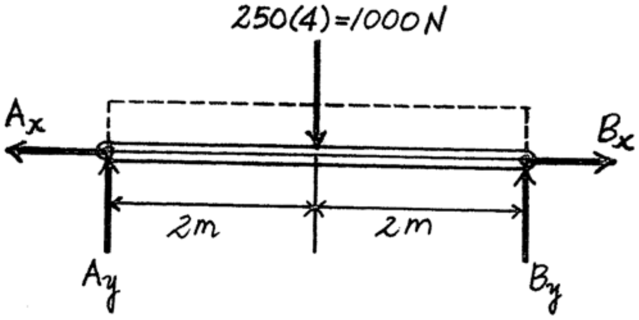
7–14. Determine the internal normal force, shear force, and moment at point *E* of the two-member frame.



Member *AB* :

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(4) - 1000(2) = 0$

$B_y = 500\text{ N}$



Member *BC* :

$\circlearrowleft + \Sigma M_C = 0; \quad -500(4) + 225(0.5) + B_x(1.5) = 0$

$B_x = 1258.33\text{ N}$

Segment *EB* :

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_E - 1258.33 - 225 = 0$

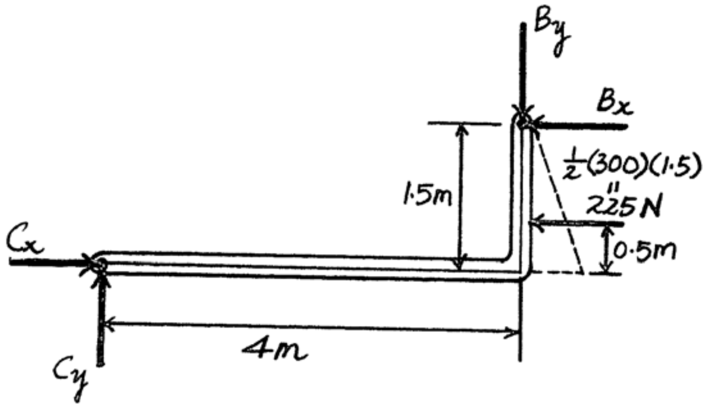
$N_E = -1.48\text{ kN}$ **Ans**

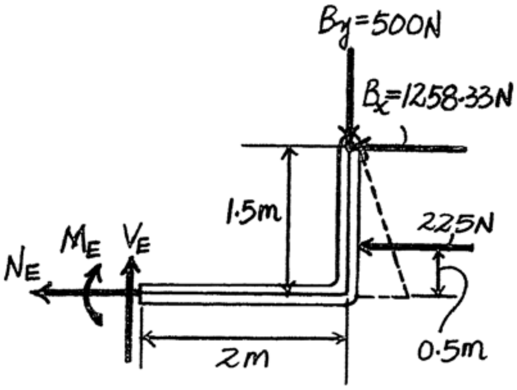
$+\uparrow \Sigma F_y = 0; \quad V_E - 500 = 0$

$V_E = 500\text{ N}$ **Ans**

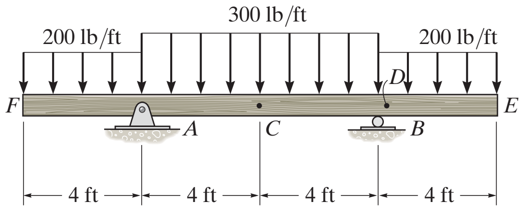
$\circlearrowleft + \Sigma M_E = 0; \quad -M_E + 225(0.5) + 1258.33(1.5) - 500(2) = 0$

$M_E = 1000\text{ N}\cdot\text{m}$ **Ans**





7–15. Determine the internal normal force, shear force, and moment acting at point C and at point D, which is located just to the right of the roller support at B.



Support Reactions : From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(8) + 800(2) - 2400(4) - 800(10) = 0$$
$$B_y = 2000 \text{ lb}$$

Internal Forces : Applying the equations of equilibrium to segment ED [FBD (b)], we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_D - 800 = 0 \quad V_D = 800 \text{ lb} \qquad \textbf{Ans}$$

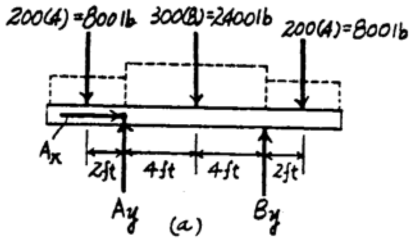
$$\circlearrowleft + \Sigma M_D = 0; \quad -M_D - 800(2) = 0$$
$$M_D = -1600 \text{ lb}\cdot\text{ft} = -1.60 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$

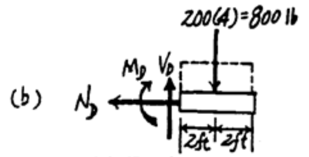
Applying the equations of equilibrium to segment EC [FBD (c)], we have

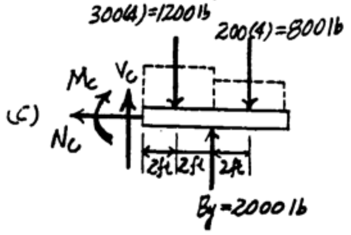
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_C + 2000 - 1200 - 800 = 0 \quad V_C = 0 \qquad \textbf{Ans}$$

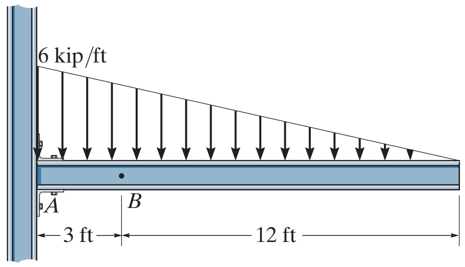
$$\circlearrowleft + \Sigma M_C = 0; \quad 2000(4) - 1200(2) - 800(6) - M_C = 0$$
$$M_C = 800 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$







***7–16.** Determine the internal normal force, shear force, and moment in the cantilever beam at point B.



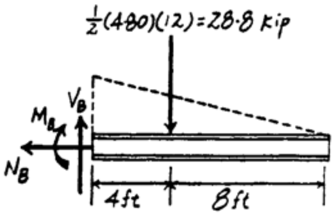
Free body Diagram : The support reactions at A need not be computed.

Internal Forces : Applying the equations of equilibrium to segment CB, we have

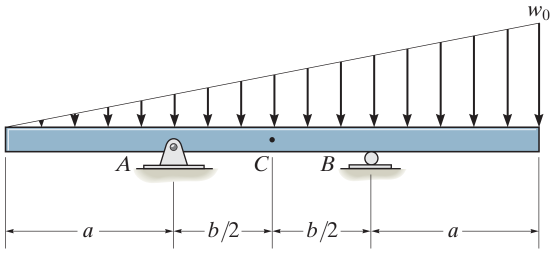
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_B = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_B - 28.8 = 0 \quad V_B = 28.8 \text{ kip} \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad -28.8(4) - M_B = 0$$
$$M_B = -115 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$



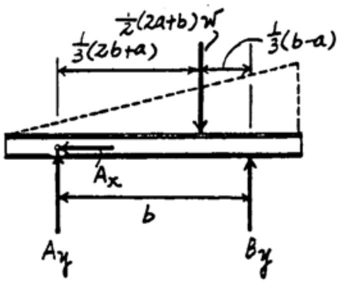
•7–17. Determine the ratio of a/b for which the shear force will be zero at the midpoint C of the double-overhang beam.



Support Reactions : . From FBD (a),

$$\curvearrowleft + \Sigma M_B = 0;\quad \frac{1}{2}(2a+b)\,w\left[\frac{1}{3}(b-a)\right] - A_y\,(b) = 0$$

$$A_y = \frac{w}{6b}(2a+b)(b-a)$$

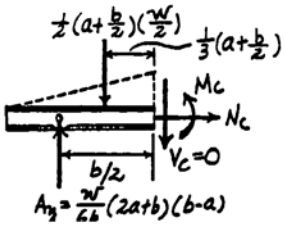


Internal Forces : This problem requires $V_C = 0$. Summing forces vertically [FBD (b)], we have

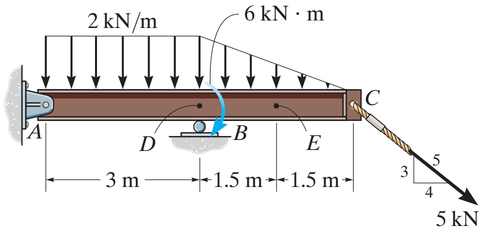
$$+\uparrow \Sigma F_y = 0;\quad \frac{w}{6b}(2a+b)(b-a) - \frac{1}{2}\left(a+\frac{b}{2}\right)\left(\frac{w}{2}\right) = 0$$

$$\frac{w}{6b}(2a+b)(b-a) = \frac{w}{8}(2a+b)$$

$$\frac{a}{b} = \frac{1}{4}$$ **Ans**


$\frac{1}{2}(a+\frac{b}{2})(\frac{w}{2})$
$\frac{1}{3}(a+\frac{b}{2})$
M_C
N_C
$V_C=0$
b/2
$A_y = \frac{w}{6b}(2a+b)(b-a)$

7–18. Determine the internal normal force, shear force, and moment at points *D* and *E* in the overhang beam. Point *D* is located just to the left of the roller support at *B*, where the couple moment acts.



The intensity of the triangular distributed load at *E* can be found using the similar triangles in Fig. *b*. With reference to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(3) - 2(3)(1.5) - 6 - \frac{1}{2}(2)(3)(4) - 5\left(\frac{3}{5}\right)(6) = 0$

$B_y = 15\text{ kN}$

Using this result and referring to Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 5\left(\frac{4}{5}\right) - N_D = 0\qquad N_D = 4\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad V_D + 15 - \frac{1}{2}(2)(3) - 5\left(\frac{3}{5}\right) = 0\qquad V_D = -9\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;\qquad -M_D - 6 - \frac{1}{2}(2)(3)(1) - 5\left(\frac{3}{5}\right)(3) = 0\qquad M_D = -18\text{ kN}\cdot\text{m}$ **Ans.**

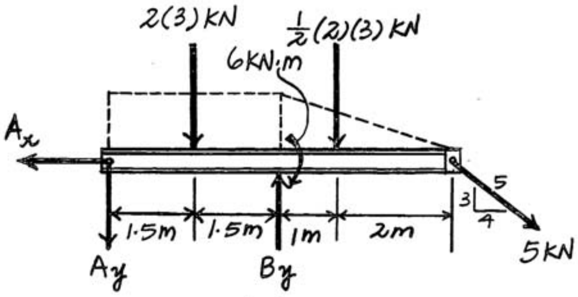
Also, by referring to Fig. *d*, we can write

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 5\left(\frac{4}{5}\right) - N_E = 0\qquad N_E = 4\text{ kN}$ **Ans.**

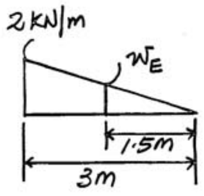
$+\uparrow \Sigma F_y = 0;\qquad V_E - \frac{1}{2}(1)(1.5) - 5\left(\frac{3}{5}\right) = 0\qquad V_E = 3.75\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;\qquad -M_E - \frac{1}{2}(1)(1.5)(0.5) - 5\left(\frac{3}{5}\right)(1.5) = 0\qquad M_E = -4.875\text{ kN}\cdot\text{m}$ **Ans.**

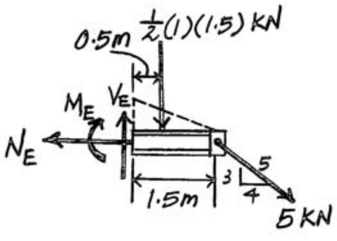
The negative sign indicates that $\mathbf{V}_D$, $\mathbf{M}_D$, and $\mathbf{M}_E$ act in the opposite sense to that shown on the free - body diagram.



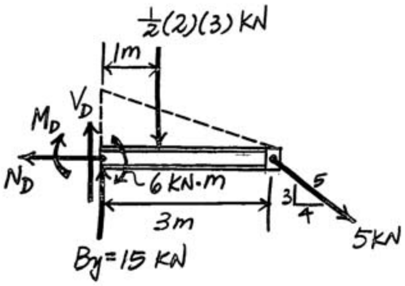
(a)



(b)

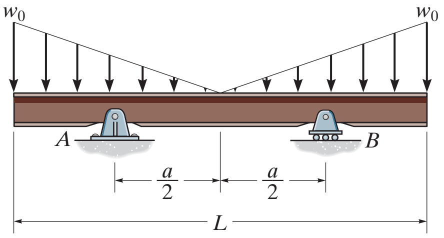


(d)



(c)

7–19. Determine the distance a in terms of the beam's length L between the symmetrically placed supports A and B so that the internal moment at the center of the beam is zero.



In this problem, it is required that the internal moment at point C be equal to zero. With reference to Fig. a,

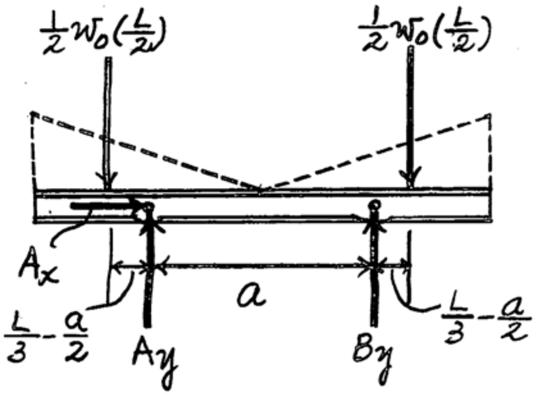
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(a) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left[a + \left(\frac{L}{3} - \frac{a}{2}\right)\right] + \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{3} - \frac{a}{2}\right) = 0$$

$$B_y = \frac{1}{4}w_0L$$

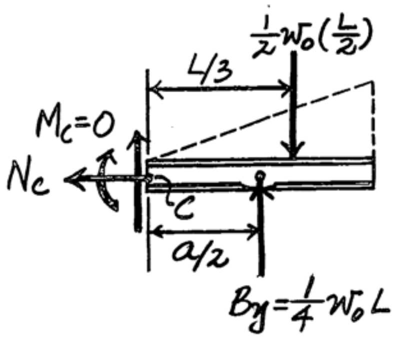
Using this result and referring to Fig. b,

$$\circlearrowleft + \Sigma M_C = 0; \qquad \frac{1}{4}w_0L\left(\frac{a}{2}\right) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = 0$$

$$a = \frac{2}{3}L$$ **Ans.**

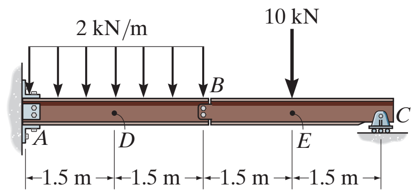

$\frac{1}{2}w_0(\frac{L}{2})$
$\frac{1}{2}w_0(\frac{L}{2})$
A_x
$\frac{L}{3}-\frac{a}{2}$
A_y
a
B_y
$\frac{L}{3}-\frac{a}{2}$

(a)



(b)

***7–20.** Determine the internal normal force, shear force, and moment at points *D* and *E* in the compound beam. Point *E* is located just to the left of the 10-kN concentrated load. Assume the support at *A* is fixed and the connection at *B* is a pin.



With reference to Fig. *b*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $B_x = 0$

$\circlearrowleft + \Sigma M_B = 0;$ $C_y(3) - 10(1.5) = 0$ $C_y = 5\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0;$ $10(1.5) - B_y(3) = 0$ $B_y = 5\text{ kN}$

Using these results and referring to Fig. *c*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $V_D - 2(1.5) - 5 = 0$ $V_D = 8\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $-M_D - 2(1.5)(0.75) - 5(1.5) = 0$ $M_D = -9.75\text{ kN}\cdot\text{m}$ **Ans.**

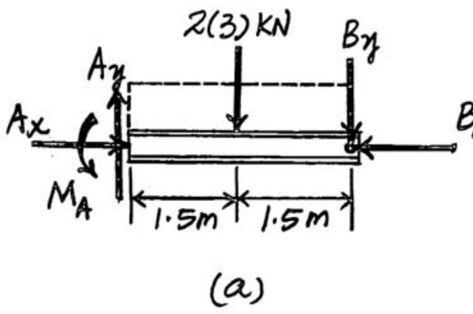
Also, by referring to Fig. *d*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $N_E = 0$ **Ans.**

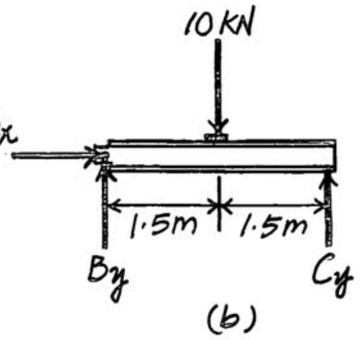
$+\uparrow \Sigma F_y = 0;$ $V_E - 10 + 5 = 0$ $V_E = 5\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $5(1.5) - M_E = 0$ $M_E = 7.5\text{ kN}\cdot\text{m}$ **Ans.**

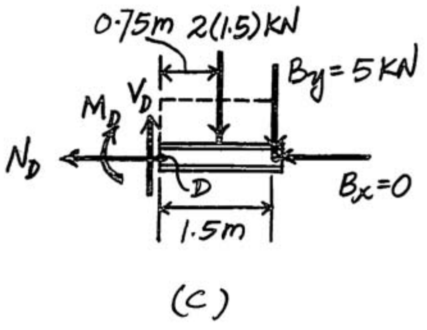
The negative sign indicates that $\mathbf{M}_P$ acts in the opposite sense to that shown in the free - body diagram.



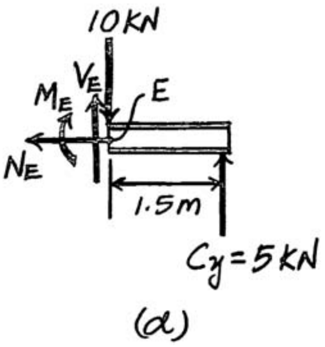
(a)



(b)

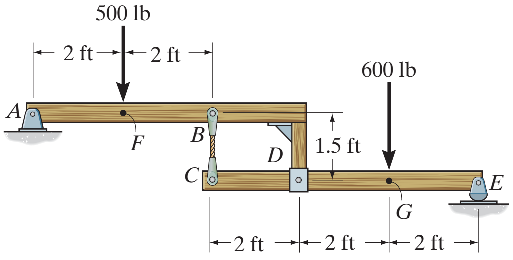


(c)



(d)

•7–21. Determine the internal normal force, shear force, and moment at points *F* and *G* in the compound beam. Point *F* is located just to the right of the 500-lb force, while point *G* is located just to the right of the 600-lb force.



With reference to Fig. *b*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad D_x = 0$

Using this result and writing the moment equation of equilibrium about point *A*, Fig. *a*, and about point *E*, Fig. *b*, we have

$\curvearrowleft + \Sigma M_A = 0;$ $\quad D_y(6) - F_{BC}(4) - 500(2) = 0 \quad (1)$

$\curvearrowleft + \Sigma M_E = 0;$ $\quad 600(2) + D_y(4) - F_{BC}(6) = 0 \quad (2)$

Solving Eqs. (1) and (2)

$F_{BC} = 560\text{ lb} \qquad D_y = 540\text{ lb}$

Using these results and referring to Fig. *b*,

$+\uparrow \Sigma F_y = 0;$ $\quad E_y - 600 - 540 + 560 = 0 \qquad E_y = 580\text{ lb}$

Again, using the results of $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{F}_{BC}$, the force equation of equilibrium written along the *x* and *y* axes, Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 500 - 560 + 540 = 0 \qquad A_y = 520\text{ lb}$

Using these results and referring to Fig. *c*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad N_F = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 520 - 500 - V_F = 0 \quad V_F = 20\text{ lb}$ **Ans.**

$\curvearrowleft + \Sigma M_F = 0;$ $\quad M_F - 520(2) = 0 \qquad M_F = 1040\text{ lb}\cdot\text{ft}$ **Ans.**

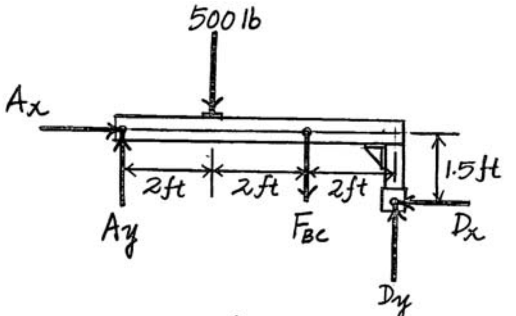
Using the result for $\mathbf{E}_y$ and referring to Fig. *d*

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad N_G = 0$ **Ans.**

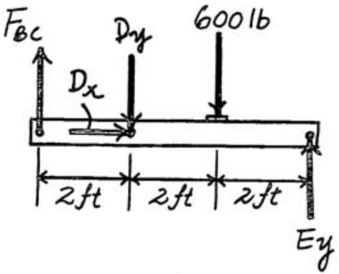
$+\uparrow \Sigma F_y = 0;$ $\quad V_G + 580 = 0 \qquad V_G = -580\text{ lb}$ **Ans.**

$\curvearrowleft + \Sigma M_G = 0;$ $\quad 580(2) - M_G = 0 \qquad M_G = 1160\text{ lb}\cdot\text{ft}$ **Ans.**

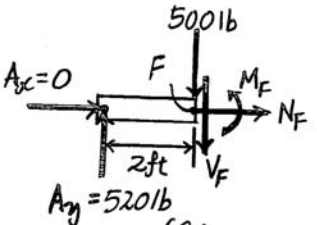
The negative sign indicates that $\mathbf{V}_G$ acts in the opposite sense to that shown in the free-body diagram.



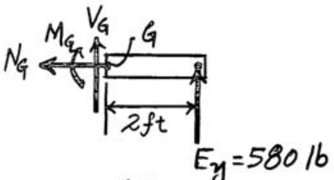
(a)



(b)

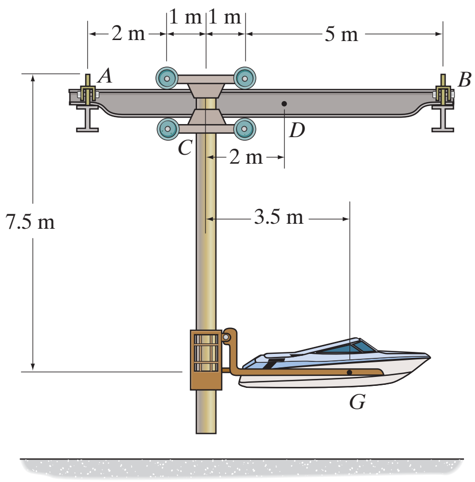


(C)



(d)

7–22. The stacker crane supports a 1.5-Mg boat with the center of mass at *G*. Determine the internal normal force, shear force, and moment at point *D* in the girder. The trolley is free to roll along the girder rail and is located at the position shown. Only vertical reactions occur at *A* and *B*.



With reference to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(9) - 1500(9.81)(3.5 + 3) = 0 \quad B_y = 10627.5\text{ N}$

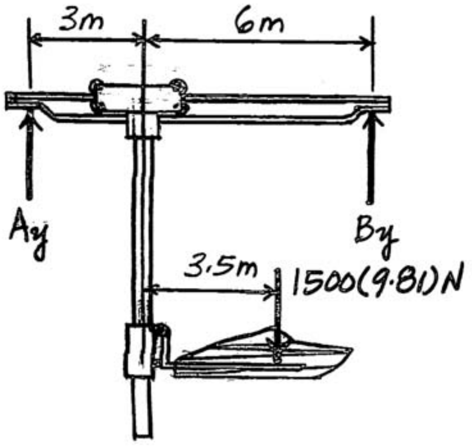
Using this result and referring to Fig. *b*,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_D = 0$ **Ans.**

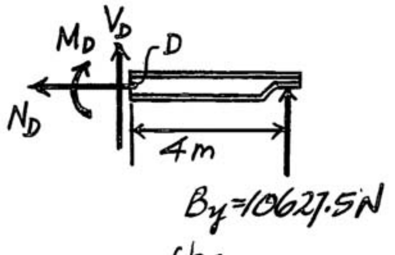
$+\uparrow \Sigma F_y = 0;$ $\quad V_D + 10627.5 = 0$ $\quad V_D = -10627.5\text{ N} = -10.6\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad 10627.5(4) - M_D = 0$ $\quad M_D = 42510\text{ N}\cdot\text{m} = 42.5\text{ kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{V}_D$ acts in the opposite sense to that shown on the free-body diagram.

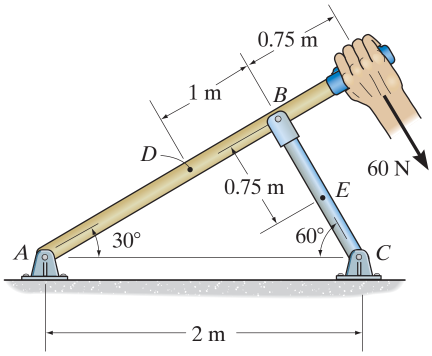


(a)



(b)

7–23. Determine the internal normal force, shear force, and moment at points *D* and *E* in the two members.



With reference to Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0; \quad F_{BC}\,(2\cos 30°) - 60(2\cos 30° + 0.75) = 0$

$F_{BC} = 85.98\text{ N}$

Using this result and referring to Fig. *b*,

$\overset{+}{\swarrow} \Sigma F_{x'} = 0; \quad N_D = 0$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0; \quad 85.98 - 60 - V_D = 0 \qquad V_D = 26.0\text{ N}$ **Ans.**

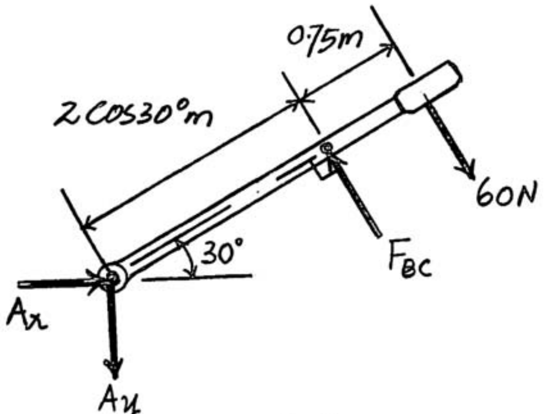
$\curvearrowleft + \Sigma M_D = 0; \quad 85.98(1) - 60(1.75) + M_D = 0 \qquad M_D = 19.0\text{ N}\cdot\text{m}$ **Ans.**

Also, be referring to Fig. *c*,

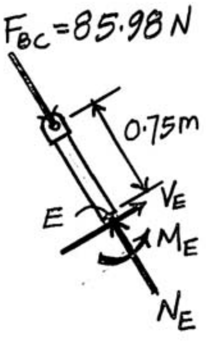
$\overset{+}{\nearrow} \Sigma F_{x'} = 0; \quad V_E = 0$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0; \quad N_E - 85.98 = 0 \qquad N_E = 86.0\text{ N}$ **Ans.**

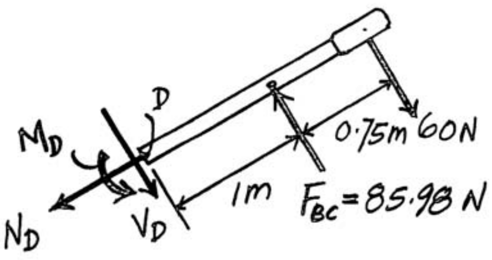
$\curvearrowleft + \Sigma M_E = 0; \quad M_E = 0$ **Ans.**



(a)

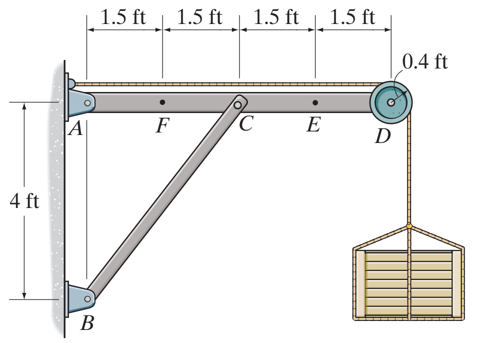


(c)



(b)

***7–24.** Determine the internal normal force, shear force, and moment at points *F* and *E* in the frame. The crate weighs 300 lb.



With reference to Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0;\quad F_{BC}\left(\frac{4}{5}\right)(3) + 300(0.4) - 300(6.4) = 0 \qquad F_{BC} = 750\text{ lb}$

Referring to Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -N_E - 300 = 0 \qquad N_E = -300\text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_E - 300 = 0 \qquad V_E = 300\text{ lb}$ **Ans.**

$\curvearrowleft + \Sigma M_E = 0;\quad -M_E + 300(0.4) - 300(1.9) = 0 \quad M_E = -450\text{ lb}\cdot\text{ft}$ **Ans.**

Using the result of $\mathbf{F}_{BC}$ and referring to Fig. *c*,

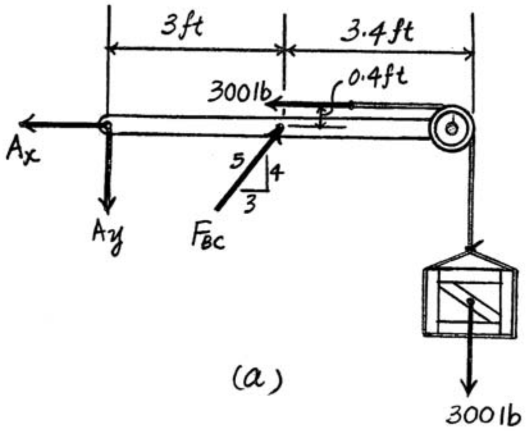
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 750\left(\frac{3}{5}\right) - 300 - N_F = 0 \qquad N_F = 150\text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_F + 750\left(\frac{4}{5}\right) - 300 = 0 \qquad V_F = -300\text{ lb}$ **Ans.**

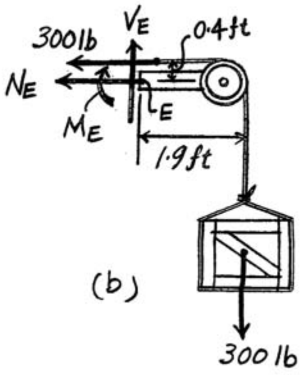
$\curvearrowleft + \Sigma M_F = 0;\quad 750\left(\frac{4}{5}\right)(1.5) + 300(0.4) - 300(4.9) - M_F = 0$

$M_F = -450\text{ lb}\cdot\text{ft}$ **Ans.**

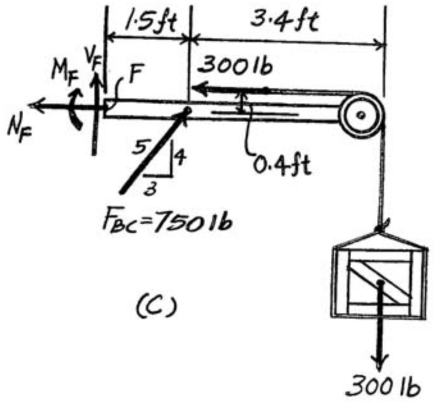
The negative sign indicates that $\mathbf{N}_E$, $\mathbf{V}_F$, and $\mathbf{M}_F$ act in the opposite sense to that shown in the free-body diagram.



(a)

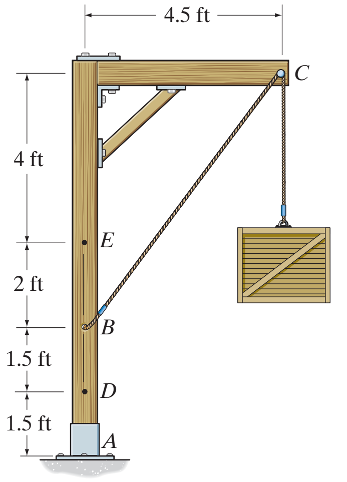


(b)



(C)

•7–25. Determine the internal normal force, shear force, and moment at points *D* and *E* of the frame which supports the 200-lb crate. Neglect the size of the smooth peg at *C*.



Referring to Fig. *a*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad V_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad N_D - 200 = 0$ $\quad N_D = 200 \text{ lb}$ **Ans.**

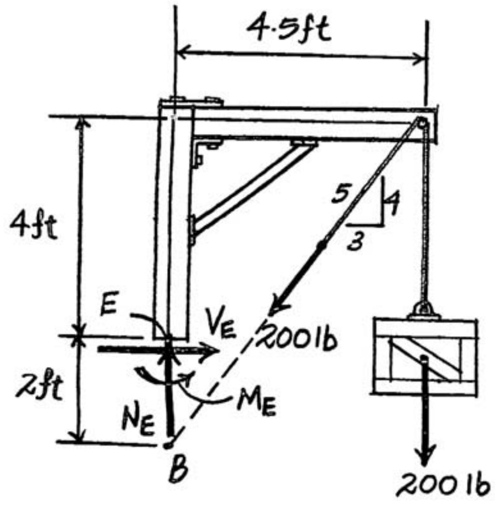
$\circlearrowleft + \Sigma M_F = 0;$ $\quad M_D - 200(4.5) = 0$ $\quad M_D = 900 \text{ lb}\cdot\text{ft}$ **Ans.**

Also, by referring to Fig. *b*,

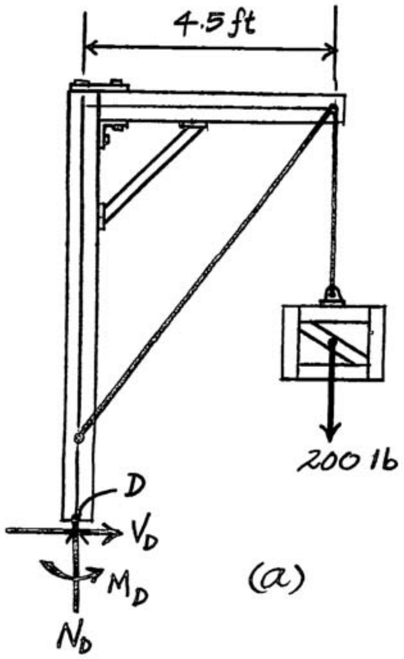
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad V_E - 200\left(\frac{3}{5}\right) = 0$ $\quad V_E = 120 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad N_E - 200\left(\frac{4}{5}\right) - 200 = 0$ $\quad N_E = 360 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad M_E - 200(4.5) - 200\left(\frac{3}{5}\right)(2) = 0$ $\quad M_E = 1140 \text{ lb}\cdot\text{ft}$ **Ans.**

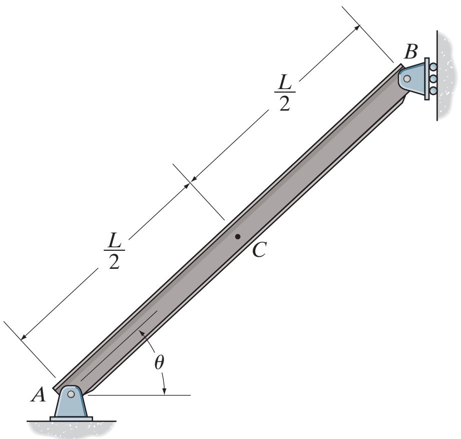


(b)



(a)

7–26. The beam has a weight *w* per unit length. Determine the internal normal force, shear force, and moment at point *C* due to its weight.



With reference to Fig. *a*,

$\curvearrowleft +\Sigma M_A = 0; \qquad B_x(L\sin\theta) - wL\cos\theta\left(\frac{L}{2}\right) = 0 \quad B_x = \frac{wL\cos\theta}{2\sin\theta}$

Using this result and referring to Fig. *b*,

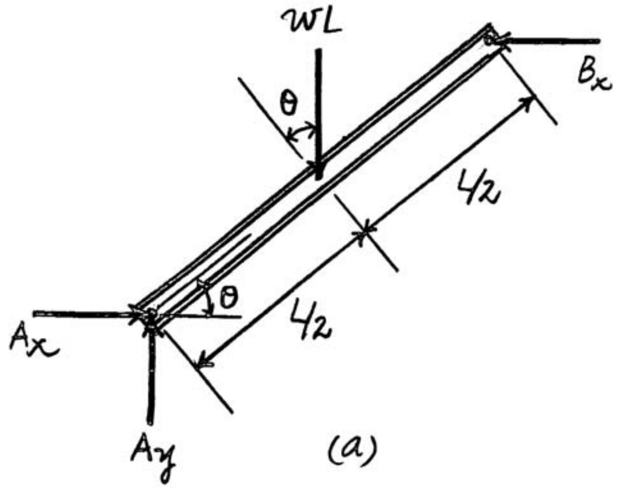
$\overset{+}{\nearrow}\Sigma F_{x'} = 0; \; -N_C - \frac{wL\cos\theta}{2\sin\theta}(\cos\theta) - w\left(\frac{L}{2}\right)\sin\theta = 0 \quad N_C = -\frac{wL}{2}\csc\theta$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0; \; V_C - w\left(\frac{L}{2}\right)\cos\theta + \frac{wL\cos\theta}{2\sin\theta}\sin\theta = 0 \qquad V_C = 0$ **Ans.**

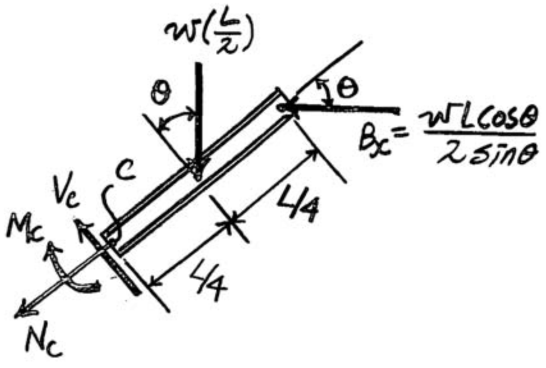
$\curvearrowleft +\Sigma M_C = 0; \qquad \frac{wL\cos\theta}{2\sin\theta}\left(\frac{L}{2}\sin\theta\right) - w\left(\frac{L}{2}\right)\cos\theta\left(\frac{L}{4}\right) - M_C = 0$

$$M_C = \frac{wL^2}{8}\cos\theta$$ **Ans.**

The negative sign indicates that $\mathbf{N}_C$ acts in the opposite sense to that shown on the free-body diagram.

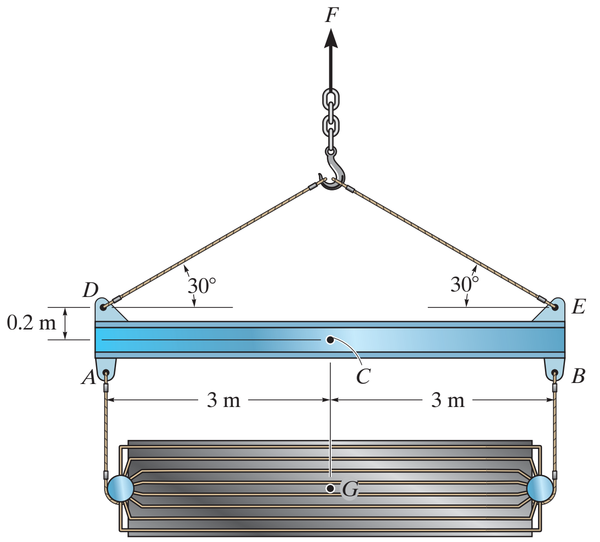


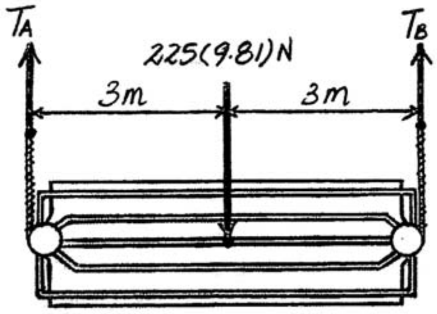
(a)



(b)

7–27. Determine the internal normal force, shear force, and moment acting at point *C*. The cooling unit has a total mass of 225 kg with a center of mass at *G*.





(a)

From FBD (a)

$\circlearrowleft + \Sigma M_A = 0;\quad T_B(6) - 225(9.81)(3) = 0 \qquad T_B = 1103.625 \text{ N}$

From FBD (b)

$\circlearrowleft + \Sigma M_D = 0;\quad T_E \sin 30°(6) - 1103.625(6) = 0 \qquad T_E = 2207.25 \text{ N}$

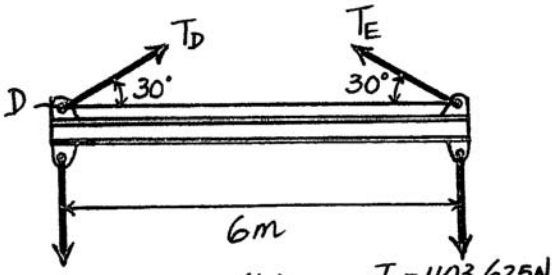
From FBD (c)

$\xrightarrow{+} \Sigma F_x = 0;\quad -N_C - 2207.25 \cos 30° = 0 \qquad N_C = -1.91 \text{ kN}$ **Ans**

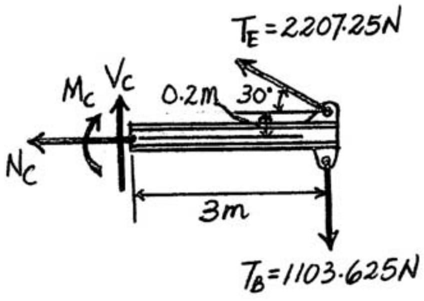
$+\uparrow \Sigma F_y = 0;\quad V_C + 2207.25 \sin 30° - 1103.625 = 0 \qquad V_C = 0$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad 2207.25 \cos 30°(0.2) + 2207.25 \sin 30°(3) - 1103.625(3) - M_C = 0$

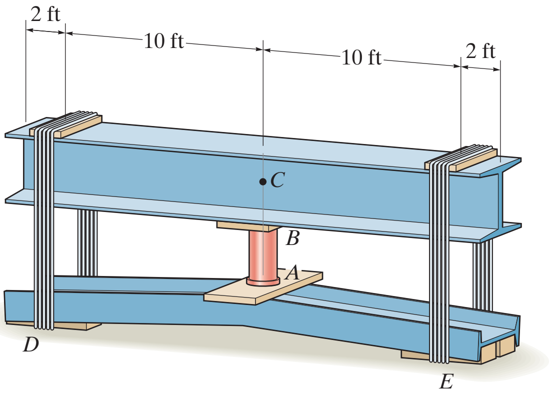
$M_C = 382 \text{ N} \cdot \text{m}$ **Ans**



(b)



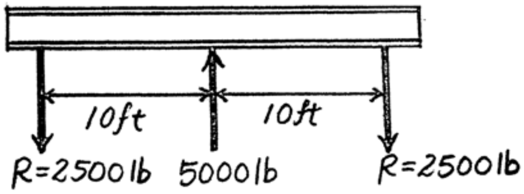
***7–28.** The jack *AB* is used to straighten the bent beam *DE* using the arrangement shown. If the axial compressive force in the jack is 5000 lb, determine the internal moment developed at point *C* of the top beam. Neglect the weight of the beams.

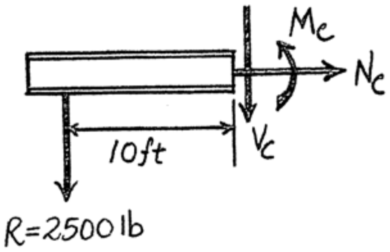


Segment :

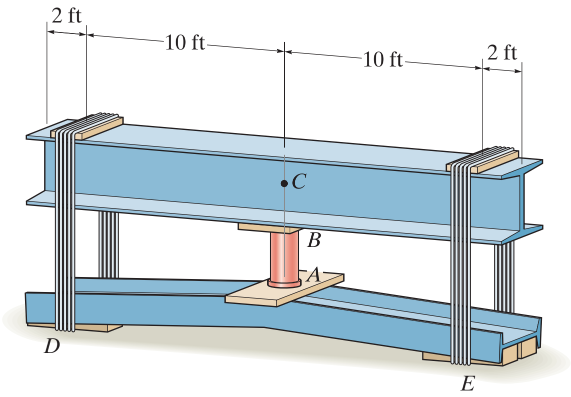
$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 2500\,(10) = 0$

$M_C = -25.0 \text{ kip} \cdot \text{ft}$ **Ans**





•7–29. Solve Prob. 7–28 assuming that each beam has a uniform weight of 150 lb/ft.



Beam :

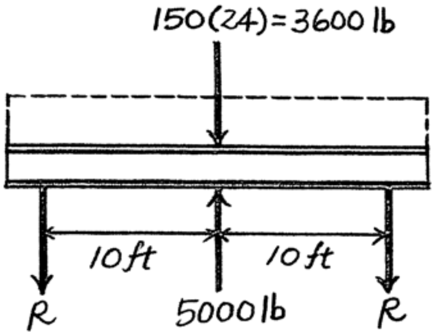
$+\uparrow \Sigma F_y = 0;$ $\quad 5000 - 3600 - 2R = 0$

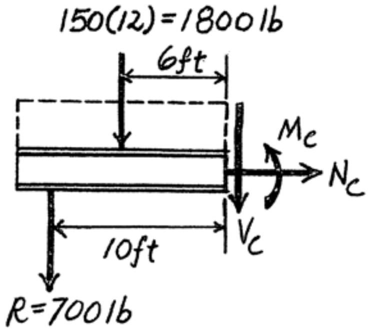
$R = 700\text{ lb}$

Segment :

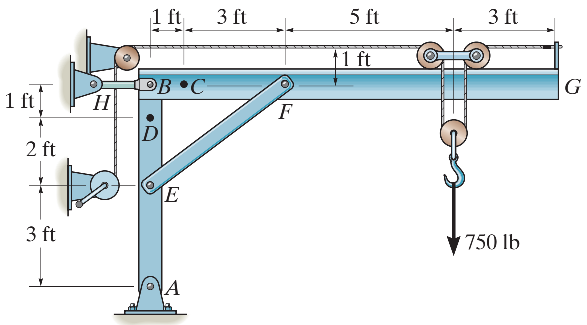
$\circlearrowleft + \Sigma M_C = 0;$ $\quad M_C + 700(10) + 1800(6) = 0$

$M_C = -17.8\text{ kip}\cdot\text{ft}$ **Ans**





7–30. The jib crane supports a load of 750 lb from the trolley which rides on the top of the jib. Determine the internal normal force, shear force, and moment in the jib at point *C* when the trolley is at the position shown. The crane members are pinned together at *B*, *E* and *F* and supported by a short link *BH*.



Member *BFG*:

$\circlearrowleft + \Sigma M_B = 0; \quad F_{EF}\left(\frac{3}{5}\right)(4) - 750(9) + 375(1) = 0$

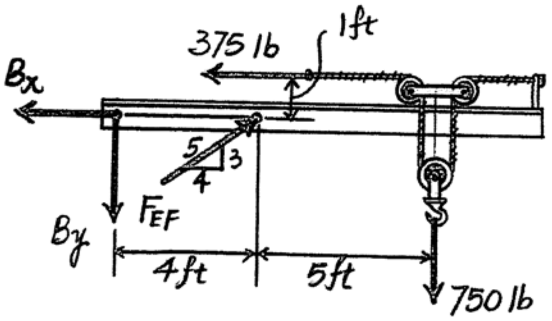
$F_{EF} = 2656.25 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad -B_x + 2656.25\left(\frac{4}{5}\right) - 375 = 0$

$B_x = 1750 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -B_y + 2656.25\left(\frac{3}{5}\right) - 750 = 0$

$B_y = 843.75 \text{ lb}$



Segment *BC*:

$\xrightarrow{+} \Sigma F_x = 0; \quad N_C - 1750 = 0$

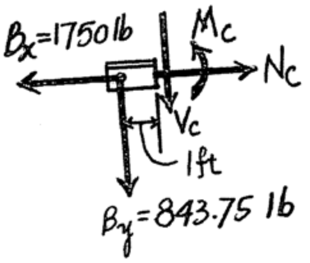
$N_C = 1.75 \text{ kip} \quad$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -843.75 - V_C = 0$

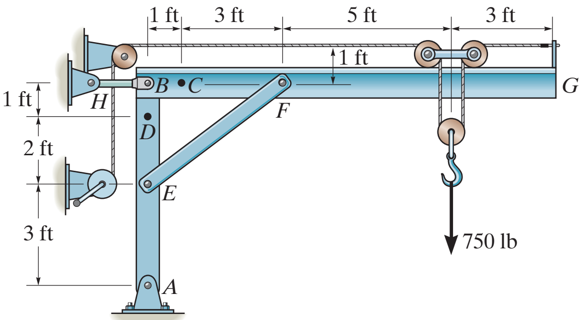
$V_C = -844 \text{ lb} \quad$ **Ans**

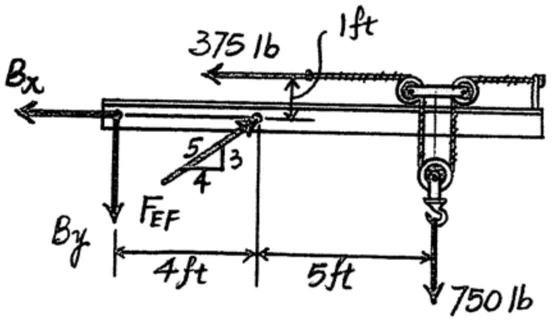
$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 843.75(1) = 0$

$M_C = -844 \text{ lb} \cdot \text{ft} \quad$ **Ans**



7–31. The jib crane supports a load of 750 lb from the trolley which rides on the top of the jib. Determine the internal normal force, shear force, and moment in the column at point *D* when the trolley is at the position shown. The crane members are pinned together at *B*, *E* and *F* and supported by a short link *BH*.



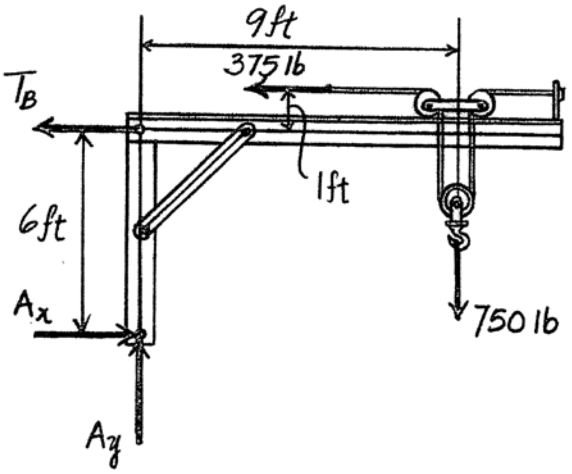


Member *BFG*:

$\circlearrowleft + \Sigma M_B = 0; \quad F_{EF}\left(\frac{3}{5}\right)(4) - 750(9) + 375(1) = 0$

$F_{EF} = 2656.25 \text{ lb}$

Entire Crane:



$\circlearrowleft + \Sigma M_A = 0; \quad T_B(6) - 750(9) + 375(7) = 0$

$T_B = 687.5 \text{ lb}$

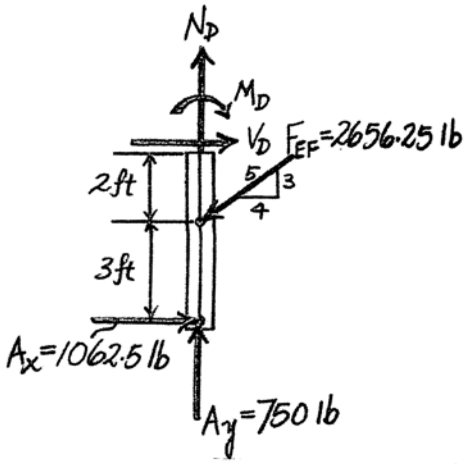
$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 687.5 - 375 = 0$

$A_x = 1062.5 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 750 = 0$

$A_y = 750 \text{ lb}$

Segment *AED*:



$+\uparrow \Sigma F_y = 0; \quad N_D + 750 - 2656.25\left(\frac{3}{5}\right) = 0$

$N_D = 844 \text{ lb}$ **Ans**

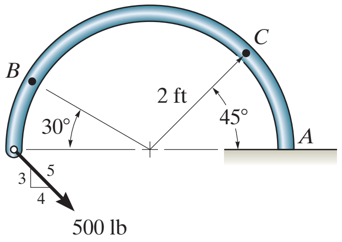
$\xrightarrow{+} \Sigma F_x = 0; \quad 1062.5 - 2656.25\left(\frac{4}{5}\right) + V_D = 0$

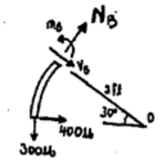
$V_D = 1.06 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0; \quad -M_D - 2656.25\left(\frac{4}{5}\right)(2) + 1062.5(5) = 0$

$M_D = 1.06 \text{ kip} \cdot \text{ft}$ **Ans**

***7–32.** Determine the internal normal force, shear force, and moment acting at points *B* and *C* on the curved rod.





$+\nearrow \Sigma F_x = 0;$ $\quad 400 \sin 30^\circ - 300 \cos 30^\circ + N_B = 0$

$N_B = 59.81 \text{ lb} = 59.8 \text{ lb}$ **Ans**

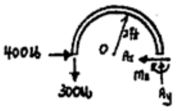
$\searrow + \Sigma F_y = 0;$ $\quad V_B + 400 \cos 30^\circ + 300 \sin 30^\circ = 0$

$V_B = -496 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_B = 0;$ $\quad M_B + 400(2 \sin 30^\circ) + 300(2 - 2 \cos 30^\circ) = 0$

$M_B = -480 \text{ lb} \cdot \text{ft}$ **Ans**

Also,



$\circlearrowleft + \Sigma M_O = 0;$ $\quad -59.81(2) + 300(2) + M_B = 0$

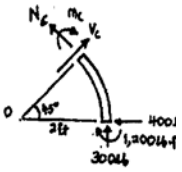
$M_B = -480 \text{ lb} \cdot \text{ft}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 400 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 300 \text{ lb}$

$\circlearrowright + \Sigma M_A = 0;$ $\quad M_A - 300(4) = 0$

$M_A = 1200 \text{ lb} \cdot \text{ft}$



$+\nwarrow \Sigma F_x = 0;$ $\quad N_C + 400 \sin 45^\circ + 300 \cos 45^\circ = 0$

$N_C = -495 \text{ lb}$ **Ans**

$+\nearrow \Sigma F_y = 0;$ $\quad V_C - 400 \cos 45^\circ + 300 \sin 45^\circ = 0$

$V_C = 70.7 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -M_C - 1200 - 400(2 \sin 45^\circ) + 300(2 - 2 \cos 45^\circ) = 0$

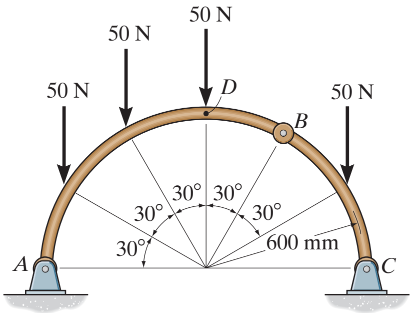
$M_C = -1590 \text{ lb} \cdot \text{ft} = -1.59 \text{ kip} \cdot \text{ft}$ **Ans**

Also,

$\circlearrowleft + \Sigma M_O = 0;$ $\quad -1200 - 495(2) + 300(2) - M_C = 0$

$M_C = -1590 \text{ lb} \cdot \text{ft} = -1.59 \text{ kip} \cdot \text{ft}$ **Ans**

•7–33. Determine the internal normal force, shear force, and moment at point *D* which is located just to the right of the 50-N force.



Referring to Figs. *a* and *b*, respectively,

$\curvearrowleft + \Sigma M_A = 0;\quad B_y(0.6+0.6\sin 30°)+B_x(0.6\cos 30°)-50(0.6-0.6\cos 30°)=0$
$-50(0.6-0.6\cos 60°)-50(0.6)=0 \qquad (1)$

$\curvearrowleft + \Sigma M_C = 0;\quad B_y(0.6-0.6\cos 60°)-B_x(0.6\sin 60°)+50(0.6-0.6\cos 30°)=0 \qquad (2)$

Solving Eqs. (1) and (2) yields

$B_x = 29.39\text{ N} \qquad B_y = 37.5\text{ N}$

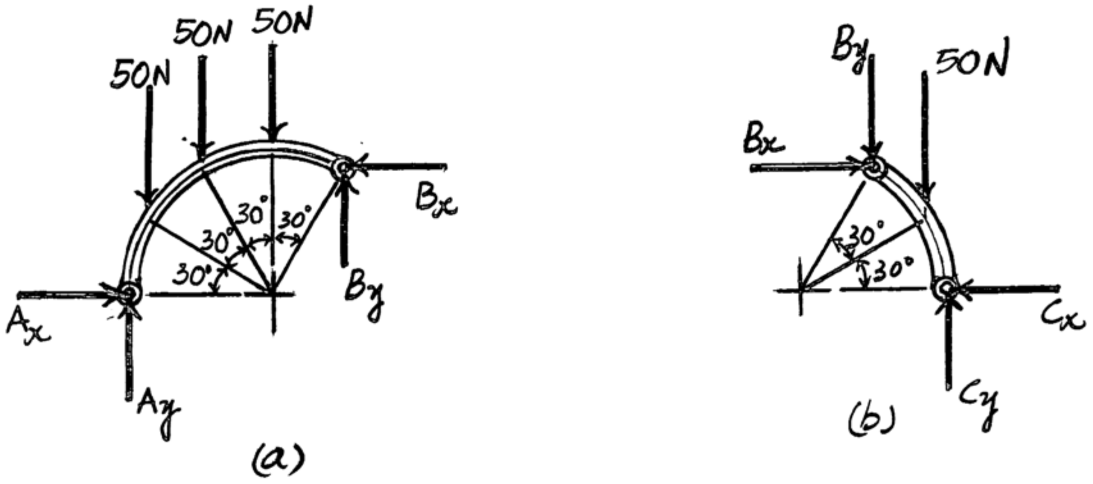
Using these results and referring to Fig. *c*,

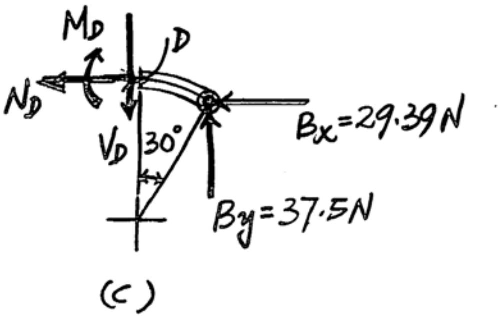
$\xrightarrow{+}\Sigma F_x = 0;\quad -N_D - 29.39 = 0 \qquad N_D = -29.4\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 37.5 - V_D = 0 \qquad V_D = 37.5\text{N}$ **Ans.**

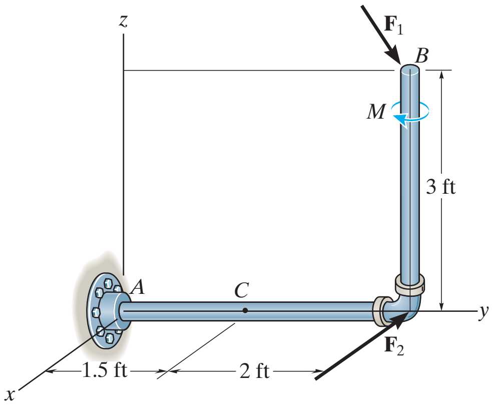
$\curvearrowleft + \Sigma M_D = 0;\quad 37.5(0.6\sin 30°) - 29.39(0.6-0.6\cos 30°) - M_D = 0$

$M_D = 8.89\text{ N}\cdot\text{m}$ **Ans.**





7–34. Determine the *x*, *y*, *z* components of internal loading at point *C* in the pipe assembly. Neglect the weight of the pipe. The load is $\mathbf{F}_1 = \{-24\mathbf{i} - 10\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{-80\mathbf{i}\}$ lb, and $\mathbf{M} = \{-30\mathbf{k}\}$ lb · ft.



Free body Diagram : The support reactions need not be computed.

Internal Forces : Applying the equations of equilibrium to segment *BC*, we have

$\Sigma F_x = 0;$ $(V_C)_x - 24 - 80 = 0$ $(V_C)_x = 104$ lb **Ans**

$\Sigma F_y = 0;$ $N_C = 0$ **Ans**

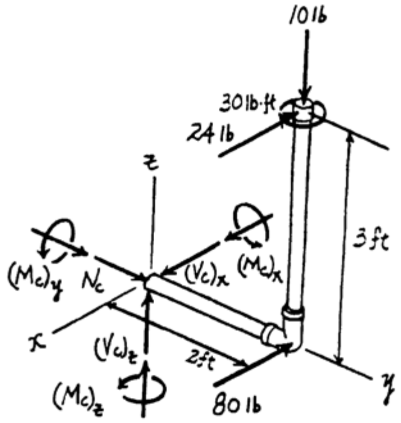
$\Sigma F_z = 0;$ $(V_C)_z - 10 = 0$ $(V_C)_z = 10.0$ lb **Ans**

$\Sigma M_x = 0;$ $(M_C)_x - 10(2) = 0$ $(M_C)_x = 20.0$ lb · ft **Ans**

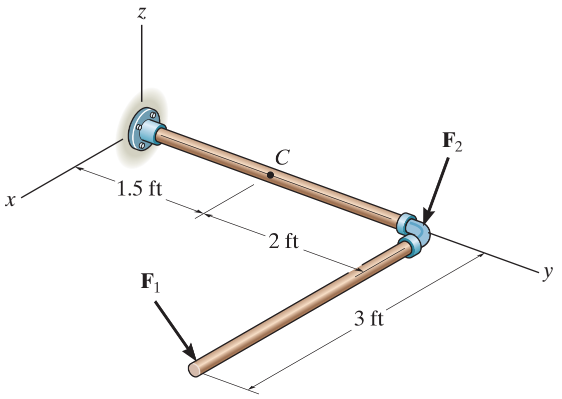
$\Sigma M_y = 0;$ $(M_C)_y - 24(3) = 0$ $(M_C)_y = 72.0$ lb · ft **Ans**

$\Sigma M_z = 0;$ $(M_C)_z + 24(2) + 80(2) - 30 = 0$

$(M_C)_z = -178$ lb · ft **Ans**



7–35. Determine the *x*, *y*, *z* components of internal loading at a section passing through point *C* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{350\mathbf{j} - 400\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{150\mathbf{i} - 300\mathbf{k}\}$ lb.



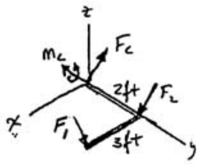
$\Sigma \mathbf{F}_R = 0;$ $\mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = 0$

$\mathbf{F}_C = \{-150\mathbf{i} - 350\mathbf{j} + 700\mathbf{k}\}$ lb

$C_x = -150$ lb **Ans**

$C_y = -350$ lb **Ans**

$C_z = 700$ lb **Ans**



$\Sigma \mathbf{M}_R = 0;$ $\mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = 0$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 0 & 350 & -400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 150 & 0 & -300 \end{vmatrix} = 0$$

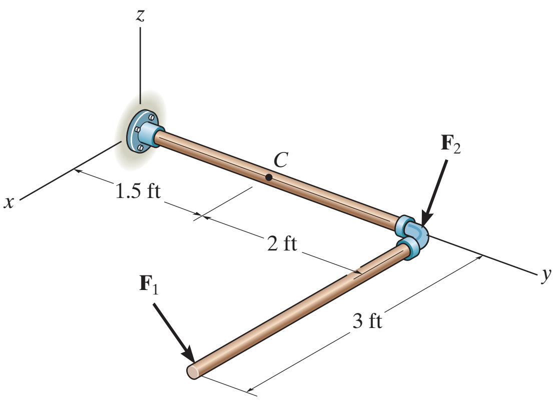
$\mathbf{M}_C = \{1400\mathbf{i} - 1200\mathbf{j} - 750\mathbf{k}\}$ lb·ft

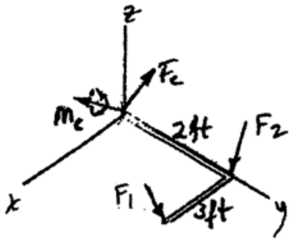
$M_{Cx} = 1.40$ kip·ft **Ans**

$M_{Cy} = -1.20$ kip·ft **Ans**

$M_{Cz} = -750$ lb·ft **Ans**

***7–36.** Determine the *x, y, z* components of internal loading at a section passing through point *C* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{-80\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{250\mathbf{i} - 150\mathbf{j} - 200\mathbf{k}\}$ lb.





$\Sigma \mathbf{F}_R = \mathbf{0};\qquad \mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$

$$\mathbf{F}_C = \{-170\mathbf{i} - 50\mathbf{j} + 500\mathbf{k}\}\ \text{lb}$$

$C_x = -170$ lb **Ans**

$C_y = -50$ lb **Ans**

$C_z = 500$ lb **Ans**

$\Sigma \mathbf{M}_R = \mathbf{0};\qquad \mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = \mathbf{0}$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ -80 & 200 & -300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 250 & -150 & -200 \end{vmatrix} = \mathbf{0}$$

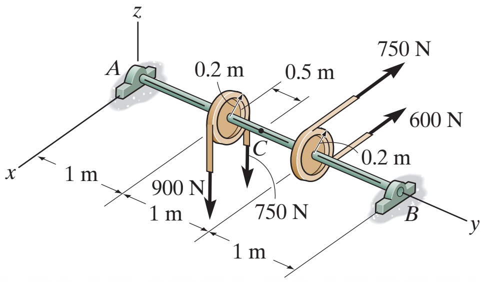
$$\mathbf{M}_C = \{1000\mathbf{i} - 900\mathbf{j} - 260\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

$M_{Cx} = 1$ kip·ft **Ans**

$M_{Cy} = 900$ lb·ft **Ans.**

$M_{Cz} = -260$ lb·ft **Ans.**

•7–37. The shaft is supported by a thrust bearing at *A* and a journal bearing at *B*. Determine the *x*, *y*, *z* components of internal loading at point *C*.



With reference to Fig. *a*,

$\Sigma M_x = 0;\quad B_z(3) - 900(1) - 750(1) = 0 \qquad B_z = 550\text{ N}$

$\Sigma M_z = 0;\quad 750(2) + 600(2) - B_x(3) = 0 \qquad B_x = 900\text{ N}$

Using these results and referring to Fig. *b*,

$\Sigma F_x = 0;\quad (V_C)_x + 900 - 750 - 600 = 0 \qquad (V_C)_x = 450\text{ N}$ **Ans.**

$\Sigma F_y = 0;\quad N_C = 0$ **Ans.**

$\Sigma F_z = 0;\quad (V_C)_z + 550 = 0 \qquad (V_C)_z = -550\text{ N}$ **Ans.**

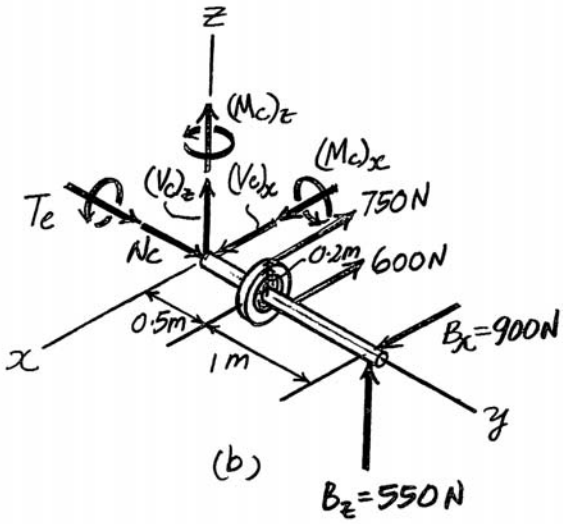
$\Sigma M_x = 0;\quad (M_C)_x + 550(1.5) = 0 \qquad (M_C)_x = -825\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0;\quad T_C + 600(0.2) - 750(0.2) = 0 \qquad T_C = 30\text{ N}\cdot\text{m}$ **Ans.**

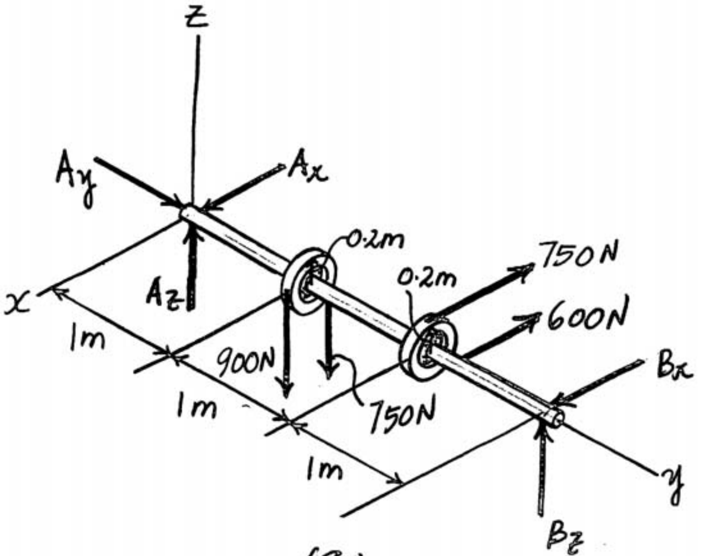
$\Sigma M_z = 0;\quad (M_C)_z + 750(0.5) + 600(0.5) - 900(1.5) = 0$

$(M_C)_z = 675\text{ N}\cdot\text{m}$ **Ans.**

The negative signs indicate that $(\mathbf{V}_C)_z$ and $(\mathbf{M}_C)_z$ act in the opposite sense to those shown in the free-body diagram.

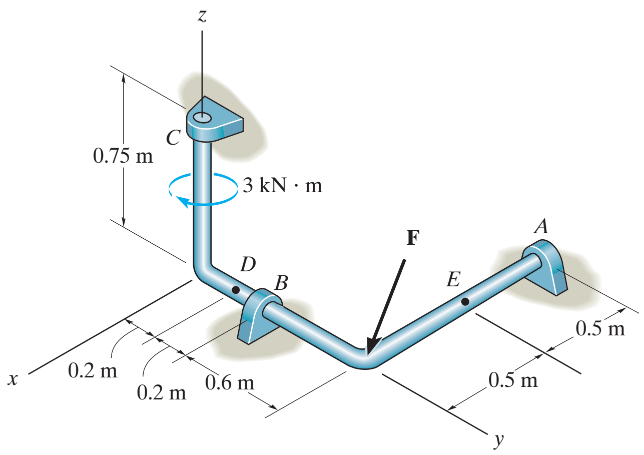


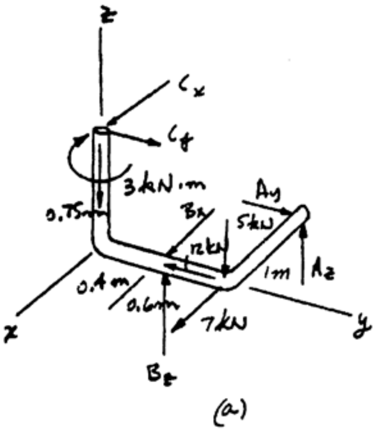
(b)

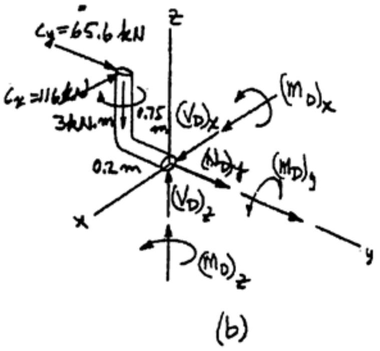


(a)

7–38. Determine the *x, y, z* components of internal loading in the rod at point *D*. There are journal bearings at *A*, *B*, and *C*. Take $\mathbf{F} = \{7\mathbf{i} - 12\mathbf{j} - 5\mathbf{k}\}$ kN.







Support reactions : FBD (a)

$\Sigma M_x = 0;\quad B_z(0.4) + A_z(1) - C_y(0.75) - 5(1) = 0 \qquad (1)$

$\Sigma M_y = 0;\quad A_z(1) + C_x(0.75) = 0 \qquad (2)$

$\Sigma M_z = 0;\quad -B_x(0.4) - A_y(1) - 7(1) - 3 = 0 \qquad (3)$

$\Sigma F_x = 0;\quad C_x + B_x + 7 = 0 \qquad (4)$

$\Sigma F_y = 0;\quad C_y + A_y - 12 = 0 \qquad (5)$

$\Sigma F_z = 0;\quad B_z + A_z - 5 = 0 \qquad (6)$

Solving Eqs. (1) to (6) yields :

$C_x = -116\text{ kN} \quad B_x = 109\text{ kN} \quad A_z = 87.0\text{ kN}$

$A_y = -53.6\text{ kN} \quad C_y = 65.6\text{ kN} \quad B_z = -82.0\text{ kN}$

Negative signs indicate that the support reactions act in the opposite sense to those shown on FBD (a).
From FBD (b)

$\Sigma F_x = 0;\quad (V_D)_x - 116 = 0;\quad (V_D)_x = 116\text{ kN}$ **Ans**

$\Sigma F_y = 0;\quad (N_D)_y + 65.6 = 0;\quad (N_D)_y = -65.6\text{ kN}$ **Ans**

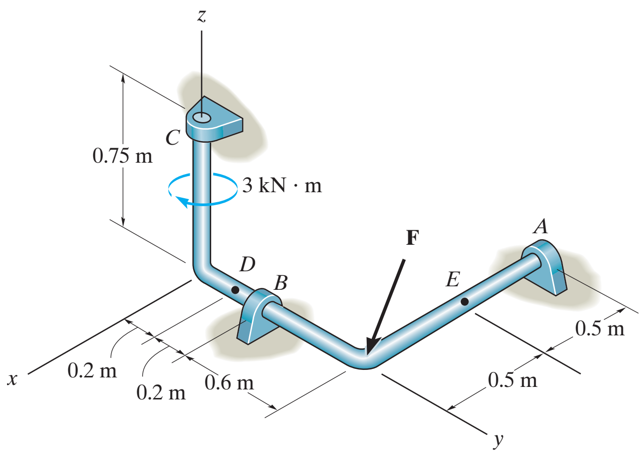
$\Sigma F_z = 0;\quad (V_D)_z = 0$ **Ans**

$\Sigma M_x = 0;\quad (M_D)_x - 65.6(0.75) = 0;\quad (M_D)_x = 49.2\text{ kN}\cdot\text{m}$ **Ans**

$\Sigma M_y = 0;\quad (M_D)_y - 116(0.75) = 0;\quad (M_D)_y = 87.0\text{ kN}\cdot\text{m}$ **Ans**

$\Sigma M_z = 0;\quad (M_D)_z - 116(0.2) - 3 = 0;\quad (M_D)_z = 26.2\text{ kN}\cdot\text{m}$ **Ans**

7–39. Determine the *x, y, z* components of internal loading in the rod at point *E*. Take $\mathbf{F} = \{7\mathbf{i} - 12\mathbf{j} - 5\mathbf{k}\}$ kN.



Support reactions : FBD (a)

$\Sigma M_x = 0;\quad B_z(0.4) + A_z(1) - C_y(0.75) - 5(1) = 0 \quad (1)$

$\Sigma M_y = 0;\quad A_x(1) + C_x(0.75) = 0 \quad (2)$

$\Sigma M_z = 0;\quad -B_x(0.4) - A_y(1) - 7(1) - 3 = 0 \quad (3)$

$\Sigma F_x = 0;\quad C_x + B_x + 7 = 0 \quad (4)$

$\Sigma F_y = 0;\quad C_y + A_y - 12 = 0 \quad (5)$

$\Sigma F_z = 0;\quad B_z + A_z - 5 = 0 \quad (6)$

Solving Eqs. (1) to (6) yields :

$C_x = -116 \text{ kN} \quad B_x = 109 \text{ kN} \quad A_z = 87.0 \text{ kN}$

$A_y = -53.6 \text{ kN} \quad C_y = 65.6 \text{ kN} \quad B_z = -82.0 \text{ kN}$

Negative signs indicate that the support reactions act in the opposite sense to those shown on FBD (a).
From FBD (b)

$\Sigma F_x = 0;\quad (N_E)_x = 0$ **Ans**

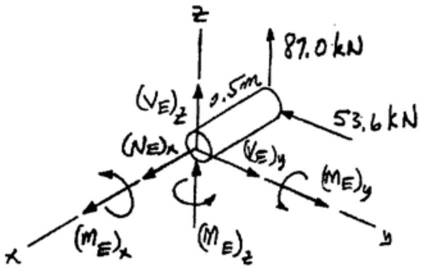
$\Sigma F_y = 0;\quad (V_E)_y - 53.6 = 0;\quad (V_E)_y = 53.6 \text{ kN}$ **Ans**

$\Sigma F_z = 0;\quad (V_E)_z + 87.0 = 0;\quad (V_E)_z = -87.0 \text{ kN}$ **Ans**

$\Sigma M_x = 0;\quad (M_E)_x = 0$ **Ans**

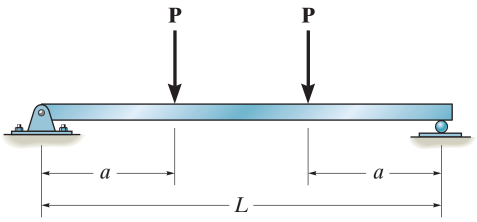
$\Sigma M_y = 0;\quad (M_E)_y + 87.0(0.5) = 0;\quad (M_E)_y = -43.5 \text{ kN} \cdot \text{m}$ **Ans**

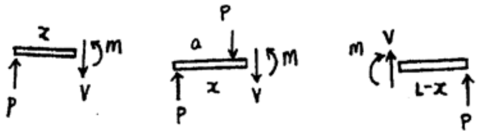
$\Sigma M_z = 0;\quad (M_E)_z + 53.6(0.5) = 0;\quad (M_E)_z = -26.8 \text{ kN} \cdot \text{m}$ **Ans**



(b)

***7–40.** Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft.





(a) For $0 \le x < a$

$+\uparrow \Sigma F_y = 0;$ $V = P$ **Ans**

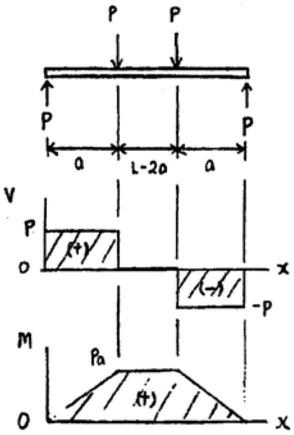
$\circlearrowleft + \Sigma M = 0;$ $M = Px$ **Ans**

For $a < x < L - a$

$+\uparrow \Sigma F_y = 0;$ $V = 0$ **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-Px + P(x - a) + M = 0$

$M = Pa$ **Ans**

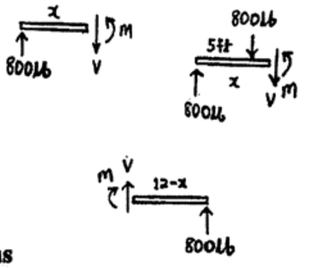


For $L - a < x \le L$

$+\uparrow \Sigma F_y = 0;$ $V = -P$ **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-M + P(L - x) = 0$

$M = P(L - x)$ **Ans**



(b) Set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft

For $0 \le x < 5$ ft

$+\uparrow \Sigma F_y = 0;$ $V = 800$ lb **Ans**

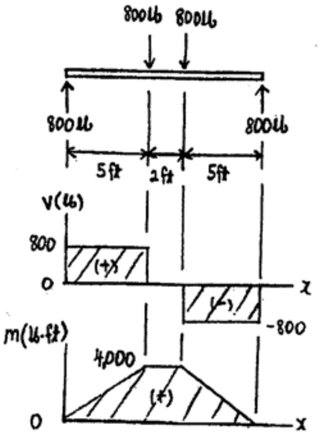
$\circlearrowleft + \Sigma M = 0;$ $M = 800x$ lb·ft **Ans**

For 5 ft $< x < 7$ ft

$+\uparrow \Sigma F_y = 0;$ $V = 0$ **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-800x + 800(x - 5) + M = 0$

$M = 4000$ lb·ft **Ans**



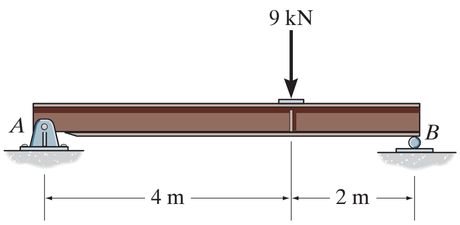
For 7 ft $< x \le 12$ ft

$+\uparrow \Sigma F_y = 0;$ $V = -800$ lb **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-M + 800(12 - x) = 0$

$M = (9600 - 800x)$ lb·ft **Ans**

•7–41. Draw the shear and moment diagrams for the simply supported beam.



Since the loading discontinues at the 9-kN concentrated force, the shear and moment equations must be written for the regions $0 \le x < 4$ m and 4 m $< x \le 6$ m of the beam. The free-body diagrams of the beam's segment sectioned through the arbitrary points in these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < 4$ m, Fig. *b*

$+\uparrow \Sigma F_y = 0; \quad 3 - V = 0 \qquad V = 3\text{kN} \qquad (1)$

$\circlearrowleft + \Sigma M = 0; \; M - 3x = 0 \qquad M = \{3x\}\text{kN}\cdot\text{m} \qquad (2)$

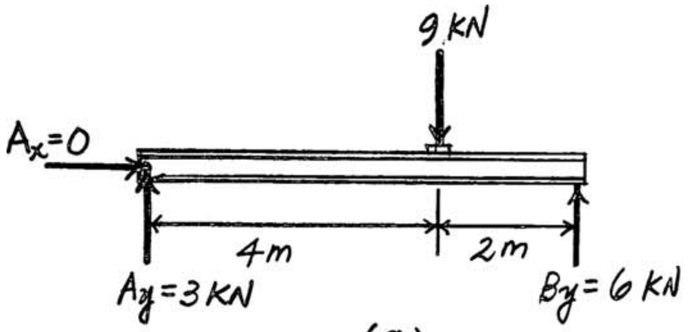
Region 4 m $< x \le 6$ m, Fig. *c*

$+\uparrow \Sigma F_y = 0; \quad V + 6 = 0 \qquad V = -6\text{ kN} \qquad (3)$

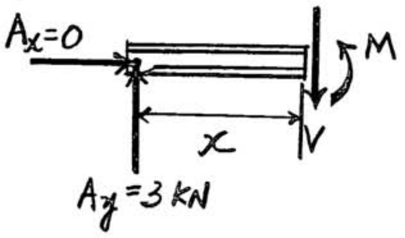
$\circlearrowleft + \Sigma M = 0; \; 6(6 - x) - M = 0 \qquad M = \{36 - 6x\}\text{ kN}\cdot\text{m} \qquad (4)$

The shear and moment diagrams in Figs. *d* and *e* are plotted using Eqs. (1) and (3), and Eqs. (3) and (4), respectively. The values of the moment at $x = 4$ m are evaluated using either Eqs. (2) or (4),

$$M\big|_{x=4\text{ m}} = 3(4) = 12\text{ kN}\cdot\text{m or } M\big|_{x=4\text{m}} = 36 - 6(4) = 12\text{ kN}\cdot\text{m}$$

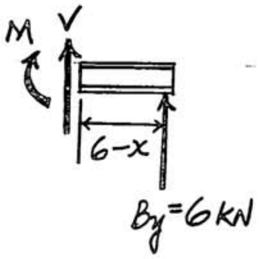


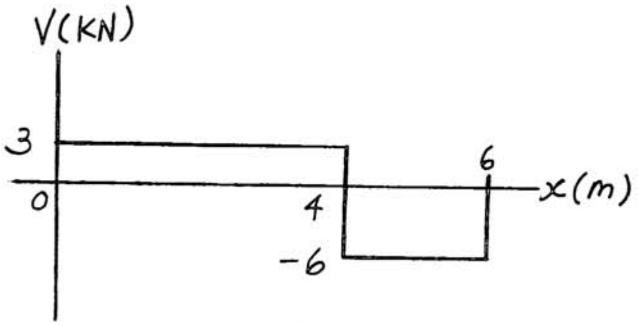
(a)



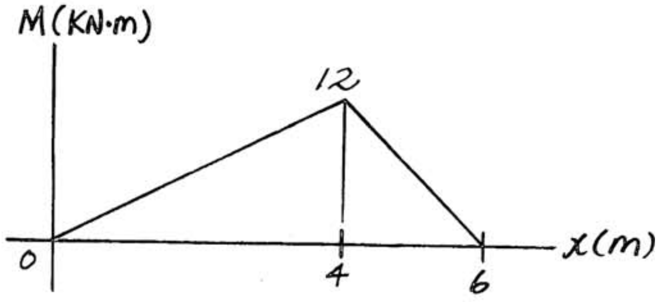
$0 \le x < 4\text{m}$ (b)

(C) $4\text{m} < x \le 6\text{m}$



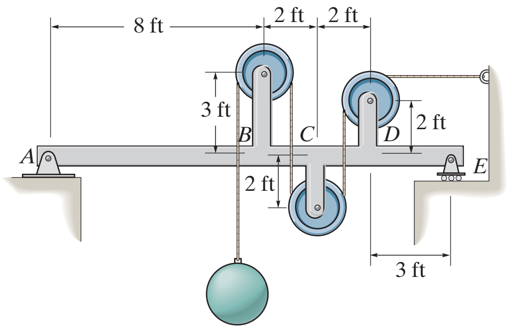


(d)



(e)

7–42. Draw the shear and moment diagrams for the beam *ABCDE*. All pulleys have a radius of 1 ft. Neglect the weight of the beam and pulley arrangement. The load weighs 500 lb.

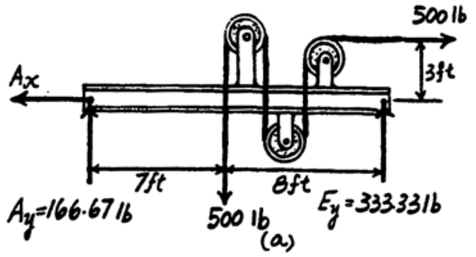


Support Reactions : From FBD (a),

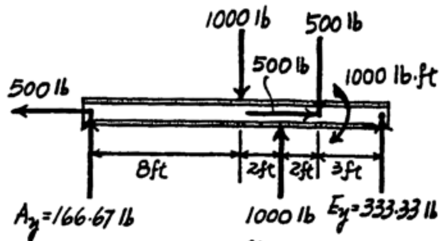
$$\curvearrowleft + \Sigma M_A = 0; \quad E_y(15) - 500(7) - 500(3) = 0 \quad E_y = 333.33 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 333.33 - 500 = 0 \quad A_y = 166.67 \text{ lb}$$

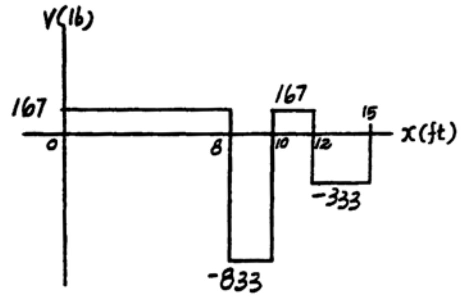
Shear and Moment Diagrams : The load on the pulley at *D* can be replaced by equivalent force and couple moment at *D* as shown on FBD (b).

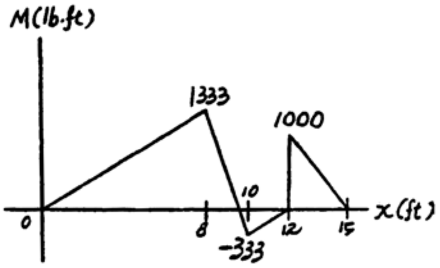


(a)

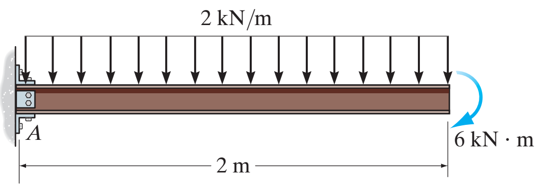


(b)





7–43. Draw the shear and moment diagrams for the cantilever beam.



The free - body diagram of the beam's right segment sectioned through an arbitrary point shown in Fig. *a* will be used to write the shear and moment equations of the beam.

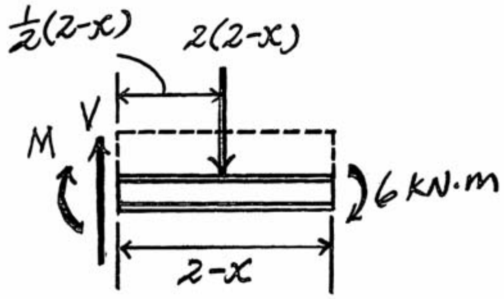
$$+\uparrow \Sigma F_y = 0; \qquad V - 2(2-x) = 0 \qquad V = \{4 - 2x\}\ \text{kN} \qquad (1)$$

$$+\Sigma M = 0; \ -M - 2(2-x)\left[\frac{1}{2}(2-x)\right] - 6 = 0 \qquad M = \{-x^2 + 4x - 10\}\ \text{kN}\cdot\text{m} \qquad (2)$$

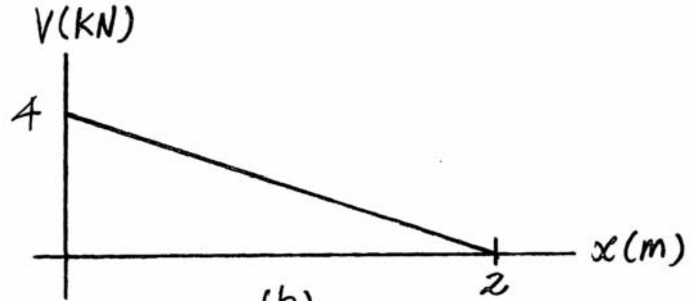
The shear and moment diagrams shown in Figs. *b* and *c* are plotted using Eqs. (1) and (2), respectively. The value of the shear and moment at $x = 0$ is evaluated using Eqs. (1) and (2).

$$V\big|_{x=0} = 4 - 2(0) = 4\ \text{kN}$$

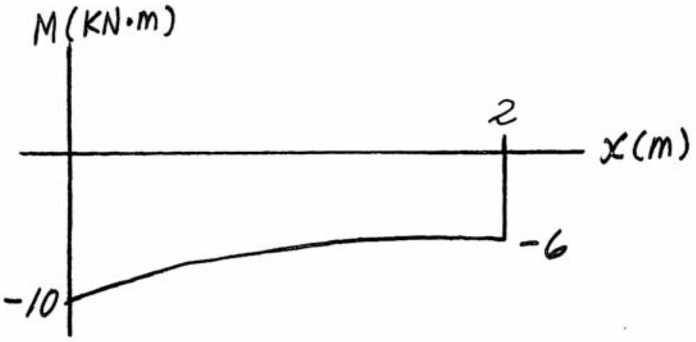
$$M\big|_{x=0} = [-0 + 4(0) - 10] = -10\ \text{kN}\cdot\text{m}$$



(a)

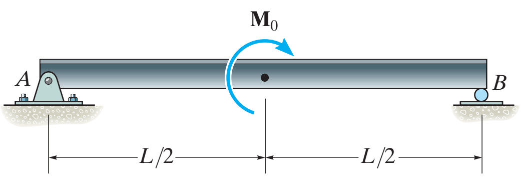


(b)



(c)

***7–44.** Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $M_0 = 500\ \text{N}\cdot\text{m}$, $L = 8$ m.



(a) $0 \le x < \frac{L}{2}$

$+\uparrow\Sigma F_y = 0;\quad -\frac{M_0}{L} - V = 0$

$V = -\frac{M_0}{L}$ **Ans**

$\circlearrowleft +\Sigma M = 0;\quad M + \frac{M_0}{L}x = 0$

$M = -\frac{M_0}{L}x$ **Ans**

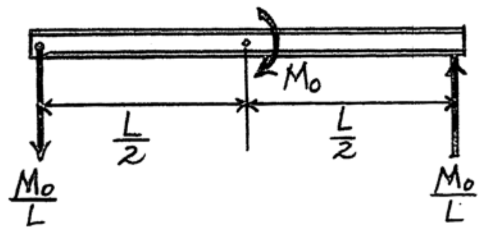
$\frac{L}{2} < x \le L$

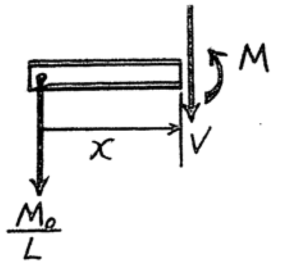
$+\uparrow\Sigma F_y = 0;\quad -\frac{M_0}{L} - V = 0$

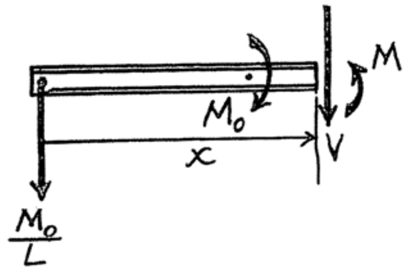
$V = -\frac{M_0}{L}$ **Ans**

$\circlearrowleft +\Sigma M = 0;\quad M + \frac{M_0}{L}x - M_0 = 0$

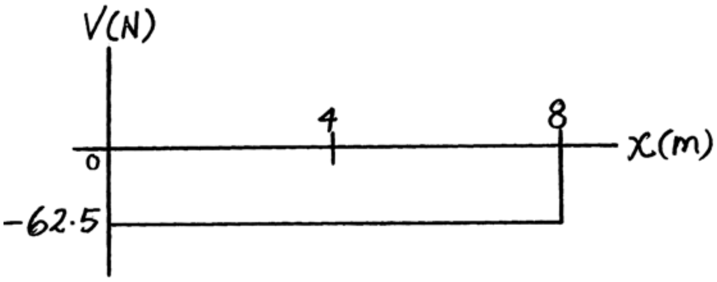
$M = M_0\left(1 - \frac{x}{L}\right)$ **Ans**

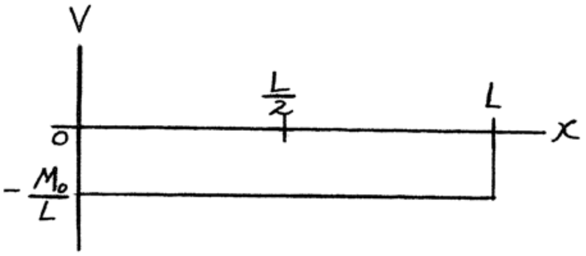


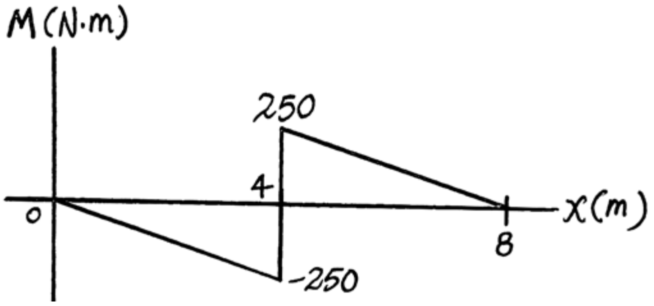


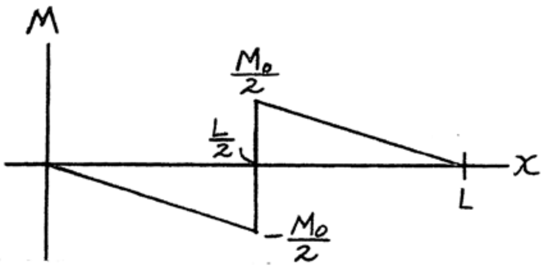


(b) When $M_0 = 500\ \text{N}\cdot\text{m}$, and $L = 8$ m

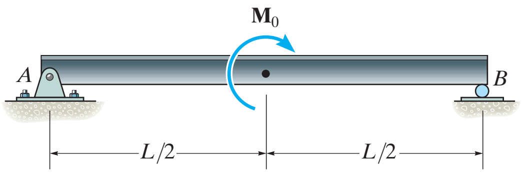




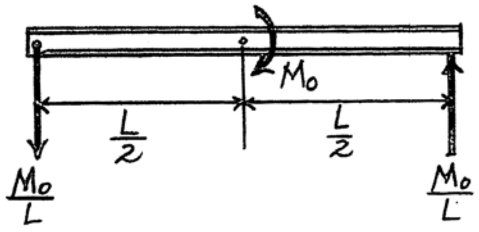




•7–45. If $L = 9$ m, the beam will fail when the maximum shear force is $V_{\max} = 5$ kN or the maximum bending moment is $M_{\max} = 22$ kN · m. Determine the largest couple moment M_0 the beam will support.



(a) $0 \le x < \frac{L}{2}$

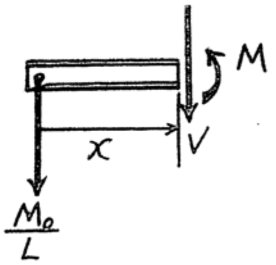


$+\uparrow \Sigma F_y = 0; \quad -\frac{M_0}{L} - V = 0$

$V = -\frac{M_0}{L}$ Ans

$\circlearrowleft + \Sigma M = 0; \quad M + \frac{M_0}{L}x = 0$

$M = -\frac{M_0}{L}x$ Ans



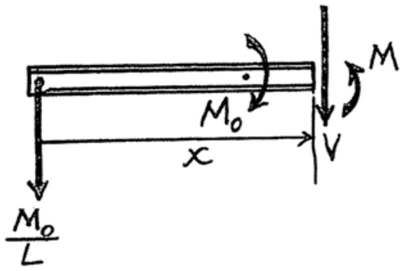
$\frac{L}{2} < x \le L$

$+\uparrow \Sigma F_y = 0; \quad -\frac{M_0}{L} - V = 0$

$V = -\frac{M_0}{L}$ Ans

$\circlearrowleft + \Sigma M = 0; \quad M + \frac{M_0}{L}x - M_0 = 0$

$M = M_0\left(1 - \frac{x}{L}\right)$ Ans



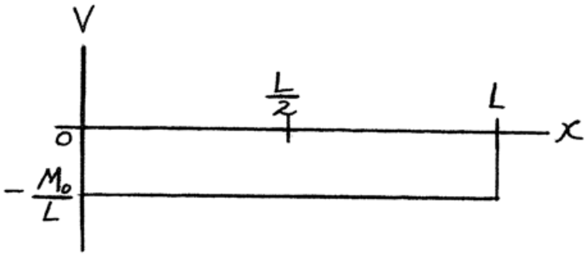
(b) When $M_0 = 500$ N · m, and $L = 8$ m

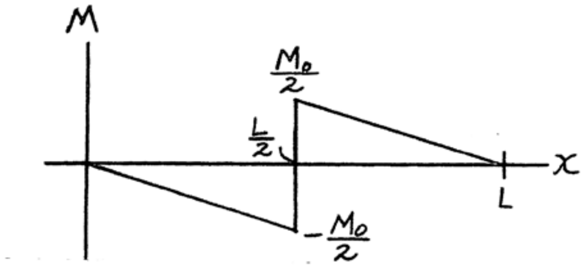
$V_{\max} = \frac{M_0}{L}; \quad 5 = \frac{M_0}{9}; \quad M_0 = 45$ kN · m

$M_{\max} = \frac{M_0}{2}; \quad 22 = \frac{M_0}{2}; \quad M_0 = 44$ kN · m

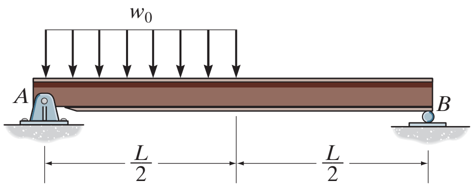
Thus,

$M_0 = 44$ kN · m Ans





7–46. Draw the shear and moment diagrams for the simply supported beam.



Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < L/2$ and $L/2 < x \le L$ of the beam. The free - body diagram of the beam's segments sectioned through arbitrary points in these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < \frac{L}{2}$, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{8}w_0L - w_0x - V = 0 \qquad V = w_0\left(\frac{3}{8}L - x\right) \quad (1)$$

$$\circlearrowleft + \Sigma M = 0; \quad M + w_0x\left(\frac{x}{2}\right) - \frac{3}{8}w_0L(x) = 0 \qquad M = \frac{w_0}{8}(3Lx - 4x^2) \quad (2)$$

Region $L/2 < x \le L$, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \quad V + \frac{w_0L}{8} = 0 \qquad V = -\frac{w_0L}{8} \quad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad \frac{w_0L}{8}(L - x) - M = 0 \qquad M = \frac{w_0L}{8}(L - x) \quad (4)$$

The shear diagram is plotted using Eqs. (1) and (3). The location at where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

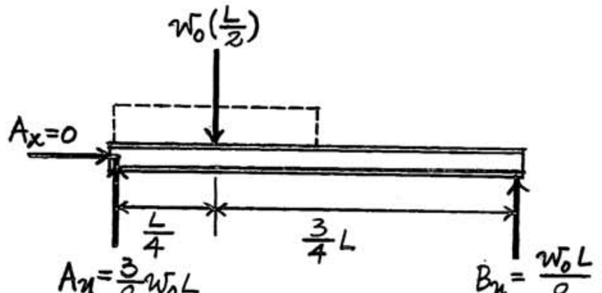
$$0 = w_0\left(\frac{3}{8}L - x\right) \qquad x = \frac{3}{8}L$$

The moment diagram is plotted using Eqs. (2) and (4). The value of the moment at $x = \frac{3}{8}L$ $(V = 0)$ can be evaluated using Eq. (2).

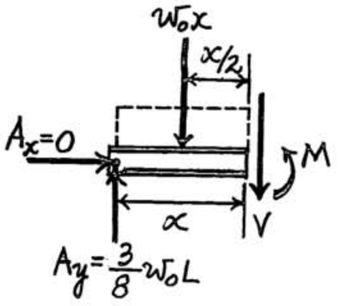
$$M\Big|_{x=\frac{3}{8}L} = \frac{w_0}{8}\left(3L\left(\frac{3}{8}L\right) - 4\left(\frac{3}{8}L\right)^2\right) = \frac{9}{128}w_0L^2$$

The value of the moment at $x = L/2$ is evaluated using either Eqs. (3) or (4).

$$M\Big|_{x=\frac{L}{2}} = \frac{w_0L}{8}\left(L - \frac{L}{2}\right) = \frac{w_0L^2}{16}$$

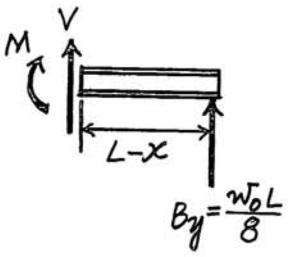


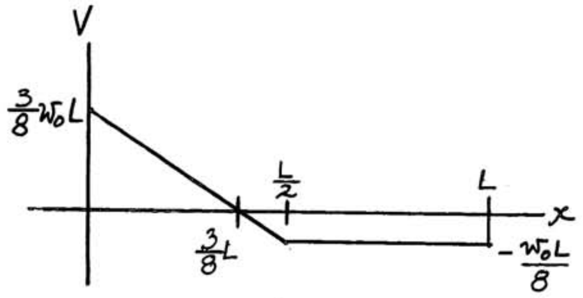
(a)



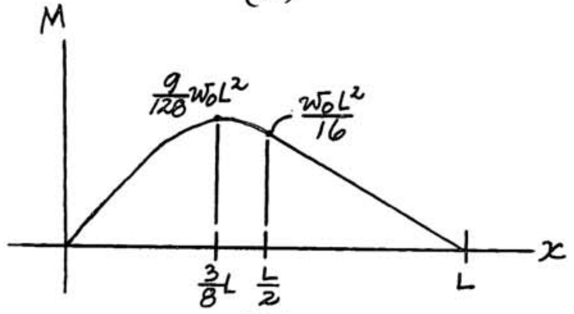
$0 \leq x < \frac{L}{2}$ (b)

(c) $\frac{L}{2} < x \leq L$



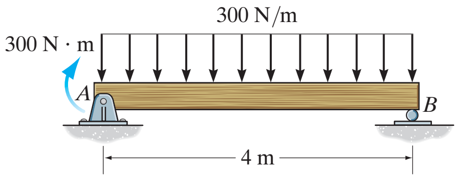


(d)



(e)

7–47. Draw the shear and moment diagrams for the simply supported beam.



The free - body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations.

$+\uparrow \Sigma F_y = 0; \qquad 525 - 300x - V = 0 \qquad V = \{525 - 300x\}\text{kN} \qquad (1)$

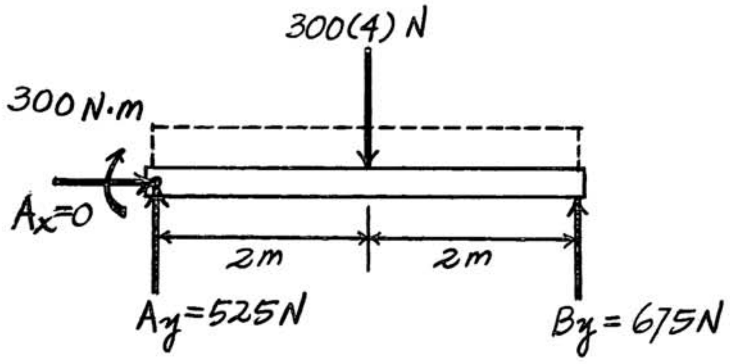
$+\Sigma M = 0; \; M + 300x\left(\frac{x}{2}\right) - 525x - 300 = 0 \quad M = \{-150x^2 + 525x + 300\}\text{N}\cdot\text{m} \qquad (2)$

The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively. The location where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

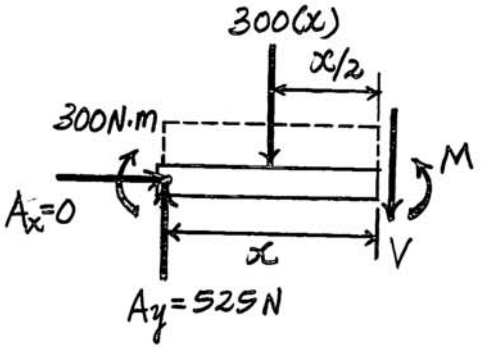
$0 = 525 - 300x \qquad x = 1.75\text{ m}$

The value of the moment at $x = 1.75\text{ m}$ ($V = 0$) can be evaluated using Eq. (2).

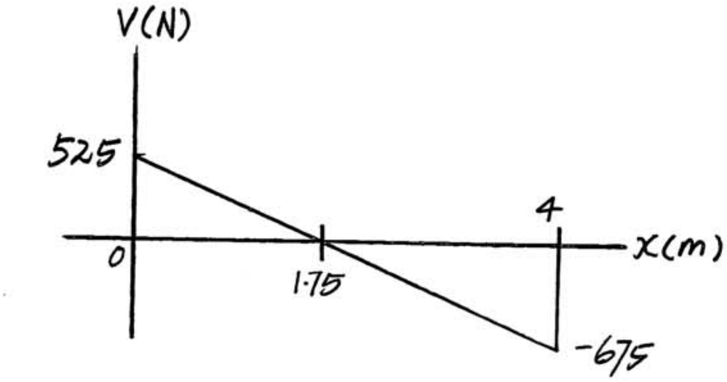
$M\big|_{x=1.75\text{ m}} = -150(1.75^2) + 525(1.75) + 300 = 759\text{ N}\cdot\text{m}$


300(4) N
300 N·m
$A_x = 0$
2m
2m
$A_y = 525N$
$B_y = 675N$

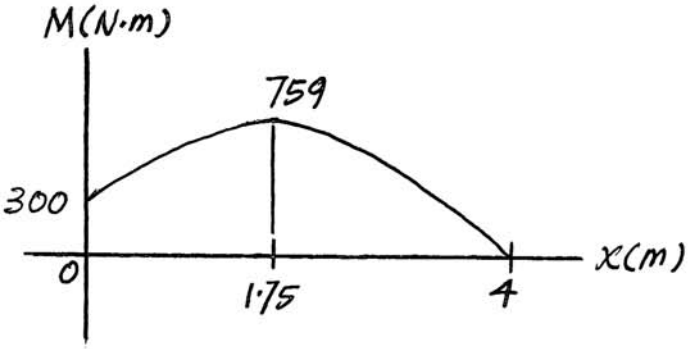
(a)


300(x)
x/2
300N·m
M
$A_x = 0$
x
V
$A_y = 525N$

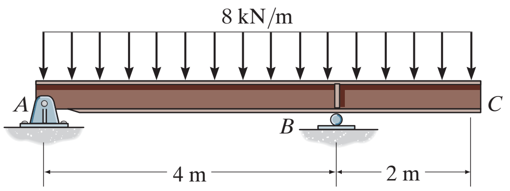
(b)


V(N)
525
4
0
x(m)
1·75
-675

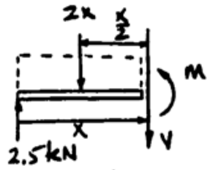
(c)


M(N·m)
759
300
0
1·75
4
x(m)

***7–48.** Draw the shear and moment diagrams for the overhang beam.

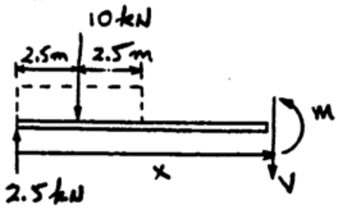


$0 \le x < 5\ \mathrm{m}$:



$+\uparrow \Sigma F_y = 0;\quad 2.5 - 2x - V = 0$

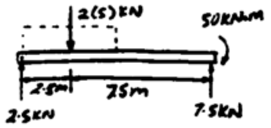
$V = 2.5 - 2x$

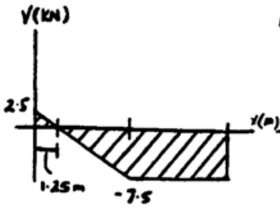


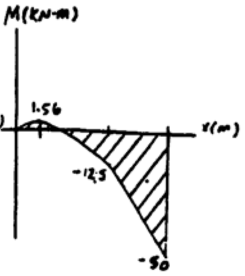
$\circlearrowleft + \Sigma M = 0;\quad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$M = 2.5x - x^2$

$5\ \mathrm{m} < x \le 10\ \mathrm{m}$:







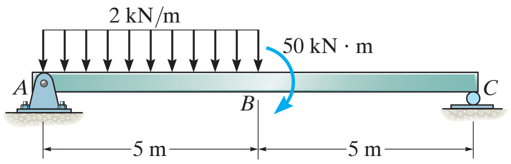
$+\uparrow \Sigma F_y = 0;\quad 2.5 - 10 - V = 0$

$V = -7.5$

$\circlearrowleft + \Sigma M = 0;\quad M + 10(x - 2.5) - 2.5x = 0$

$M = -7.5x + 25$

•7–49. Draw the shear and moment diagrams for the beam.



$0 \leq x < 5\text{ m}$:

$+\uparrow \Sigma F_y = 0; \quad 2.5 - 2x - V = 0$

$V = 2.5 - 2x$

$\circlearrowleft + \Sigma M = 0; \quad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$M = 2.5x - x^2$

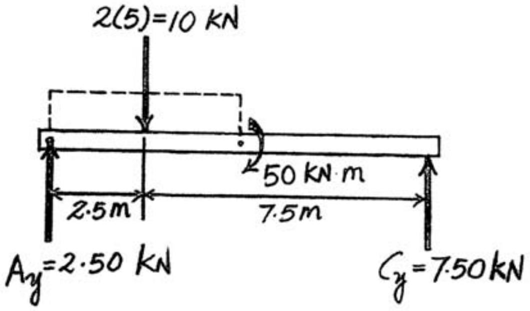
$5\text{ m} < x < 10\text{ m}$:

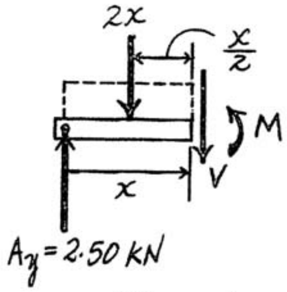
$+\uparrow \Sigma F_y = 0; \quad 2.5 - 10 - V = 0$

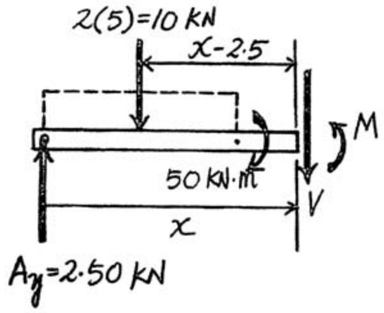
$V = -7.5$

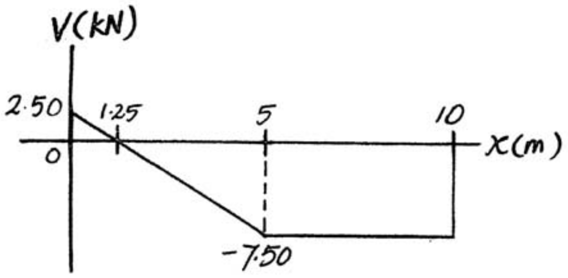
$\circlearrowleft + \Sigma M = 0; \quad M + 10(x - 2.5) - 2.5x - 50 = 0$

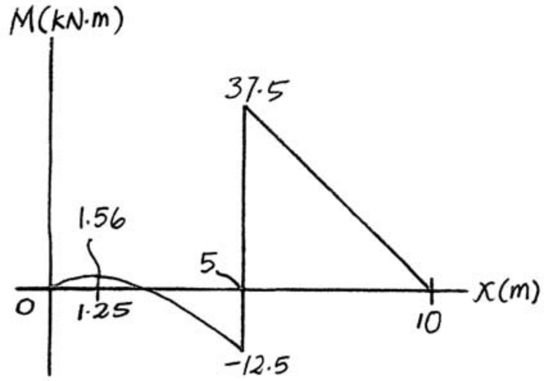
$M = -7.5x + 75$



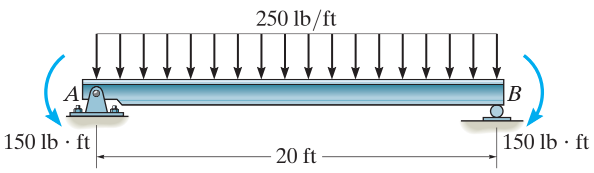








7–50. Draw the shear and moment diagrams for the beam.



$\circlearrowleft + \Sigma M_A = 0; \quad -5000(10) + B_y(20) = 0$

$B_y = 2500 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

$+\uparrow \Sigma F_y = 0; \quad A_y - 5000 + 2500 = 0$

$A_y = 2500 \text{ lb}$

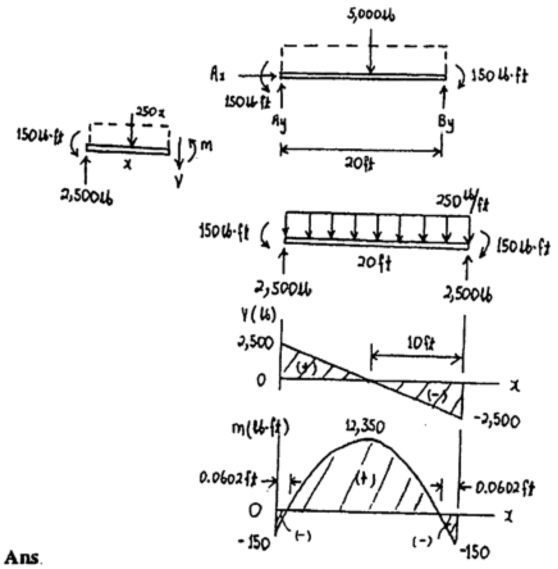
For $0 \le x \le 20 \text{ ft}$

$+\uparrow \Sigma F_y = 0; \quad 2500 - 250x - V = 0$

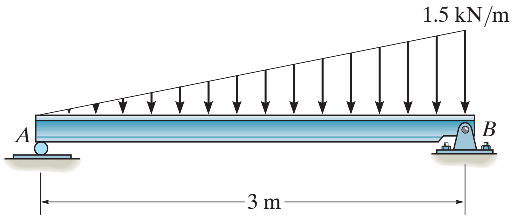
$V = 250(10 - x)$ **Ans**

$\circlearrowleft + \Sigma M = 0; \quad -2500(x) + 150 + 250x\left(\frac{x}{2}\right) + M = 0$

$M = 25(100x - 5x^2 - 6)$ **Ans.**



7–51. Draw the shear and moment diagrams for the beam.

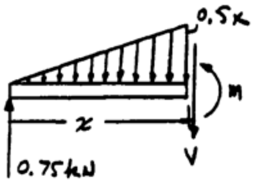


$+\uparrow \Sigma F_y = 0; \quad 0.75 - \frac{1}{2}x(0.5x) - V = 0$

$V = 0.75 - 0.25x^2$

$V = 0 = 0.75 - 0.25x^2$

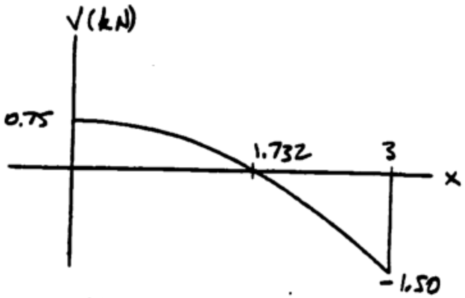
$x = 1.732 \text{ m}$

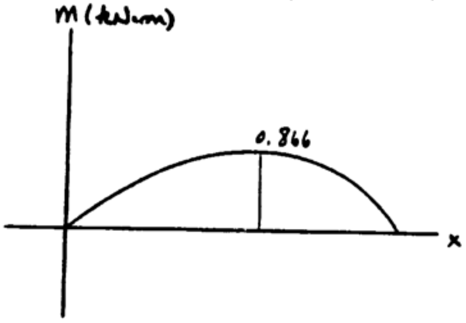


$\curvearrowleft + \Sigma M = 0; \quad M + \left(\frac{1}{2}\right)(0.5x)(x)\left(\frac{1}{3}x\right) - 0.75x = 0$

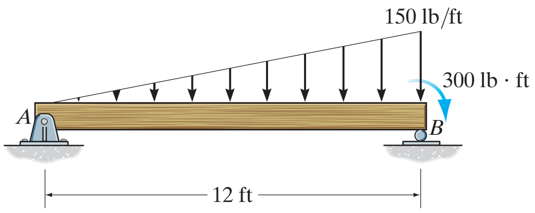
$M = 0.75x - 0.08333x^3$

$M_{max} = 0.75(1.732) - 0.08333(1.732)^3 = 0.866$





***7–52.** Draw the shear and moment diagrams for the simply supported beam.



The free - body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

$$w = 150\left(\frac{x}{12}\right) = 12.5x$$

Referring to Fig. *b*,

$+\uparrow \Sigma F_y = 0;\quad 275 - \frac{1}{2}(12.5x)(x) - V = 0\qquad V = \{275 - 6.25x^2\}\text{ lb}\qquad (1)$

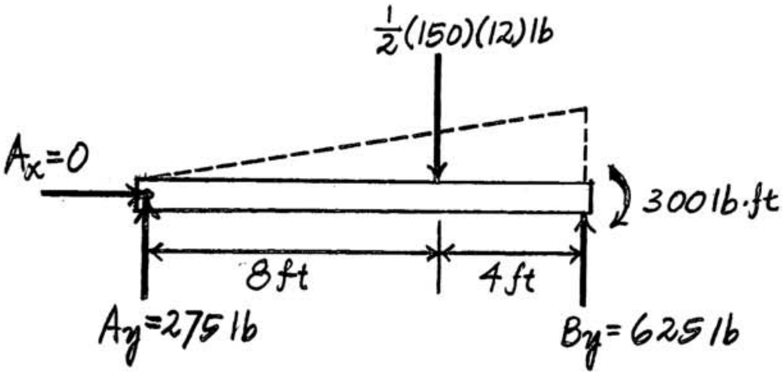
$\circlearrowleft + \Sigma M = 0;\quad M + \frac{1}{2}(12.5x)(x)\left(\frac{x}{3}\right) - 275x = 0\qquad M = \{275x - 2.083x^2\}\text{ lb}\cdot\text{ft}\qquad (2)$

The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively. The location where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

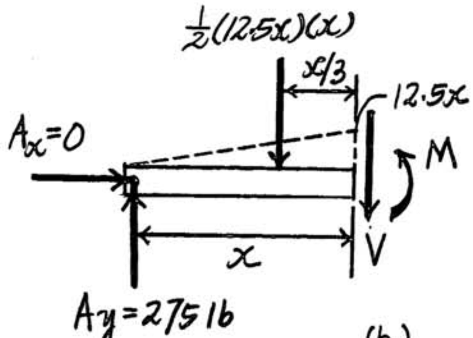
$0 = 275 - 6.25x^2\qquad x = 6.633\text{ ft}$

The value of the moment at $x = 6.633$ ft ($V = 0$) is evaluated using Eq. (2).

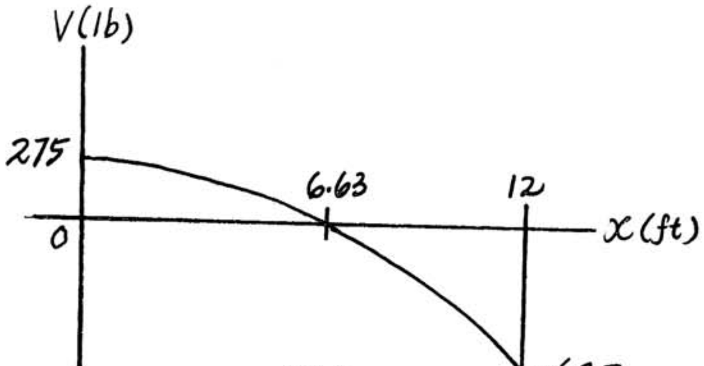
$$M\big|_{x=6.633\text{ ft}} = 275(6.633) - 2.083(6.633)^3 = 1216\text{ lb}\cdot\text{ft}$$


$\frac{1}{2}(150)(12)$ lb
$A_x = 0$
300 lb·ft
8 ft
4 ft
$A_y = 275$ lb
$B_y = 625$ lb

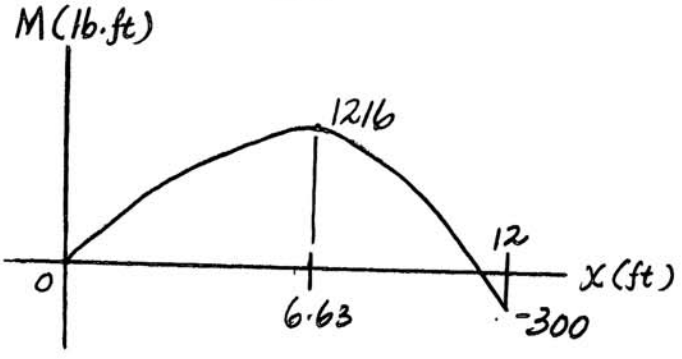
(a)


$\frac{1}{2}(12.5x)(x)$
$x/3$
$12.5x$
$A_x = 0$
M
V
x
$A_y = 275$ lb

(b)

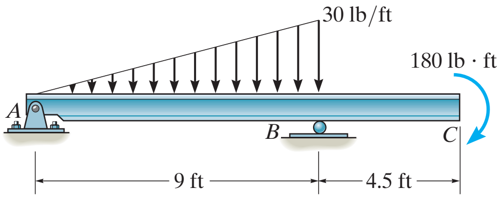

V (lb)
275
6.63
12
0
x (ft)
−625

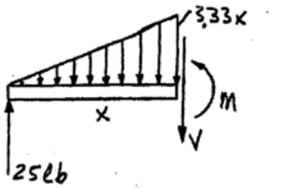
(c)


M (lb·ft)
1216
12
0
6.63
x (ft)
−300

(d)

•7–53. Draw the shear and moment diagrams for the beam.





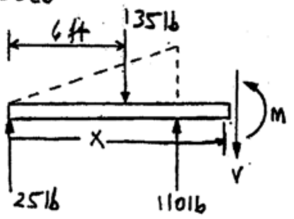
$0 \le x < 9$ ft:

$+\uparrow \Sigma F_y = 0; \quad 25 - \frac{1}{2}(3.33x)(x) - V = 0$

$V = 25 - 1.667x^2$ **Ans**

$V = 0 = 25 - 1.667x^2$

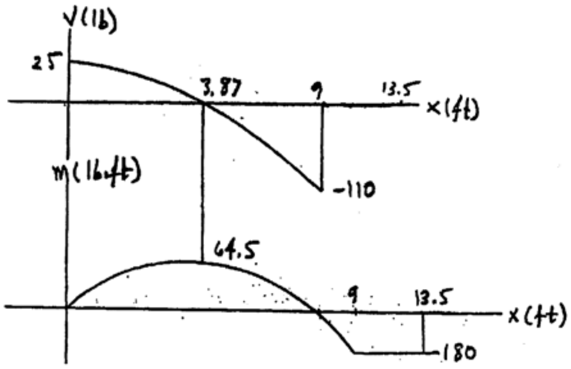
$x = 3.87$ ft



$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(3.33x)(x)\left(\frac{x}{3}\right) - 25x = 0$

$M = 25x - 0.5556x^3$ **Ans**

$M_{max} = 25(3.87) - 0.5556(3.87)^3 = 64.5$ lb · ft



9 ft $< x < 13.5$ ft:

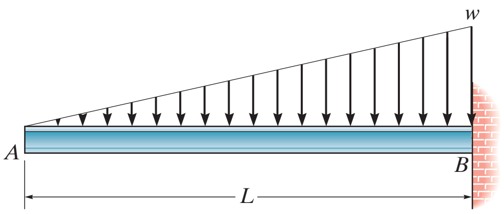
$+\uparrow \Sigma F_y = 0; \quad 25 - 135 + 110 - V = 0$

$V = 0$ **Ans**

$\circlearrowleft + \Sigma M = 0; \quad -25x + 135(x - 6) - 110(x - 9) + M = 0$

$M = -180$ **Ans**

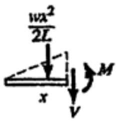
7–54. If $L = 18$ ft, the beam will fail when the maximum shear force is $V_{\max} = 800$ lb, or the maximum moment is $M_{\max} = 1200$ lb · ft. Determine the largest intensity w of the distributed loading it will support.



For $0 \leq x \leq L$

$+\uparrow \Sigma F_y = 0;$ $\quad V = -\dfrac{wx^2}{2L}$

$\circlearrowleft + \Sigma M = 0;$ $\quad M = -\dfrac{wx^3}{6L}$



$$V_{max} = \frac{-wL}{2}$$

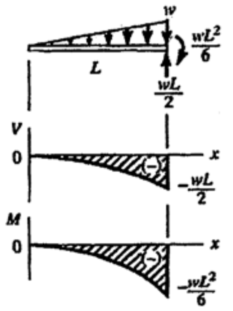
$$-800 = \frac{-w(18)}{2}$$

$w = 88.9$ lb/ft

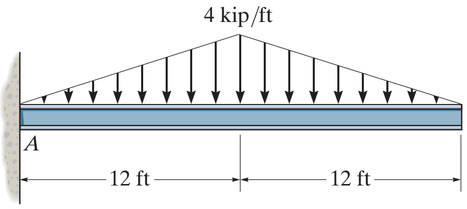
$$M_{max} = -\frac{wL^2}{6};$$

$$-1200 = \frac{-w(18)^2}{6}$$

$w = 22.2$ lb/ft **Ans**



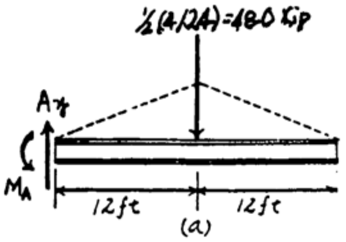
7–55. Draw the shear and moment diagrams for the beam.



Support Reactions : From FBD (a),

$\curvearrowleft + \Sigma M_A = 0; \quad M_A - 48.0(12) = 0 \quad M_A = 576 \text{ kip} \cdot \text{ft}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 48.0 = 0 \quad A_y = 48.0 \text{ kip}$



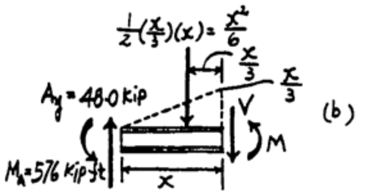
Shear and Moment Functions : For $0 \le x < 12$ ft [FBD (b)],

$+\uparrow \Sigma F_y = 0; \quad 48.0 - \frac{x^2}{6} - V = 0$

$V = \left\{48.0 - \frac{x^2}{6}\right\}$ kip **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + \frac{x^2}{6}\left(\frac{x}{3}\right) + 576 - 48.0x = 0$

$M = \left\{48.0x - \frac{x^3}{18} - 576\right\}$ kip · ft **Ans**



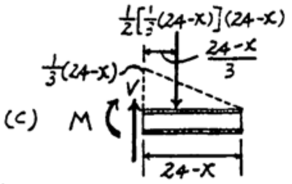
For $12 \text{ ft} < x \le 24$ ft [FBD (c)],

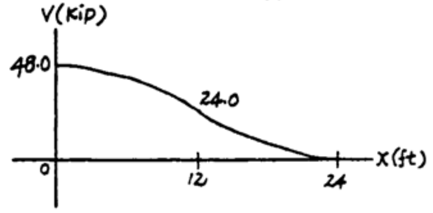
$+\uparrow \Sigma F_y = 0; \quad V - \frac{1}{2}\left[\frac{1}{3}(24-x)\right](24-x) = 0$

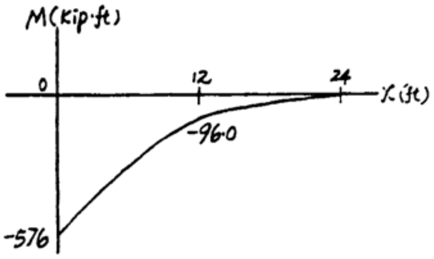
$V = \left\{\frac{1}{6}(24-x)^2\right\}$ kip **Ans**

$\curvearrowleft + \Sigma M = 0; \quad -\frac{1}{2}\left[\frac{1}{3}(24-x)\right](24-x)\left(\frac{24-x}{3}\right) - M = 0$

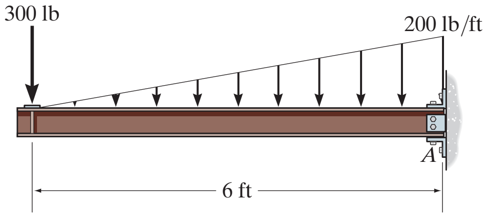
$M = \left\{-\frac{1}{18}(24-x)^3\right\}$ kip · ft **Ans**







***7–56.** Draw the shear and moment diagrams for the cantilevered beam.



The free - body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

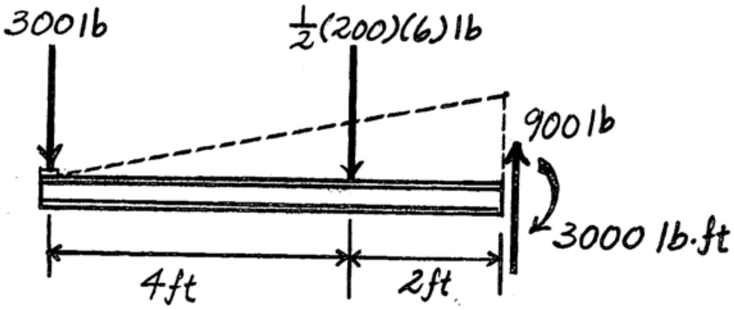
$$w = 200\left(\frac{x}{6}\right) = 33.33x$$

Referring to Fig. *b*,

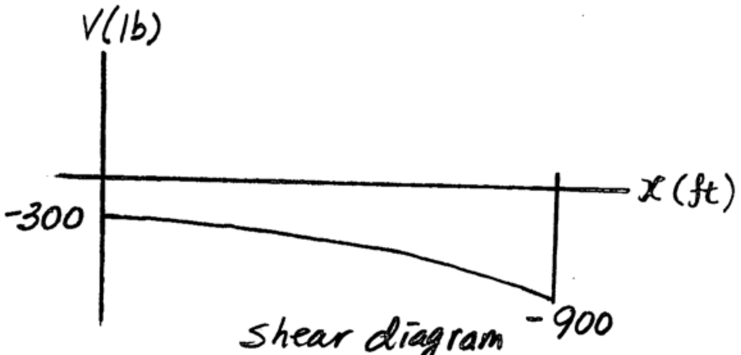
$+\uparrow \Sigma F_y = 0; \quad -300 - \frac{1}{2}(33.33x)(x) - V = 0 \qquad V = \{-300 - 16.67x^2\}\text{ lb} \qquad (1)$

$\curvearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(33.33x)(x)\left(\frac{x}{3}\right) + 300x = 0 \qquad M = \{-300x - 5.556x^3\}\text{ lb}\cdot\text{ft} \qquad (2)$

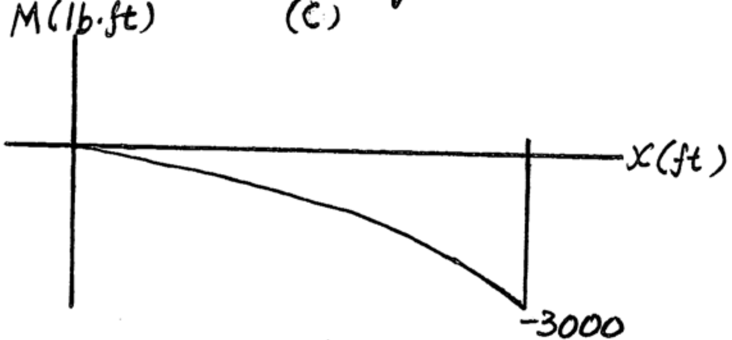
The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively.



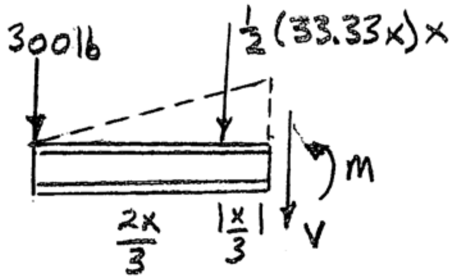
(a)



Shear diagram
(c)

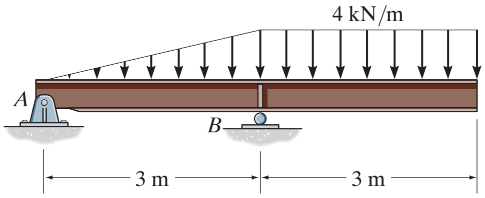


Moment diagram
(d)



(b)

•7–57. Draw the shear and moment diagrams for the overhang beam.



Since the loading is discontinuous at support *B*, the shear and moment equations must be written for regions $0 \le x < 3$ m and $3\text{ m} < x \le 6$ m of the beam. The free - body diagram of the beam' s segment sectioned through an arbitrary point within these two regions is shown in Figs. *b* and *c*.

Region $0 \le x < 3$ m, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad -4-\frac{1}{2}\left(\frac{4}{3}x\right)(x)-V=0 \qquad V=\left\{-\frac{2}{3}x^2-4\right\}\text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \quad M+\frac{1}{2}\left(\frac{4}{3}x\right)(x)\left(\frac{x}{3}\right)+4x=0 \qquad M=\left\{-\frac{2}{9}x^3-4x\right\}\text{ kN}\cdot\text{m} \qquad (2)$$

Region $3\text{ m} < x \le 6$ m, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \qquad V-4(6-x)=0 \qquad V=\{24-4x\}\text{kN} \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad -M-4(6-x)\left[\frac{1}{2}(6-x)\right]=0 \qquad M=\{-2(6-x)^2\}\text{ kN}\cdot\text{m} \qquad (4)$$

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The value of shear just to the left and just to the right of the support is evaluated using Eqs. (1) and (3), respectively.

$$V\Big|_{x=3\text{ m}-} = -\frac{2}{3}(3^2)-4=-10\text{ kN}$$

$$V\Big|_{x=3\text{ m}+} = 24-4(3)=12\text{ kN}$$

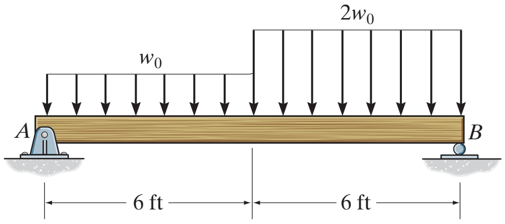
The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at support *B* is evaluated using either Eq. (2) or Eq. (4).

$$M\Big|_{x=3\text{ m}} = -\frac{2}{9}(3^3)-4(3) = -18\text{ kN}\cdot\text{m}$$

or

$$M\Big|_{x=3\text{ m}} = -2(6-3)^2 = -18\text{ kN}\cdot\text{m}$$

7–58. Determine the largest intensity w_0 of the distributed load that the beam can support if the beam can withstand a maximum shear force of $V_{\max} = 1200$ lb and a maximum bending moment of $M_{\max} = 600$ lb · ft.



Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < 6$ ft and $6\text{ ft} < x \le 12$ ft of the beam. The free - body diagram of the beam' s segment sectioned through the arbitrary point within these two regions are shown in Figs. *b* and *c*.

Region $0 \le x \le 6$ft, Fig. *b*

$+\uparrow \Sigma F_y = 0; \qquad 7.5w_0 - w_0x - V = 0 \qquad V = w_0(7.5 - x) \qquad (1)$

$\circlearrowleft + \Sigma M = 0; \; M + w_0x\left(\dfrac{x}{2}\right) - 7.5w_0x = 0 \qquad M = \dfrac{w_0}{2}(15x - x^2) \qquad (2)$

Region $6\text{ ft} < x \le 12$ft, Fig. *c*

$+\uparrow \Sigma F_y = 0; \qquad 10.5w_0 - 2w_0(12 - x) + V = 0 \qquad V = w_0(13.5 - 2x) \qquad (3)$

$\circlearrowleft + \Sigma M = 0; \; 10.5w_0(12 - x) - 2w_0(12 - x)\left[\dfrac{1}{2}(12 - x)\right] - M = 0$

$M = w_0(-x^2 + 13.5x - 18) \qquad (4)$

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The value of the shear at $x = 6$ ft is evaluated using either Eq. (1) or Eq. (3).

$$V\big|_{x=6\text{ ft}} = w_0(7.5 - 6) = 1.5w_0$$

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (3).

$$0 = w_0(13.5 - 2x) \qquad x = 6.75\text{ft}$$

The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at $x = 6$ ft is evaluated using either Eqs. (2) or (4).

$$M\big|_{x=6\text{ ft}} = \frac{w_0}{2}(15.6 - 6^2) = 27w_0$$

The value of the moment at $x = 6.75$ ft (where $V = 0$) is evaluated using Eq. (4).

$$M\big|_{x=6.75\text{ ft}} = w_0\left[-6.75^2 + 13.5(6.75) - 18\right] = 27.5625w_0$$

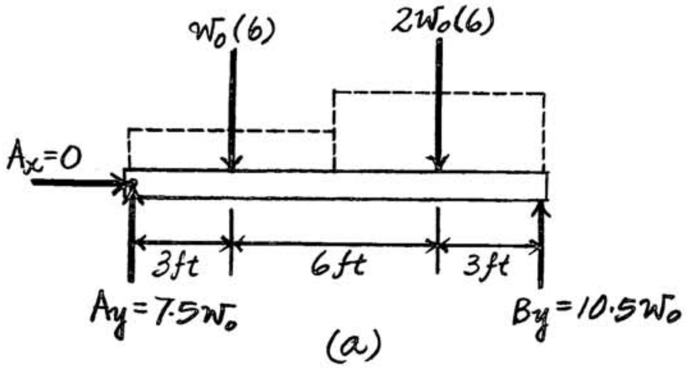
By observing the shear and moment diagrams, we notice that $V_{\max} = 10.5w_0$ and $M_{\max} = 27.56w_0$. Thus,

$V_{\max} = 1200 = 10.5w_0$

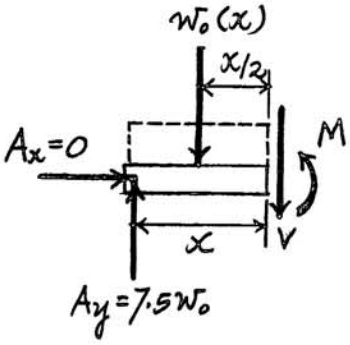
$w_0 = 114.29$ lb·ft

$M_{\max} = 600 = 27.56w_0$

$w_0 = 21.8$ lb/ft (control!) **Ans.**

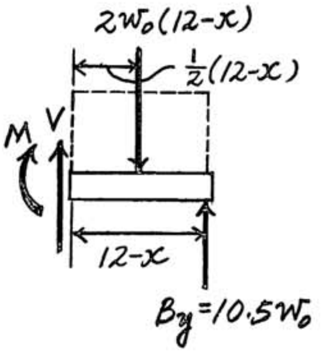


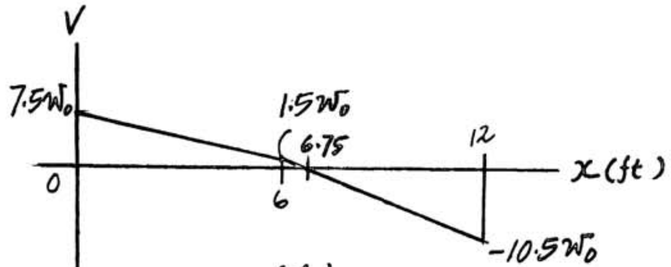
(a)



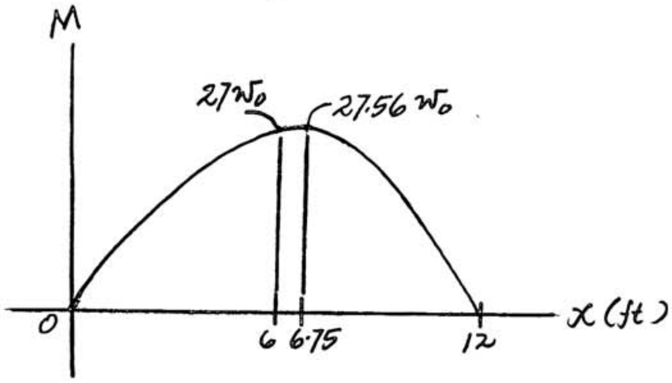
$0 \leq x < 6\text{ft}$ (b)

(c) $6\text{ft} < x \leq 12\text{ft}$



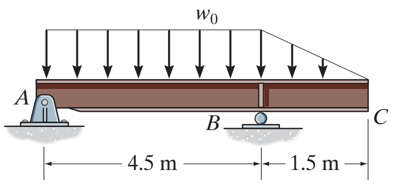


(d)



(e)

7–59. Determine the largest intensity w_0 of the distributed load that the beam can support if the beam can withstand a maximum bending moment of $M_{\max} = 20\ \text{kN}\cdot\text{m}$ and a maximum shear force of $V_{\max} = 80$ kN.



Since the loading is discontinuous at support *B*, the shear and moment equations must be written for regions $0 \le x < 4.5$ m and $4.5\text{ m} < x \le 6$ m of the beam. The free - body diagram of the beam' s segment sectioned through the arbitrary points within these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < 4.5$ m, Fig. *b*

$+\uparrow \Sigma F_y = 0; \qquad 2.167w_0 - w_0x - V = 0 \qquad V = w_0(2.167 - x) \qquad (1)$

$\circlearrowleft + \Sigma M = 0; \; M + w_0x\left(\frac{x}{2}\right) - 2.167w_0x = 0 \qquad M = w_0(2.167x - 0.5x^2) \qquad (2)$

Region $4.5\text{ m} < x \le 6$ m, Fig. *c*

$+\uparrow \Sigma F_y = 0; \qquad V - \frac{1}{2}\left[\left(\frac{6-x}{1.5}\right)w_0\right](6-x) = 0 \qquad V = \frac{w_0}{3}(6-x)^2 \qquad (3)$

$\circlearrowleft + \Sigma M = 0; \; -M - \frac{1}{2}\left[\left(\frac{6-x}{1.5}\right)w_0\right](6-x)\left[\frac{1}{3}(6-x)\right] = 0$

$$M = -\frac{w_0}{9}(6-x)^3 \qquad (4)$$

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The value of the shear just to the left and right of support *B* is evaluated using either Eq. (1) or Eq. (3), respectively.

$$V\big|_{x=4.5\text{ m}-} = w_0(2.167 - 4.5) = -2.333w_0$$

$$V\big|_{x=4.5\text{ m}+} = \frac{w_0}{3}(6-4.5)^2 = 0.75w_0$$

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

$$0 = w_0(2.167 - x) \qquad x = 2.167\text{ m}$$

The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at $x = 2.167$ m ($V = 0$) is evaluated using Eq. (2).

$$M\big|_{x=2.167\text{ m}} = w_0\left[2.167(2.167) - 0.5(2.167^2)\right] = 2.347w_0$$

The value of the moment at support *B* is evaluated using Eqs. (2) or (4).

$$M\big|_{x=4.5\text{ m}} = -\frac{w_0}{9}(6-4.5)^3 = -0.375w_0$$

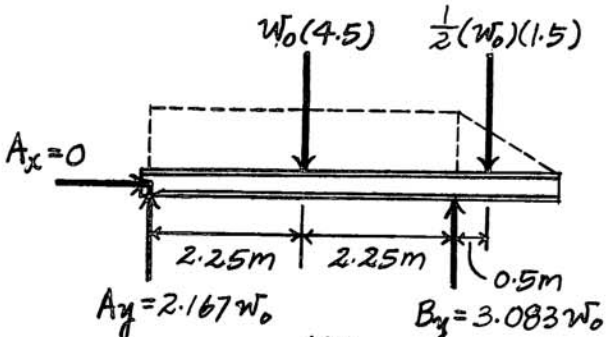
By observing the shear and moment diagrams, we notice that $V_{\max} = 2.333w_0$ and $M_{\max} = 2.347w_0$. Thus,

$V_{\max} = 80 = 2.333w_0$

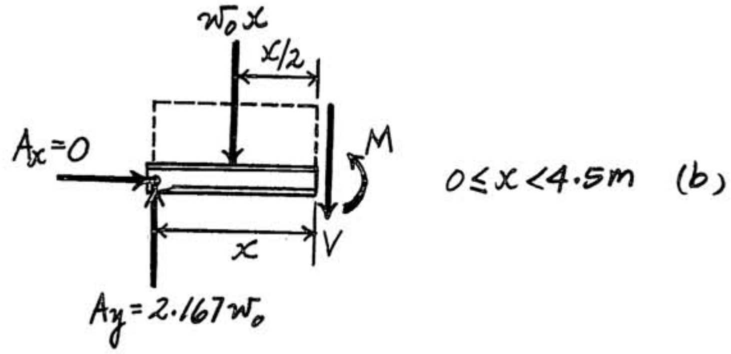
$w_0 = 34.29\text{ kN/m}$

$M_{\max} = 20 = 2.347w_0$

$w_0 = 8.52\text{ kN/m}$ (control!) **Ans.**

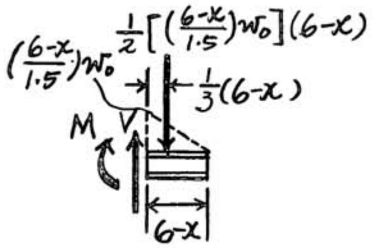


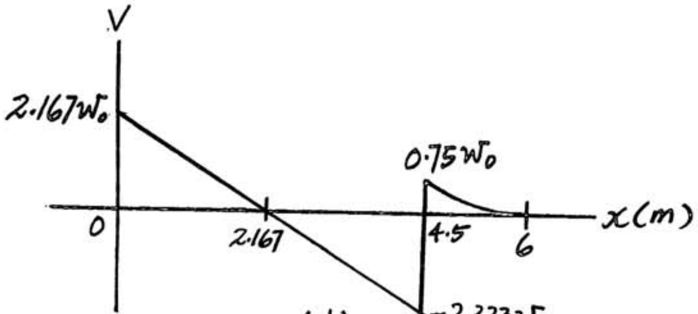
(a)



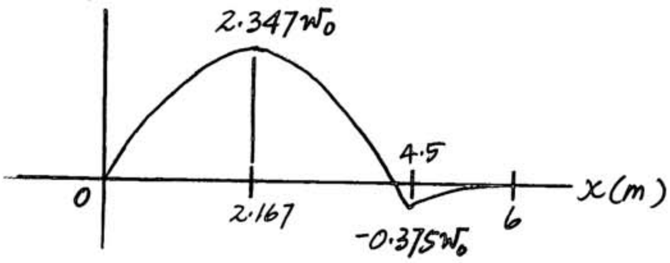
$0 \le x < 4.5\text{m}$ (b)

(C) $4.5\text{m} < x \le 6\text{m}$



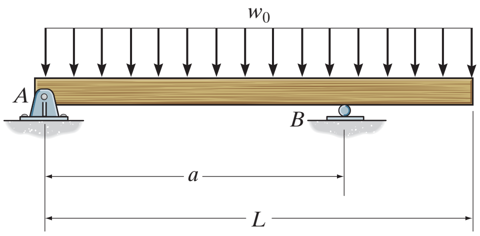


(d)



(e)

***7–60.** Determine the placement *a* of the roller support *B* so that the maximum moment within the span *AB* is equivalent to the moment at the support *B*.



Since the loading is discontinuous at support *B*, the shear and moment equations must be written for regions $0 \le x < a$ and $a < x \le L$. The free - body diagram of the beam' s segment sectioned through the arbitrary points within these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < a$, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0}{2a}(2aL - L^2) - w_0 x - V = 0 \qquad V = \frac{w_0}{2a}(2aL - L^2 - 2ax) \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \; M + w_0 x\left(\frac{x}{2}\right) - \frac{w_0}{2a}(2aL - L^2)x = 0 \qquad M = \frac{w_0}{2a}\left[(2aL - L^2)x - ax^2\right] \qquad (2)$$

Region $a < x \le L$, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \qquad V - w_0(L - x) = 0 \qquad V = w_0(L - x) \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \; -M - w_0(L - x)\left[\frac{1}{2}(L - x)\right] = 0 \quad M = -\frac{w_0}{2}(L - x)^2 \qquad (4)$$

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

$$0 = \frac{w_0}{2a}(2aL - L^2 - 2ax) \qquad x = \frac{2aL - L^2}{2a}$$

The maximum span moment occurs at the position at which $V = 0$. Thus, using Eq. (2), we obtain

$$\left(M_{\text{span}}\right)_{\max} = \frac{w_0}{2a}\left[(2aL - L^2)\left(\frac{2aL - L^2}{2a}\right) - a\left(\frac{2aL - L^2}{2a}\right)^2\right] = \frac{w_0}{8a^2}\left[\left(2aL - L^2\right)^2\right]$$

The support moment at *B* is evaluated using Eq. (2).

$$M_{\text{suppport}} = \frac{w_0}{2a}\left[(2aL - L^2)a - a^3\right] = \frac{w_0}{2}(2aL - L^2 - a^2) = -\frac{w_0}{2}(L - a)^2$$

The support moment at *B* can also be computed from Eq. (4).

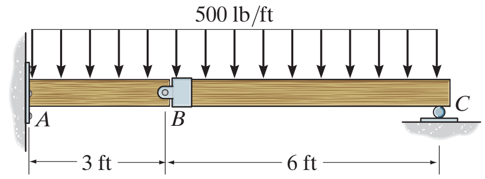
$$M_{\text{suppport}} = -\frac{w_0}{2}(L - a)^2$$

Here, we require $\left|(M_{\max})_{\text{span}}\right| = \left|M_{\text{support}}\right|$ Thus,

$$\frac{w_0}{8a^2}(2aL - L^2)^2 = \frac{w_0}{2}(L - a)^2$$

$$a = \frac{L}{\sqrt{2}} \qquad \textbf{Ans.}$$

•7–61. The compound beam is fix supported at *A*, pin connected at *B* and supported by a roller at *C*. Draw the shear and moment diagrams for the beam.



The support reactions at *A* and *C* and the interaction force at pin connection *B* are indicated on the free-body diagram of members *AB* and *BC* of the compound beam shown in Figs. *a* and *b*. Since the loading is continuous through the entire beam and the interaction force at the pin connection at *B* is internal to the beam, the shear and moment equations can be described by a single function. The free-body diagram of the beam's left segment sectioned through an arbitrary point is shown in Fig. *c*.

By referring to Fig. *c*, we have

$+\uparrow \Sigma F_y = 0; \quad 3000 - 500x - V = 0 \qquad V = \{3000 - 500x\}\ \text{lb} \qquad (1)$

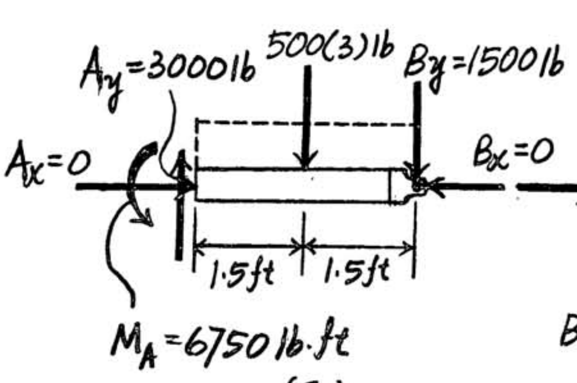
$\curvearrowleft + \Sigma M = 0; \quad M + 500x\left(\frac{x}{2}\right) + 6750 - 3000x = 0 \qquad M = \{3000x - 250x^2 - 6750\}\ \text{lb}\cdot\text{ft} \qquad (2)$

The shear and moment diagram shown in Figs. *d* and *e* are plotted using Eqs. (1) and (2), respectively. The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

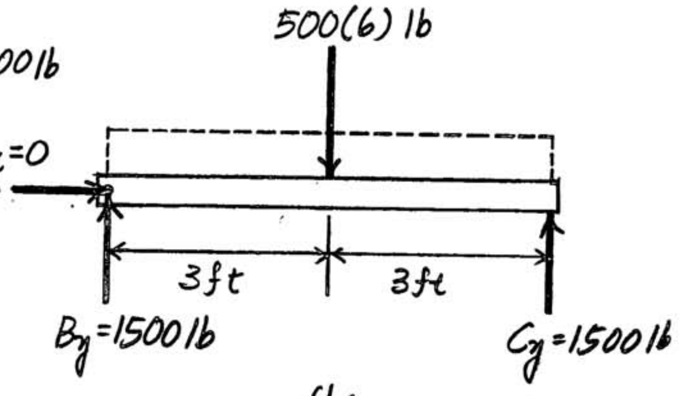
$0 = 3000 - 500x \qquad x = 6\ \text{ft}$

The value of the moment at $x = 6$ ft ($V = 0$) is computed using Eq. (2).

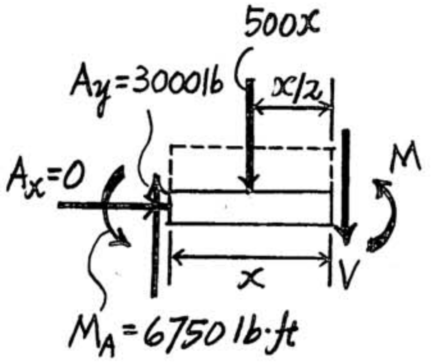
$M\big|_{x=6\ \text{ft}} = 3000(6) - 250(6^2) - 6750 = 2250\ \text{lb}\cdot\text{ft}$


$A_y = 3000$ lb
500(3) lb
$B_y = 1500$ lb
$A_x = 0$
1.5 ft
1.5 ft
$M_A = 6750$ lb·ft

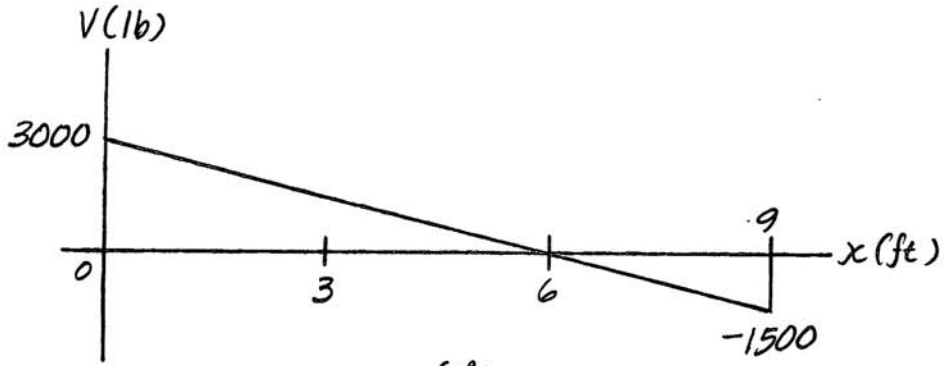
(a)


500(6) lb
$B_x = 0$
3 ft
3 ft
$B_y = 1500$ lb
$C_y = 1500$ lb

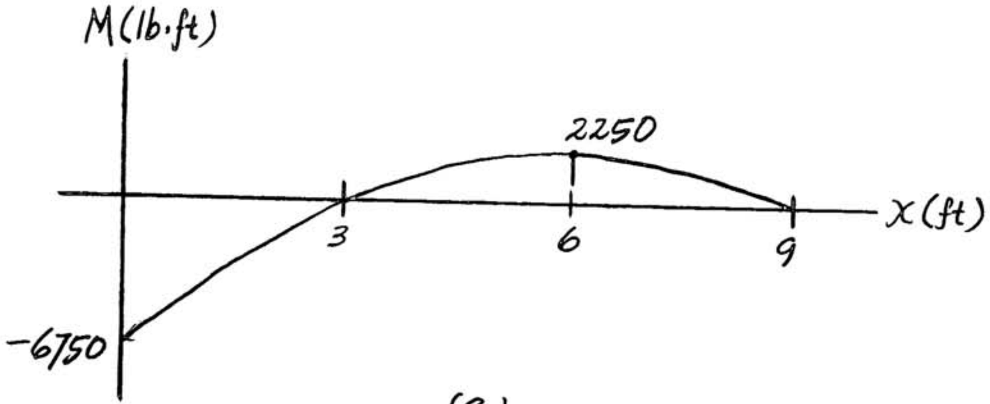
(b)


500x
$A_y = 3000$ lb
x/2
$A_x = 0$
M
x
V
$M_A = 6750$ lb·ft

(c)

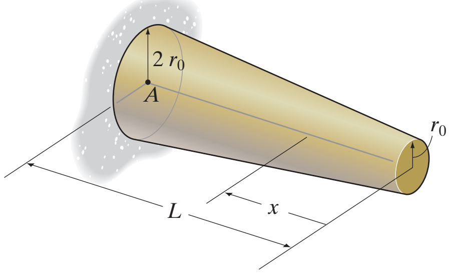

V (lb)
3000
0
3
6
9
x (ft)
−1500

(d)


M (lb·ft)
2250
3
6
9
x (ft)
−6750

(e)

7–62. The frustum of the cone is cantilevered from point A. If the cone is made from a material having a specific weight of γ, determine the internal shear force and moment in the cone as a function of x.



Using the similar triangles shown in Fig. *a*,

$$r = r_0 + \frac{r_0}{L}x = \frac{r_0}{L}(L+x)$$

$$\frac{L'}{r_0} = \frac{L+L'}{2r_0} \qquad L' = L$$

Thus, the volume of the frustrum of the cone shown shaded in Fig. *a* is

$$V = \frac{1}{3}\pi\left[\frac{r_0}{L}(L+x)\right]^2(L+x) - \frac{1}{3}\pi r_0{}^2 L$$

$$= \frac{\pi r_0{}^2}{3L^2}\left[(L+x)^3 - L^3\right]$$

The weight of the frustrum is

$$W = \gamma V = \frac{\pi\gamma r_0{}^2}{3L^2}\left[(L+x)^3 - L^3\right]$$

The location $\bar{x}$ of the center of gravity of the frustrum is

$$\bar{x} = \frac{\frac{1}{3}\pi\left[\frac{r_0}{L}(L+x)\right]^2(L+x)\left[\frac{1}{4}(L+x)\right] - \frac{1}{3}\pi r_0{}^2 L\left(x+\frac{L}{4}\right)}{\frac{\pi r_0{}^2}{3L^2}\left[(L+x)^3 - L^3\right]} = \frac{(L+x)^4 - L^3(4x+L)}{4\left[(L+x)^3 - L^3\right]}$$

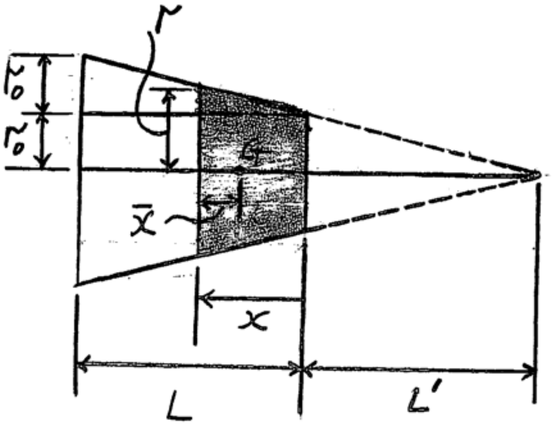
Using these results and referring to the free - body diagram of the frustrum shown in Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad V - \frac{\pi\gamma r_0{}^2}{3L^2}\left[(L+x)^3 - L^3\right] = 0$$

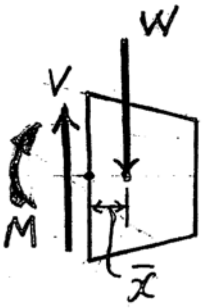
$$V = \frac{\pi\gamma r_0{}^2}{3L^2}\left[(L+x)^3 - L^3\right] \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \; -M - \left\{\frac{\pi\gamma r_0{}^2}{3L^2}\left[(L+x)^3 - L^3\right]\right\}\left\{\frac{(L+x)^4 - L^3(4x+L)}{4[(L+x)^3 - L^3]}\right\} = 0$$

$$M = -\frac{\pi\gamma r_0{}^2}{12L^2}\left[(L+x)^4 - L^3(4x+L)\right] \qquad \textbf{Ans.}$$

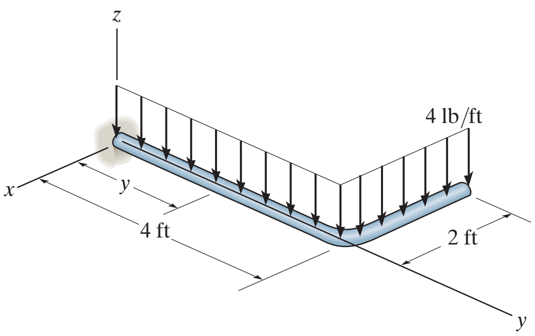


(a)



(b)

7–63. Express the internal shear and moment components acting in the rod as a function of y, where $0 \leq y \leq 4$ ft.



Shear and Moment Functions :

$\Sigma F_x = 0;$ $V_x = 0$ **Ans**

$\Sigma F_z = 0;$ $V_z - 4(4-y) - 8.00 = 0$

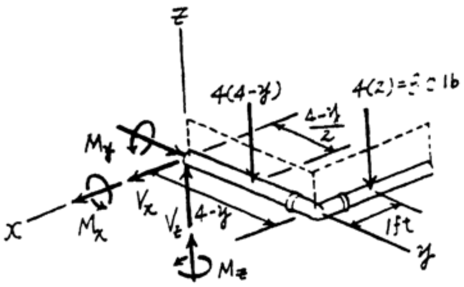
$V_z = \{24.0 - 4y\}$ lb **Ans**

$\Sigma M_x = 0;$ $M_x - 4(4-y)\left(\frac{4-y}{2}\right) - 8.00(4-y) = 0$

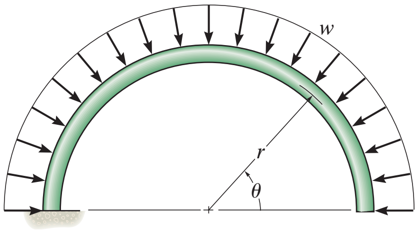
$M_x = \{2y^2 - 24y + 64.0\}$ lb · ft **Ans**

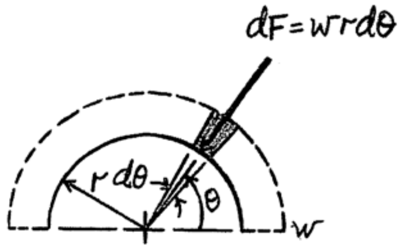
$\Sigma M_y = 0;$ $M_y - 8.00(1) = 0$ $M_y = 8.00$ lb · ft . **Ans**

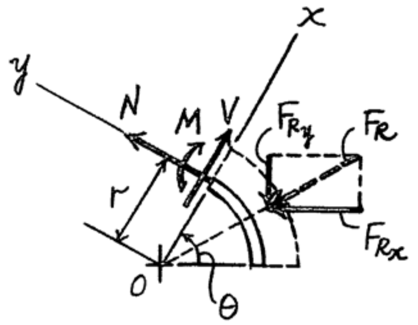
$\Sigma M_z = 0;$ $M_z = 0$ **Ans**

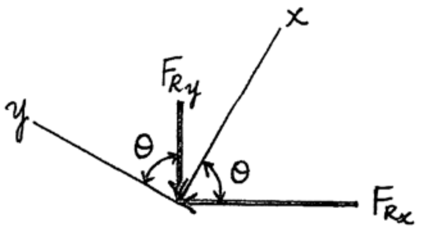


***7–64.** Determine the normal force, shear force, and moment in the curved rod as a function of θ.









$$F_{Rx} = \int_0^\theta w(r\,d\theta)\cos\theta = w\,r\sin\theta$$

$$F_{Ry} = \int_0^\theta w(r\,d\theta)\sin\theta = w\,r(1-\cos\theta)$$

$$\nearrow + \Sigma F_x = 0; \quad V - (w\,r\sin\theta)\cos\theta - r\,w(1-\cos\theta)\sin\theta = 0$$

$$V = w\,r\sin\theta \qquad \textbf{Ans}$$

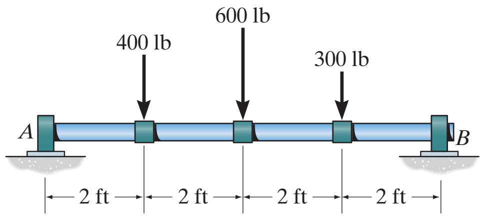
$$+\nwarrow \Sigma F_y = 0; \quad N + (w\,r\sin\theta)\sin\theta - r\,w(1-\cos\theta)\cos\theta = 0$$

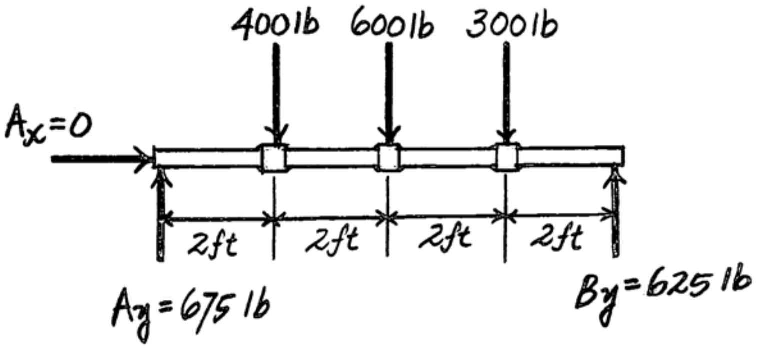
$$N = w\,r(\cos\theta - 1) \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_O = 0; \quad w\,r(\cos\theta - 1)\,r - M = 0$$

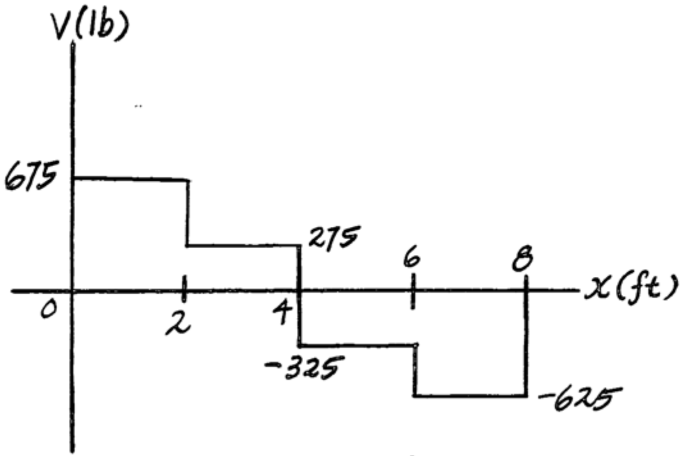
$$M = w\,r^2(\cos\theta - 1) \qquad \textbf{Ans}$$

•7–65. The shaft is supported by a smooth thrust bearing at *A* and a smooth journal bearing at *B*. Draw the shear and moment diagrams for the shaft.

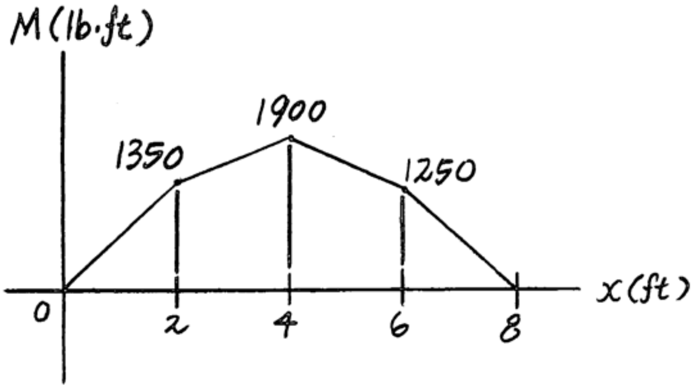




(a)

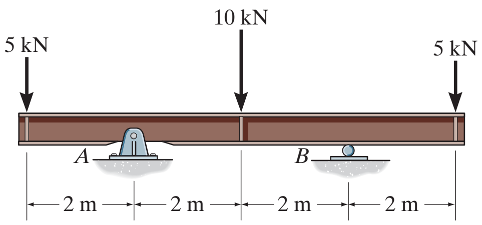


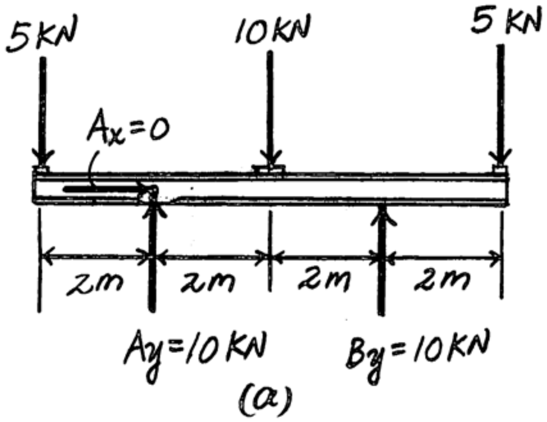
Shear diagram
(b)

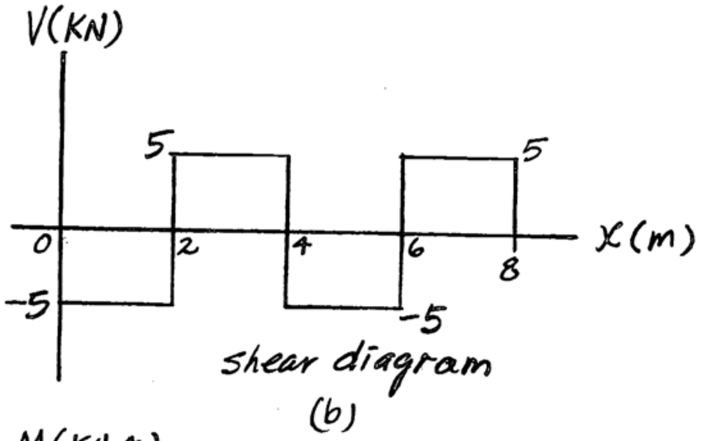


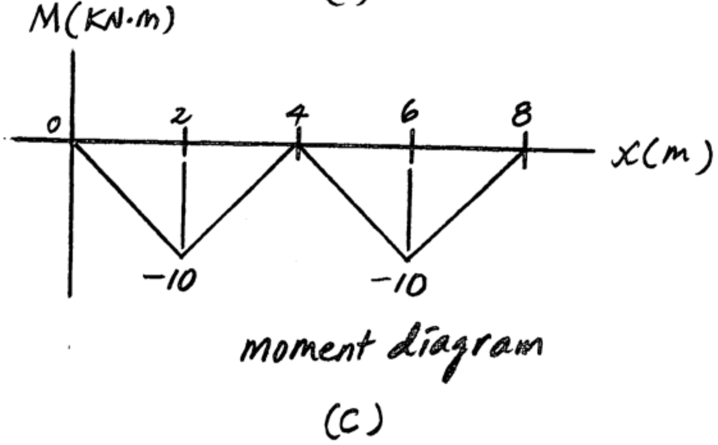
Moment diagram
(C)

7–66. Draw the shear and moment diagrams for the double overhang beam.

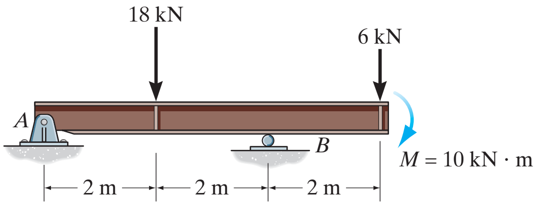


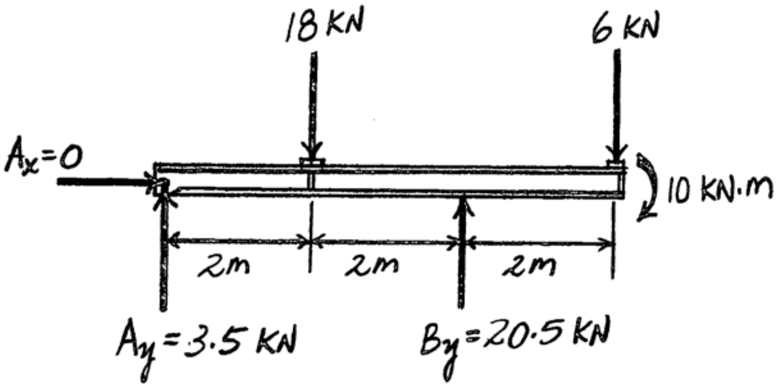




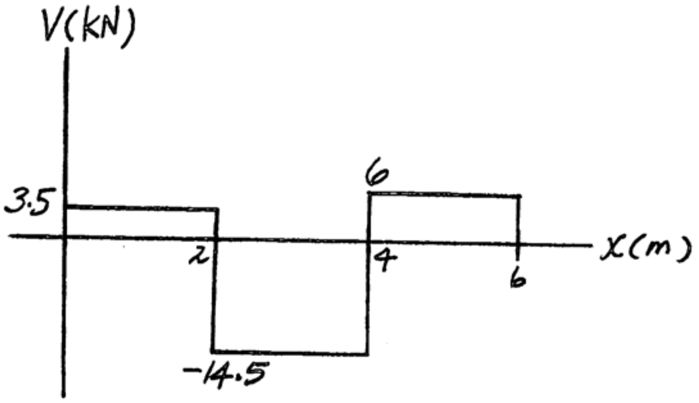


7–67. Draw the shear and moment diagrams for the overhang beam.



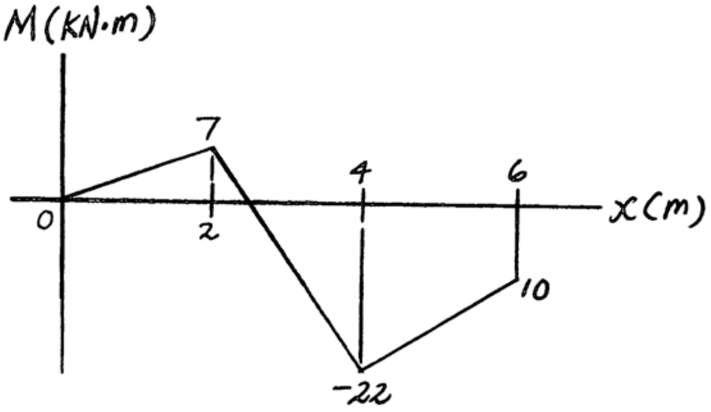


(a)



shear diagram

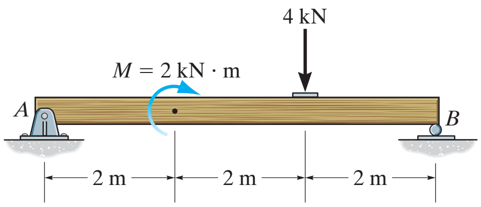
(b)

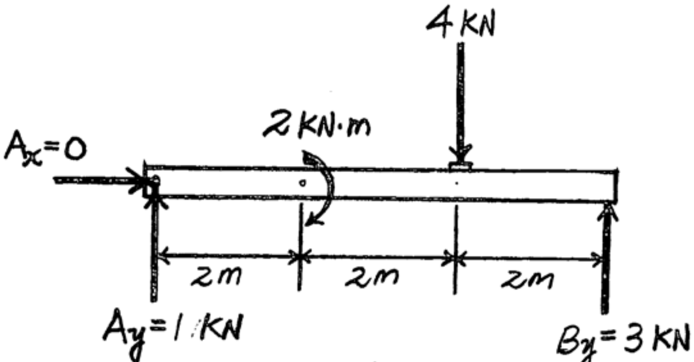


moment diagram

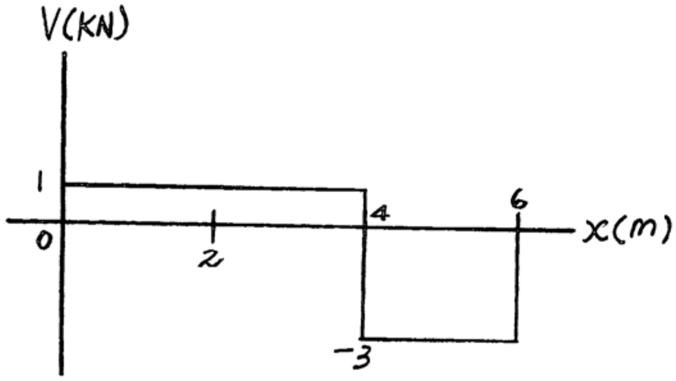
(c)

***7–68.** Draw the shear and moment diagrams for the simply supported beam.



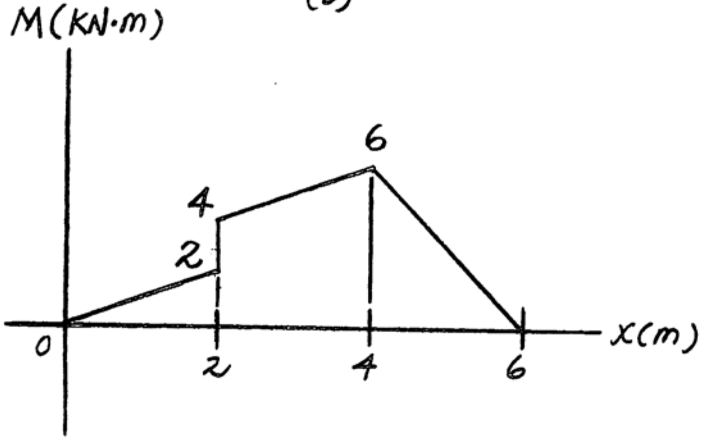


(a)



shear diagram

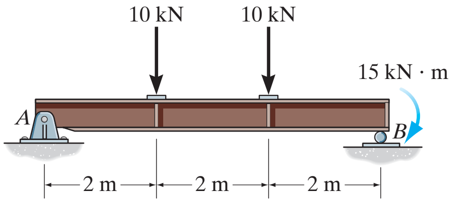
(b)

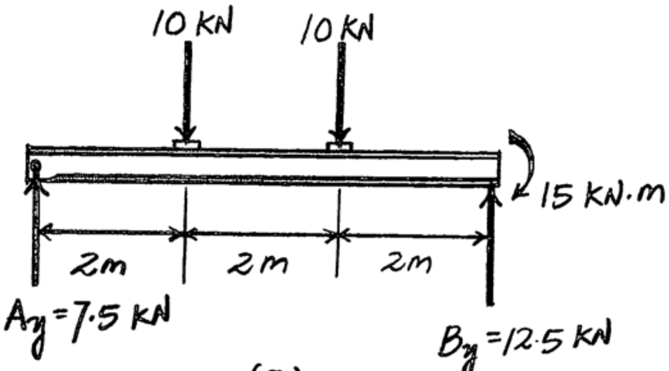


moment diagram

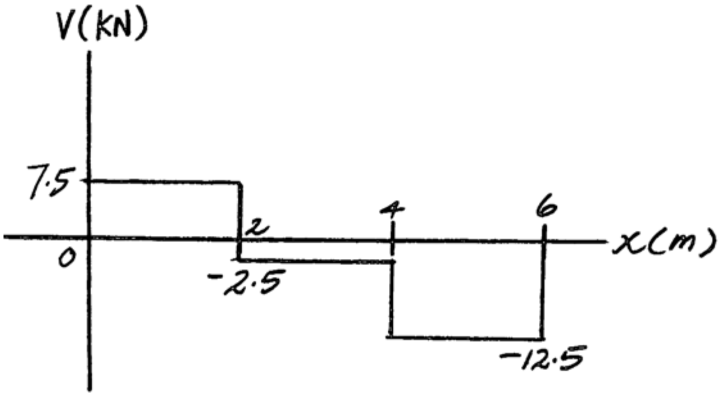
(c)

•7–69. Draw the shear and moment diagrams for the simply supported beam.

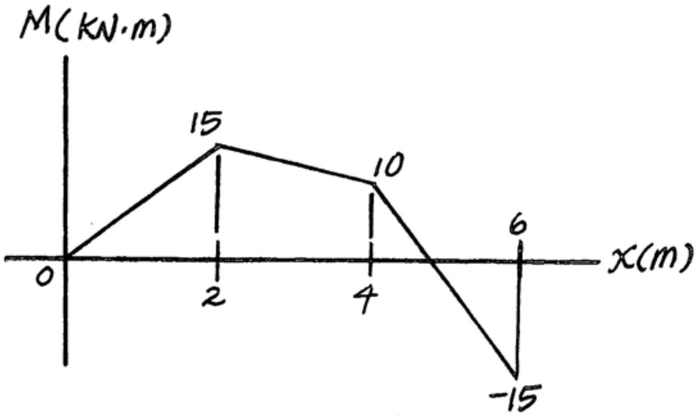




(a)

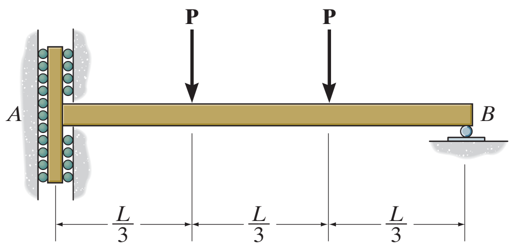


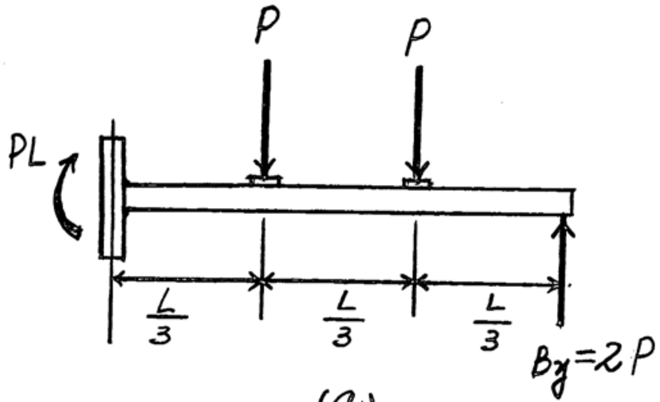
Shear diagram
(b)



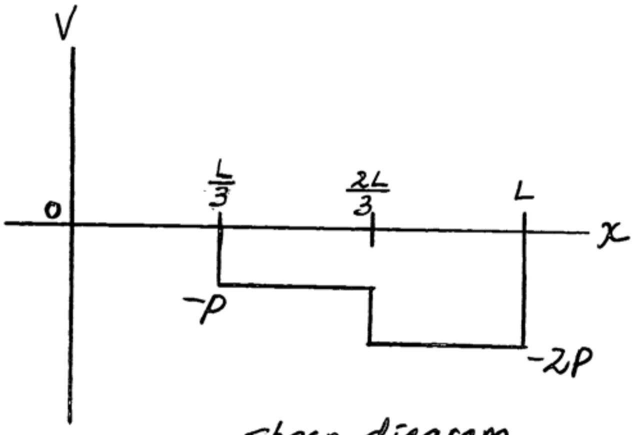
moment diagram
(c)

7–70. Draw the shear and moment diagrams for the beam. The support at A offers no resistance to vertical load.



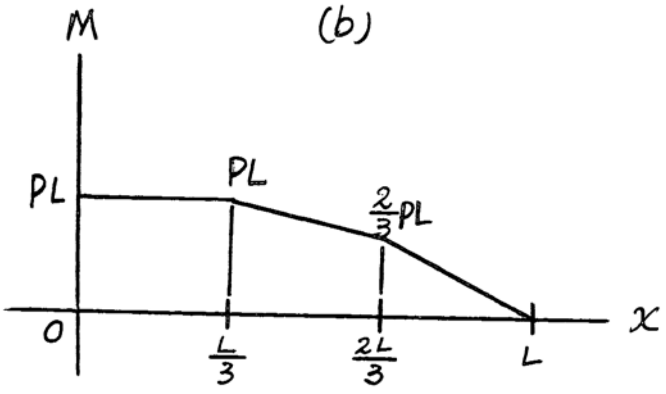


(a)



shear diagram

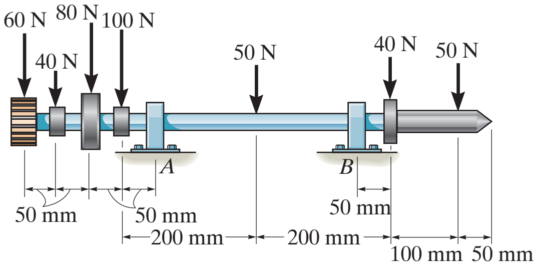
(b)

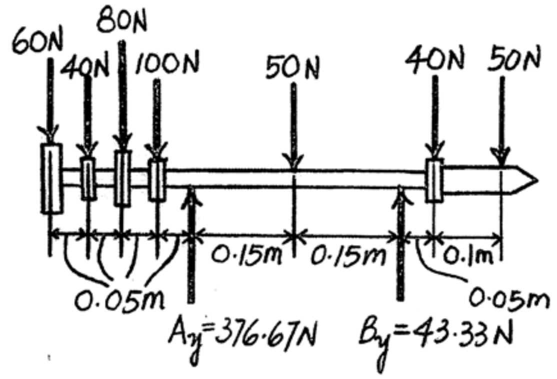


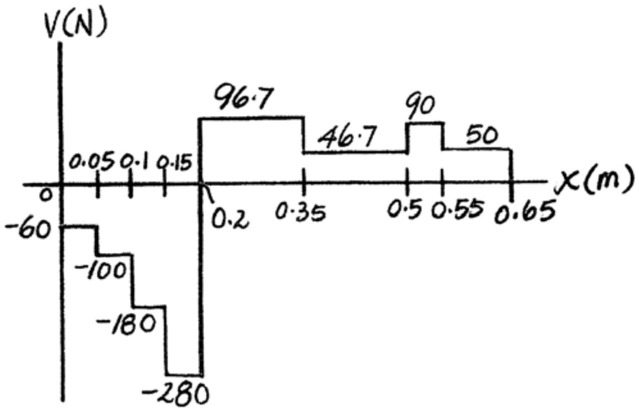
Moment diagram

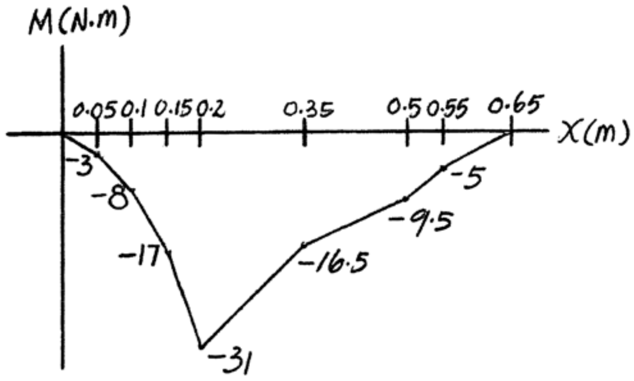
(C)

7–71. Draw the shear and moment diagrams for the lathe shaft if it is subjected to the loads shown. The bearing at *A* is a journal bearing, and *B* is a thrust bearing.

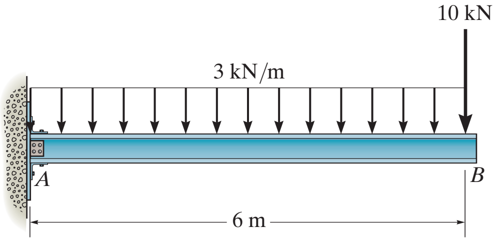


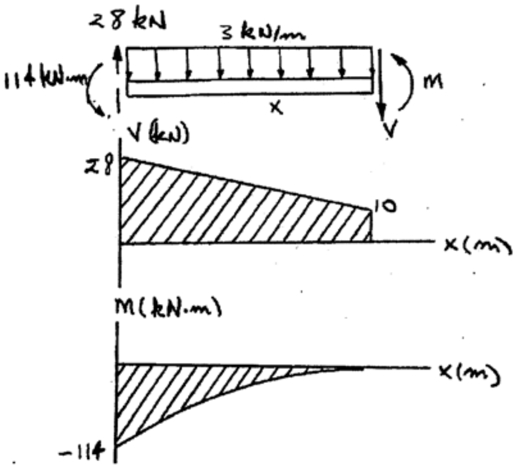




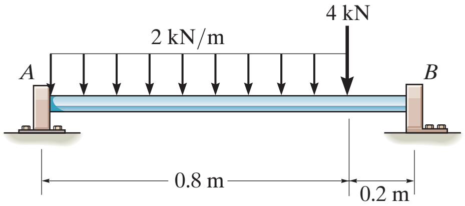


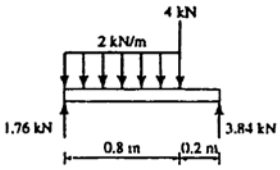
***7–72.** Draw the shear and moment diagrams for the beam.

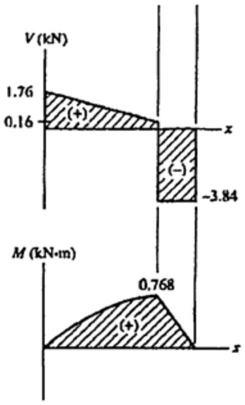




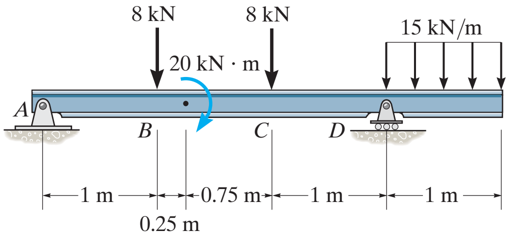
•7–73. Draw the shear and moment diagrams for the shaft. The support at *A* is a thrust bearing and at *B* it is a journal bearing.







7–74. Draw the shear and moment diagrams for the beam.



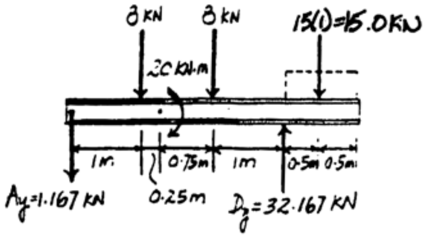
Support Reactions :

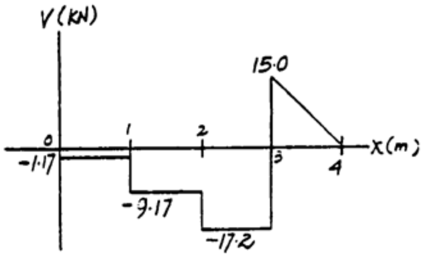
$$\curvearrowleft + \Sigma M_A = 0; \quad D_y(3) - 8(1) - 8(2) - 15.0(3.5) - 20 = 0$$

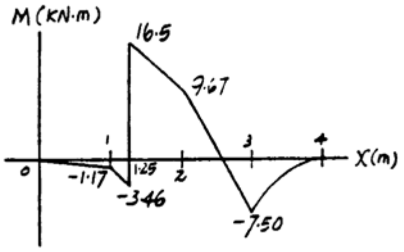
$$D_y = 32.167 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad 32.167 - 8 - 8 - 15.0 - A_y = 0$$

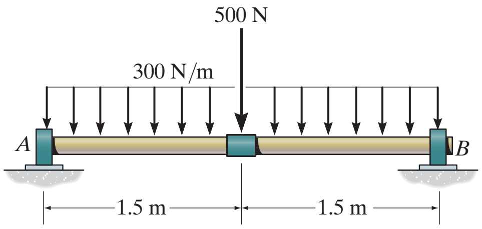
$$A_y = 1.167 \text{ kN}$$

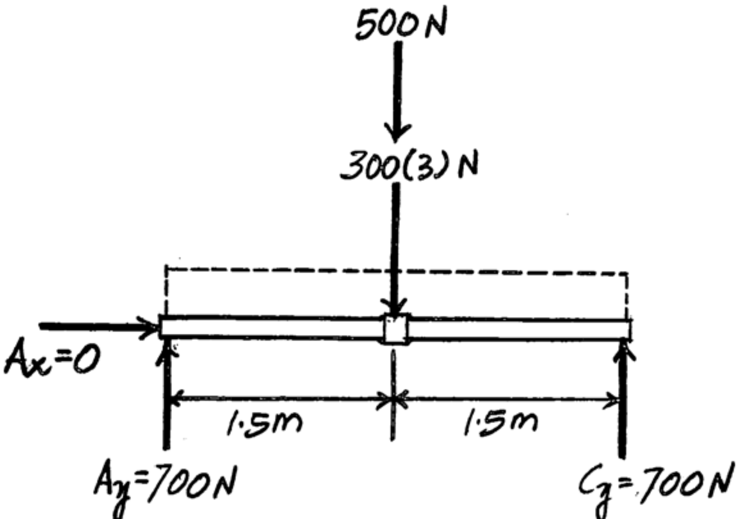




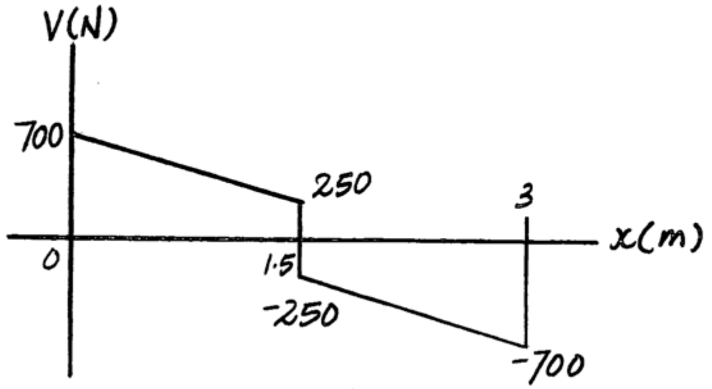


7–75. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.

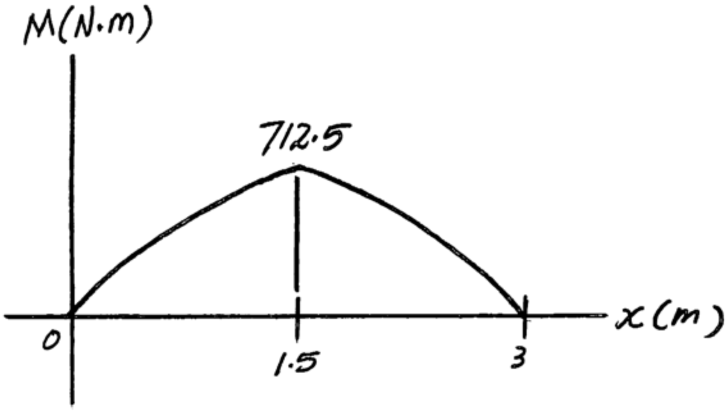




(a)

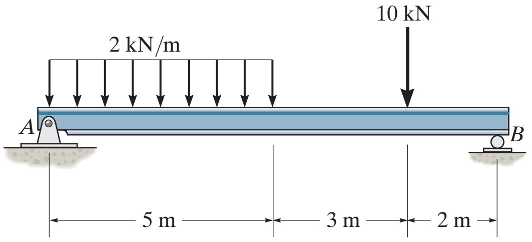


shear diagram
(b)



moment diagram
(c)

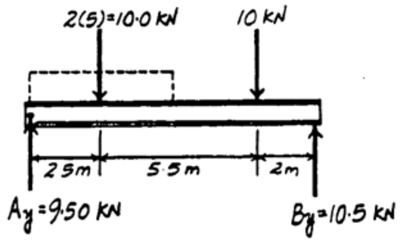
***7–76.** Draw the shear and moment diagrams for the beam.

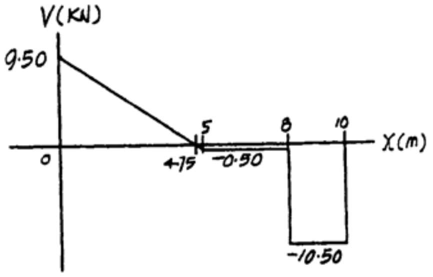


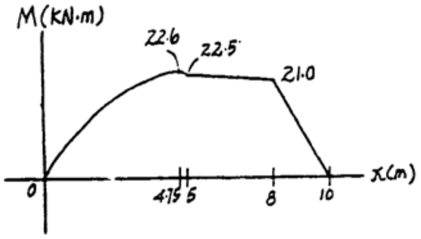
Support Reactions :

$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(10) - 10.0(2.5) - 10(8) = 0 \quad B_y = 10.5\text{ kN}$$

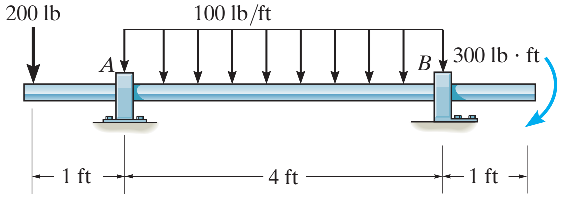
$$+\uparrow \Sigma F_y = 0; \quad A_y + 10.5 - 10.0 - 10 = 0 \quad A_y = 9.50\text{ kN}$$

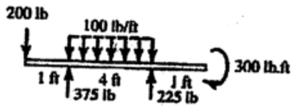


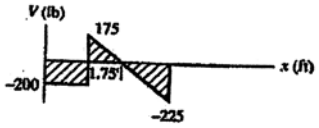


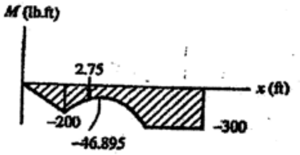


•7–77. Draw the shear and moment diagrams for the shaft. The support at *A* is a journal bearing and at *B* it is a thrust bearing.

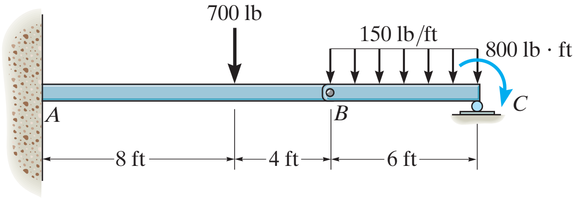


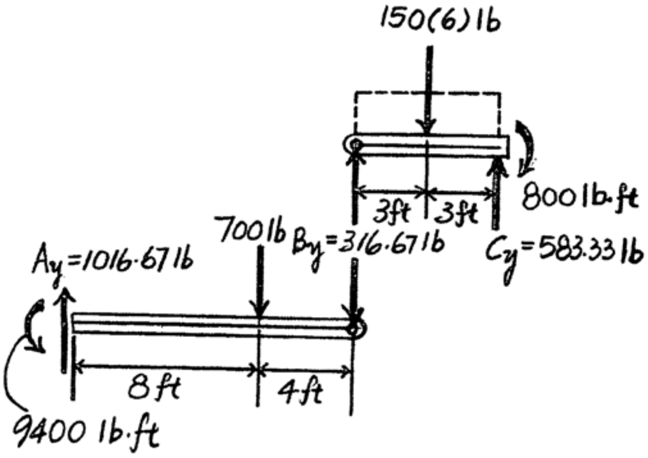


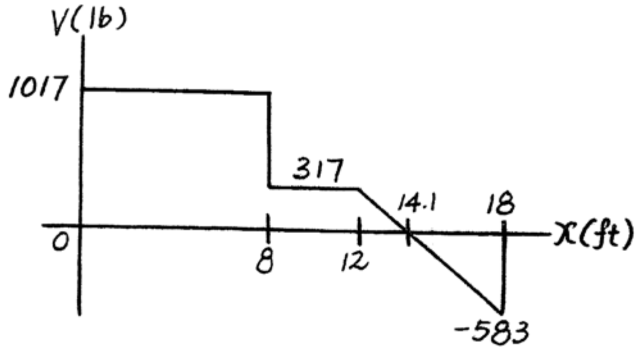


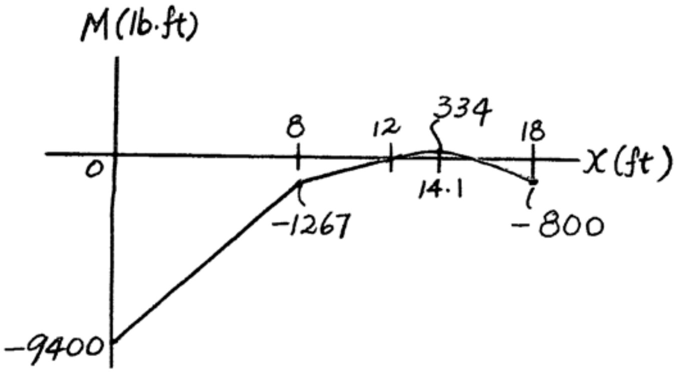


7–78. The beam consists of two segments pin connected at *B*. Draw the shear and moment diagrams for the beam.

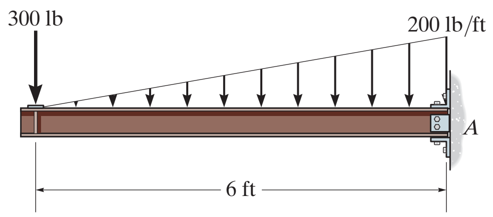


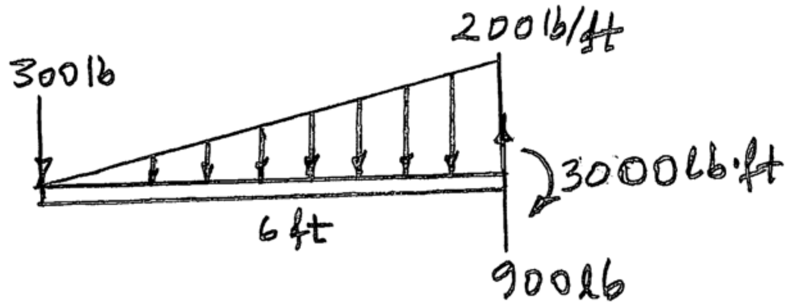


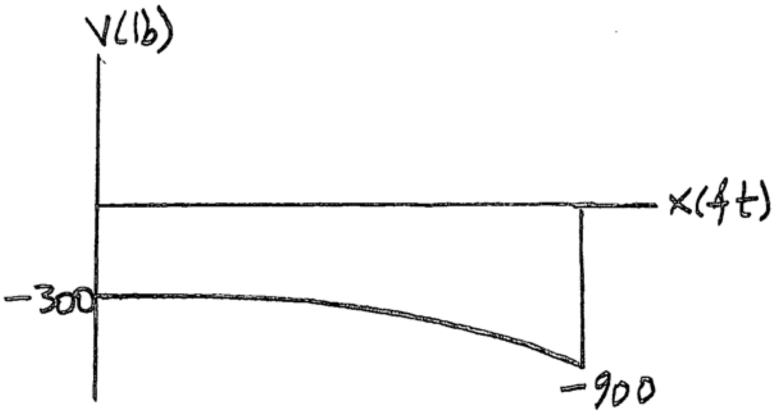


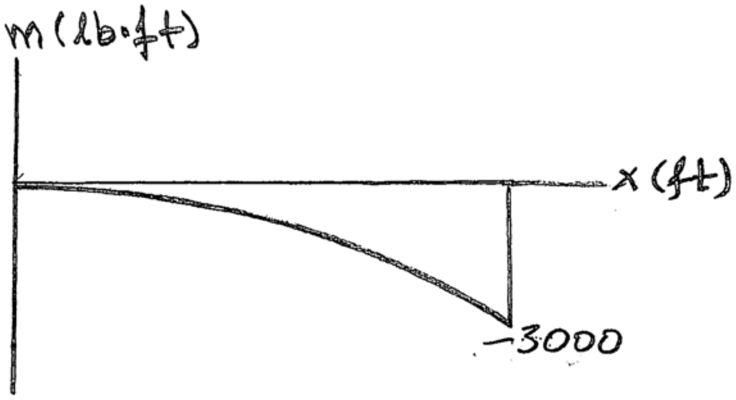


7–79. Draw the shear and moment diagrams for the cantilever beam.

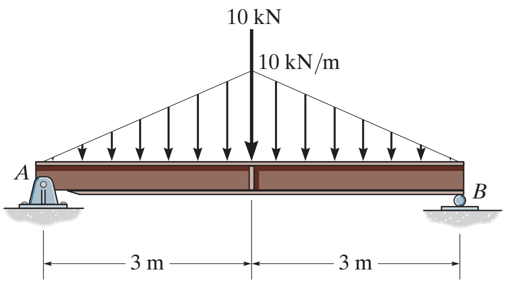






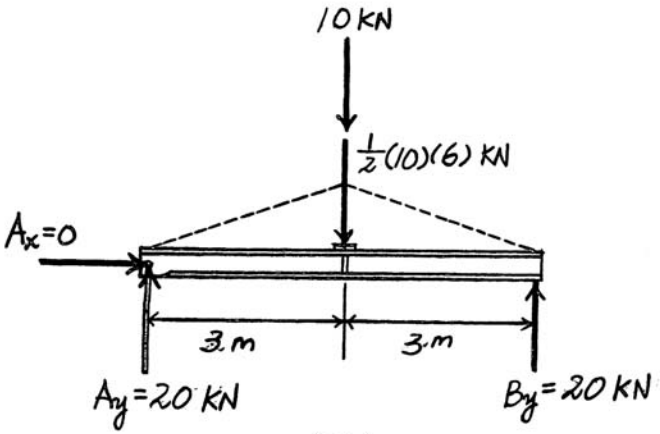


***7–80.** Draw the shear and moment diagrams for the simply supported beam.

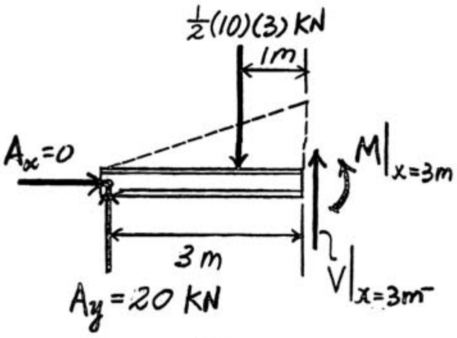


Since the area under the curved shear diagram can not be computed directly, the value of the moment at $x = 3$ m will be computed using the method of sections. By referring to the free-body diagram shown in Fig. *b*,

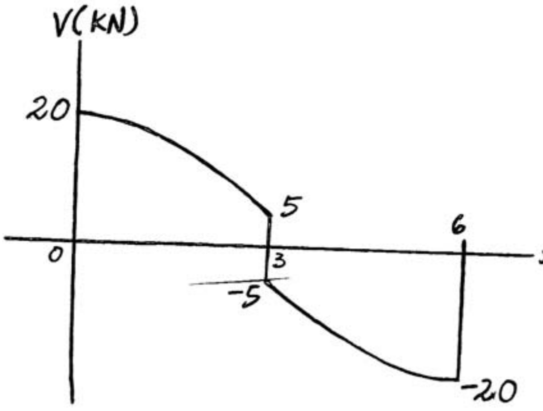
$\circlearrowleft +\Sigma M = 0;\ M\big|_{x=3\text{ m}} + \frac{1}{2}(10)(3)(1) - 20(3) = 0 \qquad M\big|_{x=3\text{ m}} = 45\text{ kN}\cdot\text{m}$ **Ans.**



(a)

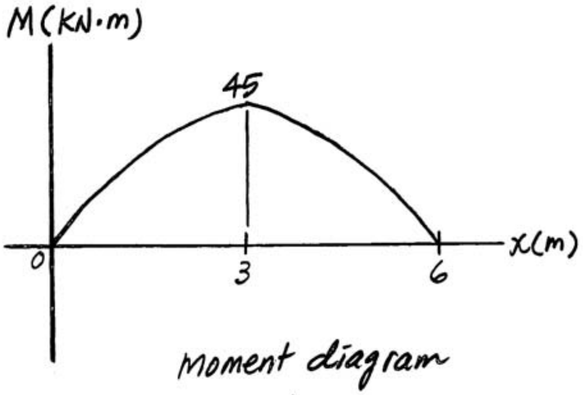


(b)



shear diagram

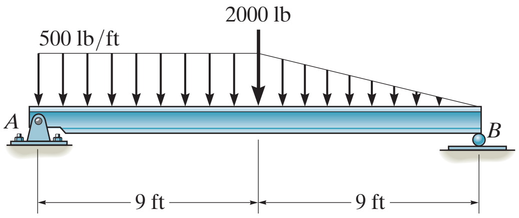
(c)

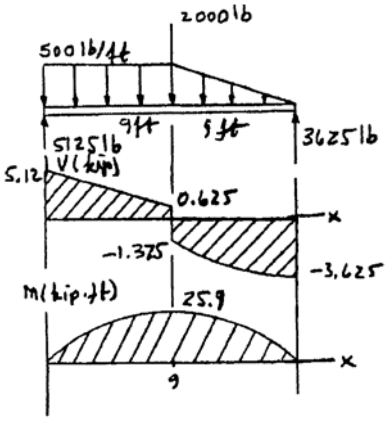


moment diagram

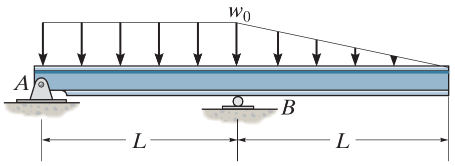
(d)

•7–81. Draw the shear and moment diagrams for the beam.





7–82. Draw the shear and moment diagrams for the beam.



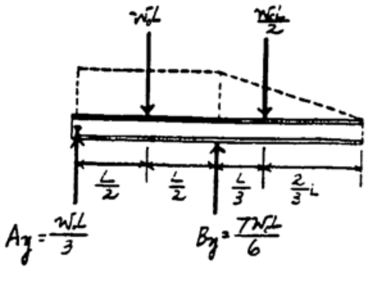
Support Reactions :

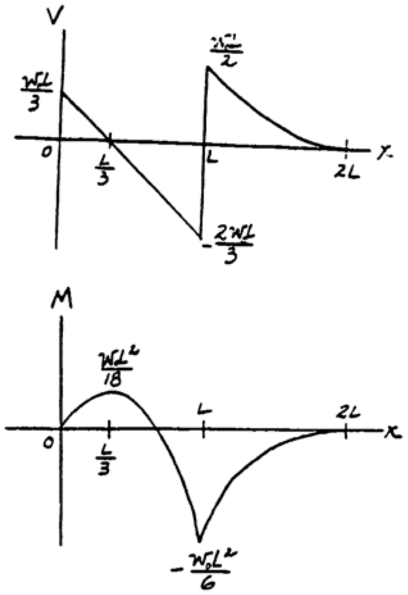
$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(L) - w_0L\left(\frac{L}{2}\right) - \frac{w_0L}{2}\left(\frac{4L}{3}\right) = 0$$

$$B_y = \frac{7w_0L}{6}$$

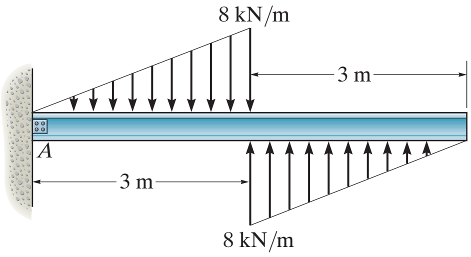
$$+\uparrow \Sigma F_y = 0; \quad A_y + \frac{7w_0L}{6} - w_0L - \frac{w_0L}{2} = 0$$

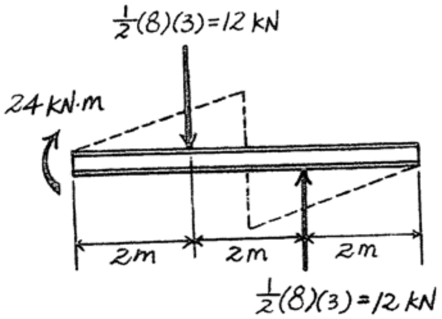
$$A_y = \frac{w_0L}{3}$$

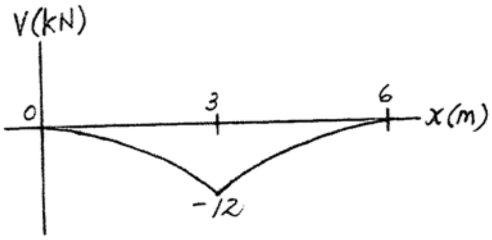


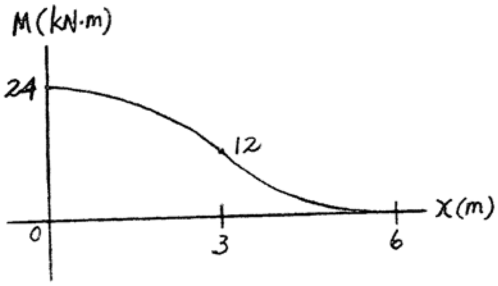


7–83. Draw the shear and moment diagrams for the beam.

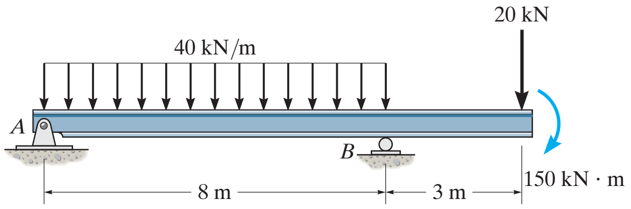


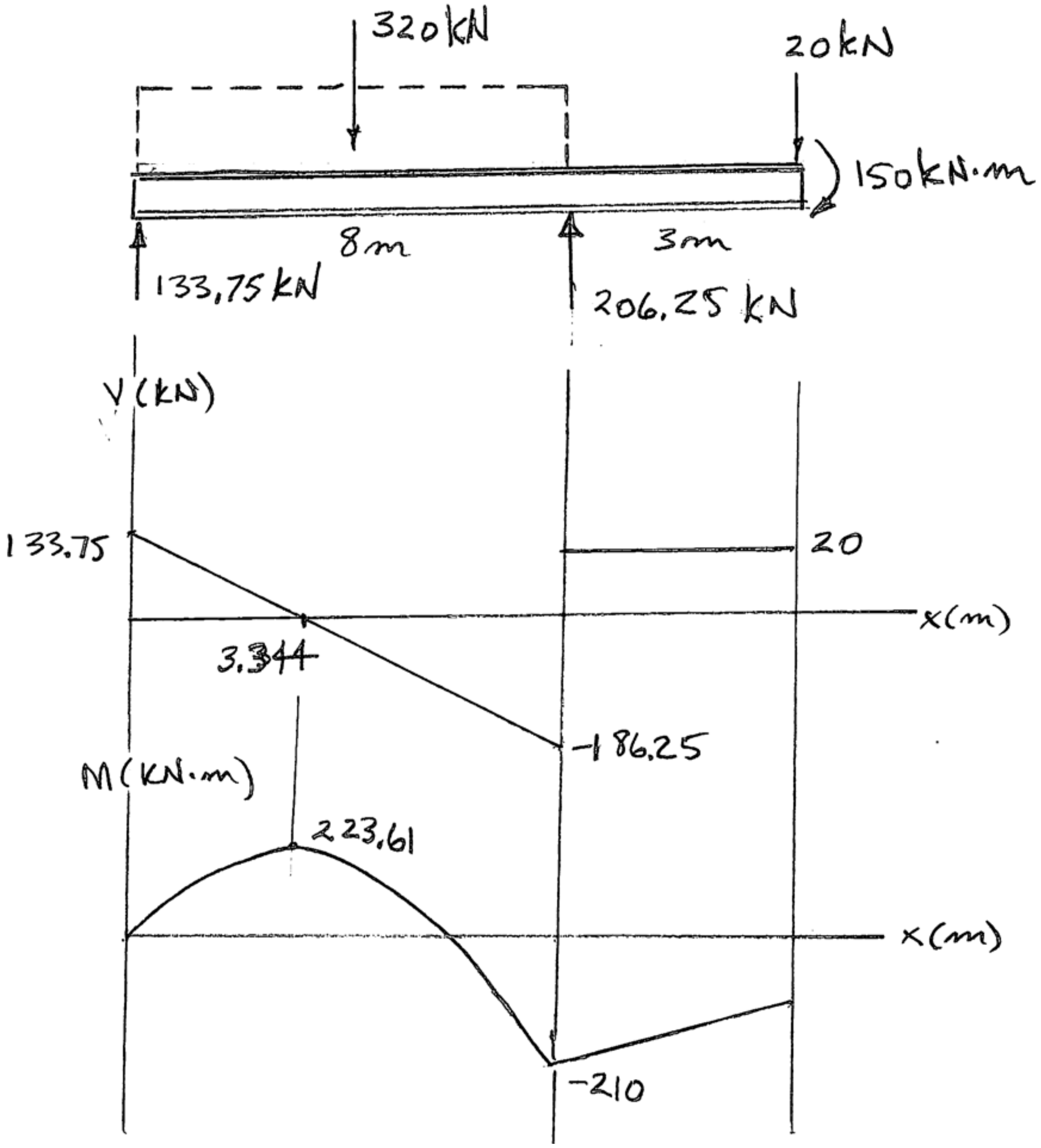




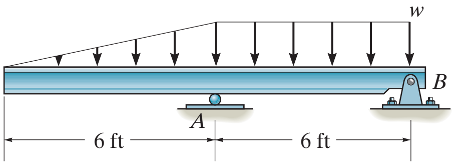


***7–84.** Draw the shear and moment diagrams for the beam.





•7–85. The beam will fail when the maximum moment is $M_{\max} = 30\ \text{kip} \cdot \text{ft}$ or the maximum shear is $V_{\max} = 8\ \text{kip}$. Determine the largest intensity w of the distributed load the beam will support.



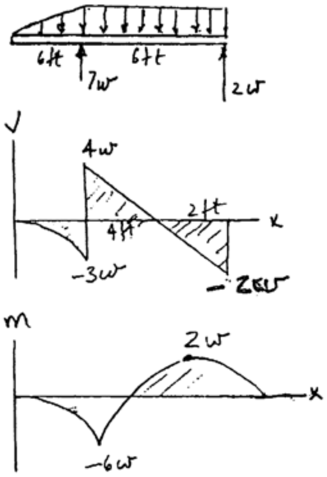
$V_{max} = 4w; \quad 8 = 4w$

$w = 2\ \text{kip/ft}$

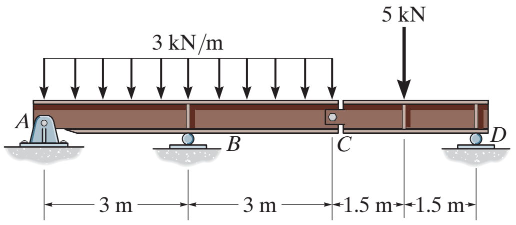
$M_{max} = -6w; \quad -30 = -6w$

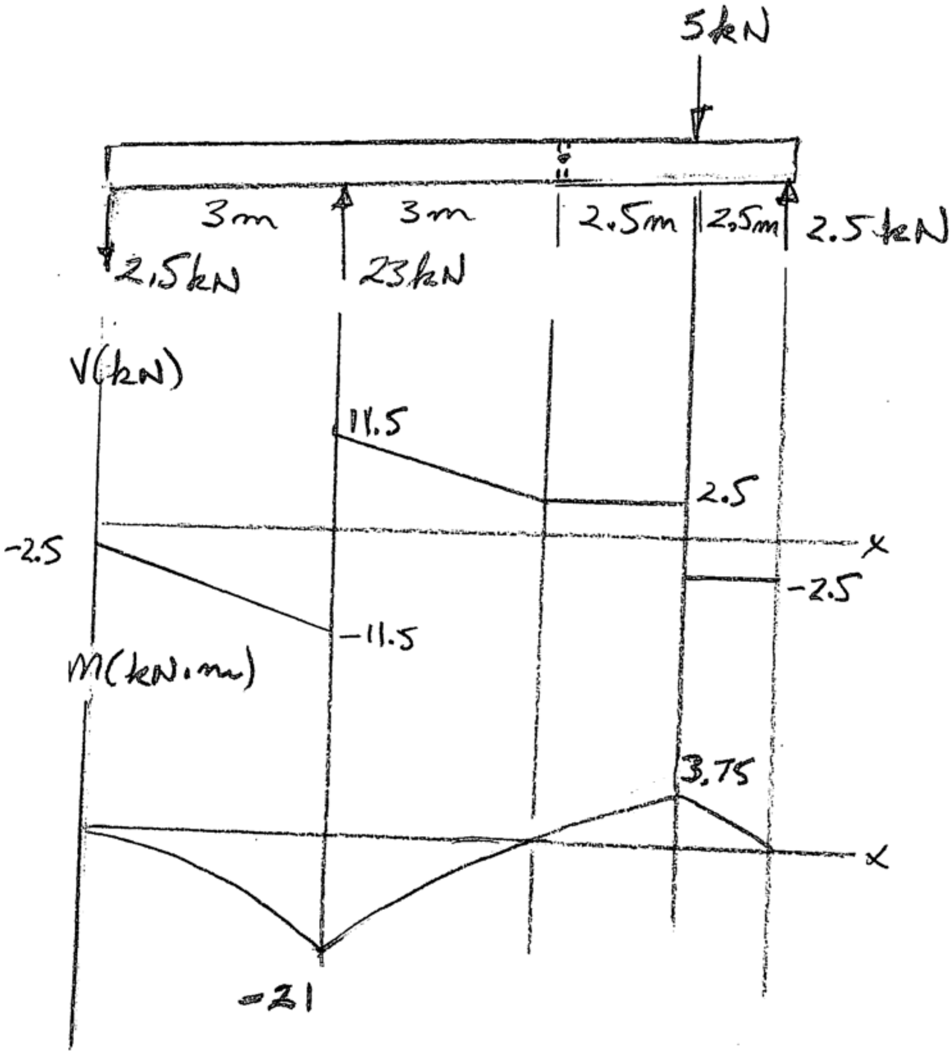
$w = 5\ \text{kip/ft}$

Thus, $\quad w = 2\ \text{kip/ft}$ **Ans**

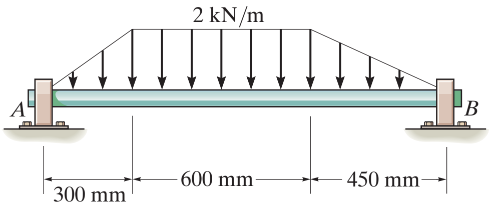


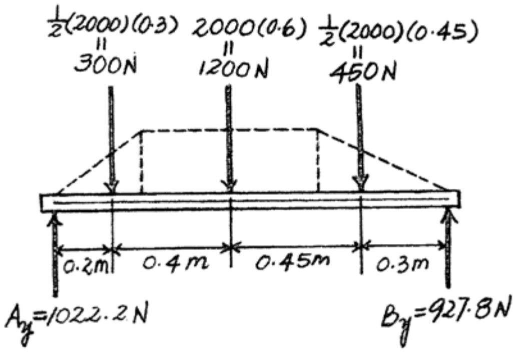
7–86. Draw the shear and moment diagrams for the compound beam.





7–87. Draw the shear and moment diagrams for the shaft. The supports at A and B are journal bearings.



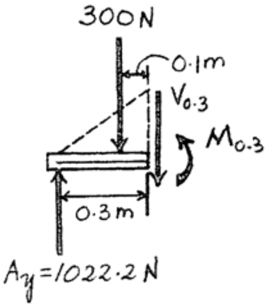


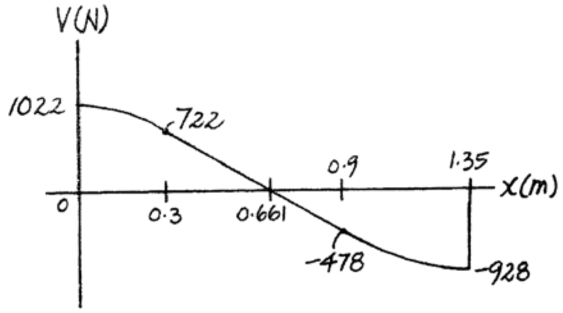
$+\uparrow \Sigma F_y = 0;\quad 1022.2 - \frac{1}{2}(2000)(0.3) - V_{0.3} = 0$

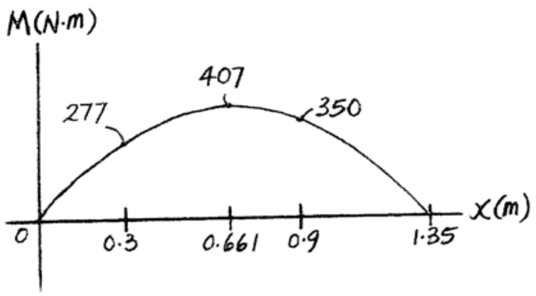
$V_{0.3} = 722\text{ N}$

$\circlearrowleft + \Sigma M = 0;\quad M_{0.3} + \frac{1}{2}(2000)(0.3)(0.1) - 1022.2(0.3) = 0$

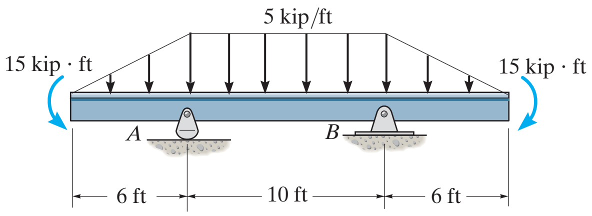
$M_{0.3} = 277\text{ N}\cdot\text{m}$







***7–88.** Draw the shear and moment diagrams for the beam.



Support Reactions : From FBD (a),

$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(10) + 15.0(2) + 15 - 50.0(5) - 15.0(12) - 15 = 0$$

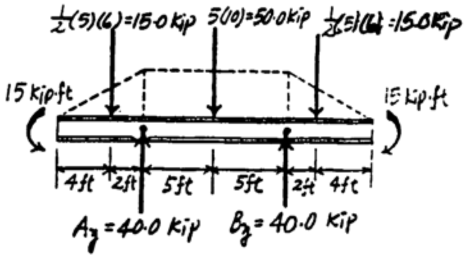
$$B_y = 40.0 \text{ kip}$$

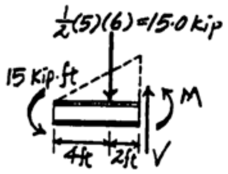
$$+\uparrow \Sigma F_y = 0; \quad A_y + 40.0 - 15.0 - 50.0 - 15.0 = 0$$

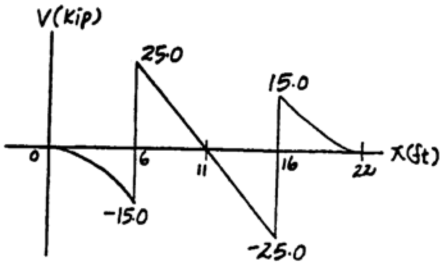
$$A_y = 40.0 \text{ kip}$$

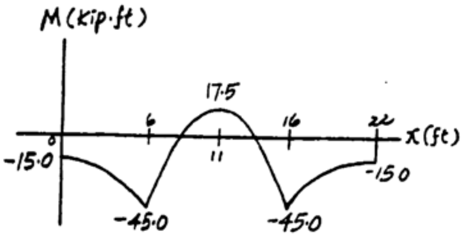
Shear and Moment Diagrams : The value of the moment at supports A and B can be evaluated using the method of sections [FBD (c)].

$$\curvearrowleft + \Sigma M = 0; \quad M + 15.0(2) + 15 = 0 \qquad M = -45.0 \text{ kip} \cdot \text{ft}$$

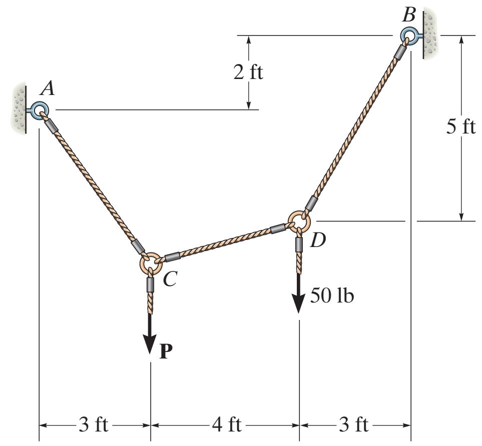


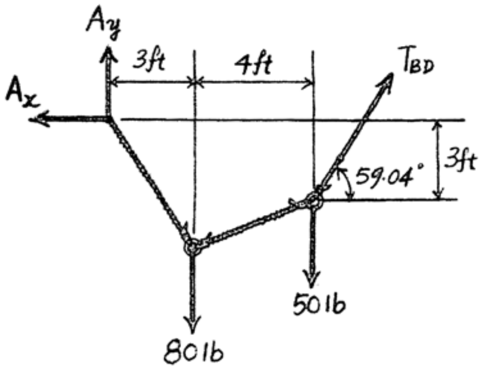






•7–89. Determine the tension in each segment of the cable and the cable's total length. Set $P = 80$ lb.





(a)

rom FBD (a)

$\circlearrowleft +\Sigma M_A = 0; \quad T_{BD}\cos 59.04^\circ(3) + T_{BD}\sin 59.04^\circ(7) - 50(7) - 80(3) = 0$

$T_{BD} = 78.188\text{ lb} = 78.2\text{ lb}$ **Ans**

$\xrightarrow{+}\Sigma F_x = 0; \quad 78.188\cos 59.04^\circ - A_x = 0 \quad A_x = 40.227\text{ lb}$

$+\uparrow\Sigma F_y = 0; \quad A_y + 78.188\sin 59.04^\circ - 80 - 50 = 0 \quad A_y = 62.955\text{ lb}$

Joint A :

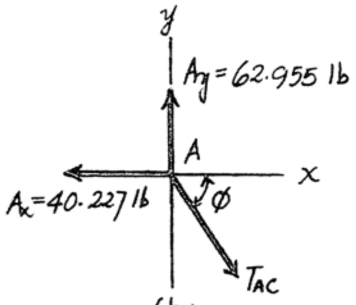
$\xrightarrow{+}\Sigma F_x = 0; \quad T_{AC}\cos\phi - 40.227 = 0$ (1)

$+\uparrow\Sigma F_y = 0; \quad -T_{AC}\sin\phi + 62.955 = 0$ (2)

Solving Eqs.(1) and (2) yields :

$\phi = 57.42^\circ$

$T_{AC} = 74.7\text{ lb}$ **Ans**



(b)

Joint D :

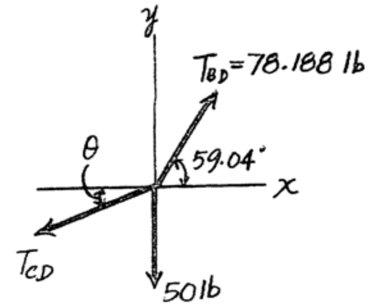
$\xrightarrow{+}\Sigma F_x = 0; \quad 78.188\cos 59.04^\circ - T_{CD}\cos\theta = 0$ (3)

$+\uparrow\Sigma F_y = 0; \quad 78.188\sin 59.04^\circ - T_{CD}\sin\theta - 50 = 0$ (4)

Solving Eqs.(3) and (4) yields :

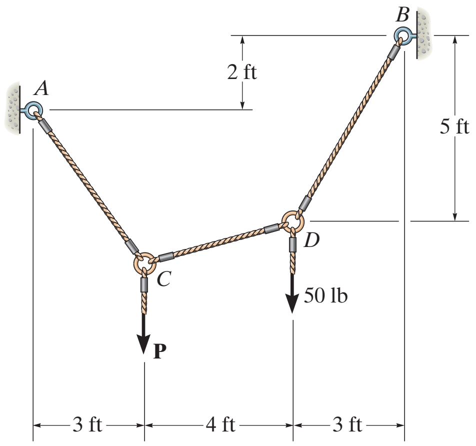
$\theta = 22.96^\circ$

$T_{CD} = 43.7\text{ lb}$ **Ans**



Total length of the cable : $\quad l_T = \dfrac{5}{\sin 59.04^\circ} + \dfrac{4}{\cos 22.96^\circ} + \dfrac{3}{\cos 57.42^\circ} = 15.7\text{ ft}$ **Ans**

7–90. If each cable segment can support a maximum tension of 75 lb, determine the largest load P that can be applied.



$\circlearrowleft + \Sigma M_A = 0;\quad -T_{BD}(\cos 59.04°)\,2 + T_{BD}(\sin 59.04°)(10) - 50(7) - P(3) = 0$

$T_{BD} = 0.39756\,P + 46.383$

$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + T_{BD}\cos 59.04° = 0$

$+\uparrow \Sigma F_y = 0;\quad A_y - P - 50 + T_{BD}\sin 59.04° = 0$

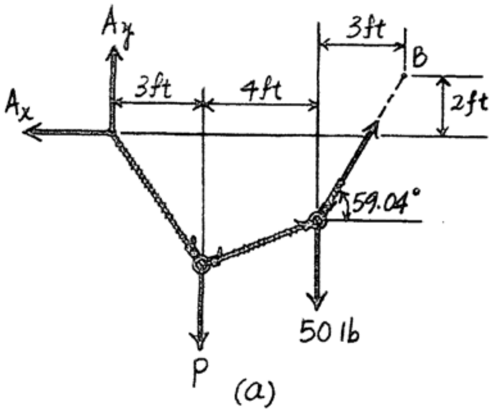
Assume maximum tension is in cable BD.

$T_{BD} = 75\text{ lb}$

$P = 71.98\text{ lb}$

$A_x = 38.59\text{ lb}$

$A_y = 57.670\text{ lb}$

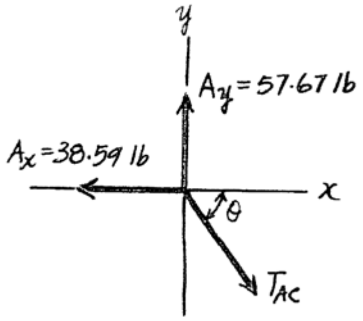


(a)

Pin A:

$T_{AC} = \sqrt{(38.59)^2 + (57.670)^2} = 69.39\text{ lb} < 75\text{ lb}$ OK

$\theta = \tan^{-1}\left(\dfrac{57.670}{38.59}\right) = 56.21°$



(b)

Joint C:

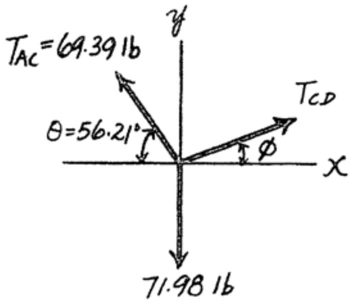
$\xrightarrow{+} \Sigma F_x = 0;\quad T_{CD}\cos\phi - 69.39\cos 56.21° = 0$

$+\uparrow \Sigma F_y = 0;\quad T_{CD}\sin\phi + 69.39\sin 56.21° - 71.98 = 0$

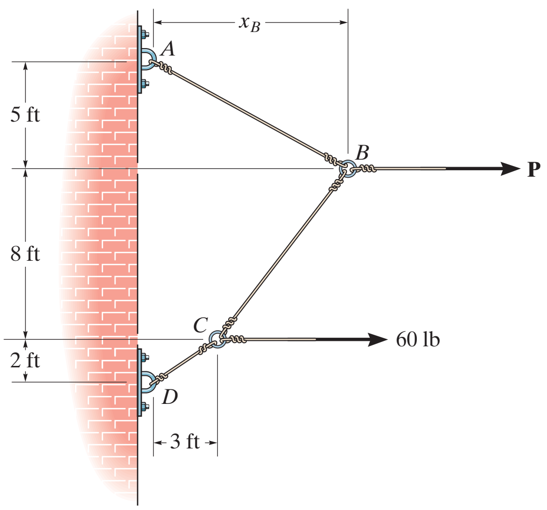
$T_{CD} = 41.2\text{ lb} < 75\text{ lb}$ OK

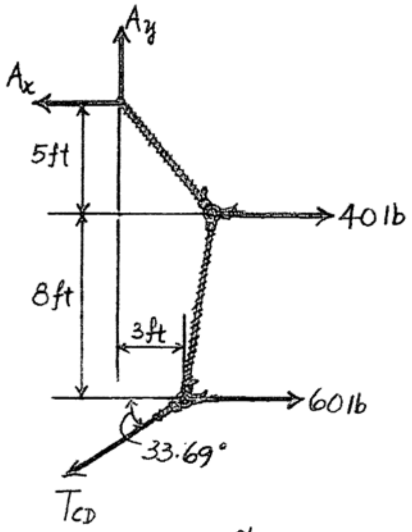
$\phi = 20.3°$

Thus, $P = 72.0\text{ lb}$ **Ans**



7–91. The cable segments support the loading shown. Determine the horizontal distance x_B from the force at B to point A. Set $P = 40$ lb.





$\circlearrowleft + \Sigma M_A = 0; \quad -T_{CD}\cos 33.69°(13) - T_{CD}\sin 33.69°(3) + 60(13) + 40(5) = 0$

$T_{CD} = 78.521$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad 40 + 60 - 78.521\cos 33.69° - A_x = 0$

$A_x = 34.667$ lb

$+\uparrow \Sigma F_y = 0; \quad A_y - 78.521\sin 33.69° = 0$

$A_y = 43.555$ lb

Joint A :

$\xrightarrow{+} \Sigma F_x = 0; \quad T_{AB}\cos\theta - 34.667 = 0 \qquad (1)$

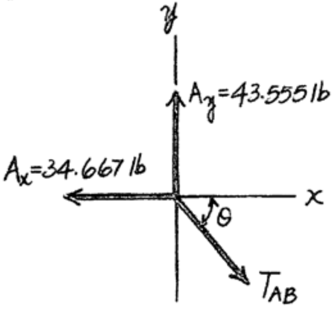
$+\uparrow \Sigma F_y = 0; \quad 43.555 - T_{AB}\sin\theta = 0 \qquad (2)$

Solving Eqs. (1) and (2) yields :

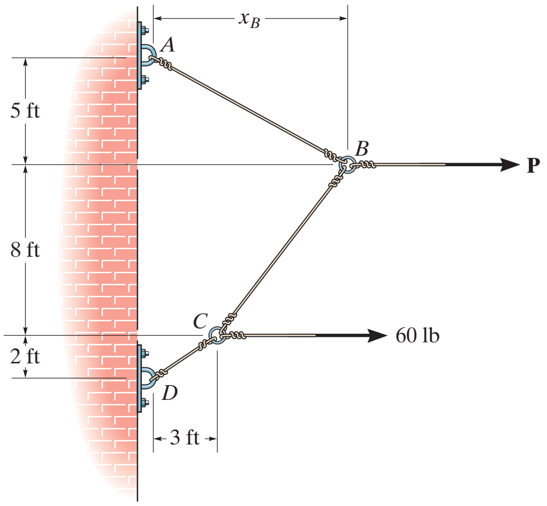
$\theta = 51.48°$

$T_{AB} = 55.67$ lb

$x_B = \dfrac{5}{\tan 51.48°} = 3.98$ ft **Ans**



***7–92.** The cable segments support the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.



$\curvearrowleft + \Sigma M_D = 0;$ $\quad T_{AB}\cos 39.81°(10) + T_{AB}\sin 39.81°(6) - 60(2) - P(10) = 0$

$11.523T_{AB} - 10P = 120 \qquad (1)$

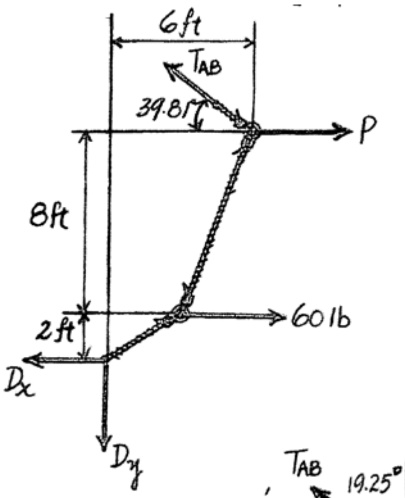
Joint B :

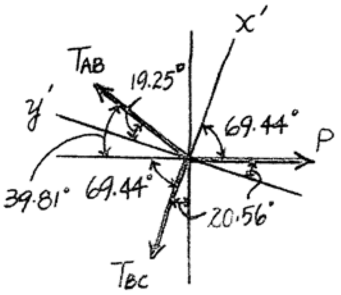
$+\nwarrow \Sigma F_{y'} = 0;$ $\quad T_{AB}\cos 19.25° - P\sin 69.44° = 0 \qquad (2)$

Solving Eqs. (1) and (2) yields :

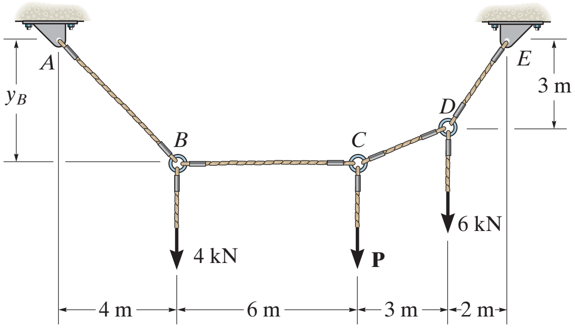
$P = 84.0$ lb Ans

$T_{AB} = 83.32$ lb





•7–93. Determine the force P needed to hold the cable in the position shown, i.e., so segment BC remains horizontal. Also, compute the sag y_B and the maximum tension in the cable.

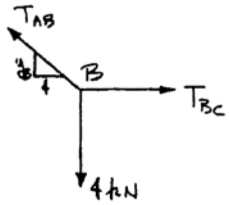


Joint B:

$$\xrightarrow{+} \Sigma F_x = 0; \quad T_{BC} - \frac{4}{\sqrt{y_B^2 + 16}} T_{AB} = 0$$

$$+\uparrow \Sigma F_y = 0, \quad \frac{y_B}{\sqrt{y_B^2 + 16}} T_{AB} - 4 = 0$$

$$y_B T_{BC} = 16 \qquad (1)$$

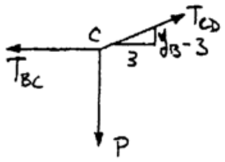


Joint C:

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - T_{BC} = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - P = 0$$

$$(y_B - 3) T_{BC} = 3P \qquad (3)$$



Combining Eqs. (1) and (2):

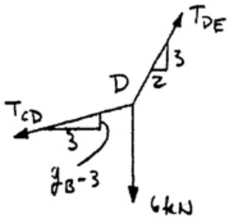
$$\frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = \frac{16}{y_B} \qquad (4)$$

Joint D:

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{2}{\sqrt{13}} T_{DE} - \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{\sqrt{13}} T_{DE} - \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - 6 = 0$$

$$\frac{15 - 2y_B}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 12 \qquad (5)$$



From Eqs. (1) and (3): $\quad 3y_B P - 16y_B + 48 = 0$

From Eqs. (4) and (5): $\quad y_B = 3.53$ m **Ans**

$P = 0.8$ kN **Ans**

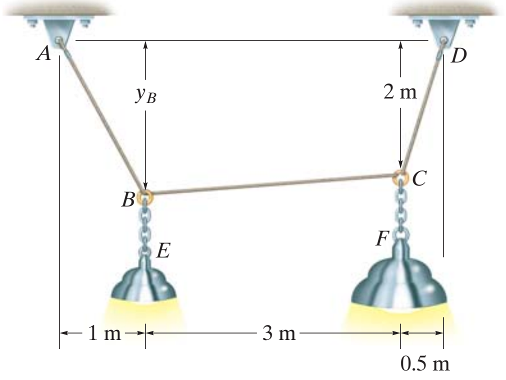
$T_{AB} = 6.05$ kN

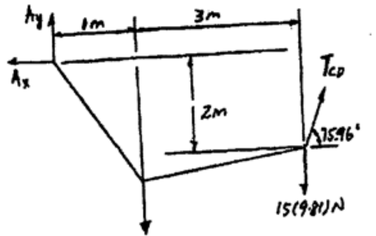
$T_{BC} = 4.53$ kN

$T_{CD} = 4.60$ kN

$T_{max} = T_{DE} = 8.17$ kN **Ans**

7–94. Cable *ABCD* supports the 10-kg lamp *E* and the 15-kg lamp *F*. Determine the maximum tension in the cable and the sag y_B of point *B*.





(a)

From FBD (a)

$\circlearrowleft + \Sigma M_A = 0; \quad T_{CD}\cos 75.96°(2) + T_{CD}\sin 75.96°(4)$

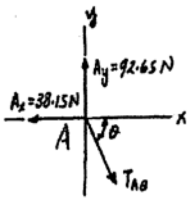
$- 15(9.81)(4) - 10(9.81)(1) = 0$

$T_{CD} = 157.30\ \text{N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 157.30\cos 75.96° - A_x = 0 \quad A_x = 38.15\ \text{N}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 157.30\sin 75.96° - 15(9.81) - 10(9.81) = 0$

$A_y = 92.65\ \text{N}$



Joint A :

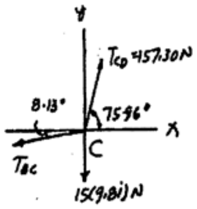
$\xrightarrow{+} \Sigma F_x = 0; \quad T_{AB}\cos\theta - 38.15 = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad 92.65 - T_{AB}\sin\theta = 0 \qquad (2)$

Solving Eqs. (1) and (2) yields :

$\theta = 67.62°$ $\qquad T_{AB} = 100.2\ \text{N}$

$y_B = (1)\tan 67.62° = 2.43\ \text{m}$ **Ans**



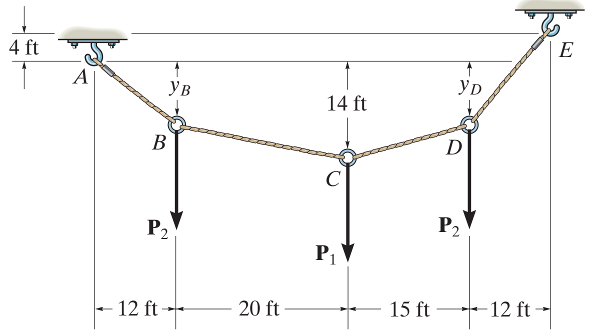
Joint C :

$\xrightarrow{+} \Sigma F_x = 0; \quad 157.30\cos 75.96° - T_{BC}\cos 8.13° = 0 \quad T_{BC} = 38.54\ \text{N}$

$+\uparrow \Sigma F_y = 0; \quad 157.3\sin 75.96° - 38.54\sin 8.13° - 15(9.81) = 0$ (Check)

$T_{max} = T_{CD} = 157\ \text{N}$ **Ans**

7–95. The cable supports the three loads shown. Determine the sags y_B and y_D of points B and D. Take $P_1 = 400$ lb, $P_2 = 250$ lb.

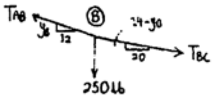


At B

$\xrightarrow{+}\Sigma F_x = 0;$ $\qquad \frac{20}{\sqrt{(14-y_B)^2+400}}T_{BC} - \frac{12}{\sqrt{y_B^2+144}}T_{AB} = 0$

$+\uparrow\Sigma F_y = 0;$ $\qquad -\frac{14-y_B}{\sqrt{(14-y_B)^2+400}}T_{BC} + \frac{y_B}{\sqrt{y_B^2+144}}T_{AB} - 250 = 0$

$$\frac{32y_B-168}{\sqrt{(14-y_B)^2+400}}T_{BC} = 3000 \qquad (1)$$

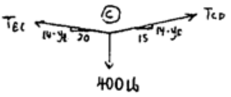


At C

$\xrightarrow{+}\Sigma F_x = 0;$ $\qquad \frac{15}{\sqrt{(14-y_D)^2+225}}T_{CD} - \frac{20}{\sqrt{(14-y_B)^2+400}}T_{BC} = 0$

$+\uparrow\Sigma F_y = 0;$ $\qquad \frac{14-y_D}{\sqrt{(14-y_D)^2+225}}T_{CD} + \frac{14-y_B}{\sqrt{(14-y_B)^2+400}}T_{BC} - 400 = 0$

$$\frac{-20y_D+490-15y_B}{\sqrt{(14-y_B)^2+400}}T_{BC} = 6000 \qquad (2)$$



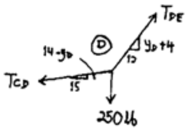
$$\frac{-20y_D+490-15y_B}{\sqrt{(14-y_D)^2+225}}T_{CD} = 8000 \qquad (3)$$

At D

$\xrightarrow{+}\Sigma F_x = 0;$ $\qquad \frac{12}{\sqrt{(4+y_D)^2+144}}T_{DE} - \frac{15}{\sqrt{(14-y_D)^2+225}}T_{CD} = 0$

$+\uparrow\Sigma F_y = 0;$ $\qquad \frac{4+y_D}{\sqrt{(4+y_D)^2+144}}T_{DE} - \frac{14-y_D}{\sqrt{(14-y_D)^2+225}}T_{CD} - 250 = 0$

$$\frac{-108+27y_D}{\sqrt{(14-y_D)^2+225}}T_{CD} = 3000 \qquad (4)$$



Combining Eqs. (1) & (2)

$$79y_B + 20y_D = 826$$

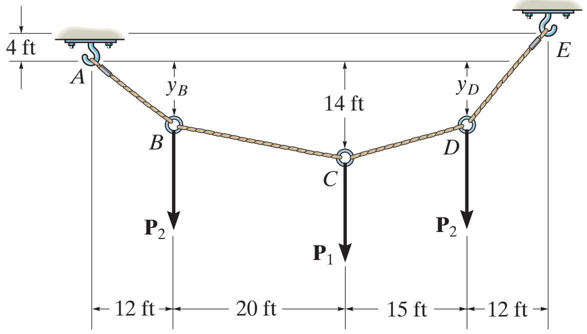
Combining Eqs. (3) & (4)

$$45y_B + 276y_D = 2334$$

$y_B = 8.67$ ft **Ans**

$y_D = 7.04$ ft **Ans**

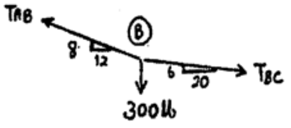
***7–96.** The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 300$ lb and $y_B = 8$ ft. Also find the sag y_D.



At *B*

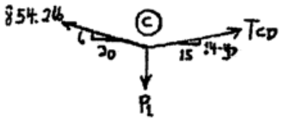
$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{20}{\sqrt{436}}T_{BC} - \frac{12}{\sqrt{208}}T_{AB} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{-6}{\sqrt{436}}T_{BC} + \frac{8}{\sqrt{208}}T_{AB} - 300 = 0$$



$$T_{AB} = 983.3 \text{ lb}$$

$$T_{BC} = 854.2 \text{ lb}$$



At *C*

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{-20}{\sqrt{436}}(854.2) + \frac{15}{\sqrt{(14-y_D)^2 + 225}}T_{CD} = 0 \qquad (1)$$

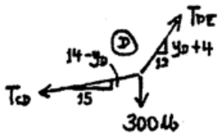
$$+\uparrow\Sigma F_y = 0; \qquad \frac{6}{\sqrt{436}}(854.2) + \frac{14 - y_D}{\sqrt{(14-y_D)^2 + 225}}T_{CD} - P_1 = 0 \qquad (2)$$

At *D*

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{12}{\sqrt{(4+y_D)^2 + 144}}T_{DE} - \frac{15}{\sqrt{(14-y_D)^2 + 225}}T_{CD} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{4 + y_D}{\sqrt{(4+y_D)^2 + 144}}T_{DE} - \frac{14 - y_D}{\sqrt{(14-y_D)^2 + 225}}T_{CD} - 300 = 0$$

$$T_{CD} = \frac{3600\sqrt{225 + (14 - y_D)^2}}{27y_D - 108}$$



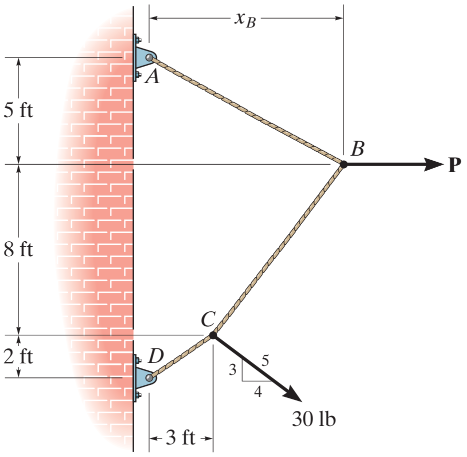
Substitute into Eq. (1) :

$$y_D = 6.44 \text{ ft} \qquad \textbf{Ans}$$

$$T_{CD} = 916.1 \text{ lb}$$

$$P_1 = 658 \text{ lb} \qquad \textbf{Ans}$$

•7–97. The cable supports the loading shown. Determine the horizontal distance x_B the force at point B acts from A. Set $P = 40$ lb.

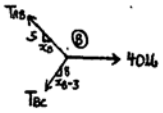


At B

$$\xrightarrow{+}\Sigma F_x = 0; \qquad 40 - \frac{x_B}{\sqrt{x_B^2 + 25}}T_{AB} - \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{5}{\sqrt{x_B^2 + 25}}T_{AB} - \frac{8}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 0$$

$$\frac{13x_B - 15}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 200 \qquad (1)$$

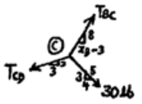


At C

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{4}{5}(30) + \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} - \frac{3}{\sqrt{13}}T_{CD} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{8}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} - \frac{2}{\sqrt{13}}T_{CD} - \frac{3}{5}(30) = 0$$

$$\frac{30 - 2x_B}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 102 \qquad (2)$$

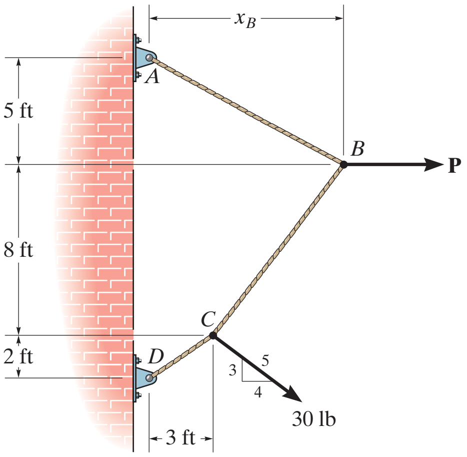


Solving Eqs. (1) & (2)

$$\frac{13x_B - 15}{30 - 2x_B} = \frac{200}{102}$$

$$x_B = 4.36 \text{ ft} \qquad \textbf{Ans}$$

7–98. The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.



At B

$$\xrightarrow{+}\Sigma F_x = 0; \qquad P - \frac{6}{\sqrt{61}}T_{AB} - \frac{3}{\sqrt{73}}T_{BC} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{5}{\sqrt{61}}T_{AB} - \frac{8}{\sqrt{73}}T_{BC} = 0$$

$$5P - \frac{63}{\sqrt{73}}T_{BC} = 0 \qquad (1)$$

At C

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{4}{5}(30) + \frac{3}{\sqrt{73}}T_{BC} - \frac{3}{\sqrt{13}}T_{CD} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{8}{\sqrt{73}}T_{BC} - \frac{2}{\sqrt{13}}T_{CD} - \frac{3}{5}(30) = 0$$

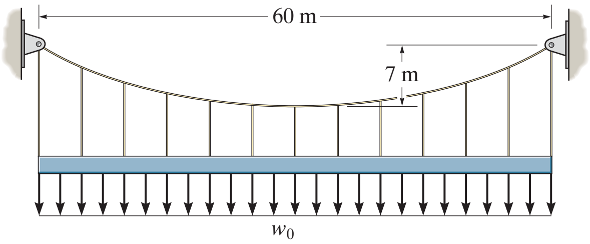
$$\frac{18}{\sqrt{73}}T_{BC} = 102 \qquad (2)$$

Solving Eqs. (1) & (2)

$$\frac{63}{18} = \frac{5P}{102}$$

$$P = 71.4 \text{ lb} \qquad \textbf{Ans}$$

7–99. Determine the maximum uniform distributed loading w_0 N/m that the cable can support if it is capable of sustaining a maximum tension of 60 kN.



The Equation of The Cable :

$$y = \frac{1}{F_H}\int\left(\int w(x)\,dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1 x + C_2\right) \qquad [1]$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0 x + C_1) \qquad [2]$$

Boundary Conditions :

$y = 0$ at $x = 0$, then from Eq.[1] $\quad 0 = \frac{1}{F_H}(C_2) \quad C_2 = 0$

$\frac{dy}{dx} = 0$ at $x = 0$, then from Eq.[2] $\quad 0 = \frac{1}{F_H}(C_1) \quad C_1 = 0$

Thus,

$$y = \frac{w_0}{2F_H}x^2 \qquad [3]$$

$$\frac{dy}{dx} = \frac{w_0}{F_H}x \qquad [4]$$

$y = 7$ m at $x = 30$ m, then from Eq.[3] $\quad 7 = \frac{w_0}{2F_H}(30^2) \quad F_H = \frac{450}{7}w_0$

$\theta = \theta_{\max}$ at $x = 30$ m and the maximum tension occurs when $\theta = \theta_{\max}$. From Eq.[4]

$$\tan\theta_{\max} = \left.\frac{dy}{dx}\right|_{x=30\text{ m}} = \frac{w_0}{\frac{450}{7}w_0}x = 0.01556(30) = 0.4667$$

$$\theta_{\max} = 25.02^\circ$$

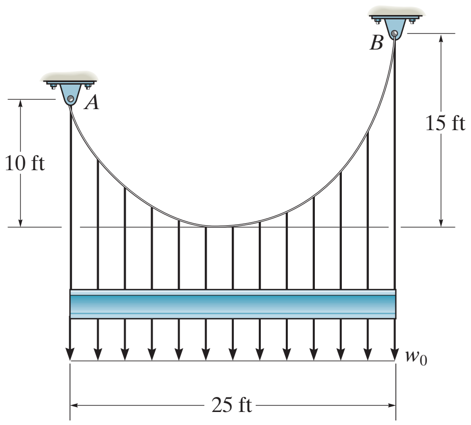
The maximum tension in the cable is

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}}$$

$$60 = \frac{\frac{450}{7}w_0}{\cos 25.02^\circ}$$

$$w_0 = 0.846\text{ kN/m} \qquad \textbf{Ans}$$

***7–100.** The cable supports the uniform distributed load of $w_0 = 600$ lb/ft. Determine the tension in the cable at each support A and B.



Use the equations of Example 7 - 12.

$$y = \frac{w_0}{2F_H}x^2$$

$$15 = \frac{600}{2F_H}x^2$$

$$10 = \frac{600}{2F_H}(25 - x)^2$$

$$\frac{600}{2(15)}x^2 = \frac{600}{2(10)}(25 - x)^2$$

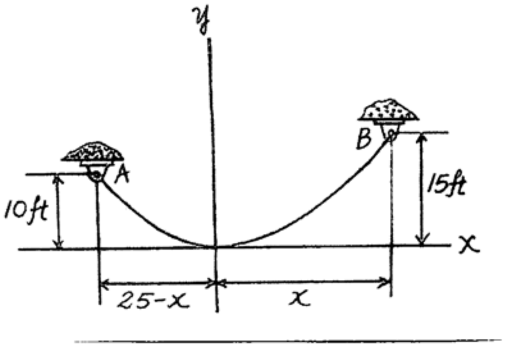
$$x^2 = 1.5(625 - 50x + x^2)$$

$$0.5x^2 - 75x + 937.50 = 0$$

Choose root < 25 ft.

$$x = 13.76 \text{ ft}$$

$$F_H = \frac{w_0}{2y}x^2 = \frac{600}{2(15)}(13.76)^2 = 3788 \text{ lb}$$



At B:

$$y = \frac{w_0}{2F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_B = 0.15838\,x\Big|_{x=13.76} = 2.180$$

$$\theta_B = 65.36^\circ$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{3788}{\cos 65.36^\circ} = 9085 \text{ lb} = 9.09 \text{ kip} \qquad \text{Ans}$$

At A

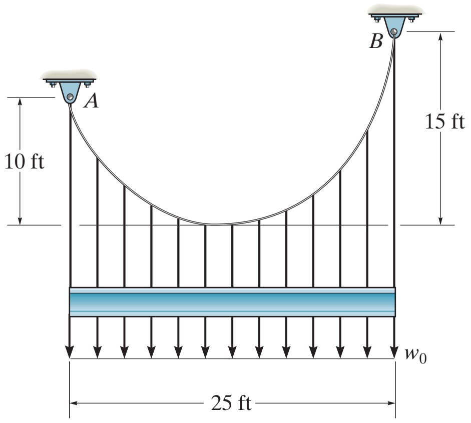
$$y = \frac{w_0}{2F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_A = 0.15838\,x\Big|_{x=(25-13.76)} = 1.780$$

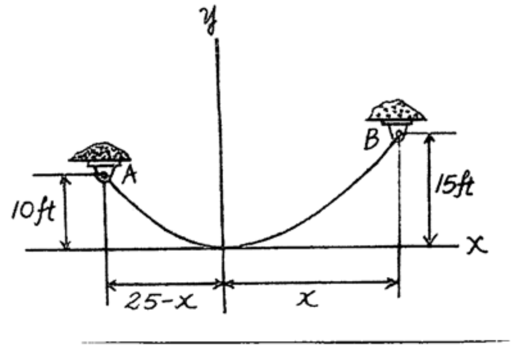
$$\theta_A = 60.67^\circ$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{3788}{\cos 60.67^\circ} = 7733 \text{ lb} = 7.73 \text{ kip} \qquad \text{Ans}$$

•7–101. Determine the maximum uniform distributed load w_0 the cable can support if the maximum tension the cable can sustain is 4000 lb.



Use the equations of Example 7 - 12.



$$y = \frac{w_0}{2\,F_H}x^2$$

$$15 = \frac{w_0}{2\,F_H}x^2$$

$$10 = \frac{w_0}{2\,F_H}(25 - x)^2$$

$$\frac{x^2}{15} = \frac{1}{10}(25 - x)^2$$

$$x^2 = 1.5\,(625 - 50x + x^2)$$

$$0.5x^2 - 75x + 937.50 = 0$$

Choose root < 25 ft.

$$x = 13.76\text{ ft}$$

$$F_H = \frac{w_0}{2\,y}x^2 = \frac{w_0}{2\,(15)}(13.76)^2 = 6.31378\,w_0$$

Maximum tension occurs at B since the slope y of the cable is greatest there.

$$y = \frac{w_0}{2\,F_H}x^2$$

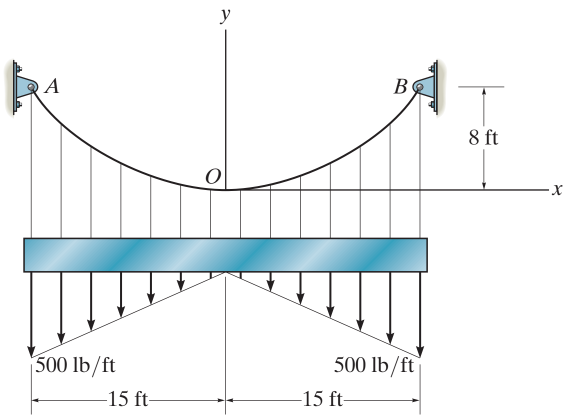
$$\left.\frac{dy}{dx}\right|_{x\,=\,13.76\text{ ft}} = \tan\theta_{max} = \frac{w_0 x}{F_H} = \frac{w_0\,(13.76)}{6.31378\,w_0}$$

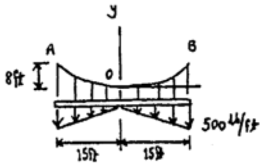
$$\theta_{max} = 65.36^\circ$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

$$4000 = \frac{6.31378\,w_0}{\cos 65.36^\circ} \qquad w_0 = 264\text{ lb/ft} \qquad \text{Ans}$$

7–102. The cable is subjected to the triangular loading. If the slope of the cable at point *O* is zero, determine the equation of the curve $y = f(x)$ which defines the cable shape *OB*, and the maximum tension developed in the cable.





$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\int\left(\int \frac{500}{15}x\,dx\right)dx$$

$$= \frac{1}{F_H}\int\left(\frac{50}{3}x^2 + C_1\right)dx$$

$$= \frac{1}{F_H}\left(\frac{50}{9}x^3 + C_1x + C_2\right)$$

$$\frac{dy}{dx} = \frac{50}{3F_H}x^2 + \frac{C_1}{F_H}$$

t $x = 0, \quad \frac{dy}{dx} = 0 \quad C_1 = 0$

t $x = 0, \quad y = 0 \quad C_2 = 0$

$$y = \frac{50}{9F_H}x^3$$

t $x = 15\text{ ft}, \quad y = 8\text{ ft} \quad F_H = 2344\text{ lb}$

$y = 2.37(10^{-3})x^3$ **Ans**

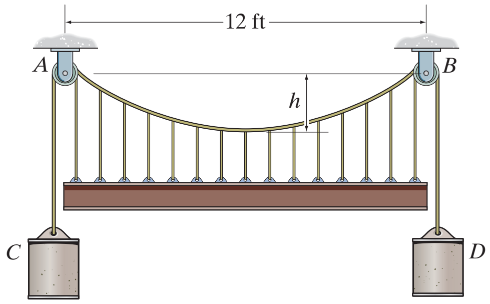
$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{50}{3(2344)}x^2\right|_{x=15\text{ ft}}$$

$$\theta_{max} = \tan^{-1}(1.6) = 57.99°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2344}{\cos 57.99°} = 4422\text{ lb}$$

$T_{max} = 4.42$ kip **Ans**

7–103. If cylinders *C* and *D* each weigh 900 lb, determine the maximum sag *h*, and the length of the cable between the smooth pulleys at *A* and *B*. The beam has a weight per unit length of 100 lb/ft.



Since the loading and system are symmetric as indicated in the free-body diagram shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2(900\sin\theta_{\max}) - 100(12) = 0$$

$$\theta_{\max} = 41.81°$$

Thus,

$$F_H = T_{\max}\cos\theta_{\max} = 900\cos 41.81° = 670.82\text{ lb}$$

As shown in Fig. *a*, the origin of the $x-y$ coordinate system will be set at the lowest point of the cable.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{100}{670.82} = 0.1491$$

Integrating the above equation,

$$\frac{dy}{dx} = 0.1491x + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus, Eq. (1) becomes

$$\frac{dy}{dx} = 0.1491x$$

Integrating,

$$y = 0.07454x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus, Eq. (1) becomes

$$y = 0.07454x^2$$

Applying another boundary condition, $y = h$, at $x = 6$ ft,

$$h = 0.07454(6^2) = 2.68\text{ ft} \qquad \textbf{Ans.}$$

The differential length of the cable is

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + 0.02222x^2}\,dx$$

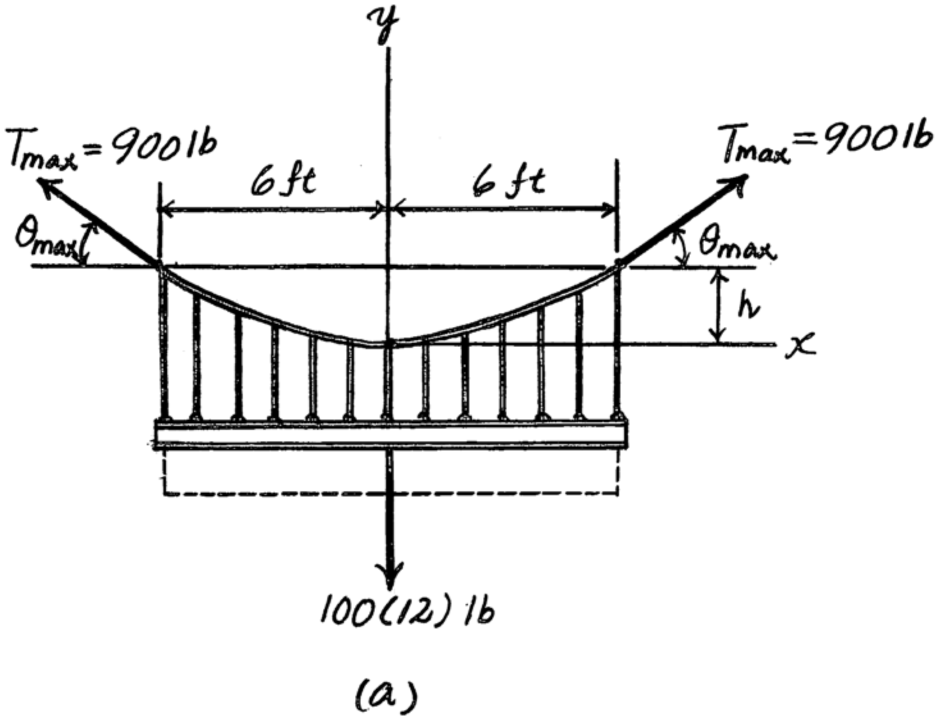
Thus, the total length of the cable is

$$L = \int ds = 2\int_0^{6\text{ ft}} \sqrt{1 + 0.02222x^2}\,dx$$

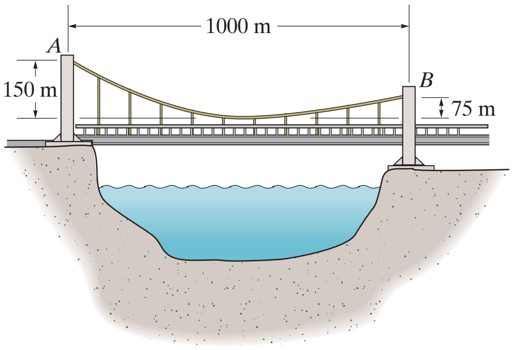
$$= 0.2981\int_0^{6\text{ ft}} \sqrt{45 + x^2}\,dx$$

$$= 0.2981\left\{\frac{1}{2}\left[x\sqrt{45 + x^2} + 45\ln\left(x + \sqrt{45 + x^2}\right)\right]\right\}\Bigg|_0^{6\text{ ft}}$$

$$= 13.4\text{ ft}$$ **Ans.**



***7–104.** The bridge deck has a weight per unit length of 80 kN/m. It is supported on each side by a cable. Determine the tension in each cable at the piers *A* and *B*.



As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Since the bridge deck is supported by two cables, $w(x) = \frac{80}{2} = 40\text{ kN / m}$.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{40(10^3)}{F_H}$$

Integrating,

$$\frac{dy}{dx} = \frac{40(10^3)}{F_H} + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{40(10^3)}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{20(10^3)}{F_H}x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{20(10^3)}{F_H}x^2$$

Applying two other boundary conditions $y = 75\text{ m}$ at $x = x_0$ and $y = 150\text{ m}$ at $x = -(1000 - x_0)$,

$$75 = \frac{20(10^3)}{F_H}x_0^2$$

$$150 = \frac{20(10^3)}{F_H}[-(1000 - x_0)]^2$$

Solving these equations

$$x_0 = 414.21\text{ m} \qquad F_H = 45.75(10^6)\text{ N}$$

Substituting the result for $\mathbf{F}_H$ into Eq. (1),

$$\frac{dy}{dx} = \frac{40(10^3)}{45.75(10^6)}x = 0.8743(10^{-3})x$$

Thus, the angles the cables make with the horizontal at A and B are

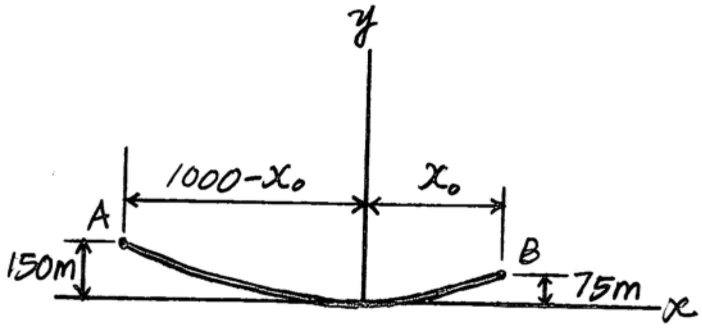
$$\theta_B = \left|\tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x_B}\right)\right| = \left|\tan^{-1}\left[0.8743(10^{-3})(414.21)\right]\right| = 19.91°$$

$$\theta_A = \left|\tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x_A}\right)\right| = \left|\tan^{-1}\left\{0.8743(10^{-3})\left[-(1000-414.21)\right]\right\}\right| = 27.12°$$

Thus,

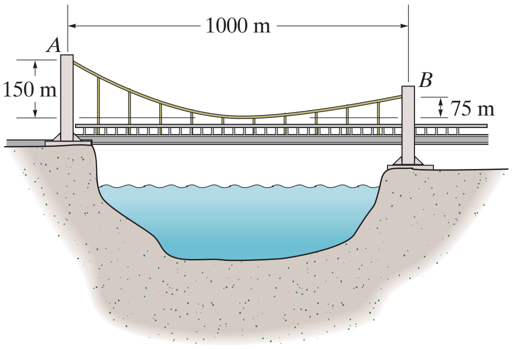
$$T_B = \frac{F_H}{\cos\theta_B} = \frac{45.75(10^6)}{\cos 19.91°} = 48.66(10^6)\text{ N} = 48.7\text{ MN} \qquad \textbf{Ans.}$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{45.75(10^6)}{\cos 27.12°} = 51.40(10^6)\text{ N} = 51.4\text{ MN} \qquad \textbf{Ans.}$$



(a)

•7–105. If each of the two side cables that support the bridge deck can sustain a maximum tension of 50 MN, determine the allowable uniform distributed load w_0 caused by the weight of the bridge deck.



As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Since the bridge deck is supported by two cables, $w(x) = \frac{w_0}{2}$.

$$\frac{d^2y}{dx^2} = \frac{w_0/2}{F_H} = \frac{w_0}{2F_H}$$

Integrating,

$$\frac{dy}{dx} = \frac{w_0}{2F_H} + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_0}{2F_H}x \qquad (1)$$

Integrating,

$$y = \frac{w_0}{4F_H}x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{w_0}{4F_H}x^2$$

Applying two other boundary conditions $y = 75$ m at $x = x_0$ and $y = 150$ m at $x = -(1000 - x_0)$,

$$75 = \frac{w_0}{4F_H}x^2$$

$$150 = \frac{w_0}{4F_H}[-(1000 - x_0)]^2$$

Solving these equations

$$x_0 = 414.21\text{ m} \qquad F_H = 571.91w_0$$

Substituting the result for $\mathbf{F}_H$ into Eq. (1),

$$\frac{dy}{dx} = \frac{w_0}{2(571.91w_0)}x = 0.8743(10^{-3})x$$

By observation, the angle the cable makes with the horizontal at $A\,(\theta_A)$ is greater than that at $B\,(\theta_B)$. Thus, the cable tension at A is the greatest.

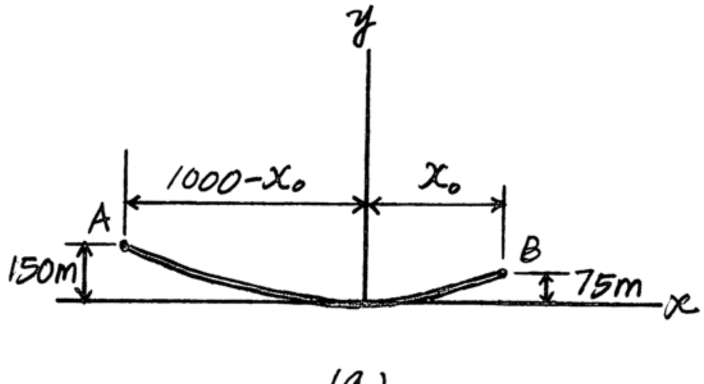
$$\theta_A = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_A}\right)\right| = \left|\tan^{-1}\left\{0.8743(10^{-3})[-(1000-414.21)]\right\}\right| = 27.12^\circ$$

By setting $T_A = 50(10^6)\,\text{N}$,

$$T_A = \frac{F_H}{\cos\theta_A}$$

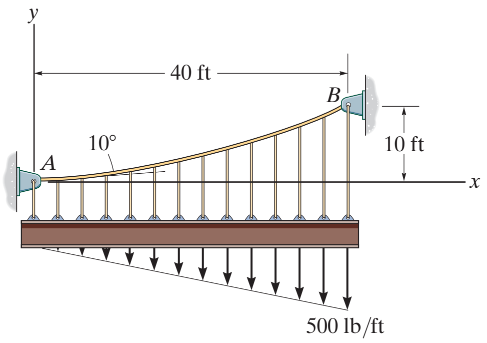
$$50(10^6) = \frac{571.91w_0}{\cos 27.12^\circ}$$

$$w_0 = 77.82(10^3)\ \text{N/m} = 77.8\ \text{kN/m}$$ **Ans.**



(a)

7–106. If the slope of the cable at support A is 10°, determine the deflection curve $y = f(x)$ of the cable and the maximum tension developed in the cable.



The triangular distributed load is described by $w(x) = \frac{500}{40}x = 12.5x$.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{12.5}{F_H}x$$

Integrating,

$$\frac{dy}{dx} = \frac{6.25}{F_H}x^2 + C_1$$

Applying the boundary condition $\frac{dy}{dx} = \tan 10°$ at $x = 0$ results in $C_1 = \tan 10°$. Thus,

$$\frac{dy}{dx} = \frac{6.25}{F_H}x^2 + \tan 10° \qquad (1)$$

Integrating,

$$y = \frac{2.0833}{F_H}x^3 + \tan 10° x + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{2.0833}{F_H}x^3 + \tan 10° x \qquad (2)$$

Applying the boundary condition $y = 10$ ft at $x = 40$ ft,

$$10 = \frac{2.0833}{F_H}(40)^3 + \tan 10°(40)$$

$$F_H = 45.245(10^3)\text{ lb}$$

Substituting the result into Eqs. (1) and (2),

$$\frac{dy}{dx} = \frac{6.25}{45.245(10^3)}x^2 + \tan 10°$$

$$= 0.1381(10^{-3})x^2 + \tan 10°$$

and

$$y = \frac{2.0833}{45.245(10^3)}x^3 + \tan 10° x$$

$$= 46.0(10^{-6})x^3 + 0.176x \qquad \textbf{Ans.}$$

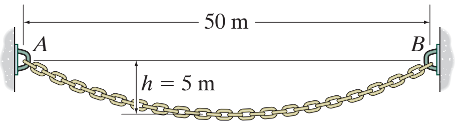
The maximum tension occurs at point B, where the cable makes the greatest angle with the horizontal. Here,

$$\theta_{\max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{40\text{ ft}}\right) = \tan^{-1}\left[\frac{6.25}{45.245(10^3)}(40^2) + \tan 10°\right] = 21.67°$$

Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{45.245(10^3)}{\cos 21.67°} = 48.69(10^3)\text{ lb} = 48.7\text{ kip} \qquad \textbf{Ans.}$$

7–107. If $h = 5$ m, determine the maximum tension developed in the chain and its length. The chain has a mass per unit length of 8 kg/m.



As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Here, $w(s) = 8(9.81)\text{ N / m} = 78.48\text{ N / m}$.

$$\frac{d^2y}{dx^2} = \frac{78.48}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{78.48}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{78.48}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{78.48}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{78.48}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{78.48}{F_H}x} - e^{-\frac{78.48}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{78.48}{F_H}x \qquad (1)$$

Integrating Eq. (1),

$$y = \frac{F_H}{78.48}\cosh\left(\frac{78.48}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 5$ m at $x = 25$ m,

$$5 = \frac{F_H}{78.48}\left\{\cosh\left(\frac{78.48}{F_H}(25)\right) - 1\right\}.$$

Solving by trial and error,

$$F_H = 4969.06\text{ N}$$

The maximum tension occurs at either points A or B where the chain makes the greatest angle with the horizontal. Here,

$$\theta_{\max} = \tan^{-1}\left(\frac{dy}{dx}\bigg|_{x=25\text{ m}}\right) = \tan^{-1}\left\{\sinh\left(\frac{78.48}{F_H}(25)\right)\right\} = 22.06°$$

Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{4969.06}{\cos 22.06°} = 5361.46\text{ N} = 5.36\text{ kN} \qquad \textbf{Ans.}$$

Referring to the free - body diagram shown in Fig. b,

$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta - 8(9.81)s = 0$

$\xrightarrow{+} \Sigma F_y = 0; \qquad T\cos\theta - 4969.06 = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = 0.015794s$$

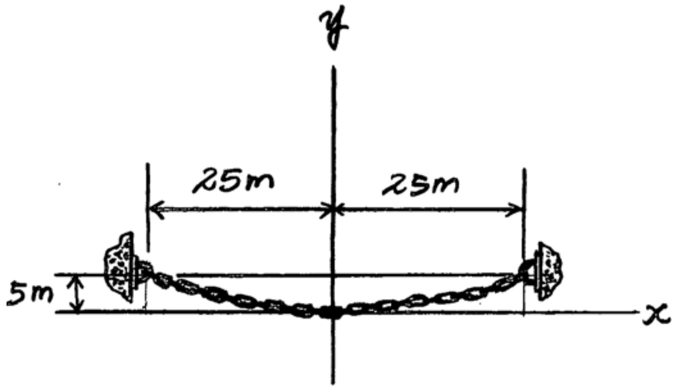
Equating Eqs. (1) and (2),

$$\sinh\left[\frac{78.48}{4969.06}x\right] = 0.015794s$$

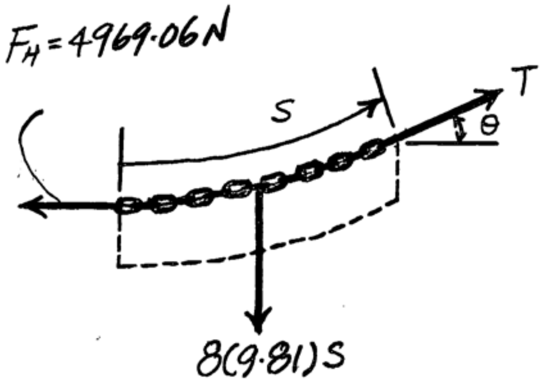
$$s = 63.32\sinh[0.01579x]$$

Thus, the length of the chain is

$$L = 2\{63.32\sinh[0.01579(25)]\} = 51.3\text{ m} \qquad \textbf{Ans.}$$

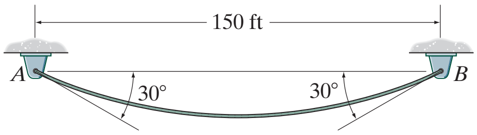


(a)



(b)

***7–108.** A cable having a weight per unit length of 5 lb/ft is suspended between supports *A* and *B*. Determine the equation of the catenary curve of the cable and the cable's length.



As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Here, $w(s) = 5$ lb / ft.

$$\frac{d^2y}{dx^2} = \frac{5}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Substituting these two values into the equation,

$$\frac{du}{\sqrt{1+u^2}} = \frac{5}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{5}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{5}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{5}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{5}{F_H}x} - e^{-\frac{5}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{5}{F_H}x \qquad (1)$$

Applying the boundary equation $\frac{dy}{dx} = \tan 30°$ at $x = 75$ ft,

$$\tan 30° = \sinh\left[\frac{5}{F_H}(75)\right]$$

$$F_H = 682.68 \text{ lb}$$

Substituting this result into Eq. (1),

$$\frac{dy}{dx} = \sinh\left[7.324(10^{-3})x\right] \qquad (2)$$

Integrating,

$$y = 136.54\cosh\left[7.324(10^{-3})x\right] + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -136.54$. Thus,

$$y = 137\left\{\cosh\left[7.324(10^{-3})x\right] - 1\right\}\text{ ft} \qquad \textbf{Ans.}$$

If we write the force equation of equilibrium along the x and y axes by referring to the free - body diagram shown in Fig. *b*, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T\cos\theta - 682.68 = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta - 5s = 0$$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = 7.324(10^{-3})s \qquad (3)$$

Equating Eqs. (2) and (3),

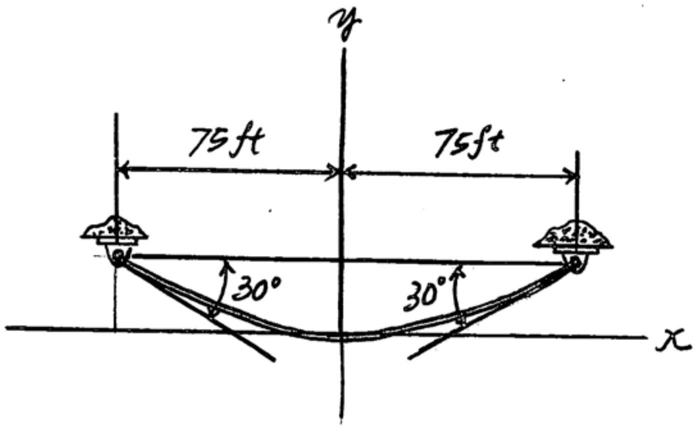
$$7.324(10^{-3})s = \sinh\left[7.324(10^{-3})x\right]$$

$$s = 136.54\sinh\left[7.324(10^{-3})x\right]\text{ft}$$

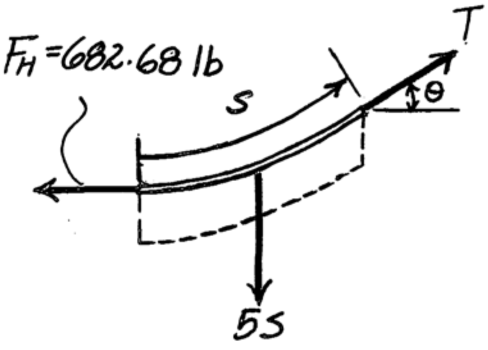
Thus, the length of the cable is

$$L = 2\left\{136.54\sinh\left[7.324(10^{-3})(75)\right]\right\}$$

$$= 157.66\text{ ft} = 158\text{ ft} \qquad \textbf{Ans.}$$

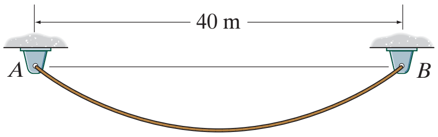


(a)



(b)

•7–109. If the 45-m-long cable has a mass per unit length of 5 kg/m, determine the equation of the catenary curve of the cable and the maximum tension developed in the cable.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 5(9.81)\text{ N/m} = 49.05\text{ N/m}$.

$$\frac{d^2y}{dx^2} = \frac{49.05}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{49.05}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{49.05}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{49.05}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{49.05}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{49.05}{F_H}x} - e^{-\frac{49.05}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{49.05}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{F_H}{49.05}\cosh\left(\frac{49.05}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -\frac{F_H}{49.05}$. Thus,

$$y = \frac{F_H}{49.05}\left[\cosh\left(\frac{49.05}{F_H}x\right) - 1\right]\text{ m}$$

If we write the force equation of equilibrium along the x and y axes by referring to the free - body diagram shown in Fig. b,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad T\cos\theta - F_H = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad T\sin\theta - 5(9.81)s = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = \frac{49.05s}{F_H} \qquad (3)$$

Equating Eqs. (1) and (3) yields

$$\frac{49.05s}{F_H} = \sinh\left(\frac{49.05}{F_H}x\right)$$

$$s = \frac{F_H}{49.05}\sinh\left(\frac{49.05}{F_H}\right)$$

Thus, the length of the cable is

$$L = 45 = 2\left\{\frac{F_H}{49.05}\sinh\left(\frac{49.05}{F_H}(20)\right)\right\}$$

Solving by trial and error,

$$F_H = 1153.41\text{ N}$$

Substituting this result into Eq. (2),

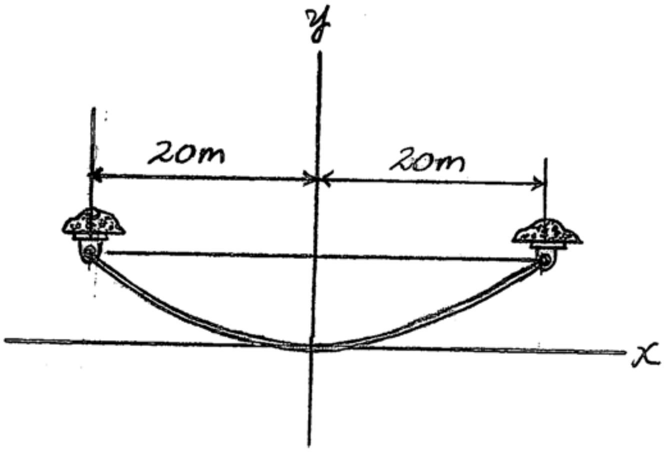
$$y = 23.5[\cos h\,0.0425x - 1]\text{ m} \qquad \textbf{Ans.}$$

The maximum tension occurs at either points A or B where the cable makes the greatest angle with the horizontal. Here

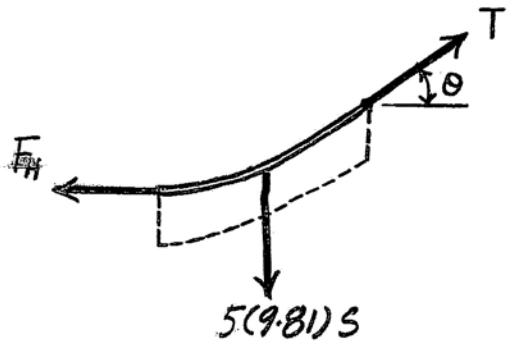
$$\theta_{\max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=20\text{ m}}\right) = \tan^{-1}\left\{\sinh\left(\frac{49.05}{F_H}(20)\right)\right\} = 43.74°$$

Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{1153.41}{\cos 43.74°} = 1596.36\text{ N} = 1.60\text{ kN} \qquad \textbf{Ans.}$$



(a)



(b)

7–110. Show that the deflection curve of the cable discussed in Example 7–13 reduces to Eq. 4 in Example 7–12 when the *hyperbolic cosine function* is expanded in terms of a series and only the first two terms are retained. (The answer indicates that the *catenary* may be replaced by a parabola in the analysis of problems in which the sag is small. In this case, the cable weight is assumed to be uniformly distributed along the horizontal.)

$$\cosh x = 1 + \frac{x^2}{2!} + \ldots$$

Substituting into

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$= \frac{F_H}{w_0}\left[1 + \frac{w_0^2 x^2}{2F_H^2} + \ldots - 1\right]$$

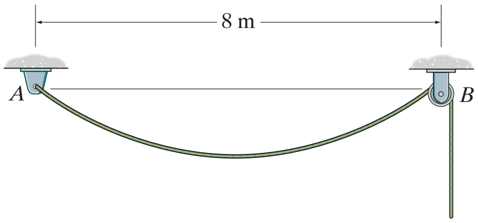
$$\cong \frac{w_0 x^2}{2F_H}$$

Using Eq. (3) in Example 7 – 12,

$$F_H = \frac{w_0 L^2}{8h}$$

We get $y = \dfrac{4h}{L^2}x^2$ **QED**

7–111. The cable has a mass per unit length of 10 kg/m. Determine the shortest total length L of the cable that can be suspended in equilibrium.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. $w(s) = 10(9.81)\text{N / m} = 98.1 \text{ N / m}$.

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{98.1}{F_H}dx$$

Integrating,

$$\ln\left(u + \sqrt{1+u^2}\right) = \frac{98.1}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u + \sqrt{1+u^2}\right) = \frac{98.1}{F_H}x$$

$$u + \sqrt{1+u^2} = e^{\frac{98.1}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{98.1}{F_H}x} - e^{-\frac{98.1}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{98.1}{F_H}x \qquad (1)$$

Referring to the free-body diagram shown in Fig. *b*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad T\cos\theta - F_H = 0$

$+\uparrow \Sigma F_y = 0;\qquad T\sin\theta - 10(9.81)s = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = \frac{98.1s}{F_H} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{98.1s}{F_H} = \sinh\left(\frac{98.1}{F_H}x\right)$$

$$s = \frac{F_H}{98.1}\sinh\left(\frac{98.1}{F_H}x\right)$$

The length of the cable between A and B is therefore

$$L' = 2\left\{\frac{F_H}{98.1}\sinh\left(\frac{98.1}{F_H}(4)\right)\right\} = 0.02039F_H\sinh\left(\frac{392.4}{F_H}\right)$$

Thus, the length of the overhanging cable is

$$L - L' = L - 0.2039F_H\sinh\left(\frac{392.4}{F_H}\right)$$

The tension developed in the cable at B is equal to the weight of the overhanging cable.

$$T_B = 10(9.81)\left[L - 0.2039F_H\sinh\left(\frac{392.4}{F_H}\right)\right]$$

$$= 98.1L - 2F_H\sinh\left(\frac{392.4}{F_H}\right) \qquad \textbf{(3)}$$

Using Eq. (1), the angle that the cable makes with the horizontal at B is

$$\tan\theta_B = \sinh\left(\frac{98.1}{F_H}(4)\right) = \sinh\left(\frac{392.4}{F_H}\right)$$

From the geometry of Fig. c,

$$\cos\theta_B = \frac{1}{\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)}}$$

$$T_B = \frac{F_H}{\cos\theta_B} = F_H\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)} \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4),

$$F_H\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)} = 98.1L - 2F_H \sin\left(\frac{392.4}{F_H}\right)$$

$$L = \frac{1}{98.1}\left[F_H\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)} + 2F_H \sin\left(\frac{392.4}{F_H}\right)\right]$$

However, $\cosh^2\left(\frac{392.4}{F_H}\right) = 1+\sinh^2\left(\frac{392.4}{F_H}\right)$. Thus,

$$L = \frac{1}{98.1}\left[F_H \cosh\left(\frac{392.4}{F_H}\right) + 2F_H \sinh\left(\frac{392.4}{F_H}\right)\right] \qquad (5)$$

In order for L to be minimum, $\frac{dL}{dF_H}$ must be equal to zero.

$$\frac{dL}{dF_H} = \frac{1}{98.1}\left[F_H \sinh\left(\frac{392.4}{F_H}\right)\left(-\frac{392.4}{F_H{}^2}\right) + \cosh\left(\frac{392.4}{F_H}\right) + 2F_H \cosh\left(\frac{392.4}{F_H}\right)\left(-\frac{392.4}{F_H{}^2}\right) + 2\sinh\left(\frac{392.4}{F_H}\right)\right]$$

$$= \frac{1}{98.1}\left[\cosh\left(\frac{392.4}{F_H}\right) + 2\sinh\left(\frac{392.4}{F_H}\right) - \frac{392.4}{F_H}\sinh\left(\frac{392.4}{F_H}\right) - \frac{784.8}{F_H}\cosh\left(\frac{392.4}{F_H}\right)\right]$$

Setting $\frac{dL}{dF_H} = 0.$

$$\sinh\left(\frac{392.4}{F_H}\right)\left[2 - \frac{392.4}{F_H}\right] + \cosh\left(\frac{392.4}{F_H}\right)\left[1 - \frac{784.8}{F_H}\right] = 0$$

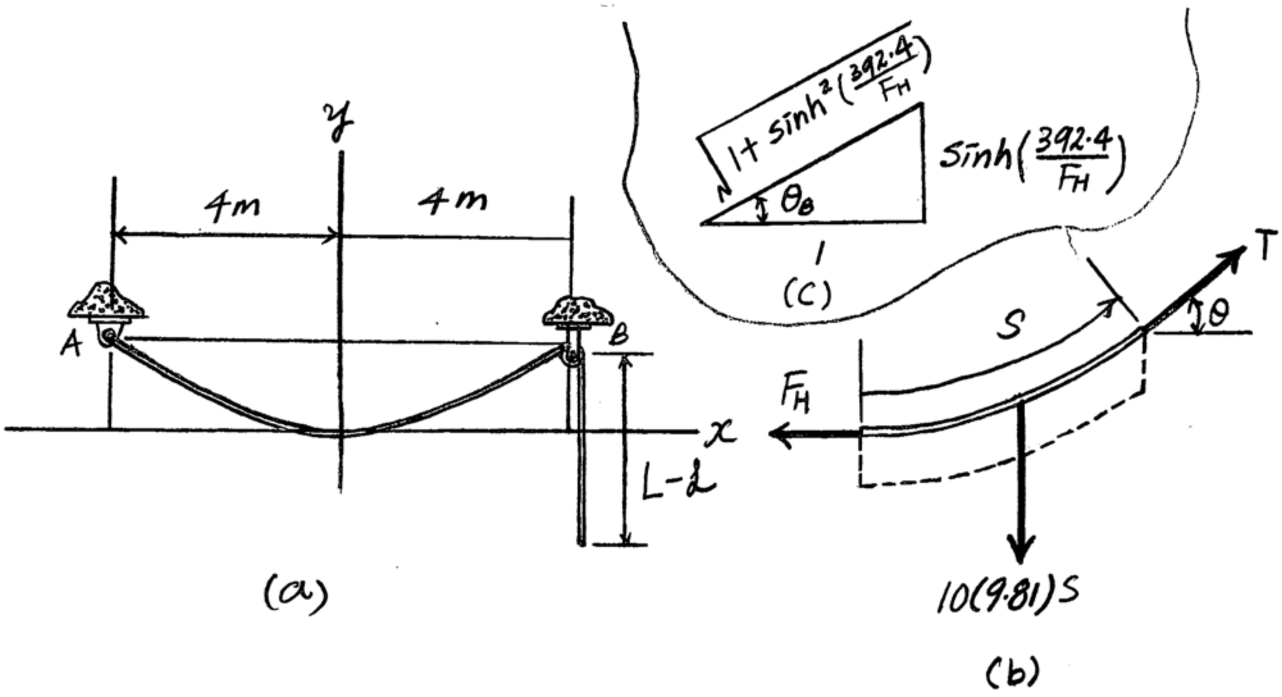
$$\tanh\left(\frac{392.4}{F_H}\right)(2F_H - 392.4) + (F_H - 784.8) = 0$$

Solving by trial and error,

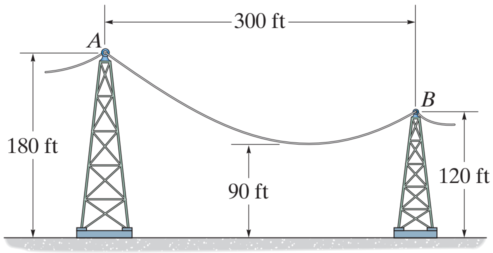
$$F_H = 438.70\,\text{N}$$

Substituting this result into Eq. (5) yields

$L = 15.5\,\text{m}$ **Ans.**


y
4m
4m
A
B
x
L-l
(a)
$1+\sinh^2\left(\frac{392.4}{F_H}\right)$
$\sinh\left(\frac{392.4}{F_H}\right)$
θ_B
1
(c)
T
s
θ
F_H
10(9.81)s
(b)

***7–112.** The power transmission cable has a weight per unit length of 15 lb/ft. If the lowest point of the cable must be at least 90 ft above the ground, determine the maximum tension developed in the cable and the cable's length between *A* and *B*.



As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Here, $w(s) = 15$ lb / ft.

$$\frac{d^2y}{dx^2} = \frac{15}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Thus,

$$\frac{du}{\sqrt{1+u^2}} = \frac{15}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{15}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{15}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{15}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{15}{F_H}x} - e^{-\frac{15}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{15}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{F_H}{15}\cosh\left(\frac{15}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -\frac{F_H}{15}$. Thus,

$$y = \frac{F_H}{15}\left[\cosh\left(\frac{15}{F_H}x\right) - 1\right]$$

Applying the boundary equation $y = 30$ ft at $x = x_0$ and $y = 90$ ft at $x = -(300 - x_0)$,

$$30 = \frac{F_H}{15}\left[\cosh\left(\frac{15x_0}{F_H}\right) - 1\right] \qquad (2)$$

$$90 = \frac{F_H}{15}\left\{\cosh\left[\frac{-15(300 - x_0)}{F_H}\right] - 1\right\}$$

Since $\cosh(a - b) = \cosh a \cosh b - \sinh a \sinh b$, then

$$90 = \frac{F_H}{15}\left(\cosh\frac{15x_0}{F_H}\cosh\frac{4500}{F_H} - \sinh\frac{15x_0}{F_H}\sinh\frac{4500}{F_H} - 1\right) \qquad (3)$$

Eq. (2) can be rewritten as

$$\cosh\frac{15x_0}{F_H} = \frac{450 + F_H}{F_H} \qquad (4)$$

Since $\sinh a = \sqrt{\cosh^2 a - 1}$, then

$$\sinh\frac{15x_0}{F_H} = \sqrt{\left(\frac{450 + F_H}{F_H}\right)^2 - 1} = \frac{1}{F_H}\sqrt{202500 + 900F_H} \qquad (5)$$

Substituting Eqs. (4) and (5) into Eq. (3),

$$1350 = (450 + F_H)\cosh\frac{4500}{F_H} - \sqrt{202500 + 900F_H}\,\sinh\frac{4500}{F_H} - F_H$$

Solving by trial and error,

$$F_H = 3169.58\text{ lb}$$

Substituting this result into Eq. (4),

$$x_0 = 111.31\text{ ft}$$

The maximum tension occurs at point A where the cable makes the greatest angle with the horizontal. Here,

$$\theta_{\max} = \left|\tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=-188.69\text{ ft}}\right)\right| = \tan^{-1}\left\{\sinh\left(\frac{15}{3169.58}(-188.69)\right)\right\} = 45.47°$$

Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{3169.58}{\cos 45.47°} = 4519.58\text{ lb} = 4.52\text{ kip}$$ **Ans.**

Referring to the free - body diagram shown in Fig. *b*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad T\cos\theta - 3169.58 = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad T\sin\theta - 15s = 0$

Eliminating *T*,

$$\frac{dy}{dx} = 4.732(10^{-3})s \qquad (6)$$

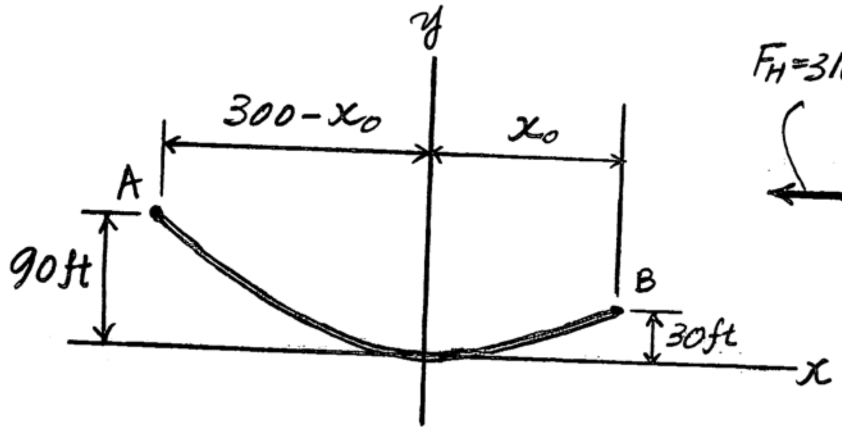
Equating Eqs. (1) and (6) yields

$$4.732(10^{-3})s = \sinh\left[4.732(10^{-3})x\right]$$

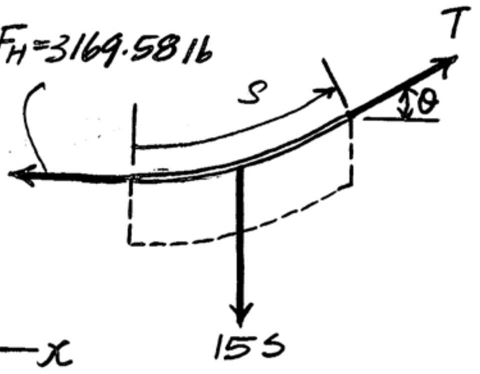
$$s = 211.31\sinh\left[4.732(10^{-3})x\right]$$

Thus, the length of the cable is

$$L = 211.31\sinh\left[4.732(10^{-3})(111.31)\right] + 211.31\sinh\left[4.732(10^{-3})(188.69)\right] = 331\text{ ft} \qquad \textbf{Ans.}$$

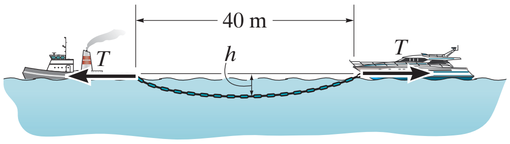


(a)



(b)

•7–113. If the horizontal towing force is $T = 20$ kN and the chain has a mass per unit length of 15 kg/m, determine the maximum sag h. Neglect the buoyancy effect of the water on the chain. The boats are stationary.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the chain.

Here, $F_H = T = 20(10^3)$ N and

$w(s) = 15(9.81)$ N / m $= 147.15$ N / m.

$$\frac{d^2y}{dx^2} = \frac{147.15}{20(10^3)}\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 7.3575(10^{-3})\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Thus,

$$\frac{du}{\sqrt{1+u^2}} = 7.3575(10^{-3})dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = 7.3575(10^{-3})x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

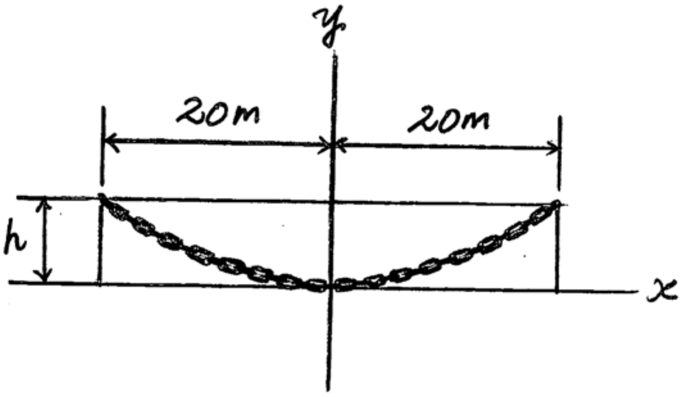
$$\ln\left(u+\sqrt{1+u^2}\right) = 7.3575(10^{-3})x$$

$$u+\sqrt{1+u^2} = e^{7.3575(10^{-3})x}$$

$$\frac{dy}{dx} = u = \frac{e^{7.3575(10^{-3})x} - e^{-7.3575(10^{-3})x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh 7.3575(10^{-3})x$$



(a)

Integrating,

$$y = 135.92\cosh 7.3575(10^{-3})x + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -135.92$. Thus,

$$y = 135.92\left[\cosh 7.3575(10^{-3})x - 1\right]$$

Applying the boundary equation $y = h$ at $x = 20$ m,

$$h = 135.92\left[\cosh 7.3575(10^{-3})(20) - 1\right] = 1.47 \text{ m}$$ **Ans.**

7–114. A 100-lb cable is attached between two points at a distance 50 ft apart having equal elevations. If the maximum tension developed in the cable is 75 lb, determine the length of the cable and the sag.

From Example 7 - 15,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = 75 \text{ lb}$$

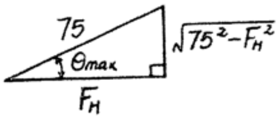
$$\cos\theta_{max} = \frac{F_H}{75}$$

For $\frac{1}{2}$ of cable,

$$w_0 = \frac{\frac{100}{2}}{s} = \frac{50}{s}$$

$$\tan\theta_{max} = \frac{w_0\, s}{F_H} = \frac{\sqrt{(75)^2 - F_H^2}}{F_H} = \frac{50}{F_H}$$

Thus,



$$\sqrt{(75)^2 - F_H^2} = 50; \qquad F_H = 55.9 \text{ lb}$$

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right) = \frac{55.9}{\left(\frac{50}{s}\right)}\sinh\left\{\left(\frac{50}{s\,(55.9)}\right)\left(\frac{50}{2}\right)\right\}$$

$s = 27.8$ ft

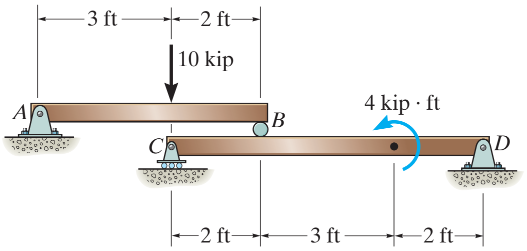
$$w_0 = \frac{50}{27.8} = 1.80 \text{ lb/ft}$$

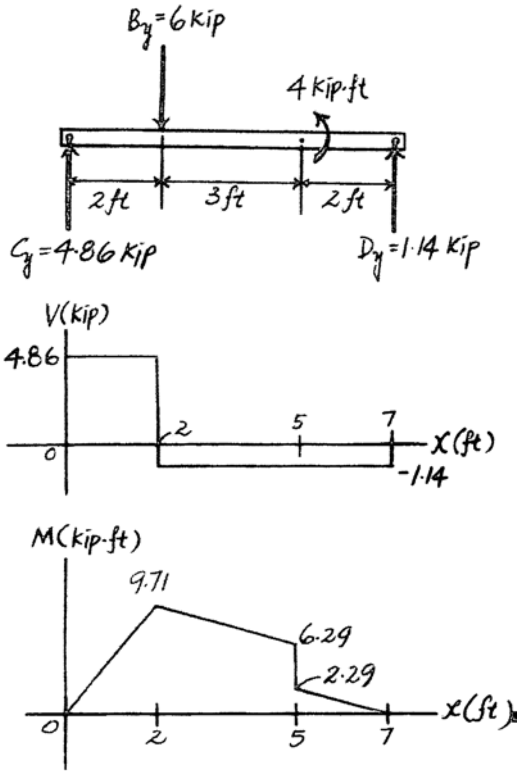
Total length $= 2s = 55.6$ ft   **Ans**

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2F_H}\right) - 1\right] = \frac{55.9}{1.80}\left[\cosh\left(\frac{1.80\,(50)}{2\,(55.9)}\right) - 1\right]$$

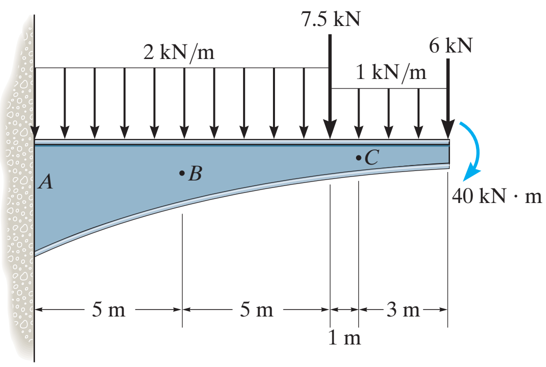
$= 10.6$ ft   **Ans**

7–115. Draw the shear and moment diagrams for beam *CD*.





***7–116.** Determine the internal normal force, shear force, and moment at points *B* and *C* of the beam.



Free body Diagram : The Support reactions need not be computed for this case.

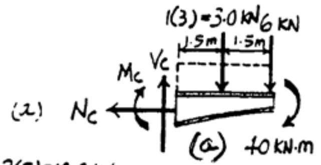
Internal Forces : Applying the equations of equilibrium to [FBD (a)], we have

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 3.00 - 6 = 0 \qquad V_C = 9.00\text{ kN}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 3.00(1.5) - 6(3) - 40 = 0$

$M_C = -62.5\text{ kN}\cdot\text{m}$ **Ans**



Applying the equations of equilibrium to segment *DB* [FBD (b)], we have

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B = 0$ **Ans**

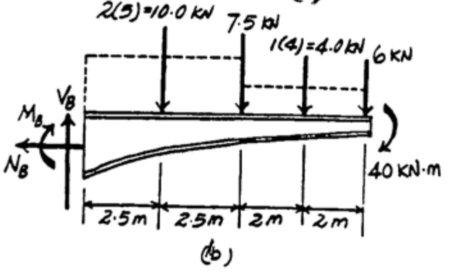
$+\uparrow \Sigma F_y = 0; \qquad V_B - 10.0 - 7.5 - 4.00 - 6 = 0$

$V_D = 27.5\text{ kN}$ **Ans**

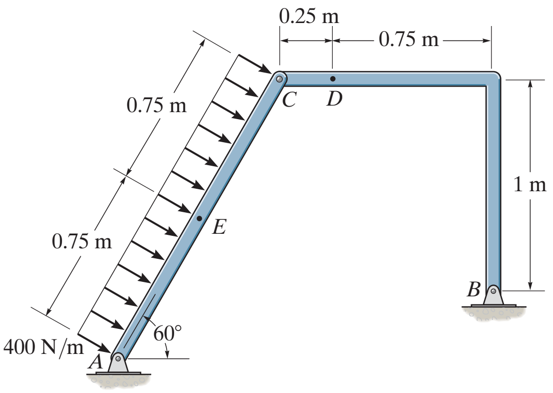
$\circlearrowleft + \Sigma M_B = 0; \qquad -M_B - 10.0(2.5) - 7.5(5)$

$-4.00(7) - 6(9) - 40 = 0$

$M_B = -184.5\text{ kN}\cdot\text{m}$ **Ans**



•7–117. Determine the internal normal force, shear force and moment at points *D* and *E* of the frame.



Support Reactions : Member *BC* is a two force member. From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_{BC} \cos 15°(1.5) - 600(0.75) = 0$$
$$F_{BC} = 310.58 \text{ N}$$

Internal Forces : Applying the equations of equilibrium to segment *CE* [FBD (b)], we have

$$\nearrow + \Sigma F_{x'} = 0; \qquad 310.58 \sin 15° - N_E = 0 \qquad N_E = 80.4 \text{ N} \qquad \textbf{Ans}$$

$$\nwarrow + \Sigma F_{y'} = 0; \qquad V_E + 310.58 \cos 15° - 300 = 0 \qquad V_E = 0 \qquad \textbf{Ans}$$

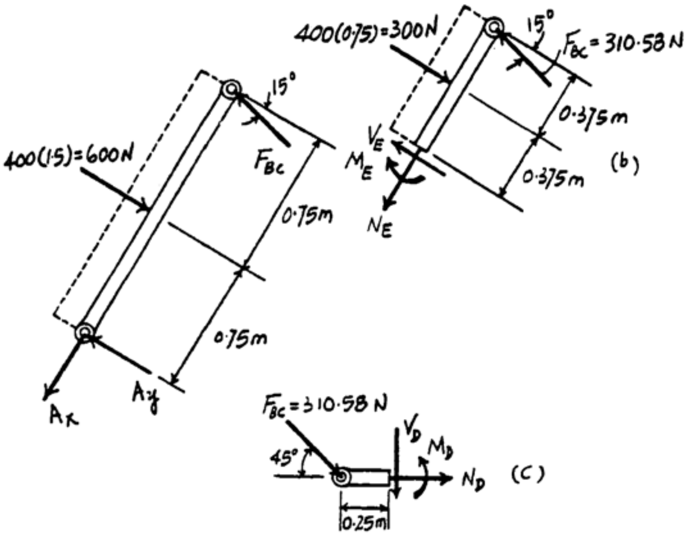
$$\circlearrowleft + \Sigma M_E = 0; \qquad 310.58 \cos 15°(0.75) - 300(0.375) - M_E = 0$$
$$M_E = 112.5 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

Applying the equations of equilibrium to segment *CD*[FBD (c)], we have

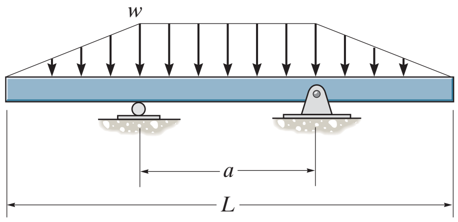
$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_D + 310.58 \cos 45° = 0 \qquad N_D = -220 \text{ N} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad -310.58 \sin 45° - V_D = 0 \qquad V_D = -220 \text{ N} \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad M_D + 310.58 \sin 45°(0.25) = 0$$
$$M_D = -54.9 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$



7–118. Determine the distance a between the supports in terms of the beam's length L so that the moment in the *symmetric* beam is zero at the beam's center.



Support Reactions : From FBD (a).

$$\circlearrowleft + \Sigma M_C = 0; \quad \frac{w}{2}(L+a)\left(\frac{a}{2}\right) - B_y(a) = 0 \qquad B_y = \frac{w}{4}(L+a)$$

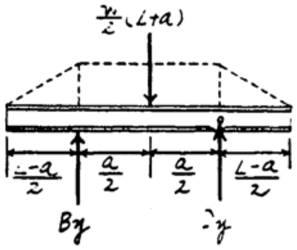
Free body Diagram : The FBD for segment AC sectioned through point C is drawn.

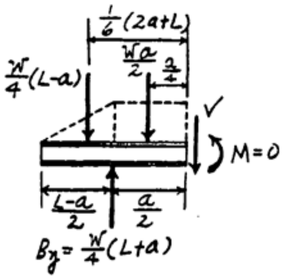
Internal Forces : This problem requires $M_C = 0$. Summing moments about point C[FBD (b)], we have

$$\circlearrowleft + \Sigma M_C = 0; \quad \frac{wa}{2}\left(\frac{a}{4}\right) + \frac{w}{4}(L-a)\left[\frac{1}{6}(2a+L)\right] - \frac{w}{4}(L+a)\left(\frac{a}{2}\right) = 0$$

$$2a^2 + 2aL - L^2 = 0$$

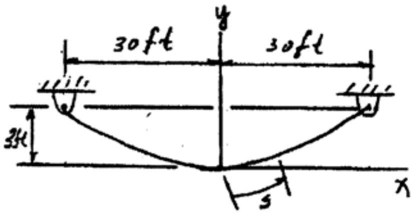
$$a = 0.366L \qquad \textbf{Ans}$$





7–119. A chain is suspended between points at the same elevation and spaced a distance of 60 ft apart. If it has a weight per unit length of 0.5 lb/ft and the sag is 3 ft, determine the maximum tension in the chain.

$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}\left(\int w_0\, ds\right)^2\right\}^{\frac{1}{2}}}$$



Performing the integration yields :

$$x = \frac{F_H}{0.5}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0.5s + C_1)\right] + C_2\right\} \qquad [1]$$

From Eq. 7-14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(0.5s + C_1)$$

At $s = 0$; $\frac{dy}{dx} = 0$ hence $C_1 = 0$

$$\frac{dy}{dx} = \tan\theta = \frac{0.5s}{F_H} \qquad [2]$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.[1] and using the result $C_1 = 0$ yields $C_2 = 0$. Hence

$$s = \frac{F_H}{0.5}\sinh\left(\frac{0.5}{F_H}x\right) \qquad [3]$$

Substituting Eq.[3] into [2] yields :

$$\frac{dy}{dx} = \sinh\left(\frac{0.5x}{F_H}\right) \qquad [4]$$

Performing the integration

$$y = \frac{F_H}{0.5}\cosh\left(\frac{0.5}{F_H}x\right) + C_3$$

Applying boundary conditions at $x = 0$; $y = 0$ yields $C_3 = -\frac{F_H}{0.5}$. Therefore

$$y = \frac{F_H}{0.5}\left[\cosh\left(\frac{0.5}{F_H}x\right) - 1\right]$$

At $x = 30$ ft; $y = 3$ ft $\qquad 3 = \frac{F_H}{0.5}\left[\cosh\left(\frac{0.5}{F_H}(30)\right) - 1\right]$

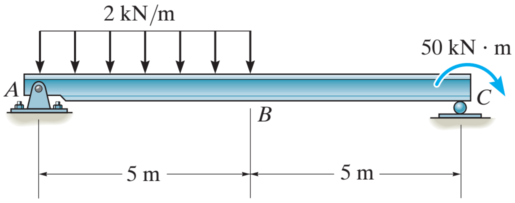
By trial and error $\qquad F_H = 75.25$ lb

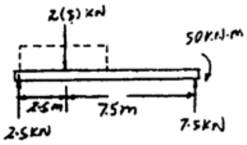
At $x = 30$ ft; $\theta = \theta_{max}$. From Eq.[4]

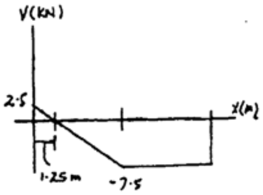
$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x=30\text{ ft}} = \sinh\left(\frac{0.5(30)}{75.25}\right) \qquad \theta_{max} = 11.346°$$

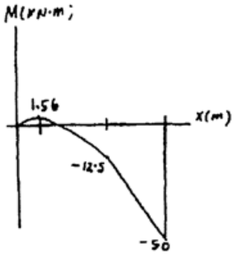
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{75.25}{\cos 11.346°} = 76.7 \text{ lb.} \qquad \textbf{Ans}$$

***7–120.** Draw the shear and moment diagrams for the beam.

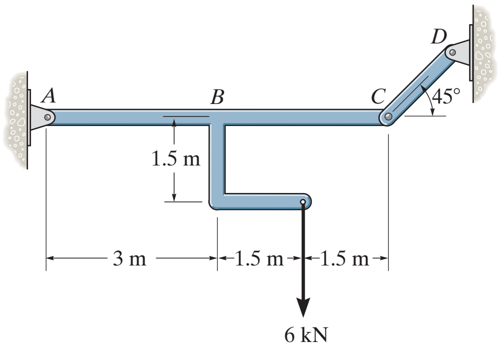








•7–121. Determine the internal shear and moment in member *ABC* as a function of *x*, where the origin for *x* is at *A*.



Support Reactions : The 6 kN load can be replaced by an equivalent force and couple moment at *B* as shown on FBD (a).

$\curvearrowleft + \Sigma M_A = 0;$ $F_{CD}\sin 45°(6) - 6(3) - 9.00 = 0$ $F_{CD} = 6.364$ kN

$+\uparrow \Sigma F_y = 0;$ $A_y + 6.364\sin 45° - 6 = 0$ $A_y = 1.50$ kN

Shear and Moment Functions : For $0 \le x < 3$ m [FBD (b)],

$+\uparrow \Sigma F_y = 0;$ $1.50 - V = 0$ $V = 1.50$ kN **Ans**

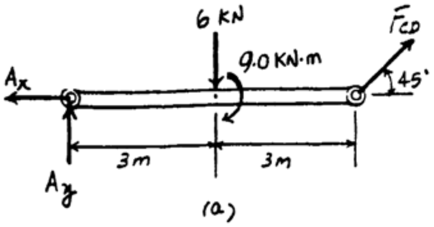
$\curvearrowleft + \Sigma M = 0;$ $M - 1.50x = 0$ $M = \{1.50x\}$ kN·m **Ans**

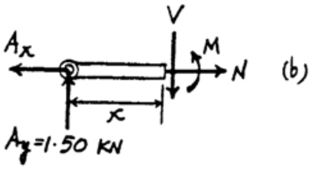
For $3\text{ m} < x \le 6$ m [FBD (c)],

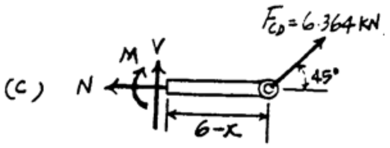
$+\uparrow \Sigma F_y = 0;$ $V + 6.364\sin °45 = 0$ $V = -4.50$ kN **Ans**

$\curvearrowleft + \Sigma M = 0;$ $6.364\sin 45°(6 - x) - M = 0$

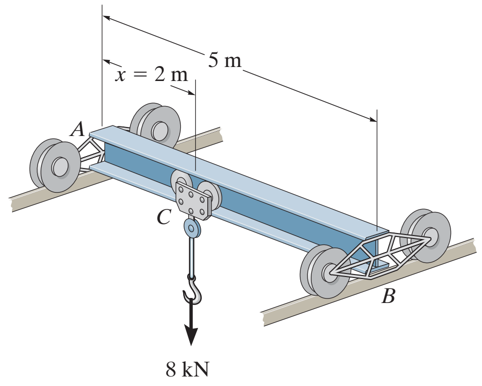
$M = \{27.0 - 4.50x\}$ kN·m **Ans**

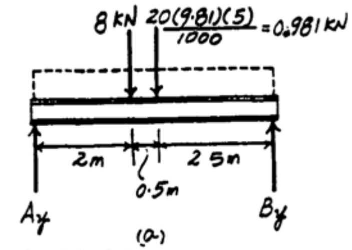


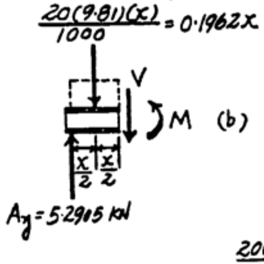


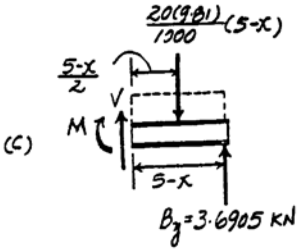


7–122. The traveling crane consists of a 5-m-long beam having a uniform mass per unit length of 20 kg/m. The chain hoist and its supported load exert a force of 8 kN on the beam when $x = 2$ m. Draw the shear and moment diagrams for the beam. The guide wheels at the ends A and B exert only vertical reactions on the beam. Neglect the size of the trolley at C.









Support Reactions : From FBD (a),

$\curvearrowleft + \Sigma M_A = 0; \quad B_y(5) - 8(2) - 0.981(2.5) = 0 \qquad B_y = 3.6905\text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 3.6905 - 8 - 0.981 = 0 \qquad A_y = 5.2905\text{ kN}$

Shear and Moment Functions : For $0 \le x < 2$ m [FBD (b)],

$+\uparrow \Sigma F_y = 0; \quad 5.2905 - 0.1962x - V = 0$

$V = \{5.29 - 0.196x\}$ kN **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + 0.1962x\left(\frac{x}{2}\right) - 5.2905x = 0$

$M = \{5.29x - 0.0981x^2\}$ kN · m **Ans**

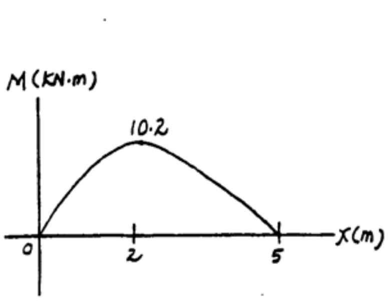
For $2\text{ m} < x \le 5$ m [FBD (c)],

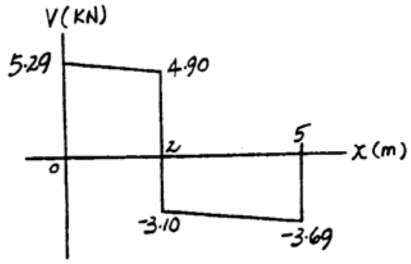
$+\uparrow \Sigma F_y = 0; \quad V + 3.6905 - \frac{20(9.81)}{1000}(5 - x) = 0$

$V = \{-0.196x - 2.71\}$ kN **Ans**

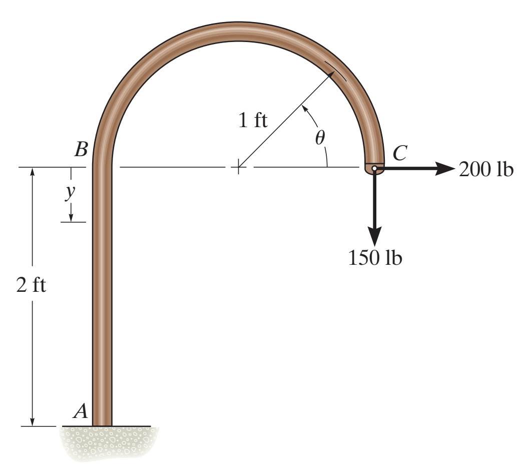
$\curvearrowleft + \Sigma M = 0; \quad 3.6905(5 - x) - \frac{20(9.81)}{1000}(5 - x)\left(\frac{5 - x}{2}\right) - M = 0$

$M = \{16.0 - 2.71x - 0.0981x^2\}$ kN · m **Ans**





***7–123.** Determine the internal normal force, shear force, and the moment as a function of $0° \leq \theta \leq 180°$ and $0 \leq y \leq 2$ ft for the member loaded as shown.



For $0° \leq \theta \leq 180°$:

$+\nearrow \Sigma F_x = 0;\quad V + 200\cos\theta - 150\sin\theta = 0$

$V = 150\sin\theta - 200\cos\theta$ **Ans**

$+\nwarrow \Sigma F_y = 0;\quad N - 200\sin\theta - 150\cos\theta = 0$

$N = 150\cos\theta + 200\sin\theta$ **Ans**

$\circlearrowleft + \Sigma M = 0;\quad -M - 150(1)(1 - \cos\theta) + 200(1)\sin\theta = 0$

$M = 150\cos\theta + 200\sin\theta - 150$ **Ans**

At section B, $\theta = 180°$, thus

$V_B = 200$ lb

$N_B = -150$ lb

$M_B = -300$ lb · ft

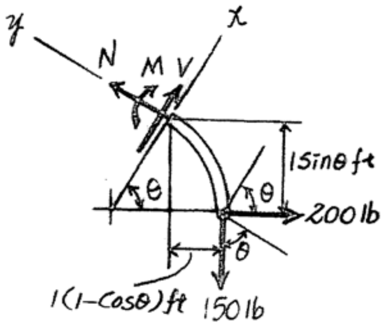
For $0 \leq y \leq 2$ ft:

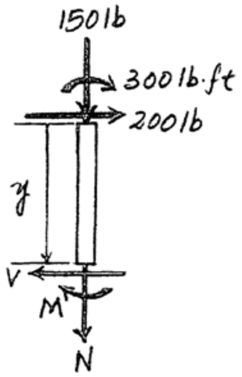
$\xrightarrow{+} \Sigma F_x = 0;\quad V = 200$ lb **Ans**

$+\uparrow \Sigma F_y = 0;\quad N = -150$ lb **Ans**

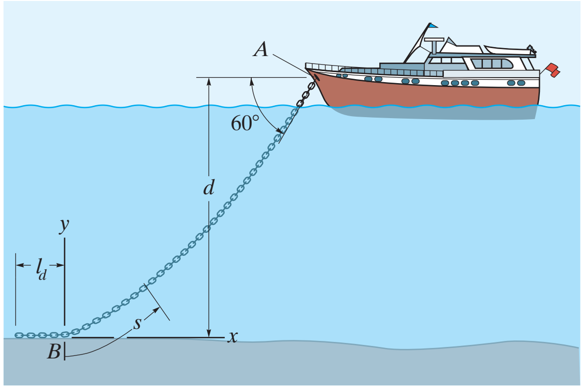
$\circlearrowleft + \Sigma M = 0;\quad -M - 300 - 200y = 0$

$M = -300 - 200y$ **Ans**





***7–124.** The yacht is anchored with a chain that has a total length of 40 m and a mass per unit length of 18 kg/m, and the tension in the chain at A is 7 kN. Determine the length of chain l_d which is lying at the bottom of the sea. What is the distance d? Assume that buoyancy effects of the water on the chain are negligible. *Hint:* Establish the origin of the coordinate system at B as shown in order to find the chain length BA.



Component of force at A is

$$F_H = T\cos\theta = 7000\cos 60^\circ = 3500\text{ N}$$

From Eq. (1) of Example 7-13

$$x = \frac{3500}{18\,(9.81)}\left(\sinh^{-1}\left[\frac{1}{3500}(18)(9.81)s + C_1\right] + C_2\right)$$

Since $\frac{dy}{dx} = 0,\ s = 0,$ then

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0 s + C_1);\quad C_1 = 0$$

Also $x = 0,\ s = 0,$ so that $C_2 = 0$ and the above equation becomes

$$x = 19.82\left(\sinh^{-1}\left(\frac{s}{19.82}\right)\right)\quad (1)$$

or,

$$s = 19.82\left(\sinh\left(\frac{x}{19.82}\right)\right)\quad (2)$$

From Example 7-13

$$\frac{dy}{dx} = \frac{w_0 s}{F_H} = \frac{18\,(9.81)}{3500}s = \frac{s}{19.82}\quad (3)$$

Substituting Eq. (2) into Eq. (3),

$$\frac{dy}{dx} = \sinh\left(\frac{x}{19.82}\right)$$

Integrating,

$$y = 19.82\cosh\left(\frac{x}{19.82}\right) + C_3$$

Since $x = 0,\ y = 0,$ then $C_3 = -19.82$

Thus,

$$y = 19.82\left(\cosh\left(\frac{x}{19.82}\right) - 1\right)\quad (4)$$

Slope of the cable at point A is

$$\frac{dy}{dx} = \tan 60^\circ = 1.732$$

Using Eq. (3),

$$s_{AB} = 19.82\,(1.732) = 34.33\text{ m}$$

Length of chain on the ground is thus

$$l_d = 40 - 34.33 = 5.67\text{ m}\qquad \text{Ans}$$

From Eq. (1), with $s = 34.33$ m

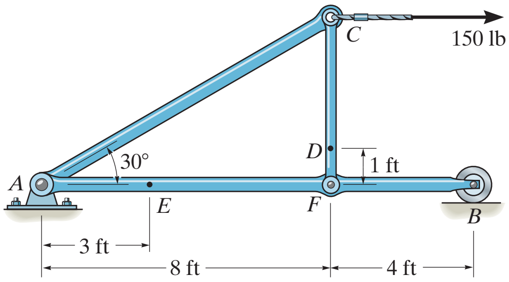
$$x = 19.82\left(\sinh^{-1}\left(\frac{34.33}{19.82}\right)\right) = 26.10\text{ m}$$

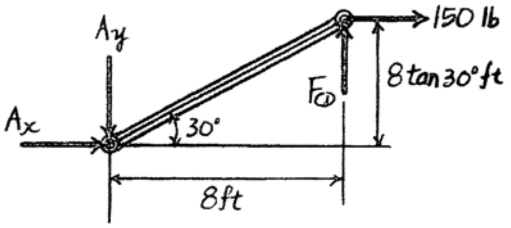
Using Eq. (4),

$$y = 19.82\left(\cosh\left(\frac{26.10}{19.82}\right) - 1\right)$$

$$d = y = 19.8\text{ m}\qquad \text{Ans}$$

•7–125. Determine the internal normal force, shear force, and moment at points *D* and *E* of the frame.





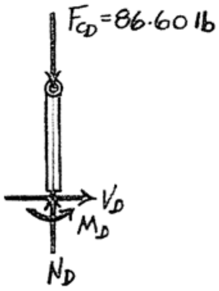
$\circlearrowleft + \Sigma M_A = 0;\quad F_{CD}(8) - 150(8\tan 30^\circ) = 0$

$F_{CD} = 86.60\text{ lb}$

Since member *CF* is a two-force member

$V_D = M_D = 0$ Ans

$N_D = F_{CD} = 86.6\text{ lb}$ Ans



$\circlearrowleft + \Sigma M_A = 0;\quad B_y(12) - 150(8\tan 30^\circ) = 0$

$B_y = 57.735\text{ lb}$

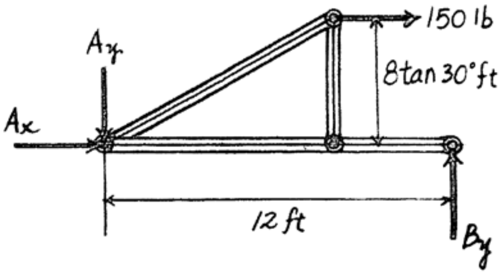
$\xrightarrow{+} \Sigma F_x = 0;\quad N_E = 0$ Ans

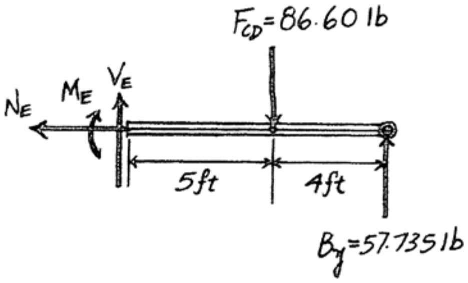
$+\uparrow \Sigma F_y = 0;\quad V_E + 57.735 - 86.60 = 0$

$V_E = 28.9\text{ lb}$ Ans

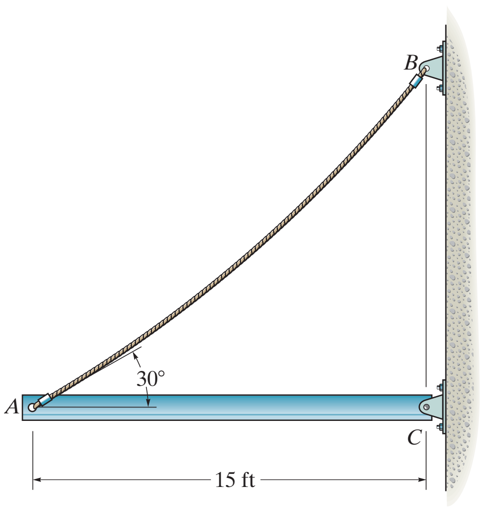
$\circlearrowleft + \Sigma M_E = 0;\quad 57.735(9) - 86.60(5) - M_E = 0$

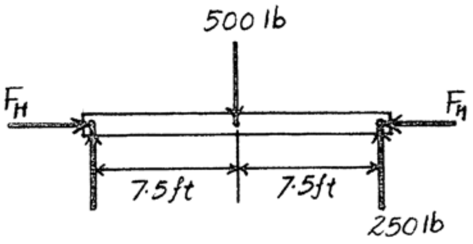
$M_E = 86.6\text{ lb}\cdot\text{ft}$ Ans





7–126. The uniform beam weighs 500 lb and is held in the horizontal position by means of cable *AB*, which has a weight of 5 lb/ft. If the slope of the cable at *A* is 30°, determine the length of the cable.





$$T = \frac{250}{\sin 30^\circ} = 500 \text{ lb}$$

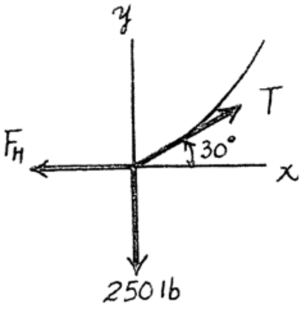
$$F_H = 500 \cos 30^\circ = 433.0 \text{ lb}$$

From Example 7-13

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0 s + C_1)$$

At $s = 0$, $\frac{dy}{dx} = \tan 30^\circ = 0.577$

$$\therefore C_1 = 433.0(0.577) = 250$$



$$x = \frac{F_H}{w_0}\left\{\sinh^{-1}\left[\frac{1}{F_H}(w_0 s + C_1)\right] + C_2\right\}$$

$$= \frac{433.0}{5}\left\{\sinh^{-1}\left[\frac{1}{433.0}(5s + 250)\right] + C_2\right\}$$

$s = 0$ at $x = 0$, $C_2 = -0.5493$

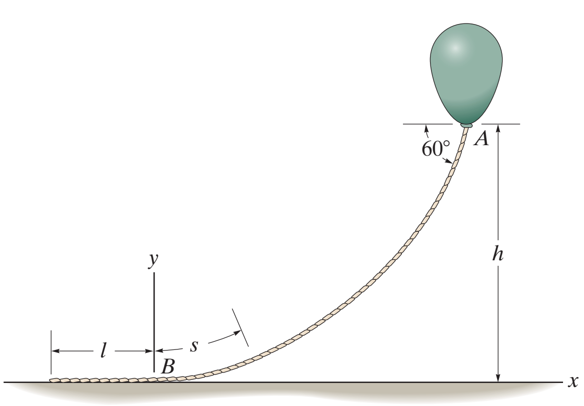
Thus,

$$x = 86.6\left\{\sinh^{-1}\left[\frac{1}{433.0}(5s + 250)\right] - 0.5493\right\}$$

When $x = 15$ ft,

$s = 18.2$ ft **Ans**

7–127. The balloon is held in place using a 400-ft cord that weighs 0.8 lb/ft and makes a 60° angle with the horizontal. If the tension in the cord at point A is 150 lb, determine the length of the cord, l, that is lying on the ground and the height h. *Hint*: Establish the coordinate system at B as shown.



Deflection Curve of The Cable :

$$x = \int \frac{ds}{\left[1 + \left(1/F_H^2\right)\left(\int w_0\, ds\right)^2\right]^{\frac{1}{2}}} \qquad \text{where } w_0 = 0.8 \text{ lb/ft}$$

Performing the integration yields

$$x = \frac{F_H}{0.8}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0.8s + C_1)\right] + C_2\right\} \qquad [1]$$

From Eq. 7 – 14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds = \frac{1}{F_H}(0.8s + C_1) \qquad [2]$$

Boundary Conditions :

$\frac{dy}{dx} = 0$ at $s = 0$. From Eq.[2] $\quad 0 = \frac{1}{F_H}(0 + C_1) \qquad C_1 = 0$

Then, Eq.[2] becomes

$$\frac{dy}{dx} = \tan\theta = \frac{0.8s}{F_H} \qquad [3]$$

$s = 0$ at $x = 0$ and use the result $C_1 = 0$. From Eq.[1]

$$x = \frac{F_H}{3}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0 + 0)\right] + C_2\right\} \qquad C_2 = 0$$

Rearranging Eq.[1], we have

$$s = \frac{F_H}{0.8}\sinh\left(\frac{0.8}{F_H}x\right) \qquad [4]$$

Substituting Eq.[4] into [3] yields

$$\frac{dy}{dx} = \sinh\left(\frac{0.8}{F_H}x\right)$$

Performing the integration

$$y = \frac{F_H}{0.8}\cosh\left(\frac{0.8}{F_H}x\right) + C_3 \qquad [5]$$

$y = 0$ at $x = 0$. From Eq.[5] $\quad 0 = \frac{F_H}{0.8}\cosh 0 + C_3$, thus, $C_3 = -\frac{F_H}{0.8}$

Then, Eq.[5] becomes

$$y = \frac{F_H}{0.8}\left[\cosh\left(\frac{0.8}{F_H}x\right) - 1\right] \qquad [6]$$

The tension developed at the end of the cord is $T = 150$ lb and $\theta = 60°$. Thus

$$T = \frac{F_H}{\cos\theta} \qquad 150 = \frac{F_H}{\cos 60°} \qquad F_H = 75.0 \text{ lb}$$

From Eq.[3]

$$\frac{dy}{dx} = \tan 60° = \frac{0.8s}{75} \qquad s = 162.38 \text{ ft}$$

Thus, $\quad l = 400 - 162.38 = 238$ ft

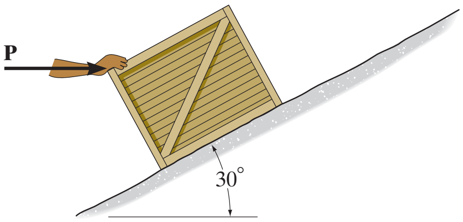
Substituting $s = 162.38$ ft into Eq.[4], **Ans**

$$162.38 = \frac{75}{0.8}\sinh\left(\frac{0.8}{75}x\right)$$
$$x = 123.46 \text{ ft}$$

$y = h$ at $x = 123.46$ ft. From Eq.[6]

$$h = \frac{75.0}{0.8}\left[\cosh\left[\frac{0.8}{75.0}(123.46)\right] - 1\right] = 93.75 \text{ ft} \qquad \textbf{Ans}$$

•8–1. Determine the minimum horizontal force P required to hold the crate from sliding down the plane. The crate has a mass of 50 kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$.



Free - Body Diagram. When the crate is on the verge of sliding down the plane, the frictional force **F** will act up the plane as indicated on the free - body diagram of the crate shown in Fig. *a*.

Equations of Equilibrium.

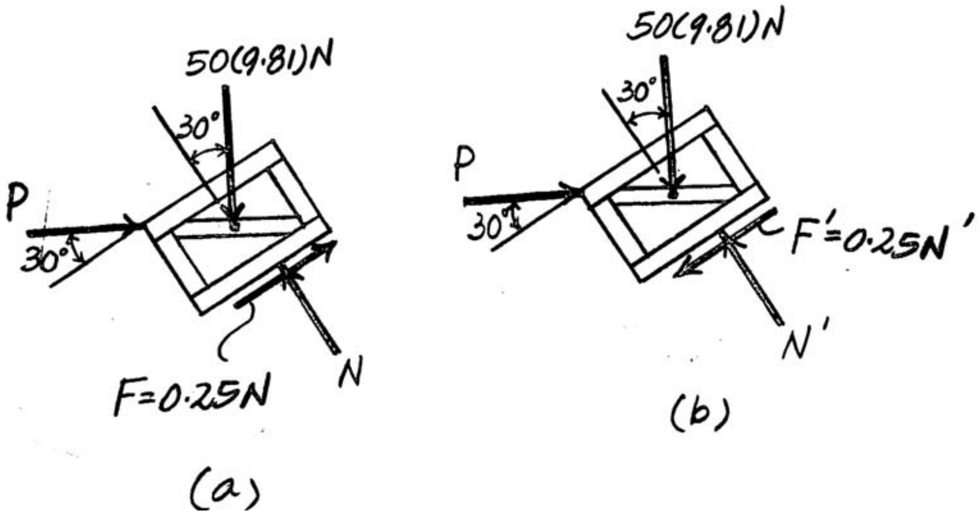
$\Sigma F_{y'} = 0;\ N - P\sin 30° - 50(9.81)\cos 30° = 0$

$\Sigma F_{x'} = 0;\ P\cos 30° + 0.25N - 50(9.81)\sin 30° = 0$

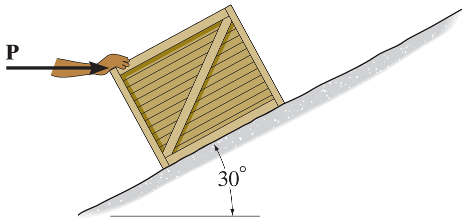
Solving

$P = 140\text{ N}$ **Ans.**

$N = 494.94\text{ N}$



8–2. Determine the minimum force P required to push the crate up the plane. The crate has a mass of 50 kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$.



When the crate is on the verge of sliding up the plane, the frictional force F' will act down the plane as indicated on the free-body diagram of the crate shown in Fig. *b*. Thus, $F = \mu_s N = 0.25N$ and $F' = \mu_s N' = 0.25N'$. By referring to Fig. *b*,

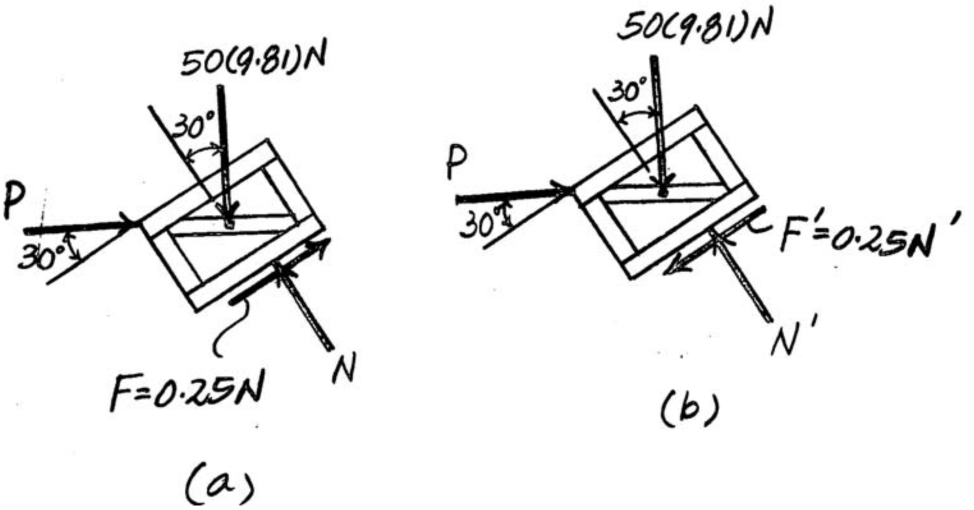
$\Sigma F_{y'} = 0;\quad N' - P\sin 30° - 50(9.81)\cos 30° = 0$

$\Sigma F_{x'} = 0;\quad P\cos 30° - 0.25N' - 50(9.81)\sin 30° = 0$

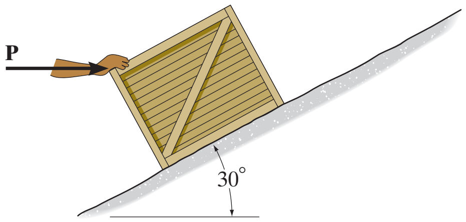
Solving,

$$P = 474\text{ N}$$ **Ans.**

$$N' = 661.92\text{ N}$$



8–3. A horizontal force of $P = 100$ N is just sufficient to hold the crate from sliding down the plane, and a horizontal force of $P = 350$ N is required to just push the crate up the plane. Determine the coefficient of static friction between the plane and the crate, and find the mass of the crate.



Free - Body Diagram. When the crate is subjected to a force of $P = 100\,\text{N}$, it is on the verge of slipping down the plane. Thus, the frictional force F will act up the plane as indicated on the free - body diagram of the crate shown in Fig. *a*. When $P = 350\,\text{N}$, it will cause the crate to be on the verge of slipping up the plane, and so the frictional force F' acts down the plane as indicated on the free - body diagram of the crate shown in Fig. *a*. Thus, $F = \mu_s N$ and $F' = \mu_s N'$.

Equations of Equilibrium.

$+\nwarrow \Sigma F_{y'} = 0;\ N - 100\sin 30° - m(9.81)\cos 30° = 0$

$\overset{+}{\nearrow} \Sigma F_{x'} = 0;\ \mu_s N + 100\cos 30° - m(9.81)\sin 30° = 0$

Eliminating N,

$$\mu_s = \frac{4.905m - 86.603}{8.496m + 50} \qquad (1)$$

Also, by referring to Fig. *b*, we can write

$+\nwarrow \Sigma F_{y'} = 0;\ N' - m(9.81)\cos 30° - 350\sin 30° = 0$

$\overset{+}{\nearrow} \Sigma F_{x'} = 0;\ 350\cos 30° - m(9.81)\sin 30° - \mu_s N' = 0$

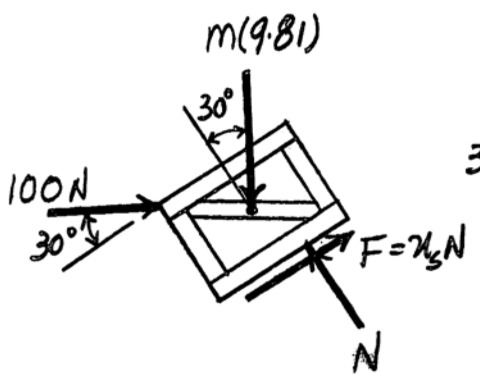
Eliminating N',

$$\mu_s = \frac{303.11 - 4.905m}{175 + 8.496m} \qquad (2)$$

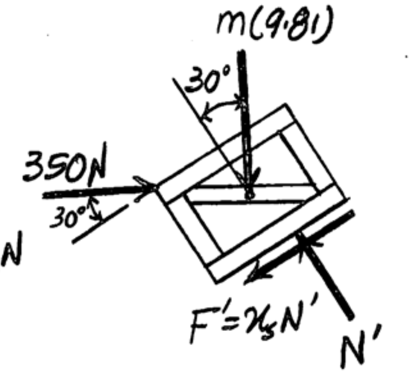
Solving Eqs. (1) and (2) yields

$m = 36.46$ kg

$\mu_s = 0.256$ **Ans.**

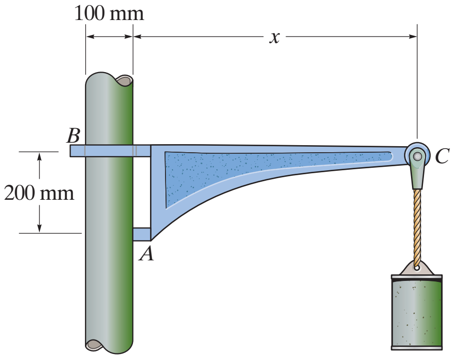


(a)



(b)

***8–4.** If the coefficient of static friction at A is $\mu_s = 0.4$ and the collar at B is smooth so it only exerts a horizontal force on the pipe, determine the minimum distance x so that the bracket can support the cylinder of any mass without slipping. Neglect the mass of the bracket.



Free - Body Diagram. The weight of cylinder tends to cause the bracket to slide downward. Thus, the frictional force $\mathbf{F}_A$ must act upwards as indicated in the free - body diagram shown in Fig. *a*. Here the bracket is required to be on the verge of slipping so that $F_A = \mu_s N_A = 0.4 N_A$.

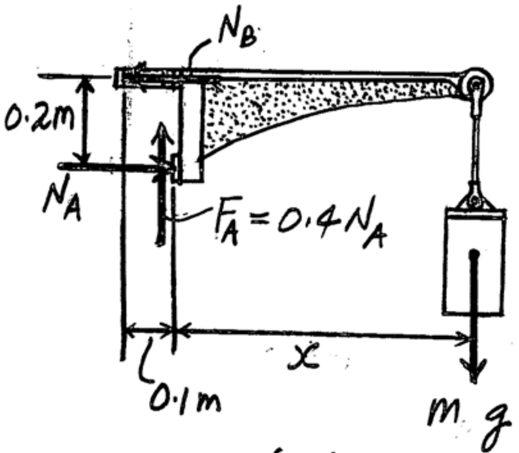
Equations of Equilibrium.

$+\uparrow \Sigma F_y = 0;$ $\quad 0.4N_A - m\,g = 0 \qquad N_A = 2.5m\,g$

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 2.5mg(0.2) + 0.4(2.5m\,g)(0.1) - m\,(g)(x + 0.1) = 0$

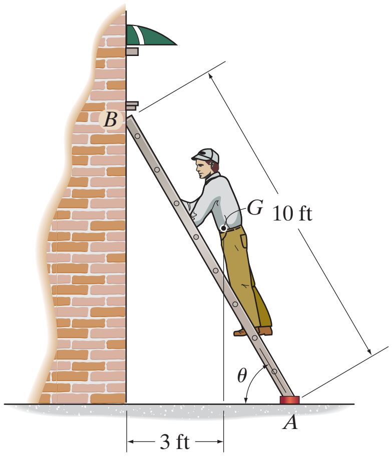
$x = 0.5\text{ m}$ **Ans.**

Note. Since x is independent of the mass of the cylinder, the bracket will not slip regardless of the mass of the cylinder provided $x > 0.5$ m.



(a)

•8–5. The 180-lb man climbs up the ladder and stops at the position shown after he senses that the ladder is on the verge of slipping. Determine the inclination θ of the ladder if the coefficient of static friction between the friction pad A and the ground is $\mu_s = 0.4$. Assume the wall at B is smooth. The center of gravity for the man is at G. Neglect the weight of the ladder.



Free - Body Diagram. Since the weight of the man tends to cause the friction pad A to slide to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free - body diagram of the ladder, Fig. *a*. Here, the ladder is on the verge of slipping. Thus, $F_A = \mu_s N_A$.

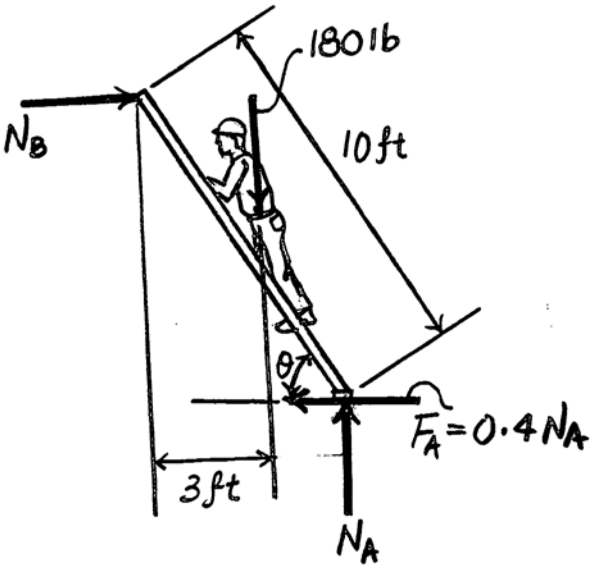
Equations of Equilibrium.

$+\uparrow \Sigma F_y = 0;$ $N_A - 180 = 0$ $N_A = 180 \text{ lb}$

$+\Sigma M_B = 0;$ $180(10\cos 60°) - \mu_s(180)(10\sin 60°) - 180(3) = 0$

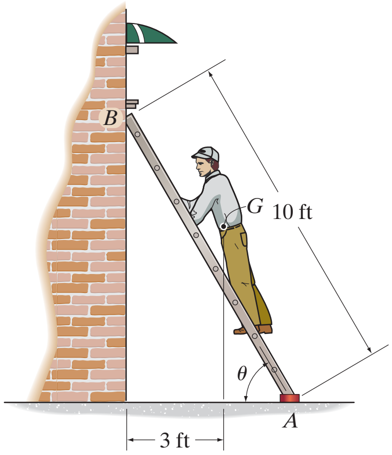
$10\cos 60° - \mu_s 10\sin 60° = 3$

$\mu_s = 0.231$ **Ans.**



(a)

8–6. The 180-lb man climbs up the ladder and stops at the position shown after he senses that the ladder is on the verge of slipping. Determine the coefficient of static friction between the friction pad at *A* and ground if the inclination of the ladder is $\theta = 60°$ and the wall at *B* is smooth. The center of gravity for the man is at *G*. Neglect the weight of the ladder.



Free - Body Diagram. Since the weight of the man tends to cause the friction pad *A* to slide to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free - body diagram of the ladder, Fig. *a*. Here, the ladder is on the verge of slipping. Thus, $F_A = \mu_s N_A$.

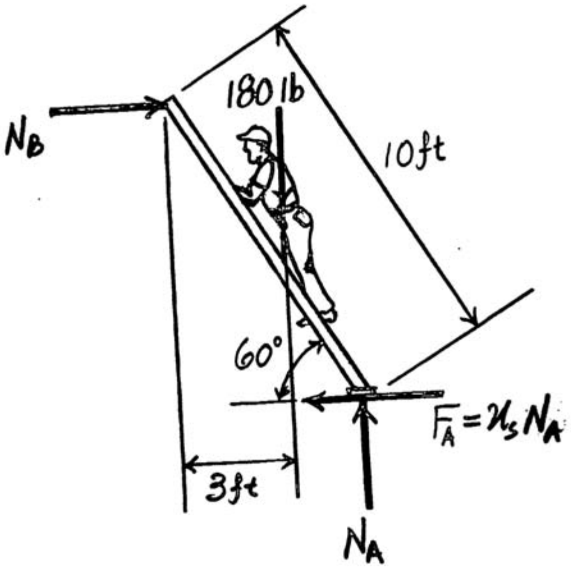
Equations of Equilibrium.

$+\uparrow \Sigma F_y = 0;$ $N_A - 180 = 0$ $N_A = 180\text{ lb}$

$\curvearrowleft + \Sigma M_B = 0;$ $180(10\cos 60°) - \mu_s(180)(10\sin 60°) - 180(3) = 0$

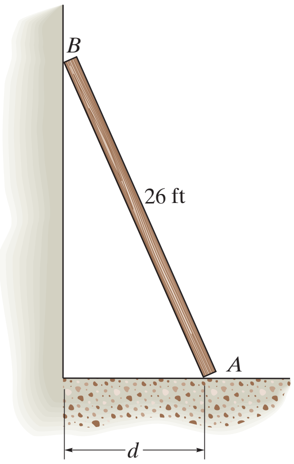
$180\cos\theta - 72\sin\theta = 54$

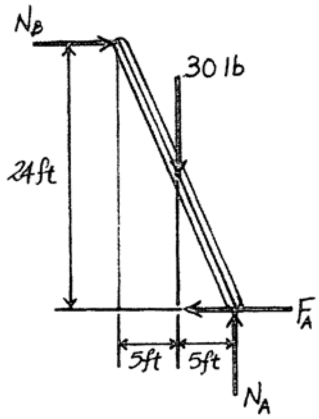
$\mu_s = 0.231$ **Ans.**



(a)

8–7. The uniform thin pole has a weight of 30 lb and a length of 26 ft. If it is placed against the smooth wall and on the rough floor in the position $d = 10$ ft, will it remain in this position when it is released? The coefficient of static friction is $\mu_s = 0.3$.





$\circlearrowleft + \Sigma M_A = 0;$ $30(5) - N_B(24) = 0$

$N_B = 6.25$ lb

$\xrightarrow{+} \Sigma F_x = 0;$ $6.25 - F_A = 0$

$F_A = 6.25$ lb

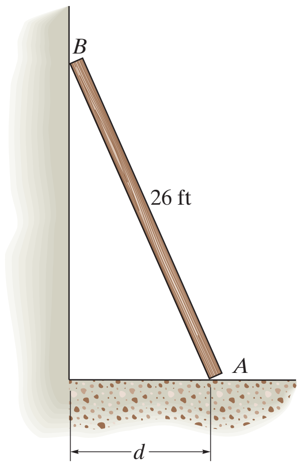
$+\uparrow \Sigma F_y = 0;$ $N_A - 30 = 0$

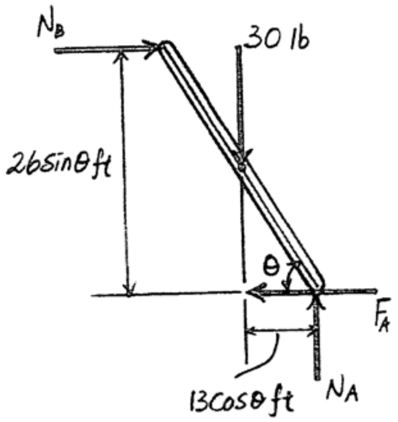
$N_A = 30$ lb

$(F_A)_{max} = 0.3(30) = 9 \text{ lb} > 6.25 \text{ lb}$

Yes, the pole will remain stationary. **Ans**

***8–8.** The uniform pole has a weight of 30 lb and a length of 26 ft. Determine the maximum distance d it can be placed from the smooth wall and not slip. The coefficient of static friction between the floor and the pole is $\mu_s = 0.3$.





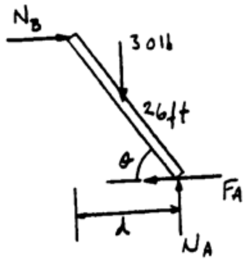
$+\uparrow \Sigma F_y = 0; \quad N_A - 30 = 0$

$N_A = 30 \text{ lb}$

$F_A = (F_A)_{max} = 0.3\,(30) = 9 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_B - 9 = 0$

$N_B = 9 \text{ lb}$

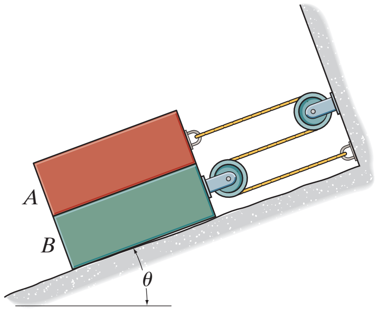


$\curvearrowleft + \Sigma M_A = 0; \quad 30\,(13\cos\theta) - 9\,(26\sin\theta) = 0$

$\theta = 59.04^\circ$

$d = 26\cos 59.04^\circ = 13.4 \text{ ft}$ **Ans**

•8–9. If the coefficient of static friction at all contacting surfaces is μ_s, determine the inclination θ at which the identical blocks, each of weight W, begin to slide.



Free - Body Diagram. Here, we will assume that the impending motion of the upper block is down the plane while the impending motion of the lower block is up the plane. Thus, the frictional force F acting on the upper block acts up the plane while the friction forces **F** and F' acting on the lower block act down the plane as indicated on the free - body diagram of the upper and lower blocks shown in Figs. *a* and *b*, respectively. Since both block are required to be on the verge of slipping, then $F = \mu_s N$ and $F' = \mu_s N'$.

Equations of Equilibrium. Referring to Fig. *a*,

$+\nwarrow \Sigma F_{y'} = 0;\quad N - W\cos\theta = 0 \qquad\qquad N = W\cos\theta$

$+\nearrow \Sigma F_{x'} = 0;\quad T + \mu_s(W\cos\theta) - W\sin\theta = 0 \qquad\qquad T = W\sin\theta - \mu_s W\cos\theta$

Using these results and referring to Fig. *b*,

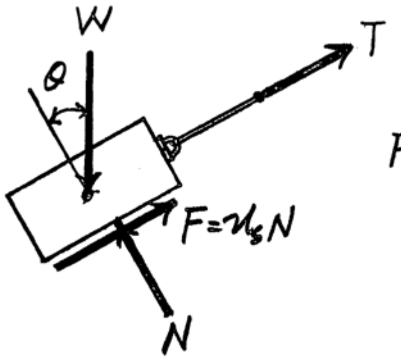
$+\nwarrow \Sigma F_{y'} = 0;\quad N' - W\cos\theta - W\cos\theta = 0 \qquad\qquad N' = 2W\cos\theta$

$+\nearrow \Sigma F_{x'} = 0;\quad 2(W\sin\theta - \mu_s W\cos\theta) - \mu_s W\cos\theta - \mu_s(2W\cos\theta) - W\sin\theta = 0$

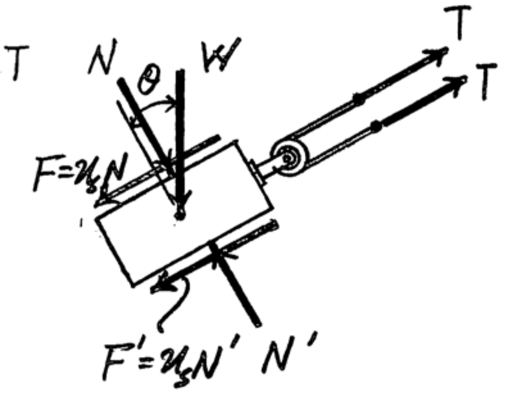
$$\sin\theta - 5\mu_s\cos\theta = 0$$

$$\theta = \tan^{-1} 5\mu_s \qquad \textbf{Ans.}$$

Since the analysis yields a positive θ, the above assumption is correct.

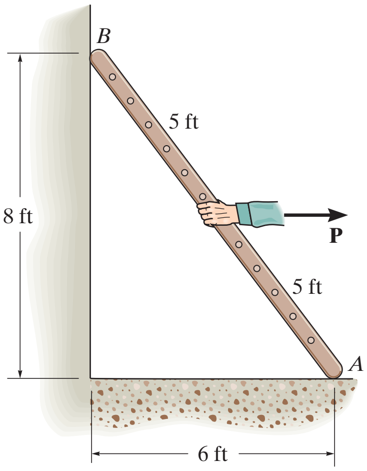


(a)



(b)

8–10. The uniform 20-lb ladder rests on the rough floor for which the coefficient of static friction is $\mu_s = 0.8$ and against the smooth wall at B. Determine the horizontal force P the man must exert on the ladder in order to cause it to move.



Assume that the ladder tips about A:

$N_B = 0$;

$\xrightarrow{+} \Sigma F_x = 0; \quad P - F_A = 0$

$+\uparrow \Sigma F_y = 0; \quad -20 + N_A = 0$

$N_A = 20 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0; \quad 20(3) - P(4) = 0$

$P = 15 \text{ lb}$

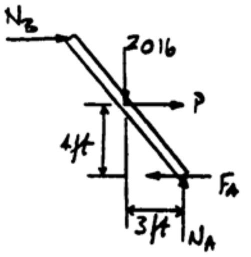
Thus

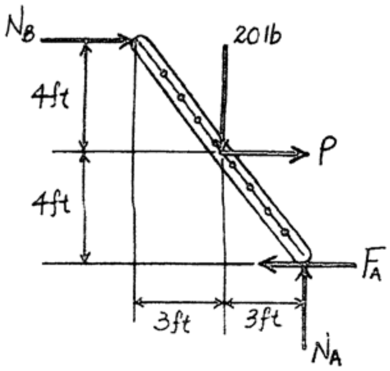
$F_A = 15 \text{ lb}$

$(F_A)_{max} = 0.8(20) = 16 \text{ lb} > 15 \text{ lb}$ OK

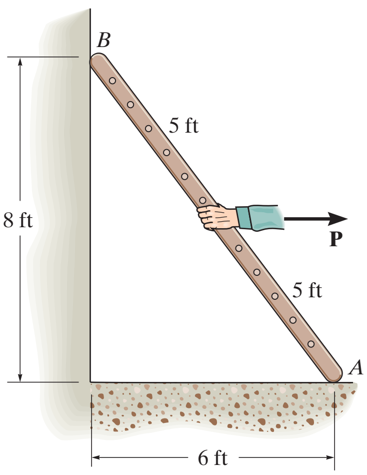
Ladder tips as assumed.

$P = 15 \text{ lb}$ **Ans**





8–11. The uniform 20-lb ladder rests on the rough floor for which the coefficient of static friction is $\mu_s = 0.4$ and against the smooth wall at B. Determine the horizontal force P the man must exert on the ladder in order to cause it to move.



Assume that the ladder slips at A:

$F_A = 0.4 N_A$

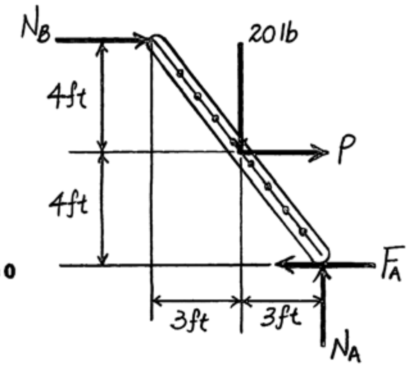
$+\uparrow \Sigma F_y = 0;\quad N_A - 20 = 0$

$N_A = 20 \text{ lb}$

$F_A = 0.4(20) = 8 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\quad P(4) - 20(3) + 20(6) - 8(8) = 0$

$P = 1 \text{ lb}$ Ans

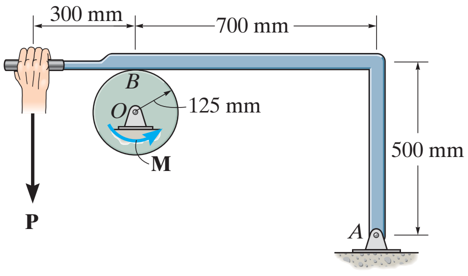


$\xrightarrow{+} \Sigma F_x = 0;\quad N_B + 1 - 8 = 0$

$N_B = 7 \text{ lb} > 0$ OK

The ladder will remain in contact with the wall.

***8–12.** The coefficients of static and kinetic friction between the drum and brake bar are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. If $M = 50\text{ N}\cdot\text{m}$ and $P = 85\text{ N}$ determine the horizontal and vertical components of reaction at the pin O. Neglect the weight and thickness of the brake. The drum has a mass of 25 kg.



Equations of Equilibrium : From FBD (b),

$$\curvearrowleft + \Sigma M_O = 0 \qquad 50 - F_B(0.125) = 0 \qquad F_B = 400\text{ N}$$

From FBD (a),

$$\curvearrowleft + \Sigma M_A = 0; \qquad 85(1.00) + 400(0.5) - N_B(0.7) = 0$$
$$N_B = 407.14\text{ N}$$

Friction : Since $F_B > (F_B)_{\max} = \mu_s N_B = 0.4(407.14) = 162.86\text{ N}$, the drum slips at point B and rotates. Therefore, the coefficient of kinetic friction should be used. Thus, $F_B = \mu_k N_B = 0.3N_B$.

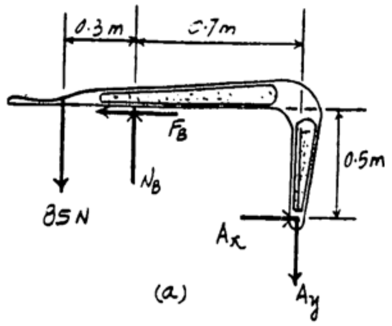
Equations of Equilibrium : From FBD (b),

$$\curvearrowleft + \Sigma M_A = 0; \qquad 85(1.00) + 0.3N_B(0.5) - N_B(0.7) = 0$$
$$N_B = 154.54\text{ N}$$

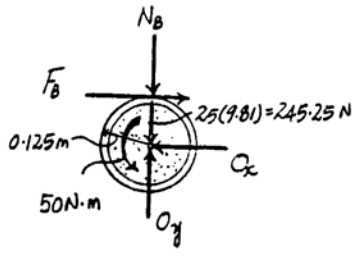
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad O_y - 245.25 - 154.54 = 0 \qquad O_y = 400\text{ N} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.3(154.54) - O_x = 0 \qquad O_x = 46.4\text{ N} \qquad \textbf{Ans}$$

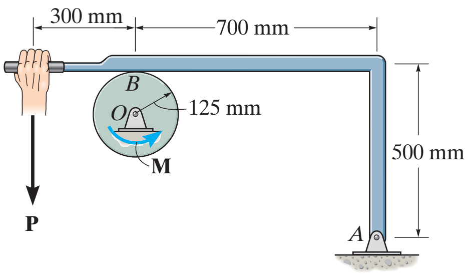


(a)



(b)

•8–13. The coefficient of static friction between the drum and brake bar is $\mu_s = 0.4$. If the moment $M = 35\ \text{N}\cdot\text{m}$, determine the smallest force P that needs to be applied to the brake bar in order to prevent the drum from rotating. Also determine the corresponding horizontal and vertical components of reaction at pin O. Neglect the weight and thickness of the brake bar. The drum has a mass of 25 kg.



Equations of Equilibrium : From FBD (b),

$$\circlearrowleft + \Sigma M_O = 0 \qquad 35 - F_B(0.125) = 0 \qquad F_B = 280\ \text{N}$$

From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(1.00) + 280(0.5) - N_B(0.7) = 0$$

Friction : When the drum is on the verge of rotating,

$$F_B = \mu_s N_B$$
$$280 = 0.4 N_B$$
$$N_B = 700\ \text{N}$$

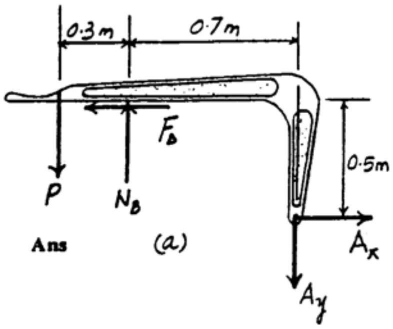
Substituting $N_B = 700$ N into Eq.[1] yields

$$P = 350\ \text{N}$$ **Ans**

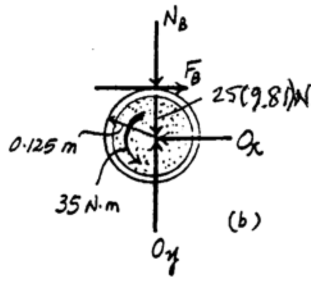
Equations of Equilibrium : From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad O_y - 245.25 - 700 = 0 \qquad O_y = 945\ \text{N}$$ **Ans**

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 280 - O_x = 0 \qquad O_x = 280\ \text{N}$$ **Ans**

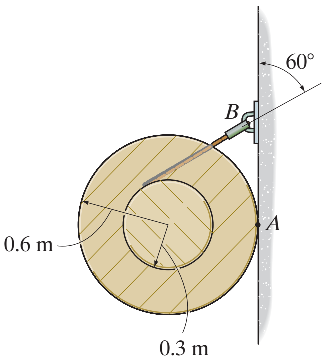


(a)



(b)

8–14. Determine the minimum coefficient of static friction between the uniform 50-kg spool and the wall so that the spool does not slip.



Free - Body Diagram. Here, the frictional force $\mathbf{F}_A$ must act upwards to produce the counterclockwise moment about the center of mass of the spool, opposing the impending clockwise rotational motion caused by force **T** as indicated on the free - body diagram of the spool, Fig. *a*. Since the spool is required to be on the verge of slipping, then $F_A = \mu_s N_A$.

Equations of Equilibrium. Referring to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad mg(0.6) - T\cos 60°(0.3\cos 60° + 0.6) - T\sin 60°(0.3\sin 60°) = 0$

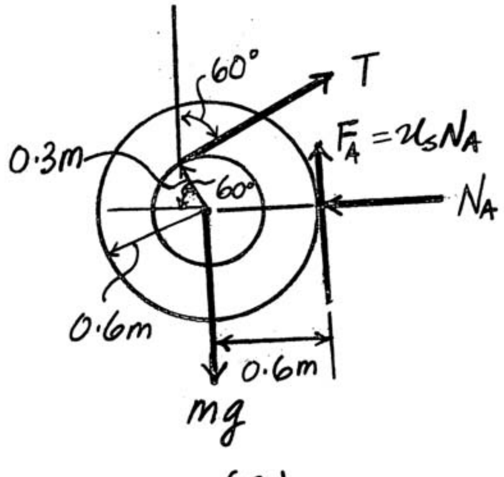
$T = mg$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad mg\sin 60° - N_A = 0 \qquad N_A = 0.8660mg$

$+\uparrow \Sigma F_y = 0;$ $\quad \mu_s(0.8660mg) + mg\cos 60° - mg = 0$

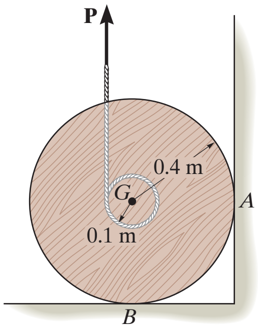
$\mu_s = 0.577$ **Ans.**

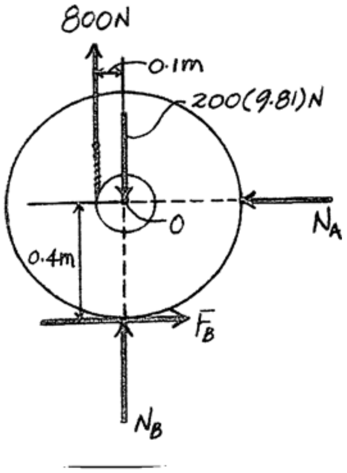
Note: Since μ_s is independent of the mass of the spool, it will not slip regardless of its mass provided $\mu_s > 0.577$.



(a)

8–15. The spool has a mass of 200 kg and rests against the wall and on the floor. If the coefficient of static friction at B is $(\mu_s)_B = 0.3$, the coefficient of kinetic friction is $(\mu_k)_B = 0.2$, and the wall is smooth, determine the friction force developed at B when the vertical force applied to the cable is $P = 800$ N.





$\xrightarrow{+}\Sigma F_x = 0;$ $F_B - N_A = 0$

$+\uparrow \Sigma F_y = 0;$ $800 - 200(9.81) + N_B = 0$

$\circlearrowleft + \Sigma M_O = 0;$ $-800(0.1) + F_B(0.4) = 0$

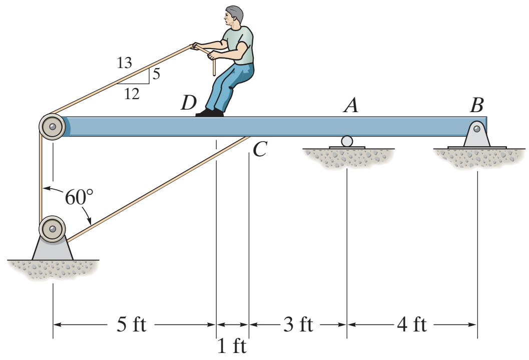
$F_B = 200\text{ N}$

$N_B = 1162\text{ N}$

$(F_B)_{max} = 0.3(1162) = 348.6\text{ N} > 200\text{ N}$

Thus, $F_B = 200\text{ N}$ **Ans**

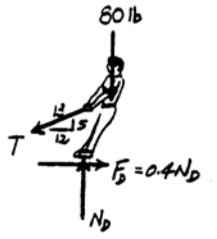
***8–16.** The 80-lb boy stands on the beam and pulls on the cord with a force large enough to just cause him to slip. If the coefficient of static friction between his shoes and the beam is $(\mu_s)_D = 0.4$, determine the reactions at A and B. The beam is uniform and has a weight of 100 lb. Neglect the size of the pulleys and the thickness of the beam.



Equations of Equilibrium and Friction : When the boy is on the verge to slipping, then $F_D = (\mu_s)_D N_D = 0.4N_D$. From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N_D - T\left(\frac{5}{13}\right) - 80 = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.4N_D - T\left(\frac{12}{13}\right) = 0 \qquad [2]$$

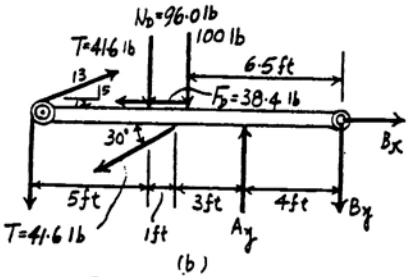


(a)

Solving Eqs. [1] and [2] yields

$$T = 41.6 \text{ lb} \qquad N_D = 96.0 \text{ lb}$$

Hence, $F_D = 0.4(96.0) = 38.4$ lb. From FBD (b),



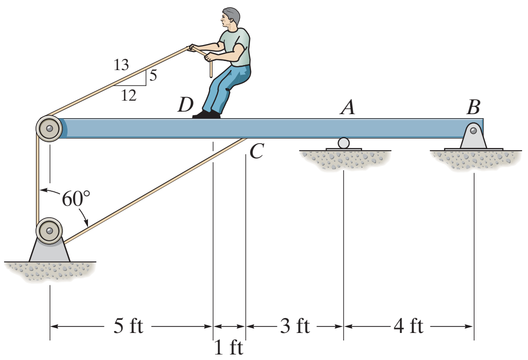
(b)

$$\circlearrowleft + \Sigma M_B = 0; \quad 100(6.5) + 96.0(8) - 41.6\left(\frac{5}{13}\right)(13)$$
$$+ 41.6(13) + 41.6 \sin 30°(7) - A_y(4) = 0$$
$$A_y = 474.1 \text{ lb} = 474 \text{ lb} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 41.6\left(\frac{12}{13}\right) - 38.4 - 41.6\cos 30° = 0$$
$$B_x = 36.0 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 474.1 + 41.6\left(\frac{5}{13}\right) - 41.6$$
$$- 41.6 \sin 30° - 96.0 - 100 - B_y = 0$$
$$B_y = 231.7 \text{ lb} = 232 \text{ lb} \qquad \textbf{Ans}$$

•8–17. The 80-lb boy stands on the beam and pulls with a force of 40 lb. If $(\mu_s)_D = 0.4$, determine the frictional force between his shoes and the beam and the reactions at A and B. The beam is uniform and has a weight of 100 lb. Neglect the size of the pulleys and the thickness of the beam.



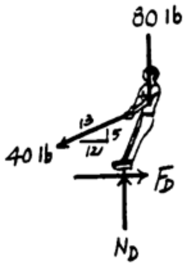
Equations of Equilibrium and Friction : From FBD (a).

$$+\uparrow \Sigma F_y = 0; \quad N_D - 40\left(\frac{5}{13}\right) - 80 = 0 \qquad N_D = 95.38 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_D - 40\left(\frac{12}{13}\right) = 0 \qquad F_D = 36.92 \text{ lb}$$

Since $(F_D)_{max} = (\mu_s)N_D = 0.4(95.38) = 38.15 \text{ lb} > F_D$, then the boy does not slip. Therefore, the friction force developed is

$$F_D = 36.92 \text{ lb} = 36.9 \text{ lb} \qquad \textbf{Ans}$$



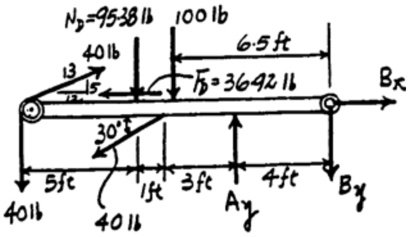
(a)

From FBD (b).

$$\circlearrowleft + \Sigma M_B = 0; \quad 100(6.5) + 95.38(8) - 40\left(\frac{5}{13}\right)(13)$$
$$+ 40(13) + 40\sin 30°(7) - A_y(4) = 0$$
$$A_y = 468.27 \text{ lb} = 468 \text{ lb} \qquad \textbf{Ans}$$

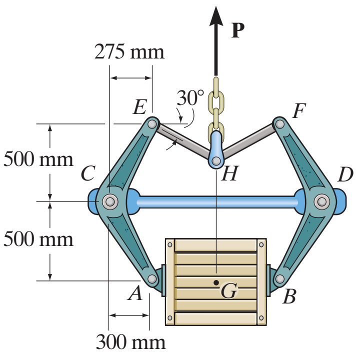
$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 40\left(\frac{12}{13}\right) - 36.92 - 40\cos 30° = 0$$
$$B_x = 34.64 \text{ lb} = 34.6 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 468.27 + 40\left(\frac{5}{13}\right) - 40$$
$$- 40\sin 30° - 95.38 - 100 - B_y = 0$$
$$B_y = 228.27 \text{ lb} = 228 \text{ lb} \qquad \textbf{Ans}$$



(b)

8–18. The tongs are used to lift the 150-kg crate, whose center of mass is at *G*. Determine the least coefficient of static friction at the pivot blocks so that the crate can be lifted.



Free - Body Diagram. Since the crate is suspended from the tongs, **P** must be equal to the weight of the crate; i.e., $P = 150(9.81)$N as indicated on the free - body diagram of joint *H* shown in Fig. *a*. Since the crate is required to be on the verge of slipping downward, $\mathbf{F}_A$ and $\mathbf{F}_B$ must act upward so that $F_A = \mu_s N_A$ and $F_B = \mu_s N_B$ as indicated on the free - body diagram of the crate shown in Fig. *c*.

Equations of Equilibrium. Referring to Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $F_{HE}\cos 30° - F_{HF}\cos 30° = 0$ $F_{HE} = F_{HF} = F$

$+\uparrow \Sigma F_y = 0;$ $150(9.81) - 2F\sin 30° = 0$ $F = 1471.5\text{ N}$

Referring to Fig. *b*,

$\circlearrowleft + \Sigma M_C = 0;$ $1471.5\cos 30°(0.5) + 1471.5\sin 30°(0.275) - N_A(0.5) - \mu_s N_A(0.3) = 0$

$0.5N_A + 0.3\mu_s N_A = 839.51$ (1)

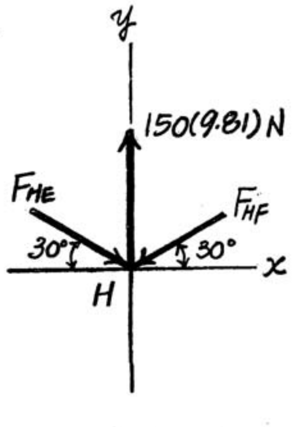
Due to the symmetry of the system and loading, $N_B = N_A$. Referring to Fig. *c*,

$+\uparrow \Sigma F_y = 0;$ $2\mu_s N_A - 150(9.81) = 0$ (2)

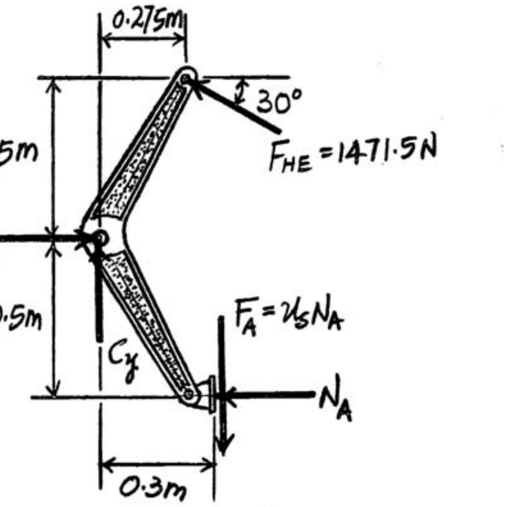
Solving Eqs. (1) and (2), yields

$N_A = 1237.57\text{ N}$

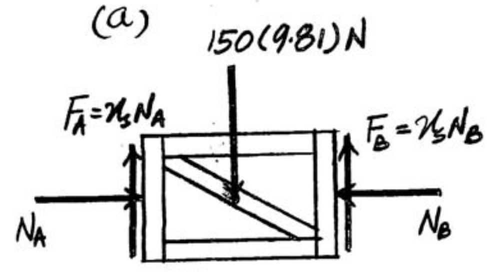
$\mu_s = 0.595$ **Ans.**



(a)

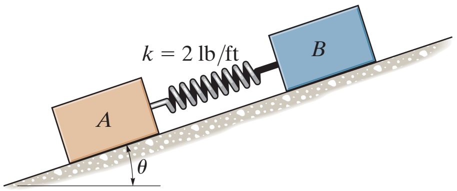


(b)



(C)

8–19. Two blocks *A* and *B* have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the incline angle θ for which both blocks begin to slide. Also find the required stretch or compression in the connecting spring for this to occur. The spring has a stiffness of $k = 2$ lb/ft.



Equations of Equilibrium : Using the spring force formula, $F_{sp} = kx = 2x$. From FBD (a),

$$\nearrow + \Sigma F_{x'} = 0; \quad 2x + F_A - 10\sin\theta = 0 \qquad [1]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_A - 10\cos\theta = 0 \qquad [2]$$

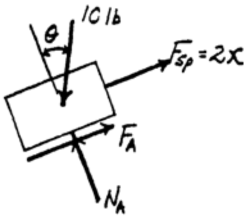
From FBD (b),

$$\nearrow + \Sigma F_{x'} = 0; \quad F_B - 2x - 6\sin\theta = 0 \qquad [3]$$

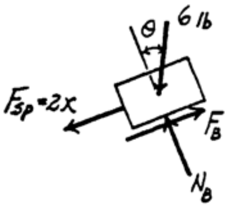
$$\nwarrow + \Sigma F_{y'} = 0; \quad N_B - 6\cos\theta = 0 \qquad [4]$$

Friction : If block *A* and *B* are on the verge to move, slipping would have to occur at point *A* and *B*. Hence, $F_A = \mu_{s_A} N_A = 0.15N_A$ and $F_B = \mu_{s_B} N_B = 0.25N_B$. Substituting these values into Eqs. [1], [2], [3] and [4] and solving, we have

$$\theta = 10.6^\circ \qquad x = 0.184 \text{ ft} \qquad \textbf{Ans}$$
$$N_A = 9.829 \text{ lb} \qquad N_B = 5.897 \text{ lb}$$

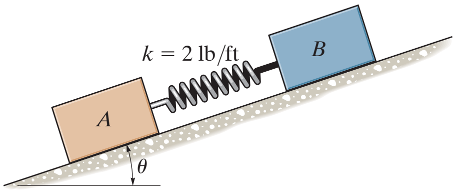


(a)



(b)

***8–20.** Two blocks *A* and *B* have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the angle θ which will cause motion of one of the blocks. What is the friction force under each of the blocks when this occurs? The spring has a stiffness of $k = 2$ lb/ft and is originally unstretched.



Equations of Equilibrium : Since Block *A* and *B* is either not moving or on the verge of moving, the spring force $F_{sp} = 0$. From FBD (a),

$$\nearrow + \Sigma F_{x'} = 0; \quad F_A - 10\sin\theta = 0 \qquad [1]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_A - 10\cos\theta = 0 \qquad [2]$$

From FBD (b),

$$\nearrow + \Sigma F_{x'} = 0; \quad F_B - 6\sin\theta = 0 \qquad [3]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_B - 6\cos\theta = 0 \qquad [4]$$

Friction : Assuming block *A* is on the verge of slipping, then

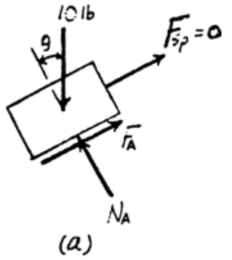
$$F_A = \mu_{s\,A} N_A = 0.15 N_A \qquad [5]$$

Solving Eqs. [1], [2], [3], [4] and [5] yields

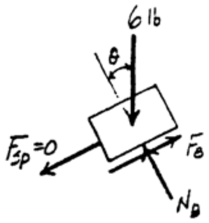
$$\theta = 8.531^\circ \quad N_A = 9.889 \text{ lb} \quad F_A = 1.483 \text{ lb}$$
$$F_B = 0.8900 \text{ lb} \quad N_B = 5.934 \text{ lb}$$

Since $(F_B)_{max} = \mu_{s\,B} N_B = 0.25(5.934) = 1.483 \text{ lb} > F_B$, block *B* does not slip. Therefore, the above assumption is correct. Thus

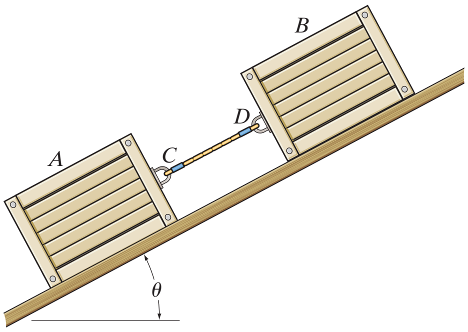
$$\theta = 8.53^\circ \quad F_A = 1.48 \text{ lb} \quad F_B = 0.890 \text{ lb} \qquad \textbf{Ans}$$



(a)



•8–21. Crates *A* and *B* weigh 200 lb and 150 lb, respectively. They are connected together with a cable and placed on the inclined plane. If the angle θ is gradually increased, determine θ when the crates begin to slide. The coefficients of static friction between the crates and the plane are $\mu_A = 0.25$ and $\mu_B = 0.35$.



Free - Body Diagram. Since both crates are required to be on the verge of sliding down the plane, the frictional forces $\mathbf{F}_A$ and $\mathbf{F}_B$ must act up the plane so that $F_A = \mu_A N_A = 0.25N_A$ and $F_B = \mu_B N_B = 0.35N_B$ as indicated on the free - body diagram of the crates shown in Figs. *a* and *b*.

Equations of Equilibrium. Referring to Fig. *a*,

$\nwarrow + \Sigma F_{y'} = 0;\quad N_A - 200\cos\theta = 0 \qquad N_A = 200\cos\theta$

$\overset{+}{\nearrow} \Sigma F_{x'} = 0;\quad F_{CD} + 0.25(200\cos\theta) - 200\sin\theta = 0 \qquad (1)$

Also, by referring to Fig. *b*,

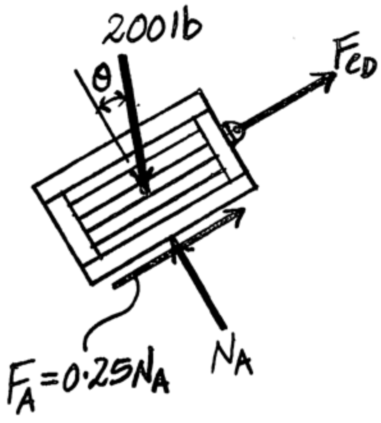
$\nwarrow + \Sigma F_{y'} = 0;\quad N_B - 150\cos\theta = 0 \qquad N_B = 150\cos\theta$

$\overset{+}{\nearrow} \Sigma F_{x'} = 0;\quad 0.35(150\cos\theta) - F_{CD} - 150\sin\theta = 0 \qquad (2)$

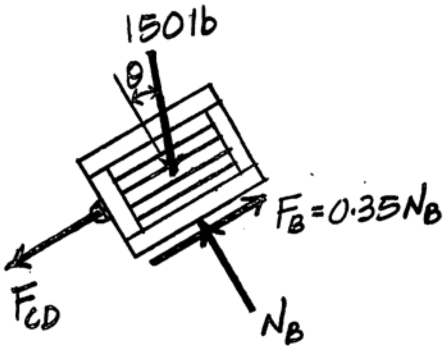
Solving Eqs. (1) and (2), yields

$\theta = 16.3°$ **Ans.**

$F_{CD} = 8.23\text{ lb}$

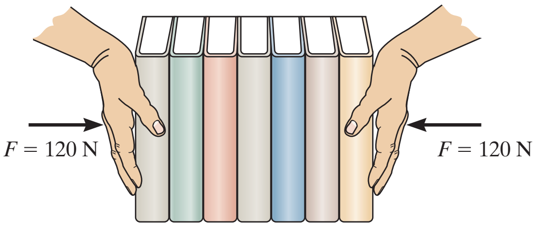


(a)



(b)

8–22. A man attempts to support a stack of books horizontally by applying a compressive force of $F = 120$ N to the ends of the stack with his hands. If each book has a mass of 0.95 kg, determine the greatest number of books that can be supported in the stack. The coefficient of static friction between the man's hands and a book is $(\mu_s)_h = 0.6$ and between any two books $(\mu_s)_b = 0.4$.



Equations of Equilibrium and Friction : Let n' be the number of books that are on the verge of sliding together between the two books at the edge. Thus, $F_b = (\mu_s)_b N = 0.4(120) = 48.0$ N. From FBD (a),

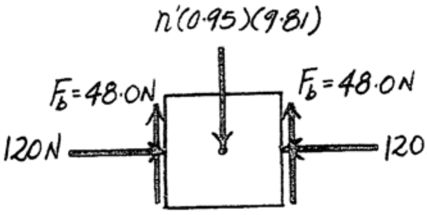
$$+\uparrow \Sigma F_y = 0; \qquad 2(48.0) - n'(0.95)(9.81) = 0 \qquad n' = 10.30$$

Let n be the number of books are on the verge of sliding together in the stack between the hands. Thus, $F_h = (\mu_s)_h N = 0.6(120) = 72.0$ N. From FBD (b),

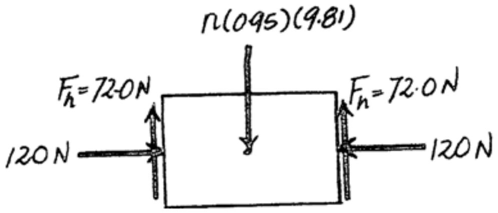
$$+\uparrow \Sigma F_y = 0; \qquad 2(72.0) - n(0.95)(9.81) = 0 \qquad n = 15.45$$

Thus, the maximum number of books can be supported in stack is

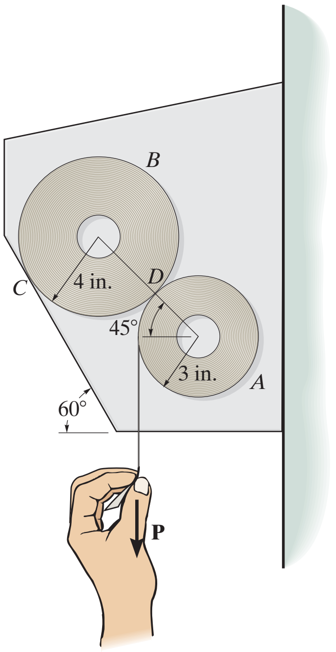
$$n = 10 + 2 = 12 \qquad \textbf{Ans}$$



(a)



8–23. The paper towel dispenser carries two rolls of paper. The one in use is called the stub roll *A* and the other is the fresh roll *B*. They weigh 2 lb and 5 lb, respectively. If the coefficients of static friction at the points of contact *C* and *D* are $(\mu_s)_C = 0.2$ and $(\mu_s)_D = 0.5$, determine the initial vertical force *P* that must be applied to the paper on the stub roll in order to pull down a sheet. The stub roll is pinned in the center, whereas the fresh roll is not. Neglect friction at the pin.



Equations of Equilibrium : From FBD (a),

$$\circlearrowleft + \Sigma M_E = 0; \qquad P(3) - F_D(3) = 0 \qquad [1]$$

From FBD (b),

$$\circlearrowleft + \Sigma M_F = 0; \qquad F_C(4) - F_D(4) = 0 \qquad [2]$$

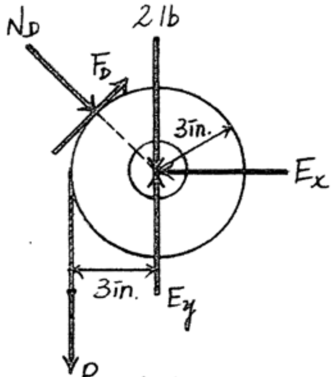
$$+\uparrow \Sigma F_y = 0; \qquad N_C \sin 30° + N_D \sin 45° - F_C \sin 60° - F_D \sin 45° - 5 = 0 \qquad [3]$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C \cos 30° + F_C \cos 60° - N_D \cos 45° - F_D \cos 45° = 0 \qquad [4]$$

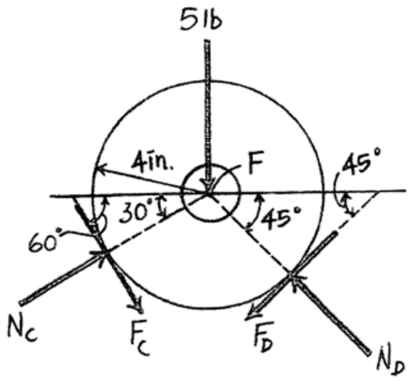
Friction : Assume slipping occurs at point *C*. Hence, $F_C = \mu_{s\,C} N_C = 0.2 N_C$. Substituting this value into Eqs. [1], [2], [3] and [4] and solving we have

$$N_D = 5.773 \text{ lb} \qquad N_C = 4.951 \text{ lb} \qquad F_D = 0.9901 \text{ lb}$$
$$P = 0.990 \text{ lb} \qquad \textbf{Ans}$$

Since $F_D < (F_D)_{\max} = (\mu_s)_D N_D = 0.5(5.773) = 2.887$ lb, then slipping does not occur at point *D*. Therefore, the above assumption is correct.

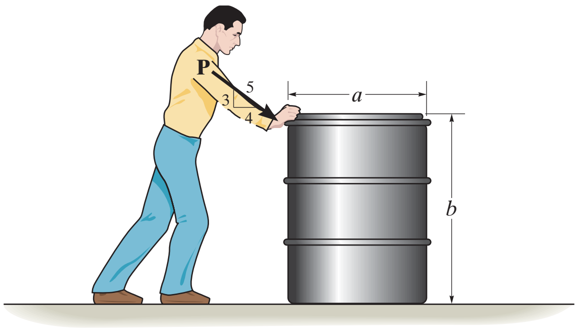


(a)



(b)

***8–24.** The drum has a weight of 100 lb and rests on the floor for which the coefficient of static friction is $\mu_s = 0.6$. If $a = 2$ ft and $b = 3$ ft, determine the smallest magnitude of the force P that will cause impending motion of the drum.



Assume that the drum tips :

$x = 1$ ft

$\circlearrowleft + \Sigma M_O = 0; \quad 100\,(1) + P\left(\frac{3}{5}\right)(2) - P\left(\frac{4}{5}\right)(3) = 0$

$P = 83.3$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad -F + 83.3\left(\frac{4}{5}\right) = 0$

$F = 66.7$ lb

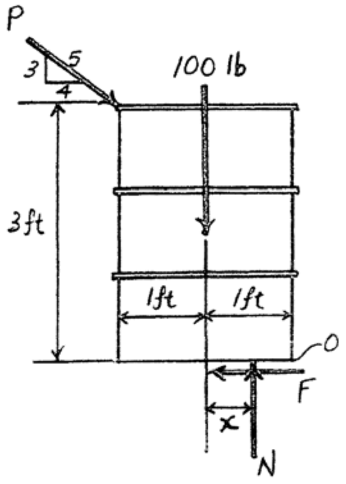
$+\uparrow \Sigma F_y = 0; \quad N - 100 - 83.3\left(\frac{3}{5}\right) = 0$

$N = 150$ lb

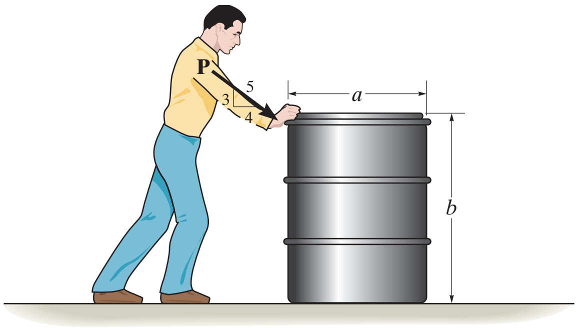
$F_{max} = 0.6\,(150) = 90 \text{ lb} > 66.7$ OK

Drum tips as assumed.

$P = 83.3$ lb **Ans**



•8–25. The drum has a weight of 100 lb and rests on the floor for which the coefficient of static friction is $\mu_s = 0.5$. If $a = 3$ ft and $b = 4$ ft, determine the smallest magnitude of the force P that will cause impending motion of the drum.



Assume that the drum slips :

$F = 0.5N$

$\xrightarrow{+} \Sigma F_x = 0; \quad -0.5\,N + P\left(\frac{4}{5}\right) = 0$

$+\uparrow \Sigma F_y = 0; \quad -P\left(\frac{3}{5}\right) - 100 + N = 0$

$P = 100$ lb

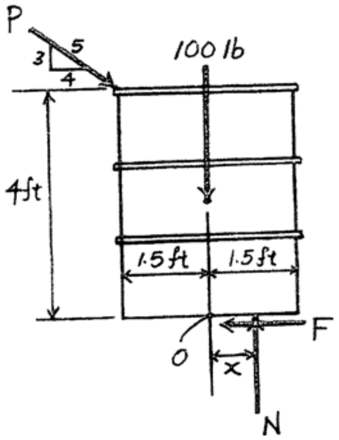
$N = 160$ lb

$\circlearrowleft + \Sigma M_O = 0; \quad 160\,(x) + 100\left(\frac{3}{5}\right)(1.5) - 100\left(\frac{4}{5}\right)(4) = 0$

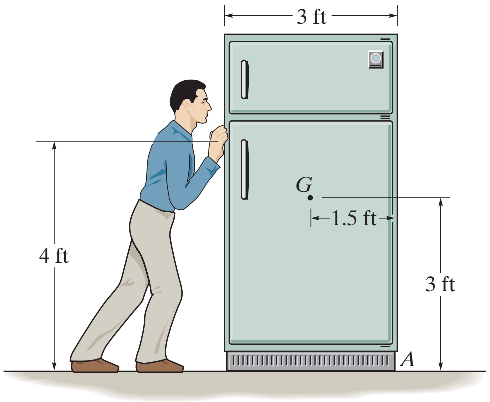
$x = 1.44 \text{ ft} < 1.5 \text{ ft}$ OK

Drum slips as assumed.

$P = 100$ lb **Ans**



8–26. The refrigerator has a weight of 180 lb and rests on a tile floor for which $\mu_s = 0.25$. If the man pushes horizontally on the refrigerator in the direction shown, determine the smallest magnitude of horizontal force needed to move it. Also, if the man has a weight of 150 lb, determine the smallest coefficient of friction between his shoes and the floor so that he does not slip.



Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N - 180 = 0 \quad N = 180 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - F = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad 180(x) - P(4) = 0 \qquad [2]$$

Friction : Assuming the refrigerator is on the verge of slipping, then $F = \mu N = 0.25(180) = 45$ lb. Substituting this value into Eqs. [1], and [2] and solving yields

$$P = 45.0 \text{ lb} \qquad x = 1.00 \text{ ft}$$

Since $x < 1.5$ ft, the refrigerator does not tip. Therefore, the above assumption is correct. Thus

$$P = 45.0 \text{ lb} \qquad \textbf{Ans}$$

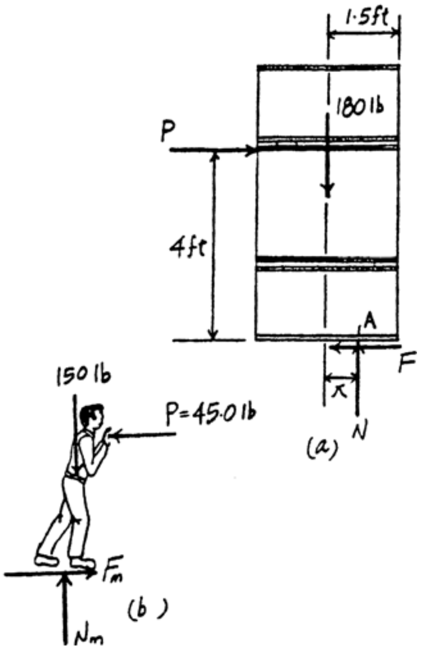
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_m - 150 = 0 \quad N_m = 150 \text{ lb}$$

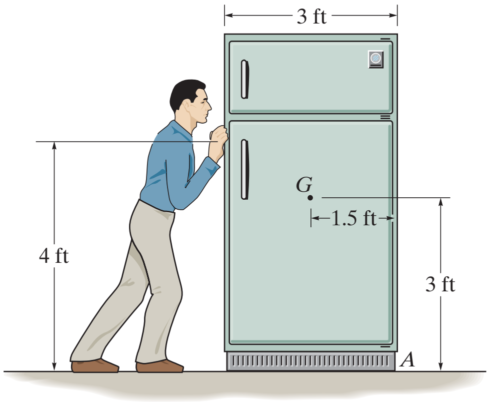
$$\xrightarrow{+} \Sigma F_x = 0; \quad F_m - 45.0 = 0 \quad F_m = 45.0 \text{ lb}$$

When the man is on the verge of slipping, then

$$F_m = \mu_s' N_m$$
$$45.0 = \mu_s'(150)$$
$$\mu_s' = 0.300 \qquad \textbf{Ans}$$



8–27. The refrigerator has a weight of 180 lb and rests on a tile floor for which $\mu_s = 0.25$. Also, the man has a weight of 150 lb and the coefficient of static friction between the floor and his shoes is $\mu_s = 0.6$. If he pushes horizontally on the refrigerator, determine if he can move it. If so, does the refrigerator slip or tip?



Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N - 180 = 0 \quad N = 180 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - F = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad 180(x) - P(4) = 0 \qquad [2]$$

Friction : Assuming the refrigerator is on the verge of slipping, then $F = \mu N = 0.25(180) = 45$ lb. Substituting this value into Eqs.[1], and [2] and solving yields

$$P = 45.0 \text{ lb} \qquad x = 1.00 \text{ ft}$$

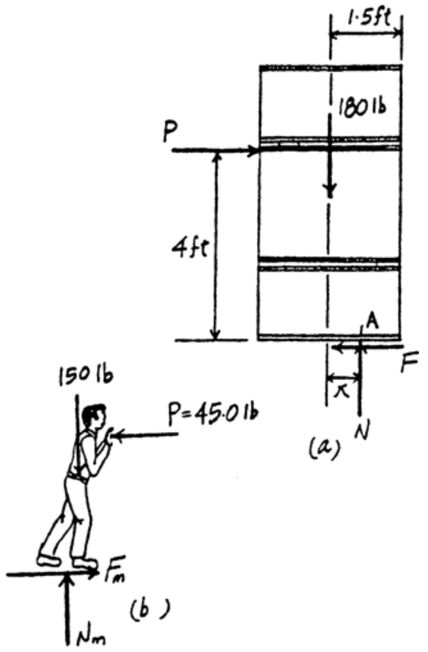
Since $x < 1.5$ ft, the refrigerator does not tip. Therefore, the above assumption is correct. Thus, **the refrigerator slips.** **Ans**

From FBD (b),

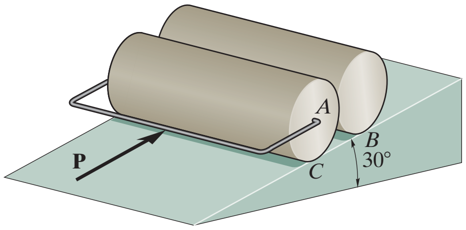
$$+\uparrow \Sigma F_y = 0; \quad N_m - 150 = 0 \quad N_m = 150 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_m - 45.0 = 0 \quad F_m = 45.0 \text{ lb}$$

Since $(F_m)_{max} = \mu_s' N_m = 0.6(150) = 90.0 \text{ lb} > F_m$, then the man does not slip. Thus, **The man is capable of moving the refrigerator.** **Ans**



***8–28.** Determine the minimum force P needed to push the two 75-kg cylinders up the incline. The force acts parallel to the plane and the coefficients of static friction of the contacting surfaces are $\mu_A = 0.3$, $\mu_B = 0.25$, and $\mu_C = 0.4$. Each cylinder has a radius of 150 mm.



Equations of Equilibrium : From FBD (a),

$$\nearrow + \Sigma F_{x'} = 0; \qquad P - N_A - F_C - 735.75\sin 30° = 0 \qquad [1]$$

$$\nwarrow + \Sigma F_{y'} = 0; \qquad N_C + F_A - 735.75\cos 30° = 0 \qquad [2]$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad F_A(r) - F_C(r) = 0 \qquad [3]$$

From FBD (b),

$$\nearrow + \Sigma F_{x'} = 0; \qquad N_A - F_B - 735.75\sin 30° = 0 \qquad [4]$$

$$\nwarrow + \Sigma F_{y'} = 0; \qquad N_B - F_A - 735.75\cos 30° = 0 \qquad [5]$$

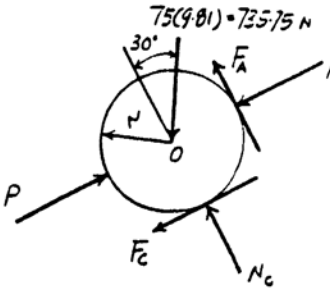
$$\circlearrowleft + \Sigma M_O = 0; \qquad F_A(r) - F_B(r) = 0 \qquad [6]$$

Friction : Assuming slipping occur at point A, then $F_A = \mu_{s\,A} N_A = 0.3N_A$. Substituting this value into Eqs. [1], [2], [3], [4], [5] and [6] and solving, we have

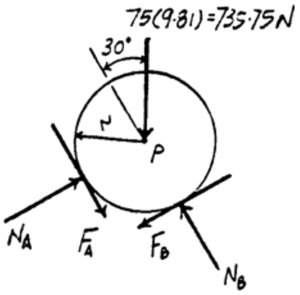
$$N_A = 525.54 \text{ N} \qquad N_B = 794.84 \text{ N}$$
$$N_C = 479.52 \text{ N} \qquad F_C = F_B = 157.66 \text{ N}$$

$$P = 1051.07 \text{ N} = 1.05 \text{ kN} \qquad \textbf{Ans}$$

Since $(F_C)_{\max} = \mu_{s\,C} N_C = 0.4(479.52) = 191.81 \text{ N} > F_C$ and $(F_B)_{\max} = \mu_{s\,B} N_B = 0.25(794.84) = 198.71 \text{ N} > F_B$, slipping do not occur at points C and B. Therefore the above assumption is correct.

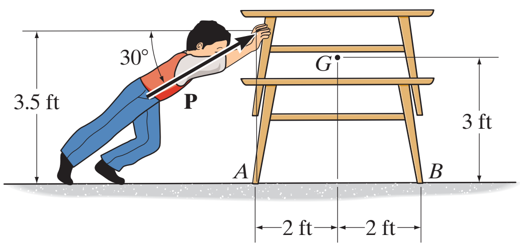


(a)



(b)

•8–29. If the center of gravity of the stacked tables is at G, and the stack weighs 100 lb, determine the smallest force P the boy must push on the stack in order to cause movement. The coefficient of static friction at A and B is $\mu_s = 0.3$. The tables are locked together.



Free - Body Diagram. The impending motion of the stack could be due to either sliding or tipping about point B. We will assume that sliding occurs. Thus, $F_A = \mu_s N_A = 0.3N_A$ and $F_B = \mu_s N_B = 0.3N_B$.

Equations of Equilibrium. Referring to the free - body diagram of the stack shown in Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad P\cos 30° - 0.3N_A - 0.3N_B = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N_A + N_B + P\sin 30° - 100 = 0$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B(4) - P\cos 30°(3.5) - 100(2) = 0$

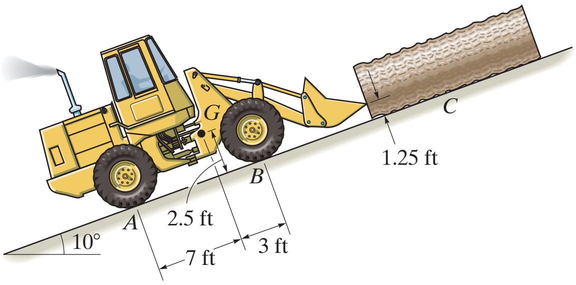
Solving,

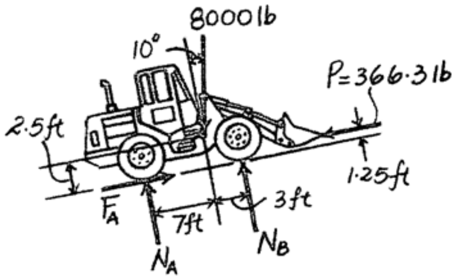
$P = 29.5\text{ N}$ **Ans.**

$N_A = 12.9\text{ N}$ $\quad N_B = 72.4\text{ N}$

Since the result for N_A is a positive quantity, the leg of the chair at A still remains in contact with the floor. This means that the stack will not tip. Thus, the above assumption is correct.

8–30. The tractor has a weight of 8000 lb with center of gravity at G. Determine if it can push the 550-lb log up the incline. The coefficient of static friction between the log and the ground is $\mu_s = 0.5$, and between the rear wheels of the tractor and the ground $\mu_s' = 0.8$. The front wheels are free to roll. Assume the engine can develop enough torque to cause the rear wheels to slip.





Log:

$+\nwarrow\Sigma F_y = 0;$ $N_C - 550\cos 10° = 0$

$N_C = 541.6\text{ lb}$

$+\nearrow\Sigma F_x = 0;$ $-0.5(541.6) - 550\sin 10° + P = 0$

$P = 366.3\text{ lb}$

Tractor:

$\circlearrowleft +\Sigma M_B = 0;$ $366.3(1.25) + 8000(\cos 10°)(3) + 8000(\sin 10°)(2.5) - N_A(10) = 0$

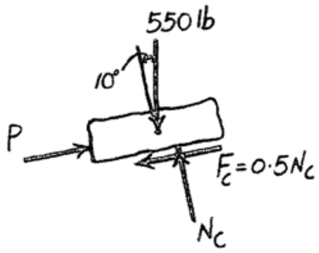
$N_A = 2757\text{ lb}$

$+\nearrow\Sigma F_x = 0;$ $F_A - 8000\sin 10° - 366.3 = 0$

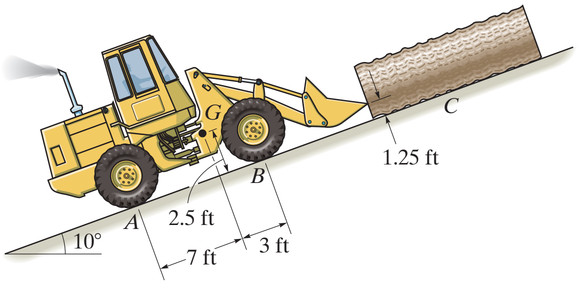
$F_A = 1756\text{ lb}$

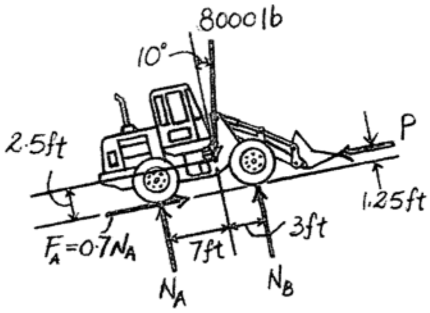
$(F_A)_{max} = 0.8(2757) = 2205\text{ lb} > 1756\text{ lb}$

Tractor can move log. **Ans**



8–31. The tractor has a weight of 8000 lb with center of gravity at G. Determine the greatest weight of the log that can be pushed up the incline. The coefficient of static friction between the log and the ground is $\mu_s = 0.5$, and between the rear wheels of the tractor and the ground $\mu_s' = 0.7$. The front wheels are free to roll. Assume the engine can develop enough torque to cause the rear wheels to slip.





Tractor :

$\circlearrowleft + \Sigma M_B = 0;\quad 8000(\cos 10^\circ)(3) + 8000(\sin 10^\circ)(2.5) + P(1.25) - N_A(10) = 0$

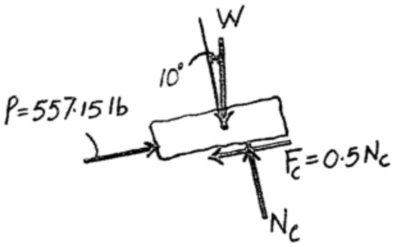
$N_A - P(0.125) = 2710.8$

$+\nearrow \Sigma F_x = 0;\quad 0.7N_A - 8000\sin 10^\circ - P = 0$

$0.7N_A - P = 1389.2$

$N_A = 2780 \text{ lb}$

$P = 557.15 \text{ lb}$



Log :

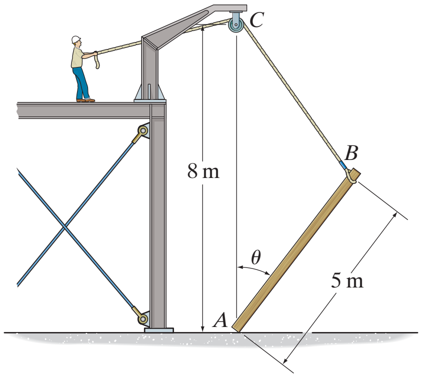
$+\nwarrow \Sigma F_y = 0;\quad N_C - W\cos 10^\circ = 0$

$+\nearrow \Sigma F_x = 0;\quad 557.15 - W\sin 10^\circ - 0.5N_C = 0$

$N_C = 824 \text{ lb}$

$W = 836 \text{ lb}$ **Ans**

***8–32.** The 50-kg uniform pole is on the verge of slipping at A when $\theta = 45°$. Determine the coefficient of static friction at A.



Free - Body Diagram. By referring to the geometry shown in Fig. a,

$$\tan\alpha = \frac{8 - 5\cos 45°}{5\sin 45°} \qquad \alpha = 51.62°$$

$$\phi = 180° - 45° - \alpha = 83.38°$$

Since the end A of the pole is on the verge of sliding to the left due to $\mathbf{T}_{BC}$, the frictional force $\mathbf{F}_A$ must act to the right such that $F_A = \mu_s N_A$ as indicated on the free - body diagram of the pole, Fig. b.

Equations of Equilibrium. Referring to Fig. b,

$$\circlearrowleft +\Sigma M_A = 0; \qquad T_{BC}\sin 83.38°(5) - 50(9.81)\sin 45°(2.5) = 0$$

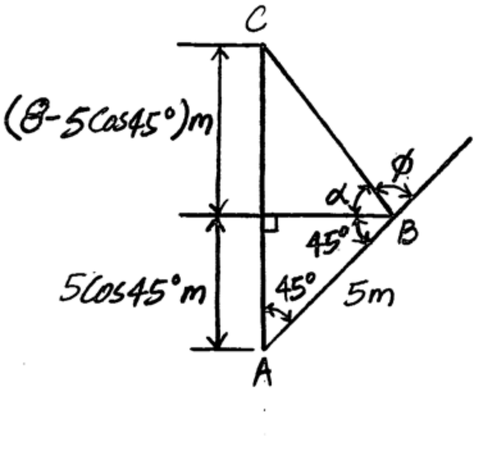
$$T_{BC} = 175\text{N}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A + 209.50\sin 51.62° - 50(9.81) = 0$$

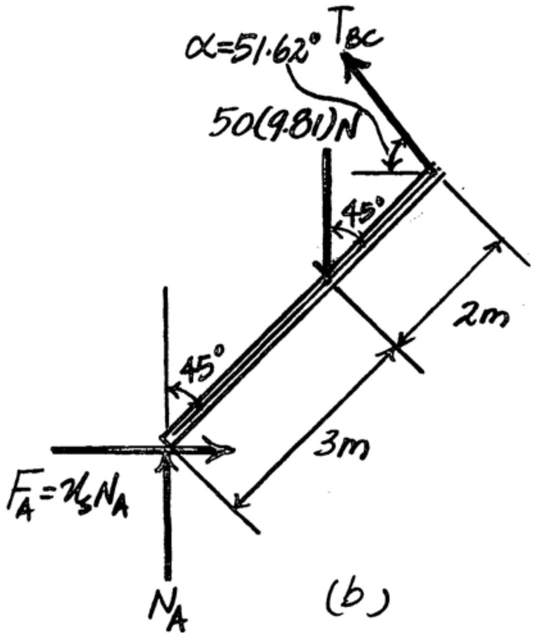
$$N_A = 353.6\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad \mu_s(326.26) - 209.50\cos 51.62° = 0$$

$$\mu_s = 0.306 \qquad \textbf{Ans.}$$

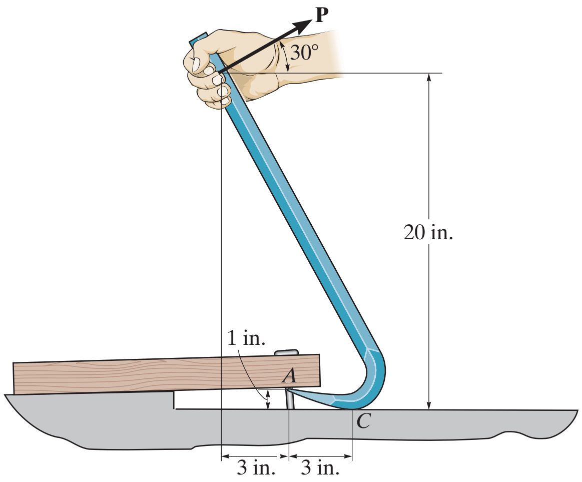


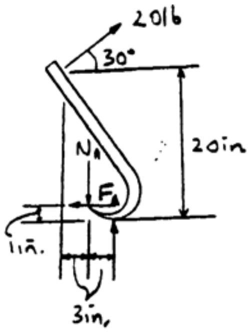
(a)

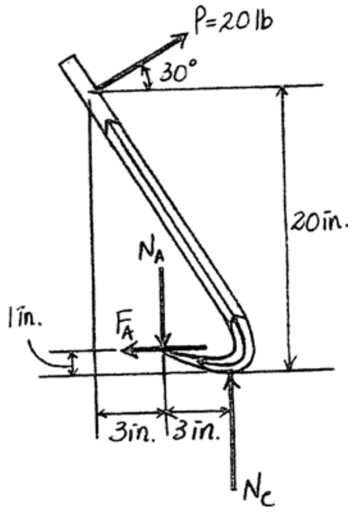


(b)

•8–33. A force of $P = 20$ lb is applied perpendicular to the handle of the gooseneck wrecking bar as shown. If the coefficient of static friction between the bar and the wood is $\mu_s = 0.5$, determine the normal force of the tines at A on the upper board. Assume the surface at C is smooth.







$\xrightarrow{+}\Sigma F_x = 0;\quad 20\cos 30° - F_A = 0$

$F_A = 17.32$ lb

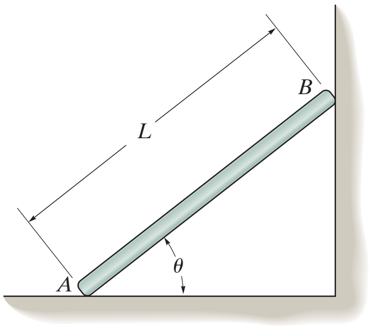
$\circlearrowleft + \Sigma M_C = 0;\quad N_A(3) + 17.32(1) - 20\cos 30°(20) - 20\sin 30°(6) = 0$

$N_A = 129.7$ lb $= 130$ lb **Ans**

$F_{max} = 0.5(129.7) = 64.8$ lb > 17.32 lb

The bar will not slip.

8–34. The thin rod has a weight W and rests against the floor and wall for which the coefficients of static friction are μ_A and μ_B, respectively. Determine the smallest value of θ for which the rod will not move.

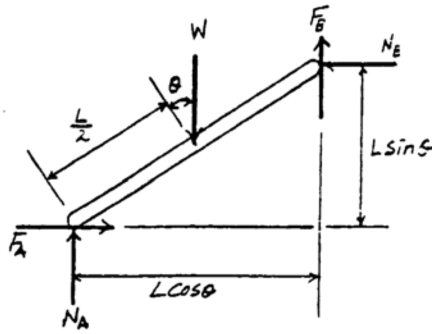


Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_A - N_B = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0 \quad N_A + F_B - W = 0 \qquad [2]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(L\sin\theta) + F_B(\cos\theta)L - W\cos\theta\left(\frac{L}{2}\right) = 0 \qquad [3]$$

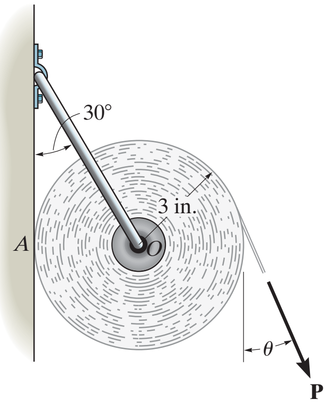


Friction : If the rod is on the verge of moving, slipping will have to occur at points A and B. Hence, $F_A = \mu_A N_A$ and $F_B = \mu_B N_B$ .Substituting these values into Eqs.[1], [2] and [3] and solving we have

$$N_A = \frac{W}{1+\mu_A\mu_B} \qquad N_B = \frac{\mu_A W}{1+\mu_A\mu_B}$$

$$\theta = \tan^{-1}\left(\frac{1-\mu_A\mu_B}{2\mu_A}\right) \qquad \textbf{Ans}$$

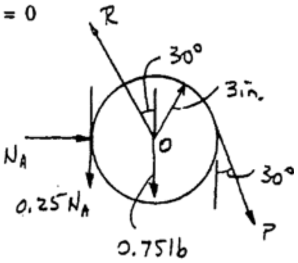
8–35. A roll of paper has a uniform weight of 0.75 lb and is suspended from the wire hanger so that it rests against the wall. If the hanger has a negligible weight and the bearing at O can be considered frictionless, determine the force P needed to start turning the roll if $\theta = 30°$. The coefficient of static friction between the wall and the paper is $\mu_s = 0.25$.



$$\xrightarrow{+} \Sigma F_x = 0; \quad N_A - R\sin 30° + P\sin 30° = 0$$

$$+\uparrow \Sigma F_y = 0; \quad R\cos 30° - 0.75 - P\cos 30° - 0.25\,N_A = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \quad 0.25\,N_A(3) - P(3) = 0$$



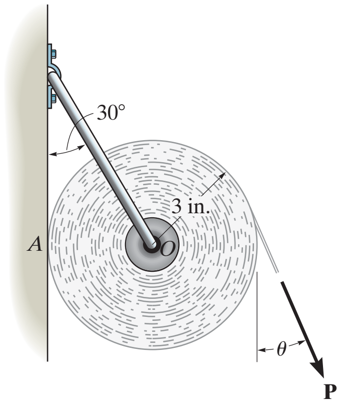
Solving for P,

$$R = 1.14 \text{ lb}$$

$$N_A = 0.506 \text{ lb}$$

$$P = 0.127 \text{ lb} \qquad \textbf{Ans}$$

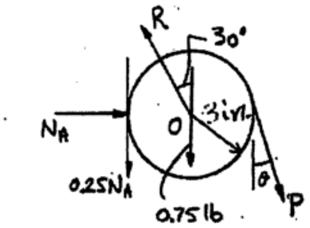
***8–36.** A roll of paper has a uniform weight of 0.75 lb and is suspended from the wire hanger so that it rests against the wall. If the hanger has a negligible weight and the bearing at O can be considered frictionless, determine the minimum force P and the associated angle θ needed to start turning the roll. The coefficient of static friction between the wall and the paper is $\mu_s = 0.25$.



$\xrightarrow{+} \Sigma F_x = 0; \quad N_A - R\sin 30° + P\sin\theta = 0$

$+\uparrow \Sigma F_y = 0; \quad R\cos 30° - 0.75 - P\cos\theta - 0.25\,N_A = 0$

$\circlearrowleft + \Sigma M_O = 0; \quad 0.25\,N_A\,(3) - P\,(3) = 0$



Solving for P,

$$P = \frac{0.433013}{(3.4226 + \sin\theta - 0.57735\cos\theta)} \qquad (1)$$

For maximum or minimum P,

$$\frac{dP}{d\theta} = \frac{0 - (0.433013)(\cos\theta + 0.57735\sin\theta)}{(3.4226 + \sin\theta - 0.57735\cos\theta)^2} = 0$$

$$\cos\theta + 0.57735\sin\theta = 0$$

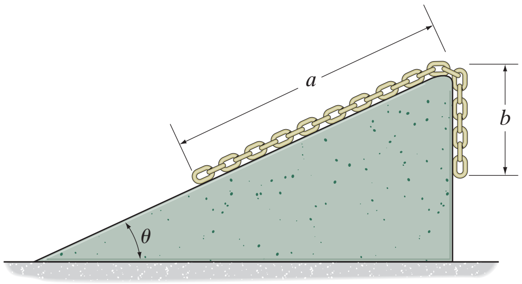
$$\tan\theta = -1.732$$

$$\theta = -60° \text{ or } 120°$$

For minimum P choose $\theta = 120°$, since N_A would be smaller than for $\theta = -60°$. Also, a comparison could be made from substitution into Eq. (1). Using $\theta = 120°$,

$$P = \frac{0.433013}{(3.4226 + \sin 120° - 0.57735\cos 120°)} = 0.0946 \text{ lb} \qquad \textbf{Ans}$$

•8–37. If the coefficient of static friction between the chain and the inclined plane is $\mu_s = \tan\theta$, determine the overhang length b so that the chain is on the verge of slipping up the plane. The chain weighs w per unit length.



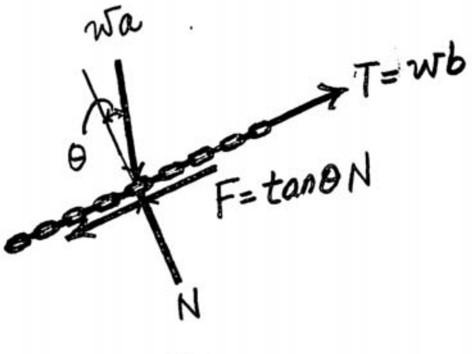
Free - Body Diagram. The tension developed in the chain at the end of the inclined plane is equal to the weight of the overhanging chain, i.e. $T = wb$. Since the chain is required to be on the verge of sliding up the plane, the frictional force **F** must act down the plane so that $F = \mu_s N = \tan\theta N$ as indicated on the free - body diagram of the chain shown in Fig. *a*.

Equations of Equilibrium.

$\nwarrow^{+}\Sigma F_{y'} = 0;\quad N - wa\cos\theta = 0 \qquad\qquad N = wa\cos\theta$

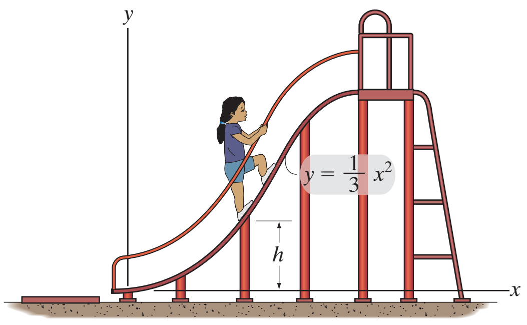
$+\nearrow\Sigma F_{x'} = 0;\quad wb - wa\sin\theta - \tan\theta(wa\cos\theta) = 0$

$b = 2a\sin\theta$ **Ans.**



(a)

8–38. Determine the maximum height h in meters to which the girl can walk up the slide without supporting herself by the rails or by her left leg. The coefficient of static friction between the girl's shoes and the slide is $\mu_s = 0.8$.



Free - Body Diagram. Since the girl is required to be on the verge of slipping down, the frictional force **F** must act upwards so that $F = \mu_s N = 0.8N$ as indicated on the free - body diagram of the girl shown in Fig. *a*.

Equations of Equilibrium. Referring to Fig. *a*,

$\nwarrow + \Sigma F_{y'} = 0;\quad N - mg\cos\theta = 0 \qquad\qquad N = mg\cos\theta$

$+\nearrow \Sigma F_{x'} = 0;\quad 0.8(mg\cos\theta) - mg\sin\theta = 0 \qquad \tan\theta = 0.8$

From the geometry of the slide, we have $\frac{dy}{dx} = \frac{2}{3}x$. Thus,

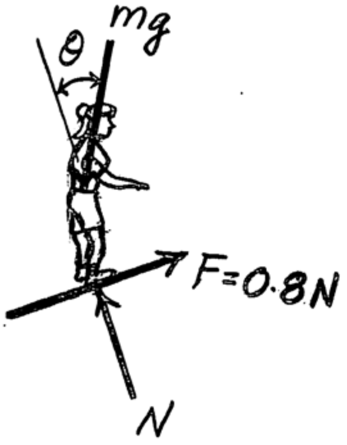
$$\frac{dy}{dx} = \tan\theta$$

$$\frac{2}{3}x = 0.8$$

$$x = 1.2\text{ m}$$

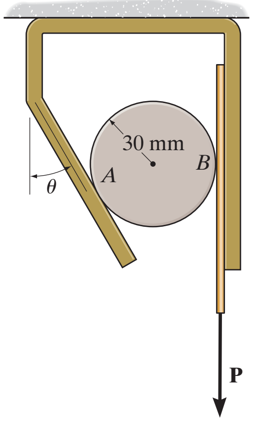
Therefore,

$$h = \frac{1}{3}(1.2)^2 = 0.48\text{ m} \qquad \textbf{Ans.}$$



(a)

8–39. If the coefficient of static friction at B is $\mu_s = 0.3$, determine the largest angle θ and the minimum coefficient of static friction at A so that the roller remains self-locking, regardless of the magnitude of force **P** applied to the belt. Neglect the weight of the roller and neglect friction between the belt and the vertical surface.



Free - Body Diagram. Since the belt is required to be on the verge of slipping downwards, the frictional force $\mathbf{F}_B$ must act downward on the rod so that $F_B = \mu_s N_B = 0.3N_B$ as indicated on the free - body diagram of the cylinder shown in Fig. *a*.

Equations of Equilibrium. Referring to Fig. *a*,

$$\circlearrowleft +\Sigma M_A = 0; \qquad N_B(0.03\sin\theta) - 0.3N_B(0.03 + 0.03\cos\theta) = 0$$

$$\sin\theta - 0.3\cos\theta = 0.3$$

$$\theta = 33.40° = 33.4° \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_A \sin 33.40° + N_A \cos 33.40° - N_B = 0$$

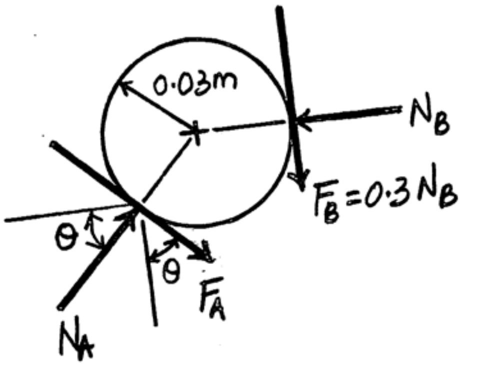
$$+\uparrow \Sigma F_y = 0; \qquad N_A \sin 33.40° - F_A \cos 33.40° - 0.3N_B = 0$$

Solving,

$$F_A = 0.3N_B \qquad N_A = N_B$$

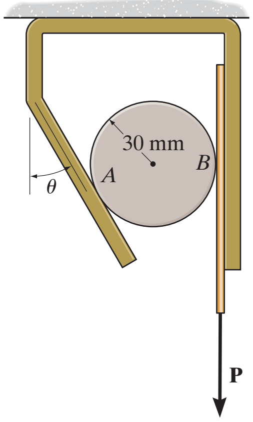
To prevent slipping at A, the coefficient of static friction at A must be at least

$$\mu_s = \frac{F_A}{N_A} = \frac{0.3N_B}{N_B} = 0.3 \qquad \textbf{Ans.}$$



(a)

***8–40.** If $\theta = 30°$, determine the minimum coefficient of static friction at A and B so that the roller remains self-locking, regardless of the magnitude of force **P** applied to the belt. Neglect the weight of the roller and neglect friction between the belt and the vertical surface.



Free - Body Diagram. Since the belt is required to be on the verge of slipping downwards, the frictional force $\mathbf{F}_B$ must act downward on the roller so that $F_B = \mu_s N_B$ as indicated on the free - body diagram of the roller shown in Fig. *a*.

Equations of Equilibrium. Referring to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B(0.03\sin 30°) - \mu_s N_B(0.03 + 0.03\cos 30°) = 0$

$\mu_s = 0.2679 = 0.268$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_A \sin 30° + N_A \sin 60° - N_B = 0$

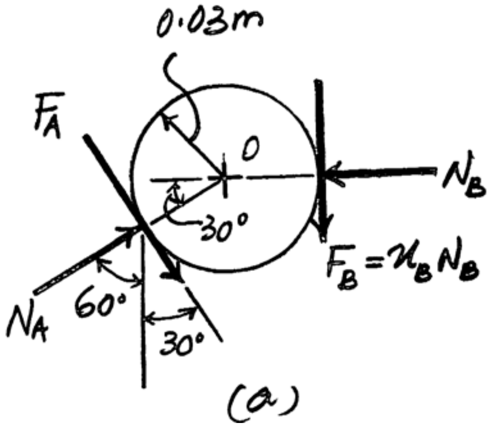
$+\uparrow \Sigma F_y = 0;$ $\quad -F_A \cos 30° + N_A \cos 60° - 0.2679 N_B = 0$

Solving,

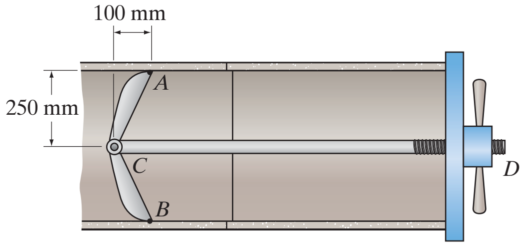
$$F_A = 0.2679 N_B \qquad N_A = N_B$$

To prevent slipping at A, the coefficient of static friction at A must be at least

$$\mu_s = \frac{F_A}{N_A} = \frac{0.2679 N_B}{N_B} = 0.268$$ **Ans.**



•8–41. The clamp is used to tighten the connection between two concrete drain pipes. Determine the least coefficient of static friction at *A* or *B* so that the clamp does not slip regardless of the force in the shaft *CD*.



Free - Body Diagram. Since member *CA* tends to move to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free - body diagram of member *CA* shown in Fig. *a*.

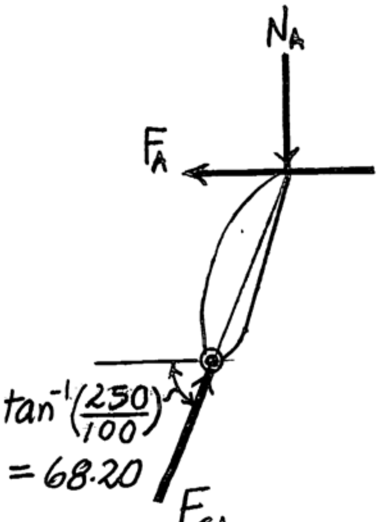
Equations of Equilibrium. Referring to Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $F_{CA}\cos 68.20° - F_A = 0$ $F_A = 0.3714F_{CA}$

$+\uparrow \Sigma F_y = 0;$ $F_{CA}\sin 68.20° - N_A = 0$ $N_A = 0.9285F_{CA}$

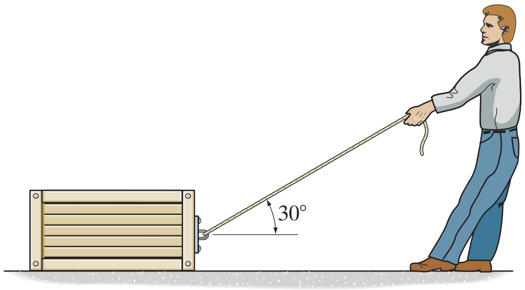
To prevent slipping at *A*, the coefficient of static friction at *A* must be at least

$$\mu_s = \frac{F_A}{N_A} = \frac{0.3714F_{CA}}{0.9285F_{CA}} = 0.4$$ **Ans.**



(a)

8–42. The coefficient of static friction between the 150-kg crate and the ground is $\mu_s = 0.3$, while the coefficient of static friction between the 80-kg man's shoes and the ground is $\mu_s' = 0.4$. Determine if the man can move the crate.



Free - Body Diagram. Since **P** tends to move the crate to the right, the frictional force $\mathbf{F}_C$ will act to the left as indicated on the free - body diagram shown in Fig. *a*. Since the crate is required to be on the verge of sliding the magnitude of $\mathbf{F}_C$ can be computed using the friction formula, i.e. $F_C = \mu_s N_C = 0.3\,N_C$. As indicated on the free - body diagram of the man shown in Fig. *b*, the frictional force $\mathbf{F}_m$ acts to the right since force **P** has the tendency to cause the man to slip to the left.

Equations of Equilibrium. Referring to Fig. *a*,

$+\uparrow \Sigma F_y = 0;\qquad N_C + P\sin 30° - 150(9.81) = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad P\cos 30° - 0.3N_C = 0$

Solving,

$$P = 434.49\text{ N}$$
$$N_C = 1254.26\text{ N}$$

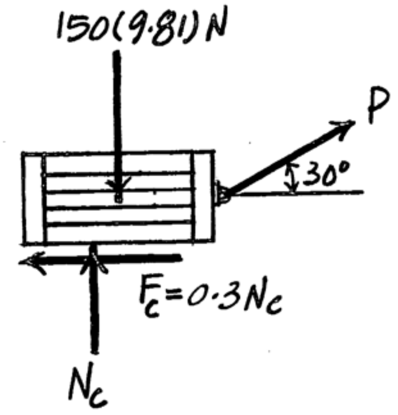
Using the result of *P* and referring to Fig. *a*, we have

$+\uparrow \Sigma F_y = 0;\qquad N_m - 434.49\sin 30° - 80(9.81) = 0\qquad N_m = 1002.04\text{ N}$

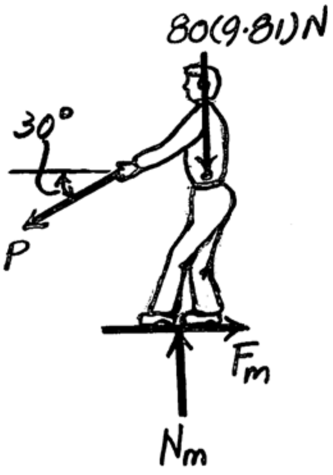
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_m - 434.49\cos 30° = 0\qquad F_m = 376.28\text{ N}$

Since $F_m < F_{\max} = \mu_s{}' N_m = 0.4(1002.04) = 400.82\text{ N}$, the man does not slip. Thus,

he can move the crate. **Ans.**

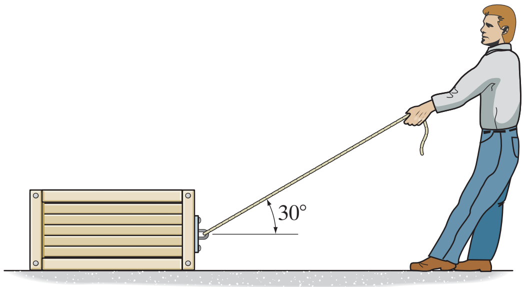


(a)



(b)

8–43. If the coefficient of static friction between the crate and the ground is $\mu_s = 0.3$, determine the minimum coefficient of static friction between the man's shoes and the ground so that the man can move the crate.



Free - Body Diagram. Since force **P** tends to move the crate to the right, the frictional force $\mathbf{F}_C$ will act to the left as indicated on the free - body diagram shown in Fig. *a*. Since the crate is required to be on the verge of sliding, $F_C = \mu_s N_C = 0.3\,N_C$. As indicated on the free - body diagram of the man shown in Fig. *b*, the frictional force $\mathbf{F}_m$ acts to the right since force **P** has the tendency to cause the man to slip to the left.

Equations of Equilibrium. Referring to Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N_C + P\sin 30^\circ - 150(9.81) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P\cos 30^\circ - 0.3N_C = 0$$

Solving yields

$$P = 434.49\text{ N}$$

$$N_C = 1254.26\text{ N}$$

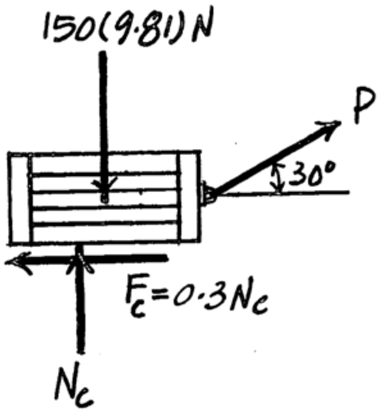
Using the result of **P** and referring to Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N_m - 434.49\sin 30^\circ - 80(9.81) = 0 \qquad N_m = 1002.04\text{ N}$$

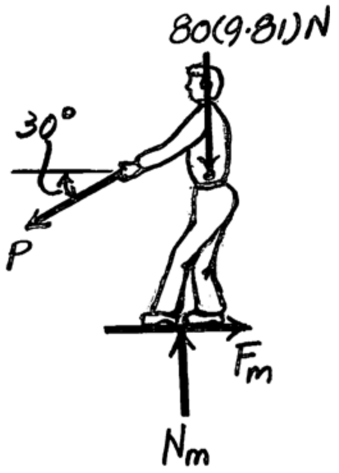
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_m - 434.49\cos 30^\circ = 0 \qquad F_m = 376.28\text{ N}$$

Thus, the required minimum coefficient of static friction between the man's shoes and the ground is given by

$$\mu_s' = \frac{F_m}{N_m} = \frac{376.28}{1002.04} = 0.376 \qquad \textbf{Ans.}$$

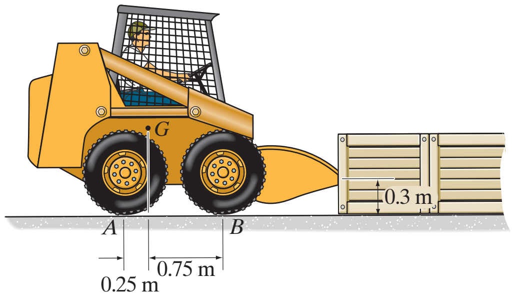


(a)



(b)

***8–44.** The 3-Mg rear-wheel-drive skid loader has a center of mass at *G*. Determine the largest number of crates that can be pushed by the loader if each crate has a mass of 500 kg. The coefficient of static friction between a crate and the ground is $\mu_s = 0.3$, and the coefficient of static friction between the rear wheels of the loader and the ground is $\mu_s' = 0.5$. The front wheels are free to roll. Assume that the engine of the loader is powerful enough to generate a torque that will cause the rear wheels to slip.



Free - Body Diagram. Since the frictional force $\mathbf{F}_A$ provides the driving force to the skid roller which is about to move to the right, it must act to the right as indicated on the free - body diagram shown in Fig. *a*. Here, $\mathbf{F}_A$ is required to be maximum, i.e., $F_A = \mu_s N_A = 0.5N_A$. Since the crates are required to be on the verge of slipping to the right, the frictional force $\mathbf{F}_C$ must act to the left so that $F_C = \mu_s N_C = 0.3N_C$ as indicated on the free - body diagram of the crate shown in Fig. *b*.

Equations of Equilibrium. Referring to Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad P(0.3) + 3000(9.81)(0.75) - N_A(1) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.5N_A - P = 0$$

Solving,

$$P = 12983.82 \text{ N} \qquad N_A = 25967.65 \text{ N}$$

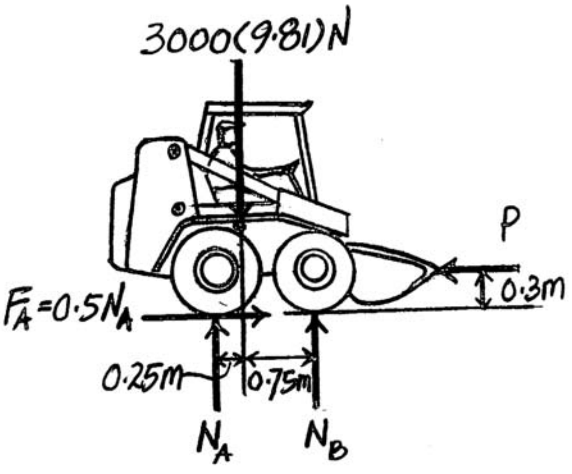
Using the result of **P** and referring to Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad N_C - n(500)(9.81) = 0 \qquad N_C = 4905n$$

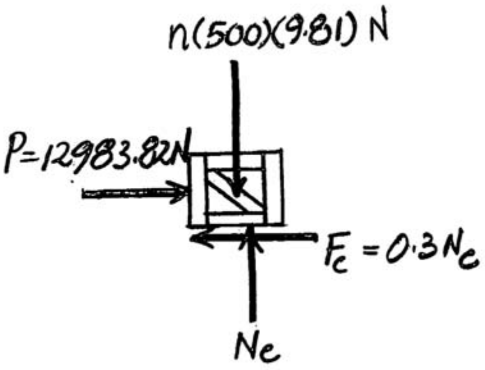
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 12983.82 - 0.3(4905n) = 0 \qquad n = 8.82$$

Thus, the largest number of crates that can be pushed by the skid roller is

$$n = 8 \qquad \textbf{Ans.}$$

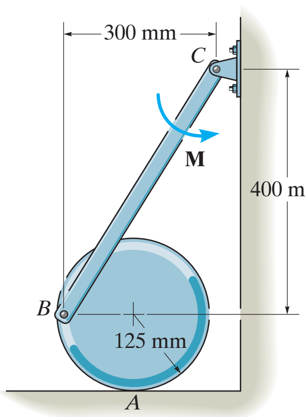


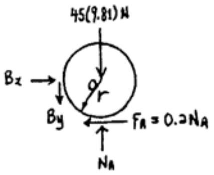
(a)



(b)

•8–45. The 45-kg disk rests on the surface for which the coefficient of static friction is $\mu_A = 0.2$. Determine the largest couple moment M that can be applied to the bar without causing motion.





$\circlearrowleft + \Sigma M_O = 0; \quad F_A = B_y = 0.2\,N_A$

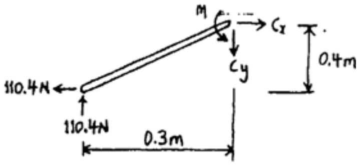
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 0.2N_A = 0$

$+\uparrow \Sigma F_y = 0; \quad N_A - B_y - 45(9.81) = 0$

$N_A = 551.8 \text{ N}$

$B_x = 110.4 \text{ N}$

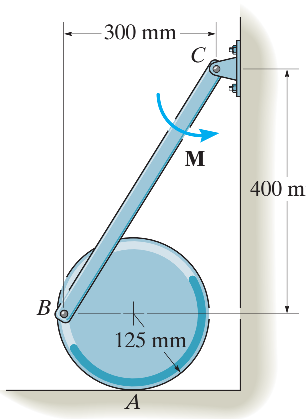
$B_y = 110.4 \text{ N}$



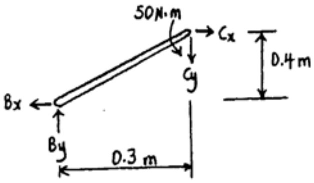
$\circlearrowleft + \Sigma M_C = 0; \quad -110.4(0.3) - 110.4(0.4) + M = 0$

$M = 77.3 \text{ N}\cdot\text{m}$ **Ans**

8–46. The 45-kg disk rests on the surface for which the coefficient of static friction is $\mu_A = 0.15$. If $M = 50 \text{ N}\cdot\text{m}$, determine the friction force at A.



Bar :



$\circlearrowleft + \Sigma M_C = 0; \quad -B_y(0.3) - B_x(0.4) + 50 = 0$

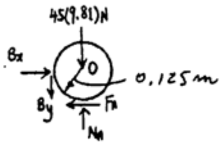
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = C_x$

$+\uparrow \Sigma F_y = 0; \quad B_y = C_y$

Disk :

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = F_A$

$+\uparrow \Sigma F_y = 0; \quad N_A - B_y - 45(9.81) = 0$



$\circlearrowleft + \Sigma M_O = 0; \quad B_y(0.125) - F_A(0.125) = 0$

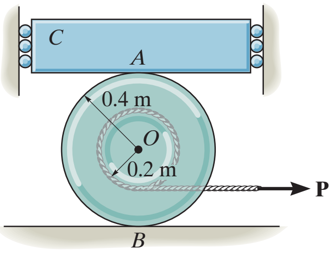
$N_A = 512.9 \text{ N}$

$F_A = 71.4 \text{ N}$ **Ans**

$(F_A)_{max} = 0.15(512.9) = 76.93 \text{ N} > 71.43 \text{ N}$

No motion of disk.

8–47. Block *C* has a mass of 50 kg and is confined between two walls by smooth rollers. If the block rests on top of the 40-kg spool, determine the minimum cable force *P* needed to move the spool. The cable is wrapped around the spool's inner core. The coefficients of static friction at *A* and *B* are $\mu_A = 0.3$ and $\mu_B = 0.6$.



$+\uparrow \Sigma F_y = 0; \quad N_B - 40(9.81) - 50(9.81) = 0$

$N_B = 882.9\text{ N}$

$\circlearrowleft + \Sigma M_O = 0; \quad F_A(0.4) - F_B(0.4) + P(0.2) = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad -F_A + P - F_B = 0$

Assume spool slips at *A*, then

$F_A = 0.3(50)(9.81) = 147.2\text{ N}$

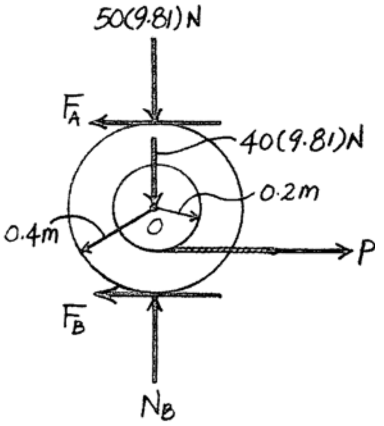
Solving,

$F_B = 441.4\text{ N}$

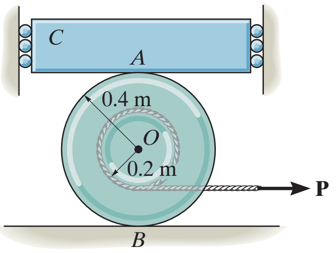
$P = 589\text{ N}$ **Ans**

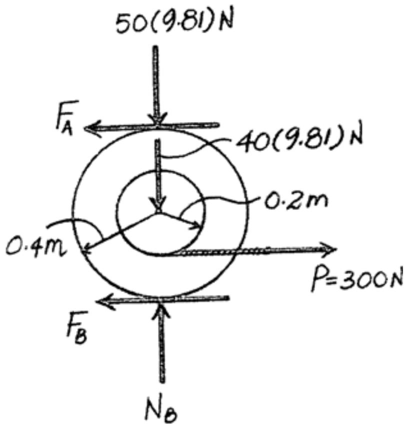
$N_B = 882.9\text{ N}$

Since $(F_B)_{max} = 0.6(882.9) = 529.7\text{ N} > 441.4\text{ N}$ **OK**



***8–48.** Block *C* has a mass of 50 kg and is confined between two walls by smooth rollers. If the block rests on top of the 40-kg spool, determine the required coefficients of static friction at *A* and *B* so that the spool slips at *A* and *B* when the magnitude of the applied force is increased to $P = 300$ N.





$\xrightarrow{+} \Sigma F_x = 0; \quad 300 - F_A - F_B = 0$

$\circlearrowleft + \Sigma M_B = 0; \quad F_A(0.8) - 300(0.2) = 0$

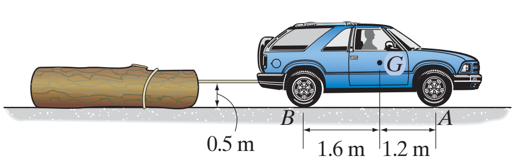
$F_A = 75\text{ N}$

$F_B = 225\text{ N}$

$\mu_A = \dfrac{F_A}{N_A} = \dfrac{75}{50(9.81)} = 0.153$ **Ans**

$\mu_B = \dfrac{F_B}{N_B} = \dfrac{225}{90(9.81)} = 0.255$ **Ans**

•8–49. The 3-Mg four-wheel-drive truck (SUV) has a center of mass at G. Determine the maximum mass of the log that can be towed by the truck. The coefficient of static friction between the log and the ground is $\mu_s = 0.8$, and the coefficient of static friction between the wheels of the truck and the ground is $\mu_s' = 0.4$. Assume that the engine of the truck is powerful enough to generate a torque that will cause all the wheels to slip.



Free - Body Diagram. Since the truck is about to move to the right, its driving force $\mathbf{F}_t$ provided by the friction of all the wheels must act to the right as indicated on the free - body diagram of the truck shown in Fig. a. Here, $\mathbf{F}_t$ is required to be maximum, thus $F_t = \mu_s'(N_A + N_B) = 0.4(N_A + N_B)$. Since the log is required to be on the verge of sliding to the right, the frictional force $\mathbf{F}_l$ must act to the left such that $F_l = \mu_s N_l = 0.8N_l$.

Equations of Equilibrium. Referring to Fig. a, we have

$+\uparrow \Sigma F_y = 0; \qquad N_A + N_B - 3000(9.81) = 0 \qquad N_A + N_B = 29430\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.4(29430) - T = 0 \qquad T = 11772\text{ N}$

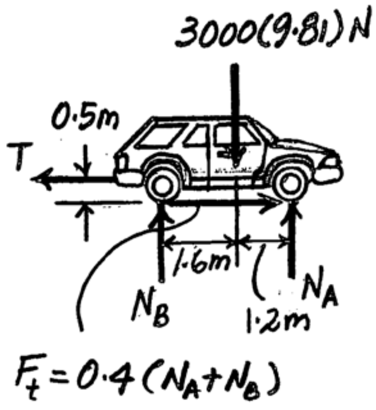
$\circlearrowleft + \Sigma M_B = 0; \qquad N_A(2.8) + 11772(0.5) - 3000(9.81)(1.6) = 0$

$N_A = 14715\text{ N} > 0\text{ (OK!)}$

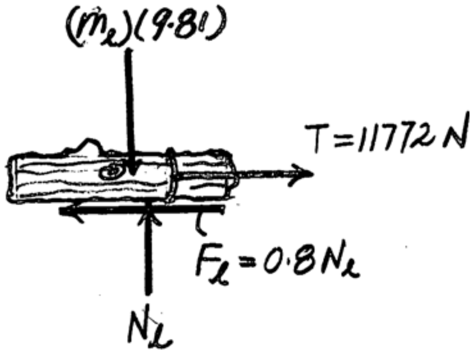
Using the result of $\mathbf{T}$ and referring to Fig. b, we have

$+\uparrow \Sigma F_y = 0; \qquad N_l - m_l(9.81) = 0 \qquad N_l = 9.81m_l$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 11772 - 0.8(9.81m_l) = 0 \qquad m_l = 1500\text{ kg}$

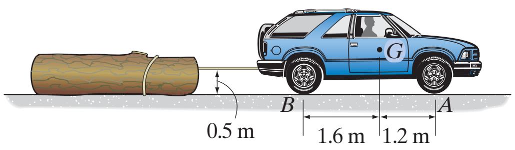


(a)



(b)

8–50. A 3-Mg front-wheel-drive truck (SUV) has a center of mass at G. Determine the maximum mass of the log that can be towed by the truck. The coefficient of static friction between the log and the ground is $\mu_s = 0.8$, and the coefficient of static friction between the front wheels of the truck and the ground is $\mu_s' = 0.4$. The rear wheels are free to roll. Assume that the engine of the truck is powerful enough to generate a torque that will cause the front wheels to slip.



Free - Body Diagram. Since the truck is about to move to the right, its driving force $\mathbf{F}_A$ provided by the friction of the front wheels must act to the right as indicated on the free - body diagram of the truck shown in Fig. *a*. Here, $\mathbf{F}_A$ is required to be maximum, so that $F_A = \mu_s' N_A = 0.4N_A$. Since the log is required to be on the verge of sliding to the right, the frictional force $\mathbf{F}_l$ must act to the left such that $F_l = \mu_s N_l = 0.8N_l$.

Equations of Equilibrium. Referring to Fig. *a*, we have

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 0.4N_A - T = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_B = 0; \qquad T(0.5) + N_A(2.8) - 3000(9.81)(1.6) = 0 \qquad (2)$

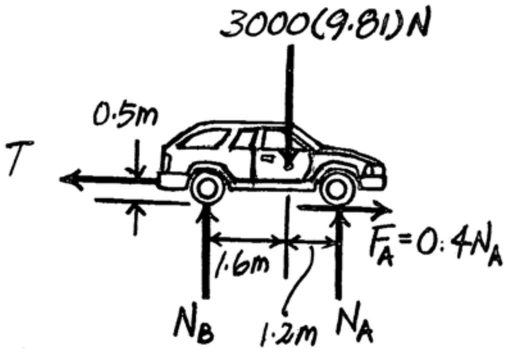
Solving Eqs. (1) and (2) yields

$$N_A = 15\,696\text{ N} \qquad T = 6278.4\text{ N}$$

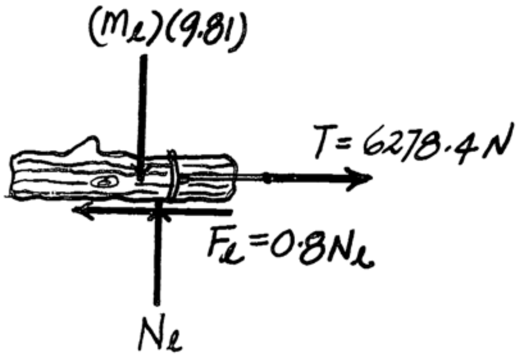
Using the result of $\mathbf{T}$ and referring to Fig. *b*, we have

$+\uparrow \Sigma F_y = 0; \qquad N_l - m_l(9.81) = 0 \qquad N_l = 9.81m_l$

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 6278.4 - 0.8(9.81m_l) = 0 \qquad m_l = 800\text{ kg}$

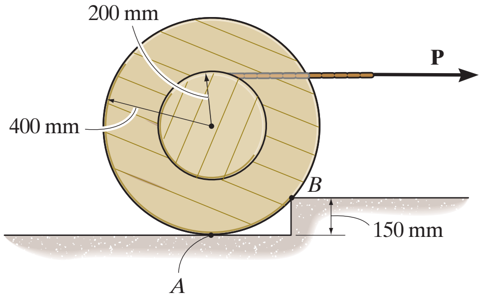


(a)



(b)

8–51. If the coefficients of static friction at contact points A and B are $\mu_s = 0.3$ and $\mu_s' = 0.4$ respectively, determine the smallest force P that will cause the 150-kg spool to have impending motion.



Free - Body Diagram. There are two possible modes of impending motion for the spool. The first mode is as the spool slips at A and B and is on the verge of rotating. The second mode is as point A of the spool just loses contact with the ground and the spool is on the verge of rolling about point B without slipping. We will assume that the first mode of motion occurs. Thus, $F_A = \mu_s N_A = 0.3N_A$ and $F_B = \mu_s' N_B = 0.4N_B$.

Equations of Equilibrium. Referring to the free - body diagram of the spool shown in Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 0.3N_A + 0.4N_B \cos 51.32° - N_B \sin 51.32° + P = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N_A + N_B \cos 51.32° + 0.4N_B \sin 51.32° - 150(9.81) = 0$

$\circlearrowleft + \Sigma M_O = 0;$ $\quad 0.3N_A(0.4) + 0.4N_B(0.4) - P(0.2) = 0$

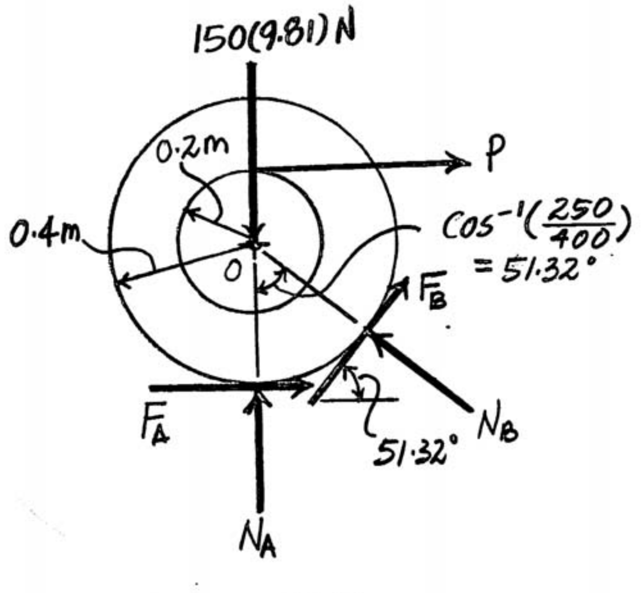
Solving,

$$N_A = -690.39\text{ N} \quad N_B = 2306.63\text{ N} \quad P = 1431.07\text{ N}$$

Since the result of $\mathbf{N}_A$ is a negative quantity, point A loses contact with the ground which indicates that the above assumption is incorrect. Thus, the solution must be reworked based on the second mode of motion. In this case, $N_A = 0$ so that $F_A = 0$. Referring to Fig. a,

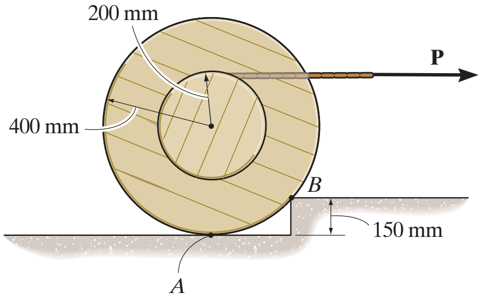
$\circlearrowleft + \Sigma M_B = 0;$ $\quad 150(9.81)(0.4 \sin 51.32°) - P(0.2 + 0.4 \cos 51.32°) = 0$

$P = 1021.05\text{ N} = 1.02\text{ kN}$ **Ans.**



(a)

***8–52.** If the coefficients of static friction at contact points A and B are $\mu_s = 0.4$ and $\mu_s' = 0.2$ respectively, determine the smallest force P that will cause the 150-kg spool to have impending motion.



Free - Body Diagram. There are two possible modes of impending motion for the spool. The first mode is as the spool slips at A and B and is on the verge of rotating. The second mode is as point A of the spool just loses contact with the ground and the spool is on the verge of rolling about point B without slipping. We will assume that the first mode of motion occurs. Thus, $F_A = \mu_s N_A = 0.4N_A$ and $F_B = \mu_s' N_B = 0.2N_B$.

Equations of Equilibrium. Referring to the free - body diagram of the spool shown in Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 0.4N_A + 0.2N_B \cos 51.32° - N_B \sin 51.32° + P = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N_A + 0.2N_B \sin 51.32° + N_B \cos 51.32° - 150(9.81) = 0$

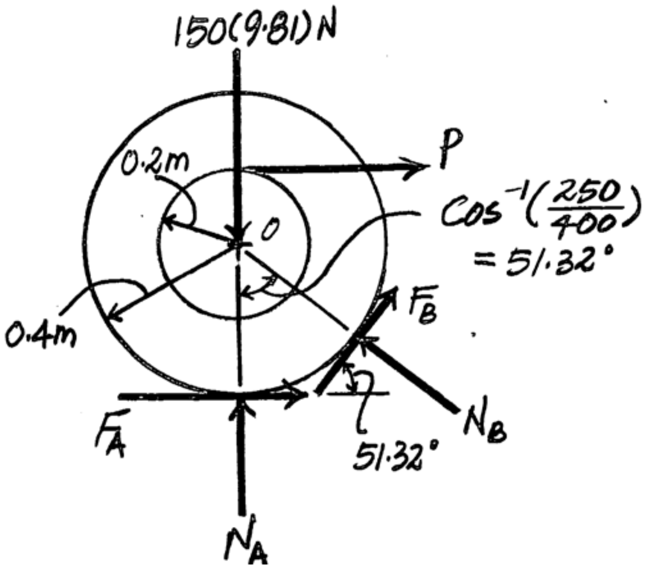
$\curvearrowleft + \Sigma M_O = 0;$ $\quad 0.4N_A(0.4) + 0.2N_B(0.4) - P(0.2) = 0$

Solving,

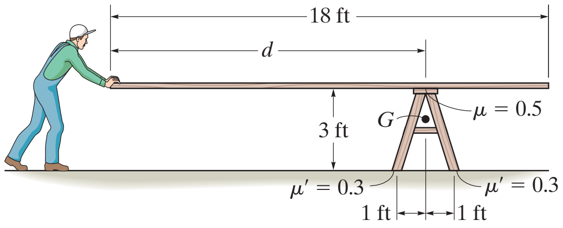
$P = 844\text{ N}$ **Ans.**

$N_A = 315.31\text{ N} \quad N_B = 1480.17\text{ N}$

Since the result of $\mathbf{N}_A$ is a positive quantity, point A will remain in contact with the ground. Thus, the above assumption is correct.



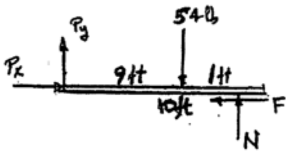
•8–53. The carpenter slowly pushes the uniform board horizontally over the top of the saw horse. The board has a uniform weight of 3 lb/ft, and the saw horse has a weight of 15 lb and a center of gravity at G. Determine if the saw horse will stay in position, slip, or tip if the board is pushed forward when $d = 10$ ft. The coefficients of static friction are shown in the figure.



Board:

$\zeta + \Sigma M_P = 0; \quad -54(9) + N(10) = 0$

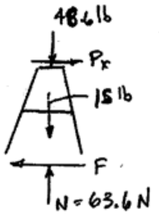
$N = 48.6 \text{ lb}$



To cause slipping of board on saw horse:

$P_x = F'_{max} = 0.5N = 24.3 \text{ lb}$

Saw horse:



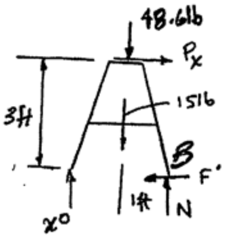
To cause slipping at ground:

$P_x = F = F_{max} = 0.3(48.6 + 15) = 19.08 \text{ lb}$

To cause tipping:

$\zeta + \Sigma M_B = 0; \quad (48.6 + 15)(1) - P_x(3) = 0$

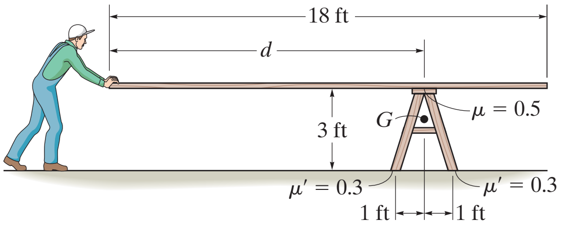
$P_x = 21.2 \text{ lb}$



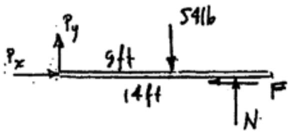
Thus, $\quad P_x = 19.1 \text{ lb}$

The saw horse will start to slip. **Ans**

8–54. The carpenter slowly pushes the uniform board horizontally over the top of the saw horse. The board has a uniform weight of 3 lb/ft, and the saw horse has a weight of 15 lb and a center of gravity at *G*. Determine if the saw horse will stay in position, slip, or tip if the board is pushed forward when $d = 14$ ft. The coefficients of static friction are shown in the figure.



Board:

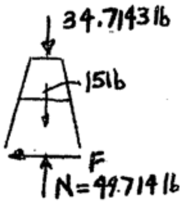


$\circlearrowleft + \Sigma M_P = 0; \qquad -54(9) + N(14) = 0$

$N = 34.714 \text{ lb}$

To cause slipping of board on saw horse:

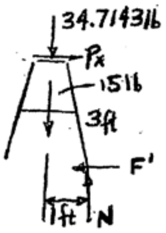
$P_x = F'_{max} = 0.5\,N = 17.36 \text{ lb}$



Saw horse:

To cause slipping at ground:

$P_x = F = F_{max} = 0.3(34.714 + 15) = 14.91 \text{ lb}$



To cause tipping:

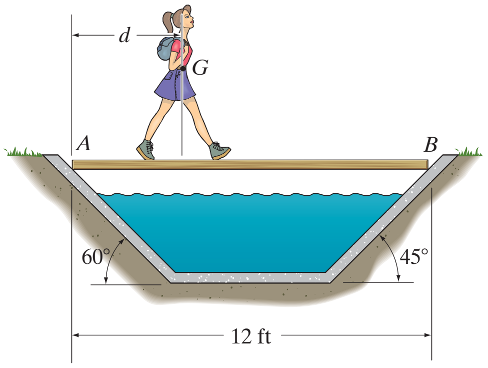
$\circlearrowleft + \Sigma M_B = 0; \qquad (34.714 + 15)(1) - P_x(3) = 0$

$P_x = 16.57 \text{ lb}$

Thus, $\quad P_x = 14.9 \text{ lb}$

The saw horse will start to slip. **Ans**

8–55. If the 75-lb girl is at position $d = 4$ ft, determine the minimum coefficient of static friction μ_s at contact points A and B so that the plank does not slip. Neglect the weight of the plank.



Free - Body Diagram. Here, we will assume that the plank is on the verge of rotating counterclockwise due to of the girl' s weight. Thus, the frictional forces $\mathbf{F}_A$ and $\mathbf{F}_B$ must act in the direction as indicated on the free - body diagram of the plank shown in Fig. *a* so that $F_A = \mu_s N_A$ and $F_B = \mu_s N_B$.

Equations of Equilibrium. Referring to Fig. *a*,

$$\circlearrowleft +\Sigma M_A = 0; \qquad N_B \sin 45°(12) - \mu_s N_B \sin 45°(12) - 75(4) = 0$$

$$\circlearrowleft +\Sigma M_B = 0; \qquad 75(8) - N_A \sin 30°(12) - \mu_s N_A \sin 60°(12) = 0$$

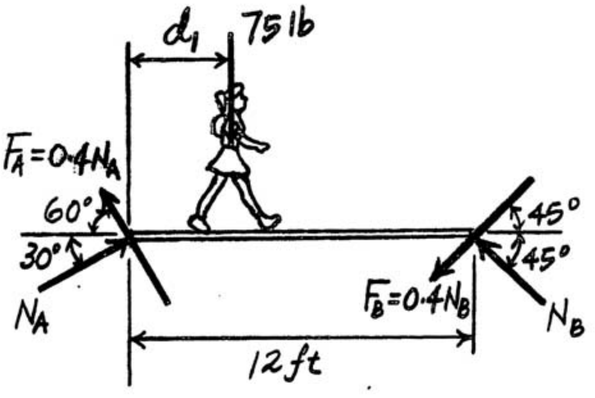
$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A \cos 30° - \mu_s N_A \cos 60° - N_B \cos 45° - \mu_s N_B \cos 45° = 0$$

Solving,

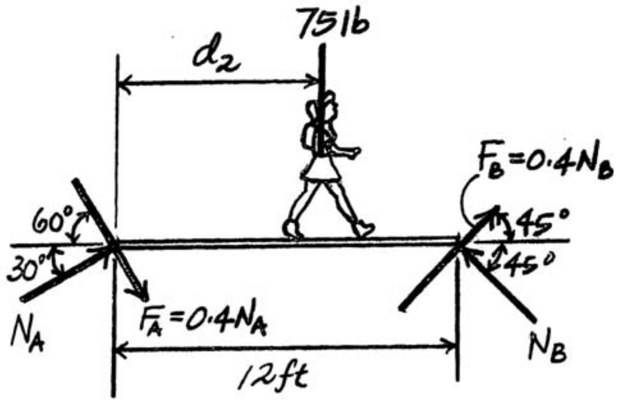
$$\mu_s = 0.304 \qquad \textbf{Ans.}$$

$$N_A = 65.5 \text{ lb} \quad N_B = 50.77 \text{ lb}$$

Since the result of μ_s is a positive quantity, the above assumption is correct.

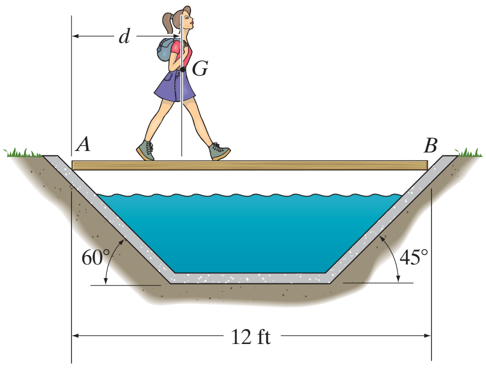


(a)



(b)

***8–56.** If the coefficient of static friction at the contact points *A* and *B* is $\mu_s = 0.4$, determine the minimum distance *d* where a 75-lb girl can stand on the plank without causing it to slip. Neglect the weight of the plank.



Free - Body Diagram. The weight of the girl tends to cause the plank to have counterclockwise and clockwise rotational motion when she is at the position $d = d_1$ and $d = d_2$, respectively. The free - body diagram of the plank for oth cases are shown in Figs. *a* and *b*. Since ends *A* and *B* of the plank are required to be on the verge of slipping the frictional forces $\mathbf{F}_A$ and $\mathbf{F}_B$ for both cases can be computed using $F_A = \mu_s N_A = 0.4N_A$ and $F_B = \mu_s N_B = 0.4N_B$.

Equations of Equilibrium. Referring to Fig. *a*, we have

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B \sin 45°(12) - 0.4N_B \sin 45°(12) - 75d = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad 75(12 - d) - N_A \sin 30°(12) - 0.4N_A \sin 60°(12) = 0 \qquad (2)$$

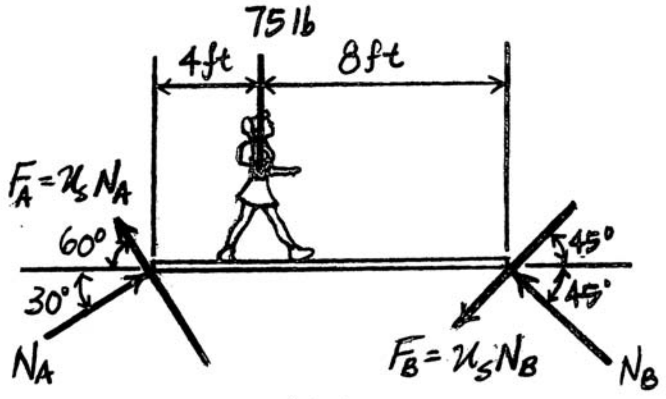
$$\rightarrow \Sigma F_x = 0; \qquad N_A \cos 30° - 0.4N_A \cos 60° - N_B \cos 45° - 0.4N_B \cos 45° = 0 \qquad (3)$$

Solving,

$d = 3.03\,\text{ft}$ **Ans.**

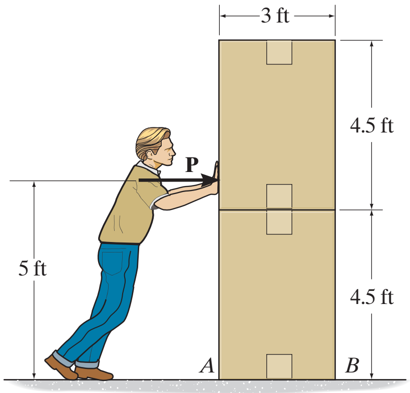
$N_A = 66.26$ lb

$N_B = 44.58$ lb



(a)

•8–57. If each box weighs 150 lb, determine the least horizontal force *P* that the man must exert on the top box in order to cause motion. The coefficient of static friction between the boxes is $\mu_s = 0.5$, and the coefficient of static friction between the box and the floor is $\mu_s' = 0.2$.



Free - Body Diagram. There are three possible motions, namely (1) the top box slides, (2) both boxes slide together as a single unit on the ground, and (3) both boxes tip as a single unit about point *B*. We will assume that both boxes slide together as a single unit such that $F = \mu_s' N = 0.2N$ as indicated on the free - body diagram shown in Fig. *a*.

Equations of Equilibrium.

$$+\uparrow \Sigma F_y = 0; \qquad N - 150 - 150 = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 0.2N = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 150(x) + 150(x) - P(5) = 0$$

Solving,

$$N = 300 \qquad x = 1\text{ ft}$$

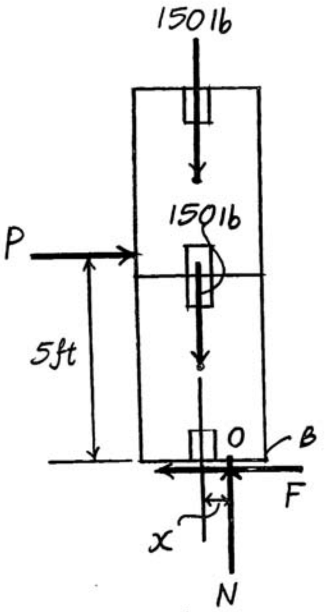
$$P = 60\text{ lb} \qquad \textbf{Ans.}$$

Since $x < 1.5$ ft, both boxes will not tip about point *B*. Using the result of **P** and considering the equilibrium of the free - body diagram shown in Fig. *b*, we have

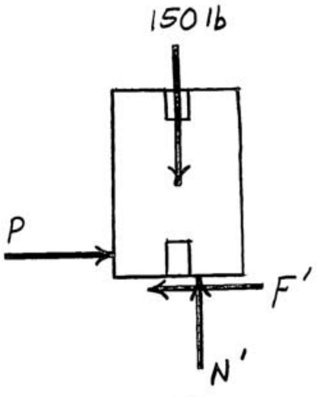
$$+\uparrow \Sigma F_y = 0; \qquad N' - 150 = 0 \qquad N' = 150\text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 60 - F' = 0 \qquad F' = 60\text{ lb}$$

Since $F' < F_{\max} = \mu_s N' = 0.5(150) = 75$ lb, the top box will not slide. Thus, the above assumption is correct.

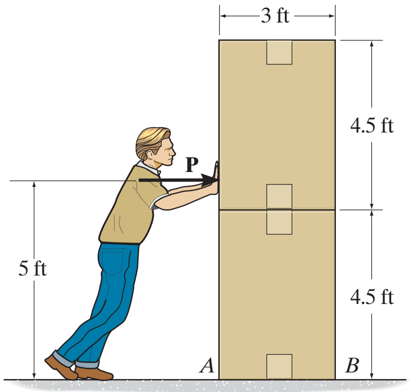


(a)



(b)

8–58. If each box weighs 150 lb, determine the least horizontal force P that the man must exert on the top box in order to cause motion. The coefficient of static friction between the boxes is $\mu_s = 0.65$, and the coefficient of static friction between the box and the floor is $\mu_s' = 0.35$.



Free - Body Diagram. There are three possible motions, namely (1) the top box slides, (2) both boxes slide together as a single unit on the ground, and (3) both boxes tip as a single unit about point B. We will assume that both boxes tip as a single unit about point B. Thus, $x = 1.5$ ft.

Equations of Equilibrium. Referring to Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad N - 150 - 150 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad 150(1.5) + 150(1.5) - P(5) = 0$

Solving,

$P = 90 \text{ lb}$ **Ans.**

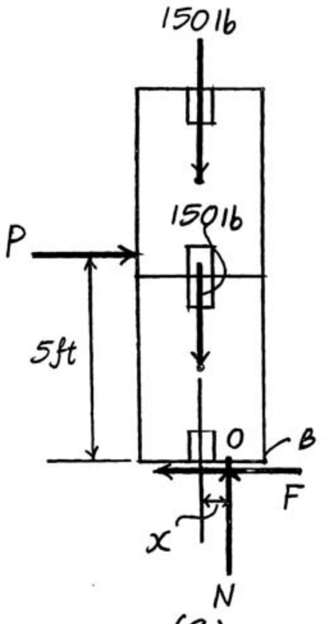
$N = 300 \text{ lb} \qquad F = 90 \text{ lb}$

Since $F < F_{\max} = \mu_s N' = 0.35(300) = 105$ lb, both boxes will not slide as a single unit on the floor. Using the result of **P** and considering the equilibrium of the free - body diagram shown in Fig. b,

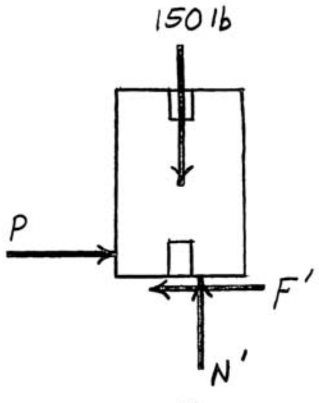
$+\uparrow \Sigma F_y = 0; \qquad N' - 150 = 0 \qquad N' = 150 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 90 - F' = 0 \qquad F' = 90 \text{ lb}$

Since $F' < F_{\max} = \mu_s' N' = 0.65(150) = 97.5$ lb, the top box will not slide. Thus, the above assumption is correct.

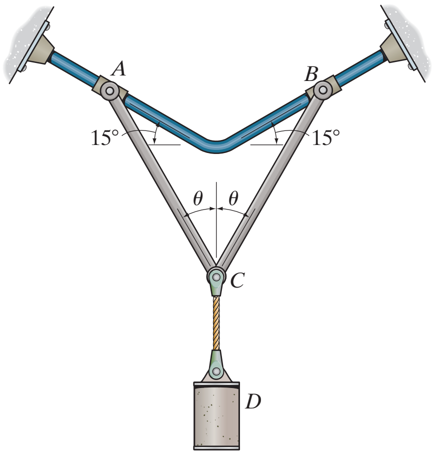


(a)



(b)

8–59. If the coefficient of static friction between the collars A and B and the rod is $\mu_s = 0.6$, determine the maximum angle θ for the system to remain in equilibrium, regardless of the weight of cylinder D. Links AC and BC have negligible weight and are connected together at C by a pin.



Free - Body Diagram. Due to the symmetrical loading and system, collars A and B will slip simultaneously. Thus, it is sufficient to consider the equilibrium of either collar. Here, the equilibrium of collar B will be considered. Since collar B is required to be on the verge of sliding down the rod the friction force $\mathbf{F}_B$ must act up the rod such that $F_B = \mu_s N_B = 0.6 N_B$ as indicated on the free - body diagram of the collar shown in Fig. a.

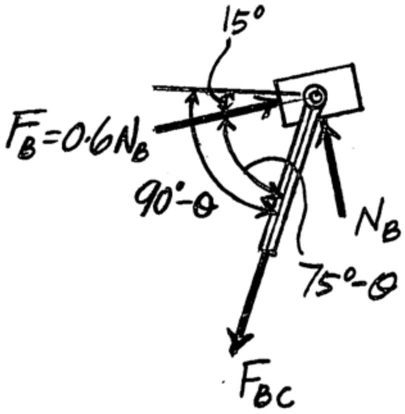
Equations of Equilibrium.

$+\nwarrow \Sigma F_y = 0;\quad N_B - F_{BC}\sin(75° - \theta) = 0 \qquad\qquad N_B = F_{BC}\sin(75° - \theta)$

$+\nearrow \Sigma F_x = 0;\quad 0.6[F_{BC}\sin(75° - \theta)] - F_{BC}\cos(75° - \theta) = 0$

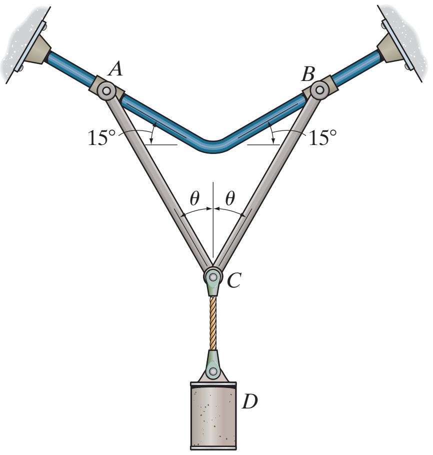
$\tan(75° - \theta) = 1.6667$

$\theta = 16.0°$ **Ans.**



(a)

***8–60.** If $\theta = 15°$, determine the minimum coefficient of static friction between the collars *A* and *B* and the rod required for the system to remain in equilibrium, regardless of the weight of cylinder *D*. Links *AC* and *BC* have negligible weight and are connected together at *C* by a pin.



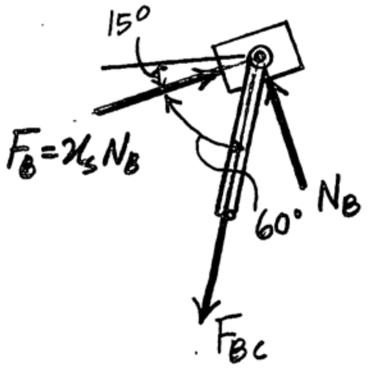
Free - Body Diagram. Due to the symmetrical loading and system, collars *A* and *B* will slip simultaneously. Thus, it is sufficient to consider the equilibrium of either collar. Here, the equilibrium of collar *B* will be considered. Since collar *B* is required to be on the verge of sliding down the rod the friction force $\mathbf{F}_B$ must up the rod such that $F_B = \mu_s N_B = 0.6 N_B$ as indicated on the free - body diagram of the collar shown in Fig. *a*.

Equations of Equilibrium.

$\Sigma F_y = 0;\quad N_B - F_{BC}\sin 60° = 0 \qquad\qquad N_B = 0.8660 F_{BC}$

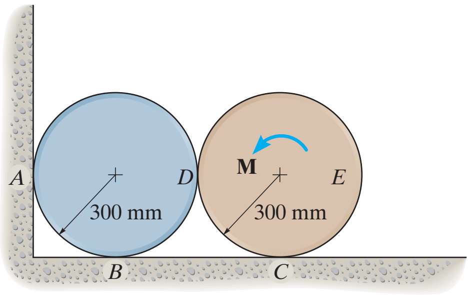
$\Sigma F_x = 0;\quad \mu_s[0.8660 F_{BC}] - F_{BC}\cos 60° = 0$

$\mu_s = 0.577$ **Ans.**



(a)

•8–61. Each of the cylinders has a mass of 50 kg. If the coefficients of static friction at the points of contact are $\mu_A = 0.5$, $\mu_B = 0.5$, $\mu_C = 0.5$, and $\mu_D = 0.6$, determine the smallest couple moment M needed to rotate cylinder E.

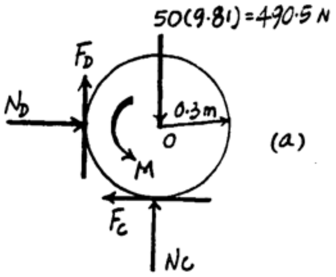


Equations of Equilibrium : From FBD (a),

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_D - F_C = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0 \qquad N_C + F_D - 490.5 = 0 \qquad [2]$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M - F_C(0.3) - F_D(0.3) = 0 \qquad [3]$$

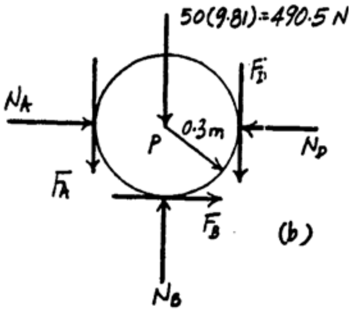


From FBD (b),

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A + F_B - N_D = 0 \qquad [4]$$

$$+\uparrow \Sigma F_y = 0 \qquad N_B - F_A - F_D - 490.5 = 0 \qquad [5]$$

$$\circlearrowleft + \Sigma M_P = 0; \qquad F_A(0.3) + F_B(0.3) - F_D(0.3) = 0 \qquad [6]$$



Friction : Assuming cylinder E slips at points C and D and cylinder F does not move, then $F_C = \mu_{s\,C} N_C = 0.5N_C$ and $F_D = \mu_{s\,D} N_D = 0.6N_D$. Substituting these values into Eqs. [1], [2] and [3] and solving, we have

$$N_C = 377.31 \text{ N} \qquad N_D = 188.65 \text{ N}$$

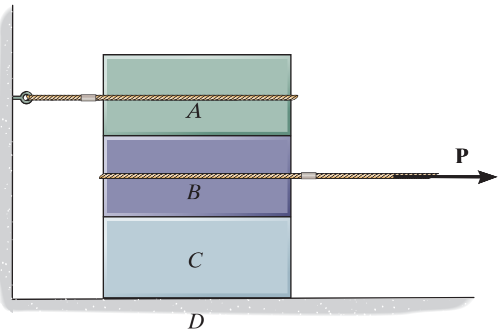
$$M = 90.55 \text{ N} \cdot \text{m} = 90.6 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

If cylinder F is on the verge of slipping at point A, then $F_A = \mu_{s\,A} N_A = 0.5N_A$. Substitute this value into Eqs. [4], [5] and [6] and solving, we have

$$N_A = 150.92 \text{ N} \qquad N_B = 679.15 \text{ N} \qquad F_B = 37.73 \text{ N}$$

Since $(F_B)_{\max} = \mu_{s\,B} N_B = 0.5(679.15) = 339.58 \text{ N} > F_B$, cylinder F does not move. Therefore the above assumption is correct.

8–62. Blocks A, B, and C have weights of 50 lb, 25 lb, and 15 lb, respectively. Determine the smallest horizontal force P that will cause impending motion. The coefficient of static friction between A and B is $\mu_s = 0.3$, between B and C, $\mu_s' = 0.4$, and between block C and the ground, $\mu_s'' = 0.35$.



Free - Body Diagram. Due to the constraint, block A will not move. Therefore, there are two possible cases of impending motion, namely (1) block B slips on top of block C or (2) blocks B and C slip on the ground and move as a single unit. For both cases, slipping occurs at the contact surface between blocks A and B. By considering the free - body diagram of block A shown in Fig. a, we obtain $N_A = 50$ lb. Thus, $F_A = \mu_s N_A = 0.3(50) = 15$ lb. We will assume that the first case of motion occurs. Thus, $F_B = \mu_s' N_B$.

Equations of Equilibrium. Referring to the free - body diagram of block B shown in Fig. b,

$+\uparrow \Sigma F_y = 0;$ $\quad N_B - 50 - 25 = 0 \quad N_B = 75$ lb

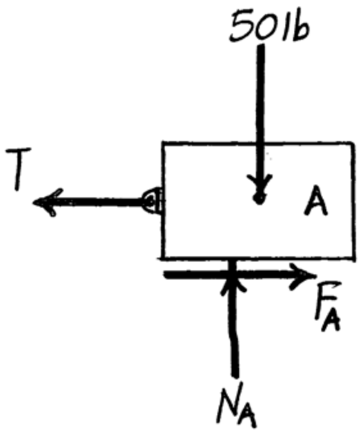
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad P - 15 - 0.4(75) = 0 \quad P = 45$ lb **Ans.**

Using this result and referring to the free - body diagram of blocks B and C shown in Fig. a,

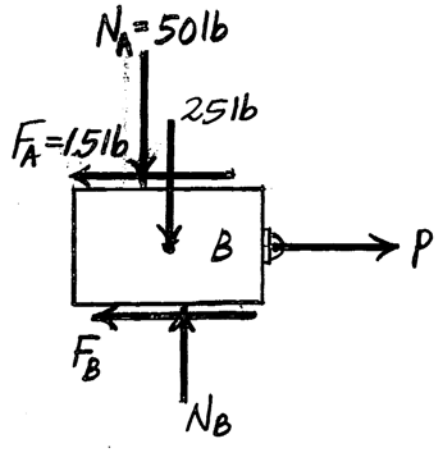
$+\uparrow \Sigma F_y = 0;$ $\quad N_C - 50 - 25 - 15 = 0 \quad N_C = 90$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 45 - 15 - F_C = 0 \quad F_C = 30$ lb

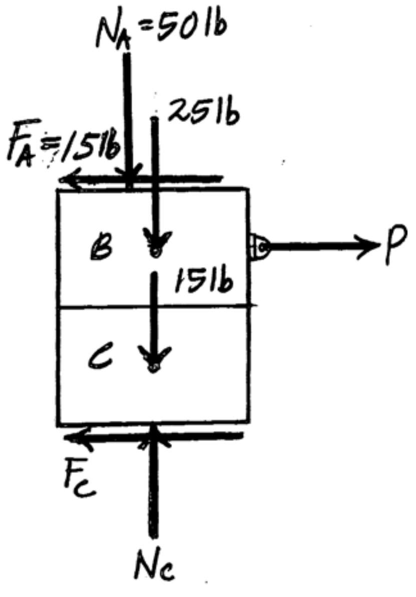
Since $F_C < (F_C)_{\max} = \mu_s'' N_C = 0.35(90) = 31.5$ lb, the system of the blocks B and C will not slip. Thus, the above assumption is correct.



(a)

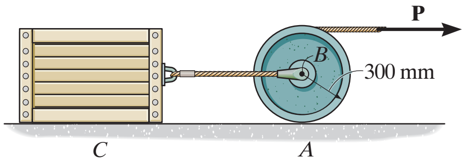


(b)



(c)

8–63. Determine the smallest force *P* that will cause impending motion. The crate and wheel have a mass of 50 kg and 25 kg, respectively. The coefficient of static friction between the crate and the ground is $\mu_s = 0.2$, and between the wheel and the ground $\mu_s' = 0.5$.



Free - Body Diagram. There are two possible motions, namely (1) the crate slips while the wheel rolls without slipping and (2) the wheel slips and rotates while the crate remains stationary. We will assume that the first mode of motion occurs. Thus, $F_C = \mu_s N_C = 0.2 N_C$.

Equations of Equilibrium. Referring to the free - body diagram of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0;\quad N_C - 50(9.81) = 0 \quad\quad N_C = 490.5\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad T - 0.2(490.5) = 0 \quad\quad T = 98.1\text{N}$

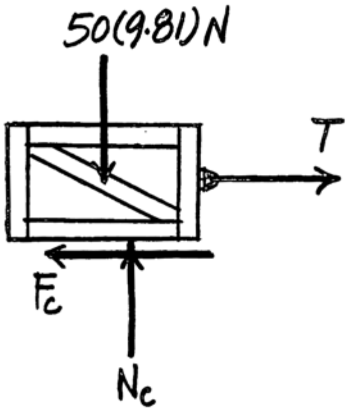
Using the result for *T* and referring to Fig. *b*,

$+\Sigma M_A = 0;\quad 98.1(0.3) - P(0.6) = 0 \quad\quad P = 49.05\text{N} = 49.0\text{N}$ **Ans.**

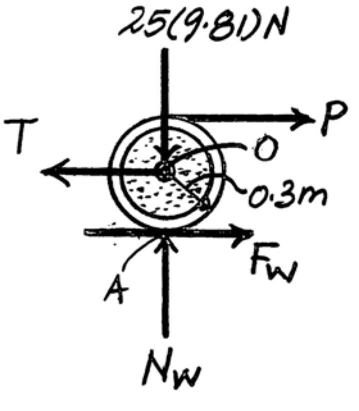
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad F_w + 49.05 - 98.1 = 0 \quad\quad F_w = 49.05\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad N_w - 25(9.81) = 0 \quad\quad N_w = 245.25$

Since $F_w < F_{\max} = \mu_s' N = 0.5(245.25) = 122.63\text{N}$, the wheel will not slip. Thus, the above assumption is correct.

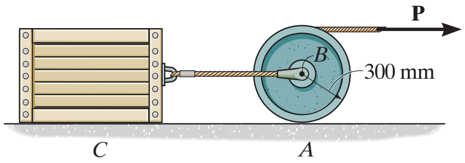


(a)



(b)

***8–64.** Determine the smallest force P that will cause impending motion. The crate and wheel have a mass of 50 kg and 25 kg, respectively. The coefficient of static friction between the crate and the ground is $\mu_s = 0.5$, and between the wheel and the ground $\mu_s' = 0.3$.



Free - Body Diagram. There are two possible motions, namely (1) the crate slips while the wheel rolls without slipping and (2) the whee slips and rotates while the crate remains stationary. We will assume that the second mode of motion occurs. Thus, $F_w = \mu_s' N_w = 0.3N_w$.

Equations of Equilibrium. Referring to the free - body diagram of the wheel shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0;$ $\quad N_w - 25(9.81) = 0 \quad N_w = 245.25\text{ N}$

$\circlearrowleft + \Sigma M_O = 0;$ $\quad 0.3(245.25)(0.3) - P(0.3) = 0 \quad P = 73.575\text{ N} = 73.6\text{ N}$ **Ans.**

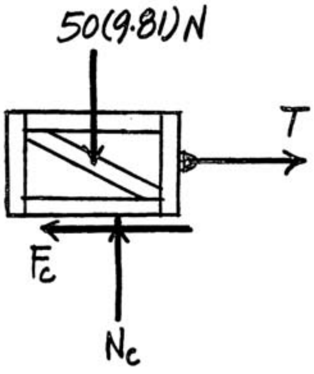
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 73.575 + 0.3(245.25) - T = 0 \quad T = 147.15\text{ N}$

Using the result for **T** and referring to the free - body diagram of the crate in Fig. *a*,

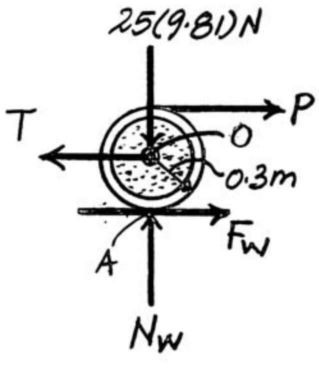
$+\uparrow \Sigma F_y = 0;$ $\quad N_C - 50(9.81) = 0 \quad N_C = 490.5\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 147.15 - F_C = 0 \quad F_C = 147.15\text{ N}$

Since $F_C < (F_E)_{\max} = \mu_s N_C = 0.5(490.5) = 245.25\text{ N}$, the crate will not slip. Thus, the above assumption is correct.

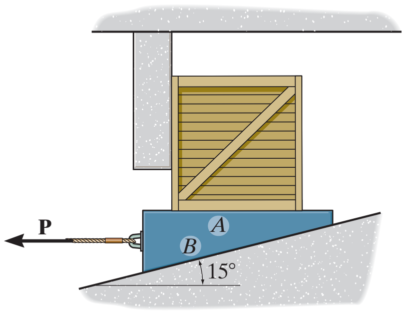


(a)



(b)

•8–65. Determine the smallest horizontal force P required to pull out wedge A. The crate has a weight of 300 lb and the coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$. Neglect the weight of the wedge.



Free - Body Diagram. Since the crate is on the verge of sliding down and the wedge is on the verge of sliding to the left, the frictional force $\mathbf{F}_B$ on the crate must act upward and forces $\mathbf{F}_C$ and $\mathbf{F}_D$ on the wedge must act to the right as indicated on the free - body diagrams as shown in Figs. a and b. Also, $F_B = \mu_s N_B = 0.3N_B$, $F_C = \mu_s N_C = 0.3N_C$, and $F_D = \mu_s N_D = 0.3N_D$.

Equations of Equilibrium. Referring to Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad N_B - 0.3N_C = 0$

$+\uparrow \Sigma F_y = 0;\qquad N_C + 0.3N_B - 300 = 0$

Solving,

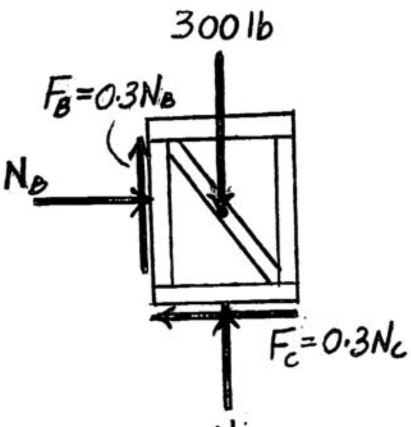
$$N_B = 82.57\text{ lb}\qquad N_C = 275.23\text{ lb}$$

Using the result of N_C and referring to Fig. b, we have

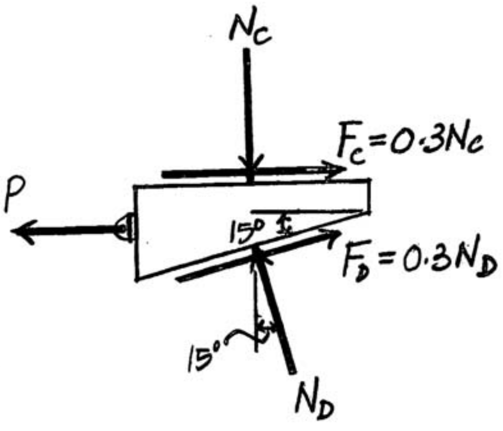
$+\uparrow \Sigma F_y = 0;\qquad N_D \cos 15° + 0.3N_D \sin 15° - 275.23 = 0;\qquad N_D = 263.74\text{ lb}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 0.3(275.23) + 0.3(263.74)\cos 15° - 263.74 \sin 15° - P = 0$

$P = 90.7\text{ lb}$ **Ans.**

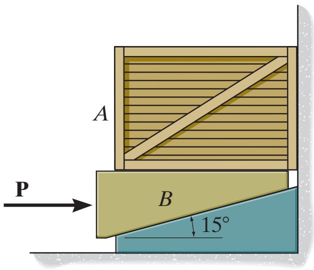


(a)



(b)

8–66. Determine the smallest horizontal force P required to lift the 200-kg crate. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$. Neglect the mass of the wedge.



Free - Body Diagram. Since the crate is on the verge of sliding up and the wedge is on the verge of sliding to the right, the frictional force $\mathbf{F}_A$ on the crate must act downward and forces $\mathbf{F}_B$ and $\mathbf{F}_C$ on the wedge must act to the left as indicated on the free - body diagrams as shown in Figs. a and b. Also, $F_A = \mu_s N_A = 0.3N_A$, $F_B = \mu_s N_B = 0.3N_B$, and $F_C = \mu_s N_C = 0.3N_C$.

Equations of Equilibrium. Referring to Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 0.3N_B - N_A = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N_B - 0.3N_A - 200(9.81) = 0$

Solving,

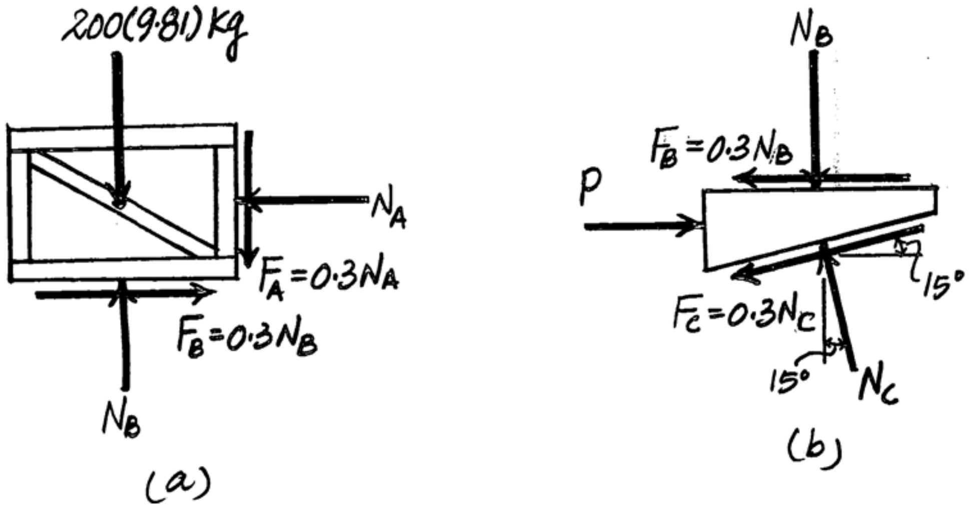
$$N_A = 646.81 \text{ N} \quad N_B = 2156.04 \text{ N}$$

Referring to Fig. b,

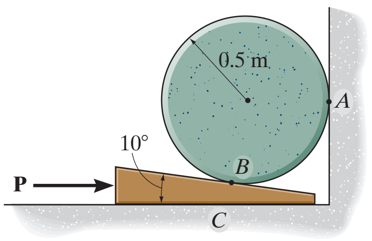
$+\uparrow \Sigma F_y = 0;$ $\quad N_C \cos 15° - 0.3N_C \sin 15° - 2156.04 = 0 \quad N_C = 2427.21 \text{ N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad P - 0.3(2156.04) - 2427.21 \sin 15° - 0.3(2427.21)\cos 15° = 0$

$P = 1978.37 \text{ N} = 1.98 \text{ N}$ **Ans.**



8–67. Determine the smallest horizontal force P required to lift the 100-kg cylinder. The coefficients of static friction at the contact points A and B are $(\mu_s)_A = 0.6$ and $(\mu_s)_B = 0.2$, respectively; and the coefficient of static friction between the wedge and the ground is $\mu_s = 0.3$.



Free - Body Diagram. There are two possible modes of motion for the cylinder, namely (1) the cylinder rolls about point A and slips at B and (2) the cylinder rolls about point B and slips at point A. We will assume that the first mode of motion occurs, thus $F_B = 0.2N_B$. This force acts to the right on the cylinder as indicated on the free - body diagram shown in Fig. *a*. The wedge is on the verge of moving to the right, Fig. *b*.

Equations of Equilibrium. Referring to Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 0.2N_B\cos 10° + N_B\sin 10° - N_A = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad N_B\cos 10° - 0.2N_B\sin 10° - F_A - 100(9.81) = 0$

$\circlearrowleft +\Sigma M_O = 0;$ $\quad 0.2N_B(0.5) - F_A(0.5) = 0$

Solving,

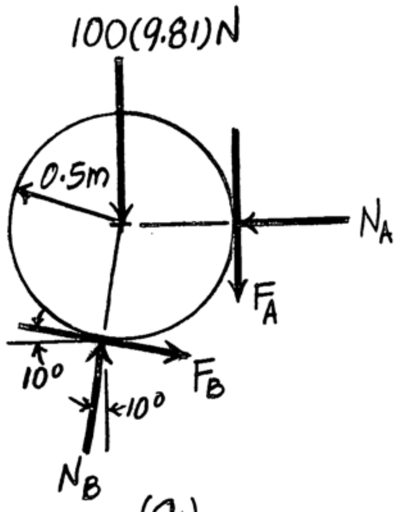
$$N_B = 1308\text{ N} \qquad N_A = 488.68\text{ N} \qquad F_A = 262\text{ N}$$

Since $F_A < (F_A)_{\max} = (\mu_s)_A N_A = 0.6(488.68) = 293$ N, the cylinder will not slip at A. Thus, the above assumption is correct. Thus, $F_C = 0.3N_C$ and $F_B = 262$ N. Referring to the free - body diagram of the wedge shown in Fig. *b*,

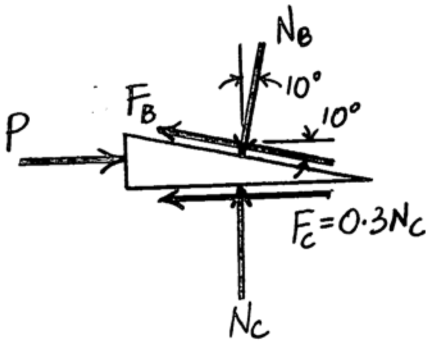
$+\uparrow\Sigma F_y = 0;$ $\quad N_C + 262\sin 10° - 1308\cos 10° = 0 \qquad N_C = 1243\text{ N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad P - 262\cos 10° - 1308\sin 10° - 0.3(1243) = 0$

$P = 863\text{N}$ **Ans.**

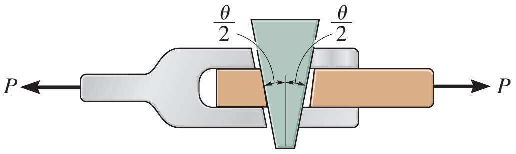


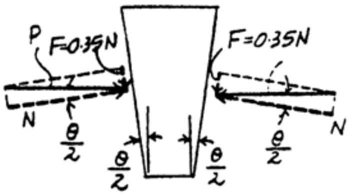
(a)



(b)

***8–68.** The wedge has a negligible weight and a coefficient of static friction $\mu_s = 0.35$ with all contacting surfaces. Determine the largest angle θ so that it is "self-locking." This requires no slipping for any magnitude of the force **P** applied to the joint.



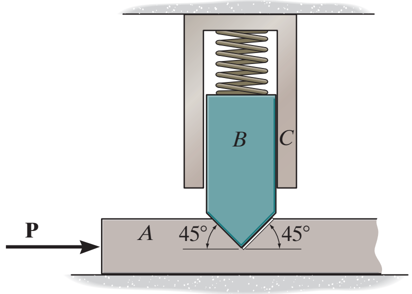


Friction : **When the wedge is on the verge of slipping, then $F = \mu N = 0.35N$. From the force diagram (P is the 'locking' force.),**

$$\tan\frac{\theta}{2} = \frac{0.35N}{N} = 0.35$$

$$\theta = 38.6° \qquad \textbf{Ans}$$

•8–69. Determine the smallest horizontal force P required to just move block A to the right if the spring force is 600 N and the coefficient of static friction at all contacting surfaces on A is $\mu_s = 0.3$. The sleeve at C is smooth. Neglect the mass of A and B.



Free - Body Diagram. Since block A is required to be on the verge of sliding to the right, the frictional forces $\mathbf{F}_A$ and $\mathbf{F}_C$ on block A must act to the left such that $F_A = \mu_s N_A = 0.3N_A$ and $F_C = \mu_s N_C = 0.3N_C$.

Equations of Equilibrium. Referring to the free-body diagram of block B shown in Fig. a,

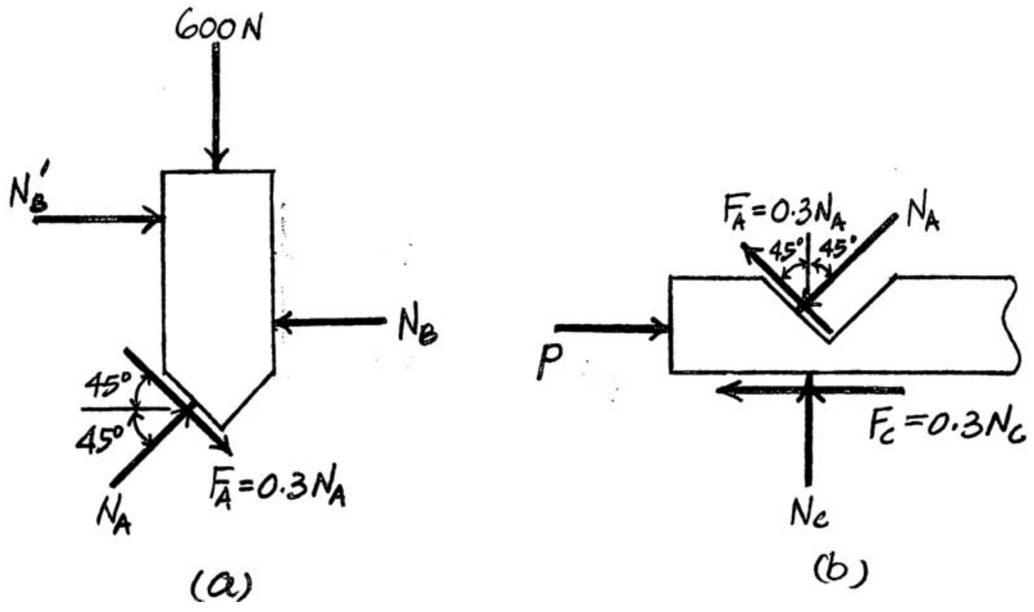
$+\uparrow \Sigma F_y = 0;$ $\quad N_A \sin 45° - 0.3N_A \sin 45° - 600 = 0;$ $\quad N_A = 1212.18\text{ N}$

Using the result of $\mathbf{N}_A$ and referring to the free - body diagram of block A shown in Fig. a,

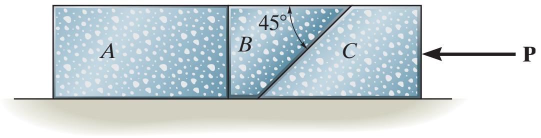
$+\uparrow \Sigma F_y = 0;$ $\quad N_C + 0.3(1212.18)\cos 45° - 1212.18 \cos 45° = 0;$ $\quad N_C = 600\text{ N}$

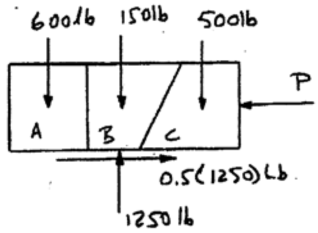
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad P - 0.3(1212.18)\sin 45° - 1212.18 \sin 45° - 0.3(600) = 0$

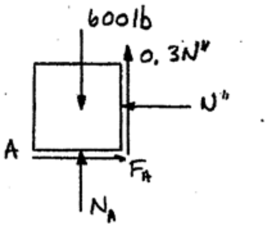
$P = 1294.29\text{ N} = 1.29\text{ kN}$ **Ans.**



8–70. The three stone blocks have weights of $W_A = 600$ lb, $W_B = 150$ lb, and $W_C = 500$ lb. Determine the smallest horizontal force P that must be applied to block C in order to move this block. The coefficient of static friction between the blocks is $\mu_s = 0.3$, and between the floor and each block $\mu_s' = 0.5$.







$\xrightarrow{+} \Sigma F_x = 0; \quad -P + 0.5\,(1250) = 0$

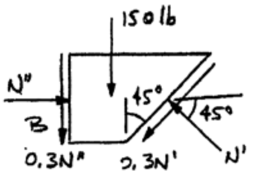
$P = 625$ lb

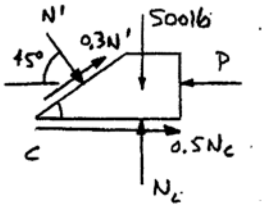
Assume block B slips up, block A does not move.

Block A:

$\xrightarrow{+} \Sigma F_x = 0; \quad F_A - N'' = 0$

$+\uparrow \Sigma F_y = 0; \quad N_A - 600 + 0.3\,N'' = 0$





Block B:

$\xrightarrow{+} \Sigma F_x = 0; \quad N'' - N' \cos 45° - 0.3\,N' \sin 45° = 0$

$+\uparrow \Sigma F_y = 0; \quad N' \sin 45° - 0.3\,N' \cos 45° - 150 - 0.3\,N'' = 0$

Block C:

$\xrightarrow{+} \Sigma F_x = 0; \quad 0.3\,N' \cos 45° + N' \cos 45° + 0.5\,N_C - P = 0$

$+\uparrow \Sigma F_y = 0; \quad N_C - N' \sin 45° + 0.3\,N' \sin 45° - 500 = 0$

Solving,

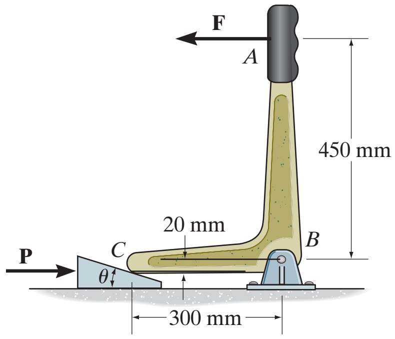
$N'' = 629.0$ lb, $\quad N' = 684.3$ lb, $\quad N_C = 838.7$ lb, $\quad P = 1048$ lb,

$N_A = 411.3$ lb

$F_A = 629.0 \text{ lb} > 0.5\,(411.3) = 205.6 \text{ lb}$ No good

All blocks slip at the same time; $\quad P = 625$ lb **Ans**

8–71. Determine the smallest horizontal force P required to move the wedge to the right. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$. Set $\theta = 15°$ and $F = 400$ N. Neglect the weight of the wedge.



Free - Body Diagram. Since the wedge is required to be on the verge of sliding to the right, the frictional forces $\mathbf{F}_C$ and $\mathbf{F}_D$ on the wedge must act to the left such that $F_C = \mu_s N_C = 0.3N_C$ and $F_D = \mu_s N_D = 0.3N_D$.

Equations of Equilibrium. Referring to the free - body diagram of the crank shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \quad 400(0.45) + 0.3N_C \cos 15°(0.02) + 0.3N_C \sin 15°(0.3) + N_C \cos 15°(0.3) - N_C \sin 15°(0.02) = 0$$

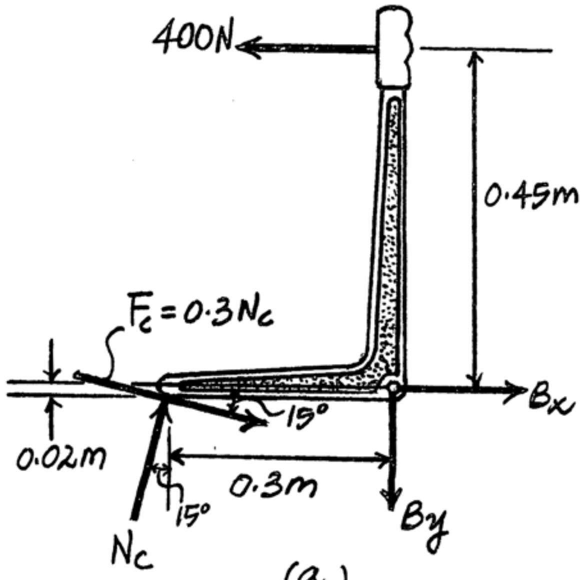
$$N_C = 704.47 \text{ N}$$

Referring to the free - body diagram of the wedge shown in Fig. *b*,

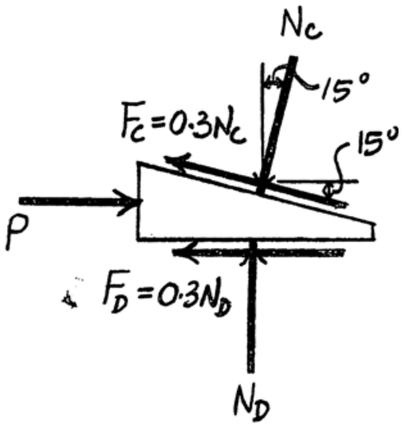
$$+\uparrow \Sigma F_y = 0; \quad N_D + 0.3(704.47)\sin 15° - 704.47\cos 15° = 0 \qquad N_D = 625.76 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P - 0.3(704.47)\cos 15° - 0.3(625.76) - 704.47\sin 15° = 0$$

$$P = 574 \text{ N}$$ **Ans.**

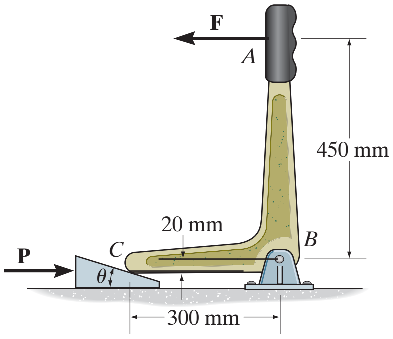


(a)



(b)

***8–72.** If the horizontal force **P** is removed, determine the largest angle θ that will cause the wedge to be self-locking regardless of the magnitude of force **F** applied to the handle. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$.



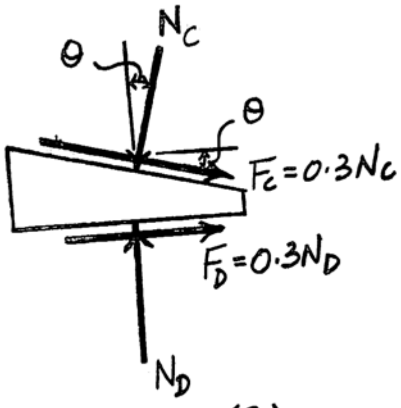
Free - Body Diagram. Since the wedge is required to be on the verge of sliding to the left (just self locking), the frictional forces $\mathbf{F}_C$ and $\mathbf{F}_D$ must act to the right such that $F_C = \mu_s N_C = 0.3N_C$ and $F_D = \mu_s N_D = 0.3N_D$ as indicated on the free - body diagram of the wedge shown in Fig. *a*.

Equations of Equilibrium. Referring to Fig. *a*,

$+\uparrow \Sigma F_y = 0;\qquad N_D - 0.3N_C \sin\theta - N_C \cos\theta = 0 \qquad N_D = N_C(0.3\sin\theta + \cos\theta)$

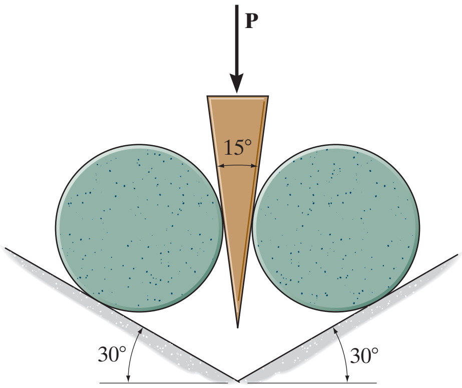
$\xrightarrow{+} \Sigma F_x = 0;\qquad 0.3N_C \cos\theta + 0.3[N_C(0.3\sin\theta + \cos\theta)] - N_C \sin\theta = 0$

$\theta = 33.4°$ **Ans.**



(a)

•8–73. Determine the smallest vertical force P required to hold the wedge between the two identical cylinders, each having a weight of W. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.1$.



Free - Body Diagram. Since the wedge is required to be on the verge of moving upward, the frictional force $\mathbf{F}_A$ on the wedge must act downward. Here, there are two possible modes of motion for the cylinder, namely (1) the cylinder rolls about B and slips at A or (2) the cylinder rolls about A and slips at B. We will assume that the first mode of motion occurs. Thus, $F_A = \mu_s N_A = 0.1 N_A$.

Equations of Equilibrium. Referring to the free - body diagram of the cylinder shown in Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_A \cos 7.5° + 0.1 N_A \sin 7.5° - N_B \sin 30° + F_B \cos 30° = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N_B \cos 30° + F_B \sin 30° + 0.1 N_A \cos 7.5° - N_A \sin 7.5° - W = 0$

$\circlearrowleft + \Sigma M_O = 0;$ $\quad F_B(r) - 0.1 N_A(r) = 0$

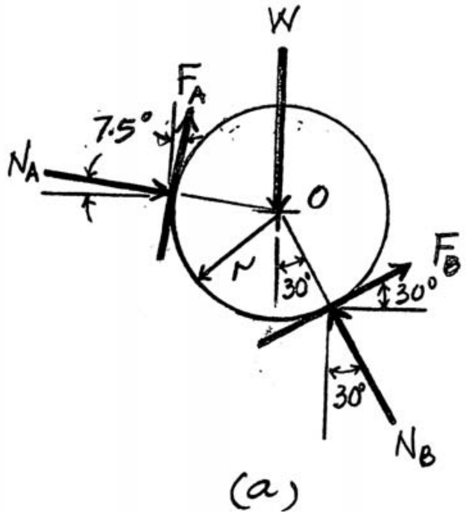
Solving,

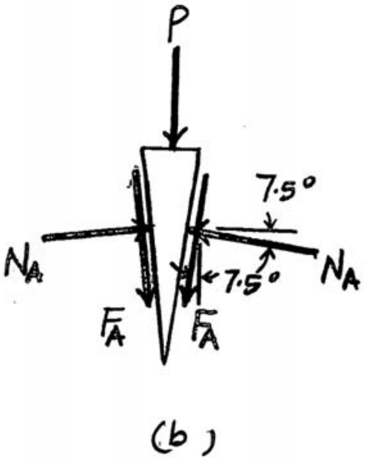
$$N_A = 0.5240W \quad N_B = 1.1435W \quad F_B = 0.05240W$$

Since $F_B < (F_B)_{\max} = \mu_s N_B = 0.1(1.1435W) = 0.11435W$, slipping will not occur at B. Thus, the above assumption is correct. Using the result of N_A, we find that $F_A = 0.1(0.5240W) = 0.05240W$. Referring to the free - body diagram of the wedge shown in Fig. b,

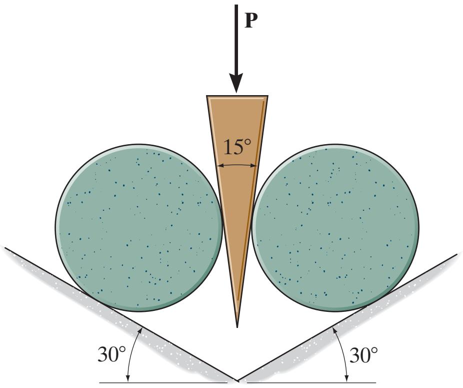
$+\uparrow \Sigma F_y = 0;$ $\quad 2(0.5240W)\sin 7.5° - 2(0.05240W \cos 7.5°) - P = 0$

$P = 0.0329W$ **Ans.**





8–74. Determine the smallest vertical force P required to push the wedge between the two identical cylinders, each having a weight of W. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$.



Free - Body Diagram. Since the wedge is required to be on the verge of moving downward, the frictional force $\mathbf{F}_A$ on the wedge must act upward. Here, there are two possible modes of motion for the cylinder, namely (1) the cylinder rolls about B and slips at A or (2) the cylinder rolls about A and slips at B. We will assume that the first mode of motion occurs. Thus, the magnitude of $\mathbf{F}_A$ can be computed using the friction formula; i.e., $F_A = \mu_s N_A = 0.3N_A$.

Equations of Equilibrium. Referring to the free - body diagram of the cylinder shown in Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N_A \cos 7.5° - 0.3N_A \sin 7.5° - F_B \cos 30° - N_B \sin 30° = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B \cos 30° - F_B \sin 30° - 0.3N_A \cos 7.5° - N_A \sin 7.5° - W = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 0.3N_A(r) - F_B(r) = 0$$

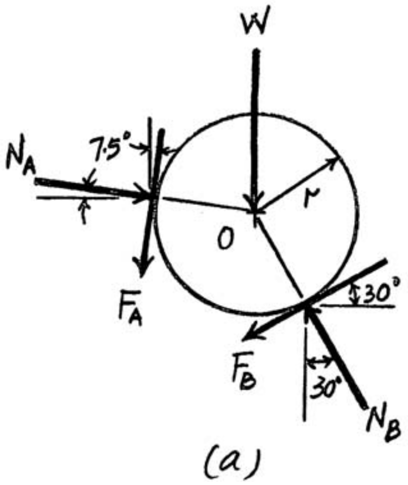
Solving,

$$N_A = 1.609W \quad N_B = 2.229W \quad F_B = 0.4827W$$

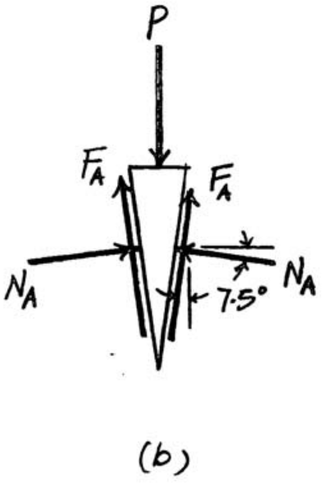
Since $F_B < (F_B)_{\max} = \mu_s N_B = 0.3(2.229W) = 0.669W$, slipping will not occur at B. Thus, the above assumption is correct. Using the result of N_A, we find that $F_A = 0.3(1.609W) = 0.4827W$. Referring to the free - body diagram of the wedge shown in Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad 2(1.609W \sin 7.5°) + 2(0.4827W \cos 7.5°) - P = 0$$

$$P = 1.38W \qquad \textbf{Ans.}$$

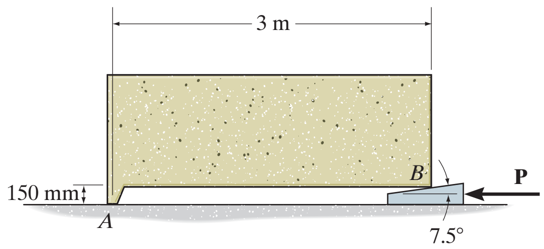


(a)



(b)

8–75. If the uniform concrete block has a mass of 500 kg, determine the smallest horizontal force P needed to move the wedge to the left. The coefficient of static friction between the wedge and the concrete and the wedge and the floor is $\mu_s = 0.3$. The coefficient of static friction between the concrete and floor is $\mu_s' = 0.5$.



Free - Body Diagram. Since the wedge is required to be on the verge of sliding to the left, the frictional forces $\mathbf{F}_B$ and $\mathbf{F}_C$ on the wedge must act to the right such that $F_B = \mu_s N_B = 0.3 N_B$ and $F_C = \mu_s N_C = 0.3 N_C$.

Equations of Equilibrium. Referring to the free - body diagram of the concrete block shown in Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 0.3N_B \cos 7.5°(0.15) - 0.3N_B \sin 7.5°(3) + N_B \cos 7.5°(3) + N_B \sin 7.5°(0.15) - 500(9.81)(1.5) = 0$

$N_B = 2518.78 \text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad F_A - 0.3(2518.78)\cos 7.5° - 2518.78 \sin 7.5° = 0$ $\quad F_A = 1077.94 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_A + 2518.78 \cos 7.5° - 0.3(2518.78)\sin 7.5° - 500(9.81) = 0$

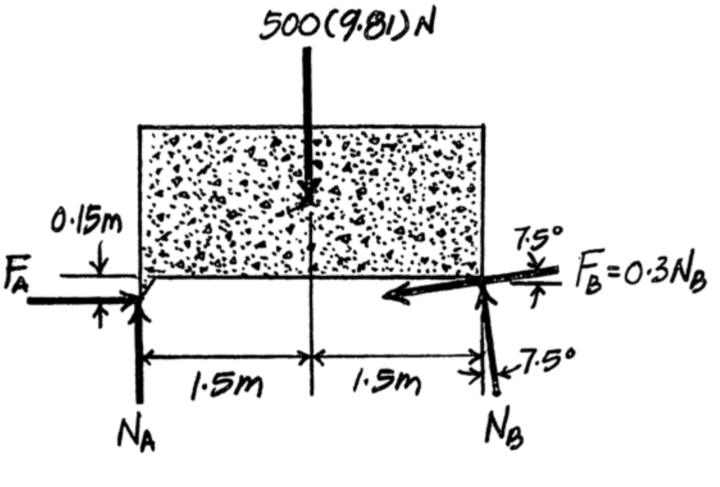
$N_A = 2506.40 \text{ N}$

Since $F_A < (F_A)_{\max} = \mu'_s N_A = 0.5(2506.40) = 1253.20 \text{ N}$, the concrete block will not slip at A. Using the result of $\mathbf{N}_B$ and referring to the free - body diagram of the wedge shown in Fig. b,

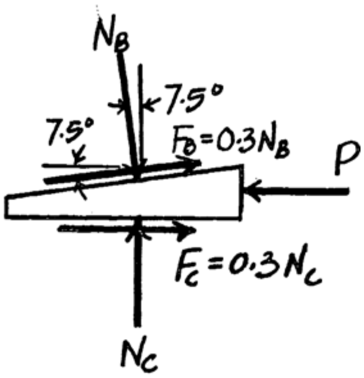
$+\uparrow \Sigma F_y = 0;$ $\quad N_C + 0.3(2518.78)\sin 7.5° - 2518.78 \cos 7.5° = 0$ $\quad N_C = 2398.60 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 0.3(2518.78)\cos 7.5° + 2518.78 \sin 7.5° + 0.3(2398.60) - P = 0$

$P = 1797.52 \text{ N} = 1.80 \text{ kN}$ **Ans.**

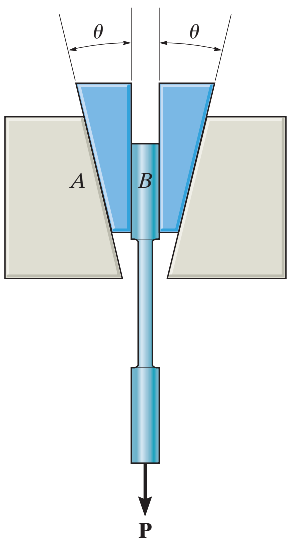


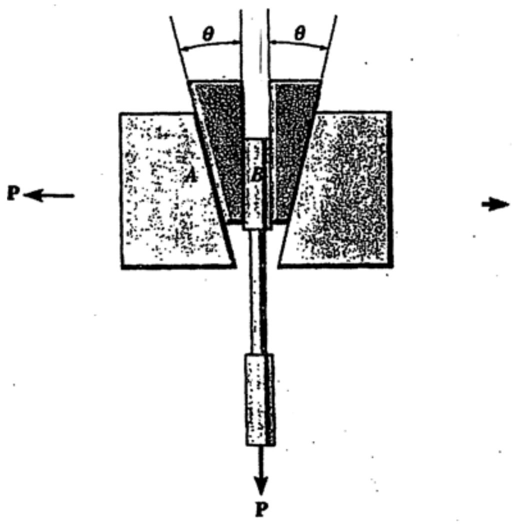
(a)

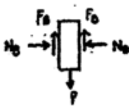


(b)

***8–76.** The wedge blocks are used to hold the specimen in a tension testing machine. Determine the largest design angle θ of the wedges so that the specimen will not slip regardless of the applied load. The coefficients of static friction are $\mu_A = 0.1$ at A and $\mu_B = 0.6$ at B. Neglect the weight of the blocks.







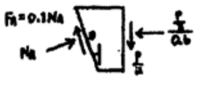
Specimen:

$+\uparrow\Sigma F_y = 0;\qquad F_B = \frac{P}{2}$

Wedge:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad N_A\cos\theta - 0.1N_A\sin\theta - \frac{\frac{P}{2}}{0.6} = 0$

$+\uparrow\Sigma F_y = 0;\qquad 0.1N_A\cos\theta + N_A\sin\theta - \frac{P}{2} = 0$



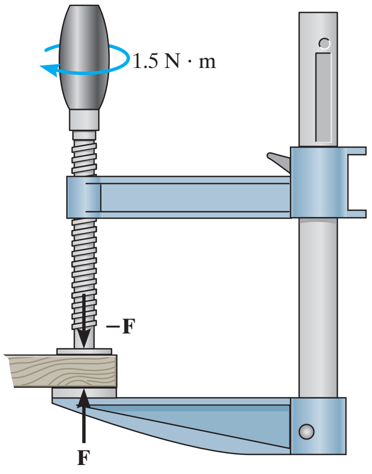
$P = 2N_A(0.1\cos\theta + \sin\theta)$

$0.6N_A(\cos\theta - 0.1\sin\theta) - N_A(0.1\cos\theta + \sin\theta) = 0$

$0.5\cos\theta - 1.06\sin\theta = 0$

$\theta = \tan^{-1}(\frac{0.5}{1.06}) = 25.3°$ **Ans**

•8–77. The square threaded screw of the clamp has a mean diameter of 14 mm and a lead of 6 mm. If $\mu_s = 0.2$ for the threads, and the torque applied to the handle is 1.5 N · m, determine the compressive force F on the block.



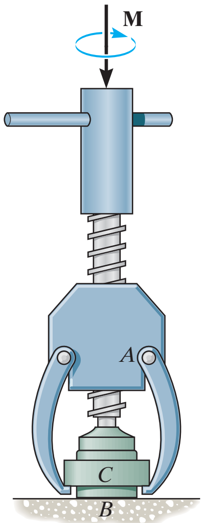
Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{6}{2\pi(7)}\right] = 7.768°$, $W = F$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310°$. Applying Eq. 8–3, we have

$$M = Wr\tan(\theta + \phi)$$
$$1.5 = F(0.007)\tan(7.768° + 11.310°)$$

$$F = 620\text{ N} \qquad \textbf{Ans}$$

Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if the moment is removed.

8–78. The device is used to pull the battery cable terminal *C* from the post of a battery. If the required pulling force is 85 lb, determine the torque *M* that must be applied to the handle on the screw to tighten it. The screw has square threads, a mean diameter of 0.2 in., a lead of 0.08 in., and the coefficient of static friction is $\mu_s = 0.5$.

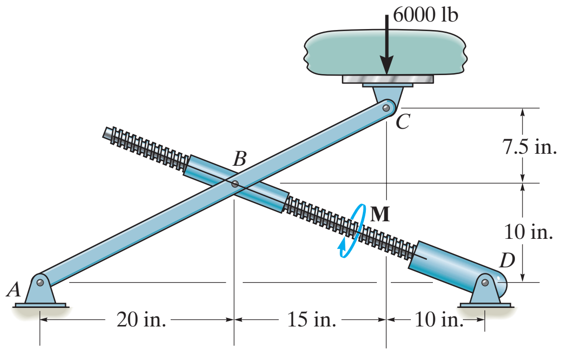


Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{0.08}{2\pi(0.1)}\right] = 7.256°$, $W = 85$ lb and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Applying Eq. 8–3, we have

$$\begin{aligned} M &= Wr\tan(\theta+\phi) \\ &= 85(0.1)\tan(7.256° + 26.565°) \\ &= 5.69 \text{ lb}\cdot\text{in} \qquad \textbf{Ans} \end{aligned}$$

Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if the moment is removed.

8–79. The jacking mechanism consists of a link that has a square-threaded screw with a mean diameter of 0.5 in. and a lead of 0.20 in., and the coefficient of static friction is $\mu_s = 0.4$. Determine the torque *M* that should be applied to the screw to start lifting the 6000-lb load acting at the end of member *ABC*.



$$\alpha = \tan^{-1}\left(\frac{10}{25}\right) = 21.80°$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -6000\,(35) + F_{BD}\cos 21.80°\,(10) + F_{BD}\sin 21.80°\,(20) = 0$$

$$F_{BD} = 12\,565 \text{ lb}$$

8-79

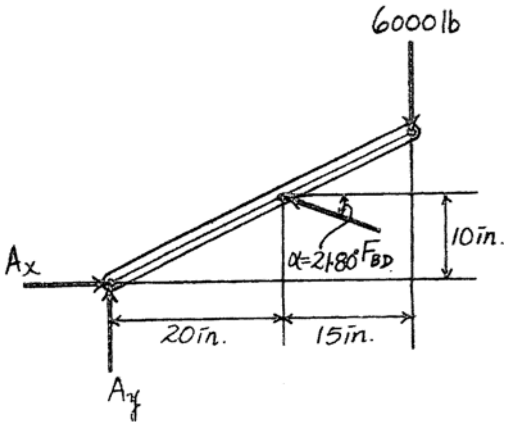
$$\phi_s = \tan^{-1}(0.4) = 21.80°$$

$$\theta = \tan^{-1}\left(\frac{0.2}{2\pi\,(0.25)}\right) = 7.256°$$

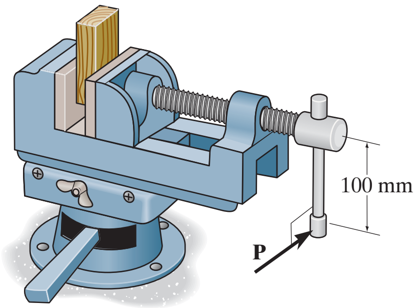
$$M = Wr\tan(\theta + \phi)$$

$$M = 12\,565\,(0.25)\tan(7.256° + 21.80°)$$

$$M = 1745 \text{ lb}\cdot\text{in.} = 145 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$



***8–80.** Determine the magnitude of the horizontal force **P** that must be applied to the handle of the bench vise in order to produce a clamping force of 600 N on the block. The single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction is $\mu_s = 0.25$.



Here, $M = P(0.1)$

$$\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$

$W = 600\,\text{N}$

Thus

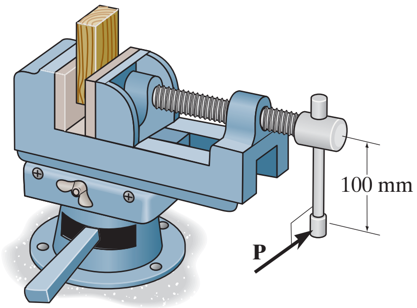
$$M = Wr\tan(\phi_s + \theta)$$

$$P(0.1) = 600(0.0125)\tan(14.036° + 5.455°)$$

$$P = 26.5\,\text{N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self - locking.

•8–81. Determine the clamping force exerted on the block if a force of $P = 30$ N is applied to the lever of the bench vise. The single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction is $\mu_s = 0.25$.



Here, $M = 30(0.1) = 3\,\text{N}\cdot\text{m}$

$$\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$

$W = F$

Thus

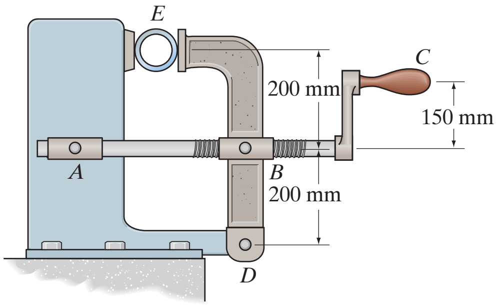
$$M = Wr\tan(\phi_s + \theta)$$

$$3 = F(0.0125)\tan(14.036° + 5.455°)$$

$$F = 678\,\text{N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self - locking.

8–82. Determine the required horizontal force that must be applied perpendicular to the handle in order to develop a 900-N clamping force on the pipe. The single square-threaded screw has a mean diameter of 25 mm and a lead of 5 mm. The coefficient of static friction is $\mu_s = 0.4$. *Note:* The screw is a two-force member since it is contained within pinned collars at *A* and *B*.



Referring to the free-body diagram of member *ED* shown in Fig. *a*,

$\circlearrowleft +\Sigma M_D = 0;\qquad F_{AB}(0.2) - 900(0.4) = 0\qquad F_{AB} = 1800\text{ N}$

Here, $\theta = \tan^{-1}\left(\dfrac{L}{2\pi r}\right) = \tan^{-1}\left[\dfrac{5}{2\pi(12.5)}\right] = 3.643°$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$

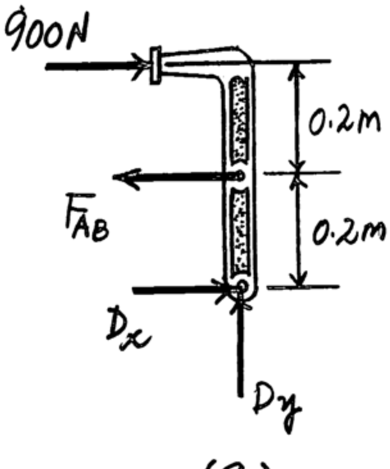
$M = F(0.15)$; and $W = F_{AB} = 1800\text{ N}$

$$M = Wr\tan(\phi_s + \theta)$$

$$F(0.15) = 1800(0.0125)\tan(21.801° + 3.643°)$$

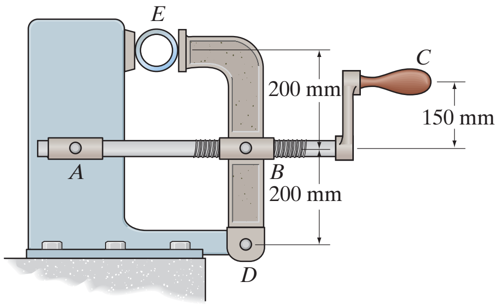
$$F = 71.4\text{ N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self-locking.



(a)

8–83. If the clamping force on the pipe is 900 N, determine the horizontal force that must be applied perpendicular to the handle in order to loosen the screw. The single square-threaded screw has a mean diameter of 25 mm and a lead of 5 mm. The coefficient of static friction is $\mu_s = 0.4$. *Note:* The screw is a two-force member since it is contained within pinned collars at *A* and *B*.



Referring to the free-body diagram of member *ED* shown in Fig. *a*,

$+\Sigma M_D = 0;$ $F_{AB}(0.2) - 900(0.4) = 0$ $F_{AB} = 1800\,\text{N}$

Here, $\theta = \tan^{-1}\left(\dfrac{L}{2\pi r}\right) = \tan^{-1}\left[\dfrac{5}{2\pi(12.5)}\right] = 3.643°$

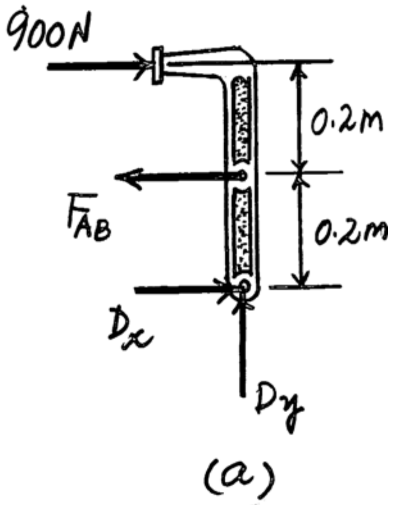
$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$

$M = F(0.15)$; and $W = F_{AB} = 1800\,\text{N}$

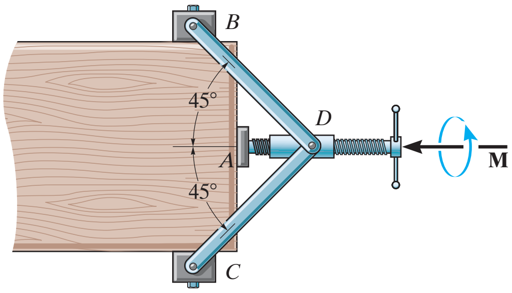
$$M = Wr\tan(\phi_s - \theta)$$

$$F(0.15) = 1800(0.0125)\tan(21.801° - 3.643°)$$

$$F = 49.2\,\text{N}$$ **Ans.**

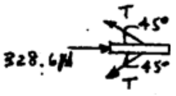


***8–84.** The clamp provides pressure from several directions on the edges of the board. If the square-threaded screw has a lead of 3 mm, mean radius of 10 mm, and the coefficient of static friction is $\mu_s = 0.4$, determine the horizontal force developed on the board at A and the vertical forces developed at B and C if a torque of $M = 1.5\ \text{N}\cdot\text{m}$ is applied to the handle to tighten it further. The blocks at B and C are pin connected to the board.



$$\phi_s = \tan^{-1}(0.4) = 21.801^\circ$$

$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734^\circ$$



$$M = W(r)\tan(\phi_s + \theta_p)$$

$$1.5 = A_x(0.01)\tan(21.801^\circ + 2.734^\circ)$$

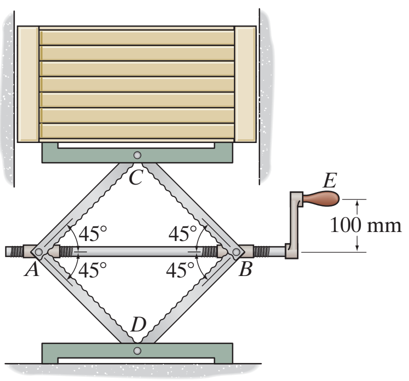
$A_x = 328.6\ \text{N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad 328.6 - 2T\cos 45^\circ = 0$

$T = 232.36\ \text{N}$

$B_y = C_y = 232.36\sin 45^\circ = 164\ \text{N}$ **Ans**

•8–85. If the jack supports the 200-kg crate, determine the horizontal force that must be applied perpendicular to the handle at *E* to lower the crate. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction is $\mu_s = 0.25$.



The force in rod *AB* can be obtained by first analyzing the equilibrium of joint *C* followed by joint *B*. Referring to the free-body diagram of joint *C* shown in Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{CA}\sin 45° - F_{CB}\sin 45° = 0\qquad F_{CA} = F_{CB} = F$

$+\uparrow \Sigma F_y = 0;\quad 2F\cos 45° - 200(9.81) = 0\qquad F = 1387.34\text{ N}$

Using the result of *F* and referring to the free-body diagram of joint *B* shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0;\quad F_{BD}\sin 45° - 1387.34\sin 45° = 0\qquad F_{BD} = 1387.34\text{ N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 1387.34\cos 45° + 1387.34\cos 45° - F_{AB} = 0\qquad F_{AB} = 1962\text{ N}$

Here, $\theta = \tan^{-1}\left(\dfrac{L}{2\pi r}\right) = \tan^{-1}\left[\dfrac{7.5}{2\pi(12.5)}\right] = 5.455°$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$

$M = F(0.1)$ and $W = F_{AB} = 1962\text{ N}$

Since **M** must overcome the friction of two screws,

$$M = 2\left[Wr\tan(\phi_s - \theta)\right]$$

$$F(0.1) = 2\left[1962(0.0125)\tan(14.036° - 5.455°)\right]$$

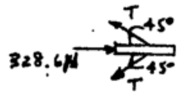
$$F = 74.0\text{ N}$$

Ans.

Note: Since $\phi_s > \theta$, the screws are self - locking.

$\phi_s = \tan^{-1}(0.4) = 21.801°$

$\theta_p = \tan^{-1}\left[\dfrac{3}{2\pi(10)}\right] = 2.734°$



$M = W(r)\tan(\phi_s + \theta_p)$

$1.5 = A_x(0.01)\tan(21.801° + 2.734°)$

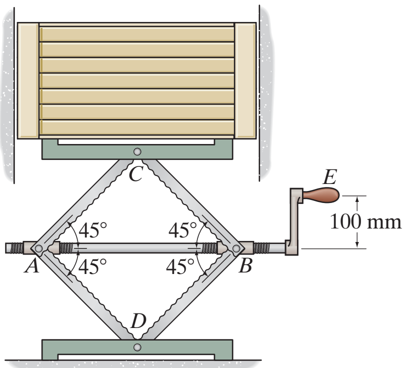
$A_x = 328.6\text{ N}$ **Ans**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 328.6 - 2T\cos 45° = 0$

$T = 232.36\text{ N}$

$B_y = C_y = 232.36\sin 45° = 164\text{ N}$ **Ans**

8–86. If the jack is required to lift the 200-kg crate, determine the horizontal force that must be applied perpendicular to the handle at E. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction is $\mu_s = 0.25$.



The force in rod AB can be obtained by first analyzing the equilibrium of joint C followed by joint B. Referring to the free-body diagram of joint C shown in Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $F_{CA}\sin 45° - F_{CB}\sin 45° = 0$ $F_{CA} = F_{CB} = F$

$+\uparrow \Sigma F_y = 0;$ $2F\cos 45° - 200(9.81) = 0$ $F = 1387.34\text{ N}$

Using the result of F and referring to the free-body diagram of joint B shown in Fig. b,

$+\uparrow \Sigma F_y = 0;$ $F_{BD}\sin 45° - 1387.34\sin 45° = 0$ $F_{BD} = 1387.34\text{ N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $1387.34\cos 45° + 1387.34\cos 45° - F_{AB} = 0$ $F_{AB} = 1962\text{ N}$

Here, $\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455°$

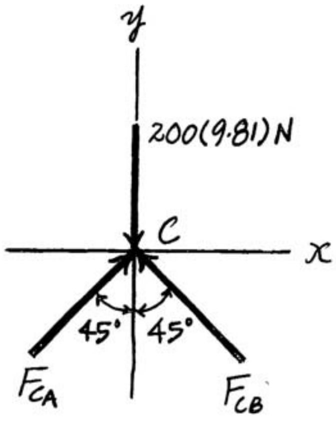
$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$

$M = F(0.1)$ and $W = F_{AB} = 1962\text{ N}$

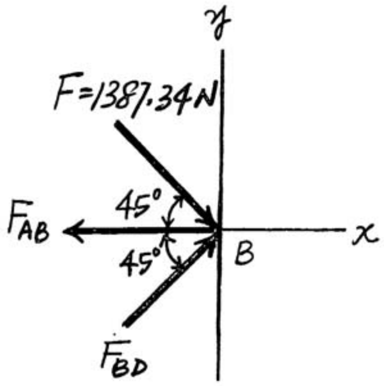
Since $\mathbf{M}$ must overcome the friction of two screws,

$$M = 2[Wr\tan(\phi_s + \theta)]$$
$$F(0.1) = 2[1962(0.0125)\tan(14.036° + 5.455°)]$$
$$F = 174\text{ N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screws are self-locking.

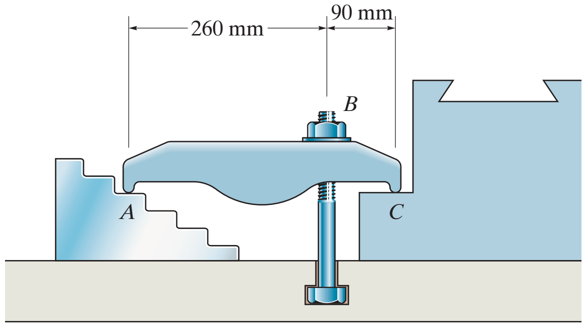


(a)



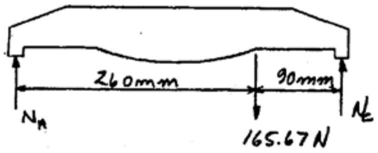
(b)

8–87. The machine part is held in place using the double-end clamp. The bolt at B has square threads with a mean radius of 4 mm and a lead of 2 mm, and the coefficient of static friction with the nut is $\mu_s = 0.5$. If a torque of $M = 0.4\ \text{N}\cdot\text{m}$ is applied to the nut to tighten it, determine the normal force of the clamp at the smooth contacts A and C.



$\phi = \tan^{-1}(0.5) = 26.565°$

$\theta = \tan^{-1}\left(\dfrac{2}{2\pi(4)}\right) = 4.550°$



$M = Wr\tan(\theta + \phi)$

$0.4 = W(0.004)\tan(4.550° + 26.565°)$

$W = 165.67\ \text{N}$

$\circlearrowleft + \Sigma M_A = 0;\quad N_C(350) - 165.67(260) = 0$

$N_C = 123.1 = 123\ \text{N}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad N_A - 165.67 + 123.1 = 0$

$N_A = 42.6\ \text{N}$ **Ans**

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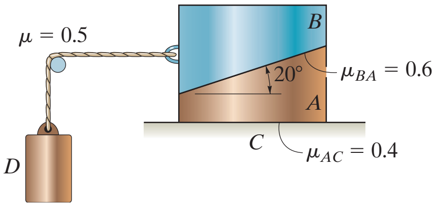
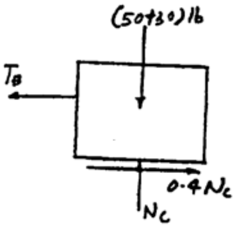
*__8–88.__ Blocks A and B weigh 50 lb and 30 lb, respectively. Using the coefficients of static friction indicated, determine the greatest weight of block D without causing motion.



For block A and B : Assuming block B does not slip



$+\uparrow \Sigma F_y = 0;\quad N_C - (50+30) = 0 \qquad N_C = 80\ \text{lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.4(80) - T_B = 0 \qquad T_B = 32\ \text{lb}$

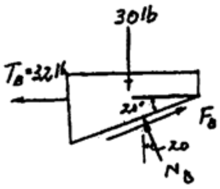
For block B :

$+\uparrow \Sigma F_y = 0;\quad N_B\cos 20° + F_B\sin 20° - 30 = 0 \qquad [1]$

$\xrightarrow{+} \Sigma F_x = 0;\quad F_B\cos 20° - N_B\sin 20° - 32 = 0 \qquad [2]$

Solving Eqs.[1] and [2] yields :

$F_B = 40.32\ \text{lb} \qquad N_B = 17.25\ \text{lb}$

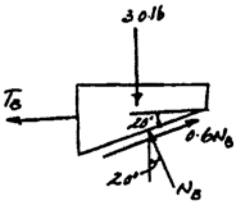


Since $F_B = 40.32\ \text{lb} > \mu N_B = 0.6(17.25) = 10.35\ \text{lb}$, slipping does occur between A and B. Therefore, the assumption is no good .

Since slipping occurs, $F_B = 0.6\,N_B$.

$+\uparrow \Sigma F_y = 0;\quad N_B\cos 20° + 0.6N_B\sin 20° - 30 = 0 \qquad N_B = 26.20\ \text{lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.6(26.20)\cos 20° - 26.20\sin 20° - T_B = 0 \qquad T_B = 5.812\ \text{lb}$

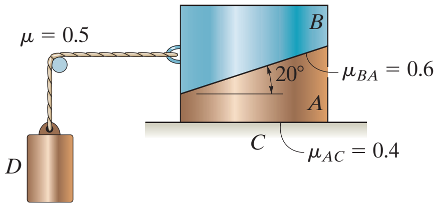


$T_2 = T_1 e^{\mu\beta}$ Where $T_2 = W_D,\ T_1 = T_B = 5.812\ \text{lb},\ \beta = 0.5\pi$ rad

$W_D = 5.812e^{0.5(0.5\pi)}$

$= 12.7\ \text{lb}$ **Ans**

•8–89. Blocks *A* and *B* weigh 75 lb each, and *D* weighs 30 lb. Using the coefficients of static friction indicated, determine the frictional force between blocks *A* and *B* and between block *A* and the floor *C*.



For the rope, $T_2 = T_1 e^{\mu\beta}$, where $T_2 = 30$ lb, $T_1 = T_B$, and $\beta = 0.5\pi$ rad.

$30 = T_B e^{0.5(0.5\pi)}$

$T_B = 13.678$ lb

$F_C = 13.7$ lb **Ans**

For block *B*:

$+\uparrow \Sigma F_y = 0;\ N_B \cos 20^\circ + F_B \sin 20^\circ - 75 = 0$ [1]

$\overset{+}{\rightarrow} \Sigma F_x = 0;\ F_B \cos 20^\circ - N_B \sin 20^\circ - 13.678 = 0$ [2]

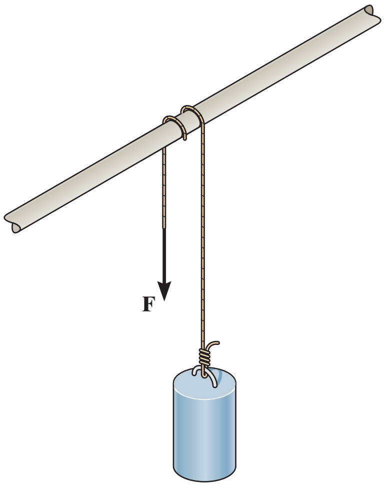
Solving Eqs. [1] and [2] yields:

$N_B = 65.8$ lb

$F_B = 38.5$ lb **Ans**

Since $F_B = 38.5 \text{ lb} < \mu N_B = 0.6(65.8) = 39.5$ lb, slipping between *A* and *B* does not occur.

8–90. A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the smallest vertical force *F* needed to support the load if the cord passes (a) once over the pipe, $\beta = 180^\circ$, and (b) two times over the pipe, $\beta = 540^\circ$. Take $\mu_s = 0.2$.



Frictional Force on Flat Belt : Here, $T_1 = F$ and $T_2 = 250(9.81) = 2452.5$ N. Applying Eq. 8 – 6, we have

a) If $\beta = 180^\circ = \pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$2452.5 = Fe^{0.2\pi}$$

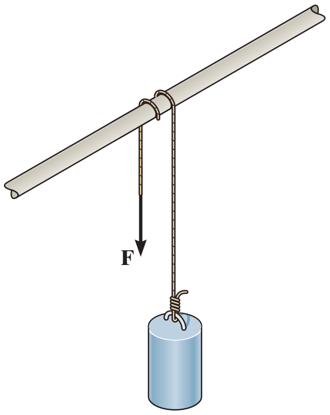
$F = 1308.38 \text{ N} = 1.31 \text{ kN}$ **Ans**

b) If $\beta = 540^\circ = 3\pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$2452.5 = Fe^{0.2(3\pi)}$$

$F = 372.38 \text{ N} = 372 \text{ N}$ **Ans**

8–91. A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the largest vertical force F that can be applied to the cord without moving the cylinder. The cord passes (a) once over the pipe, $\beta = 180^\circ$, and (b) two times over the pipe, $\beta = 540^\circ$. Take $\mu_s = 0.2$.



Frictional Force on Flat Belt : Here, $T_1 = 250(9.81) = 2452.5$ N and $T_2 = F$. Applying Eq. 8 – 6, we have

a) If $\beta = 180^\circ = \pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$F = 2452.5e^{0.2\pi}$$

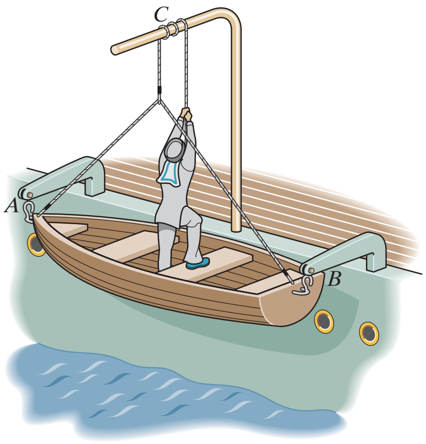
$$F = 4597.10 \text{ N} = 4.60 \text{ kN} \qquad \textbf{Ans}$$

b) If $\beta = 540^\circ = 3\pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$F = 2452.5e^{0.2(3\pi)}$$

$$F = 16152.32 \text{ N} = 16.2 \text{ kN} \qquad \textbf{Ans}$$

***8–92.** The boat has a weight of 500 lb and is held in position off the side of a ship by the spars at A and B. A man having a weight of 130 lb gets in the boat, wraps a rope around an overhead boom at C, and ties it to the end of the boat as shown. If the boat is disconnected from the spars, determine the *minimum number of half turns* the rope must make around the boom so that the boat can be safely lowered into the water at constant velocity. Also, what is the normal force between the boat and the man? The coefficient of kinetic friction between the rope and the boom is $\mu_s = 0.15$. *Hint*: The problem requires that the normal force between the man's feet and the boat be as small as possible.

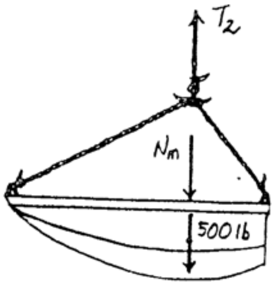


Frictional Force on Flat Belt : If the normal force between the man and the boat is equal to zero, then, $T_1 = 130$ lb and $T_2 = 500$ lb. Applying Eq. 8 – 6, we have

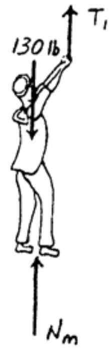
$$T_2 = T_1 e^{\mu\beta}$$
$$500 = 130e^{0.15\beta}$$
$$\beta = 8.980 \text{ rad}$$

The least number of half turns of the rope required is $\frac{8.980}{\pi} = 2.86$ turns. Thus

Use $n = 3$ half turns **Ans**



(a)



(b)

Equations of Equilibrium : From FBD (a),

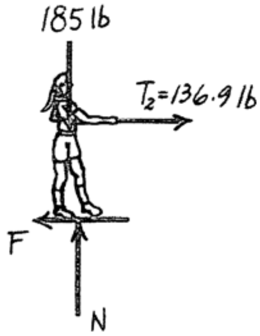
$$+\uparrow \Sigma F_y = 0; \quad T_2 - N_m - 500 = 0 \quad T_2 = N_m + 500$$

From FBD (b),

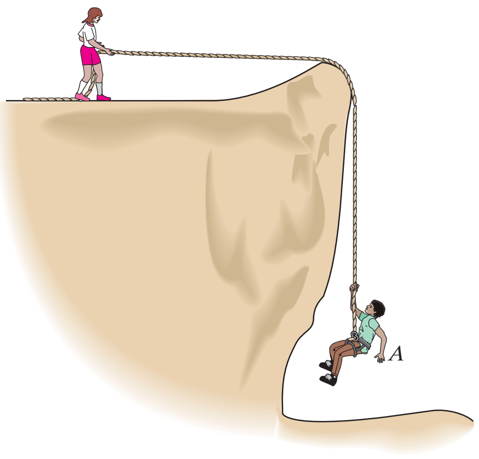
$$+\uparrow \Sigma F_y = 0; \quad T_1 + N_m - 130 = 0 \quad T_1 = 130 - N_m$$

Frictional Force on Flat Belts : Here, $\beta = 3\pi$ rad. Applying Eq. 8 – 6, we have

$$T_2 = T_1 e^{\mu\beta}$$
$$N_m + 500 = (130 - N_m)\, e^{0.15(3\pi)}$$
$$N_m = 6.74 \text{ lb} \qquad \textbf{Ans}$$



•8–93. The 100-lb boy at A is suspended from the cable that passes over the quarter circular cliff rock. Determine if it is possible for the 185-lb woman to hoist him up; and if this is possible, what smallest force must she exert on the horizontal cable? The coefficient of static friction between the cable and the rock is $\mu_s = 0.2$, and between the shoes of the woman and the ground $\mu_s' = 0.8$.



$\beta = \frac{\pi}{2}$

$T_2 = T_1 e^{\mu\beta} = 100 e^{0.2\frac{\pi}{2}} = 136.9 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad N - 185 = 0$

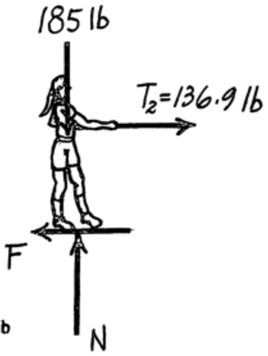
$N = 185 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 136.9 - F = 0$

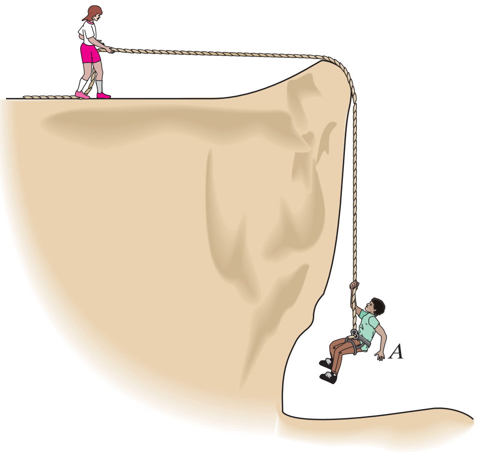
$F = 136.9 \text{ lb}$

$F_{max} = 0.8(185) = 148 \text{ lb} > 136.9 \text{ lb}$

Yes, just barely. **Ans**



8–94. The 100-lb boy at A is suspended from the cable that passes over the quarter circular cliff rock. What horizontal force must the woman at A exert on the cable in order to let the boy descend at constant velocity? The coefficients of static and kinetic friction between the cable and the rock are $\mu_s = 0.4$ and $\mu_k = 0.35$, respectively.

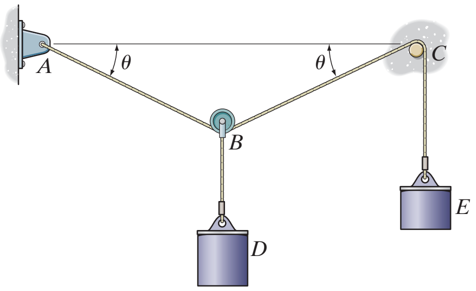


$\beta = \frac{\pi}{2}$

$T_2 = T_1 e^{\mu\beta}; \quad 100 = T_1 e^{0.35\frac{\pi}{2}}$

$T_1 = 57.7 \text{ lb}$ **Ans**

8–95. A 10-kg cylinder D, which is attached to a small pulley B, is placed on the cord as shown. Determine the smallest angle θ so that the cord does not slip over the peg at C. The cylinder at E has a mass of 10 kg, and the coefficient of static friction between the cord and the peg is $\mu_s = 0.1$.



Since pulley B is smooth, the tension in the cord between pegs A and C remains constant. Referring to the free-body diagram of the joint B shown in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2T\sin\theta - 10(9.81) = 0 \qquad T = \frac{49.05}{\sin\theta}$$

In the case where cylinder E is on the verge of ascending, $T_2 = T = \frac{49.05}{\sin\theta}$ and $T_1 = 10(9.81)$ N. Here, $\frac{\pi}{2} + \theta$, Fig. b. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$

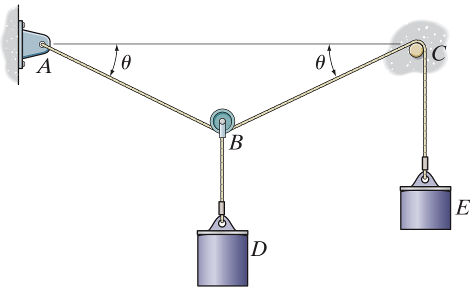
$$\frac{49.05}{\sin\theta} = 10(9.81)e^{0.1\left(\frac{\pi}{2}+\theta\right)}$$

$$\ln\frac{0.5}{\sin\theta} = 0.1\left(\frac{\pi}{2}+\theta\right)$$

Solving by trial and error, yields

$$\theta = 0.4221 \text{ rad} = 24.2° \qquad \textbf{Ans.}$$

***8–96.** A 10-kg cylinder D, which is attached to a small pulley B, is placed on the cord as shown. Determine the largest angle θ so that the cord does not slip over the peg at C. The cylinder at E has a mass of 10 kg, and the coefficient of static friction between the cord and the peg is $\mu_s = 0.1$.



In the case where cylinder E is on the verge of descending, $T_2 = 10(9.81)$ N and $T_1 = \frac{49.05}{\sin\theta}$. Here, $\frac{\pi}{2} + \theta$. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$

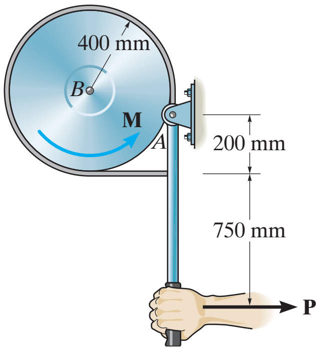
$$10(9.81) = \frac{49.05}{\sin\theta} e^{0.1\left(\frac{\pi}{2}+\theta\right)}$$

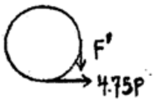
$$\ln(2\sin\theta) = 0.1\left(\frac{\pi}{2}+\theta\right)$$

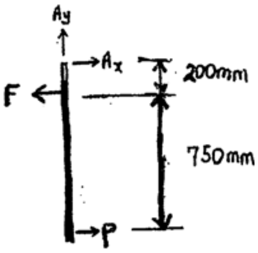
Solving by trial and error, yields

$$\theta = 0.6764 \text{ rad} = 38.8° \qquad \textbf{Ans.}$$

•8–97. Determine the smallest lever force P needed to prevent the wheel from rotating if it is subjected to a torque of $M = 250\ \text{N}\cdot\text{m}$. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$. The wheel is pin connected at its center, B.



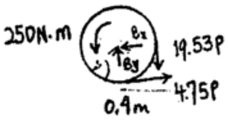




$\circlearrowleft + \Sigma M_A = 0;\qquad -F(200) + P(950) = 0$

$F = 4.75\,P$

$T_2 = T_1\, e^{\mu\beta}$

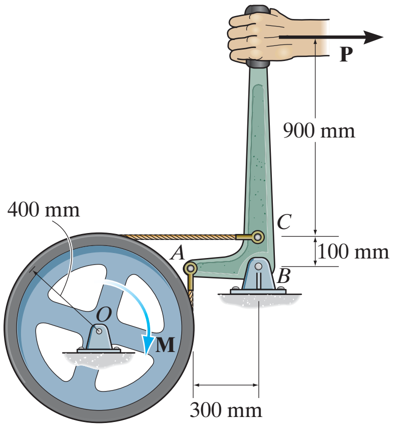


$F' = 4.75\,P\, e^{0.3(\frac{3\pi}{2})} = 19.53\,P$

$\circlearrowleft + \Sigma M_B = 0;\qquad -19.53\,P\,(0.4) + 250 + 4.75P(0.4) = 0$

$P = 42.3\ \text{N}$ **Ans**

8–98. If a force of $P = 200$ N is applied to the handle of the bell crank, determine the maximum torque M that can be resisted so that the flywheel is not on the verge of rotating clockwise. The coefficient of static friction between the brake band and the rim of the wheel is $\mu_s = 0.3$.



Referring to the free-body diagram of the bell crank shown in Fig. *a* and the flywheel shown in Fig. *b*,

$\circlearrowleft + \Sigma M_B = 0; \qquad T_A(0.3) + T_C(0.1) - 200(1) = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_O = 0; \qquad T_A(0.4) - T_C(0.4) - M = 0 \qquad (2)$

By considering the friction between the brake band and the rim of the wheel where $\beta = \dfrac{270^\circ}{180^\circ}\pi = 1.5\pi$ rad and $T_A > T_C$, we can write

$$T_A = T_C e^{\mu_s \beta}$$

$$T_A = T_C e^{0.3(1.5\pi)}$$

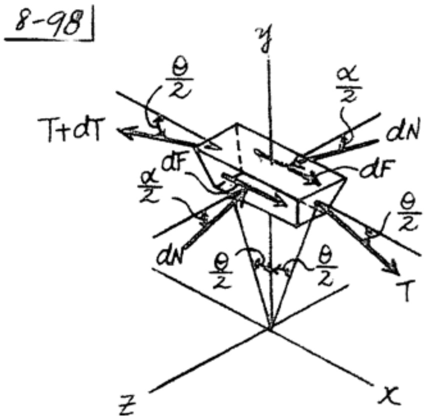
$$T_A = 4.1112 T_C \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

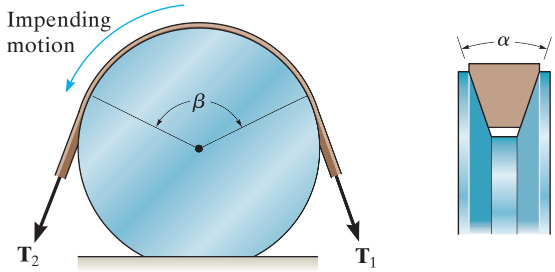
$M = 187\ \text{N}\cdot\text{m}$ **Ans.**

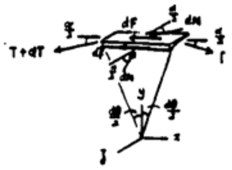
$T_A = 616.67\ \text{N} \qquad T_C = 150.00\ \text{N}$

8-98



8–99. Show that the frictional relationship between the belt tensions, the coefficient of friction μ, and the angular contacts α and β for the V-belt is $T_2 = T_1 e^{\mu\beta/\sin(\alpha/2)}$.





F.B.D of a section of the belt is shown.
Proceeding in the general manner :

$$\Sigma F_x = 0; \qquad -(T+dT)\cos\frac{d\theta}{2} + T\cos\frac{d\theta}{2} + 2\,dF = 0$$

$$\Sigma F_y = 0; \qquad -(T+dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} + 2\,dN\sin\frac{\alpha}{2} = 0$$

Replace $\sin\frac{d\theta}{2}$ by $\frac{d\theta}{2}$,

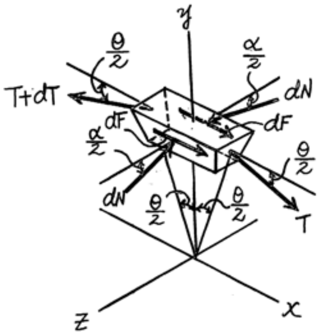
$\cos\frac{d\theta}{2}$ by 1,

$dF = \mu\,dN$

Using this and $(dT)(d\theta) \to 0$, the above relations become

$$dT = 2\mu\,dN$$

$$T\,d\theta = 2\left(dN\sin\frac{\alpha}{2}\right)$$



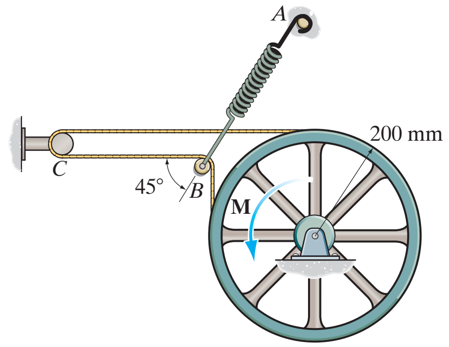
Combine $\qquad \dfrac{dT}{T} = \mu\dfrac{d\theta}{\sin\frac{\alpha}{2}}$

Integrate from $\theta = 0,\ T = T_1$
to $\theta = \beta,\ T = T_2$

we get,

$$T_2 = T_1\,e^{\left(\frac{\mu\beta}{\sin\frac{\alpha}{2}}\right)} \qquad \textbf{Q.E.D}$$

***8–100.** Determine the force developed in spring *AB* in order to hold the wheel from rotating when it is subjected to a couple moment of $M = 200\ \text{N}\cdot\text{m}$. The coefficient of static friction between the belt and the rim of the wheel is $\mu_s = 0.2$, and between the belt and peg *C*, $\mu_s' = 0.4$. The pulley at *B* is free to rotate.



Referring to the free - body diagram of the wheel shown in Fig. *a*, we have

$$\circlearrowleft +\Sigma M_O = 0; \qquad T_1(0.2) + 200 - T_2(0.2) = 0 \qquad (1)$$

In this case, the belt could slip over the wheel or peg *C*. We will assume it slips over the wheel. Here, $\beta_1 = \left(\frac{270°}{180°}\right)\pi = 1.5\pi$ rad. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$
$$T_2 = T_1 e^{0.2(1.5\pi)}$$
$$T_2 = 2.5663T_1 \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$T_1 = 638.43\ \text{N} \qquad T_2 = 1638.43\text{N}$$

Using these results and considering the friction between the belt and peg *C*, where $\beta_2 = \pi$ rad,

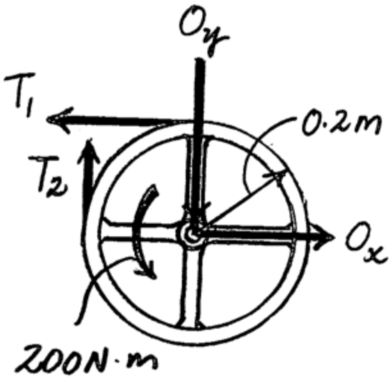
$$T_2 = T_1 e^{(\mu_s)_{\text{req}}\beta_2}$$
$$1638.43 = 638.43 e^{(\mu_s)_{\text{req}}(\pi)}$$
$$(\mu_s)_{\text{req}} = 0.3$$

Since the coefficient of static friction between the belt and peg *C* is greater than $(\mu_s)_{\text{req}}$ $(\mu_s' > 0.3)$, the belt will not slip over peg *C*. Thus, the above assumption is correct. Using the results of $\mathbf{T_2}$ and referring to the free - body diagram of joint *B* shown in Fig. *b*,

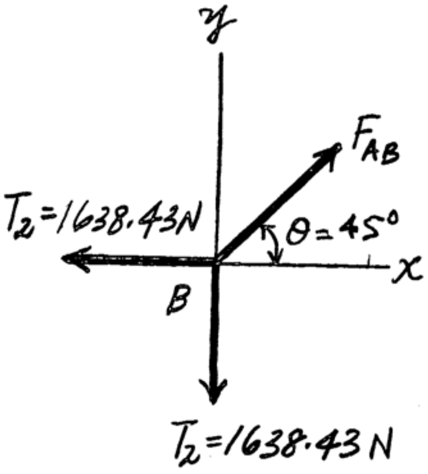
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\cos 45° - 1638.43 = 0$$

Solving

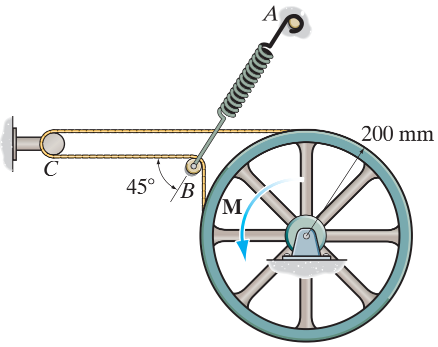
$$F_{AB} = 2317.10\text{N} = 2.32\ \text{kN} \qquad \textbf{Ans.}$$



(a)



•8–101. If the tension in the spring is $F_{AB} = 2.5$ kN, determine the largest couple moment that can be applied to the wheel without causing it to rotate. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.2$, and between the belt the peg $\mu_s' = 0.4$. The pulley B free to rotate.



Referring to the free-body diagram of joint B shown in Fig. a,

$$\xrightarrow{+}\Sigma F_x = 0; \qquad 2500\cos 45° - T_2 = 0$$

Solving,

$$T_2 = 1767.77 \text{ N}$$

In this case, the belt could slip over the wheel or peg C. We will assume that it slips over the wheel. Here, $\beta_1 = \left(\frac{270°}{180°}\right)\pi = 1.5\pi$ rad and $T_1 > T_2$. Thus,

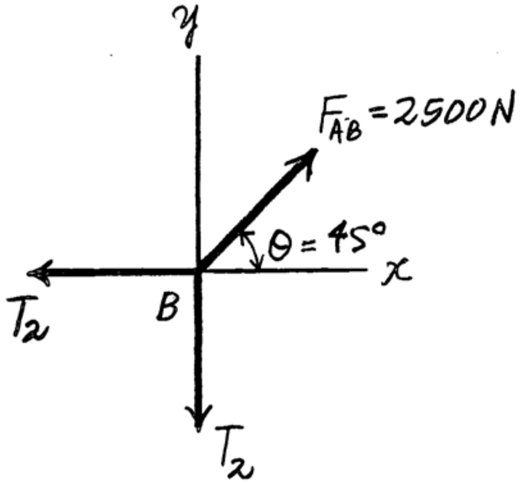
$$T_2 = T_1 e^{\mu_s \beta_1}$$
$$1767.77 = T_1 e^{0.2(1.5\pi)}$$
$$T_1 = 688.83$$

Using the results for T_1 and T_2 and considering the friction between the belt and peg C, where $\beta_2 = \pi$ rad,

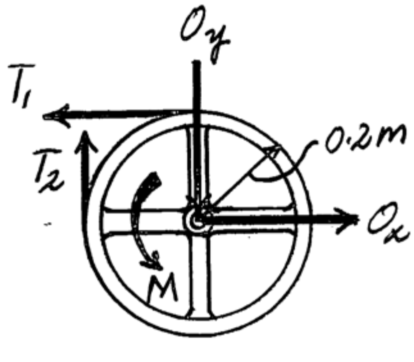
$$T_2 = T_1 e^{(\mu_s)_{req}\beta_2}$$
$$1767.77 = 688.83 e^{(\mu_s)_{req}(\pi)}$$
$$(\mu_s)_{req} = 0.3$$

Since the coefficient of static friction between the belt and peg C is greater than $(\mu_s)_{req}$ $(\mu_s' > 0.3)$, the belt will not slip over peg C. Thus, the above assumption is correct. Using the results of T_1 and T_2 and referring to the free-body diagram of the wheel shown in Fig. b,

$$\circlearrowleft +\Sigma M_O = 0; \qquad 688.83(0.2) + M - 1767.77(0.2) = 0$$
$$M = 216 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

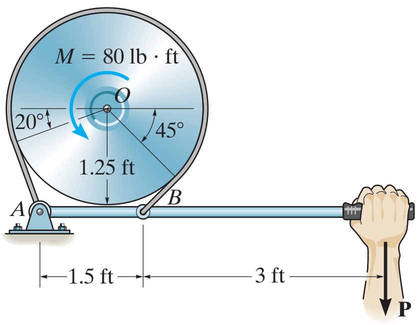


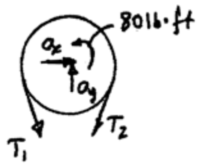
(a)



(b)

8–102. The simple band brake is constructed so that the ends of the friction strap are connected to the pin at *A* and the lever arm at *B*. If the wheel is subjected to a torque of $M = 80 \text{ lb} \cdot \text{ft}$, determine the smallest force *P* applied to the lever that is required to hold the wheel stationary. The coefficient of static friction between the strap and wheel is $\mu_s = 0.5$.



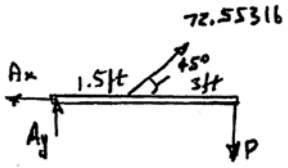


$\beta = 20° + 180° + 45° = 245°$

$\circlearrowleft + \Sigma M_O = 0; \quad T_1(1.25) + 80 - T_2(1.25) = 0$

$T_2 = T_1 e^{\mu\beta}; \quad T_2 = T_1 e^{0.5(245°)(\frac{\pi}{180°})} = 8.4827 T_1$

Solving;



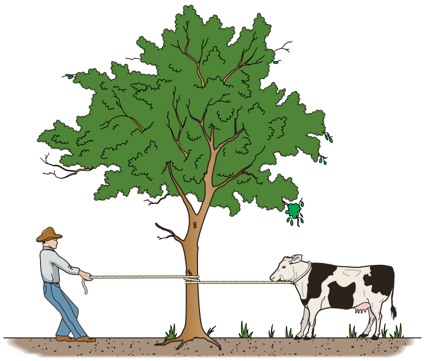
$T_1 = 8.553 \text{ lb}$

$T_2 = 72.553 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0; \quad -72.553(\sin 45°)(1.5) - 4.5P = 0$

$P = 17.1 \text{ lb}$ **Ans**

8–103. A 180-lb farmer tries to restrain the cow from escaping by wrapping the rope two turns around the tree trunk as shown. If the cow exerts a force of 250 lb on the rope, determine if the farmer can successfully restrain the cow. The coefficient of static friction between the rope and the tree trunk is $\mu_s = 0.15$, and between the farmer's shoes and the ground $\mu_s' = 0.3$.



Since the cow is on the verge of moving, the force it exerts on the rope is $T_2 = 250$ lb and the force exerted by the man on the rope is T_1. Here, $\beta = 2(2\pi) = 4\pi$ rad. Thus,

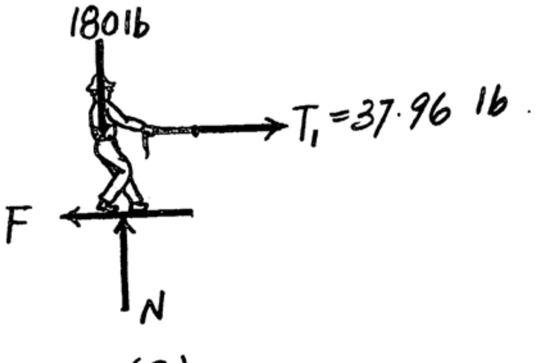
$$T_2 = T_1 e^{\mu_s \beta}$$
$$250 = T_1 e^{0.15(4\pi)}$$
$$T_1 = 37.96 \text{ lb}$$

Using this result and referring to the free - body diagram of the man shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0;$ $\quad N - 180 = 0$ $\quad N = 180$ lb

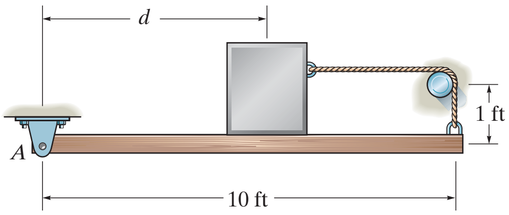
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 37.96 - F = 0$ $\quad F = 37.96$ lb

Since $F < F_{\max} = \mu_s' N = 0.3(180) = 54$ lb, the man will not slip, and he will successfully restrain the cow.



(a)

***8–104.** The uniform 50-lb beam is supported by the rope which is attached to the end of the beam, wraps over the rough peg, and is then connected to the 100-lb block. If the coefficient of static friction between the beam and the block, and between the rope and the peg, is $\mu_s = 0.4$, determine the maximum distance that the block can be placed from *A* and still remain in equilibrium. Assume the block will not tip.



Block:

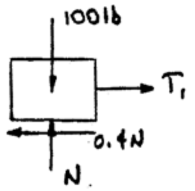
$+\uparrow\Sigma F_y = 0;\quad N - 100 = 0$

$N = 100\text{ lb}$

$\xrightarrow{+}\Sigma F_x = 0;\quad T_1 - 0.4(100) = 0$

$T_1 = 40\text{ lb}$

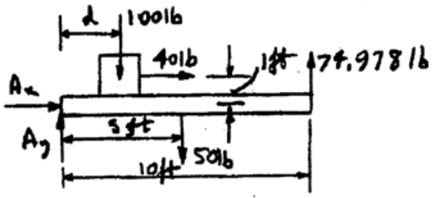
$T_2 = T_1 e^{\mu\beta};\quad T_2 = 40e^{0.4\left(\frac{\pi}{2}\right)} = 74.978\text{ lb}$



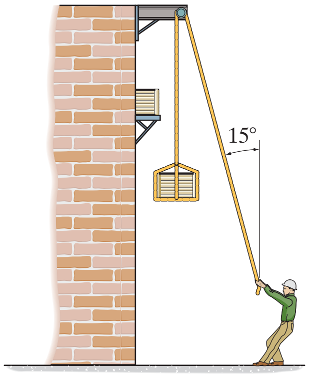
System:

$\curvearrowleft +\Sigma M_A = 0;\quad -100(d) - 40(1) - 50(5) + 74.978(10) = 0$

$d = 4.60\text{ ft}$ **Ans**



•8–105. The 80-kg man tries to lower the 150-kg crate using a rope that passes over the rough peg. Determine the least number of full turns in addition to the basic wrap (165°) around the peg to do the job. The coefficients of static friction between the rope and the peg and between the man's shoes and the ground are $\mu_s = 0.1$ and $\mu_s' = 0.4$, respectively.



If the man is on the verge of slipping, $F = \mu_s' N = 0.4N$. Referring to the free-body diagram of the man shown in Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 0.4N - T\sin 15° = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N + T\cos 15° - 80(9.81) = 0$

Solving,

$$T = 486.55\text{ N} \qquad N = 314.82\text{ N}$$

Using the result for **T** and considering the friction between the rope and the peg, where $T_2 = 150(9.81)$ N, $T_1 = T = 486.55$ N and $\beta = n(2\pi) + \left[\left(\frac{90° + 75°}{180°}\right)\pi\right] = (2n + 0.9167)\pi$ rad, Fig. *b*,

$$T_2 = T_1 e^{\mu_s \beta}$$

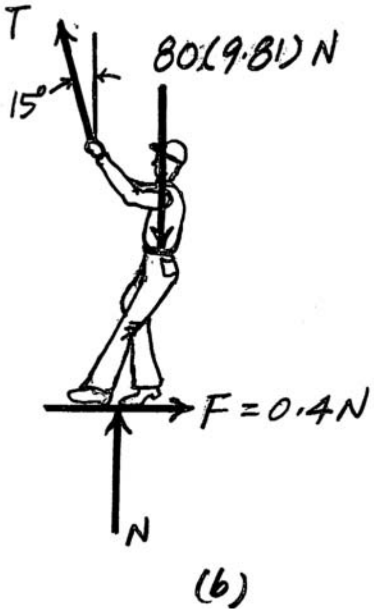
$$150(9.81) = 486.55e^{0.1(2n+0.9167)\pi}$$

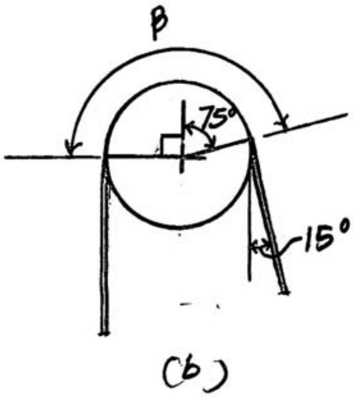
$$\ln 3.024 = 0.1(2n + 0.9167)\pi$$

$$n = 1.303$$

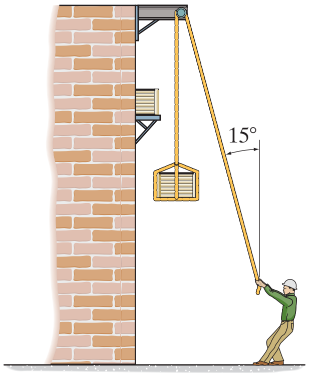
Thus, the required number of full turns is

$$n = 2 \qquad \textbf{Ans.}$$





8–106. If the rope wraps three full turns plus the basic wrap (165°) around the peg, determine if the 80-kg man can keep the 300-kg crate from moving. The coefficients of static friction between the rope and the peg and between the man's shoes and the ground are $\mu_s = 0.1$ and $\mu_s' = 0.4$, respectively.



If the man is on the verge of slipping, $F = \mu_s' N = 0.4N$. Referring to the free-body diagram of the man shown in Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\qquad 0.4N - T\sin 15° = 0$

$+\uparrow \Sigma F_y = 0;$ $\qquad N + T\cos 15° - 80(9.81) = 0$

Solving,

$$T = 486.55\text{ N} \qquad N = 314.82\text{ N}$$

Using the result for **T** and considering the friction between the rope and the peg, where $T_2 = 300(9.81)$ N, $T_1 = T = 486.55$ N and $\beta = n(2\pi) + \left[\left(\frac{90° + 75°}{180°}\right)\pi\right] = (2n + 0.9167)\pi$ rad, Fig. *b*,

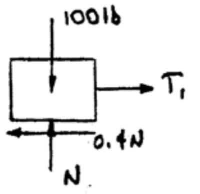
$$T_2 = T_1 e^{\mu_s \beta}$$

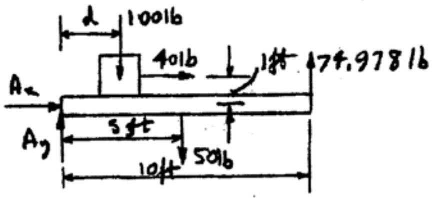
$$300(9.81) = 486.55e^{0.1(2n+0.9167)\pi}$$

$$\ln 6.049 = 0.1(2n + 0.9167)\pi$$

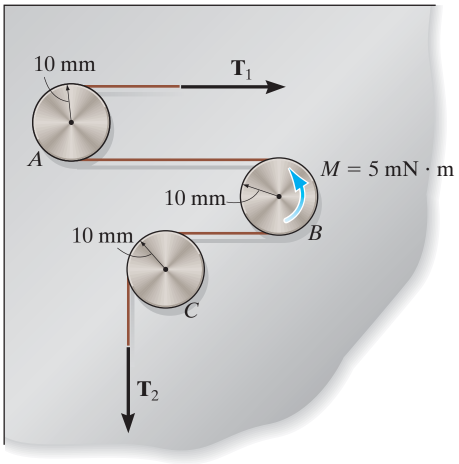
$$n = 2.406$$

Since $n > 3$, the man can hold the crate in equilibrium. **Ans.**





8–107. The drive pulley *B* in a video tape recorder is on the verge of slipping when it is subjected to a torque of $M = 0.005\ \text{N}\cdot\text{m}$. If the coefficient of static friction between the tape and the drive wheel and between the tape and the fixed shafts *A* and *C* is $\mu_s = 0.1$, determine the tensions T_1 and T_2 developed in the tape for equilibrium.



Here $\mathbf{T}_3$ must overcome $\mathbf{T}_4$ and **M**, so $\mathbf{T}_3 > \mathbf{T}_4$. Also, $\beta = \pi$ rad. Thus,

$$T_3 = T_4 e^{\mu_s \beta}$$
$$T_3 = T_4 e^{0.1(\pi)}$$
$$T_3 = 1.3691 T_4 \qquad (1)$$

Referring to the free-body diagram of pulley *B* in Fig. *a*,

$$+\Sigma M_O = 0; \qquad 0.005 + T_4(0.01) - T_3(0.01) = 0 \qquad (2)$$

Solving Eqs. (1) and (2), yields

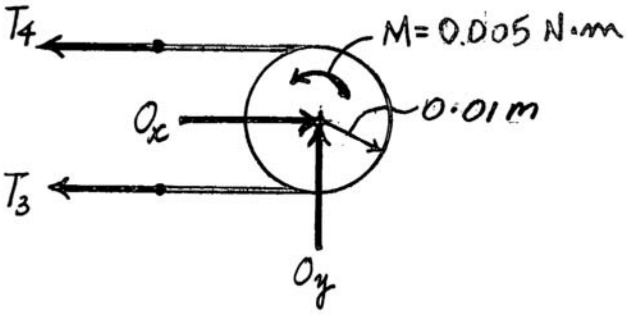
$$T_4 = 1.3546\ \text{N} \qquad T_3 = 1.8546\ \text{N}$$

Using the result of $\mathbf{T}_4$ and considering the friction on the fixed shaft *A*, where $\mathbf{T}_1 > \mathbf{T}_4$ and $\beta = \pi$ rad,

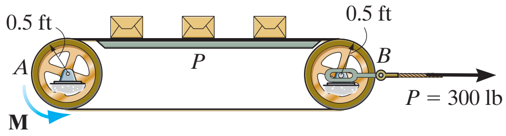
$$T_1 = T_4 e^{\mu_s \beta}$$
$$= 1.3546 e^{0.1\pi}$$
$$= 1.85\ \text{N} \qquad \textbf{Ans.}$$

Using the result of $\mathbf{T}_3$ and considering the friction on the fixed shaft *C*, where $\mathbf{T}_3 > \mathbf{T}_2$ and $\beta = \dfrac{\pi}{2}$ rad,

$$T_3 = T_2 e^{\mu_s \beta}$$
$$1.8546 = T_2 e^{0.1(\pi/2)}$$
$$T_2 = 1.59\ \text{N} \qquad \textbf{Ans.}$$



(a)



***8–108.** Determine the maximum number of 50-lb packages that can be placed on the belt without causing the belt to slip at the drive wheel *A* which is rotating with a constant angular velocity. Wheel *B* is free to rotate. Also, find the corresponding torsional moment **M** that must be supplied to wheel *A*. The conveyor belt is pre-tensioned with the 300-lb horizontal force. The coefficient of kinetic friction between the belt and platform *P* is $\mu_k = 0.2$, and the coefficient of static friction between the belt and the rim of each wheel is $\mu_s = 0.35$.

The maximum tension T_2 of the conveyor belt can be obtained by considering the equilibrium of the free-body diagram of the top belt shown in Fig. *a*.

$+\uparrow \Sigma F_y = 0; \qquad n(50) - N = 0 \qquad N = 50n \quad (1)$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 150 + 0.2(50n) - T_2 = 0 \qquad T_2 = 150 + 10n \qquad (2)$

By considering the case when the drive wheel *A* is on the verge of slipping, where $\beta = \pi$ rad, $T_2 = 150 + 10n$ and $T_1 = 150$ lb,

$$T_2 = T_1 e^{\mu\beta}$$

$$150 + 10n = 150e^{0.35(\pi)}$$

$$n = 30.04$$

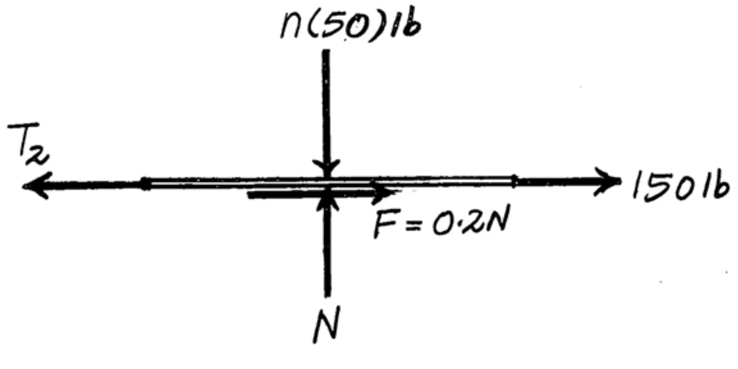
Thus, the maximum allowable number of boxes on the belt is

$n = 30$ **Ans.**

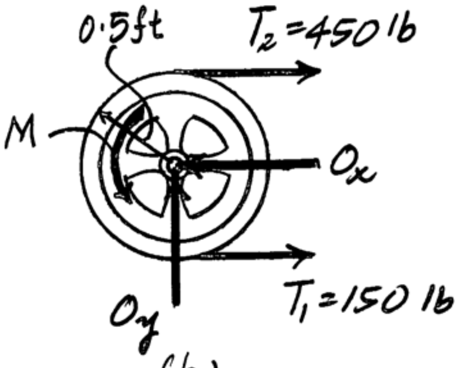
Substituting $n = 30$ into Eq. (2) gives $T_2 = 450$ lb. Referring to the free-body diagram of the wheel *A* shown in Fig. *b*,

$\circlearrowleft + \Sigma M_O = 0; \qquad M + 150(0.5) - 450(0.5) = 0$

$M = 150 \text{ lb} \cdot \text{ft}$ **Ans.**

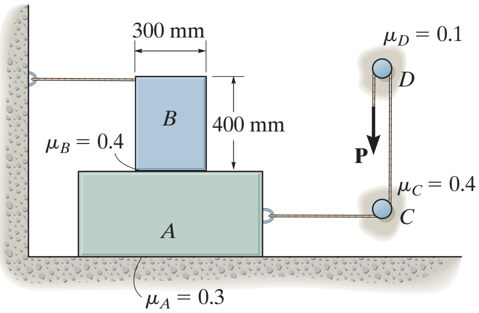


(a)



(b)

•8–109. Blocks *A* and *B* have a mass of 7 kg and 10 kg, respectively. Using the coefficients of static friction indicated, determine the largest vertical force *P* which can be applied to the cord without causing motion.



Frictional Forces on Flat Belts : When the cord pass over peg *D*, $\beta = 180° = \pi$ rad and $T_2 = P$. Applying Eq. 8 – 6, $T_2 = T_1 e^{\mu\beta}$, we have

$$P = T_1 e^{0.1\pi} \qquad T_1 = 0.7304P$$

When the cord pass over peg *C*, $\beta = 90° = \frac{\pi}{2}$ rad and $T_2{}' = T_1 = 0.7304P$. Applying Eq. 8 – 6, $T_2{}' = T_1{}' e^{\mu\beta}$, we have

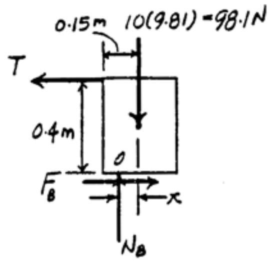
$$0.7304P = T_1{}' e^{0.4(\pi/2)} \qquad T_1{}' = 0.3897P$$

Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 98.1 = 0 \qquad N_B = 98.1 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_B - T = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad T(0.4) - 98.1(x) = 0 \qquad [2]$$

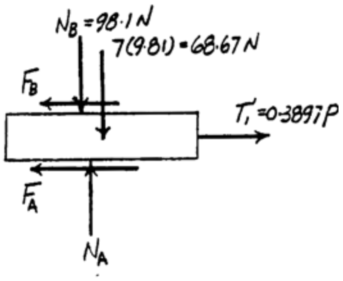


(a)

From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 98.1 - 68.67 = 0 \qquad N_A = 166.77 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3897P - F_B - F_A = 0 \qquad [3]$$



(b)

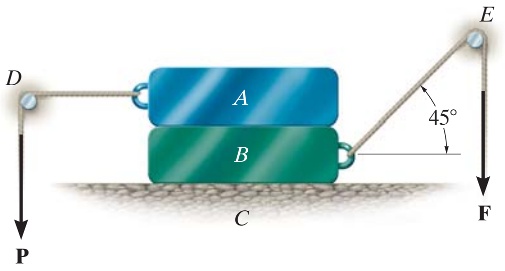
Friction : Assuming the block *B* is on the verge of tipping, then $x = 0.15$ m. For motion to occur, block *A* will have slip. Hence, $F_A = (\mu_s)_A N_A = 0.3(166.77) = 50.031$ N. Substituting these values into Eqs. [1], [2] and [3] and solving yields

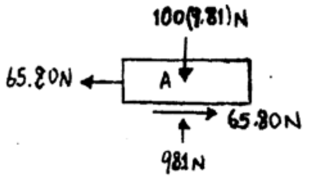
$$P = 222.81 \text{ N} = 223 \text{ N} \qquad \textbf{Ans}$$

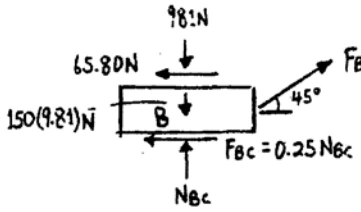
$$F_B = T = 36.79 \text{ N}$$

Since $(F_B)_{max} = (\mu_s)_B N_B = 0.4(98.1) = 39.24 \text{ N} > F_B$, block *B* does not slip but tips. Therefore, the above assumption is correct.

8–110. Blocks *A* and *B* have a mass of 100 kg and 150 kg, respectively. If the coefficient of static friction between *A* and *B* and between *B* and *C* is $\mu_s = 0.25$, and between the ropes and the pegs *D* and *E* $\mu_s' = 0.5$, determine the smallest force **F** needed to cause motion of block *B* if $P = 30$ N.







Assume no slipping between *A* and *B*.

Peg *D* :

$$T_2 = T_1 e^{\mu\beta}; \qquad F_{AD} = 30\, e^{0.5\left(\frac{\pi}{2}\right)} = 65.80\text{ N}$$

Block *B* :

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -65.80 - 0.25\, N_{BC} + F_{BE} \cos 45^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad N_{BC} - 981 + F_{BE} \sin 45^\circ - 150\,(9.81) = 0$$

$$F_{BE} = 768.1\text{ N}$$

$$N_{BC} = 1909.4\text{ N}$$

Peg *E* :

$$T_2 = T_1 e^{\mu\beta}; \qquad F = 768.1 e^{0.5\left(\frac{3\pi}{4}\right)} = 2.49\text{ kN} \qquad \text{Ans}$$

Note : Since *B* moves to the right,

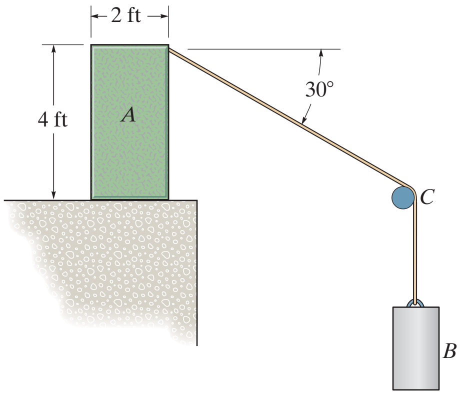
$$(F_{AB})_{max} = 0.25\,(981) = 245.25\text{ N}$$

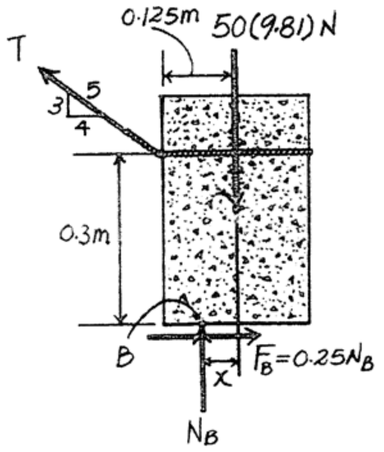
$$245.25 = P_{max}\, e^{0.5\left(\frac{\pi}{2}\right)}$$

$$P_{max} = 112\text{ N} > 30\text{ N}$$

Hence, no slipping occurs between *A* and *B* as originally assumed

8–111. Block A has a weight of 100 lb and rests on a surface for which $\mu_s = 0.25$. If the coefficient of static friction between the cord and the fixed peg at C is $\mu_s = 0.3$, determine the greatest weight of the suspended cylinder B without causing motion.





Frictional Force on Flat Belt : Here, $\beta = 60° = \frac{\pi}{3}$ rad and $T_2 = W$.

Applying Eq. 8–6, $T_2 = T_1 e^{\mu\beta}$, we have

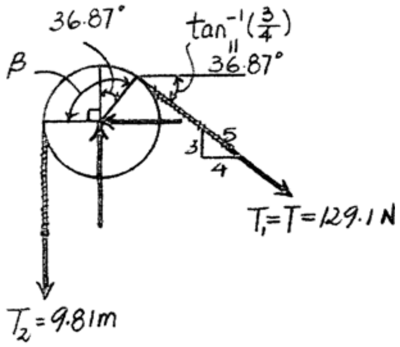
$$W = T_1 e^{0.3(\pi/3)} \qquad T_1 = 0.7304W$$

Equations of Equilibrium : From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad N - 0.7304W\sin 30° - 100 = 0 \qquad [1]$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.7304W\cos 30° - F = 0 \qquad [2]$$

$$\curvearrowleft + \Sigma M_A = 0; \qquad 100(x) - 0.7304W\cos 30°(4) - 0.7304W\sin 30°(1-x) = 0 \qquad [3]$$



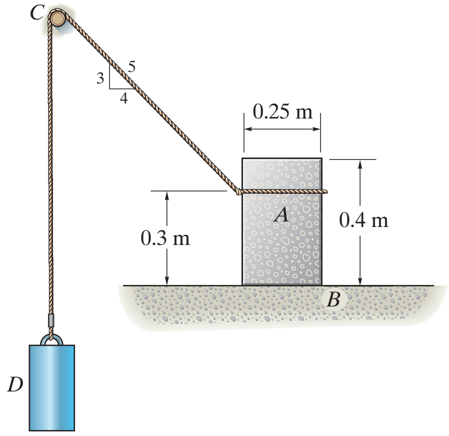
Friction : Assuming the block is on the verge of tipping, then $x = 1$ ft. Substituting this value into Eqs.[1], [2] and [3] and solving yields

$$W = 39.5 \text{ lb} \qquad \textbf{Ans}$$

$$F = 25.0 \text{ lb} \qquad N = 114.43 \text{ lb}$$

Since $F_{max} = \mu_s N = 0.25(114.43) = 28.61$ lb $> F$, the block does not slip but tips. Therefore, the above assumption is correct.

***8–112.** Block *A* has a mass of 50 kg and rests on surface *B* for which $\mu_s = 0.25$. If the coefficient of static friction between the cord and the fixed peg at *C* is $\mu_s' = 0.3$, determine the greatest mass of the suspended cylinder *D* without causing motion.



Block *A* :

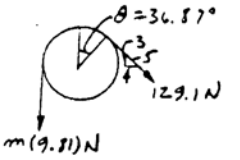
Assume block *A* slips and does not tip.

$+\uparrow \Sigma F_y = 0; \quad N_B + \frac{3}{5}T - 50(9.81) = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad 0.25 N_B - \frac{4}{5}T = 0$

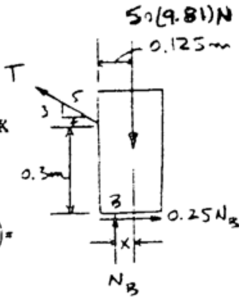
$N_B = 413.1 \text{ N}$

$T = 129.1 \text{ N}$



$\circlearrowleft + \Sigma M_B = 0; \quad -50(9.81)x + \frac{4}{5}(129.1)(0.3) - \frac{3}{5}(129.1)(0.125 - x) = 0$

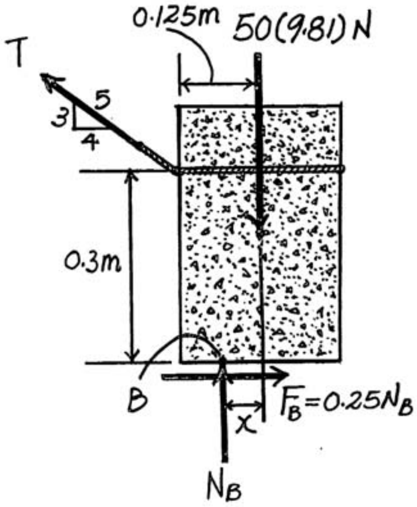
$x = 0.0516 \text{ m} < 0.125 \text{ m} \quad \text{OK}$

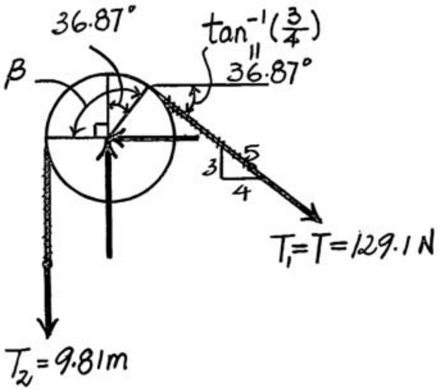


Peg :

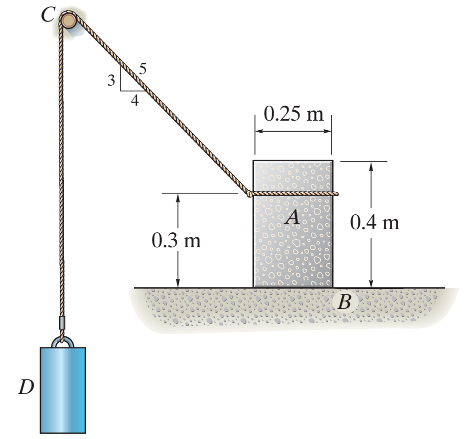
$T_2 = T_1 e^{\mu\beta}; \quad 9.81\, m = 129.1\, e^{0.3\left(\frac{90^\circ + 36.87^\circ}{180^\circ}\right)\pi}$

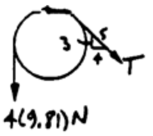
$m = 25.6 \text{ kg} \quad \text{Ans}$





•8–113. Block *A* has a mass of 50 kg and rests on surface *B* for which $\mu_s = 0.25$. If the mass of the suspended cylinder *D* is 4 kg, determine the frictional force acting on *A* and check if motion occurs. The coefficient of static friction between the cord and the fixed peg at *C* is $\mu_s' = 0.3$.





$$T_2 = T_1\,e^{\mu\beta}; \qquad 4\,(9.81) = T\,e^{0.3\left(\frac{90+36.87}{180}\right)\pi}$$

$$T = 20.19\text{ N}$$

Block *A* :

$$\xrightarrow{+}\Sigma F_x = 0; \qquad F_A - \frac{4}{5}(20.19) = 0$$

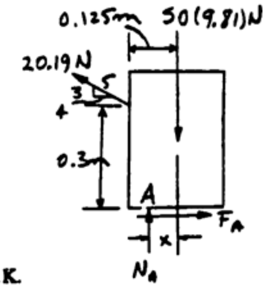
$$F_A = 16.2\text{ N} \qquad \textbf{Ans}$$

$$+\uparrow\Sigma F_y = 0; \qquad N_A - 50\,(9.81) + \frac{3}{5}(20.19) = 0$$

$$N_A = 478.4\text{ N}$$

$$(F_A)_{max} = 0.25\,(478.4) = 119.6\text{ N} > 16.16\text{ N} \qquad \text{O.K.}$$

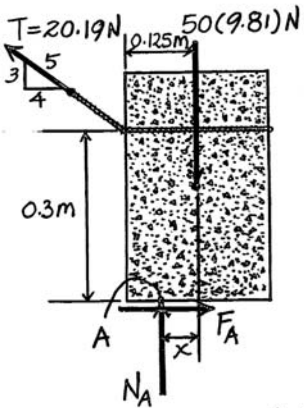
Block does not slip.

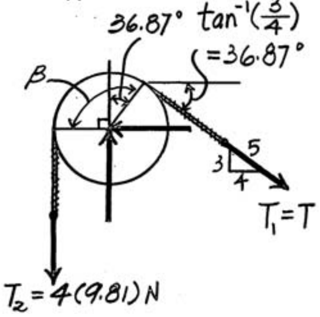


$$\circlearrowleft + \Sigma M_B = 0; \qquad -50\,(9.81)\,x + \frac{4}{5}(20.19)\,(0.3) - \frac{3}{5}(20.19)\,(0.125 - x) = 0$$

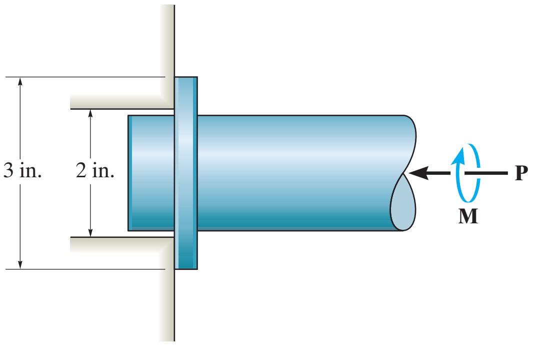
$$x = 0.00697\text{ m} < 0.125\text{ m} \qquad \text{O.K.}$$

No tipping occurs. **Ans**





8–114. The collar bearing uniformly supports an axial force of $P = 800$ lb. If the coefficient of static friction is $\mu_s = 0.3$, determine the torque M required to overcome friction.

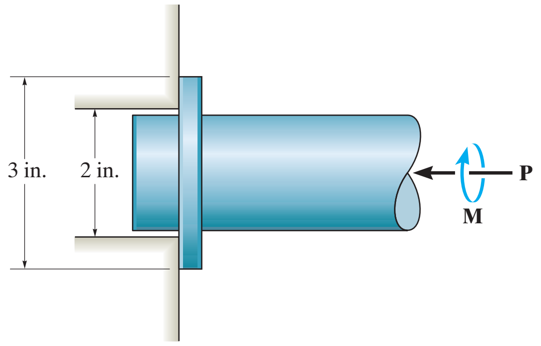


$$M = \frac{2}{3}\mu_s\,P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$= \frac{2}{3}(0.3)(800)\left[\frac{(1.5)^3 - 1^3}{(1.5)^2 - 1^2}\right]$$

$$= 304\text{ lb·in.} \qquad \textbf{Ans}$$

8–115. The collar bearing uniformly supports an axial force of $P = 500$ lb. If a torque of $M = 3$ lb · ft is applied to the shaft and causes it to rotate at constant velocity, determine the coefficient of kinetic friction at the surface of contact.

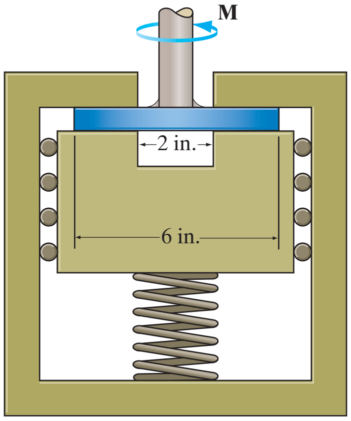


$$M = \frac{2}{3}\mu_k P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$3(12) = \frac{2}{3}\mu_k(500)\left[\frac{(1.5)^3 - 1^3}{(1.5)^2 - 1^2}\right]$$

$\mu_k = 0.0568$ **Ans**

***8–116.** If the spring exerts a force of 900 lb on the block, determine the torque M required to rotate the shaft. The coefficient of static friction at all contacting surfaces is $\mu_s = 0.3$.



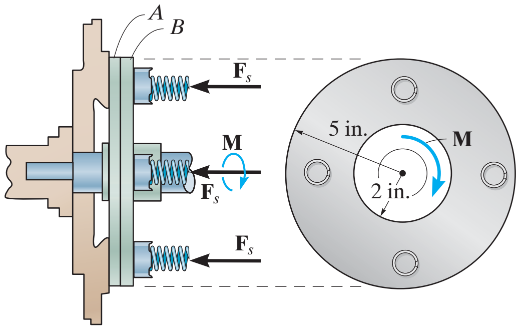
Here, $R_1 = \frac{2\text{ in.}}{2} = 1\text{ in.}$, $R_2 = \frac{6\text{ in.}}{2} = 3\text{ in.}$, $\mu_s = 0.3$ and $P = 900$ lb, since **M** is required to overcome the friction of two contacting surfaces. Eq. 8-7 becomes

$$M = 2\left[\frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)\right]$$

$$= \frac{4}{3}(0.3)(900)\left(\frac{3^3 - 1^3}{3^2 - 1^2}\right)$$

$$= 1170\text{ lb}\cdot\text{in} = 97.5\text{ lb}\cdot\text{ft}$$ **Ans.**

•8–117. The *disk clutch* is used in standard transmissions of automobiles. If four springs are used to force the two plates A and B together, determine the force in each spring required to transmit a moment of $M = 600$ lb · ft across the plates. The coefficient of static friction between A and B is $\mu_s = 0.3$.



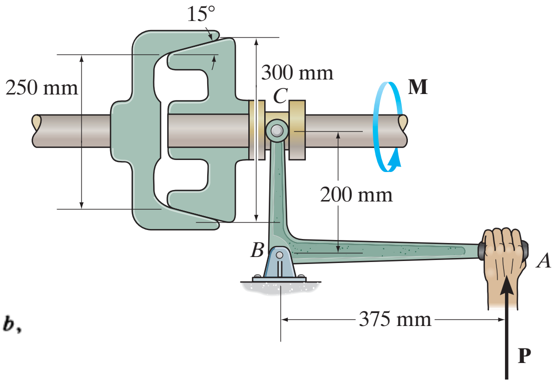
Bearing Friction : Applying Eq. 8–7 with $R_2 = 5$ in., $R_1 = 2$ in., $M = 600(12) = 7200$ lb · in, $\mu_s = 0.3$ and $P = 4F_{sp}$, we have

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$7200 = \frac{2}{3}(0.3)(4F_{sp})\left(\frac{5^3 - 2^3}{5^2 - 2^2}\right)$$

$F_{sp} = 1615.38\text{ lb} = 1.62\text{ kip}$ **Ans**

8–118. If $P = 900$ N is applied to the handle of the bell crank, determine the maximum torque M the cone clutch can transmit. The coefficient of static friction at the contacting surface is $\mu_s = 0.3$.



Referring to the free-body diagram of the bellcrank shown in Fig. *a*, we have

$$+\Sigma M_B = 0; \qquad 900(0.375) - F_C(0.2) = 0 \qquad F_C = 1687.5\text{ N}$$

Using this result and referring to the free-body diagram of the cone clutch shown in Fig. *b*,

$$\xrightarrow{+}\Sigma F_x = 0; \qquad 2\left(\frac{N}{2}\sin 15°\right) - 1687.5 = 0 \qquad N = 6520.00\text{ N}$$

The area of the differential element shown shaded in Fig. *c* is $dA = 2\pi r\ ds = 2\pi r\dfrac{dr}{\sin 15°} = \dfrac{2\pi}{\sin 15°}r\,dr$. Thus,

$A = \displaystyle\int_A dA = \int_{0.125\text{ m}}^{0.15\text{ m}} \frac{2\pi}{\sin 15°}r\,dr = 0.08345\text{ m}^2$. The pressure acting on the cone surface is

$$p = \frac{N}{A} = \frac{6520.00}{0.08345} = 78.13(10^3)\text{ N/m}^2$$

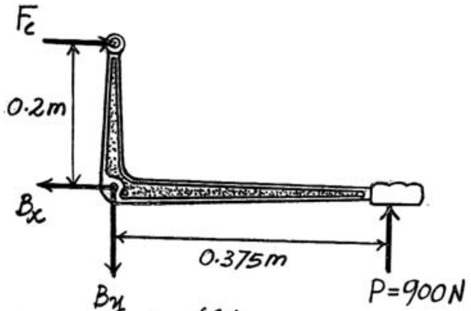
The normal force acting on the differential element dA is $dN = p\,dA = 78.13(10^3)\left[\dfrac{2\pi}{\sin 15°}\right]r\,dr = 1896.73(10^3)r\,dr$.

Thus, the frictional force acting on this differential element is given by $dF = \mu_s dN = 0.3(1896.73)(10^3)r\,dr$ $= 569.02(10^3)r\,dr$. The moment equation about the axle of the cone clutch gives

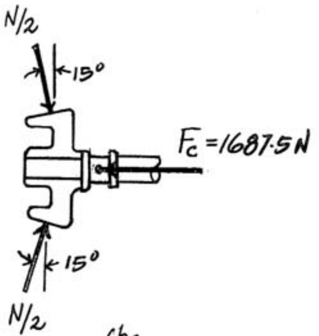
$$\Sigma M = 0; \quad M - \int rdF = 0$$

$$M = \int rdF = 569.02(10^3)\int_{0.125\text{ m}}^{0.15\text{ m}} r^2\,dr$$

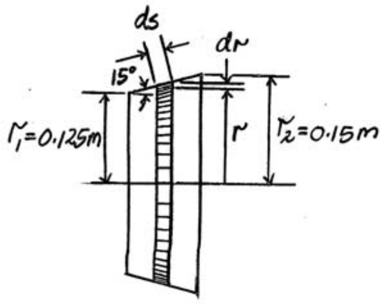
$$M = 270\text{ N}\cdot\text{m}$$ **Ans.**



(a)

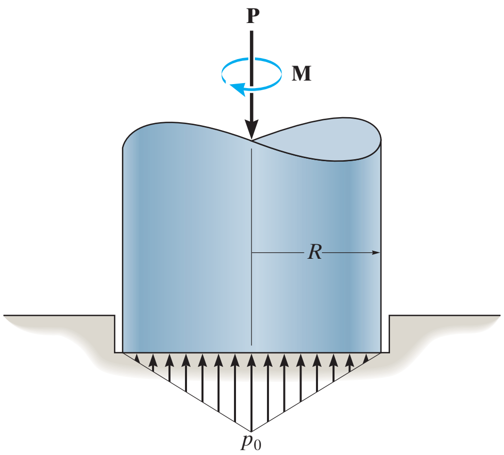


(b)



(c)

8–119. Because of wearing at the edges, the pivot bearing is subjected to a conical pressure distribution at its surface of contact. Determine the torque M required to overcome friction and turn the shaft, which supports an axial force $\mathbf{P}$. The coefficient of static friction is μ_s. For the solution, it is necessary to determine the peak pressure p_0 in terms of P and the bearing radius R.



Equations of Equilibrium and Bearing Friction : Using similar triangles, $\frac{p}{R-r} = \frac{p_0}{R}$, $p = \frac{p_0}{R}(R-r)$. Also, $dA = 2\pi r dr$, $dN = pdA$ and $dF = \mu_s dN = \mu_s pdA$.

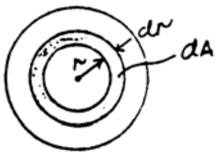
$+\uparrow \Sigma F_y = 0;$

$$\int pdA - P = 0$$

$$\int \frac{p_0}{R}(R-r)(2\pi r dr) - P = 0$$

$$\frac{2\pi p_0}{R}\int_0^R r(R-r)\,dr - P = 0$$

$$p_0 = \frac{3P}{\pi R^2} \qquad [1]$$



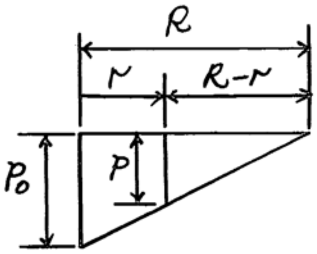
$\circlearrowleft + \Sigma M_z = 0;$

$$\int (\mu_s pdA)\,r - M = 0$$

$$\int_0^R \frac{\mu_s p_0}{R}(R-r)(2\pi r dr)\,r - M = 0$$

$$\frac{2\pi\mu_s p_0}{R}\int_0^R r^2(R-r)\,dr - M = 0$$

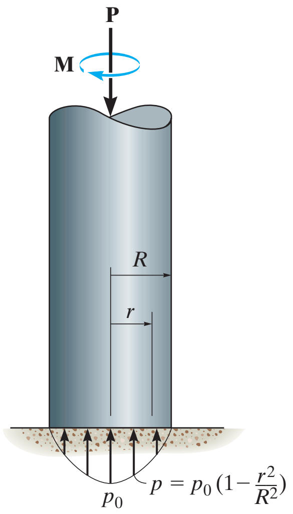
$$M = \frac{\pi\mu_s R^3 p_0}{6} \qquad [2]$$



Substituting Eq. [1] into [2] yields

$$M = \frac{\pi\mu_s R^3}{6}\left(\frac{3P}{\pi R^2}\right) = \frac{\mu_s PR}{2} \qquad \textbf{Ans}$$

***8–120.** The pivot bearing is subjected to a parabolic pressure distribution at its surface of contact. If the coefficient of static friction is μ_s, determine the torque M required to overcome friction and turn the shaft if it supports an axial force **P**.



The differential area $dA = (r\,d\theta)(dr)$

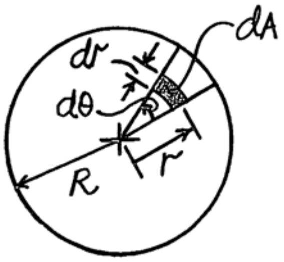
$$P=\int p\,dA=\int p_0\left(1-\frac{r^2}{R^2}\right)(r\,d\theta)(dr)=p_0\int_0^{2\pi}d\theta\int_0^R r\left(1-\frac{r^2}{R^2}\right)dr$$

$$P=\frac{\pi R^2 p_0}{2}\qquad p_0=\frac{2P}{\pi R^2}$$

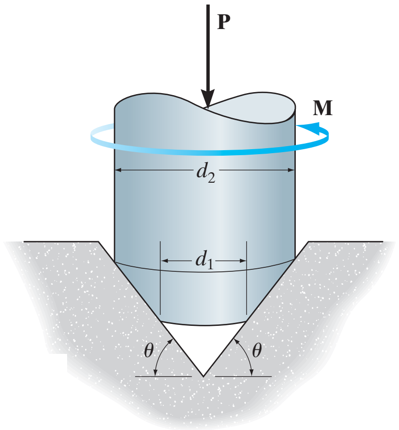
$$dN=p\,dA=\frac{2P}{\pi R^2}\left(1-\frac{r^2}{R^2}\right)(r\,d\theta)(dr)$$

$$M=\int r\,dF=\int \mu_s r\,dN=\frac{2\mu_s P}{\pi R^2}\int_0^{2\pi}d\theta\int_0^R r^2\left(1-\frac{r^2}{R^2}\right)dr$$

$$=\frac{8}{15}\mu_s PR \qquad \text{Ans}$$



•8–121. The shaft is subjected to an axial force **P**. If the reactive pressure on the conical bearing is uniform, determine the torque *M* that is just sufficient to rotate the shaft. The coefficient of static friction at the contacting surface is μ_s.



Referring to the free-body diagram of the shaft shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2\left(\frac{N}{2}\cos\theta\right) - P \qquad\qquad N = \frac{P}{\cos\theta}$$

The area of the differential element shown shaded in Fig. *b* is $dA = 2\pi r ds = \dfrac{2\pi}{\cos\theta} r\,dr$. Thus,

$$A = \int_A dA = \int_{d_1/2}^{d_2/2} \frac{2\pi}{\cos\theta} r\,dr = \frac{\pi}{4\cos\theta}\left(d_2^{\,2} - d_1^{\,2}\right)$$

Therefore, the pressure acting on the cone surface is

$$p = \frac{N}{A} = \frac{P/\cos\theta}{\dfrac{\pi}{4\cos\theta}\left(d_2^{\,2} - d_1^{\,2}\right)} = \frac{4P}{\pi\left(d_2^{\,2} - d_1^{\,2}\right)}$$

The normal force acting on the differential element *dA* is

$$dN = p\,dA = \frac{4P}{\pi\left(d_2^{\,2} - d_1^{\,2}\right)}\left(\frac{2\pi}{\cos\theta} r\,dr\right) = \frac{8P}{\left(d_2^{\,2} - d_1^{\,2}\right)\cos\theta} r\,dr$$

Thus, the frictional force acting on this differential element is given by

$$dF = \mu_s dN = \frac{8\mu_s P}{\left(d_2^{\,2} - d_1^{\,2}\right)\cos\theta} r\,dr$$

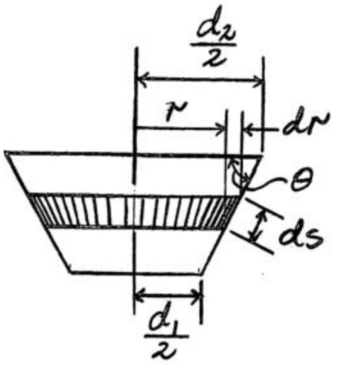
The moment equation about the axle of the shaft gives

$$\Sigma M = 0; \quad M - \int r dF = 0$$

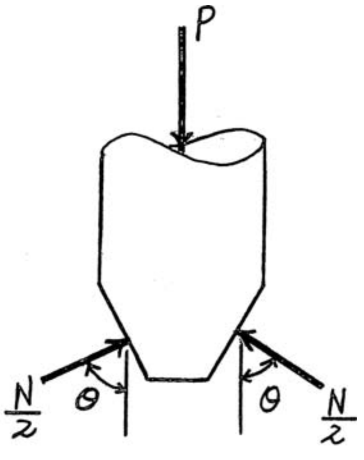
$$M = \int r dF = \frac{8\mu_s P}{\left(d_2^{\,2} - d_1^{\,2}\right)\cos\theta}\int_{d_1/2}^{d_2/2} r^2\,dr$$

$$= \frac{\mu_s P}{3\cos\theta}\left(\frac{d_2^{\,3} - d_1^{\,2}}{d_2^{\,2} - d_1^{\,2}}\right)$$

Ans.

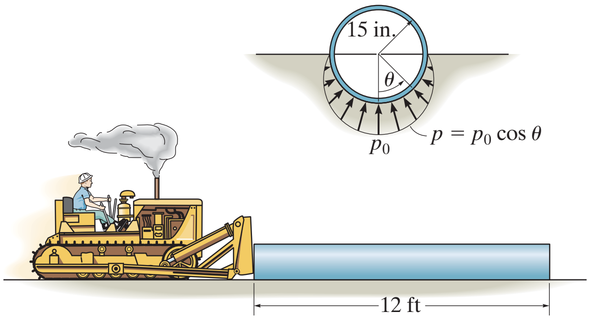


(b)



(a)

8–122. The tractor is used to push the 1500-lb pipe. To do this it must overcome the frictional forces at the ground, caused by sand. Assuming that the sand exerts a pressure on the bottom of the pipe as shown, and the coefficient of static friction between the pipe and the sand is $\mu_s = 0.3$, determine the horizontal force required to push the pipe forward. Also, determine the peak pressure p_0.



$$+\uparrow\Sigma F_y = 0; \quad 2l\int_0^{\pi/2} p_0 \cos\theta\,(r\,d\theta)\cos\theta - W = 0$$

$$2p_0 l r\int_0^{\pi/2}\cos^2\theta\,d\theta = W$$

$$2p_0\,rl\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^{\frac{\pi}{2}} = W$$

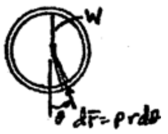
$$2(p_0)\,rl\left(\frac{\pi}{4}\right) = W$$

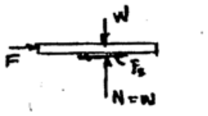
$$2\,p_0(15)(12)(12)\left(\frac{\pi}{4}\right) = 1500$$

$$p_0 = 0.442\text{ psi} \qquad \text{Ans}$$

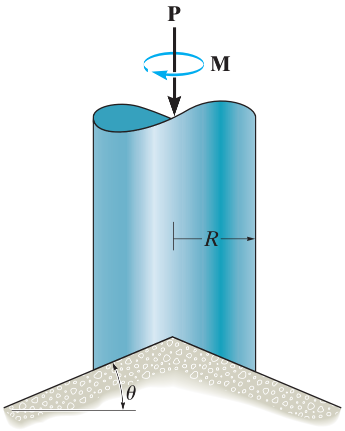
$$F = \int_{-\pi/2}^{\pi/2}(0.3)(0.442\text{ lb/in}^2)\overset{\cos\theta}{\Big\{}(12\text{ ft})(12\text{ in./ft})(15\text{ in.})\,d\theta$$

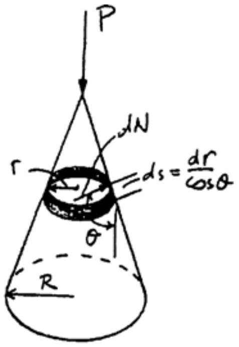
$$F = 573\text{ lb} \qquad \text{Ans}$$





8–123. The conical bearing is subjected to a constant pressure distribution at its surface of contact. If the coefficient of static friction is μ_s, determine the torque M required to overcome friction if the shaft supports an axial force **P**.





The differential area (shaded) $dA = 2\pi r\left(\frac{dr}{\cos\theta}\right) = \frac{2\pi r\,dr}{\cos\theta}$

$$P = \int p\cos\theta\, dA = \int p\cos\theta\left(\frac{2\pi r\,dr}{\cos\theta}\right) = 2\pi p\int_0^R r\,dr$$

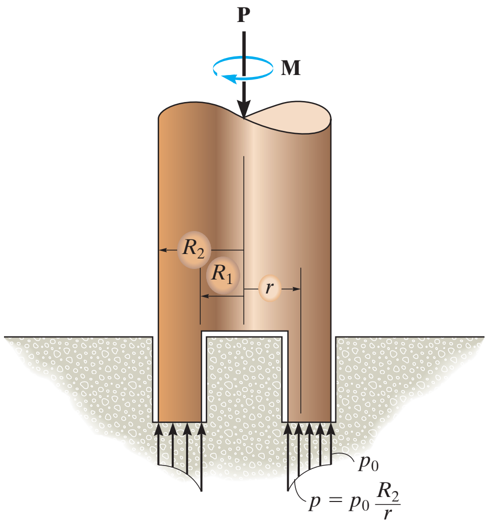
$$P = \pi p R^2 \qquad p = \frac{P}{\pi R^2}$$

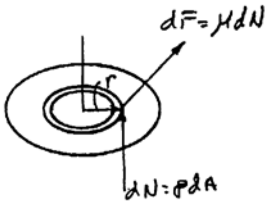
$$dN = p\,dA = \frac{P}{\pi R^2}\left(\frac{2\pi r\,dr}{\cos\theta}\right) = \frac{2P}{R^2\cos\theta}r\,dr$$

$$M = \int r\,dF = \int \mu_s r\,dN = \frac{2\mu_s P}{R^2\cos\theta}\int_0^R r^2\,dr$$

$$= \frac{2\mu_s P}{R^2\cos\theta}\,\frac{R^3}{3} = \frac{2\mu_s PR}{3\cos\theta} \qquad \textbf{Ans}$$

***8–124.** Assuming that the variation of pressure at the bottom of the pivot bearing is defined as $p = p_0(R_2/r)$, determine the torque M needed to overcome friction if the shaft is subjected to an axial force **P**. The coefficient of static friction is μ_s. For the solution, it is necessary to determine p_0 in terms of P and the bearing dimensions R_1 and R_2.





$$\Sigma F_z = 0; \quad P = \int_A dN = \int_0^{2\pi}\int_{R_1}^{R_2} pr\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_{R_1}^{R_2} p_0\left(\frac{R_2}{r}\right) r\,dr\,d\theta$$

$$= 2\pi p_0 R_2 (R_2 - R_1)$$

Thus, $$p_0 = \frac{P}{[2\pi R_2 (R_2 - R_1)]}$$

$$\Sigma M_z = 0; \quad M = \int_A r\,dF = \int_0^{2\pi}\int_{R_1}^{R_2} \mu_s pr^2\,dr\,d\theta$$

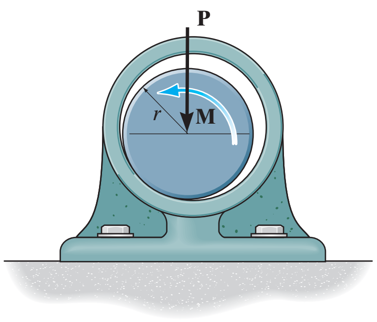
$$= \int_0^{2\pi}\int_{R_1}^{R_2} \mu_s p_0\left(\frac{R_2}{r}\right) r^2\,dr\,d\theta$$

$$= \mu_s (2\pi p_0) R_2 \frac{1}{2}(R_2^2 - R_1^2)$$

Using Eq. (1):

$$M = \frac{1}{2}\mu_s P (R_2 + R_1) \quad \textbf{Ans}$$

•8–125. The shaft of radius r fits loosely on the journal bearing. If the shaft transmits a vertical force **P** to the bearing and the coefficient of kinetic friction between the shaft and the bearing is μ_k, determine the torque M required to turn the shaft with constant velocity.



From the geometry of the free-body diagram of the shaft shown in Fig. *a*,

$$\tan\phi_k = \frac{\mu_k N}{N} = \mu_k$$

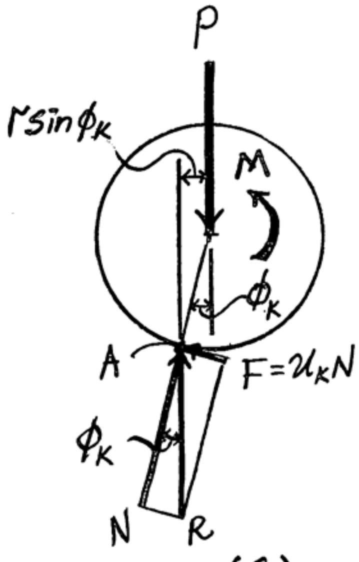
Thus, referring to Fig. *b*, we obtain

$$\sin\phi_k = \frac{\mu_k}{\sqrt{1+\mu_k^2}}$$

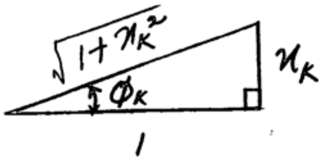
Referring to the free-body diagram of the shaft shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad M - Pr\left(\frac{\mu_k}{\sqrt{1+\mu_k^2}}\right) = 0$$

$$M = \left(\frac{\mu_k}{\sqrt{1+\mu_k^2}}\right)Pr \qquad \textbf{Ans.}$$

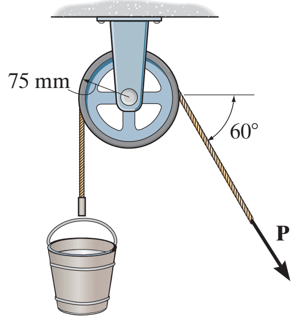


(a)



(b)

8–126. The pulley is supported by a 25-mm-diameter pin. If the pulley fits loosely on the pin, determine the smallest force P required to raise the bucket. The bucket has a mass of 20 kg and the coefficient of static friction between the pulley and the pin is $\mu_s = 0.3$. Neglect the mass of the pulley and assume that the cable does not slip on the pulley.



Referring to the free-body diagram of the pulley shown in Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad P\cos 60° - R_x = 0 \qquad R_x = 0.5P$

$+\uparrow \Sigma F_y = 0;$ $\quad R_y - P\sin 60° - 20(9.81) = 0 \qquad R_y = 0.8660P + 196.2$

Thus, the magnitude of $\mathbf{R}$ is

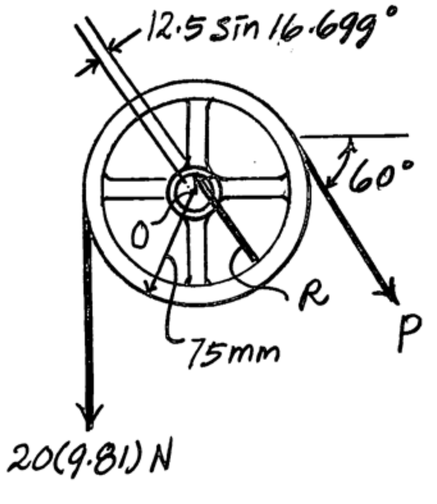
$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(0.5P)^2 + (0.8660P + 196.2)^2}$$
$$= \sqrt{P^2 + 339.83P + 38494.44}$$

By referring to the geometry shown in Fig. b, we find that $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of $\mathbf{R}$ from point O is $(12.5\sin 16.699°)$ mm. Using these results and writing the moment equation about point O, Fig. a,

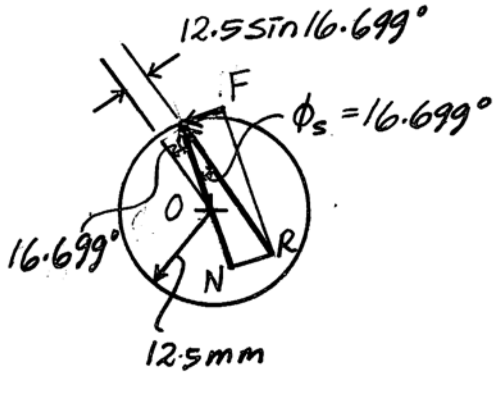
$\circlearrowleft + \Sigma M_O = 0;$ $\quad 20(9.81)(75) + \sqrt{P^2 + 339.83P + 38494.44}\,(12.5\sin 16.699°) - P(75) = 0$

Choosing the root $P > 20(9.81)$ N,

$P = 215\text{ N}$ **Ans.**

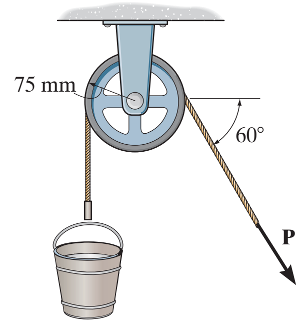


(a)



(b)

8–127. The pulley is supported by a 25-mm-diameter pin. If the pulley fits loosely on the pin, determine the largest force P that can be applied to the rope and yet lower the bucket. The bucket has a mass of 20 kg and the coefficient of static friction between the pulley and the pin is $\mu_s = 0.3$. Neglect the mass of the pulley and assume that the cable does not slip on the pulley.



Referring to the free-body diagram of the pulley shown in Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad P\cos 60° - R_x = 0 \quad R_x = 0.5P$

$+\uparrow \Sigma F_y = 0;$ $\quad R_y - P\sin 60° - 20(9.81) = 0 \quad R_y = 0.8660P + 196.2$

Thus, the magnitude of $\mathbf{R}$ is

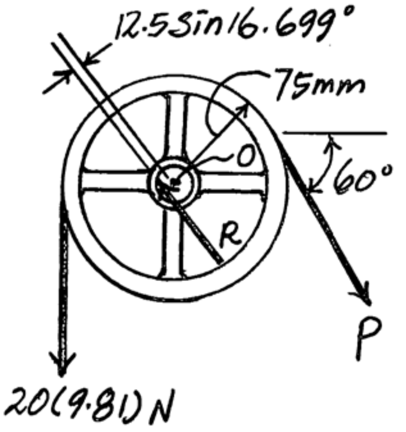
$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{(0.5P)^2 + (0.8660P + 196.2)^2}$$
$$= \sqrt{P^2 + 339.83P + 38494.44}$$

By referring to the geometry shown in Fig. b, we find that $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of $\mathbf{R}$ from point O is $(12.5\sin 16.699°)$ mm. Using these results and writing the moment equation about point O, Fig. a,

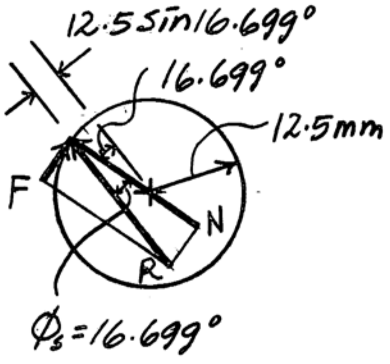
$\circlearrowleft + \Sigma M_O = 0;$ $\quad 20(9.81)(75) - P(75) - \sqrt{P^2 + 339.83P + 38494.44}\,(12.5\sin 16.699°) = 0$

Choosing the root $P < 20(9.81)$ N,

$$P = 179\text{ N}$$ **Ans.**

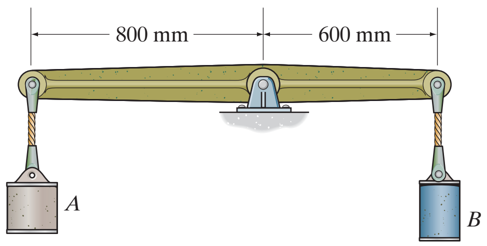


(a)



(b)

***8–128.** The cylinders are suspended from the end of the bar which fits loosely into a 40-mm-diameter pin. If *A* has a mass of 10 kg, determine the required mass of *B* which is just sufficient to keep the bar from rotating clockwise. The coefficient of static friction between the bar and the pin is $\mu_s = 0.3$. Neglect the mass of the bar.

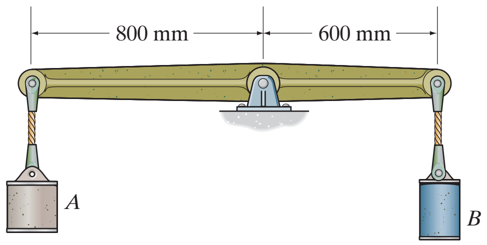


By referring to the geometry, we find that $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of **R** from point *O* is $(20\sin 16.699°)$ mm.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 10(9.81)(800 + 20\sin 16.699°) - m_B(9.81)(600 - 20\sin 16.699°) = 0$

$m_B = 13.6\text{ kg}$ **Ans.**

•8–129. The cylinders are suspended from the end of the bar which fits loosely into a 40-mm-diameter pin. If *A* has a mass of 10 kg, determine the required mass of *B* which is just sufficient to keep the bar from rotating counterclockwise. The coefficient of static friction between the bar and the pin is $\mu_s = 0.3$. Neglect the mass of the bar.

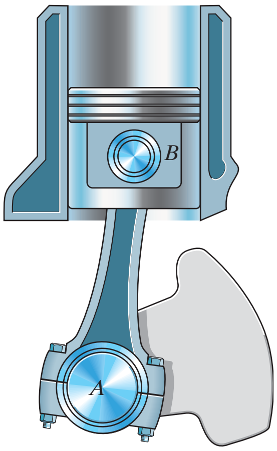


By referring to the geometry, we find that $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of **R** from point *O* is $(20\sin 16.699°)$ mm.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 10(9.81)(800 - 20\sin 16.699°) - m_B(9.81)(600 + 20\sin 16.699°) = 0$

$m_B = 13.1\text{ kg}$ **Ans.**

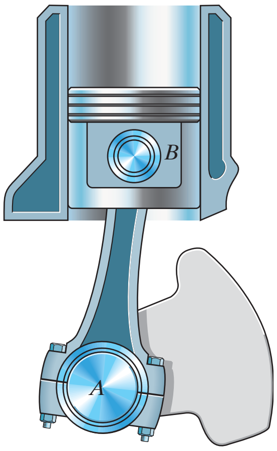
8–130. The connecting rod is attached to the piston by a 0.75-in.-diameter pin at *B* and to the crank shaft by a 2-in.-diameter bearing *A*. If the piston is moving downwards, and the coefficient of static friction at the contact points is $\mu_s = 0.2$, determine the radius of the friction circle at each connection.



$(r_f)_A = r_A \mu_s = 0.2\text{ in.}$ **Ans**

$(r_f)_B = r_B \mu_s = \dfrac{0.75(0.2)}{2} = 0.075\text{ in.}$ **Ans**

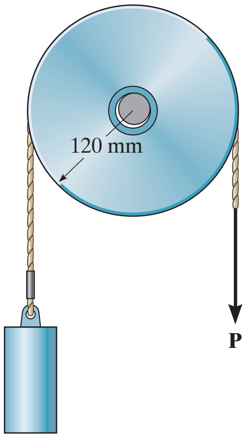
8–131. The connecting rod is attached to the piston by a 20-mm-diameter pin at B and to the crank shaft by a 50-mm-diameter bearing A. If the piston is moving upwards, and the coefficient of static friction at the contact points is $\mu_s = 0.3$, determine the radius of the friction circle at each connection.



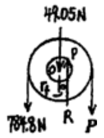
$(r_f)_A = r_A \mu_s = 25\,(0.3) = 7.50$ mm **Ans**

$(r_f)_B = r_B \mu_s = 10\,(0.3) = 3$ mm **Ans**

***8–132.** The 5-kg pulley has a diameter of 240 mm and the axle has a diameter of 40 mm. If the coefficient of kinetic friction between the axle and the pulley is $\mu_k = 0.15$, determine the vertical force P on the rope required to lift the 80-kg block at constant velocity.



$\mu = 0.15$



$\phi_k = \tan^{-1}(0.15) = 8.531°$

$r_f = r \sin\phi_k = 20 \sin 8.531° = 2.967$ mm

By approximation,

$r_f = r\mu = 20(0.15) = 3.00$ mm

$\circlearrowleft + \Sigma M_P = 0;$ $-784.8(120 + r_f) - 49.05\, r_f + P(120 - r_f) = 0$

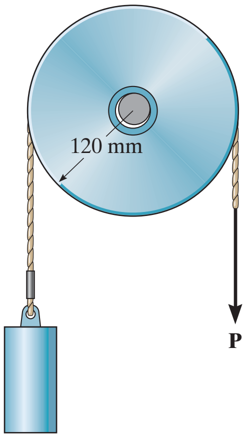
If exact value of r_f (2.967 mm) is used,

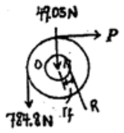
Thus $P = 826$ N **Ans**

If approximate value of r_f (3.00 mm) is used,

also $P = 826$ N **Ans** (approx.)

•8–133. Solve Prob. 8–132 if the force **P** is applied horizontally to the right.





$\phi_k = \tan^{-1}(0.15) = 8.531°$

$r_f = r\sin\phi_k = 20\sin 8.531° = 2.967$ mm

By approximation,

$r_f = r\mu = 20(0.15) = 3.00$ mm

$\circlearrowleft +\Sigma M_O = 0;\qquad 784.8(0.120) + R(r_f) - P(0.120) = 0$

$\xrightarrow{+}\Sigma F_x = 0;\qquad R_x = P$

$+\uparrow\Sigma F_y = 0;\qquad R_y = 833.85$

$$R = \sqrt{P^2 + (833.85)^2}$$

$$-94.18 + 0.12P = \sqrt{P^2 + (833.85)^2}\,r_f$$

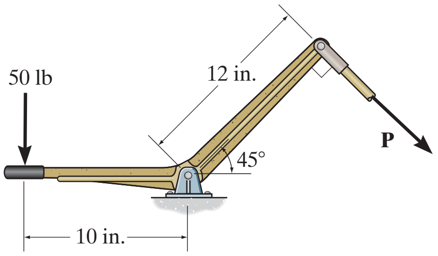
If exact value of r_f (0.002967 m) is used, then

$P = 814$ N **Ans**

If approximate value of r_f (0.003 m) is used, then

$P = 814$ N **Ans** (approx.)

8–134. The bell crank fits loosely into a 0.5-in-diameter pin. Determine the required force P which is just sufficient to rotate the bell crank clockwise. The coefficient of static friction between the pin and the bell crank is $\mu_s = 0.3$.



$\xrightarrow{+}\Sigma F_x = 0;\qquad P\cos 45° - R_x = 0\qquad R_x = 0.7071P$

$+\uparrow\Sigma F_y = 0;\qquad R_y - P\sin 45° - 50 = 0\qquad R_y = 0.7071P + 50$

Thus, the magnitude of **R** is

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(0.7071P)^2 + (0.7071P + 50)^2}$$
$$= \sqrt{P^2 + 70.71P + 2500}$$

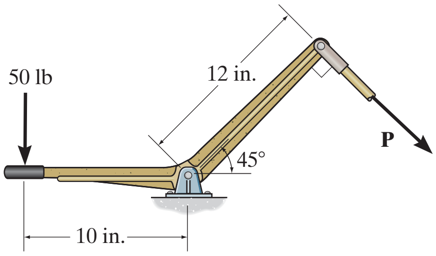
We find that $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of **R** from point O is $(0.25\sin 16.699°)$ mm. Using these results and writing the moment equation about point O, Fig. a,

$\circlearrowleft +\Sigma M_O = 0;\qquad 50(10) + \sqrt{P^2 + 70.71P + 2500}(0.25\sin 16.699°) - P(12) = 0$

Choosing the larger root,

$P = 42.2$ lb **Ans.**

8–135. The bell crank fits loosely into a 0.5-in-diameter pin. If $P = 41$ lb, the bell crank is then on the verge of rotating counterclockwise. Determine the coefficient of static friction between the pin and the bell crank.



$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad 41\cos 45° - R_x = 0 \quad R_x = 28.991\text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad R_y - 41\sin 45° - 50 = 0 \quad R_y = 78.991\text{ lb}$

Thus, the magnitude of **R** is

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{28.991^2 + 78.991^2} = 84.144\text{ lb}$$

We find that the moment arm of **R** from point O is $0.25\sin\phi_s$.

Using these results and writing the moment equation about point O, Fig. a,

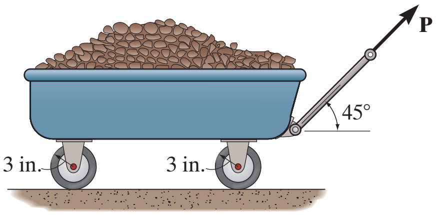
$\circlearrowleft + \Sigma M_O = 0;$ $\quad 50(10) - 41(12) - 84.144(0.25\sin\phi_s) = 0$

$\phi_s = 22.35°$

Thus,

$$\mu_s = \tan\phi_s = \tan 22.35° = 0.411 \qquad \textbf{Ans.}$$

***8–136.** The wagon together with the load weighs 150 lb. If the coefficient of rolling resistance is $a = 0.03$ in., determine the force P required to pull the wagon with constant velocity.



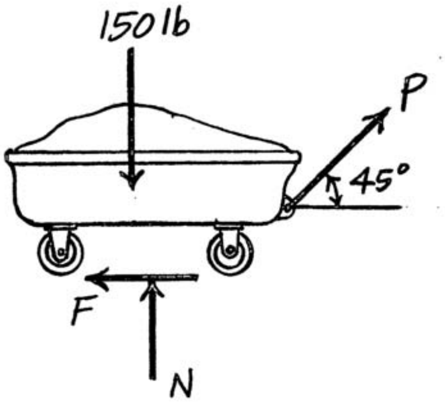
The normal reaction **N** on the wheels can be obtained by referring to the free - body diagram of the wagon shown in Fig. *a*.

$+\uparrow \Sigma F_y = 0; \qquad N + P\sin 45° - 150 = 0 \qquad N = 150 - 0.7071P$

Since the rolling resistance of the wheels is $F = \dfrac{Wa}{r}$, where $W = N = 150 - 0.7071P$, $a = 0.03$ in. and $r = 3$ in., then

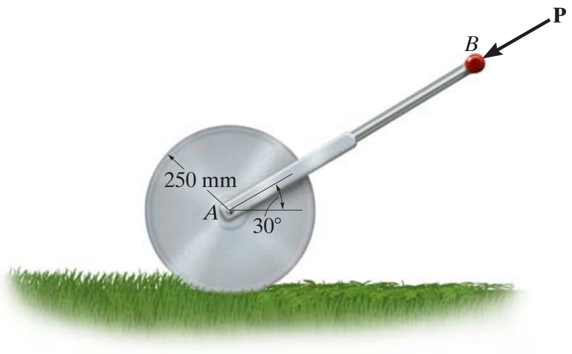
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P\cos 45° - \frac{(150 - 0.7071P)(0.03)}{3} = 0$$

$$P = 2.10 \text{ lb}$$ **Ans.**



(a)

•8–137. The lawn roller has a mass of 80 kg. If the arm *BA* is held at an angle of 30° from the horizontal and the coefficient of rolling resistance for the roller is 25 mm, determine the force *P* needed to push the roller at constant speed. Neglect friction developed at the axle, *A*, and assume that the resultant force **P** acting on the handle is applied along arm *BA*.

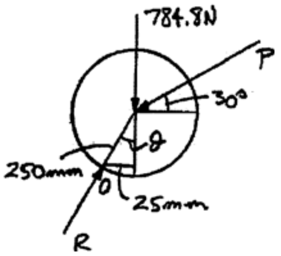


$$\theta = \sin^{-1}\left(\frac{25}{250}\right) = 5.74^\circ$$

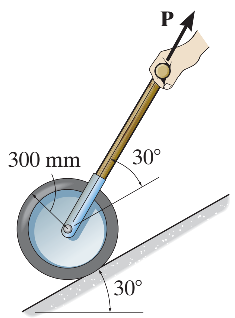
$$\circlearrowleft + \Sigma M_O = 0; \qquad -25\,(784.8) - P\sin 30^\circ\,(25) + P\cos 30^\circ\,(250\cos 5.74^\circ) = 0$$

Solving,

$$P = 96.7\text{ N} \qquad \textbf{Ans}$$



8–138. Determine the force P required to overcome rolling resistance and pull the 50-kg roller up the inclined plane with constant velocity. The coefficient of rolling resistance is $a = 15$ mm.



From the geometry indicated on the free - body diagram of the roller shown in Fig. *a*, $\theta = \sin^{-1}\left(\frac{15}{300}\right) = 2.866°$.

We have

$\overset{+}{\nearrow} \Sigma F_{x'} = 0; \quad P\cos 30° - 50(9.81)\sin 30° - R\sin 2.866° = 0$

$+\nwarrow \Sigma F_{y'} = 0; \quad P\sin 30° + R\cos 2.866° - 50(9.81)\cos 30° = 0$

Solving,

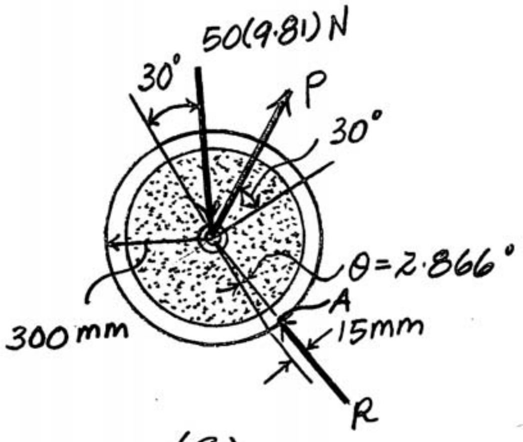
$$P = 299\text{ N} \qquad \textbf{Ans.}$$

$$R = 275.58\text{ N}$$

P can also be obtained directly by writing the moment equation of equilibrium about point A. Referring to Fig. *a*,

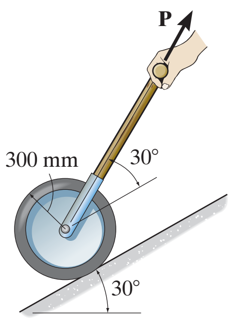
$\circlearrowleft + \Sigma M_A = 0; \quad 50(9.81)\sin(30° + 2.866°)(300) - P\cos(30° - 2.866°)(300) = 0$

$$P = 299\text{ N} \qquad \textbf{Ans.}$$



(a)

8–139. Determine the force P required to overcome rolling resistance and support the 50-kg roller if it rolls down the inclined plane with constant velocity. The coefficient of rolling resistance is $a = 15$ mm.



From the geometry indicated on the free - body diagram of the roller shown in Fig. a, $\theta = \sin^{-1}\left(\dfrac{15}{300}\right) = 2.866°$.

$\nearrow^{+} \Sigma F_{x'} = 0;\quad P\cos 30° + R\sin 2.866° - 50(9.81)\sin 30° = 0$

$+\nwarrow \Sigma F_{y'} = 0;\quad P\sin 30° + R\cos 2.866° - 50(9.81)\cos 30° = 0$

Solving,

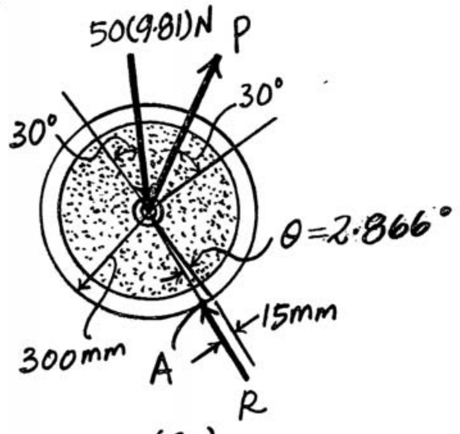
$P = 266\text{ N}$ **Ans.**

$R = 291.98\text{ N}$

P can also be obtained directly by writing the moment equation of equilibrium about point A. Referring to Fig. a,

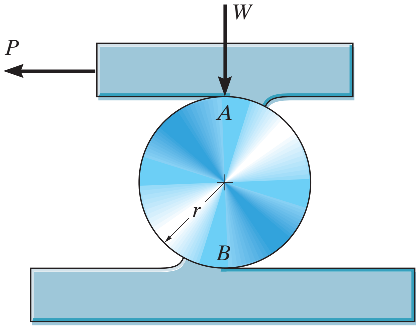
$\circlearrowleft + \Sigma M_A = 0;\quad 50(9.81)\sin(30° - 2.866°)(300) - P\cos(30° + 2.866°)(300) = 0$

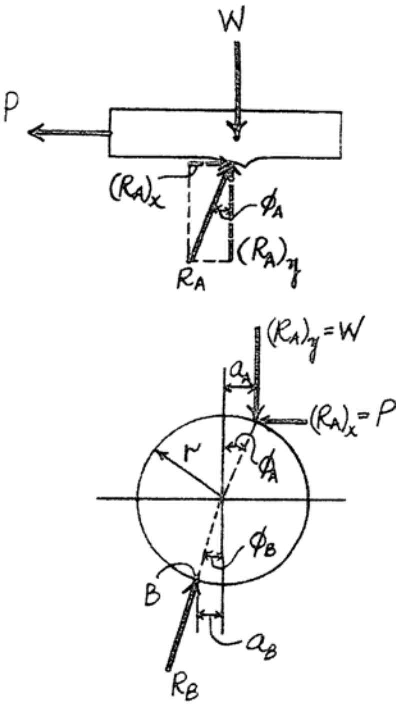
$P = 266\text{ N}$ **Ans.**



(a)

***8–140.** The cylinder is subjected to a load that has a weight W. If the coefficients of rolling resistance for the cylinder's top and bottom surfaces are a_A and a_B, respectively, show that a horizontal force having a magnitude of $P = [W(a_A + a_B)]/2r$ is required to move the load and thereby roll the cylinder forward. Neglect the weight of the cylinder.





$\xrightarrow{+} \Sigma F_x = 0;\qquad (R_A)_x - P = 0 \qquad (R_A)_x = P$

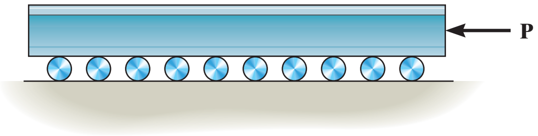
$+\uparrow \Sigma F_y = 0;\qquad (R_A)_y - W = 0 \qquad (R_A)_y = W$

$\circlearrowleft + \Sigma M_B = 0;\qquad P(r\cos\phi_A + r\cos\phi_B) - W(a_A + a_B) = 0 \qquad (1)$

Since ϕ_A and ϕ_B are very small, $\cos\phi_A \approx \cos\phi_B = 1$. Hence, from Eq. (1)

$$P = \frac{W(a_A + a_B)}{2r} \qquad \text{(QED)}$$

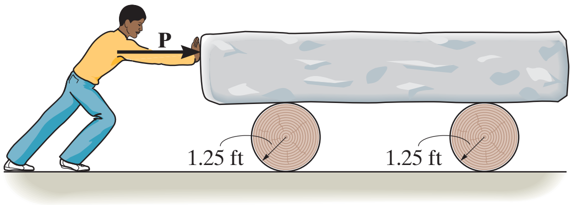
•8–141. The 1.2-Mg steel beam is moved over a level surface using a series of 30-mm-diameter rollers for which the coefficient of rolling resistance is 0.4 mm at the ground and 0.2 mm at the bottom surface of the beam. Determine the horizontal force P needed to push the beam forward at a constant speed. *Hint:* Use the result of Prob. 8–140.



$$P = \frac{W(a_A + a_B)}{2r} = \frac{(1200)(9.81)(0.2 + 0.4)}{2(15)}$$

$P = 235\text{ N}$ **Ans**

8–142. Determine the smallest horizontal force P that must be exerted on the 200-lb block to move it forward. The rollers each weigh 50 lb, and the coefficient of rolling resistance at the top and bottom surfaces is $a = 0.2$ in.



In general :

$$\circlearrowleft + \Sigma M_B = 0; \quad P\,(r\cos\phi_A + r\cos\phi_B) - W_1\,(a_A + a_B) - W_2\,a_B = 0$$

Since ϕ_A and ϕ_B are very small, $\cos\phi_A = \cos\phi_B = 1$. Hence,

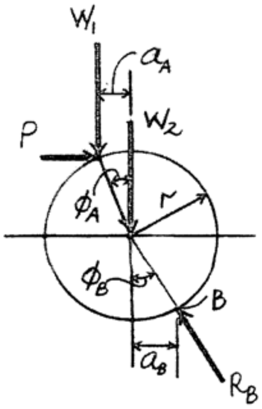
$$P\,(2r) = W_1\,(a_A + a_B) + W_2\,a_B$$

$$P = \frac{W_1\,(a_A + a_B) + W_2\,a_B}{2r}$$

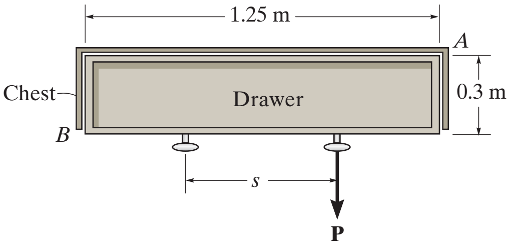
Thus, for the problem,

$$P = \left(\frac{200\,(0.2 + 0.2) + 2\,(50)\,(0.2)}{2\,(1.25)}\right)$$

$P = 40$ lb **Ans**



8–143. A single force **P** is applied to the handle of the drawer. If friction is neglected at the bottom and the coefficient of static friction along the sides is $\mu_s = 0.4$, determine the largest spacing s between the symmetrically placed handles so that the drawer does not bind at the corners A and B when the force **P** is applied to one of the handles.



Equations of Equilibrium and Friction : If the drawer does not bind at corners A and B, slipping would have to occur at points A and B. Hence, $F_A = \mu N_A = 0.4N_A$ and $F_B = \mu N_B = 0.4N_B$.

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B - N_A = 0 \quad N_A = N_B = N$$

$$+\uparrow \Sigma F_y = 0; \quad 0.4N + 0.4N - P = 0 \quad P = 0.8N$$

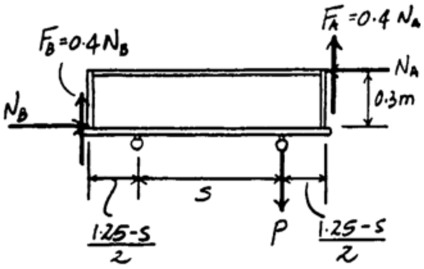
$$\circlearrowleft + \Sigma M_B = 0; \quad N(0.3) + 0.4N(1.25) - 0.8N\left(\frac{s + 1.25}{2}\right) = 0$$

$$N\left[0.3 + 0.5 - 0.8\left(\frac{s + 1.25}{2}\right)\right] = 0$$

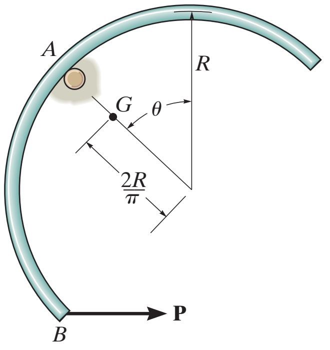
–Since $N \neq 0$, then

$$0.3 + 0.5 - 0.8\left(\frac{s + 1.25}{2}\right) = 0$$

$s = 0.750$ m **Ans**



***8–144.** The semicircular thin hoop of weight W and center of gravity at G is suspended by the small peg at A. A horizontal force **P** is slowly applied at B. If the hoop begins to slip at A when $\theta = 30°$, determine the coefficient of static friction between the hoop and the peg.



$\xrightarrow{+} \Sigma F_x = 0;\quad P + F_A \cos 30° - N_A \sin 30° = 0$

$+\uparrow \Sigma F_y = 0;\quad F_A \sin 30° + N_A \cos 30° - W = 0$

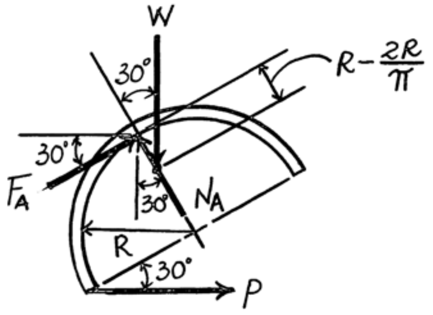
$\circlearrowleft + \Sigma M_A = 0;\quad -W \sin 30° \left(R - \frac{2R}{\pi}\right) + P \sin 30° (R) + P \cos 30° (R) = 0$

$P = 0.1330\,W$

$0.1330\,(F_A \sin 30° + N_A \cos 30°) + F_A \cos 30° - N_A \sin 30° = 0$

$F_A\,(0.9325) - N_A\,(0.3848) = 0$

$\mu_A = \frac{F_A}{N_A} = \frac{0.3848}{0.9325} = 0.413$ **Ans**



Also,

$+\nwarrow \Sigma F_x = 0;\quad N_A - W \cos 30° - P \sin 30° = 0$

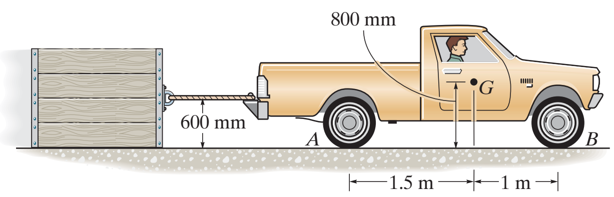
$+\nearrow \Sigma F_y = 0;\quad \mu_A N_A - W \sin 30° + P \cos 30° = 0$

$\circlearrowleft + \Sigma M_A = 0;\quad -W \sin 30° (R) \left(1 - \frac{2}{\pi}\right) + P \cos 30° (R) + P \sin 30° (R) = 0$

$P = 0.133\,W$

$\mu_A = 0.413$ **Ans**

•8–145. The truck has a mass of 1.25 Mg and a center of mass at G. Determine the greatest load it can pull if (a) the truck has rear-wheel drive while the front wheels are free to roll, and (b) the truck has four-wheel drive. The coefficient of static friction between the wheels and the ground is $\mu_s = 0.5$, and between the crate and the ground, it is $\mu_s' = 0.4$.



a) The truck with rear wheel drive.

Equations of Equilibrium and Friction : It is required that the rear wheels of the truck slip. Hence $F_A = \mu_s N_A = 0.5N_A$. From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \quad 1.25\left(10^3\right)(9.81)(1) + T(0.6) - N_A(2.5) = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.5N_A - T = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields

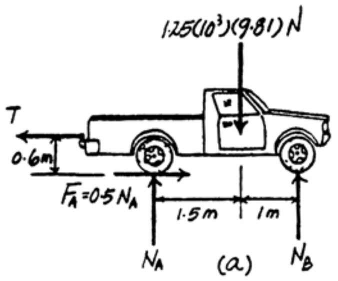
$$N_A = 5573.86 \text{ N} \qquad T = 2786.93 \text{ N}$$

Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

$$+\uparrow \Sigma F_y = 0; \quad N_C - W = 0 \quad N_C = W$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 2786.93 - 0.4W = 0$$

$$W = 6967.33 \text{ N} = 6.97 \text{ kN} \qquad \textbf{Ans}$$



b) The truck with four wheel drive.

Equations of Equilibrium and Friction : It is required that the rear wheels and front wheels of the truck slip. Hence $F_A = \mu_s N_A = 0.5N_A$ and $F_B = \mu_s N_B = 0.5N_B$. From FBD (b),

$$\circlearrowleft + \Sigma M_B = 0; \quad 1.25\left(10^3\right)(9.81)(1) + T(0.6) - N_A(2.5) = 0 \qquad [3]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(2.5) + T(0.6) - 1.25\left(10^3\right)(9.81)(1.5) = 0 \qquad [4]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.5N_A + 0.5N_B - T = 0 \qquad [5]$$

Solving Eqs. [3], [4] and [5] yields

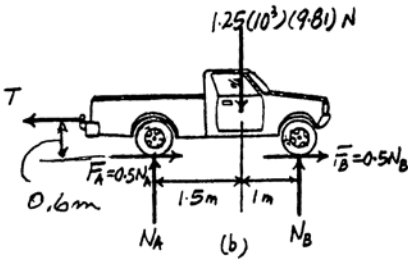
$$N_A = 6376.5 \text{ N} \qquad N_B = 5886.0 \text{ N} \qquad T = 6131.25 \text{ N}$$

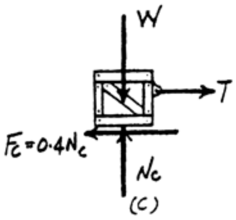
Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

$$+\uparrow \Sigma F_y = 0; \quad N_C - W = 0 \quad N_C = W$$

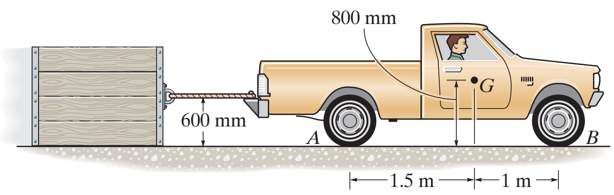
$$\xrightarrow{+} \Sigma F_x = 0; \quad 6131.25 - 0.4W = 0$$

$$W = 15328.125 \text{ N} = 15.3 \text{ kN} \qquad \textbf{Ans}$$





8–146. Solve Prob. 8–145 if the truck and crate are traveling up a 10° incline.



a) **The truck with rear wheel drive.**

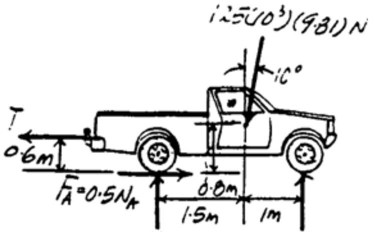
Equations of Equilibrium and Friction : It is required that the rear wheel of the truck slip hence $F_A = \mu_s N_A = 0.5N_A$. From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \quad 1.25(10^3)(9.81)\cos 10°(1) + 1.25(10^3)(9.81)\sin 10°(0.8) + T(0.6) - N_A(2.5) = 0 \qquad [1]$$

$$\nearrow + \Sigma F_{x'} = 0; \quad 0.5N_A - 1.25(10^3)(9.81)\sin 10° - T = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$N_A = 5682.76\text{ N} \qquad T = 712.02\text{ N}$$



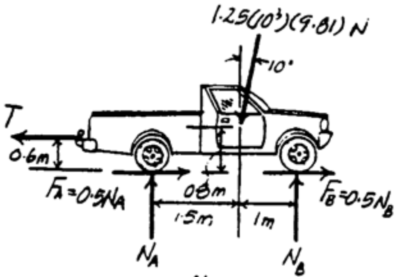
(a)

Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_C - W\cos 10° = 0 \qquad N_C = 0.9848W$$

$$\nearrow + \Sigma F_x = 0; \quad 712.02 - W\sin 10° - 0.4(0.9848W) = 0$$

$$W = 1254.50\text{ N} = 1.25\text{ kN} \qquad \textbf{Ans}$$



(b)

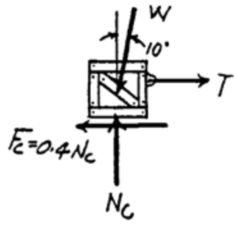
b) **The truck with four wheel drive.**

Equations of Equilibrium and Friction : It is required that the rear wheels of the truck slip hence $F_A = \mu_s N_A = 0.5N_A$. From FBD (b),

$$\circlearrowleft + \Sigma M_B = 0; \quad 1.25(10^3)(9.81)\cos 10°(1) + 1.25(10^3)(9.81)\sin 10°(0.8) + T(0.6) - N_A(2.5) = 0 \qquad [3]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -1.25(10^3)(9.81)\cos 10°(1.5) + 1.25(10^3)(9.81)\sin 10°(0.8) + T(0.6) + N_B(2.5) = 0 \qquad [4]$$

$$\nearrow + \Sigma F_{x'} = 0; \quad 0.5N_A + 0.5N_B - 1.25(10^3)(9.81)\sin 10° - T = 0 \qquad [5]$$



(c)

Solving Eqs. [3], [4] and [5] yields

$$N_A = 6449.98\text{ N} \qquad N_B = 5626.23\text{ N} \qquad T = 3908.74\text{ N}$$

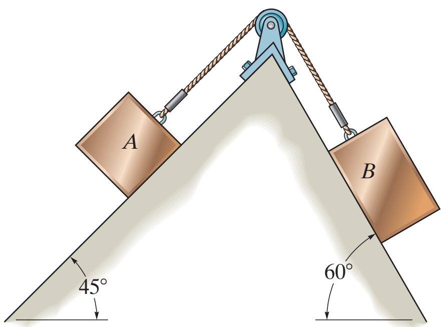
Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

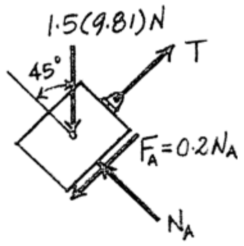
$$\uparrow + \Sigma F_{y'} = 0; \quad N_C - W\cos 10° = 0 \qquad N_C = 0.9848W$$

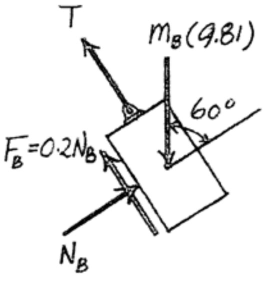
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 3908.74 - W\sin 10° - 0.4(0.9848W) = 0$$

$$W = 6886.79\text{ N} = 6.89\text{ kN} \qquad \textbf{Ans}$$

8–147. If block *A* has a mass of 1.5 kg, determine the largest mass of block *B* without causing motion of the system. The coefficient of static friction between the blocks and inclined planes is $\mu_s = 0.2$.







By inspection, *B* will tend to move down the plane.

Block *A* :

$+\nearrow \Sigma F_x = 0; \quad T - 0.2N_A - 1.5(9.81)\sin 45° = 0$

$+\nwarrow \Sigma F_y = 0; \quad N_A - 1.5(9.81)\cos 45° = 0$

Block *B* :

$+\nwarrow \Sigma F_x = 0; \quad T + 0.2N_B - 9.81(m_B)\sin 60° = 0$

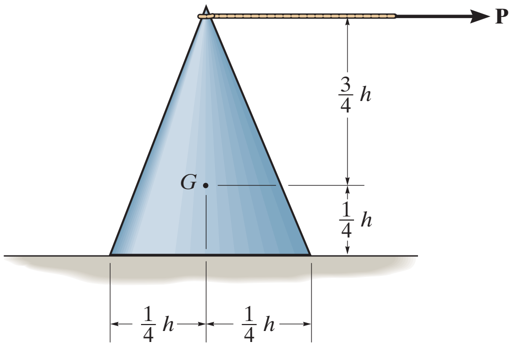
$+\nearrow \Sigma F_y = 0; \quad N_B - 9.81(m_B)\cos 60° = 0$

Solving,

$N_A = 10.4\text{ N}; \quad N_B = 8.15\text{ N}; \quad T = 12.5\text{ N};$

$m_B = 1.66\text{ kg}$ **Ans**

***8–148.** The cone has a weight W and center of gravity at G. If a horizontal force **P** is gradually applied to the string attached to its vertex, determine the maximum coefficient of static friction for slipping to occur.



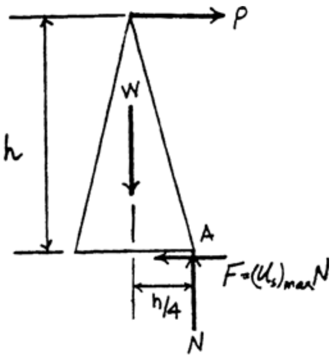
Equations of Equilibrium : In this case, it is required that the cone slips and about to tip about point A. Hence, $F = (\mu_s)_{\max} N$.

$$\circlearrowleft + \Sigma M_A = 0; \quad W\left(\frac{h}{4}\right) - P(h) = 0 \quad P = \frac{W}{4}$$

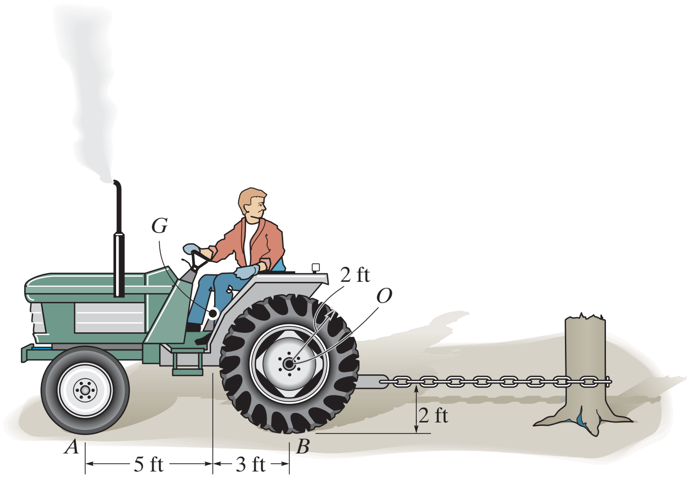
$$+\uparrow \Sigma F_y = 0; \quad N - W = 0 \quad N = W$$

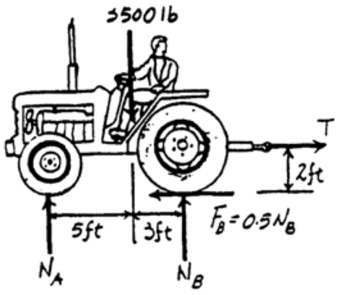
$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{W}{4} - (\mu_s)_{\max} W = 0$$

$$(\mu_s)_{\max} = 0.250 \qquad \textbf{Ans}$$



•8–149. The tractor pulls on the fixed tree stump. Determine the torque that must be applied by the engine to the rear wheels to cause them to slip. The front wheels are free to roll. The tractor weighs 3500 lb and has a center of gravity at G. The coefficient of static friction between the rear wheels and the ground is $\mu_s = 0.5$.





(a)

Equations of Equilibrium and Friction : Assume that the rear wheels B slip. Hence $F_B = \mu_s N_B = 0.5N_B$.

$$\circlearrowleft + \Sigma M_A = 0 \qquad N_B(8) - T(2) - 3500(5) = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B + N_A - 3500 = 0 \qquad [2]$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T - 0.5N_B = 0 \qquad [3]$$

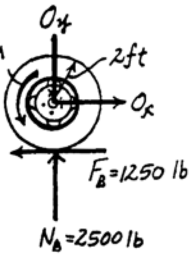
Solving Eqs. [1], [2] and [3] yields

$$N_A = 1000 \text{ lb} \qquad N_B = 2500 \text{ lb} \qquad T = 1250 \text{ lb}$$

Since $N_A > 0$, the front wheels do not lift up. Therefore the rear wheels slip as assumed. Thus, $F_B = 0.5(2500) = 1250$ lb. From FBD (b),

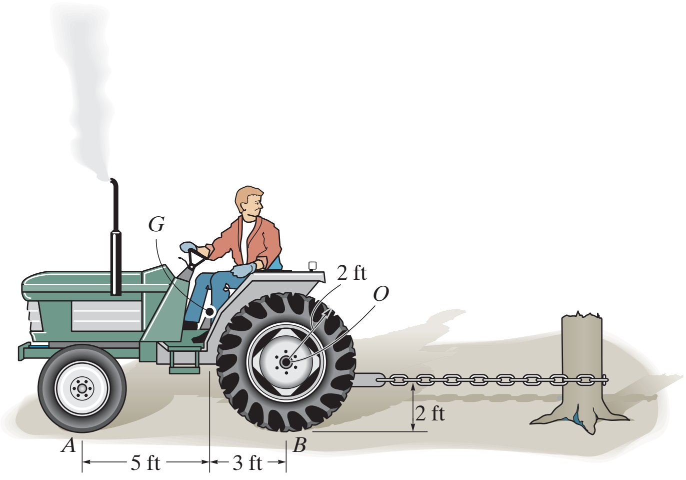
$$\circlearrowleft + \Sigma M_O = 0, \qquad M - 1250(2) = 0$$

$$M = 2500 \text{ lb} \cdot \text{ft} = 2.50 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$



(b)

8–150. The tractor pulls on the fixed tree stump. If the coefficient of static friction between the rear wheels and the ground is $\mu_s = 0.6$, determine if the rear wheels slip or the front wheels lift off the ground as the engine provides torque to the rear wheels. What is the torque needed to cause this motion? The front wheels are free to roll. The tractor weighs 2500 lb and has a center of gravity at G.

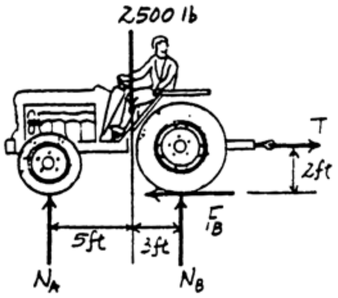


Equations of Equilibrium and Friction : Assume that the rear wheels B slip. Hence $F_B = \mu_s N_B = 0.6N_B$.

$$\curvearrowleft + \Sigma M_A = 0 \qquad N_B(8) - T(2) - 2500(5) = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B + N_A - 2500 = 0 \qquad [2]$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T - 0.6N_B = 0 \qquad [3]$$

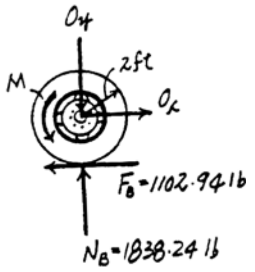


(a)

Solving Eqs. [1], [2] and [3] yields

$$N_A = 661.76 \text{ lb} \qquad N_B = 1838.24 \text{ lb} \qquad T = 1102.94 \text{ lb}$$

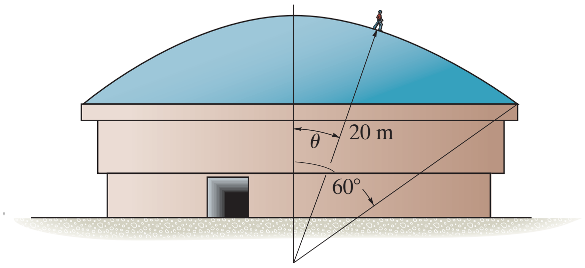
Since $N_A > 0$, **the front wheels do not lift off the ground.** Therefore the rear wheels slip as assumed. Thus, $F_B = 0.6(1838.24) = 1102.94$ lb. From FBD (b),

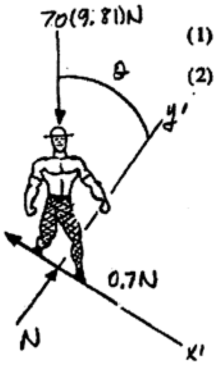


$$+\Sigma M_O = 0, \qquad M - 1102.94(2) = 0$$

$$M = 2205.88 \text{ lb} \cdot \text{ft} = 2.21 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

8–151. A roofer, having a mass of 70 kg, walks slowly in an upright position down along the surface of a dome that has a radius of curvature of $r = 20$ m. If the coefficient of static friction between his shoes and the dome is $\mu_s = 0.7$, determine the angle θ at which he first begins to slip.





$$+\nearrow \Sigma F_{y'} = 0; \quad N - 70(9.81)\cos\theta = 0 \qquad (1)$$

$$+\searrow \Sigma F_{x'} = 0; \quad 70(9.81)\sin\theta - 0.7N = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$\theta = 35.0°$ **Ans**

$N = 562.6$ N

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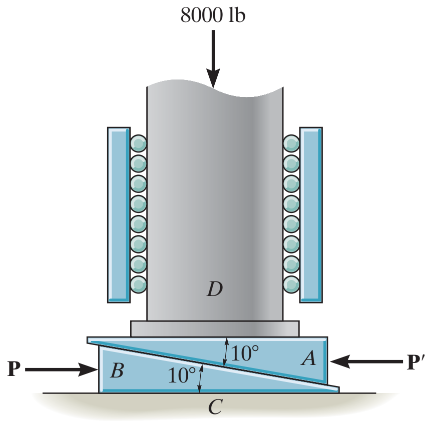
*__8–152.__ Column D is subjected to a vertical load of 8000 lb. It is supported on two identical wedges A and B for which the coefficient of static friction at the contacting surfaces between A and B and between B and C is $\mu_s = 0.4$. Determine the force P needed to raise the column and the equilibrium force P' needed to hold wedge A stationary. The contacting surface between A and D is smooth.



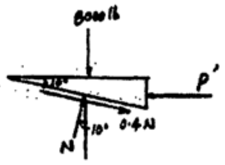
Wedge A :

$$+\uparrow \Sigma F_y = 0; \quad N\cos 10° - 0.4N\sin 10° - 8000 = 0$$

$$N = 8739.8 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.4(8739.8)\cos 10° + 8739.8\sin 10° - P' = 0$$

$P' = 4960 \text{ lb} = 4.96$ kip **Ans**



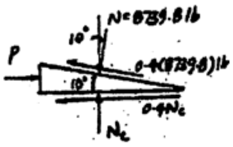
Wedge B :

$$+\uparrow \Sigma F_y = 0; \quad N_C + 0.4(8739.8)\sin 10° - 8739.8\cos 10° = 0$$

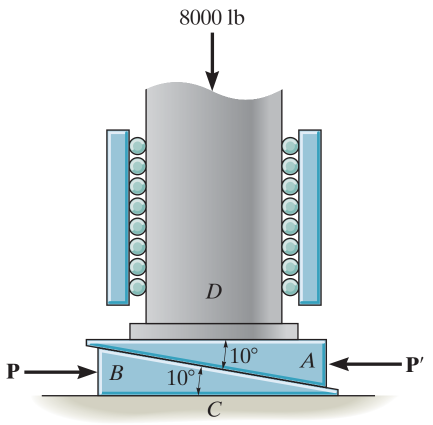
$$N_C = 8000 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.4(8000) - 8739.8\sin 10° - 0.4(8739.8)\cos 10° = 0$$

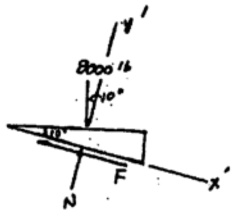
$P = 8160 \text{ lb} = 8.16$ kip **Ans**



•8–153. Column *D* is subjected to a vertical load of 8000 lb. It is supported on two identical wedges *A* and *B* for which the coefficient of static friction at the contacting surfaces between *A* and *B* and between *B* and *C* is $\mu_s = 0.4$. If the forces **P** and **P′** are removed, are the wedges self-locking? The contacting surface between *A* and *D* is smooth.



Wedge *A* :



$\nwarrow\!+\ \Sigma F_{y'} = 0;\qquad N - 8000\cos 10^\circ = 0$

$N = 7878.5\ \text{lb}$

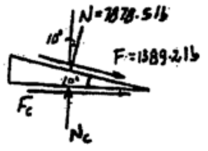
$\searrow\!+\ \Sigma F_{x'} = 0;\qquad 8000\sin 10^\circ - F = 0$

$F = 1389.2\ \text{lb}$

Since $F = 1389.2\ \text{lb} < 0.4(7878.5) = 3151.4\ \text{lb}$, the wedges do not slip at contact surface *AB*.

lip at

Wedge *B* :



$+\uparrow \Sigma F_y = 0;\qquad N_C - 1389.2\sin 10^\circ - 7878.5\cos 10^\circ = 0$

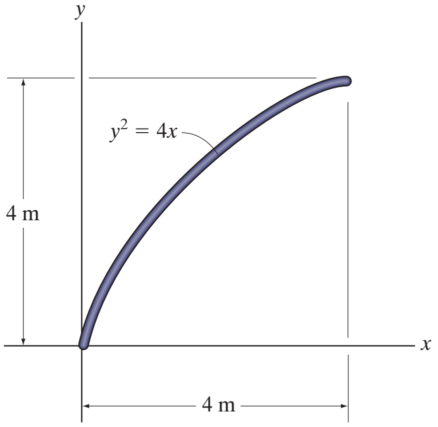
$N_C = 8000\ \text{lb}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_C + 1389.2\cos 10^\circ - 7878.5\sin 10^\circ = 0$

$F_C = 0$

Since $F_C = 0$, no slipping occurs at contact surface *BC*. Therefore, the wedges are self-locking. **Ans**

•9–1. Determine the mass and the location of the center of mass $(\bar{x}, \bar{y})$ of the uniform parabolic-shaped rod. The mass per unit length of the rod is 2 kg/m.



Differential Element. The length of the element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1+\left(\frac{dx}{dy}\right)^2}\right) dy$$

Here, $\frac{dx}{dy} = \frac{y}{2}$. Thus,

$$dL = \left(\sqrt{1+\left(\frac{y}{2}\right)^2}\right) dy = \frac{1}{2}\sqrt{y^2+4}\, dy$$

The mass of the element is

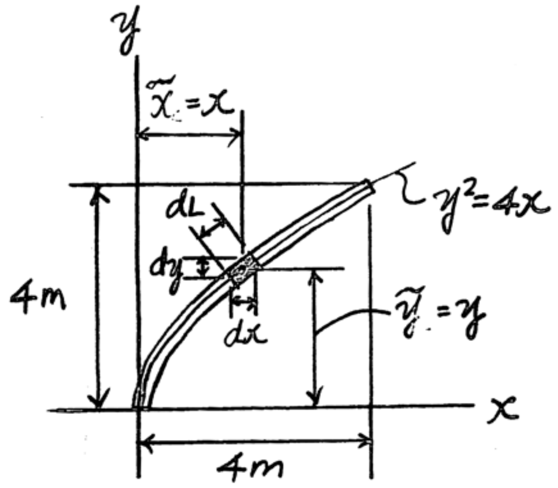
$$dm = \rho\, dL = 2\left(\frac{1}{2}\sqrt{y^2+4}\right) dy = \sqrt{y^2+4}\, dy$$

The centroid of the element is located at $\tilde{x} = x = \frac{y^2}{4}$ and $\tilde{y} = y$. Integrating,

$$m = \int_m dm = \int_0^{4\text{ m}} \sqrt{y^2+4}\, dy = 11.832 \text{ kg} = 11.8 \text{ kg} \qquad \textbf{Ans.}$$

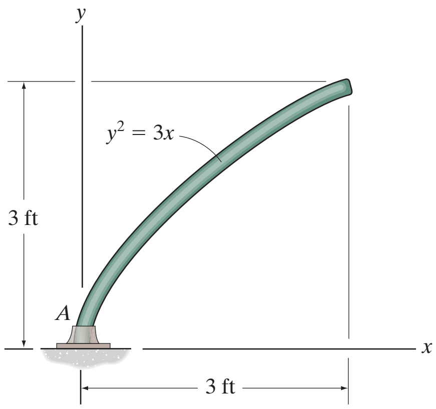
$$\bar{x} = \frac{\int_m \tilde{x}\, dm}{\int_m dm} = \frac{\int_0^{4\text{ m}} \frac{y^2}{4}\sqrt{y^2+4}\, dy}{\int_0^{4\text{ m}} \sqrt{y^2+4}\, dy} = \frac{19.403}{11.832} = 1.6399 \text{ m} = 1.64 \text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_m \tilde{y}\, dm}{\int_m dm} = \frac{\int_0^{4\text{ m}} y\sqrt{y^2+4}\, dy}{\int_0^{4\text{ m}} \sqrt{y^2+4}\, dy} = \frac{27.148}{11.832} = 2.2945 \text{ m} = 2.29 \text{ m} \qquad \textbf{Ans.}$$



(a)

9–2. The uniform rod is bent into the shape of a parabola and has a weight per unit length of 6 lb/ft. Determine the reactions at the fixed support A.



Differential Element. The length of the element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\right) dy$$

Here, $\frac{dx}{dy} = \frac{2}{3}y$. Thus,

$$dL = \left(\sqrt{1 + \left(\frac{2}{3}y\right)^2}\right) dy = \frac{1}{3}\sqrt{9^2 + 4y^2}\, dy$$

The weight of the element is therefore

$$dW = \gamma dL = 6\left[\frac{1}{3}\sqrt{9 + 4y^2}\, dy\right] = 2\sqrt{9 + 4y^2}\, dy$$

The centroid of the element is located at $\tilde{x} = x = \frac{y^2}{3}$. Integrating,

$$W = \int_W dW = \int_0^{3\text{ ft}} 2\sqrt{9 + 4y^2}\, dy = 26.621 \text{ lb}$$

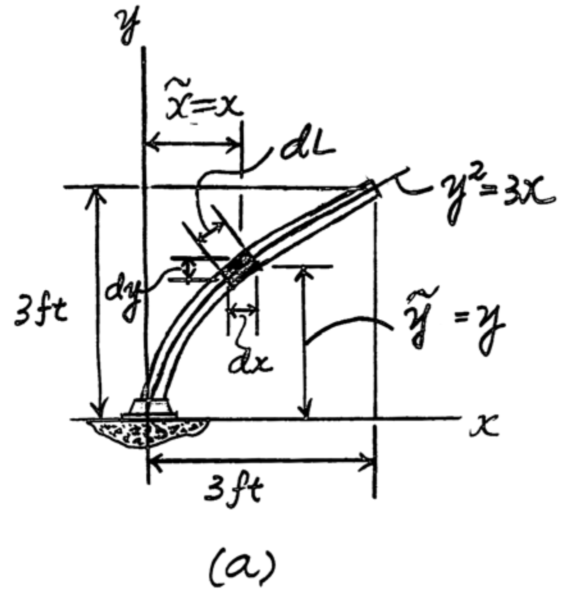
$$\bar{x} = \frac{\int_W \tilde{x}\, dW}{\int_W dW} = \frac{\int_0^{3\text{ ft}} \frac{y^2}{3}\left(2\sqrt{9 + 4y^2}\right) dy}{\int_0^{3\text{ ft}} 2\sqrt{9 + 4y^2}\, dy} = \frac{\frac{2}{3}\int_0^{3\text{ ft}} y^2\sqrt{9 + 4y^2}\, dy}{2\int_0^{3\text{ ft}} \sqrt{9 + 4y^2}\, dy} = \frac{32.742}{26.62} = 1.2299 \text{ ft}$$

Equations of Equilibrium: By referring to the free - body diagram of the rod shown in Fig. *b*, yields

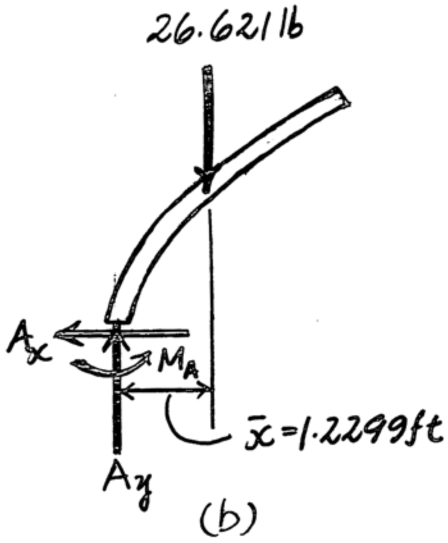
$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $A_y - 26.621 = 0$ $A_y = 26.621 \text{ lb} = 26.6 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;$ $M_A - 26.621(1.229) = 0$ $M_A = 32.74 \text{ lb}\cdot\text{ft} = 32.7 \text{ lb}\cdot\text{ft}$ **Ans.**

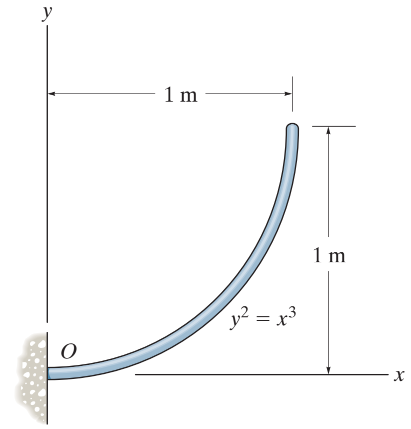


(a)



(b)

9–3. Determine the distance $\bar{x}$ to the center of mass of the homogeneous rod bent into the shape shown. If the rod has a mass per unit length of 0.5 kg/m, determine the reactions at the fixed support O.

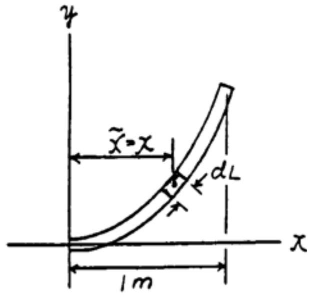


Length and Moment Arm : The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{x} = x$. Here, $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$.

Performing the integration, we have

$$L = \int dL = \int_0^{1\text{m}} \left(\sqrt{1 + \frac{9}{4}x}\right)dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}}\Bigg|_0^{1\text{m}} = 1.4397\text{ m}$$

$$\int_L \tilde{x}\,dL = \int_0^{1\text{m}} x\sqrt{1 + \frac{9}{4}x}\,dx$$
$$= \left[\frac{8}{27}x\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}} - \frac{64}{1215}\left(1 + \frac{9}{4}x\right)^{\frac{5}{2}}\right]\Bigg|_0^{1\text{m}}$$
$$= 0.7857$$



Centroid : Applying Eq. 9 – 7 , we have

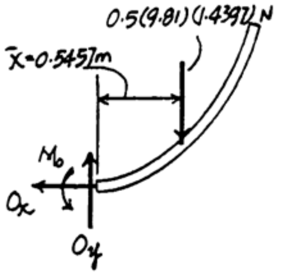
$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{0.7857}{1.4397} = 0.5457\text{ m} = 0.546\text{ m} \qquad \textbf{Ans}$$

Equations of Equilibrium :

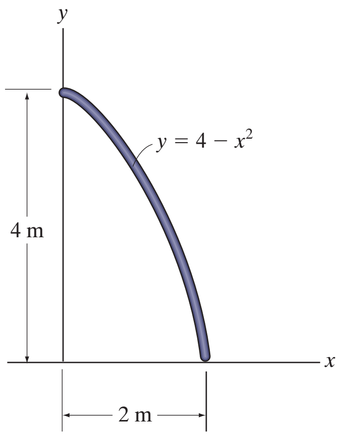
$$\xrightarrow{+} \Sigma F_x = 0; \qquad O_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad O_y - 0.5(9.81)(1.4397) = 0$$
$$O_y = 7.06\text{ N} \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M_O - 0.5(9.81)(1.4397)(0.5457) = 0$$
$$M_O = 3.85\text{ N}\cdot\text{m} \qquad \textbf{Ans}$$



***9–4.** Determine the mass and locate the center of mass $(\bar{x}, \bar{y})$ of the uniform rod. The mass per unit length of the rod is 3 kg/m.



Differential Element. The length of the element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Here, $\frac{dy}{dx} = -2x$. Thus,

$$dL = \sqrt{1 + (-2x)^2}\,dx = \sqrt{1 + 4x^2}\,dx = 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx$$

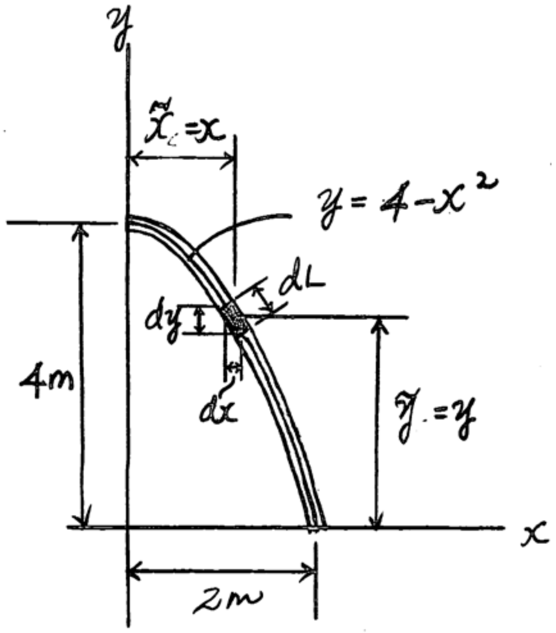
$$m = \int_0^{2\text{ m}} (3\text{ kg/m})\,2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx = 3(4.6468) = 13.9\text{ kg} \qquad \textbf{Ans.}$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y$.

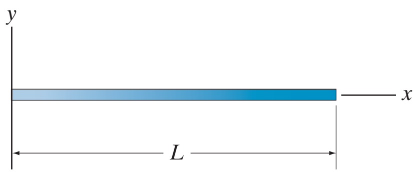
$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{2\text{ m}} x\left[2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx\right]}{\int_0^{2\text{ m}} 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx} = \frac{5.7577}{4.6468} = 1.24\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{2\text{ m}} (4 - x^2)\left[2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx\right]dx}{\int_0^{2\text{ m}} 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx} = \frac{\int_0^{2\text{ m}} \left[8\sqrt{\left(\frac{1}{2}\right)^2 + x^2} - 2x^2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\right]dx}{\int_0^{2\text{ m}} 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx}$$

$$= \frac{10.1160}{4.6468} = 2.18\text{ m} \qquad \textbf{Ans.}$$



•9–5. Determine the mass and the location of the center of mass $\bar{x}$ of the rod if its mass per unit length is $m = m_0(1 + x/L)$.

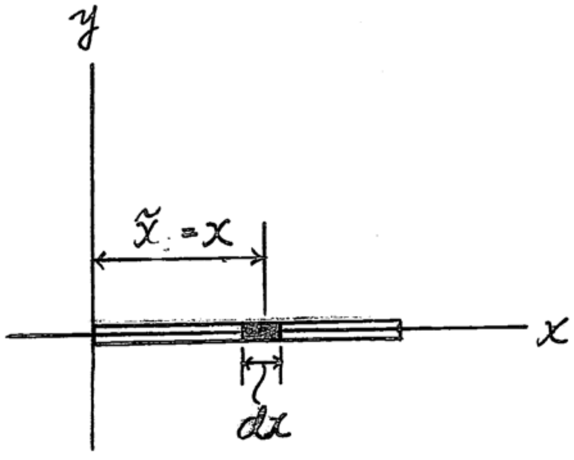


Differential Element. The element shown shaded in Fig. *a* has a mass of

$$\int_m dm = \int_0^L m_0\left(1+\frac{x}{L}\right)dx = \frac{3}{2}m_0L \qquad \textbf{Ans.}$$

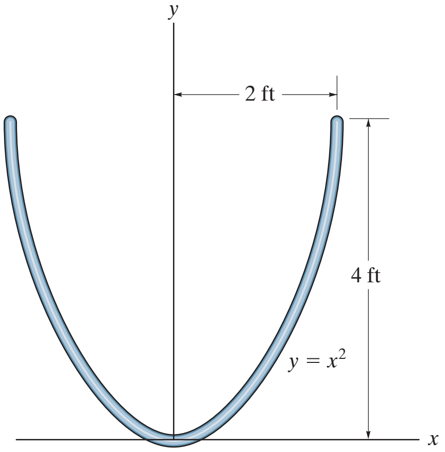
The centroid of the differential element is located at $x_c = x$.

$$\bar{x} = \frac{\int_m \tilde{x}\,dm}{\int_m dm} = \frac{\int_0^L x\left[m_0\left(1+\frac{x}{L}\right)dx\right]}{\int_0^L m_0\left(1+\frac{x}{L}\right)dx} = \frac{\int_0^L \left(x+\frac{x^2}{L}\right)dx}{\int_0^L \left(1+\frac{x}{L}\right)dx} = \frac{5}{9}L \qquad \textbf{Ans.}$$



(a)

9–6. Determine the location $(\overline{x}, \overline{y})$ of the centroid of the wire.



Length and Moment Arm : The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{y} = y = x^2$. Here, $\frac{dy}{dx} = 2x$.

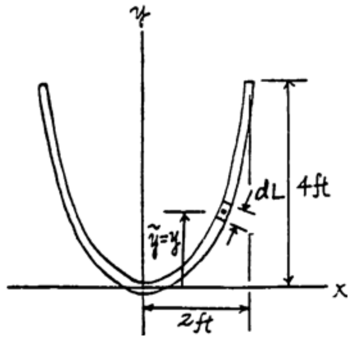
Centroid : Due to symmetry

$$\bar{x} = 0 \qquad \textbf{Ans}$$

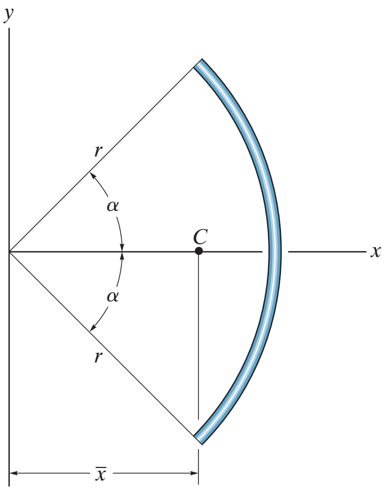
Applying Eq. 9–5 and performing the integration, we have

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_{-2\text{ft}}^{2\text{ft}} x^2\sqrt{1+4x^2}\,dx}{\int_{-2\text{ft}}^{2\text{ft}} \sqrt{1+4x^2}\,dx}$$

$$= \frac{16.9423}{9.2936} = 1.82\text{ ft} \qquad \textbf{Ans}$$



9–7. Locate the centroid $\overline{x}$ of the circular rod. Express the answer in terms of the radius r and semiarc angle α.



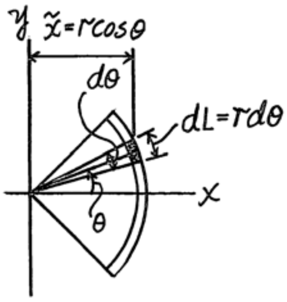
$$L = 2r\alpha$$

$$\tilde{x} = r\cos\theta$$

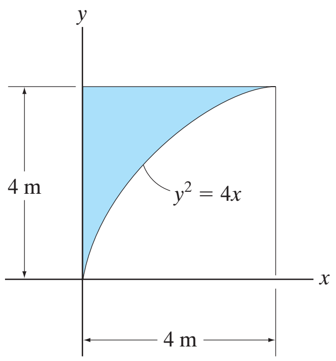
$$\int \tilde{x}\,dL = \int_{-\alpha}^{\alpha} r\cos\theta\; r\,d\theta$$

$$= 2r^2\sin\alpha$$

$$\bar{x} = \frac{2r^2\sin\alpha}{2r\alpha} = \frac{r\sin\alpha}{\alpha} \qquad \textbf{Ans}$$



***9–8.** Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the x axis shown shaded in Fig. *a* will be considered. The area of the element is

$$dA = x\,dy = \frac{y^2}{4}\,dy$$

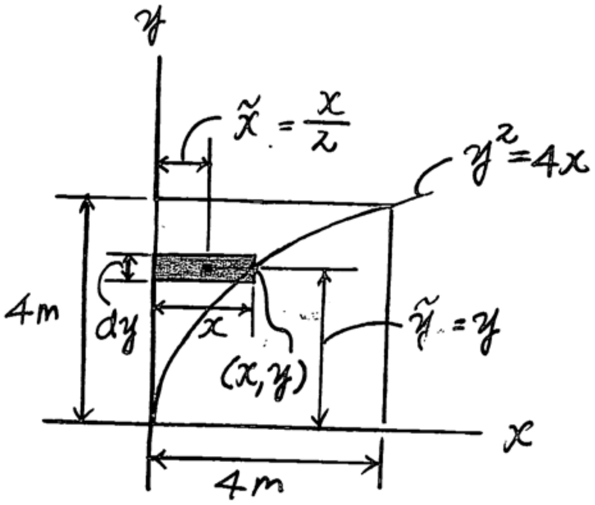
Centroid: The centroid of the element is located at $\tilde{x} = x/2 = \dfrac{(y^2/4)}{2} = \dfrac{y^2}{8}$ and $y_c = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^{4\text{ m}} \frac{y^2}{4}\,dy = \frac{y^3}{12}\bigg|_0^{4\text{ m}} = 5.333\text{ m}^2 = 5.33\text{ m}^2 \qquad \textbf{Ans.}$$

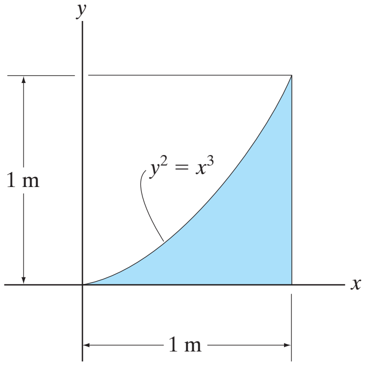
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} \frac{y^2}{8}\left(\frac{y^2}{4}\,dy\right)}{5.333} = \frac{\int_0^{4\text{ m}} \frac{y^4}{32}\,dy}{5.333} = \frac{\left(\frac{y^5}{160}\right)\Big|_0^{4\text{ m}}}{5.333} = 1.2\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} y\left(\frac{y^2}{4}\,dy\right)}{5.333} = \frac{\int_0^{4\text{ m}} \frac{y^3}{4}\,dy}{5.333} = \frac{\left(\frac{y^4}{16}\right)\Big|_0^{4\text{ m}}}{5.333} = 3\text{ m} \qquad \textbf{Ans.}$$



(a)

•9–9. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = x^{3/2}\,dx$$

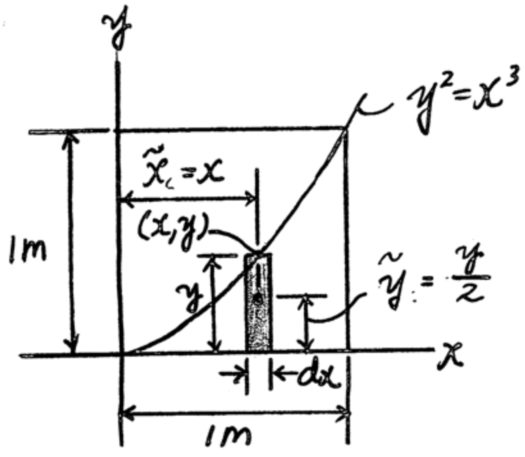
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \dfrac{x^{3/2}}{2}$.

Area: Integrating,

$$A = \int_A dA = \int_0^{1\text{ m}} x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^{1\text{ m}} = \frac{2}{5}\text{ m}^2 = 0.4\text{ m}^2 \qquad \textbf{Ans.}$$

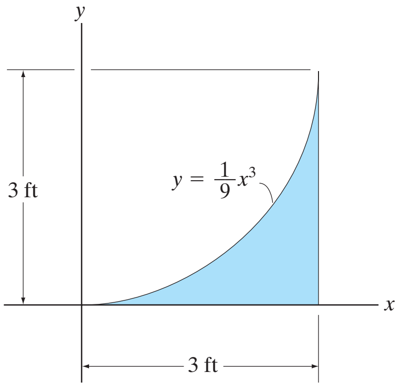
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} x\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} x^{5/2}\,dx}{2/5} = \frac{\left(\frac{2}{7}x^{7/2}\right)\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{7}\text{ m} = 0.714\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} \left(\frac{x^{3/2}}{2}\right)\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} \frac{x^3}{2}\,dx}{2/5} = \frac{\frac{x^4}{8}\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{16}\text{ m} = 0.3125\text{ m} \qquad \textbf{Ans.}$$



(a)

9–10. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{1}{9}x^3\,dx$$

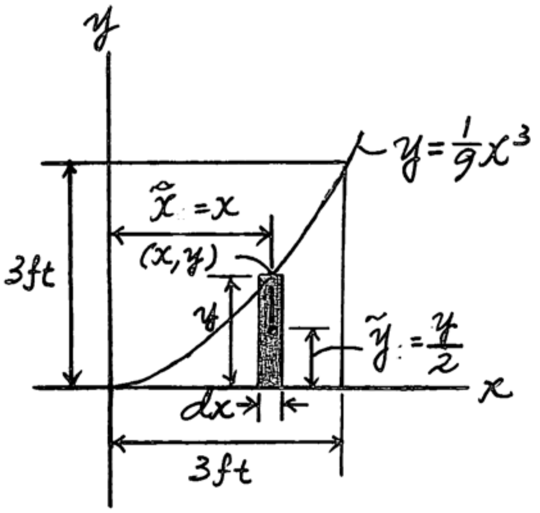
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \frac{1}{18}x^3$.

Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \frac{1}{9}x^3\,dx = \frac{1}{36}x^4\,dx\Big|_0^{3\text{ ft}} = 2.25\text{ft}^2$$ **Ans.**

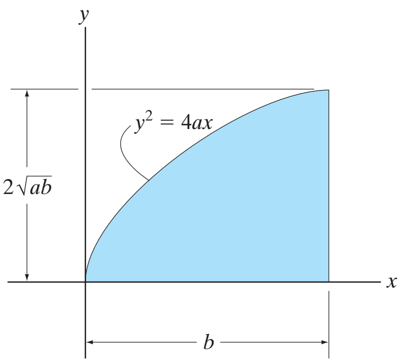
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} x\left(\frac{1}{9}x^3\,dx\right)}{2.25} = \frac{\int_0^{3\text{ ft}} \frac{1}{9}x^4\,dx}{2.25} = \frac{\frac{1}{45}x^5\Big|_0^{3\text{ ft}}}{2.25} = 2.4\text{ft}$$ **Ans.**

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} \left(\frac{1}{18}x^3\right)\left(\frac{1}{9}x^3\,dx\right)}{2.25} = \frac{\int_0^{3\text{ ft}} \frac{1}{162}x^6\,dx}{2.25} = \frac{\frac{1}{1134}x^7\Big|_0^{3\text{ ft}}}{2.25} = 0.857\text{ft}$$ **Ans.**



(a)

9–11. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = 2a^{1/2}x^{1/2}\,dx$$

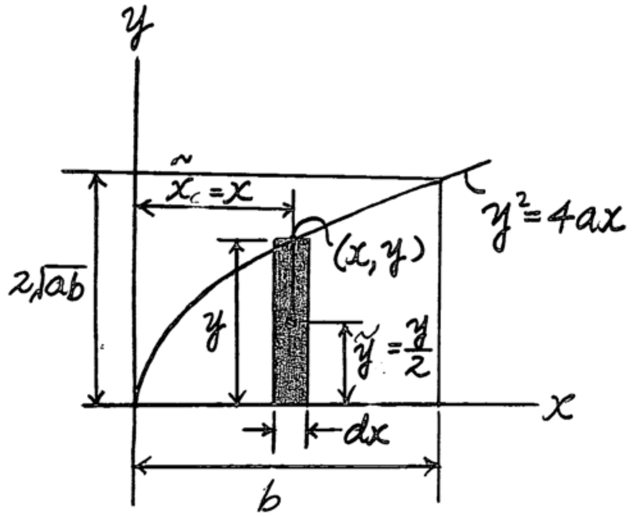
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = a^{1/2}x^{1/2}$.

Area: Integrating,

$$A = \int_A dA = \int_0^b 2a^{1/2}x^{1/2}\,dx = \frac{4}{3}a^{1/2}x^{3/2}\Big|_0^b = \frac{4}{3}a^{1/2}b^{3/2}$$ **Ans.**

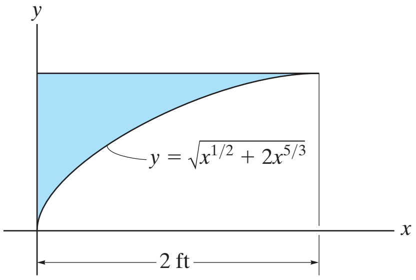
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b x\left(2a^{1/2}x^{1/2}\,dx\right)}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\int_0^b 2a^{1/2}x^{3/2}\,dx}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\frac{4}{5}a^{1/2}x^{5/2}\Big|_0^b}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{3}{5}b$$ **Ans.**

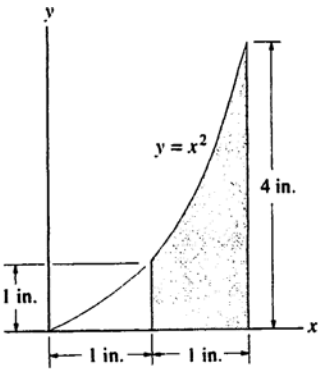
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \left(a^{1/2}x^{1/2}\right)\left(2a^{1/2}x^{1/2}\,dx\right)}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\int_0^b 2ax\,dx}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{ax^2\Big|_0^b}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{3}{4}\sqrt{ab}$$ **Ans.**



(a)

***■9–12.** Locate the centroid $\bar{x}$ of the area.

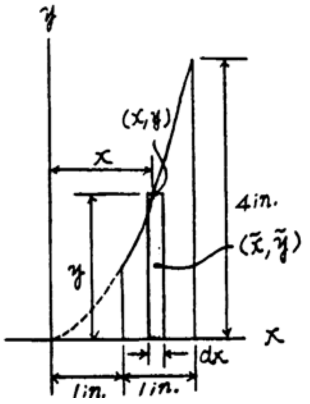




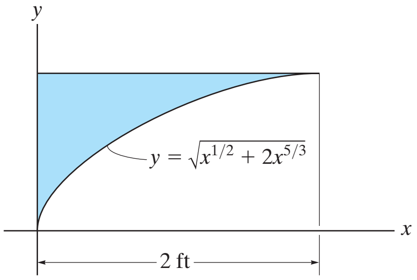
Area and Moment Arm : The area of the differential element is $dA = y\,dx = x^2 dx$ and its centroid is $\tilde{x} = x$.

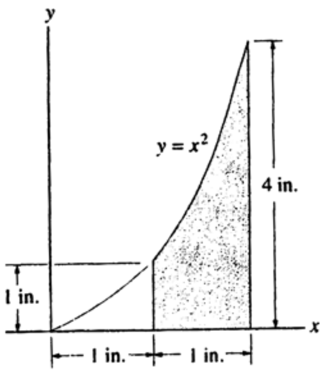
Centroid : Applying Eq. 9-4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{1\text{in}}^{2\text{in}} x(x^2 dx)}{\int_{1\text{in}}^{2\text{in}} x^2 dx} = \frac{\left.\frac{x^4}{4}\right|_{1\text{in}}^{2\text{in}}}{\left.\frac{x^3}{3}\right|_{1\text{in}}^{2\text{in}}} = 1.61 \text{ in} \qquad \textbf{Ans}$$



•■**9–13.** Locate the centroid $\overline{y}$ of the area.

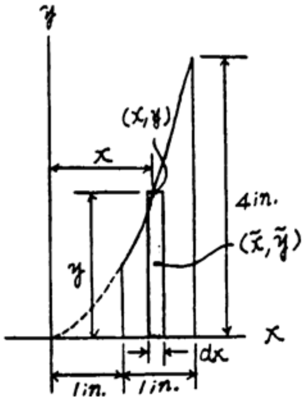




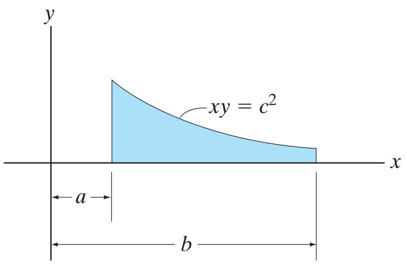
Area and Moment Arm : The area of the differential element is $dA = y\,dx = x^2\,dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}x^2$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{1\text{in.}}^{2\text{in.}} \frac{1}{2}x^2(x^2\,dx\ )}{\int_{1\text{in.}}^{2\text{in.}} x^2\,dx} = \frac{\left.\frac{x^5}{10}\right|_{1\text{in.}}^{2\text{in.}}}{\left.\frac{x^3}{3}\right|_{1\text{in.}}^{2\text{in.}}} = 1.33\text{ in.} \qquad \textbf{Ans}$$



9–14. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{c^2}{x}\,dx$$

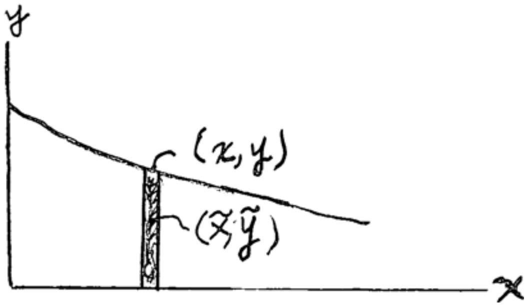
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2} = \frac{c^2}{2x}$.

Area: Integrating,

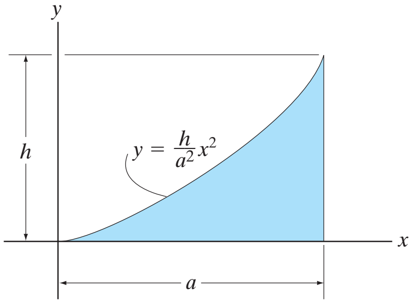
$$A = \int_A dA = \int_a^b \frac{c^2}{x}\,dx = c^2 \ln x\Big|_a^b = c^2 \ln\frac{b}{a} \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_a^b x\left(\frac{c^2}{x}\,dx\right)}{c^2 \ln\frac{b}{a}} = \frac{\int_a^b c^2\,dx}{c^2 \ln\frac{b}{a}} = \frac{c^2 x\Big|_a^b}{c^2 \ln\frac{b}{a}} = \frac{b-a}{\ln\frac{b}{a}} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_a^b \left(\frac{c^2}{2x}\right)\left(\frac{c^2}{x}\,dx\right)}{c^2 \ln\frac{b}{a}} = \frac{\int_a^b \frac{c^4}{2x^2}\,dx}{c^2 \ln\frac{b}{a}} = \frac{-\frac{c^4}{2x}\Big|_a^b}{c^2 \ln\frac{b}{a}} = \frac{c^2(b-a)}{2ab \ln\frac{b}{a}} \qquad \textbf{Ans.}$$



9–15. Determine the area and the centroid ($\bar{x}$, $\bar{y}$) of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{h}{a^2}x^2\,dx$$

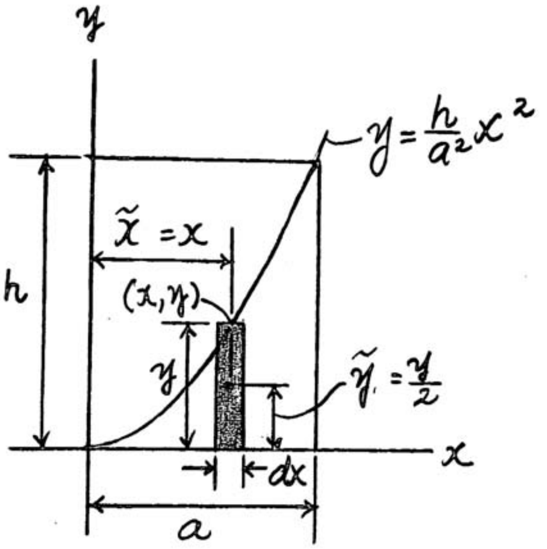
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \frac{1}{2}\left(\frac{h}{a^2}x^2\right) = \frac{h}{2a^2}x^2$.

Area: Integrating,

$$A = \int_A dA = \int_0^a \frac{h}{a^2}x^2\,dx = \frac{h}{a^2}\left(\frac{x^3}{3}\right)\Bigg|_0^a = \frac{1}{3}ah \qquad \textbf{Ans.}$$

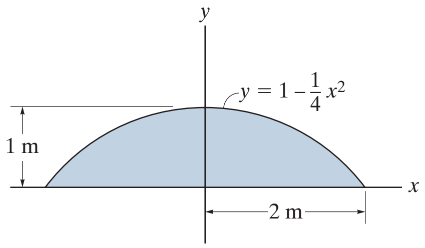
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left(\frac{h}{a^2}x^2\,dx\right)}{\frac{1}{3}ah} = \frac{\int_0^a \frac{h}{a^2}x^3\,dx}{\frac{1}{3}ah} = \frac{\frac{h}{a^2}\left(\frac{x^4}{4}\right)\Big|_0^a}{\frac{1}{3}ah} = \frac{3}{4}a \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{h}{2a^2}x^2\left(\frac{h}{a^2}x^2\,dx\right)}{\frac{1}{3}ah} = \frac{\int_0^a \frac{h^2}{2a^4}x^4\,dx}{\frac{1}{3}ah} = \frac{\frac{h^2}{2a^4}\left(\frac{x^5}{5}\right)\Big|_0^a}{\frac{1}{3}ah} = \frac{3}{10}h \qquad \textbf{Ans.}$$



(a)

***9–16.** Locate the centroid $(\bar{x}, \bar{y})$ of the area.



Area and Moment Arm : The area of the differential element is $dA = y\,dx = \left(1 - \frac{1}{4}x^2\right)dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)$.

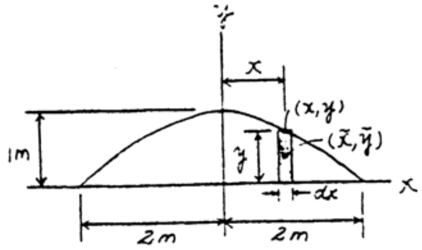
Centroid : Due to symmetry

$$\bar{x} = 0 \qquad \textbf{Ans}$$

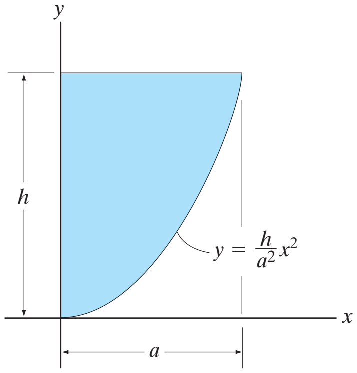
Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-2\,\text{m}}^{2\,\text{m}} \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)\left(1 - \frac{1}{4}x^2\right)dx}{\int_{-2\,\text{m}}^{2\,\text{m}} \left(1 - \frac{1}{4}x^2\right)dx}$$

$$= \frac{\left(\frac{x}{2} - \frac{x^3}{12} + \frac{x^5}{160}\right)\Big|_{-2\,\text{m}}^{2\,\text{m}}}{\left(x - \frac{x^3}{12}\right)\Big|_{-2\,\text{m}}^{2\,\text{m}}} = \frac{2}{5}\ \text{m} \qquad \textbf{Ans}$$



•9–17. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the x axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = x\,dy = \frac{a}{h^{1/2}}y^{1/2}\,dy$$

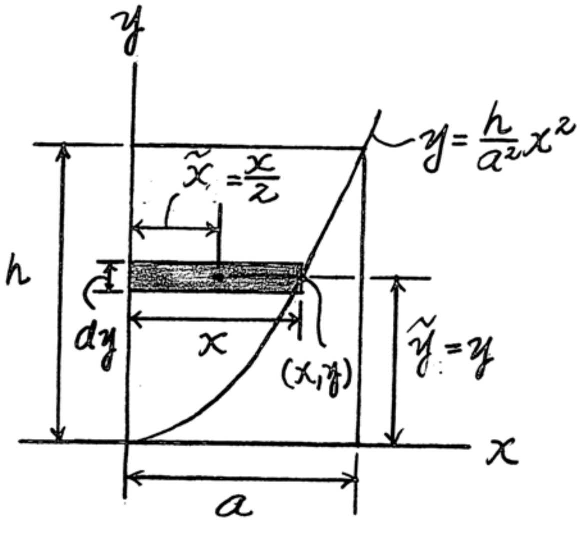
Centroid: The centroid of the element is located at $\tilde{x} = \frac{x}{2} = \frac{a}{2h^{1/2}}y^{1/2}$ and $\tilde{y} = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^h \frac{a}{h^{1/2}}y^{1/2}\,dy = \frac{2a}{3h^{1/2}}\left(y^{3/2}\right)\Big|_0^h = \frac{2}{3}ah \qquad \textbf{Ans.}$$

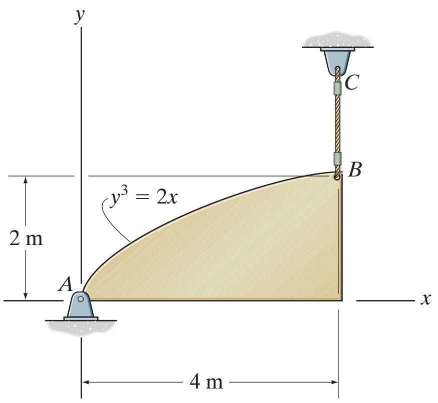
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{2h^{1/2}}y^{1/2}\right)\left(\frac{a}{h^{1/2}}y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a^2}{2h}y\,dy}{\frac{2}{3}ah} = \frac{\frac{a^2}{2h}\left(\frac{y^2}{2}\right)\Big|_0^h}{\frac{2}{3}ah} = \frac{3}{8}a \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(\frac{a}{h^{1/2}}y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a}{h^{1/2}}y^{3/2}\,dy}{\frac{2}{3}ah} = \frac{\frac{2a}{5h^{1/2}}y^{5/2}\Big|_0^h}{\frac{2}{3}ah} = \frac{3}{5}h \qquad \textbf{Ans.}$$



(a)

9–18. The plate is made of steel having a density of $7850\ \text{kg/m}^3$. If the thickness of the plate is 10 mm, determine the horizontal and vertical components of reaction at the pin A and the tension in cable BC.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of this element is given by

$$dA = y\,dx = 1.2599x^{1/3}\,dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2$.

Area: Integrating,

$$A = \int_A dA = \int_0^{4\text{ m}} 1.2599x^{1/3}\,dx = 0.9449x^{4/3}\Big|_0^{4\text{ m}} = 6\ \text{m}^2$$

Thus, the mass of the plate can be obtained from

$$m = \rho A t = 7850(6)(0.01) = 471\ \text{kg}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} x\left(1.2599x^{1/3}\,dx\right)}{6} = \frac{\int_0^{4\text{ m}} 1.2599x^{4/3}\,dx}{6} = \frac{0.5399x^{7/3}\Big|_0^{4\text{ m}}}{6} = 2.2857\ \text{m}$$

Since the plate has a uniform thickness, its center of gravity coincides with its centroid.

Equations of Equilibrium: By referring to the free body diagram shown in Fig. b,

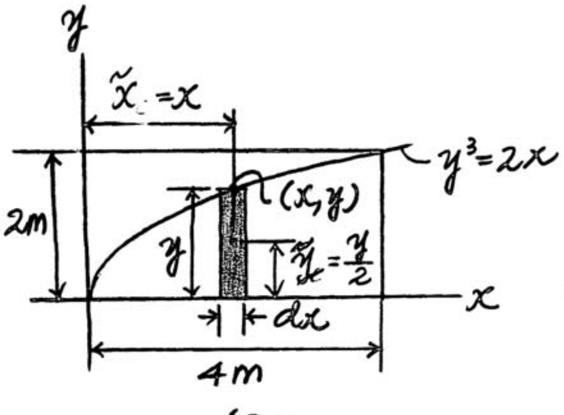
$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC}(4) - 471(9.81)(2.2857) = 0$

$F_{BC} = 2640.27\ \text{N} = 2.64\ \text{kN}$ **Ans.**

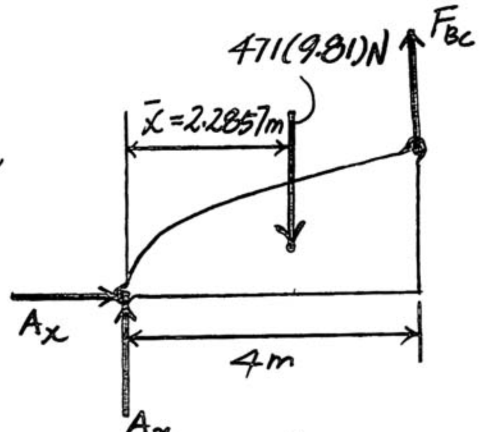
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 2640.27 - 471(9.81) = 0$

$A_y = 1980.24\ \text{N} = 1.98\ \text{kN}$ **Ans.**

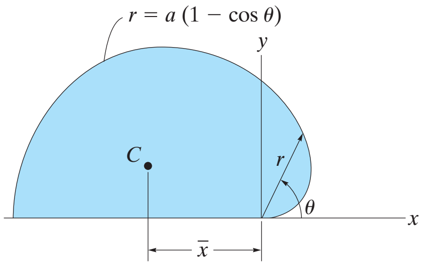


(a)



(b)

9–19. Determine the location $\bar{x}$ to the centroid C of the upper portion of the cardioid, $r = a(1 - \cos\theta)$.



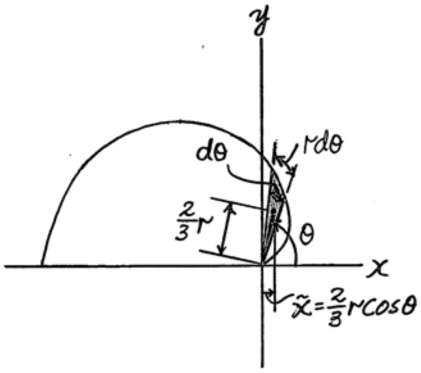
$$dA = \frac{1}{2} r^2 \, d\theta$$

$$A = \int_0^{\pi} \frac{1}{2} (a^2)(1 - \cos\theta)^2 d\theta = \frac{3}{4}\pi a^2$$

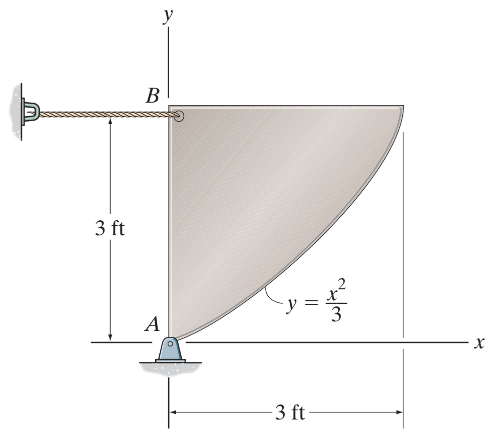
$$\int_A \tilde{x}\, dA = \int_0^{\pi} \left(\frac{2}{3} r \cos\theta\right)\left(\frac{1}{2}\right)(a^2)(1-\cos\theta)^2 \, d\theta$$

$$= \frac{2}{6} a^3 \int_0^{\pi} \cos\theta (1-\cos\theta)^3 \, d\theta = -1.9635\, a^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{-1.9635\, a^3}{\frac{3}{4}\pi a^2} = -0.833\, a \qquad \textbf{Ans}$$



***9–20.** The plate has a thickness of 0.5 in. and is made of steel having a specific weight of 490 lb/ft^3. Determine the horizontal and vertical components of reaction at the pin *A* and the force in the cord at *B*.



Differential Element: The element parallel to the x axis shown shaded in Fig. *a* will be considered. The area of this differential element is given by

$$dA = x\,dy = \sqrt{3}y^{1/2}\,dy$$

Centroid: The centroid of the element is located at $\tilde{x} = x/2 = \frac{\sqrt{3}}{2}y^{1/2}$ and $y_c = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \sqrt{3}y^{1/2}\,dy = \frac{2\sqrt{3}}{3}y^{3/2}\Big|_0^{3\text{ ft}} = 6\text{ ft}^2$$

Thus, the weight of the plate can be obtained from

$$W = \gamma At = 490(6)\left(\frac{0.5}{12}\right) = 122.5\text{ lb}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}}\left(\frac{\sqrt{3}}{2}y^{1/2}\right)\left(\sqrt{3}y^{1/2}\,dy\right)}{6} = \frac{\int_0^{3\text{ ft}}\frac{3}{2}y\,dy}{6} = \frac{\frac{3}{4}y^2\Big|_0^{3\text{ ft}}}{6} = 1.125\text{ ft}$$

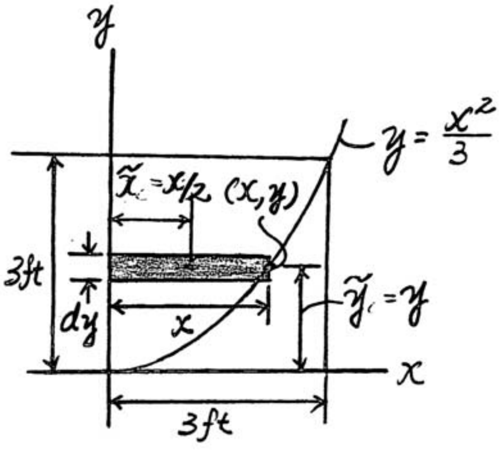
Since the plate has a uniform thickness, its center of gravity coincides with its centroid.

Equations of Equilibrium: By referring to the free body diagram shown in Fig. *b*,

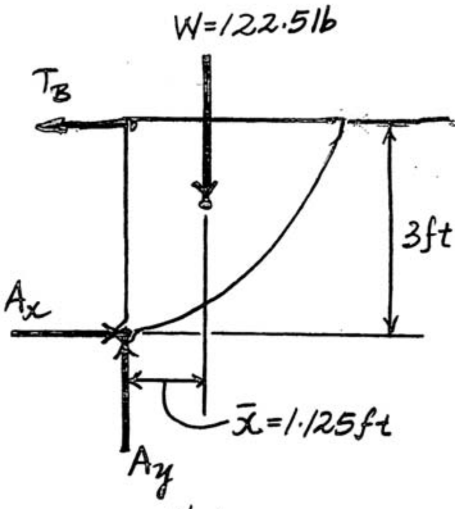
$+\Sigma M_A = 0;$ $T_B(3) - 122.5(1.125) = 0$ $T_B = 45.94\text{ lb} = 45.9\text{ lb}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $A_x - 45.94\text{ lb} = 0$ $A_x = 45.94\text{ lb} = 45.9\text{ lb}$ **Ans.**

$+\uparrow\Sigma F_y = 0;$ $A_y - 122.5 = 0$ $A_y = 122.5\text{ lb}$ **Ans.**

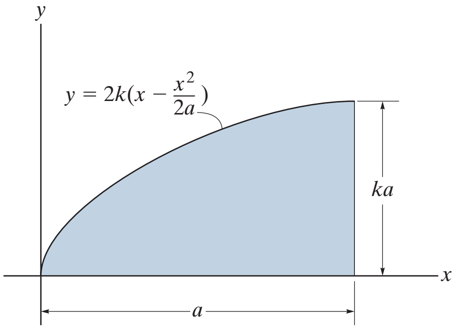


(a)



(b)

•9–21. Locate the centroid $\bar{x}$ of the shaded area.

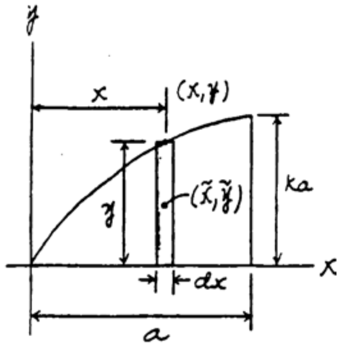


Area and Moment Arm : The area of the differential element is $dA = y\,dx = 2k\left(x - \frac{x^2}{2a}\right)dx$ and its centroid is $\tilde{x} = x$.

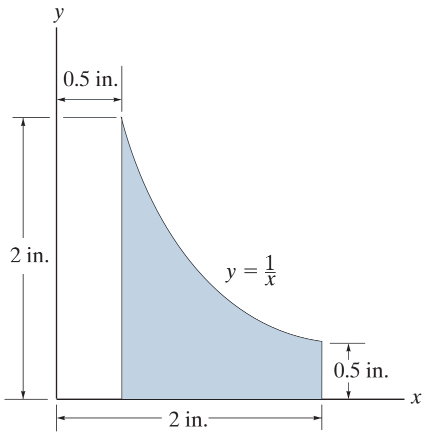
Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left[2k\left(x - \frac{x^2}{2a}\right)dx\right]}{\int_0^a 2k\left(x - \frac{x^2}{2a}\right)dx}$$

$$= \frac{2k\left(\frac{x^3}{3} - \frac{x^4}{8a}\right)\Big|_0^a}{2k\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\Big|_0^a} = \frac{5a}{8}$$ **Ans**



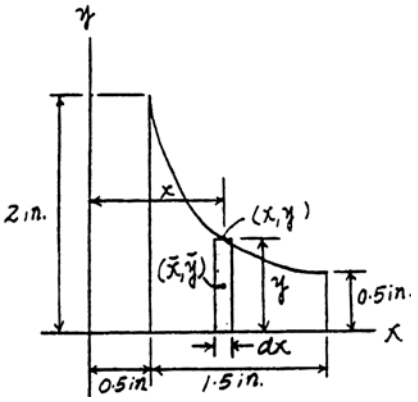
9–22. Locate the centroid $\bar{x}$ of the area.



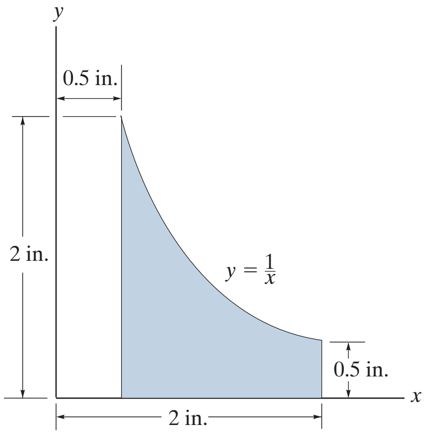
Area and Moment Arm : The area of the differential element is $dA = y\,dx = \frac{1}{x}dx$ and its centroid is $\tilde{x} = x$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{0.5\text{ in}}^{2\text{ in}} x\left(\frac{1}{x}dx\right)}{\int_{0.5\text{ in}}^{2\text{ in}} \frac{1}{x}dx} = \frac{x\big|_{0.5\text{ in}}^{2\text{ in}}}{\ln x\big|_{0.5\text{ in}}^{2\text{ in}}} = 1.08\text{ in.}$$ **Ans**



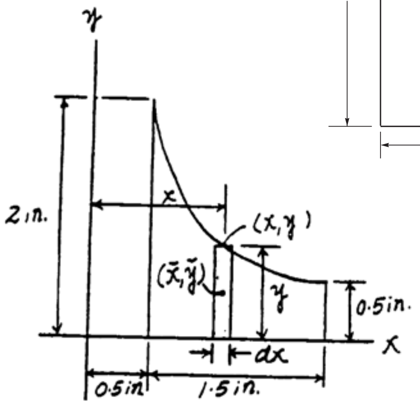
9–23. Locate the centroid $\overline{y}$ of the area.



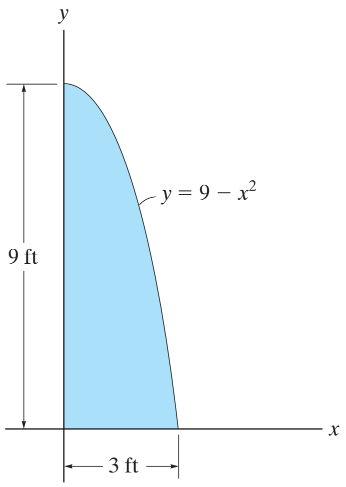
Area and Moment Arm : The area of the differential element is $dA = y dx = \frac{1}{x} dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2x}$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y} dA}{\int_A dA} = \frac{\int_{0.5\text{in}}^{2\text{in}} \frac{1}{2x}\left(\frac{1}{x}dx\right)}{\int_{0.5\text{in}}^{2\text{in}} \frac{1}{x}dx} = \frac{-\frac{1}{2x}\Big|_{0.5\text{in}}^{2\text{in}}}{\ln x\Big|_{0.5\text{in}}^{2\text{in}}} = 0.541 \text{ in} \qquad \text{Ans}$$



***9–24.** Locate the centroid $(\overline{x}, \overline{y})$ of the area.



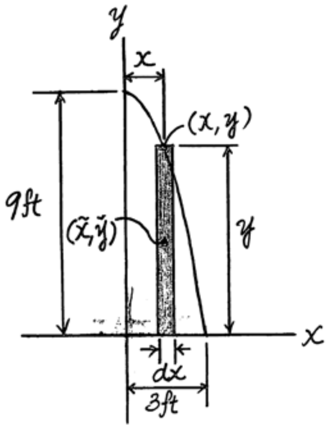
$dA = y\,dx = (9 - x^2)dx$

$\tilde{x} = x$

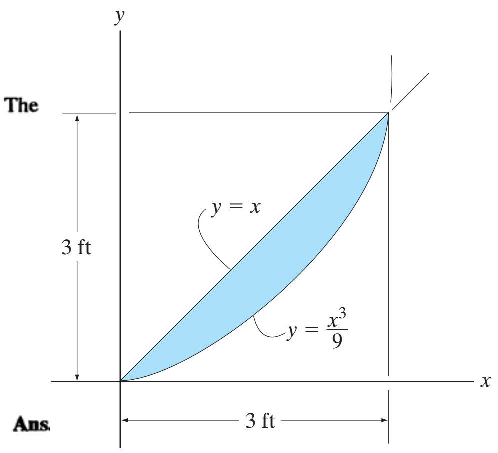
$\tilde{y} = \frac{y}{2} = \frac{1}{2}(9 - x^2)dx$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^3 x(9 - x^2)\,dx}{\int_0^3 (9 - x^2)\,dx} = 1.125 \qquad \text{Ans}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^3 (9 - x^2)^2\,dx}{\int_0^3 (9 - x^2)\,dx} = 3.60 \text{ ft} \qquad \text{Ans}$$



•9–25. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = (y_1 - y_2)\,dx = \left(x - \frac{x^3}{9}\right)dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}\left(x + \frac{x^3}{9}\right)$.

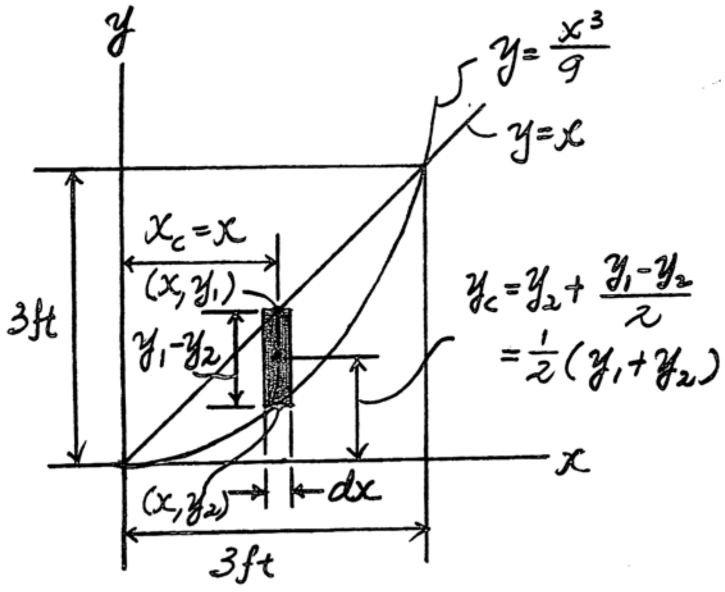
Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \left(x - \frac{x^3}{9}\right)dx = \left(\frac{x^2}{2} - \frac{x^4}{36}\right)\Bigg|_0^{3\text{ ft}} = 2.25\text{ ft}^2 \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} x\left(x - \frac{x^3}{9}\right)dx}{2.25} = \frac{\int_0^{3\text{ ft}} \left(x^2 - \frac{x^4}{9}\right)dx}{2.25} = \frac{\left(\frac{x^3}{3} - \frac{x^5}{45}\right)\Big|_0^{3\text{ ft}}}{2.25} = 1.6\text{ ft} \qquad \textbf{Ans.}$$

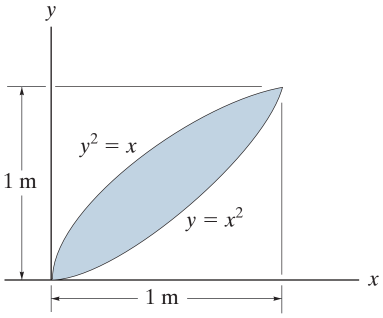
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} \frac{1}{2}\left(x + \frac{x^3}{9}\right)\left(x - \frac{x^3}{9}\right)dx}{2.25} = \frac{\int_0^{3\text{ ft}} \frac{1}{2}\left(x^2 - \frac{x^6}{81}\right)dx}{2.25}$$

$$= \frac{\frac{1}{2}\left(\frac{x^3}{3} - \frac{x^7}{567}\right)\Big|_0^{3\text{ ft}}}{2.25} = 1.143\text{ ft} = 1.14\text{ ft} \qquad \textbf{Ans.}$$



(a)

9–26. Locate the centroid $\overline{x}$ of the area.

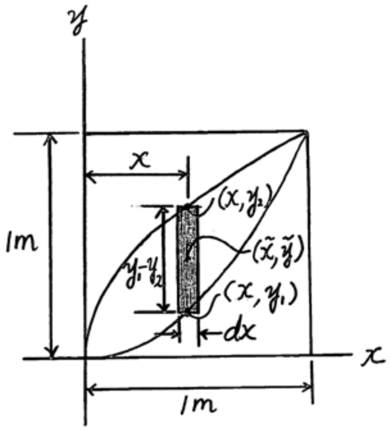


Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{x} = x$.

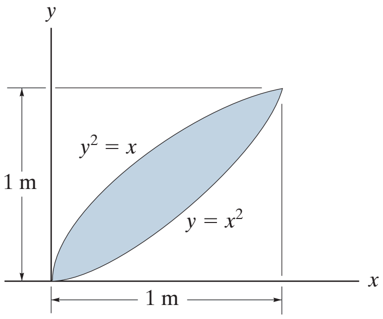
Centroid : Applying Eq. 9 – 4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} x\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$

$$= \frac{\left(\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^4\right)\Big|_0^{1\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\text{m}}} = \frac{9}{20}\text{ m} = 0.45\text{ m} \qquad \textbf{Ans}$$



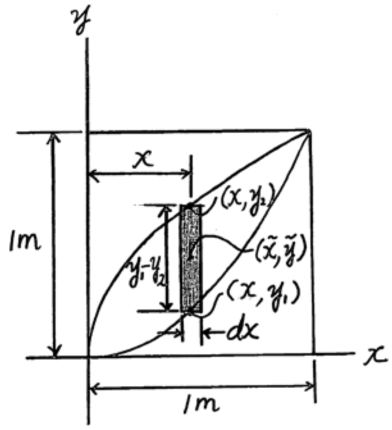
9–27. Locate the centroid $\overline{y}$ of the area.



Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{y} = y_2 + \frac{y_1 - y_2}{2} = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)$.

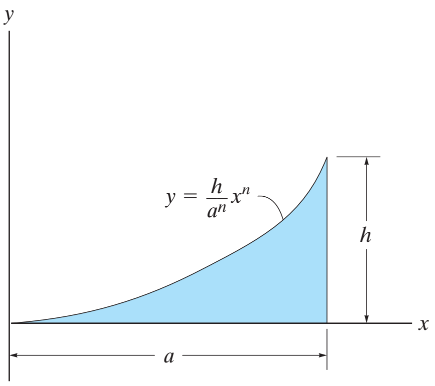
Centroid : Applying Eq. 9 – 4 and performing the integration, we have

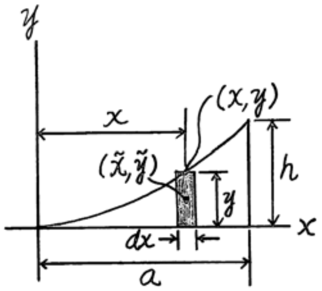
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$



$$= \frac{\frac{1}{2}\left(\frac{1}{2}x^2 - \frac{1}{5}x^5\right)\Big|_0^{1\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\text{m}}} = \frac{9}{20}\text{ m} = 0.45\text{ m} \qquad \textbf{Ans}$$

***9–28.** Locate the centroid $\bar{x}$ of the area.



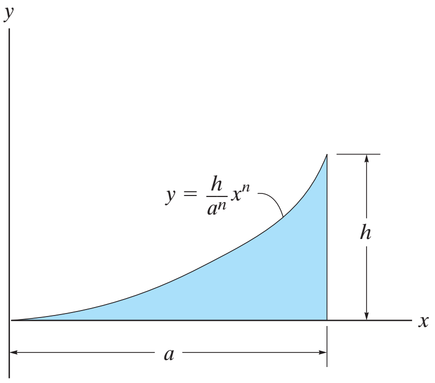


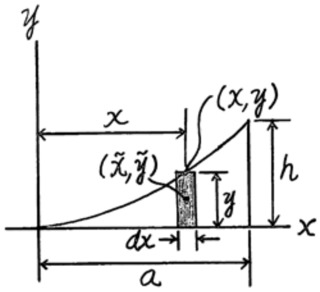
$dA = y\,dx$

$\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a \frac{h}{a^n}x^{n+1}\,dx}{\int_0^a \frac{h}{a^n}x^n\,dx} = \frac{\frac{h(a^{n+2})}{a^n(n+2)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{n+1}{n+2}a$$

•9–29. Locate the centroid $\bar{y}$ of the area.



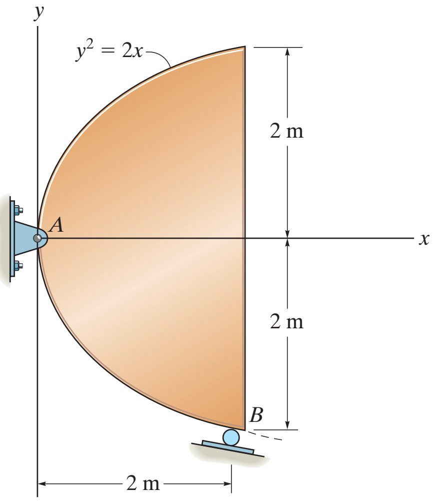


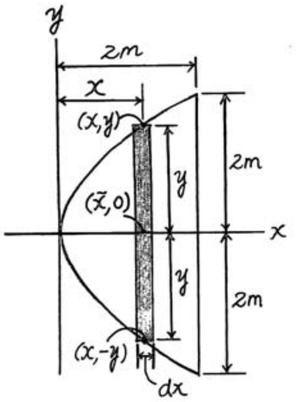
$dA = y\,dx$

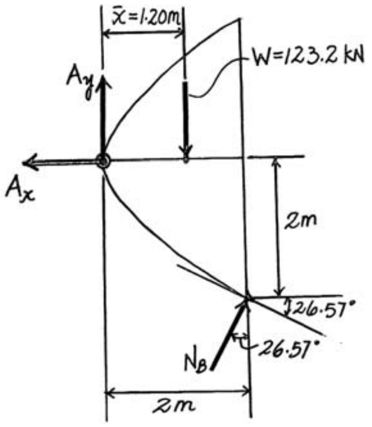
$\tilde{y} = \frac{y}{2}$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \frac{h^2}{a^{2n}}x^{2n}\,dx}{\int_0^a \frac{h}{a^n}x^n\,dx} = \frac{\frac{h^2(a^{2n+1})}{2a^{2n}(2n+1)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{n+1}{2(2n+1)}h \qquad \text{Ans}$$

9–30. The steel plate is 0.3 m thick and has a density of 7850 kg/m^3. Determine the location of its center of mass. Also determine the horizontal and vertical reactions at the pin and the reaction at the roller support. *Hint:* The normal force at B is perpendicular to the tangent at B, which is found from $\tan\theta = dy/dx$.







$dA = 2y\,dx$

$\tilde{x} = x$

$\tilde{y} = 0$

$$\int_A dA = \int_0^2 2\sqrt{2x}\,dx = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}}\Big|_0^2 = 5.333\text{ m}^2$$

$$\int_A \tilde{x}\,dA = \int_0^2 2\sqrt{2x}\,x\,dx = \frac{4\sqrt{2}}{5}x^{\frac{5}{2}}\Big|_0^2 = 6.40\text{ m}^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{6.40}{5.333} = 1.20\text{ m} \quad \text{Ans}$$

$\bar{y} = 0$ Ans (by symmetry)

$y^2 = 2x$

$2y\,dy = 2\,dx$

$\tan\theta = \frac{dy}{dx}\Big|_{(2,-2)} = -0.5$

$\theta = \tan^{-1}(-0.5) = -26.57^\circ$

$W = 7850(0.3)(5.333)(9.81) = 123.2\text{ kN}$

$\circlearrowleft + \Sigma M_A = 0;\quad -123.2(1.2) + N_B(\cos 26.57^\circ)(2) + N_B(\sin 26.57^\circ)(2) = 0$

$N_B = 55.10 = 55.1\text{ kN}$ Ans

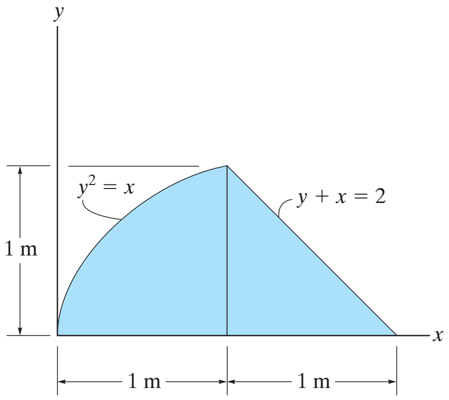
$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + 55.10 \sin 26.57^\circ = 0$

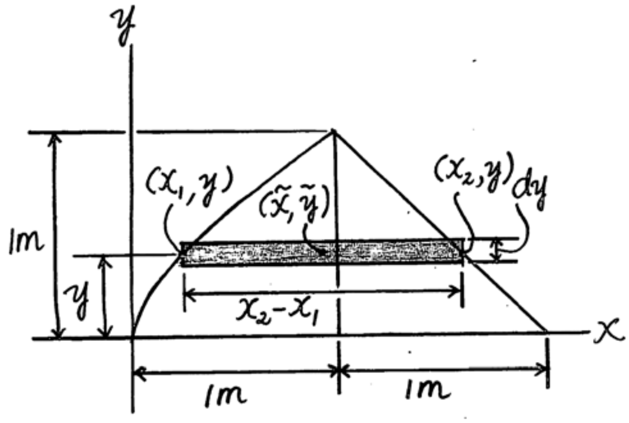
$A_x = 24.6\text{ kN}$ Ans

$+\uparrow \Sigma F_y = 0;\quad A_y - 123.2 + 55.10 \cos 26.57^\circ = 0$

$A_y = 73.9\text{ kN}$ Ans

9–31. Locate the centroid of the area. *Hint:* Choose elements of thickness dy and length $[(2 - y) - y^2]$.





$x_1 = y^2$

$x_2 = 2 - y$

$dA = (x_2 - x_1)\,dy = (2 - y - y^2)\,dy$

$\tilde{x} = \dfrac{x_2 + x_1}{2} = \dfrac{2 - y + y^2}{2}$

$\tilde{y} = y$

$$\int_A dA = \int_0^1 (2 - y - y^2)\,dy = \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_0^1 = 1.167 \text{ m}^2$$

$$\int_A \tilde{x}\,dA = \int_0^1 \frac{1}{2}(2 - y + y^2)(2 - y - y^2)\,dy$$

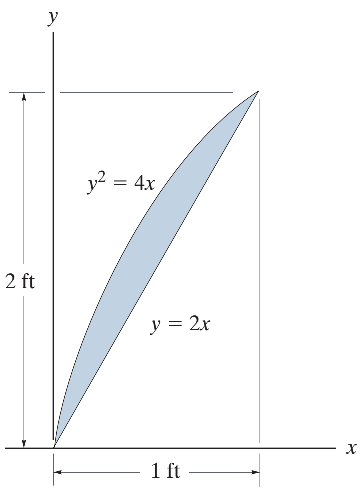
$$= \frac{1}{2}\left[4y - 2y^2 + \frac{y^3}{3} - \frac{y^5}{5}\right]_0^1 = 1.067 \text{ m}^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{1.067}{1.167} = 0.914 \text{ m} \qquad \textbf{Ans}$$

$$\int_A \tilde{y}\,dA = \int_0^1 y(2 - y - y^2)\,dy = \left[y^2 - \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = 0.4167 \text{ m}^3$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{0.4167}{1.167} = 0.357 \text{ m} \qquad \textbf{Ans}$$

***9–32.** Locate the centroid $\bar{x}$ of the area.

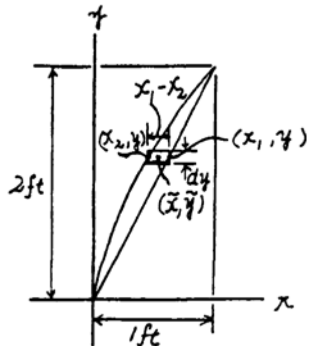


Area and Moment Arm : Here, $x_1 = \frac{y}{2}$ and $x_2 = \frac{y^2}{4}$. The area of the differential element is $dA = (x_1 - x_2)\,dy = \left(\frac{y}{2} - \frac{y^2}{4}\right)dy$ and its centroid is $\tilde{x} = x_2 + \frac{x_1 - x_2}{2} = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}\left(\frac{y}{2} + \frac{y^2}{4}\right)$.

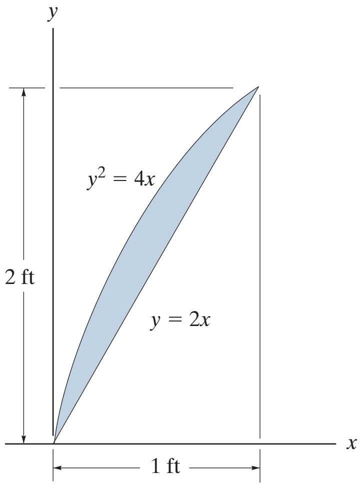
Centroid : Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{2\text{ ft}} \frac{1}{2}\left(\frac{y}{2} + \frac{y^2}{4}\right)\left[\left(\frac{y}{2} - \frac{y^2}{4}\right)dy\right]}{\int_0^{2\text{ ft}} \left(\frac{y}{2} - \frac{y^2}{4}\right)dy}$$

$$= \frac{\left[\frac{1}{2}\left(\frac{1}{12}y^3 - \frac{1}{80}y^5\right)\right]\Big|_0^{2\text{ ft}}}{\left(\frac{1}{4}y^2 - \frac{1}{12}y^3\right)\Big|_0^{2\text{ ft}}} = \frac{2}{5}\text{ ft} = 0.4\text{ ft} \qquad \textbf{Ans}$$



•**9–33.** Locate the centroid $\bar{y}$ of the area.

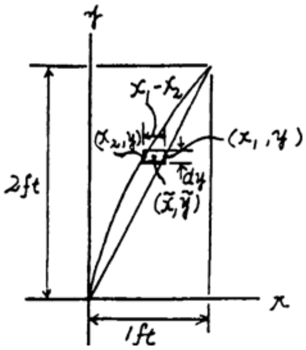


Area and Moment Arm : Here, $x_1 = \frac{y}{2}$ and $x_2 = \frac{y^2}{4}$. The area of the differential element is $dA = (x_1 - x_2)\,dy = \left(\frac{y}{2} - \frac{y^2}{4}\right)dy$ and its centroid is $\tilde{y} = y$.

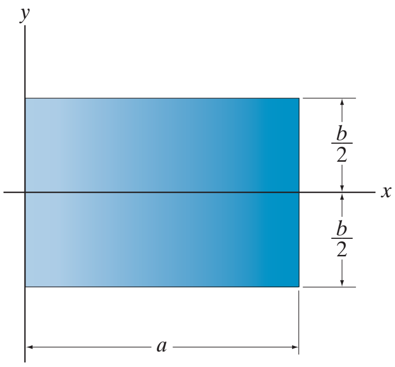
Centroid : Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{2\text{ ft}} y\left[\left(\frac{y}{2} - \frac{y^2}{4}\right)dy\right]}{\int_0^{2\text{ ft}} \left(\frac{y}{2} - \frac{y^2}{4}\right)dy}$$

$$= \frac{\left(\frac{1}{6}y^3 - \frac{1}{16}y^4\right)\Big|_0^{2\text{ ft}}}{\left(\frac{1}{4}y^2 - \frac{1}{12}y^3\right)\Big|_0^{2\text{ ft}}} = 1\text{ ft} \qquad \textbf{Ans}$$



9–34. If the density at any point in the rectangular plate is defined by $\rho = \rho_0(1 + x/a)$, where ρ_0 is a constant, determine the mass and locate the center of mass $\bar{x}$ of the plate. The plate has a thickness t.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The mass of this element is

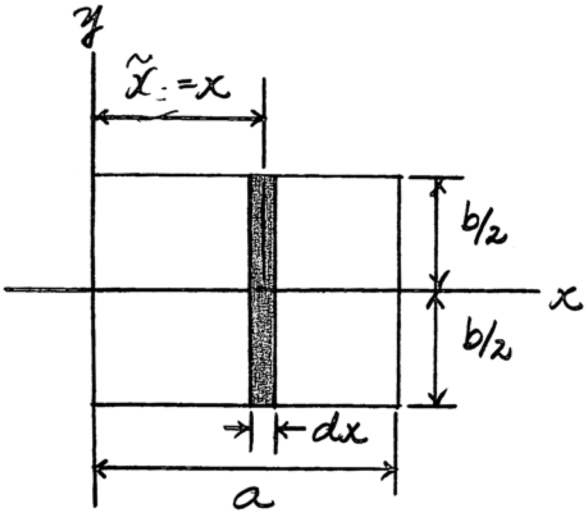
$$dm = \rho\, dV = \rho_0\left(1+\frac{x}{a}\right)t(b\,dx) = \rho_0 tb\left(1+\frac{x}{a}\right)dx$$

Mass: Integrating,

$$m = \int_m dm = \int_0^a \rho_0 tb\left(1+\frac{x}{a}\right)dx = \rho_0 tb\left(x+\frac{x^2}{2a}\right)dx\Bigg|_0^a = \frac{3}{2}\rho_0 abt \qquad \textbf{Ans.}$$

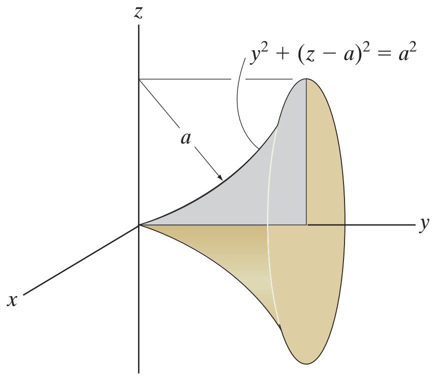
Center of Mass: The center of mass of the element is located at $\tilde{x} = x$.

$$\bar{x} = \frac{\int_m \tilde{x}\,dm}{\int_m dm} = \frac{\int_0^a x\left[\rho_0 tb\left(1+\frac{x}{a}\right)dx\right]}{\frac{3}{2}\rho_0 abt} = \frac{\int_0^a \rho_0 tb\left(x+\frac{x^2}{a}\right)dx}{\frac{3}{2}\rho_0 abt} = \frac{\rho_0 tb\left[\frac{x^2}{2}+\frac{x^3}{3a}\right]\Big|_0^a}{\frac{3}{2}\rho_0 abt} = \frac{5}{9}a \qquad \textbf{Ans.}$$



(a)

9–35. Locate the centroid $\overline{y}$ of the homogeneous solid formed by revolving the shaded area about the y axis.



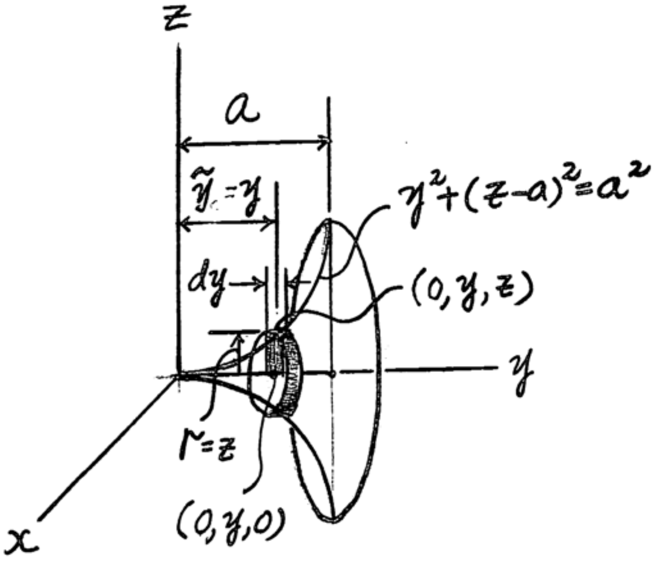
Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is $dV = \pi z^2\,dy$. Here, $z = a - \sqrt{a^2 - y^2}$. Thus,

$$dV = \pi\left(a - \sqrt{a^2 - y^2}\right)^2 dy = \pi\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy$$

Centroid: The centroid of the element is located at $\tilde{y} = y$.

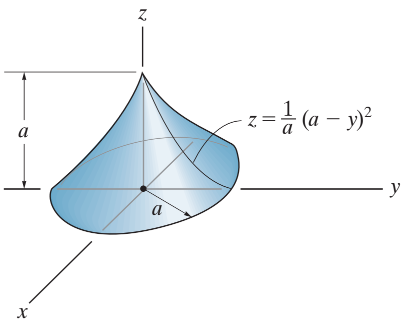
$$\bar{y} = \frac{\int_A \tilde{y}\,dV}{\int_A dV} = \frac{\int_0^a y\left[\pi\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy\right]}{\int_0^a \pi\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy} = \frac{\pi\int_0^a\left(2a^2y - y^3 - 2ay\sqrt{a^2 - y^2}\right)dy}{\pi\int_0^a\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy}$$

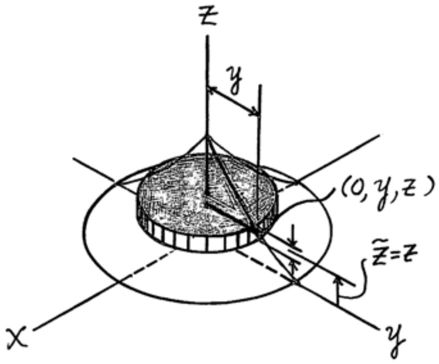
$$= \frac{\pi\left(a^2y^2 - \frac{y^4}{4} + \frac{2a}{3}\sqrt{\left(a^2 - y^2\right)^3}\right)\Bigg|_0^a}{\pi\left[2a^2y - \frac{y^3}{3} - a\left(y\sqrt{a^2 - y^2} + a^2\sin^{-1}\frac{y}{a}\right)\right]\Bigg|_0^a} = \frac{\frac{1}{12}a^4}{\left(\frac{10 - 3\pi}{6}\right)a^3} = \frac{a}{2(10 - 3\pi)}$$ **Ans.**



(a)

***9–36.** Locate the centroid $\bar{z}$ of the solid.



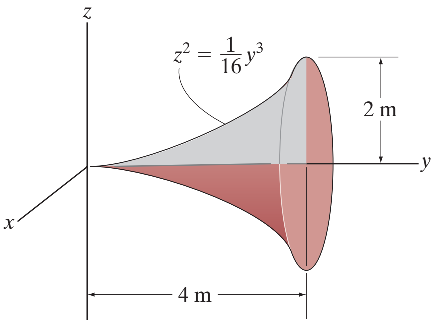


The volume of the differential thin- disk element $dV = \pi y^2 dz = \pi\left(a - \sqrt{az}\right)^2 dz$

$dV = \pi\left(a^2 + az - 2a^{\frac{3}{2}}z^{\frac{1}{2}}\right)dz$ and $\tilde{z} = z$.

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^a z\left[\pi\left(a^2 + az - 2a^{\frac{3}{2}}z^{\frac{1}{2}}\right)dz\right]}{\int_0^a \pi\left(a^2 + az - 2a^{\frac{3}{2}}z^{\frac{1}{2}}\right)dz} = \frac{1}{5}a \qquad \textbf{Ans}$$

•9–37. Locate the centroid $\bar{y}$ of the homogeneous solid formed by revolving the shaded area about the y axis.

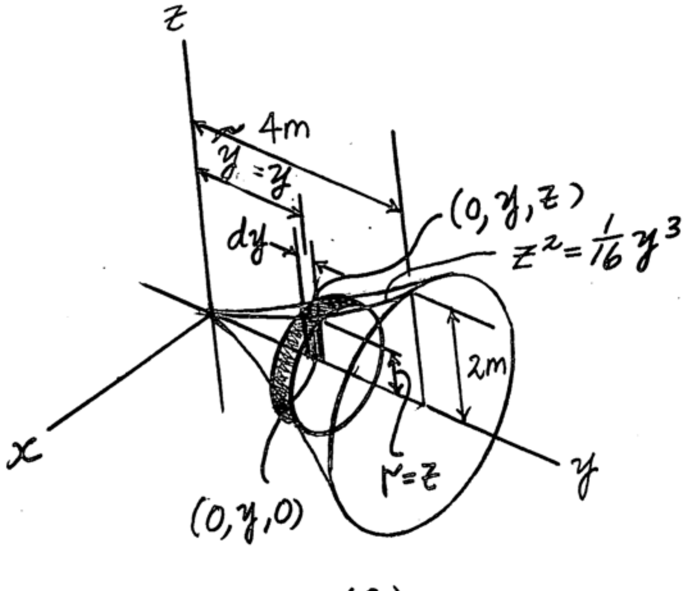


Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\,dy = \pi\left(\frac{1}{16}y^3\right)dy = \frac{\pi}{16}y^3\,dy$$

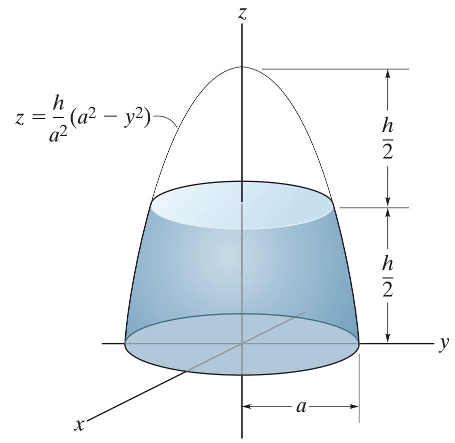
Centroid: The centroid of the element is located at $\tilde{y} = y$.

$$\bar{y} = \frac{\int_A \tilde{y}\,dV}{\int_A dV} = \frac{\int_0^{4\text{ m}} y\left(\frac{\pi}{16}y^3\,dy\right)}{\int_0^{4\text{ m}} \frac{\pi}{16}y^3\,dy} = \frac{\int_0^{4\text{ m}} \frac{\pi}{16}y^4\,dy}{\int_0^{4\text{ m}} \frac{\pi}{16}y^3\,dy} = \frac{\frac{\pi}{16}\left(\frac{y^5}{5}\right)\Big|_0^{4\text{ m}}}{\frac{\pi}{16}\left(\frac{y^4}{4}\right)\Big|_0^{4\text{ m}}} = 3.2\text{ m} \qquad \textbf{Ans.}$$



(a)

9–38. Locate the centroid $\bar{z}$ of the homogeneous solid frustum of the paraboloid formed by revolving the shaded area about the z axis.

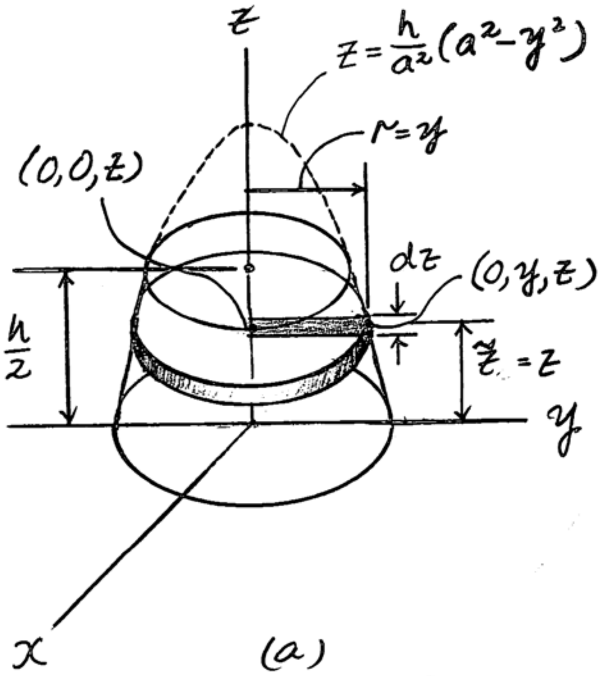


Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi y^2\, dz = \pi\left(a^2 - \frac{a^2}{h}z\right)dz$$

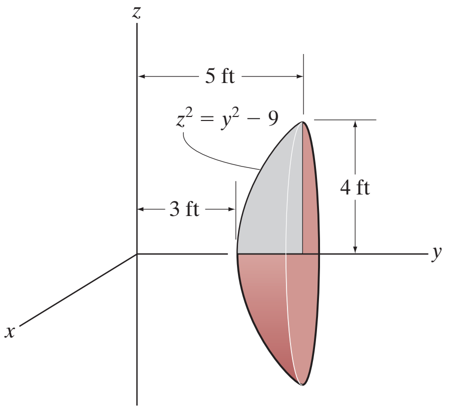
Centroid: The centroid of the element is located at $z_c = z$.

$$\bar{z} = \frac{\int_A \tilde{z}_c\, dV}{\int_A dV} = \frac{\int_0^{h/2} z\left[\pi\left(a^2 - \frac{a^2}{h}z\right)dz\right]}{\int_0^{h/2}\pi\left(a^2 - \frac{a^2}{h}z\right)dz} = \frac{\int_0^{h/2}\pi\left(a^2 z - \frac{a^2}{h}z^2\right)dz}{\int_0^{h/2}\pi\left(a^2 - \frac{a^2}{h}z\right)dz} = \frac{\pi\left(\frac{a^2}{2}z^2 - \frac{a^2}{3h}z^3\right)\Big|_0^{h/2}}{\pi\left(a^2 z - \frac{a^2}{2h}z^2\right)\Big|_0^{h/2}} = \frac{2}{9}h$$ **Ans.**



(a)

9–39. Locate the centroid $\bar{y}$ of the homogeneous solid formed by revolving the shaded area about the y axis.

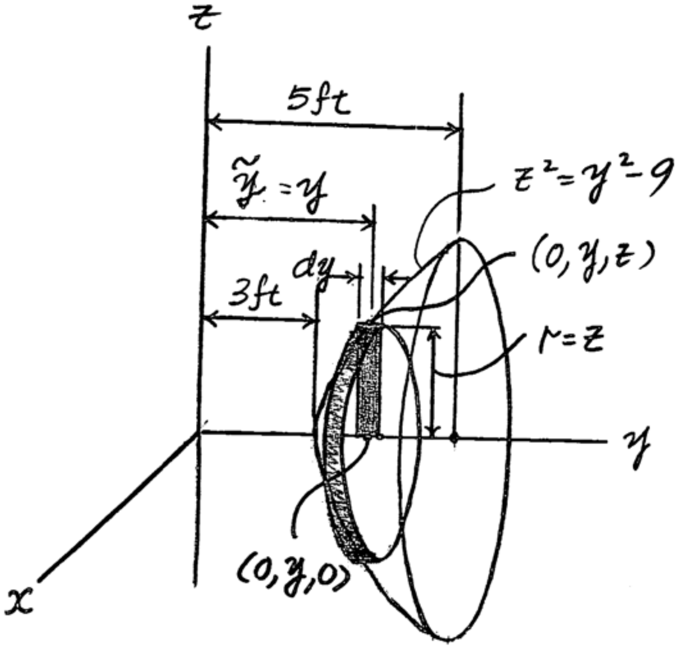


Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\, dy = \pi\left(y^2 - 9\right) dy$$

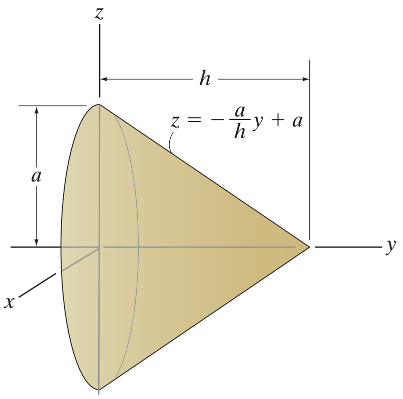
Centroid: The centroid of the element is located at $\tilde{y} = y$.

$$\bar{y} = \frac{\int_A \tilde{y}\, dV}{\int_A dV} = \frac{\int_{3\text{ ft}}^{5\text{ ft}} y\left[\pi\left(y^2-9\right)dy\right]}{\int_{3\text{ ft}}^{5\text{ ft}} \pi\left(y^2-9\right)dy} = \frac{\int_{3\text{ ft}}^{5\text{ ft}} \pi\left(y^3-9y\right)dy}{\int_{3\text{ ft}}^{5\text{ ft}} \pi\left(y^2-9\right)dy} = \frac{\pi\left(\frac{y^4}{4} - \frac{9y^2}{2}\right)\Big|_{3\text{ ft}}^{5\text{ ft}}}{\pi\left(\frac{y^3}{3} - 9y\right)\Big|_{3\text{ ft}}^{5\text{ ft}}} = 4.36\text{ ft} \qquad \textbf{Ans.}$$



(a)

***9–40.** Locate the center of mass $\bar{y}$ of the circular cone formed by revolving the shaded area about the y axis. The density at any point in the cone is defined by $\rho = (\rho_0/h)y$, where ρ_0 is a constant.



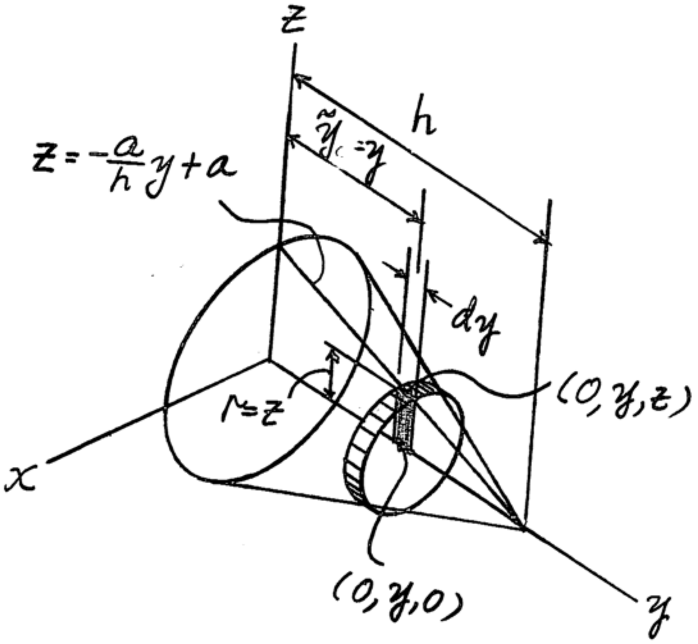
Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The mass of the element is

$$dm = \rho\, dV = \rho\pi z^2\, dy = \left(\frac{\rho_0}{h}\right)y\left[\pi\left(\frac{-a}{h}y + a\right)^2 dy\right] = \frac{\pi a^2\rho_0}{h}\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy$$

Centroid: The centroid of the element is located at $\tilde{y} = y$.

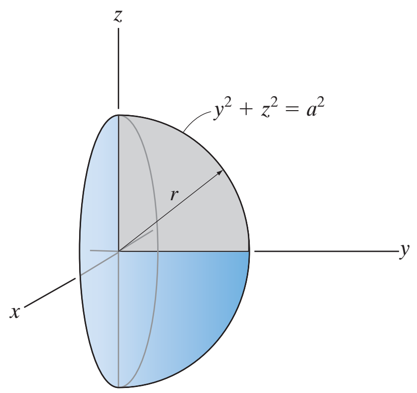
$$\bar{y} = \frac{\int_m \tilde{y}\,dm}{\int_m dm} = \frac{\int_0^h y\left[\frac{\pi a^2\rho_0}{h}\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy\right]}{\int_0^h \frac{\pi a^2\rho_0}{h}\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy} = \frac{\frac{\pi a^2\rho_0}{h}\int_0^h\left(\frac{y^4}{h^2} + y^2 - \frac{2y^3}{h}\right)dy}{\frac{\pi a^2\rho_0}{h}\int_0^h\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy} = \frac{\left(\frac{y^5}{5h^2} + \frac{y^3}{3} - \frac{y^4}{2h}\right)\Big|_0^h}{\left(\frac{y^4}{4h^2} + \frac{y^2}{2} - \frac{2y^3}{3h}\right)\Big|_0^h}$$

$$= \frac{2}{5}h \qquad \textbf{Ans.}$$



(a)

•9–41. Determine the mass and locate the center of mass $\bar{y}$ of the hemisphere formed by revolving the shaded area about the y axis. The density at any point in the hemisphere can be defined by $\rho = \rho_0(1 + y/a)$, where ρ_0 is a constant.



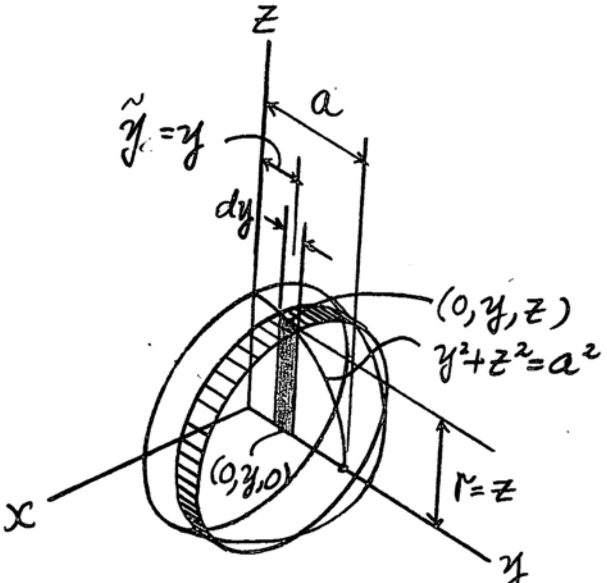
Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The mass of the element is

$$dm = \rho\, dV = \rho\pi z^2\, dy = \pi\rho_0\left(1+\frac{y}{a}\right)\left(a^2-y^2\right)dy = \pi\rho_0\left(a^2-y^2+ay-\frac{y^3}{a}\right)dy$$

Centroid: The centroid of the element is located at $\tilde{y} = y$.

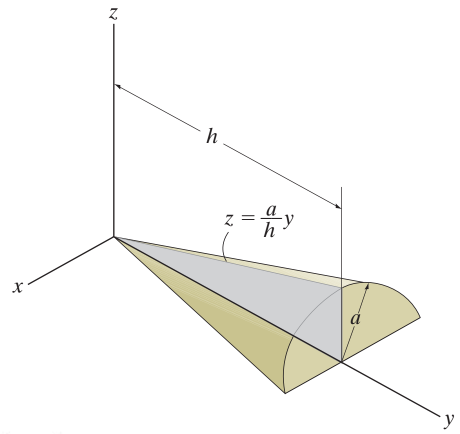
$$\bar{y} = \frac{\int_m \tilde{y}\, dm}{\int_m dm} = \frac{\int_0^a y\left[\pi\rho_0\left(a^2-y^2+ay-\frac{y^3}{a}\right)dy\right]}{\int_0^a \pi\rho_0\left(a^2-y^2+ay-\frac{y^3}{a}\right)dy} = \frac{\pi\rho_0\int_0^a\left(a^2y-y^3+ay^2-\frac{y^4}{a}\right)dy}{\pi\rho_0\int_0^a\left(a^2-y^2+ay-\frac{y^3}{a}\right)dy} = \frac{\left.\left(\frac{a^2y^2}{2}-\frac{y^4}{4}+\frac{ay^3}{3}-\frac{y^5}{5a}\right)\right|_0^a}{\left.\left(a^2y-\frac{y^3}{3}+\frac{ay^2}{2}-\frac{y^4}{4a}\right)\right|_0^a}$$

$$= \frac{23}{55}a \quad \textbf{Ans.}$$



(a)

9–42. Determine the volume and locate the centroid $(\bar{y}, \bar{z})$ of the homogeneous conical wedge.



Differential Element: The half - thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \frac{\pi}{2}z^2\,dy = \frac{\pi}{2}\left(\frac{a^2}{h^2}y^2\right)dy = \frac{a^2\pi}{2h^2}y^2\,dy$$

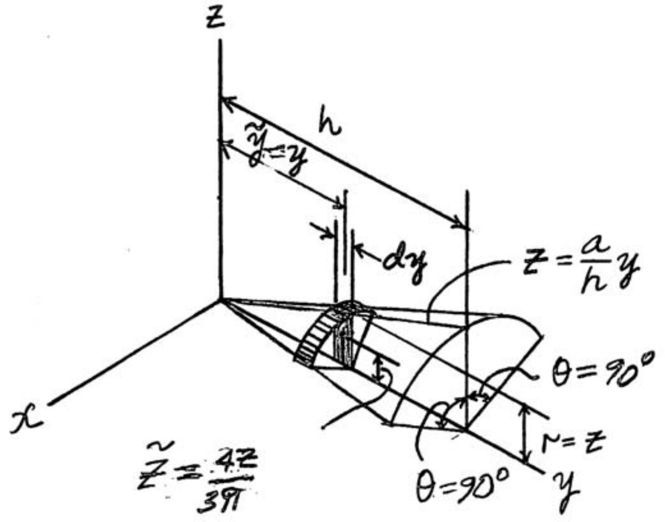
Volume: Integrating,

$$V = \int_V dV = \int_0^h \frac{a^2\pi}{2h^2}y^2\,dy = \frac{a^2\pi}{2h^2}\left(\frac{y^3}{3}\right)\Bigg|_0^h = \frac{\pi a^2 h}{6} \qquad \textbf{Ans.}$$

Centroid: The centroid of the element is located at $\tilde{y} = y$ and $\tilde{z} = \frac{4z}{3\pi} = \frac{4a}{3\pi h}y$.

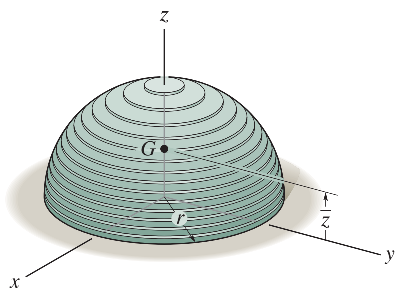
$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^h y\left(\frac{a^2\pi}{2h^2}y^2\,dy\right)}{\frac{\pi a^2 h}{6}} = \frac{\frac{a^2\pi}{2h^2}\int_0^h y^3\,dy}{\frac{\pi a^2 h}{6}} = \frac{\frac{a^2\pi}{2h^2}\left(\frac{y^4}{4}\right)\Big|_0^h}{\frac{\pi a^2 h}{6}} = \frac{3}{4}h \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^h\left(\frac{4a}{3\pi h}y\right)\left(\frac{\pi a^2}{2h^2}y^2\,dy\right)}{\frac{\pi a^2 h}{6}} = \frac{\frac{2a^3}{3h^3}\int_0^h y^3\,dy}{\frac{\pi a^2 h}{6}} = \frac{\frac{2a^3}{3h^3}\left(\frac{y^4}{4}\right)\Big|_0^h}{\frac{\pi a^2 h}{6}} = \frac{a}{\pi} \qquad \textbf{Ans.}$$



(a)

9–43. The hemisphere of radius r is made from a stack of very thin plates such that the density varies with height, $\rho = kz$, where k is a constant. Determine its mass and the distance $\bar{z}$ to the center of mass G.



Mass and Moment Arm : The density of the material is $\rho = kz$. The mass of the thin disk differential element is $dm = \rho dV = \rho\pi y^2 dz = kz\left[\pi\left(r^2 - z^2\right) dz\right]$ and its centroid $\tilde{z} = z$. Evaluating the integrals, we have

$$m = \int_m dm = \int_0^r kz\left[\pi\left(r^2 - z^2\right) dz\right]$$

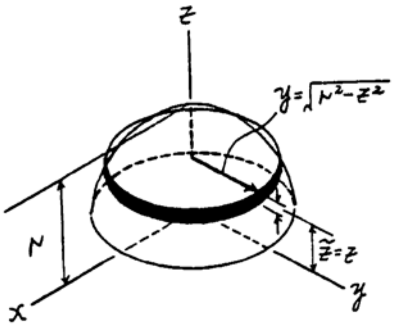
$$= \pi k\left(\frac{r^2 z^2}{2} - \frac{z^4}{4}\right)\Bigg|_0^r = \frac{\pi k r^4}{4} \qquad \textbf{Ans}$$

$$\int_m \tilde{z}\, dm = \int_0^r z\left\{kz\left[\pi\left(r^2 - z^2\right) dz\right]\right\}$$

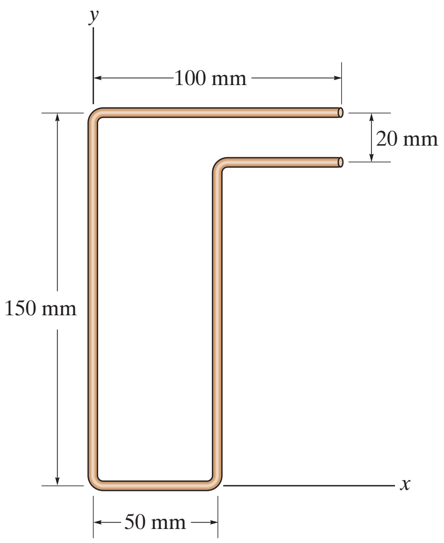
$$= \pi k\left(\frac{r^2 z^3}{3} - \frac{z^5}{5}\right)\Bigg|_0^r = \frac{2\pi k r^5}{15}$$

Centroid : Applying Eq. 9–3, we have

$$\bar{z} = \frac{\int_m \tilde{z}\, dm}{\int_m dm} = \frac{2\pi k r^5/15}{\pi k r^4/4} = \frac{8}{15} r \qquad \textbf{Ans}$$



***9–44.** Locate the centroid ($\bar{x}, \bar{y}$) of the uniform wire bent in the shape shown.



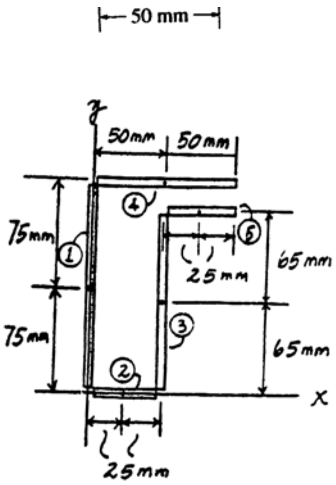
Centroid : The length of each segment and its respective centroid are tabulated below.

Segment	L(mm)	$\tilde{x}$(mm)	$\tilde{y}$(mm)	$\tilde{x}L$(mm^2)	$\tilde{y}L$(mm^2)
1	150	0	75	0	11250
2	50	25	0	1250	0
3	130	50	65	6500	8450
4	100	50	150	5000	15000
5	50	75	130	3750	6500
Σ	480			16500	41200

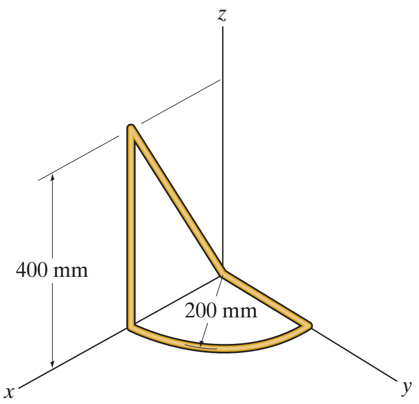
Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{16500}{480} = 34.375 \text{ mm} = 34.4 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{41200}{480} = 85.83 \text{ mm} = 85.8 \text{ mm} \qquad \textbf{Ans}$$



•9–45. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.

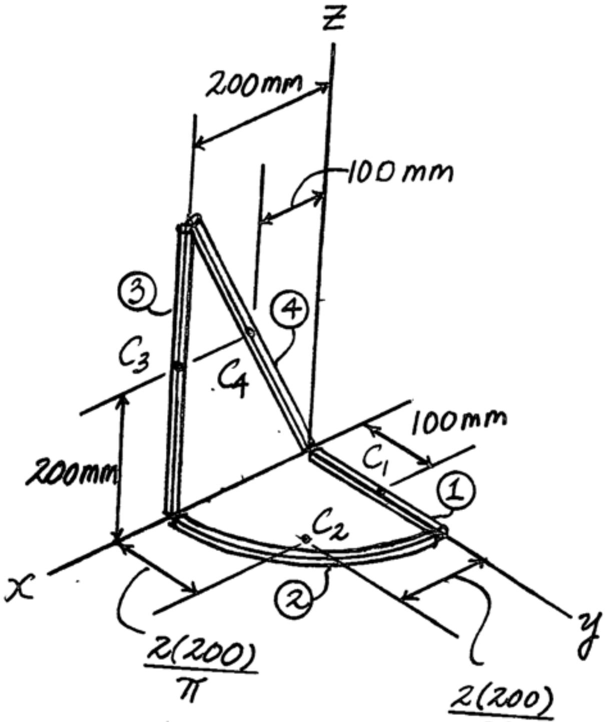


Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{0(200) + \frac{2(200)}{\pi}\left(\frac{\pi(200)}{2}\right) + 200(400) + 100\left(\sqrt{200^2+400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2+400^2}} = \frac{164.72(10^3)}{1361.37} = 121 \text{ mm} \qquad \textbf{Ans.}$$

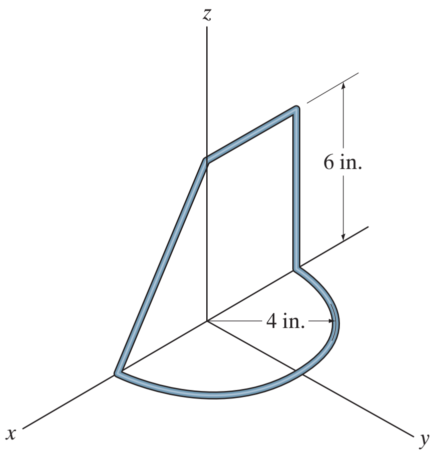
$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{100(200) + \frac{2(200)}{\pi}\left(\frac{\pi(200)}{2}\right) + 0(400) + 0\left(\sqrt{200^2+400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2+400^2}} = \frac{60(10^3)}{1361.37} = 44.1 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{0(200) + 0\left(\frac{\pi(200)}{2}\right) + 200(400) + 200\left(\sqrt{200^2+400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2+400^2}} = \frac{169.44(10^3)}{1361.37} = 124 \text{ mm} \qquad \textbf{Ans.}$$



(a)

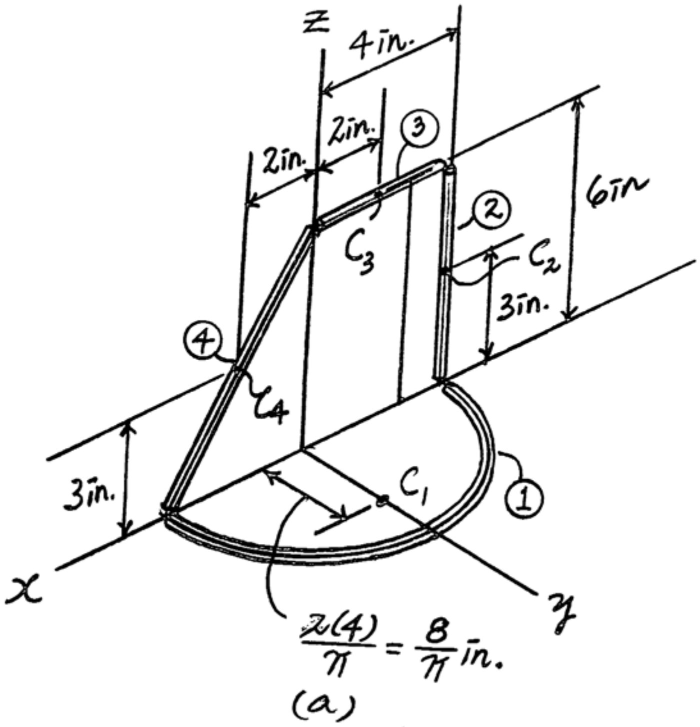
9–46. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.



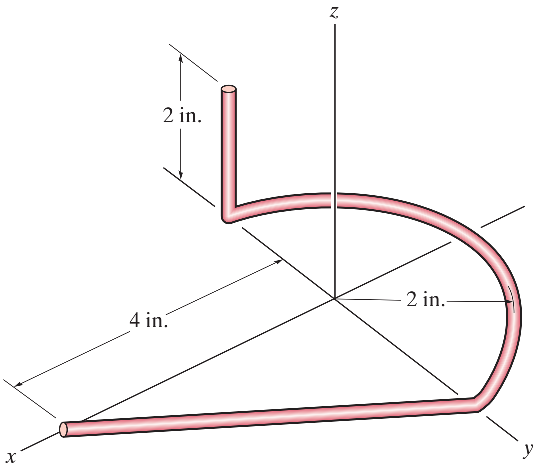
$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{(0)\pi(4)+(-4)(6)+(-2)(4)+(2)\left(\sqrt{4^2+6^2}\right)}{\pi(4)+6+4+\sqrt{4^2+6^2}} = \frac{-17.58}{29.78} = -0.590\text{ in.} \qquad \textbf{Ans.}$$

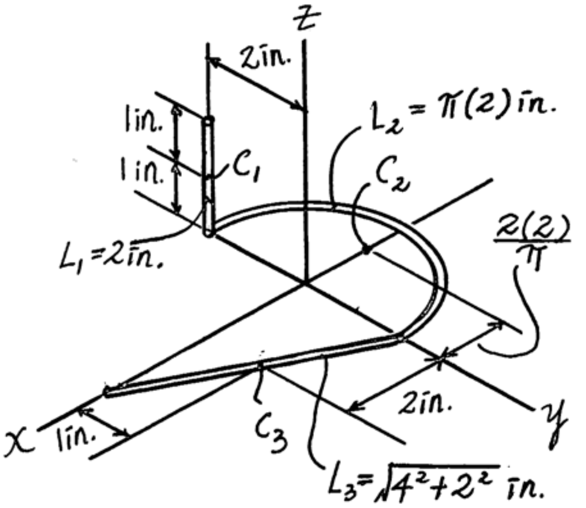
$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{(8/\pi)\pi(4)+0(6)+0(4)+0\left(\sqrt{4^2+6^2}\right)}{\pi(4)+6+4+\sqrt{4^2+6^2}} = \frac{32}{29.78} = 1.07\text{ in.} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{(0)\pi(4)+3(6)+6(4)+3\left(\sqrt{4^2+6^2}\right)}{\pi(4)+6+4+\sqrt{4^2+6^2}} = \frac{63.63}{29.78} = 2.14\text{ in.} \qquad \textbf{Ans.}$$



9–47. Locate the centroid ($\bar{x}, \bar{y}, \bar{z}$) of the wire which is bent in the shape shown.





$$\Sigma L = 2 + \pi(2) + \sqrt{4^2 + 2^2} = 12.7553 \text{ in.}$$

$$\Sigma \tilde{x} L = 0(2) - \frac{2(2)}{\pi}(\pi 2) + 2\left(\sqrt{4^2 + 2^2}\right) = 0.94427 \text{ in}^2$$

$$\Sigma \tilde{y} L = (-2)(2) - 0(\pi 2) + 1\left(\sqrt{4^2 + 2^2}\right) = 0.47214 \text{ in}^2$$

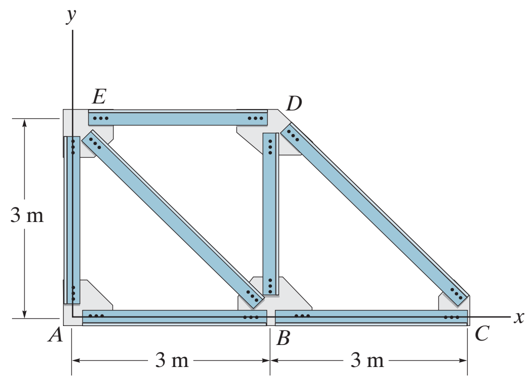
$$\Sigma \tilde{z} L = 1(2) - 0(\pi 2) + 0\left(\sqrt{4^2 + 2^2}\right) = 2 \text{ in}^2$$

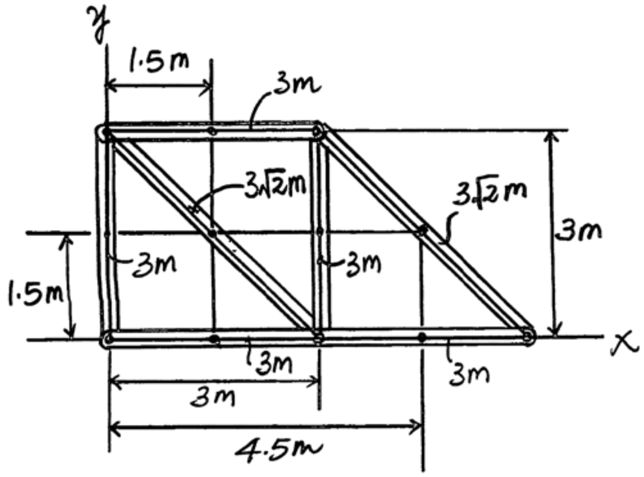
$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{0.94427}{12.7553} = 0.0740 \text{ in.} \qquad \text{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{0.47214}{12.7553} = 0.0370 \text{ in.} \qquad \text{Ans}$$

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{2}{12.7553} = 0.157 \text{ in.} \qquad \text{Ans}$$

***9–48.** The truss is made from seven members, each having a mass per unit length of 6 kg/m. Locate the position $(\bar{x}, \bar{y})$ of the center of mass. Neglect the mass of the gusset plates at the joints.





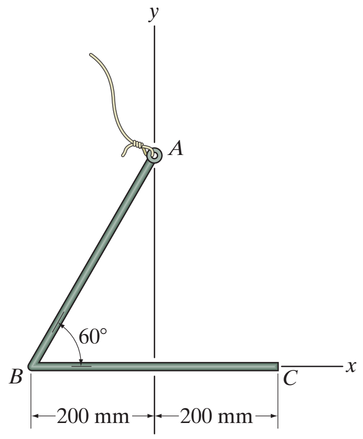
$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{(1.5)(3) + 4.5(3) + 4.5(3\sqrt{2}) + 1.5(3) + 3(3) + 1.5(3\sqrt{2})}{5(3) + 2(3\sqrt{2})}$$

$= 2.43$ m **Ans**

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{(1.5)(3) + 3(3) + 1.5(3) + 1.5(3\sqrt{2}) + 1.5(3\sqrt{2})}{5(3) + 2(3\sqrt{2})}$$

$= 1.31$ m **Ans**

•9–49. Locate the centroid $(\bar{x}, \bar{y})$ of the wire. If the wire is suspended from A, determine the angle segment AB makes with the vertical when the wire is in equilibrium.



Centroid: The length of segment AB is $l_{AB} = \dfrac{200}{\cos 60°} = 400$ mm. The centroid C_{AB} of this segment is located at $\tilde{x} = -\dfrac{400\cos 60°}{2} = -100$ mm and $\tilde{y} = \dfrac{400\sin 60°}{2} = 173.21$ mm as indicated in Fig. a. The centroid of segment BC is located at the origin of the coordinate axes.

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{400(-100) + 400(0)}{400 + 400} = -50 \text{ mm} \qquad \textbf{Ans.}$$

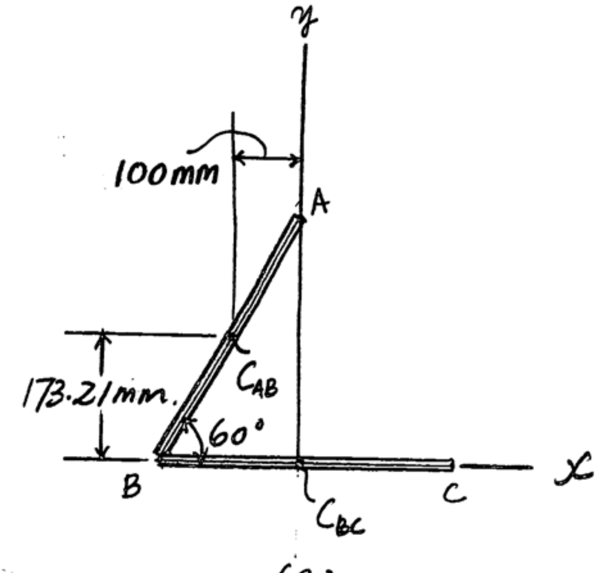
$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{400(173.21) + 400(0)}{400 + 400} = 86.60 \text{ mm} = 86.6 \text{ mm} \qquad \textbf{Ans.}$$

Geometry: When the bent wire hangs freely from A line AG will be vertical as shown in Fig. b. From the geometry of this figure, we have

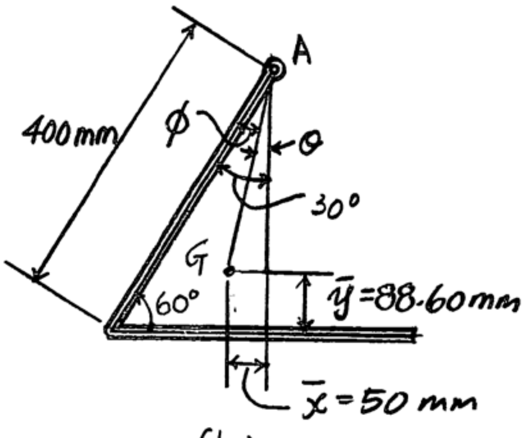
$$\tan\theta = \frac{50}{400\sin 60° - 86.60} \qquad \theta = 10.89°$$

Thus, the angle ϕ that segment AB makes with the vertical is

$$\phi = 30° - 10.89° = 19.1° \qquad \textbf{Ans.}$$

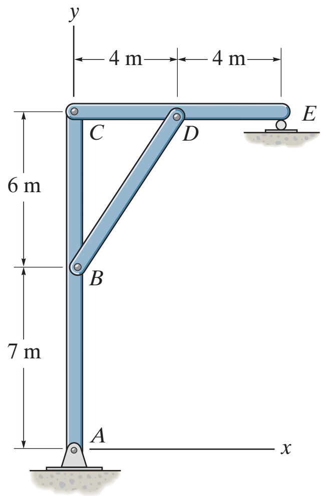


(a)



(b)

9–50. Each of the three members of the frame has a mass per unit length of 6 kg/m. Locate the position $(\overline{x}, \overline{y})$ of the center of mass. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the pin A and roller E.



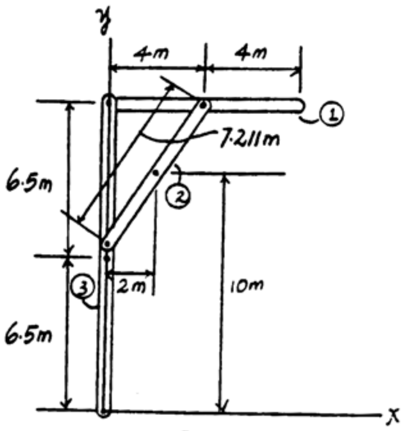
Centroid : The length of each segment and its respective centroid are tabulated below.

Segment	L(m)	$\bar{x}$(m)	$\bar{y}$(m)	$\bar{x}L$(m^2)	$\bar{y}L$(m^2)
1	8	4	13	32.0	104.0
2	7.211	2	10	14.42	72.11
3	13	0	6.5	0	84.5
Σ	28.211			46.42	260.61

Thus,

$$\bar{x} = \frac{\Sigma \bar{x}L}{\Sigma L} = \frac{46.42}{28.211} = 1.646 \text{ m} = 1.65 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{260.61}{28.211} = 9.238 \text{ m} = 9.24 \text{ m} \qquad \textbf{Ans}$$

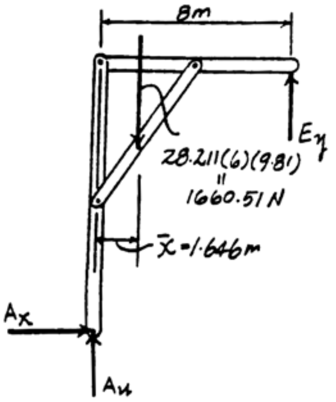


Equations of Equilibrium : The total weight of the frame is $W = 28.211(6)(9.81) = 1660.51$ N.

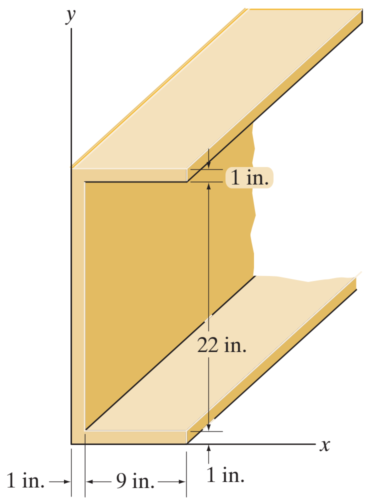
$$+\Sigma M_A = 0; \quad E_y(8) - 1660.51(1.646) = 0$$
$$E_y = 341.55 \text{ N} = 342 \text{ N} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 341.55 - 1660.51 = 0$$
$$A_y = 1318.95 \text{ N} = 1.32 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0 \qquad \textbf{Ans}$$



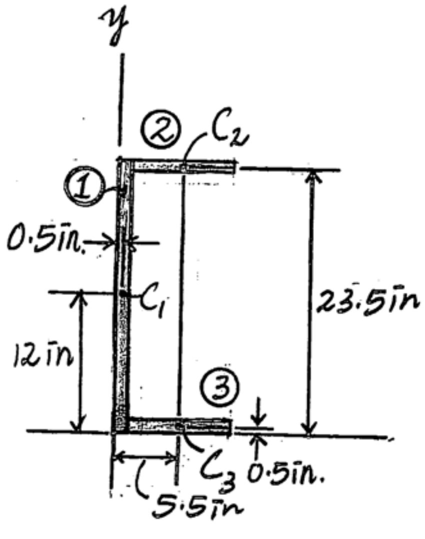
9–51. Locate the centroid $(\bar{x}, \bar{y})$ of the cross-sectional area of the channel.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

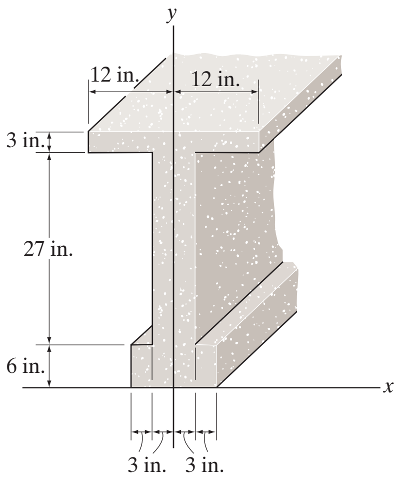
$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{0.5(24(1)) + 5.5(9(1)) + 5.5(9(1))}{24(1) + 9(1) + 9(1)} = \frac{111}{42} = 2.64 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{12(24(1)) + 23.5(9(1)) + 0.5(9(1))}{24(1) + 9(1) + 9(1)} = \frac{504}{42} = 12 \text{ in.} \qquad \textbf{Ans.}$$



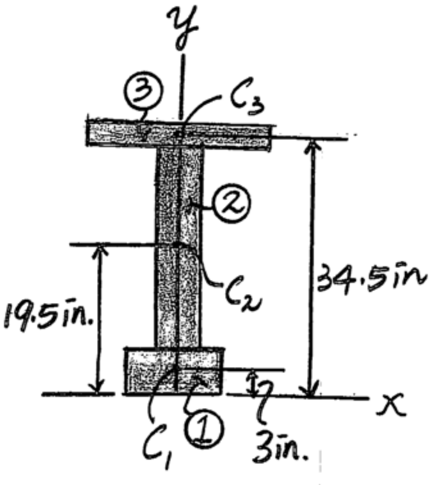
(a)

***9–52.** Locate the centroid $\bar{y}$ of the cross-sectional area of the concrete beam.



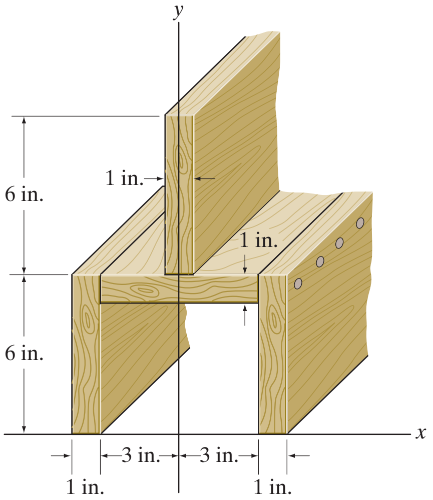
Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{3(12)(6) + 19.5(27)(6) + 34.5(24)(3)}{12(6) + 27(6) + 24(3)} = 19.1 \text{ in.}$$ **Ans.**



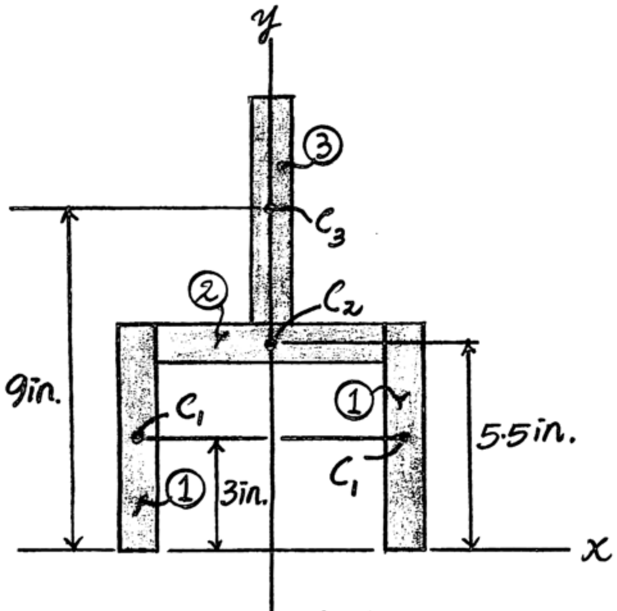
(a)

•9–53. Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



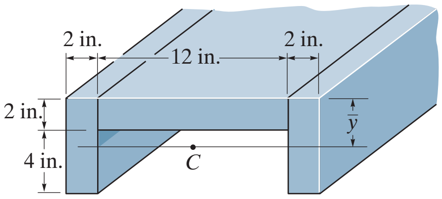
Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{3[2(6)(1)] + 5.5(6)(1) + 9(6)(1)}{2(6)(1) + 6(1) + 6(1)} = 5.125\text{ in.}$$ **Ans.**



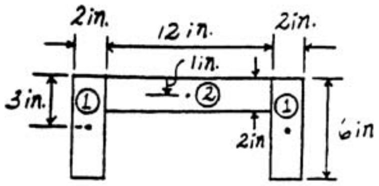
(a)

9–54. Locate the centroid $\bar{y}$ of the channel's cross-sectional area.



Centroid : The area of each segment and its respective centroid are tabulated below.

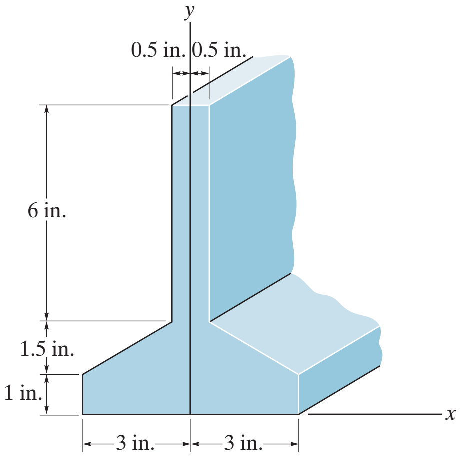
Segment	A (in^2)	$\tilde{y}$ (in.)	$\tilde{y}A$ (in^3)
1	6(4)	3	72.0
2	12(2)	1	24.0
Σ	48.0		96.0

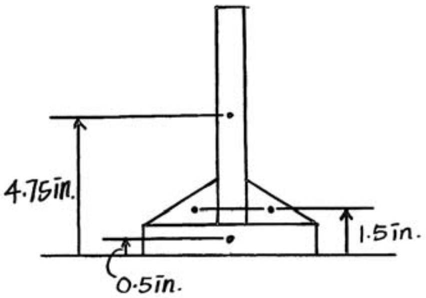


Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{96.0}{48.0} = 2.00 \text{ in.} \qquad \textbf{Ans}$$

9–55. Locate the distance $\bar{y}$ to the centroid of the member's cross-sectional area.





$$\Sigma\tilde{y}A = 0.5(6)(1) + 2(1.5)\left(\frac{1}{2}\right)(2.5)(1.5) + 4.75(7.5)(1)$$

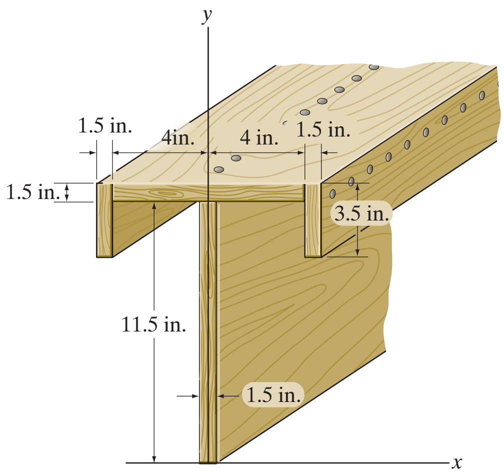
$$= 44.25 \text{ in}^3$$

$$\Sigma A = 6(1) + (2)\left(\frac{1}{2}\right)(2.5)(1.5) + 7.5(1)$$

$$= 17.25 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{44.25}{17.25} = 2.57 \text{ in.} \qquad \textbf{Ans}$$

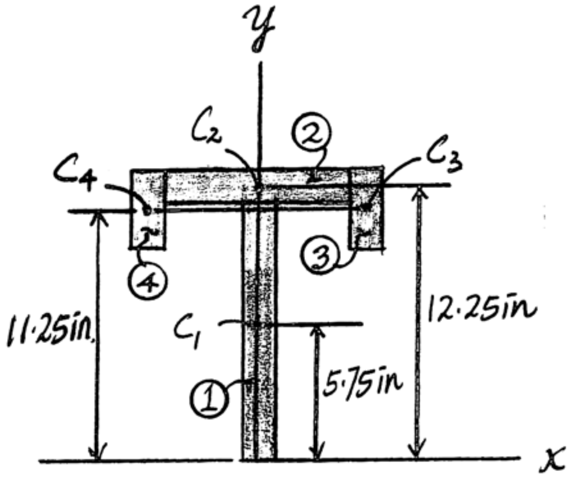
***9–56.** Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

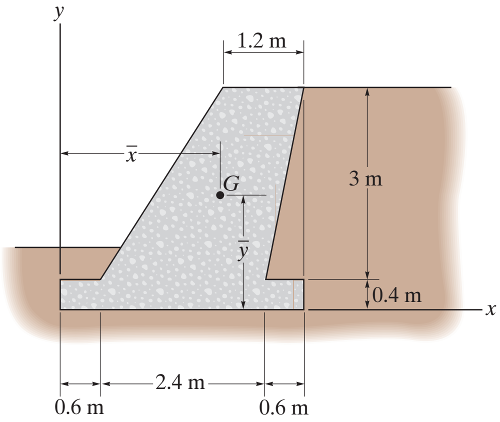
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{5.75(11.5)(1.5) + 12.25(8)(1.5) + 11.25(3.5)(1.5) + 11.25(3.5)(1.5)}{11.5(1.5) + 8(1.5) + 3.5(1.5) + 3.5(1.5)}$$

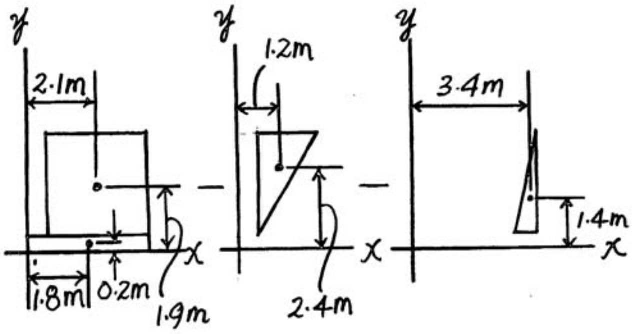
$= 9.17$ in. **Ans.**



(a)

•9–57. The gravity wall is made of concrete. Determine the location $(\bar{x}, \bar{y})$ of the center of mass G for the wall.





$$\Sigma\tilde{x}A = 1.8(3.6)(0.4) + 2.1(3)(3) - 3.4\left(\frac{1}{2}\right)(3)(0.6) - 1.2\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 15.192\ \text{m}^3$$

$$\Sigma\tilde{y}A = 0.2(3.6)(0.4) + 1.9(3)(3) - 1.4\left(\frac{1}{2}\right)(3)(0.6) - 2.4\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 9.648\ \text{m}^3$$

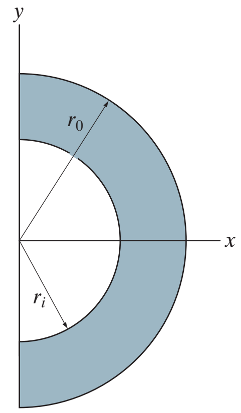
$$\Sigma A = 3.6(0.4) + 3(3) - \frac{1}{2}(3)(0.6) - \frac{1}{2}(1.8)(3)$$

$$= 6.84\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{15.192}{6.84} = 2.22\ \text{m} \qquad \text{Ans}$$

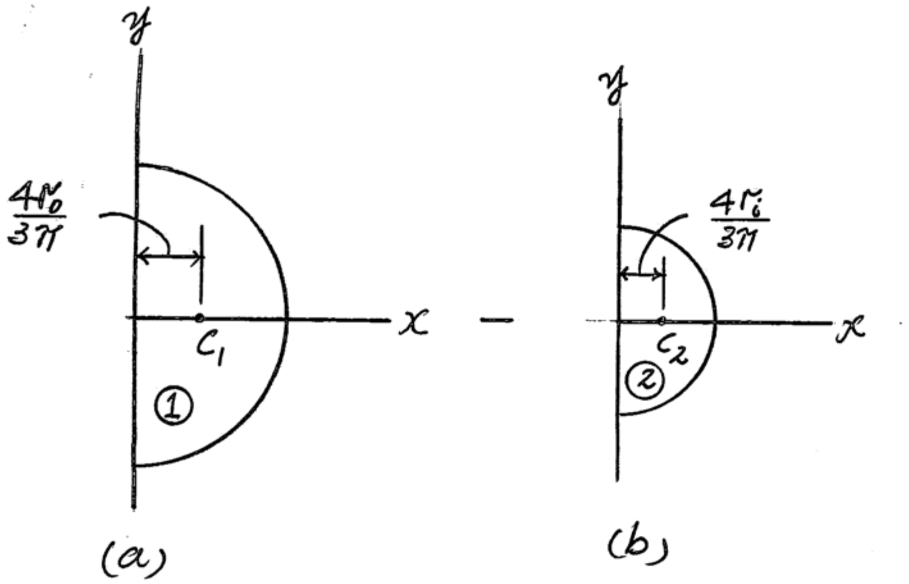
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{9.648}{6.84} = 1.41\ \text{m} \qquad \text{Ans}$$

9–58. Locate the centroid $\bar{x}$ of the composite area.

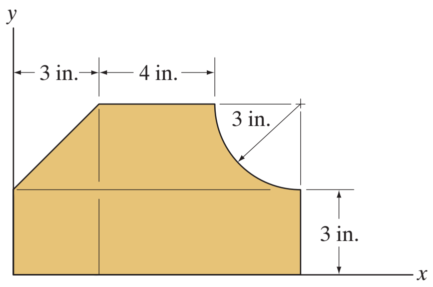


Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. Since segment (2) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{\left(\frac{4r_o}{3\pi}\right)\left(\frac{\pi r_o^2}{2}\right) + \left(\frac{4r_i}{3\pi}\right)\left(-\frac{\pi r_i^2}{2}\right)}{\frac{\pi r_o^2}{2} + \left(-\frac{\pi r_i^2}{2}\right)} = \frac{4\left(r_o^3 - r_i^3\right)}{3\pi\left(r_o^2 - r_i^2\right)} \qquad \textbf{Ans.}$$



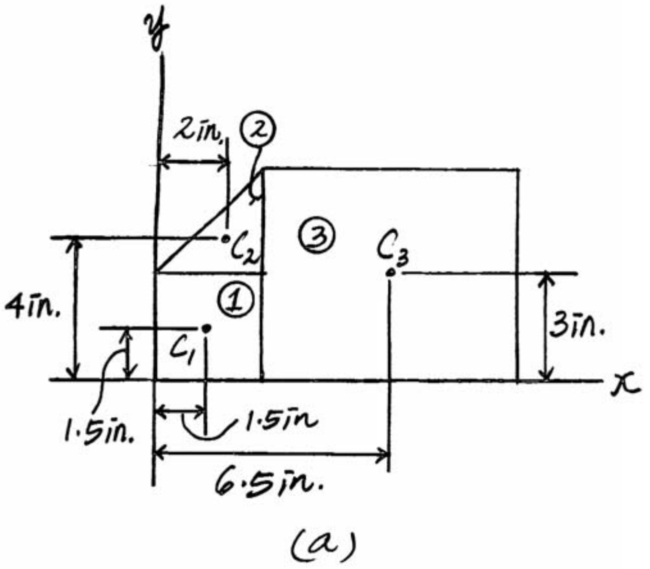
9–59. Locate the centroid $(\bar{x}, \bar{y})$ of the composite area.



Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. since segment (4) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{1.5(3(3)) + 2\left(\frac{1}{2}(3)(3)\right) + 6.5(7(6)) + \left(10 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{233.81}{48.43} = 4.83 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 3(7(6)) + \left(6 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{124.09}{48.43} = 2.56 \text{ in.} \qquad \textbf{Ans.}$$



(a)

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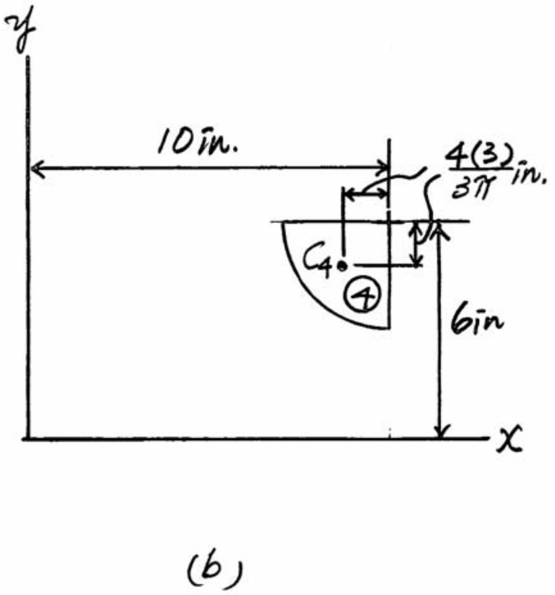
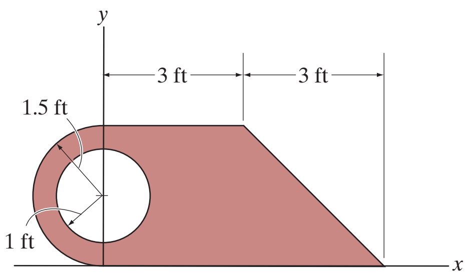


(b)

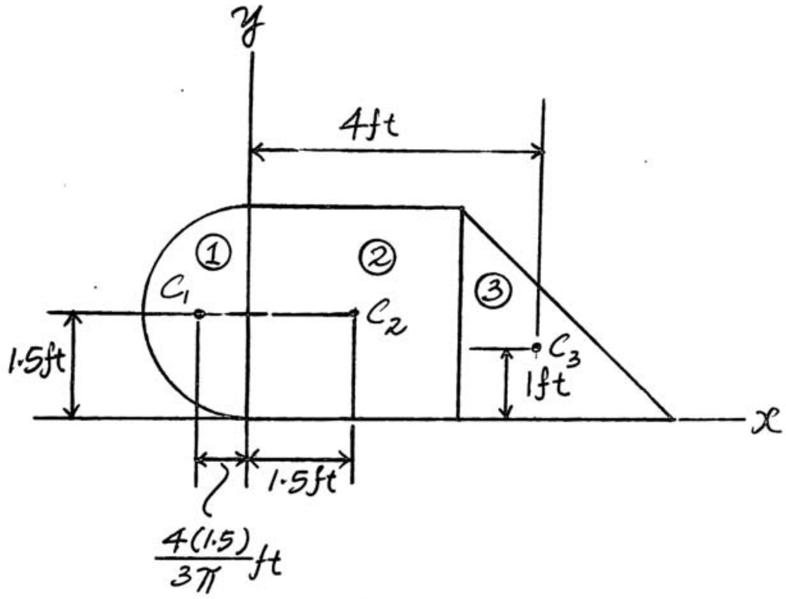
***9–60.** Locate the centroid $(\bar{x}, \bar{y})$ of the composite area.



Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. Since segment (4) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{\left(-\frac{4(1.5)}{3\pi}\right)\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 0\left(-\frac{\pi(1^2)}{4}\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\frac{\pi(1^2)}{4}\right)} = \frac{29.25}{13.89} = 2.11\text{ ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{1.5\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 1\left(\frac{1}{2}(3)(3)\right) + 1.5\left(-\frac{\pi(1^2)}{4}\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\frac{\pi(1^2)}{4}\right)} = \frac{18.59}{13.89} = 1.34\text{ ft} \qquad \textbf{Ans.}$$



(a)

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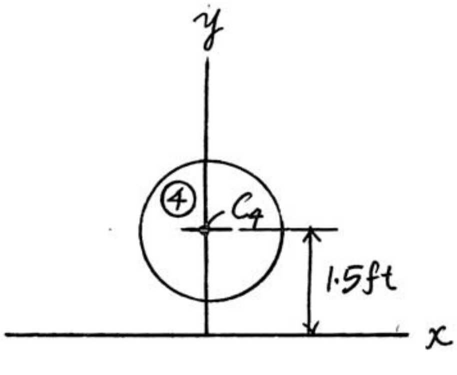
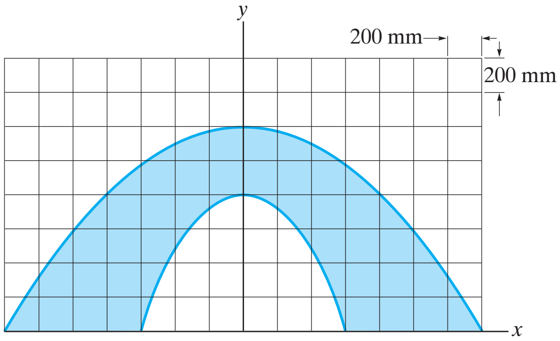


(b)

•9–61. Divide the plate into parts, and using the grid for measurement, determine approximately the location $(\bar{x}, \bar{y})$ of the centroid of the plate.

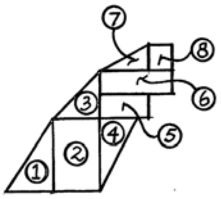


Due to symmetry.

$\bar{x} = 0$ Ans

Divide half the area into 8 segments as shown.

	A (Approx. 10^4)	$\tilde{y}$ (Approx. 10^2)	$\tilde{y}A$ (10^6)
1)	$\frac{1}{2}(6)(4)$	2	24
2)	4(6)	3	72
3)	$\frac{1}{2}(4)(4)$	7.32	58.56
4)	$\frac{1}{2}(3)(6)$	4	36
5)	8	7	56
6)	6(2)	9	108
7)	$\frac{1}{2}(4)(2)$	10.66	42.64
8)	(2)(2)	11	44



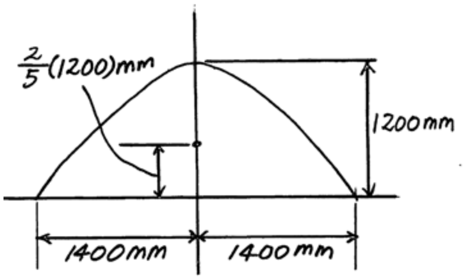
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{441.2(10^6)}{81(10^4)} = 544 \text{ mm} \quad \text{Ans}$$

A simpler solution consists of dividing the area into two parabolas.
For parabola :

$$\Sigma \tilde{y}A = \frac{2}{5}(1200)\left(\frac{4}{3}\right)(2800)(1200) - \frac{2}{5}(800)\left(\frac{4}{3}\right)(1200)(800)$$

$$\Sigma A = \frac{4}{3}(2800)(1200) - \frac{4}{3}(1200)(800)$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = 544 \text{ mm} \quad \text{Ans}$$



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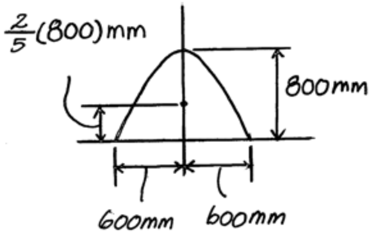
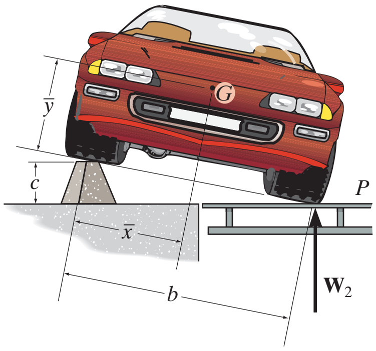


9–62. To determine the location of the center of gravity of the automobile it is first placed in a *level position*, with the two wheels on one side resting on the scale platform P. In this position the scale records a reading of W_1. Then, one side is elevated to a convenient height c as shown. The new reading on the scale is W_2. If the automobile has a total weight of W, determine the location of its center of gravity $G(\bar{x}, \bar{y})$.



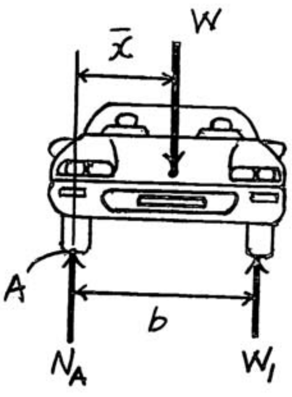
Equation of Equilibrium: First, we will consider the case in which the automobile is in a level position. Referring to the free-body diagram in Fig. *a* and writing the moment equation of equilibrium about point A,

$$\circlearrowleft +\Sigma M_A = 0; \qquad W_1(b) - W(\bar{x}) = 0 \qquad \bar{x} = \frac{W_1}{W}b$$ **Ans.**

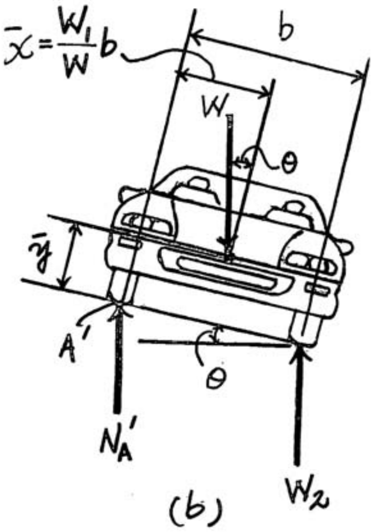
From the geometry in Fig. *c*, $\sin\theta = \frac{c}{b}$ and $\cos\theta = \frac{\sqrt{b^2-c^2}}{b}$. Using the result of $\bar{x}$ and referring to the free-body diagram in Fig. *b*, we can write the moment equation of equilibrium about point A'.

$$\circlearrowleft +\Sigma M_{A'} = 0; \qquad W_2\left[b\left(\frac{\sqrt{b^2-c^2}}{b}\right)\right] - W\left(\frac{\sqrt{b^2-c^2}}{b}\right)\left(\frac{W_1}{W}b\right) - W\left(\frac{c}{b}\right)\bar{y} = 0$$

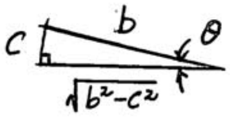
$$\bar{y} = \frac{b(W_2-W_1)\sqrt{b^2-c^2}}{cW}$$ **Ans.**



(a)

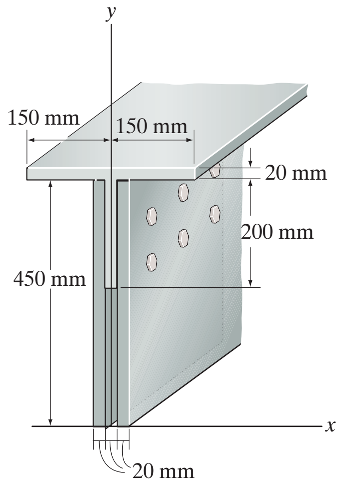


(b)



(c)

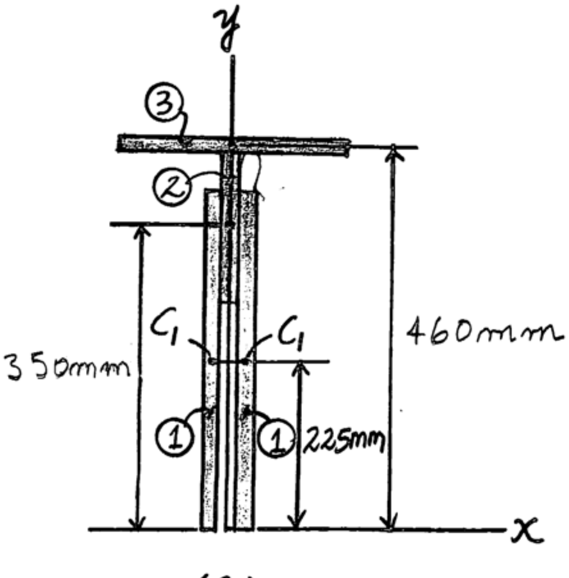
9–63. Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

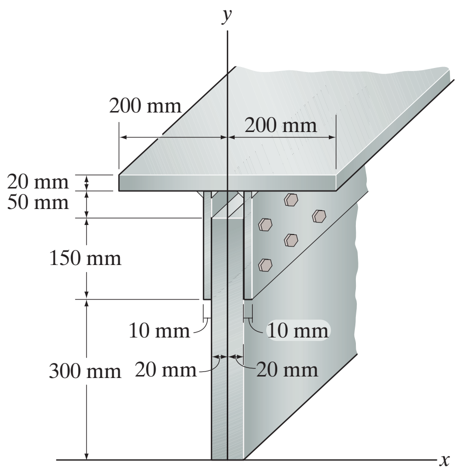
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[225(450)(20)] + 350(200)(20) + 460(300)(20)}{2(450)(20) + 200(20) + 300(20)}$$

$$= 293 \text{ mm}$$ **Ans.**



(a)

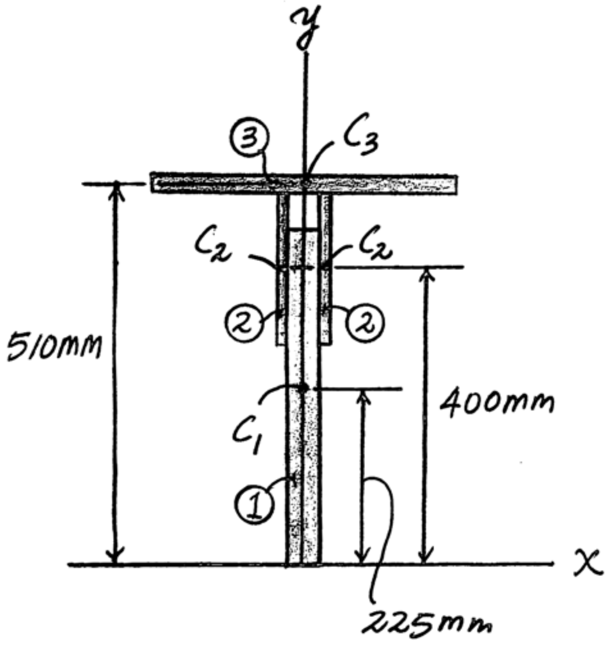
***9–64.** Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

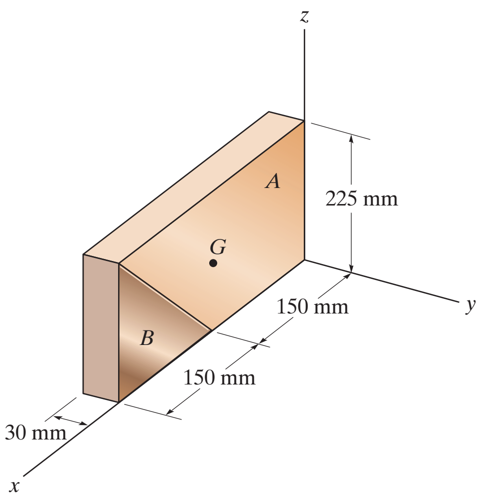
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{225(450)(40) + 2[400(200)(10)] + 510(400)(20)}{450(40) + 2(200)(10) + 400(20)}$$

$$= 324\text{ mm} \qquad \textbf{Ans.}$$



(a)

•9–65. The composite plate is made from both steel (A) and brass (B) segments. Determine the mass and location $(\bar{x}, \bar{y}, \bar{z})$ of its mass center G. Take $\rho_{st} = 7.85\ \text{Mg/m}^3$ and $\rho_{br} = 8.74\ \text{Mg/m}^3$.



$$\Sigma m = \Sigma \rho V = \left[8.74\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right] + \left[7.85\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right]$$

$$+ [7.85(0.15)(0.225)(0.03)]$$

$$= \left[4.4246\left(10^{-3}\right)\right] + \left[3.9741\left(10^{-3}\right)\right] + \left[7.9481\left(10^{-3}\right)\right]$$

$$= 16.347\left(10^{-3}\right) = 16.4\ \text{kg} \qquad \textbf{Ans}$$

$$\Sigma \tilde{x} m = \left(0.150 + \frac{2}{3}(0.150)\right)(4.4246)\left(10^{-3}\right) + \left(0.150 + \frac{1}{3}(0.150)\right)(3.9741)\left(10^{-3}\right)$$

$$+ \frac{1}{2}(0.150)(7.9481)\left(10^{-3}\right) = 2.4971\left(10^{-3}\right)\text{kg}\cdot\text{m}$$

$$\Sigma \tilde{z} m = \left(\frac{1}{3}(0.225)\right)(4.4246)\left(10^{-3}\right) + \left(\frac{2}{3}(0.225)\right)(3.9471)\left(10^{-3}\right) + \left(\frac{0.225}{2}\right)(7.9481)\left(10^{-3}\right)$$

$$= 1.8221\left(10^{-3}\right)\text{kg}\cdot\text{m}$$

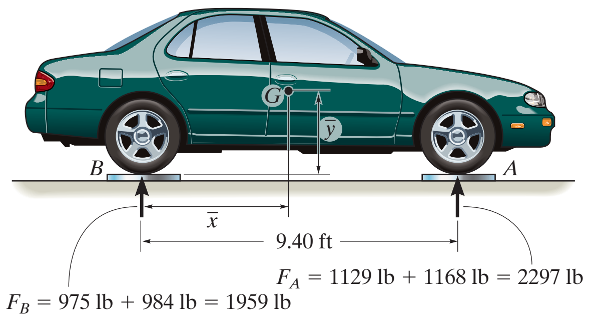
$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{2.4971(10^{-3})}{16.347(10^{-3})} = 0.153\ \text{m} = 153\ \text{mm} \qquad \textbf{Ans}$$

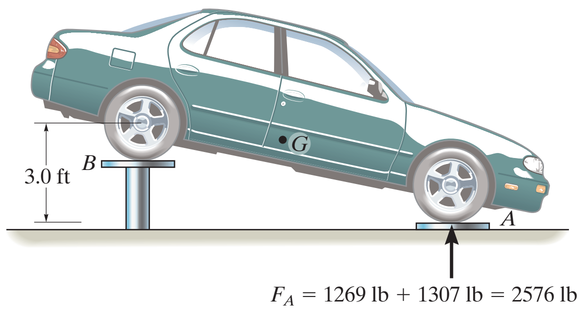
Due to symmetry :

$$\bar{y} = -15\ \text{mm} \qquad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{1.8221(10^{-3})}{16.347(10^{-3})} = 0.1115\ \text{m} = 111\ \text{mm} \qquad \textbf{Ans}$$

9–66. The car rests on four scales and in this position the scale readings of both the front and rear tires are shown by F_A and F_B. When the rear wheels are elevated to a height of 3 ft above the front scales, the new readings of the front wheels are also recorded. Use this data to compute the location $\bar{x}$ and $\bar{y}$ to the center of gravity G of the car. The tires each have a diameter of 1.98 ft.


G
ȳ
B
A
x̄
9.40 ft
F_A = 1129 lb + 1168 lb = 2297 lb
F_B = 975 lb + 984 lb = 1959 lb



In horizontal position

$W = 1959 + 2297 = 4256$ lb

$\circlearrowleft + \Sigma M_B = 0; \quad 2297(9.40) - 4256\,\bar{x} = 0$

$\bar{x} = 5.0733 = 5.07$ ft **Ans**

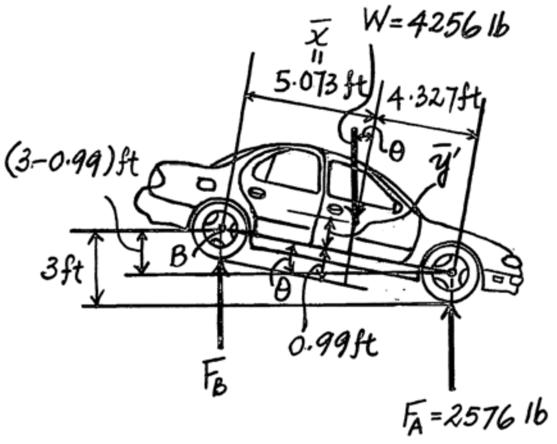
$\theta = \sin^{-1}\left(\dfrac{3 - 0.990}{9.40}\right) = 12.347°$

$\circlearrowleft + \Sigma M_B = 0; \quad 2576(9.40 \cos 12.347°) - 4256 \cos 12.347°(5.0733)$

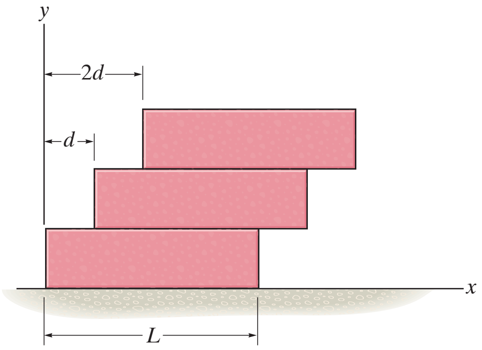
$- 4256 \sin 12.347°\,\bar{y}' = 0$

$\bar{y}' = 2.815$ ft

$\bar{y} = 2.815 + 0.990 = 3.81$ ft **Ans**



9–67. Uniform blocks having a length L and mass m are stacked one on top of the other, with each block overhanging the other by a distance d, as shown. If the blocks are glued together, so that they will not topple over, determine the location $\bar{x}$ of the center of mass of a pile of n blocks.



$n = 1$: $\quad \bar{x} = \frac{L}{2} = \frac{L}{2} + 0\left(\frac{d}{2}\right)$

$n = 2$: $\quad \bar{x} = \dfrac{\frac{L}{2}(W) + \left(\frac{L}{2} + d\right)W}{2W}$

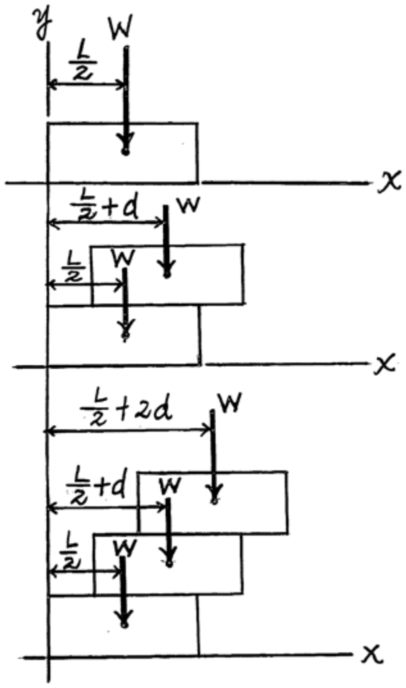
$$= \frac{L}{4} + \frac{L}{4} + \frac{d}{2} = \frac{L}{2} + (1)\frac{d}{2}$$

$n = 3$: $\quad \bar{x} = \dfrac{\frac{L}{2}(W) + \left(\frac{L}{2} + d\right)W + \left(\frac{L}{2} + 2d\right)W}{3W}$

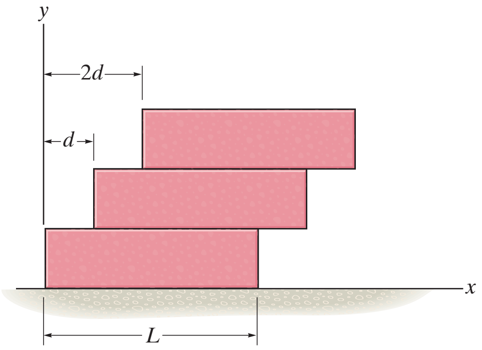
$$= \frac{L}{6} + \frac{L}{6} + \frac{d}{3} + \frac{L}{6} + \frac{2}{3}d = \frac{L}{2} + 2\left(\frac{d}{2}\right)$$

In general :

$$\bar{x} = \frac{L}{2} + (n-1)\left(\frac{d}{2}\right) = \frac{L + (n-1)d}{2} \qquad \text{Ans}$$



***9–68.** Uniform blocks having a length L and mass m are stacked one on top of the other, with each block overhanging the other by a distance d, as shown. Show that the maximum number of blocks which can be stacked in this manner is $n < L/d$.



$n = 2$: $\quad \bar{x} = \frac{L}{2} + d = \frac{L}{2} + 2\left(\frac{d}{2}\right)$

$n = 3$: $\quad \bar{x} = \dfrac{\left(d + \frac{L}{2}\right)W + \left(2d + \frac{L}{2}\right)W}{2W} = \frac{L}{2} + 3\left(\frac{d}{2}\right)$

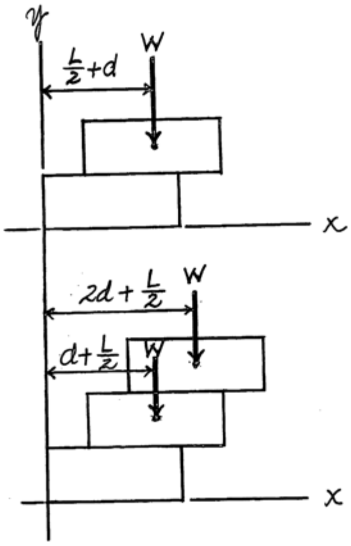
In general :

$$\bar{x} = \frac{L}{2} + n\left(\frac{d}{2}\right)$$

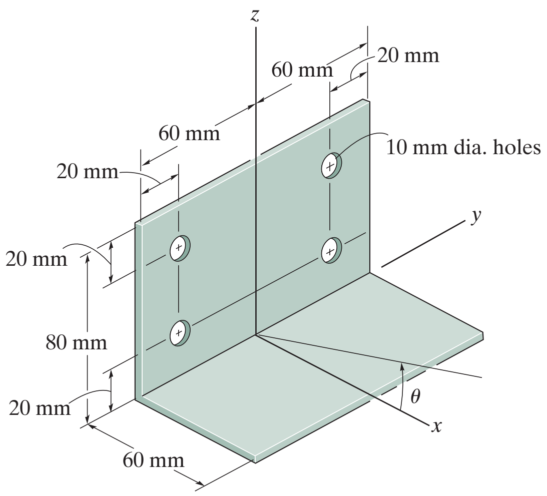
For stable stack :

$$\bar{x} = \frac{L}{2} + n\left(\frac{d}{2}\right) \leq L$$

$n \leq \frac{L}{d}$ **Ans**

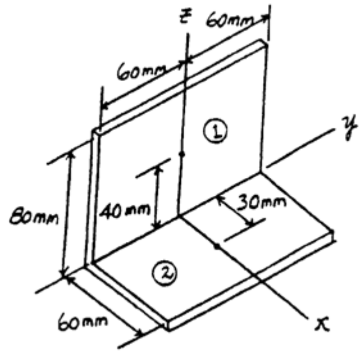


•9–69. Locate the center of gravity ($\bar{x}$, $\bar{z}$) of the sheet-metal bracket if the material is homogeneous and has a constant thickness. If the bracket is resting on the horizontal x–y plane shown, determine the maximum angle of tilt θ which it can have before it falls over, i.e., begins to rotate about the y axis.



Centroid : The area of each segment and its respective centroid are tabulated below.

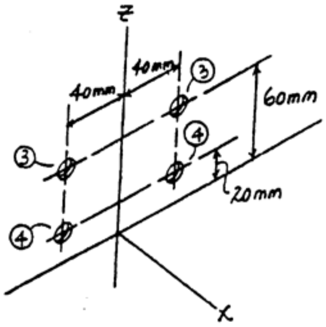
Segment	$A\,(\text{mm}^2)$	$\tilde{x}\,(\text{mm})$	$\tilde{z}\,(\text{mm})$	$\tilde{x}A\,(\text{mm}^3)$	$\tilde{z}A\,(\text{mm}^3)$
1	120(80)	0	40	0	384 000
2	120(60)	30	0	216 000	0
3	$-2\left[\frac{\pi}{4}(10^2)\right]$	0	60	0	−9424.78
4	$-2\left[\frac{\pi}{4}(10^2)\right]$	0	20	0	−3141.59
Σ	16 485.84			216 000	371 433.63



Thus,

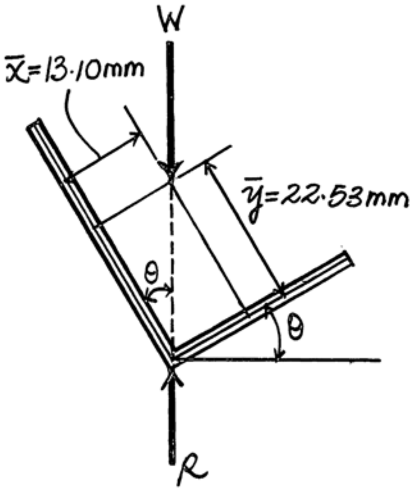
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{216\,000}{16\,485.84} = 13.10 \text{ mm} = 13.1 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma\tilde{z}A}{\Sigma A} = \frac{371\,433.63}{16\,485.84} = 22.53 \text{ mm} = 22.5 \text{ mm} \qquad \textbf{Ans}$$

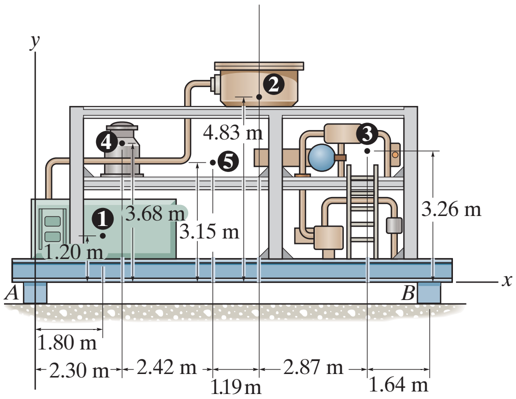


Equilibrium : In order for the bracket not to rotate about y axis, the weight of the bracket must coincide with the reaction. From the FBD,

$$\theta = \tan^{-1}\frac{13.10}{22.53} = 30.2° \qquad \textbf{Ans}$$



9–70. Locate the center of mass for the compressor assembly. The locations of the centers of mass of the various components and their masses are indicated and tabulated in the figure. What are the vertical reactions at blocks A and B needed to support the platform?



❶	Instrument panel	230 kg
❷	Filter system	183 kg
❸	Piping assembly	120 kg
❹	Liquid storage	85 kg
❺	Structural framework	468 kg

Centroid : The mass of each component of the compressor and its respective centroid are tabulated below.

Component	m (kg)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}m$ (kg · m)	$\bar{y}m$ (kg · m)
1	230	1.80	1.20	414.00	276.00
2	183	5.91	4.83	1081.53	883.89
3	120	8.78	3.26	1053.60	391.20
4	85	2.30	3.68	195.50	312.80
5	468	4.72	3.15	2208.96	1474.20
Σ	1086			4953.59	3338.09

Thus,

$$\bar{x} = \frac{\Sigma \bar{x}m}{\Sigma m} = \frac{4953.59}{1086} = 4.561 \text{ m} = 4.56 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y}m}{\Sigma m} = \frac{3338.09}{1086} = 3.074 \text{ m} = 3.07 \text{ m} \qquad \textbf{Ans}$$

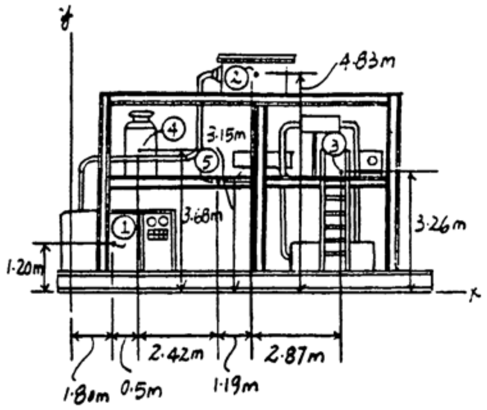
Equations of Equilibrium :

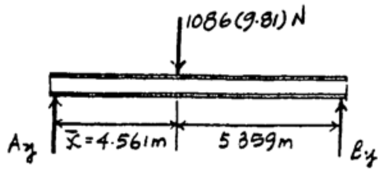
$$+\Sigma M_A = 0; \qquad B_y(10.42) - 1086(9.81)(4.561) = 0$$

$$B_y = 4663.60 \text{ N} = 4.66 \text{ kN} \qquad \textbf{Ans}$$

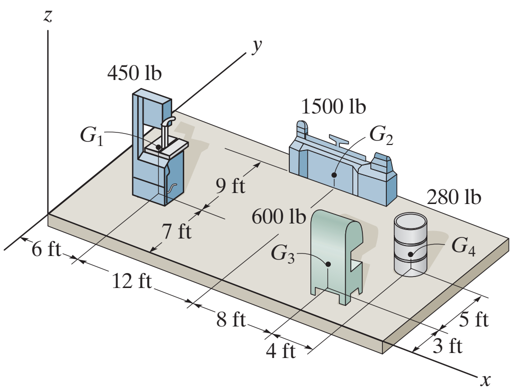
$$+\uparrow \Sigma F_y = 0; \qquad A_y + 4663.60 - 1086(9.81) = 0$$

$$A_y = 5990.06 \text{ N} = 5.99 \text{ kN} \qquad \textbf{Ans}$$



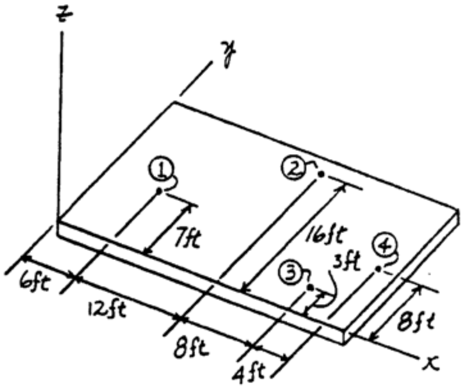


9–71. Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity G. Locate the center of gravity $(\bar{x}, \bar{y})$ of all these components.



Centroid : The floor loadings on the floor and its respective centroid are tabulated below.

Loading	W (lb)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}W$ (lb·ft)	$\tilde{y}W$ (lb·ft)
1	450	6	7	2700	3150
2	1500	18	16	27000	24000
3	600	26	3	15600	1800
4	280	30	8	8400	2240
Σ	2830			53700	31190

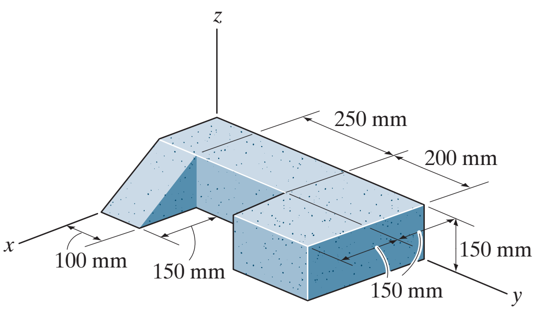


Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}W}{\Sigma W} = \frac{53700}{2830} = 18.98\text{ ft} = 19.0\text{ ft} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}W}{\Sigma W} = \frac{31190}{2830} = 11.02\text{ ft} = 11.0\text{ ft} \qquad \textbf{Ans}$$

***9–72.** Locate the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly.

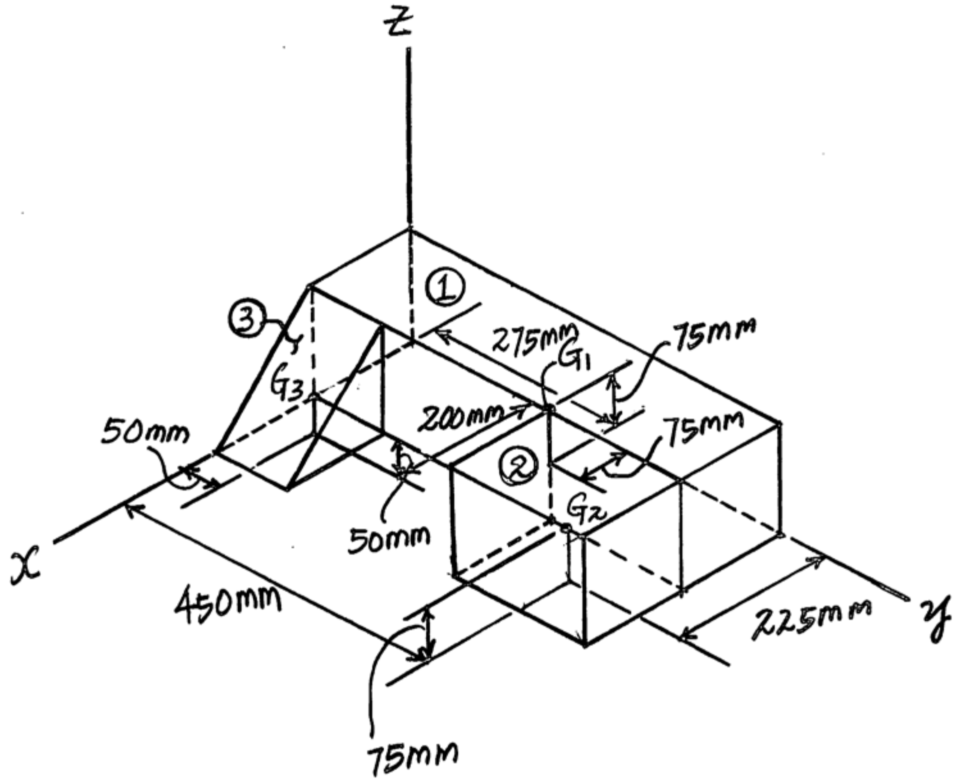


Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{(75)(150)(150)(550) + (225)(150)(150)(200) + (200)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{2.165625(10^9)}{18(10^6)} = 120 \text{ mm} \qquad \textbf{Ans.}$$

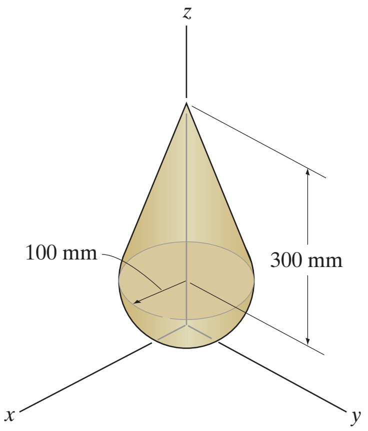
$$\bar{y} = \frac{\Sigma \tilde{y} V}{\Sigma V} = \frac{(275)(150)(150)(550) + (450)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{5.484375(10^9)}{18(10^6)} = 305 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{(75)(150)(150)(550) + (75)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{1.321875(10^9)}{18(10^6)} = 73.4 \text{ mm} \qquad \textbf{Ans.}$$



(a)

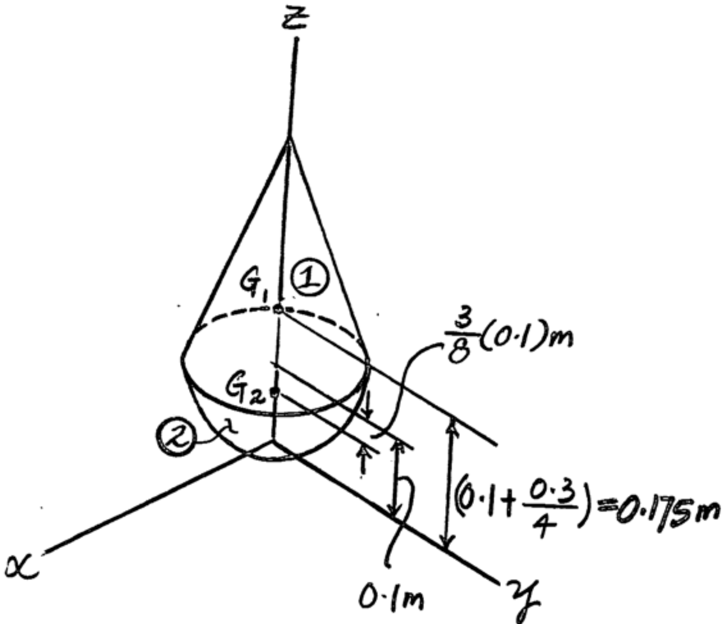
•9–73. Locate the center of mass $\bar{z}$ of the assembly. The hemisphere and the cone are made from materials having densities of 8 Mg/m^3 and 4 Mg/m^3, respectively.



Centroid: The center of mass of each composite segment is shown in Fig. *a*.

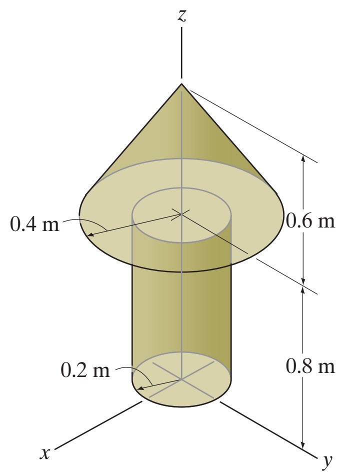
$$\bar{z} = \frac{\Sigma\tilde{z}m}{\Sigma m} = \frac{4000(0.175)\left[\frac{1}{3}\pi(0.1^2)(0.3)\right] + 8000\left(0.1 - \frac{3}{8}(0.1)\right)\left[\frac{2}{3}\pi(0.1^3)\right]}{4000\left[\frac{1}{3}\pi(0.1^2)(0.3)\right] + 8000\left[\frac{2}{3}\pi(0.1^3)\right]}$$

$$= \frac{1.0333\pi}{9.3333\pi} = 0.1107\text{ m} = 111\text{ mm} \qquad \textbf{Ans.}$$



(a)

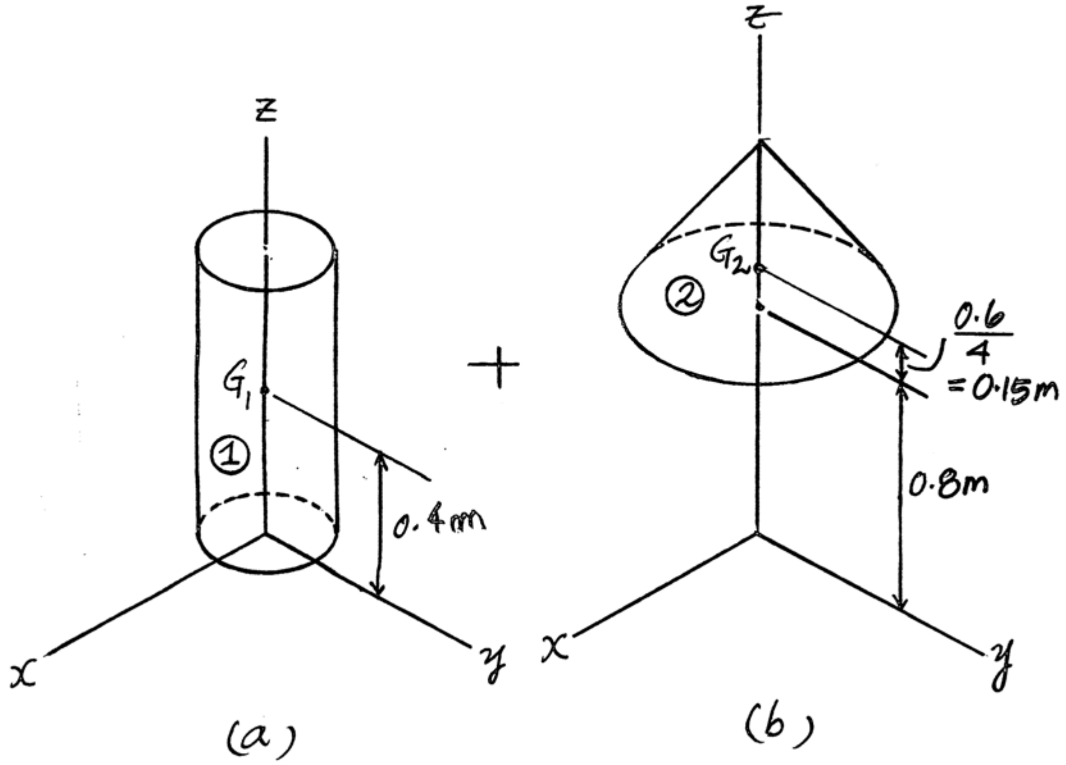
9–74. Locate the center of mass $\bar{z}$ of the assembly. The cylinder and the cone are made from materials having densities of 5 Mg/m^3 and 9 Mg/m^3, respectively.



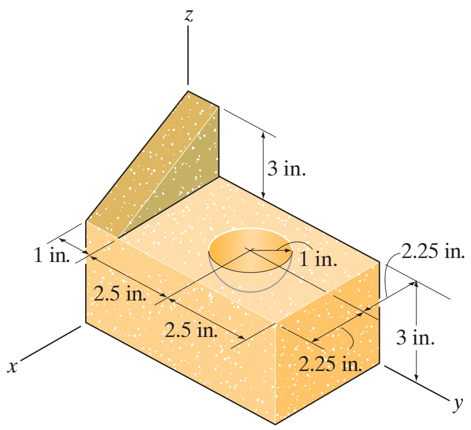
Center of mass: The assembly is broken into two composite segments, as shown in Figs. *a* and *b*.

$$\bar{z} = \frac{\Sigma\tilde{z}m}{\Sigma m} = \frac{5000(0.4)\left[\pi(0.2^2)(0.8)\right] + 9000(0.8+0.15)\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}{5000\left[\pi(0.2^2)(0.8)\right] + 9000\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}$$

$$= \frac{1060.60}{1407.4} = 0.754 \text{ m} = 754 \text{ mm}$$ **Ans.**



9–75. Locate the center of gravity $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly having a hemispherical hole.

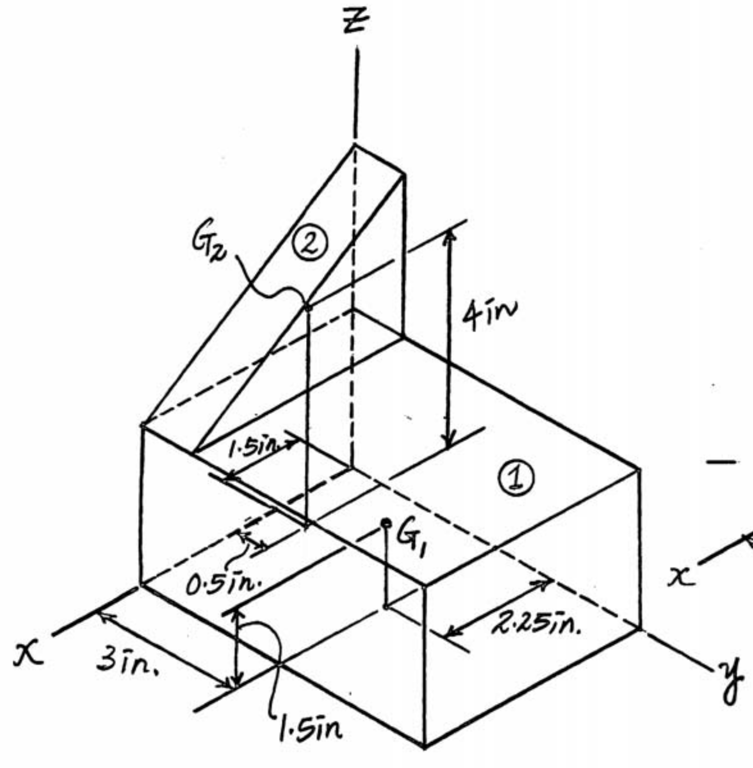


Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Figs. *a* and *b*. Since segment (3) is a hole, its volume should be considered negative.

$$\bar{x} = \frac{\Sigma\tilde{x}V}{\Sigma V} = \frac{2.25(3)(4.5)(6) + (1.5)\left(\frac{1}{2}\right)(3)(4.5)(1) + 2.25\left(-\frac{2}{3}\pi(1^3)\right)}{(3)(4.5)(6) + \frac{1}{2}(3)(4.5)(1) + \left(-\frac{2}{3}\pi(1^3)\right)} = \frac{187.663}{85.656} = 2.19 \text{ in.} \qquad \textbf{Ans.}$$

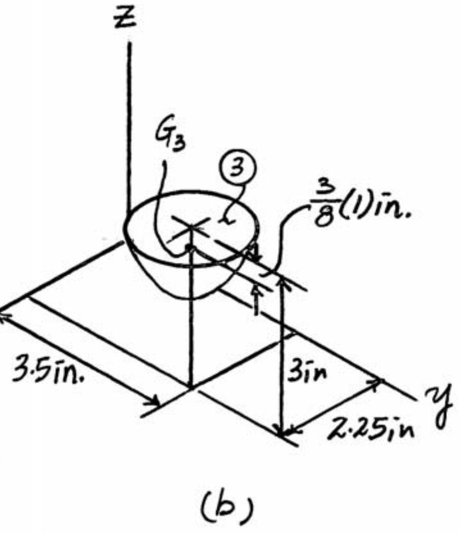
$$\bar{y} = \frac{\Sigma\tilde{y}V}{\Sigma V} = \frac{3(3)(4.5)(6) + (0.5)\left(\frac{1}{2}\right)(3)(4.5)(1) + 3.5\left(-\frac{2}{3}\pi(1^3)\right)}{(3)(4.5)(6) + \frac{1}{2}(3)(4.5)(1) + \left(-\frac{2}{3}\pi(1^3)\right)} = \frac{239.045}{85.656} = 2.79 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{1.5(3)(4.5)(6) + (4)\left(\frac{1}{2}\right)(3)(4.5)(1) + \left(3 - \frac{3}{8}(1)\right)\left(-\frac{2}{3}\pi(1^3)\right)}{(3)(4.5)(6) + \frac{1}{2}(3)(4.5)(1) + \left(-\frac{2}{3}\pi(1^3)\right)} = \frac{143.002}{85.656} = 1.67 \text{ in.} \qquad \textbf{Ans.}$$



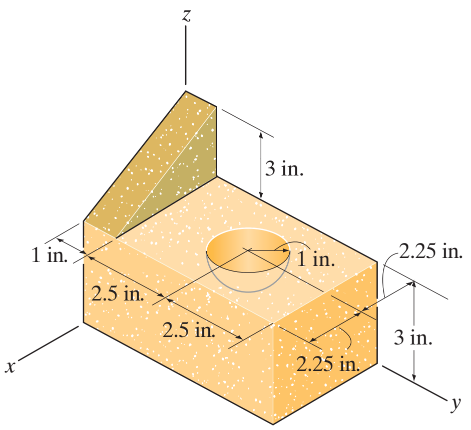
(a)

−



(b)

***9–76.** Locate the center of gravity $(\bar{x}, \bar{y}, \bar{z})$ of the assembly. The triangular and the rectangular blocks are made from materials having specific weights of 0.25 lb/in^3 and 0.1 lb/in^3, respectively.

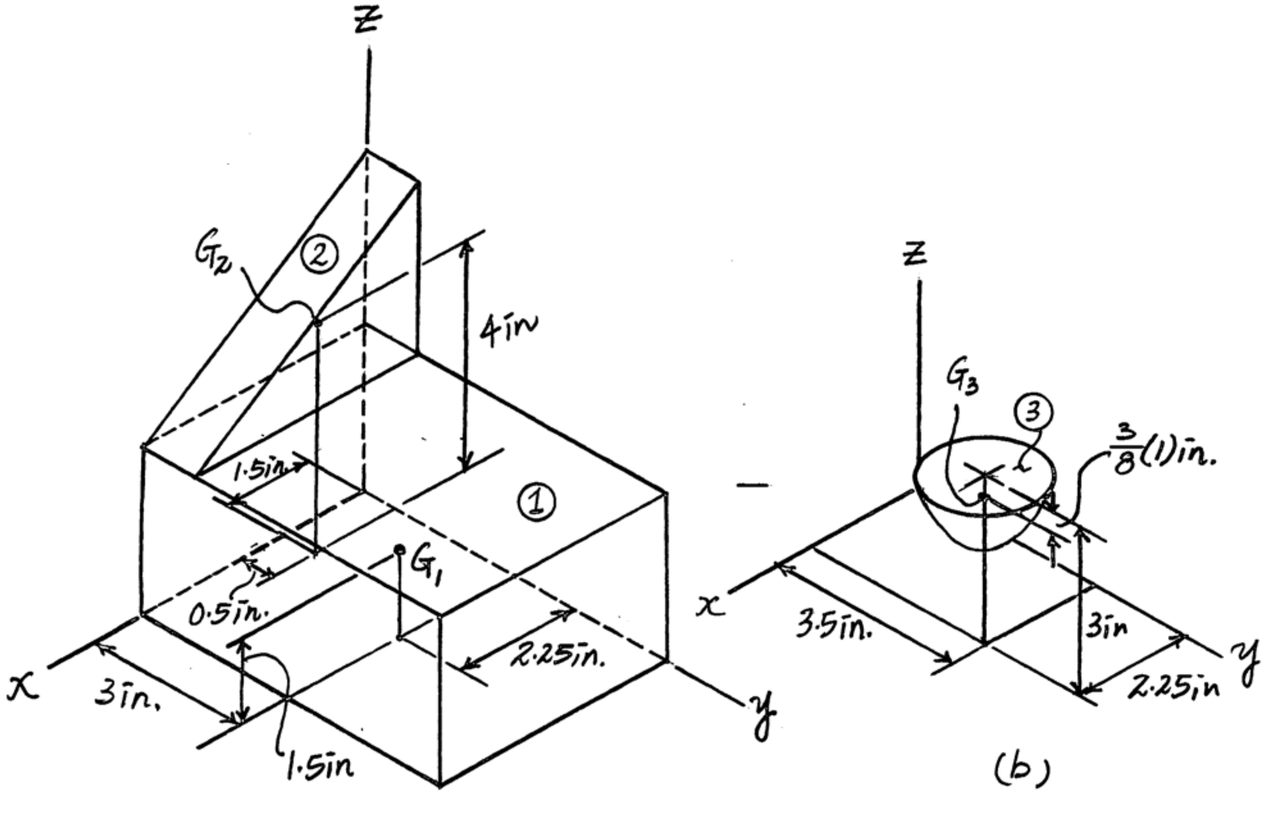


Center of Gravity: The center of gravity for each composite segment is shown in Figs. *a* and *b*. Since segment (3) is a hole, its weight should be considered negative.

$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{(2.25)(0.1)(3)(4.5)(6) + 0.25(1.5)\left[\frac{1}{2}(3)(4.5)(1)\right] + 2.25\left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]}{(0.1)(3)(4.5)(6) + 0.25\left[\frac{1}{2}(3)(4.5)(1)\right] + \left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]} = \frac{20.2850}{9.5781} = 2.12 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}W}{\Sigma W} = \frac{3(0.1)(3)(4.5)(6) + 0.25(0.5)\left[\frac{1}{2}(3)(4.5)(1)\right] + 3.5\left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]}{(0.1)(3)(4.5)(6) + 0.25\left[\frac{1}{2}(3)(4.5)(1)\right] + \left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]} = \frac{24.4107}{9.5781} = 2.55 \text{ in.} \qquad \textbf{Ans.}$$

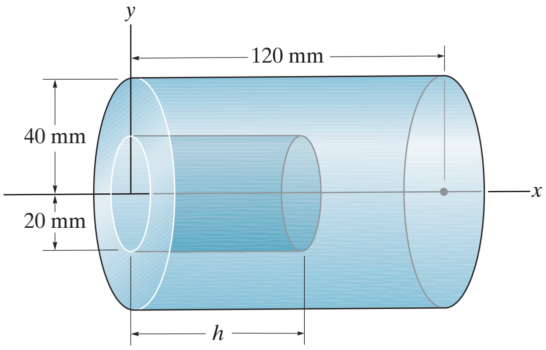
$$\bar{z} = \frac{\Sigma\tilde{z}W}{\Sigma W} = \frac{1.5(0.1)(3)(4.5)(6) + 0.25(4)\left[\frac{1}{2}(3)(4.5)(1)\right] + \left(3 - \frac{3}{8}(1)\right)\left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]}{(0.1)(3)(4.5)(6) + 0.25\left[\frac{1}{2}(3)(4.5)(1)\right] + \left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]} = \frac{18.3502}{9.5781} = 1.92 \text{ in.} \qquad \textbf{Ans.}$$

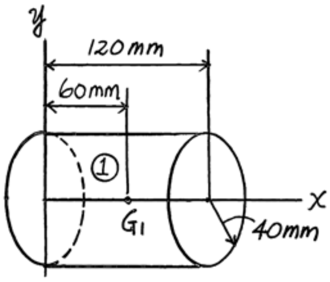


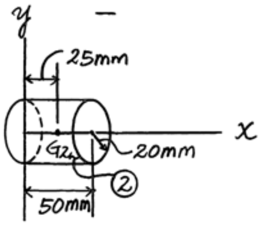
(a)

(b)

•9–77. Determine the distance $\bar{x}$ to the centroid of the solid which consists of a cylinder with a hole of length $h = 50$ mm bored into its base.





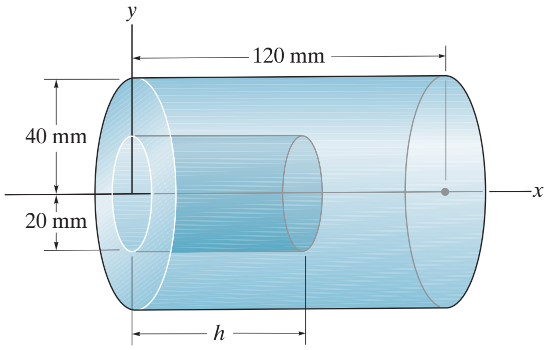


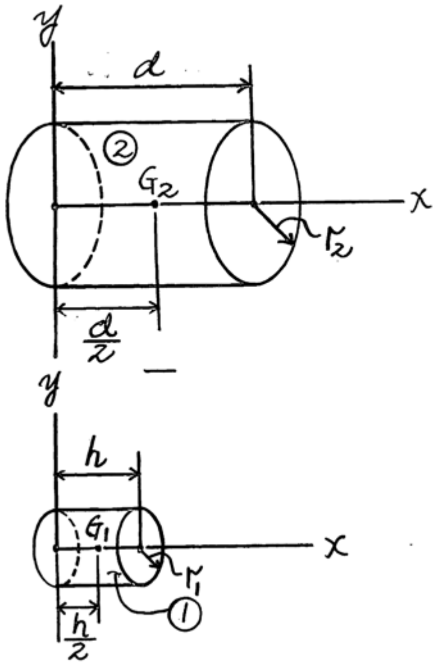
$$\Sigma V = \pi(40)^2(120) - \pi(20)^2(50) = 172\left(10^3\right)\pi \text{ mm}^3$$

$$\Sigma \tilde{x}V = 60(\pi)(40)^2(120) - 25(\pi)(20)^2(50) = 11.02\left(10^6\right)\pi \text{ mm}^4$$

$$\bar{x} = \frac{\Sigma \tilde{x}V}{\Sigma V} = \frac{11.02(10^6)\pi}{172(10^3)\pi} = 64.1 \text{ mm} \qquad \textbf{Ans}$$

9–78. Determine the distance h to which a hole must be bored into the cylinder so that the center of mass of the assembly is located at $\bar{x} = 64$ mm. The material has a density of 8 Mg/m^3.





$$\Sigma V = \pi r_2^2 d - \pi r_1^2 h$$

$$\Sigma \tilde{x} V = \frac{d}{2}(\pi)\left(r_2^2\right) d - \frac{h}{2}(\pi)\left(r_1^2\right) h$$

$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{\frac{d^2}{2}(\pi)\left(r_2^2\right) - \frac{h^2}{2}(\pi)\left(r_1^2\right)}{\pi r_2^2 d - \pi r_1^2 h}$$

$$2\bar{x}\left(r_2^2 d\right) - 2\bar{x}\left(r_1^2 h\right) = d^2 r_2^2 - h^2 r_1^2$$

$$h^2 - 2\bar{x}h + d\,(2\bar{x} - d)\left(\frac{r_2}{r_1}\right)^2 = 0$$

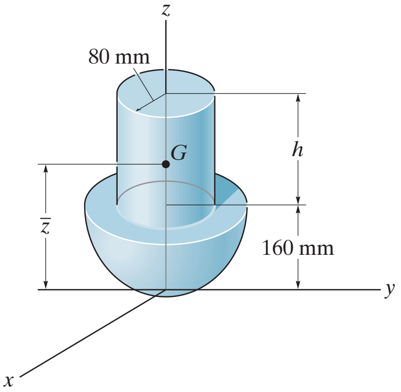
Set $\bar{x} = 64$ mm, $r_2 = 40$ mm, $r_1 = 20$ mm, $d = 120$ mm

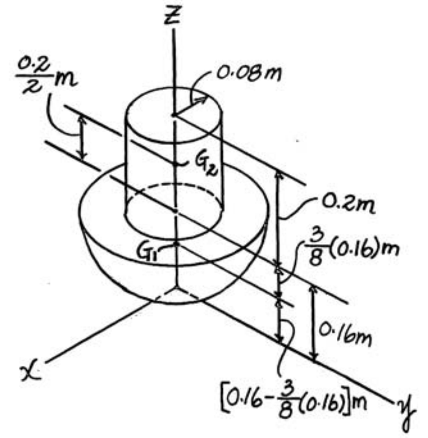
$$h^2 - 128h + 3840 = 0$$

Solving,

$h = 80$ mm **Ans** or $h = 48$ mm **Ans**

9–79. The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the mass center of the assembly if the height of the cylinder is $h = 200$ mm.



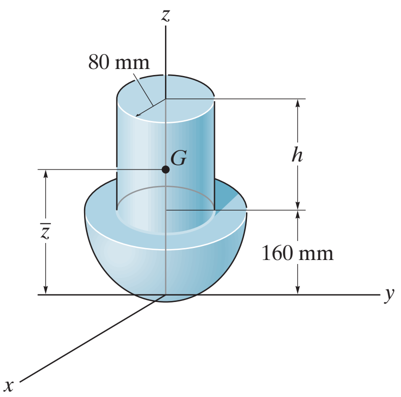


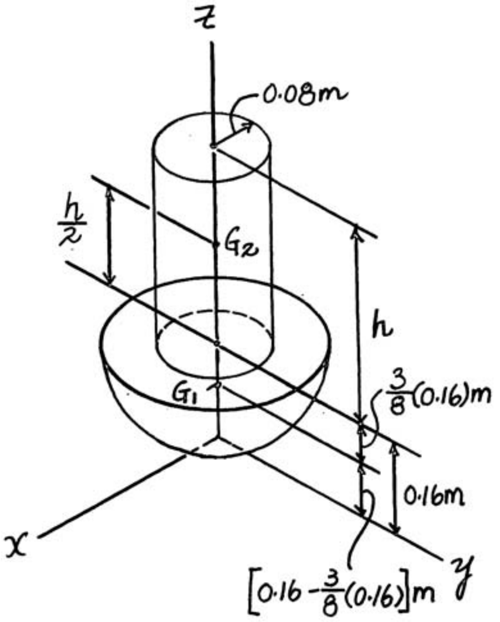
$$\Sigma \tilde{z} m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{0.2}{2}\right)\pi(0.2)(0.08)^2(2.70)$$
$$= 9.51425(10^{-3})\ \text{Mg} \cdot \text{m}$$

$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(0.2)(0.08)^2(2.70)$$
$$= 77.7706(10^{-3})\ \text{Mg}$$

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{9.51425(10^{-3})}{77.7706(10^{-3})} = 0.122\ \text{m} = 122\ \text{mm} \qquad \textbf{Ans}$$

***9–80.** The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the height h of the cylinder so that the mass center of the assembly is located at $\bar{z} = 160$ mm.





$$\Sigma \tilde{z} m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{h}{2}\right)\pi(h)(0.08)^2(2.70)$$
$$= 6.691(10^{-3}) + 8.686(10^{-3})\,h + 27.143(10^{-3})\,h^2$$

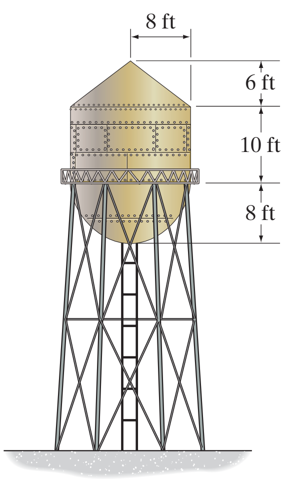
$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(h)(0.08)^2(2.70)$$
$$= 66.91(10^{-3}) + 54.29(10^{-3})\,h$$

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{6.691(10^{-3}) + 8.686(10^{-3})\,h + 27.143(10^{-3})\,h^2}{66.91(10^{-3}) + 54.29(10^{-3})\,h} = 0.160$$

Solving

$h = 0.385$ m $= 385$ mm **Ans**

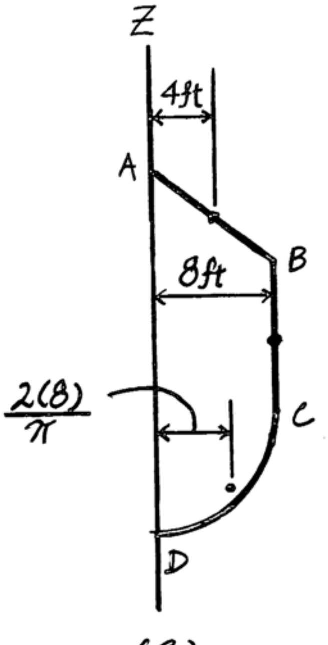
•9–81. The elevated water storage tank has a conical top and hemispherical bottom and is fabricated using thin steel plate. Determine how many square feet of plate is needed to fabricate the tank.



Surface Area: The perpendicular distance measured from the *z* axis to the centroid of each line segment is indicated in Fig. *a*.

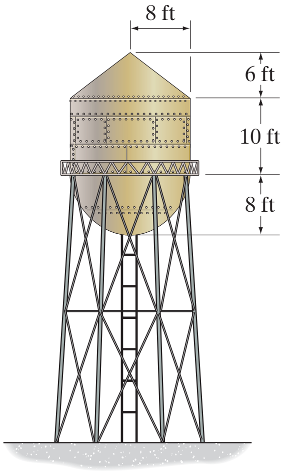
$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[4\left(\sqrt{8^2+6^2}\right) + 8(10) + \left(\frac{2(8)}{\pi}\right)\left(\frac{\pi(8)}{2}\right)\right]$$

$$= 2\pi(184) = 1156\,\text{ft}^2$$ **Ans.**



(a)

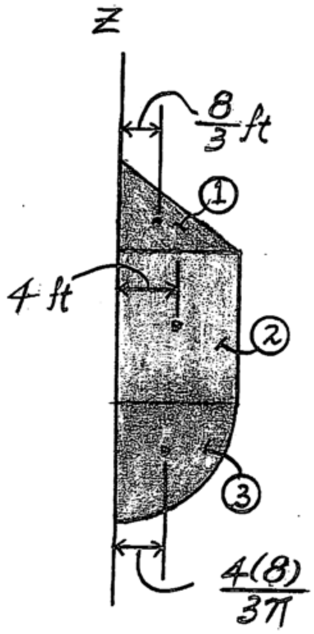
9–82. The elevated water storage tank has a conical top and hemispherical bottom and is fabricated using thin steel plate. Determine the volume within the tank.



Volume: The perpendicular distance measured from the z axis to the centroid of each area segment is indicated in Fig. *a*.

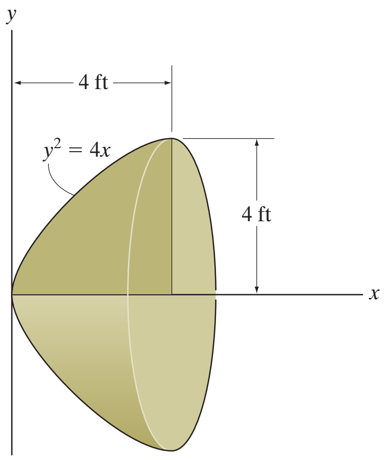
$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(\frac{8}{3}\right)\left(\frac{1}{2}\right)(6)(8) + 4(10)(8) + \left(\frac{4(8)}{3\pi}\right)\left(\frac{\pi(8^2)}{4}\right)\right]$$

$$= 2\pi(554.67) = 3485\text{ ft}^3$$ **Ans.**



(a)

9–83. Determine the volume of the solid formed by revolving the shaded area about the x axis using the second theorem of Pappus–Guldinus. The area and centroid $\overline{y}$ of the shaded area should first be obtained by using integration.



Area and Centroid: The differential element parallel to the x axis is shown shaded in Fig. a. The area of this element is given by

$$dA = (4 - x)\,dy = \left(4 - \frac{y^2}{4}\right)dy$$

Integrating,

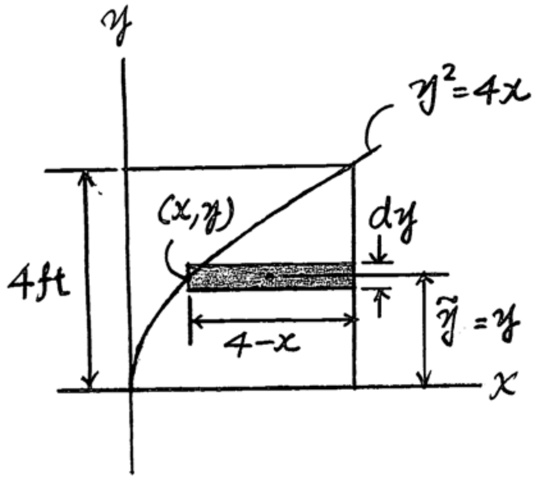
$$A = \int_A dA = \int_0^{4\text{ ft}} \left(4 - \frac{y^2}{4}\right)dy = 4y - \frac{y^3}{12}\bigg|_0^{4\text{ ft}} = 10.67\text{ ft}^2$$

With $\tilde{y} = y$,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} y\left[\left(4 - \frac{y^2}{4}\right)dy\right]}{10.67} = \frac{\int_0^{4\text{ ft}} \left(4y - \frac{y^3}{4}\right)dy}{10.67} = \frac{\left(2y^2 - \frac{y^4}{16}\right)\bigg|_0^{4\text{ ft}}}{10.67} = 1.5\text{ ft}$$

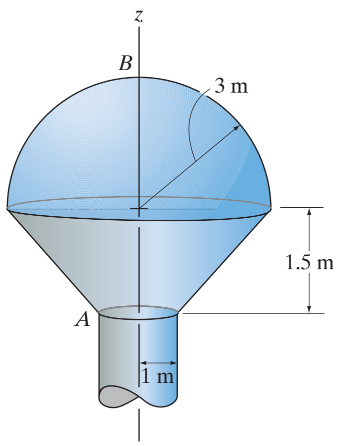
Volume: Applying the second theorem of Pappus - Guldinus and using the results obtained above,

$$V = 2\pi\bar{r}A = 2\pi(1.5)(10.67) = 101\text{ ft}^3$$ **Ans.**



(a)

***9–84.** Determine the surface area from *A* to *B* of the tank.

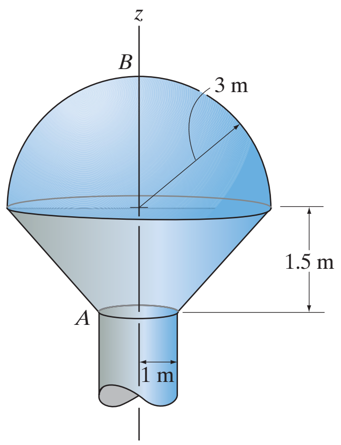


Surface Area: The perpendicular distance to the centroid of each of three line segments.

$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[\left(\frac{2(3)}{\pi}\right)\frac{2\pi(3)}{4} + 2\sqrt{1.5^2 + 2^2}\right]$$

$$= 88.0\text{ m}^2$$ **Ans**

•9–85. Determine the volume within the thin-walled tank from *A* to *B*.

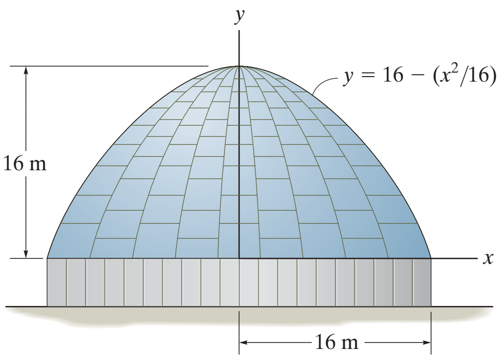


Volume: The perpendicular distance measured to the centroid of each of three area segments.

$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(\frac{4(3)}{3\pi}\right)\left(\frac{\pi(3^2)}{4}\right) + 0.5(1.5)(1) + 1.667\left(\frac{2(1.5)}{2}\right)\right]$$

$$= 77.0\text{ m}^3$$ **Ans.**

9–86. Determine the surface area of the roof of the structure if it is formed by rotating the parabola about the y axis.



Centroid : The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{x} = x$. Here, $\frac{dy}{dx} = -\frac{x}{8}$. Evaluating the integrals, we have

$$L = \int dL = \int_0^{16\text{m}} \left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 23.663 \text{ m}$$

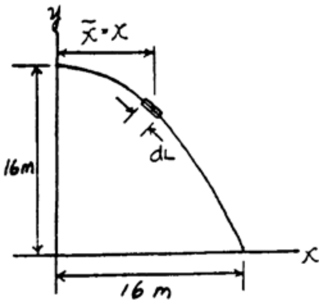
$$\int_L \tilde{x}\, dL = \int_0^{16\text{m}} x\left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 217.181 \text{ m}^2$$

Applying Eq. 9–5, we have

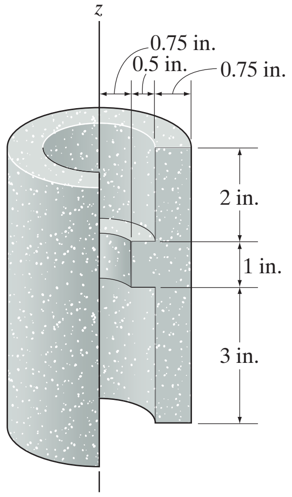
$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{217.181}{23.663} = 9.178 \text{ m}$$

Surface Area : Applying the theorem of Pappus and Guldinus, Eq. 9–7, with $\theta = 2\pi$, $L = 23.663$ m, $\bar{r} = \bar{x} = 9.178$, we have

$$A = \theta \bar{r} L = 2\pi(9.178)(23.663) = 1365 \text{ m}^2 \qquad \textbf{Ans}$$



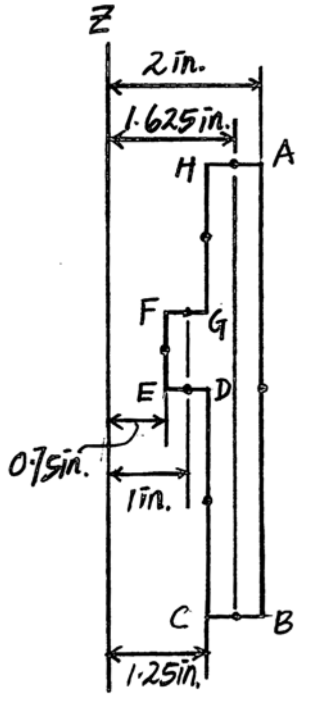
9–87. Determine the surface area of the solid formed by revolving the shaded area 360° about the z axis.



Surface Area: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. a.

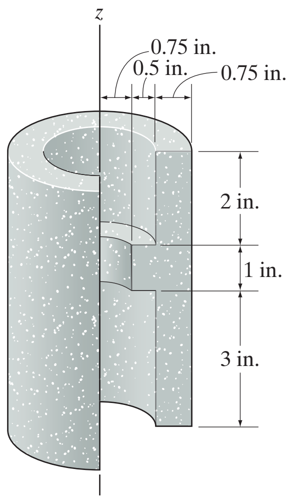
$$A = 2\pi\Sigma\bar{r}L = 2\pi[2(6) + 1.625(0.75) + 1.625(0.75) + 1.25(3) + 1.25(2) + 1(0.5) + 1(0.5) + 0.75(1)]$$

$$= 2\pi(22.4375) = 44.875\pi\,\text{in}^2 = 141\,\text{in}^2$$ **Ans.**



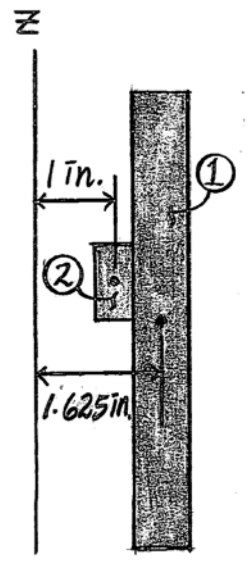
(a)

***9–88.** Determine the volume of the solid formed by revolving the shaded area 360° about the z axis.



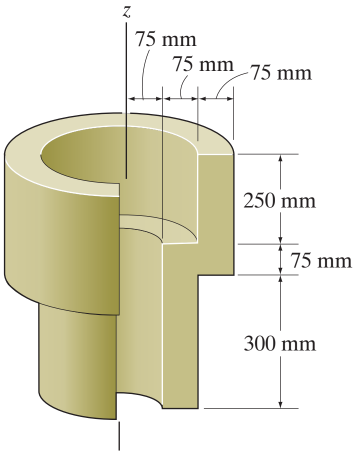
Volume: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. *a*.

$$V = 2\pi\Sigma\bar{r}A = 2\pi[1.625(6)(0.75) + 1(1)(0.5)] = 2\pi(7.8125) = 49.1\text{ in}^3$$ **Ans.**



(a)

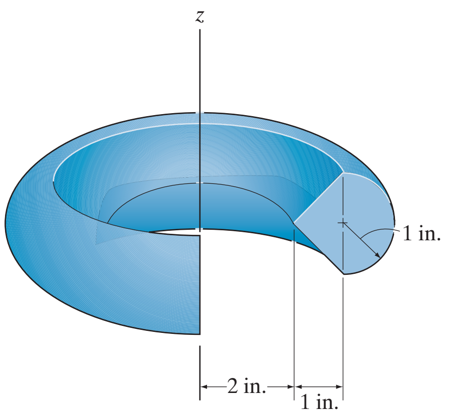
•9–89. Determine the volume of the solid formed by revolving the shaded area 360° about the z axis.



Volume: The perpendicular distance measured to the centroid of each of two area segments.

$$V = 2\pi\Sigma\bar{r}A = 2\pi[(112.5)(75)(375) + (187.5)(325)(75)]$$
$$= 0.0486\text{ m}^3$$ **Ans.**

9–90. Determine the surface area and volume of the solid formed by revolving the shaded area 360° about the *z* axis.



Surface Area: The perpendicular distance measured from the *z* axis to the centroid of each line segment is indicated in Fig. *a*.

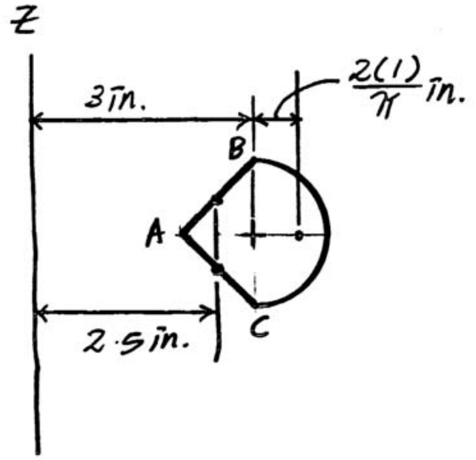
$$A = 2\pi\Sigma\tilde{r}L = 2\pi\left[(2.5)\left(\sqrt{1^2+1^2}\right)+(2.5)\left(\sqrt{1^2+1^2}\right)+\left(3+\frac{2(1)}{\pi}\right)\pi(1)\right]$$

$$= 2\pi(18.4958) = 116\text{ in}^2$$ **Ans.**

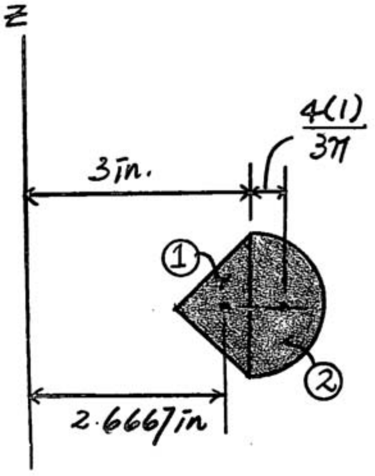
Volume: The perpendicular distance measured from the *z* axis to the centroid of each area segment is indicated in Fig. *a*.

$$V = 2\pi\Sigma\tilde{r}A = 2\pi\left[(2.667)\left(\frac{1}{2}(2)(1)\right)+\left(3+\frac{4(1)}{3\pi}\right)\left(\frac{\pi(1)}{2}\right)\right]$$

$$= 2\pi(8.0457) = 50.6\text{ in}^3$$ **Ans.**

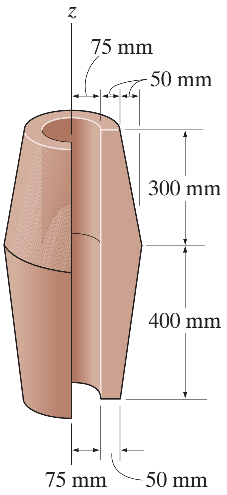


(a)



(b)

9–91. Determine the surface area and volume of the solid formed by revolving the shaded area 360° about the *z* axis.



Surface Area: The perpendicular distance measured from the *x* axis to the centroid of each of four line segments..

$$A = 2\pi\left[150\sqrt{(400)^2+(50)^2}+150\sqrt{(300)^2+(50)^2}+75(700)+2(100)(50)\right]$$

$= 1.06\ \text{m}^2$ **Ans.**

Volume: The perpendicular distance measured from the *x* axis to the centroid of each of two area segments.

$$V = 2\pi\left[(100)(700)(50)+141.667\left(\frac{1}{2}(700)(50)\right)\right]$$

$= 0.0376\ \text{m}^3$ **Ans.**

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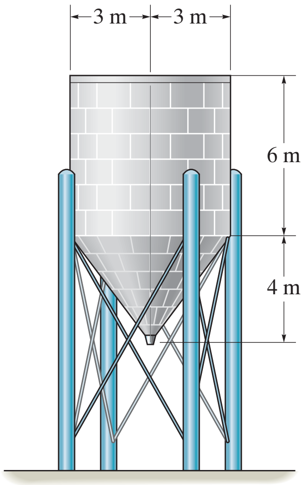
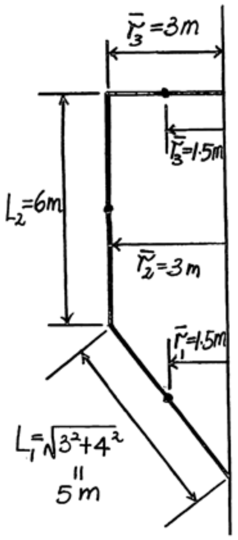
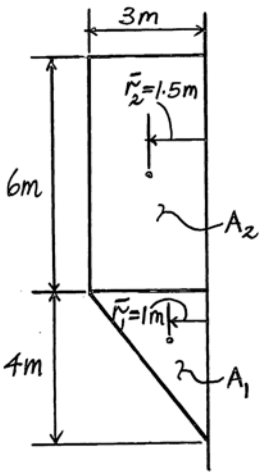
*__9–92.__ The process tank is used to store liquids during manufacturing. Estimate both the volume of the tank and its surface area. The tank has a flat top and a thin wall.



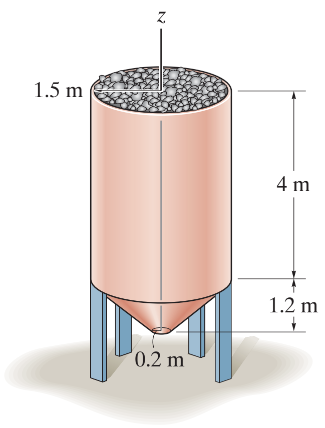




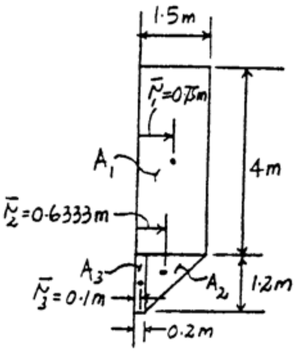
$$V = \Sigma\theta\,\bar{r}A = 2\pi\left[1\left(\frac{1}{2}\right)(3)(4) + 1.5(3)(6)\right] = 207\ \text{m}^3$$ **Ans**

$$A = \Sigma\theta\,\bar{r}L = 2\pi[1.5(5) + 3(6) + 1.5(3)] = 188\ \text{m}^2$$ **Ans**

•9–93. The hopper is filled to its top with coal. Estimate the volume of coal if the voids (air space) are 35 percent of the volume of the hopper.



Volume **:** The volume of the hopper can be obtained by applying the theorem of Pappus and Guldinus, Eq. 9–18 with $\theta = 2\pi$, $\bar{r}_1 = 0.75$ m, $\bar{r}_2 = 0.6333$ m, $\bar{r}_3 = 0.1$ m, $A_1 = 1.5(4) = 6.00\ \text{m}^2$, $A_2 = \frac{1}{2}(1.3)(1.2) = 0.780\ \text{m}^2$ and $A_3 = (0.2)(1.2) = 0.240\ \text{m}^2$.

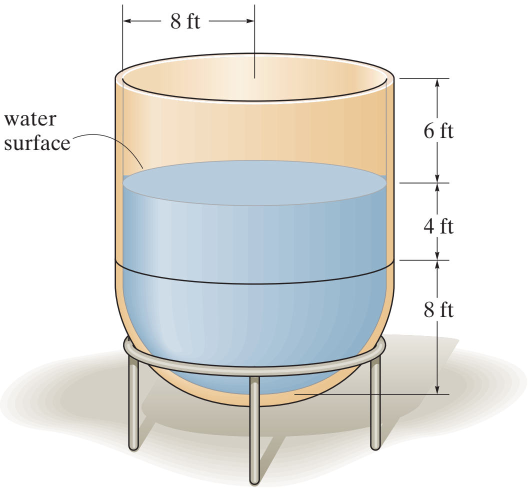


$$V_h = \theta\Sigma\bar{r}A = 2\pi[0.75(6.00) + 0.6333(0.780) + 0.1(0.240)]$$
$$= 10.036\pi\ \text{m}^3$$

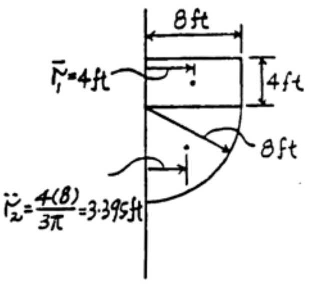
The volume of the coal is

$$V_c = 0.65V_h = 0.65(10.036\pi) = 20.5\ \text{m}^3 \qquad \textbf{Ans}$$

9–94. The thin-wall tank is fabricated from a hemisphere and cylindrical shell. Determine the vertical reactions that each of the four symmetrically placed legs exerts on the floor if the tank contains water which is 12 ft deep in the tank. The specific gravity of water is 62.4 lb/ft^3. Neglect the weight of the tank.



Volume **:** The volume of the water can be obtained by applying the theorem of Pappus and Guldinus, Eq. 9–10, with $\theta = 2\pi$, $\bar{r}_1 = 4$ ft, $\bar{r}_2 = 3.395$ ft, $A_1 = 8(4) = 32.0\ \text{ft}^2$ and $A_2 = \frac{1}{4}\pi(8^2) = 50.27\ \text{ft}^2$.



$$V = \theta\Sigma\bar{r}A = 2\pi[4(32.0) + 3.395(50.27)] = 1876.58\ \text{ft}^3$$

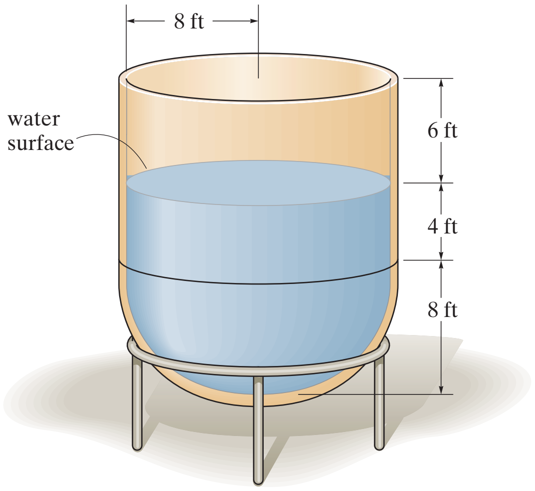
The weight of the water is

$$W = \gamma_w V = 62.4(1876.58) = 117098.47\ \text{lb}$$

Thus, the reaction of each leg on the floor is

$$R = \frac{W}{4} = \frac{117098.47}{4} = 29274.62\ \text{lb} = 29.3\ \text{kip} \qquad \textbf{Ans}$$

9–95. Determine the approximate amount of paint needed to cover the outside surface of the open tank. Assume that a gallon of paint covers 400 ft^2.

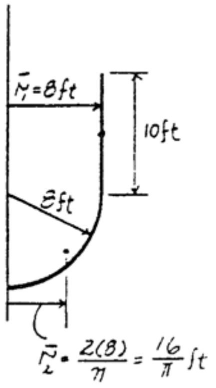


Surface Area : Applying the theorem of Pappus and Guldinus, Eq. 9 – 9 , with $\theta = 2\pi$, $L_1 = 10$ ft, $L_2 = \dfrac{\pi(8)}{2} = 4\pi$ ft, $\bar{r}_1 = 8$ ft and $\bar{r}_2 = \dfrac{16}{\pi}$ ft , we have

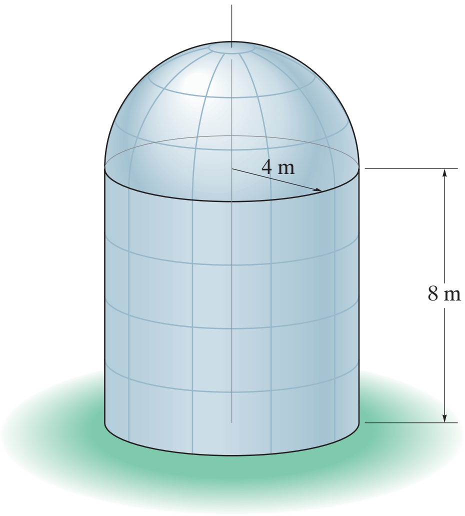
$$A = \theta\Sigma\bar{r}L = 2\pi\left[8(10) + \frac{16}{\pi}(4\pi)\right] = 288\pi \text{ ft}^2$$

Thus,

$$\text{The required amount paint} = \frac{288\pi}{400} = 2.26 \text{ gallon} \qquad \textbf{Ans}$$

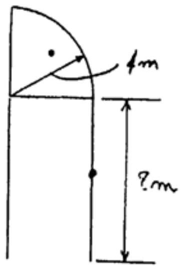


***9–96.** Determine the surface area of the tank, which consists of a cylinder and hemispherical cap.

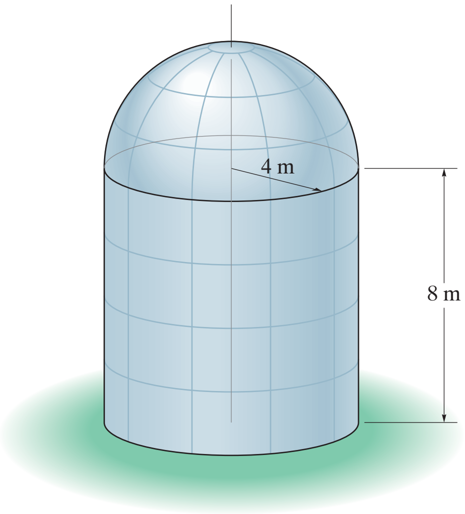


$$A = \Sigma\theta\,\bar{r}\,L = 2\pi\left[(4)(8) + \left(\frac{2(4)}{\pi}\right)\left(\frac{1}{4}(2\pi(4))\right)\right]$$

$$= 302 \text{ m}^2 \qquad \textbf{Ans}$$

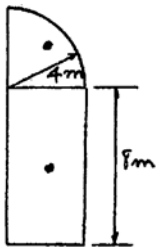


•9–97. Determine the volume of the thin-wall tank, which consists of a cylinder and hemispherical cap.

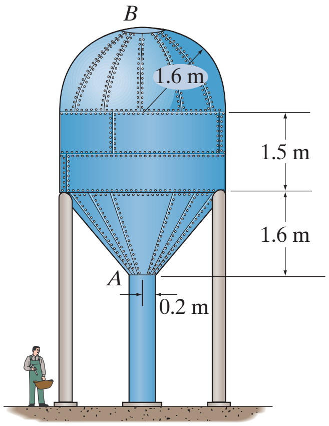


$$V = \Sigma\theta\bar{r}A = 2\pi\left[\left(\frac{4(4)}{3\pi}\right)\left(\frac{1}{4}\pi(4)^2\right) + (2)(8)(4)\right]$$

$$= 536\text{ m}^3 \qquad \textbf{Ans}$$



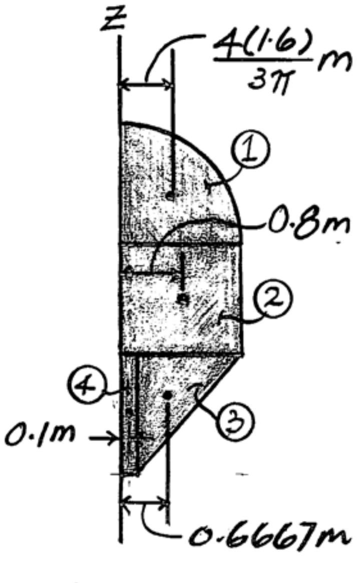
9–98. The water tank *AB* has a hemispherical top and is fabricated from thin steel plate. Determine the volume within the tank.



Volume: The perpendicular distance measured from the z axis to the centroid of each area segment is indicated in Fig. *a*.

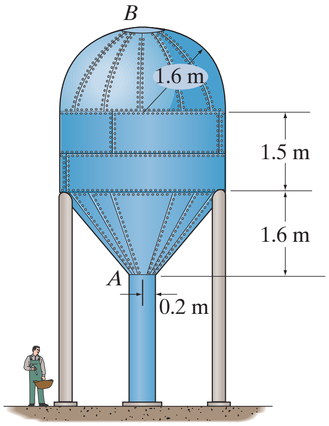
$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(\frac{4(1.6)}{3\pi}\right)\left(\frac{\pi(1.6^2)}{4}\right) + 0.8(1.6)(1.5) + 0.6667\left(\frac{1}{2}\right)(1.4)(1.6) + 0.1(0.2)(1.6)\right]$$

$$= 2\pi(4.064) = 25.5 \text{ m}^3 \qquad \textbf{Ans.}$$



(a)

9–99. The water tank *AB* has a hemispherical roof and is fabricated from thin steel plate. If a liter of paint can cover 3 m^2 of the tank's surface, determine how many liters are required to coat the surface of the tank from *A* to *B*.



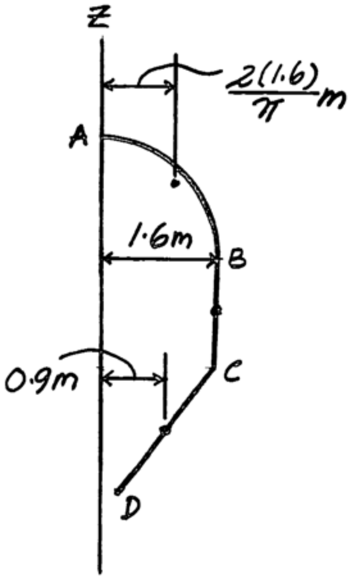
Surface Area: The perpendicular distance measured from the *z* axis to the centroid of each line segment is indicated in Fig. *a*.

$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[\left(\frac{2(1.6)}{\pi}\right)\left(\frac{\pi(1.6)}{2}\right) + 1.6(1.5) + 0.9\left(\sqrt{1.4^2 + 1.6^2}\right)\right]$$

$$= 2\pi(6.8734) = 43.18\ \text{m}^2$$

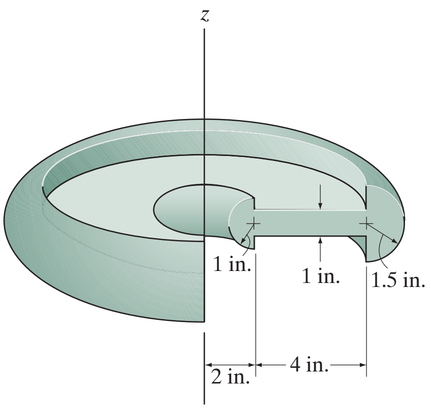
Thus, the amount of paint required is

$$\text{Number of liters} = \frac{43.18}{3} = 14.4\ \text{liters} \qquad \textbf{Ans.}$$



(a)

***9–100.** Determine the surface area and volume of the wheel formed by revolving the cross-sectional area 360° about the *z* axis.



Surface Area: The perpendicular distance measured from the *z* axis to the centroid of each line segment is indicated in Fig. *a*.

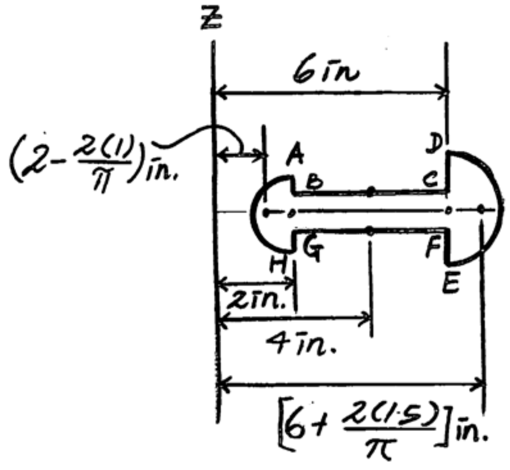
$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[\left(2-\frac{2(1)}{\pi}\right)\pi(1)+2(1)+4(2)(4)+6(2)+\left(6+\frac{2(1.5)}{\pi}\right)\pi(1.5)\right]$$

$$= 2\pi(83.0575) = 522\text{ in}^2$$ **Ans.**

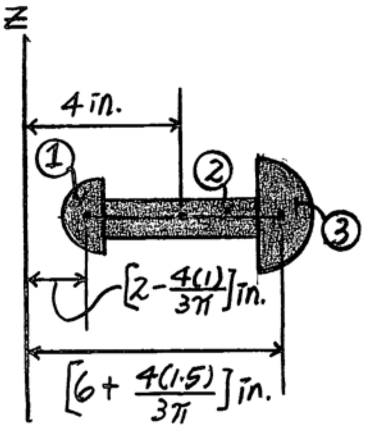
Volume: The perpendicular distance measured from the *z* axis to the centroid of each area segment is indicated in Fig. *a*.

$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(2-\frac{4(1)}{3\pi}\right)\left(\frac{\pi(1^2)}{2}\right)+4(4)(1)+\left(6+\frac{4(1.5)}{3\pi}\right)\left(\frac{\pi(1.5^2)}{2}\right)\right]$$

$$= 2\pi(41.9307) = 263\text{ in}^3$$ **Ans.**

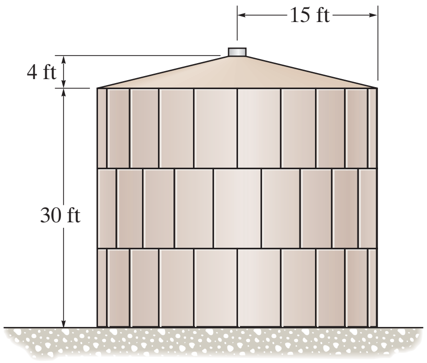


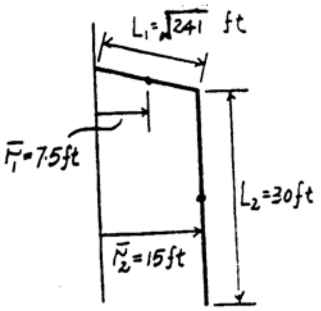
(a)



(b)

•9–101. Determine the outside surface area of the storage tank.

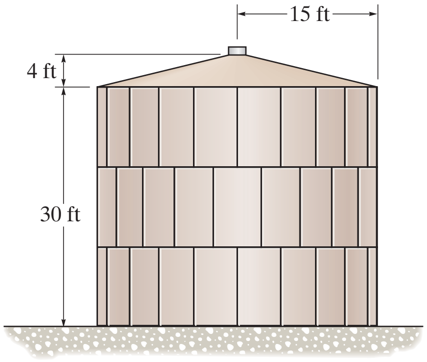


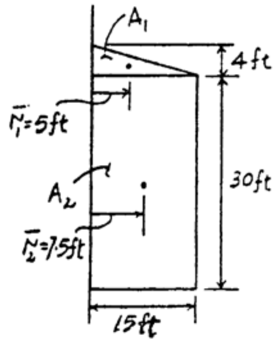


Surface Area : Applying the theorem of Pappus and Guldinus, Eq. 9 – 9, with $\theta = 2\pi$, $L_1 = \sqrt{15^2 + 4^2} = \sqrt{241}$ ft, $L_2 = 30$ ft, $\bar{r}_1 = 7.5$ ft and $\bar{r}_2 = 15$ ft, we have

$$A = \theta \Sigma \bar{r} L = 2\pi\left[7.5\left(\sqrt{241}\right) + 15(30)\right] = 3.56\left(10^3\right)\ \text{ft}^2 \qquad \textbf{Ans}$$

9–102. Determine the volume of the thin-wall storage tank.

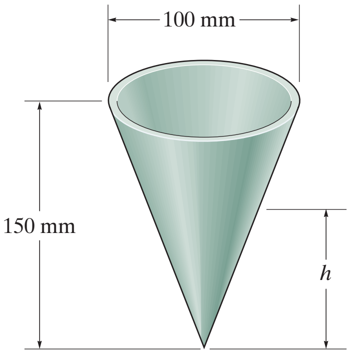




Volume : Applying the theorem of Pappus and Guldinus, Eq. 9 – 10, with $\theta = 2\pi$, $\bar{r}_1 = 5$ ft, $\bar{r}_2 = 7.5$ ft, $A_1 = \frac{1}{2}(15)(4) = 30.0$ ft^2 and $A_2 = 30(15) = 450$ ft^2, we have

$$V = \theta \Sigma \bar{r} A = 2\pi[5(30.0) + 7.5(450)] = 22.1\left(10^3\right)\ \text{ft}^3 \qquad \textbf{Ans}$$

9–103. Determine the height h to which liquid should be poured into the conical paper cup so that it contacts half the surface area on the inside of the cup.

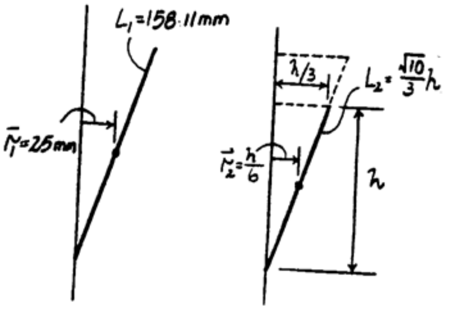


Surface Area : This problem requires that $\frac{1}{2}A_1 = A_2$. Applying the theorem of Pappus and Guldinus, Eq. 9–7, with $\theta = 2\pi$, $L_1 = \sqrt{50^2 + 150^2} = 158.11$ mm, $L_2 = \sqrt{h^2 + \left(\frac{h}{3}\right)^2} = \frac{\sqrt{10}}{3}h$, $\bar{r}_1 = 25$ mm and $\bar{r}_2 = \frac{h}{6}$, we have

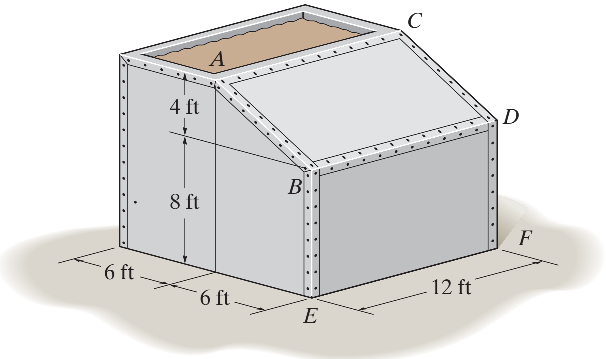
$$\frac{1}{2}(\theta \bar{r}_1 L_1) = \theta \bar{r}_2 L_2$$

$$\frac{1}{2}[2\pi(25)(158.11)] = 2\pi\left(\frac{h}{6}\right)\left(\frac{\sqrt{10}}{3}h\right)$$

$$h = 106 \text{ mm} \qquad \textbf{Ans}$$



***9–104.** The tank is used to store a liquid having a specific weight of 80 lb/ft^3. If it is filled to the top, determine the magnitude of the force the liquid exerts on each of its two sides *ABDC* and *BDFE*.



Fluid Pressure : The fluid pressure at points *B* and *E* can be determined using Eq. 9–13, $p = \gamma z$.

$$p_B = 80(4) = 320 \text{ lb/ft}^2 \qquad p_E = 80(12) = 960 \text{ lb/ft}^2$$

Thus,

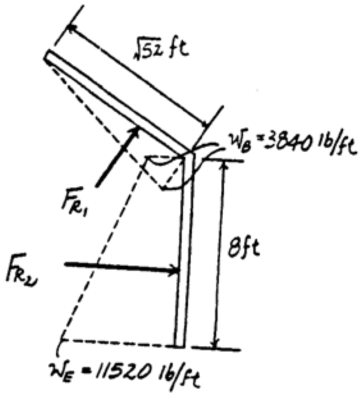
$$w_B = 320(12) = 3840 \text{ lb/ft} \qquad w_E = 960(12) = 11520 \text{ lb/ft}$$

Resultant Forces : The resultant force acts on suface *ABCD* is

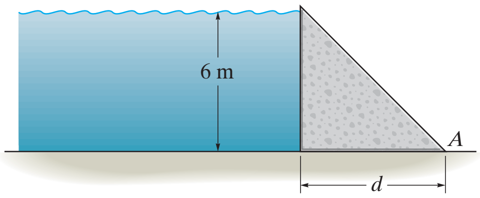
$$F_{R_1} = \frac{1}{2}(3840)\left(\sqrt{52}\right) = 13\,845.31 \text{ lb} = 13.8 \text{ kip} \qquad \textbf{Ans}$$

and acts on surface *BDFE* is

$$F_{R_2} = \frac{1}{2}(3840 + 11520)(8) = 61\,440 \text{ lb} = 61.4 \text{ kip} \qquad \textbf{Ans}$$



•9–105. The concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \text{Mg/m}^3$, and water has a density of $\rho_w = 1.0\ \text{Mg/m}^3$, determine the smallest dimension d that will prevent the dam from overturning about its end A.



Consider a 1 – m width of dam.

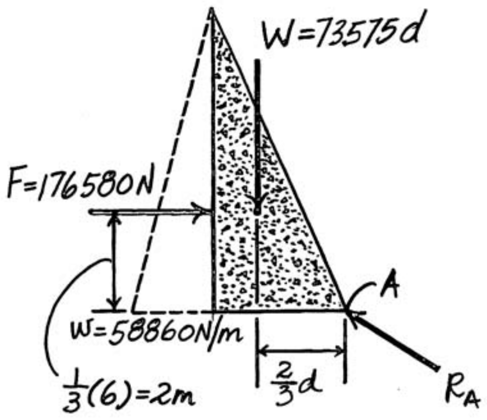
$w = 1000(9.81)(6)(1) = 58\,860\ \text{N/m}$

$F = \frac{1}{2}(58\,860)(6) = 176\,580\ \text{N}$

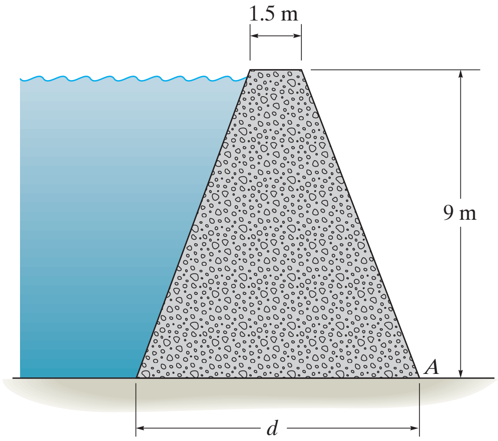
$W = \frac{1}{2}(d)(6)(1)(2500)(9.81) = 73\,575d\ \text{N}$

$\circlearrowleft +\Sigma M_A = 0; \quad -176\,580(2) + 73\,575d\left(\frac{2}{3}d\right) = 0$

$d = 2.68\ \text{m}$ **Ans**



9–106. The symmetric concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \mathrm{Mg/m^3}$, and water has a density of $\rho_w = 1.0\ \mathrm{Mg/m^3}$, determine the smallest distance d at its base that will prevent the dam from overturning about its end A. The dam has a width of 8 m.



$$w = b\rho_w gh = 8(1000)(9.81)(9) = 706.32(10^3)\ \mathrm{N/m}$$

$$F_h = \tfrac{1}{2}\left[706.32(10^3)\right](9) = 3178.44(10)^3\ \mathrm{N}$$

$$F_v = 1000(9.81)\left(\tfrac{1}{2}\right)\left(\tfrac{d-1.5}{2}\right)(9)(8) = (176.58d - 264.87)(10^3)$$

$$W = 2.5(10^3)(9.81)\left[\tfrac{1}{2}(d+1.5)(9)(8)\right] = (882.9d + 1324.35)(10^3)$$

$$x_1 = d - \tfrac{1}{3}\left(\tfrac{d-1.5}{2}\right) = \tfrac{5}{6}d + 0.25$$

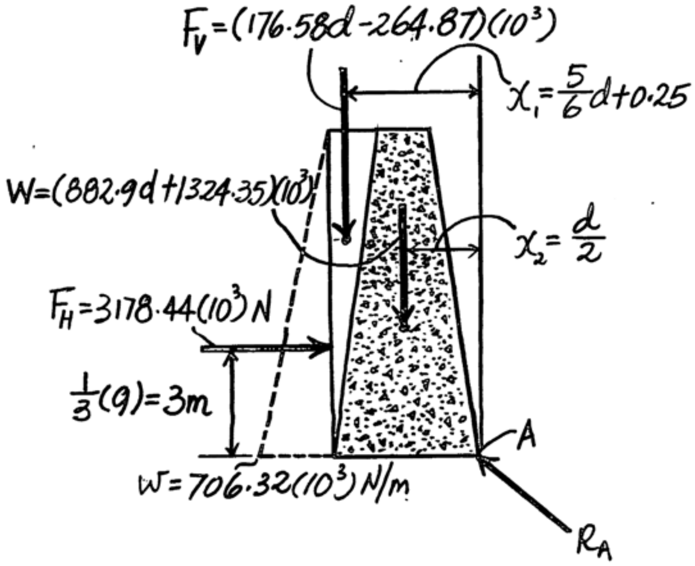
$$x_2 = \tfrac{d}{2}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad (176.58d - 264.87)(10^3)\left(\tfrac{5}{6}d + 0.25\right)$$

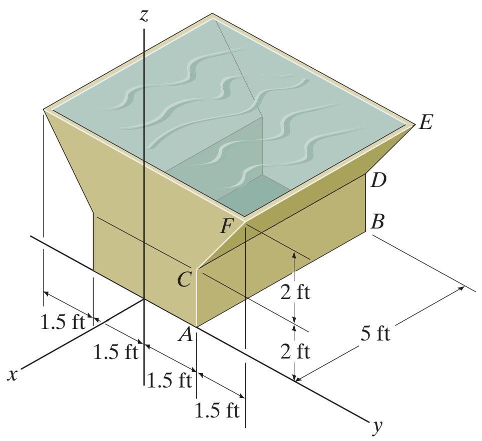
$$+ (882.9d + 1324.35)(10^3)\left(\tfrac{d}{2}\right) - 3178.44(10^3)(3) = 0$$

$$588.6d^2 + 485.595d - 9601.54 = 0$$

$$d = 3.65\ \mathrm{m}$$



9–107. The tank is used to store a liquid having a specific weight of 60 lb/ft^3. If the tank is full, determine the magnitude of the hydrostatic force on plates *CDEF* and *ABDC*.



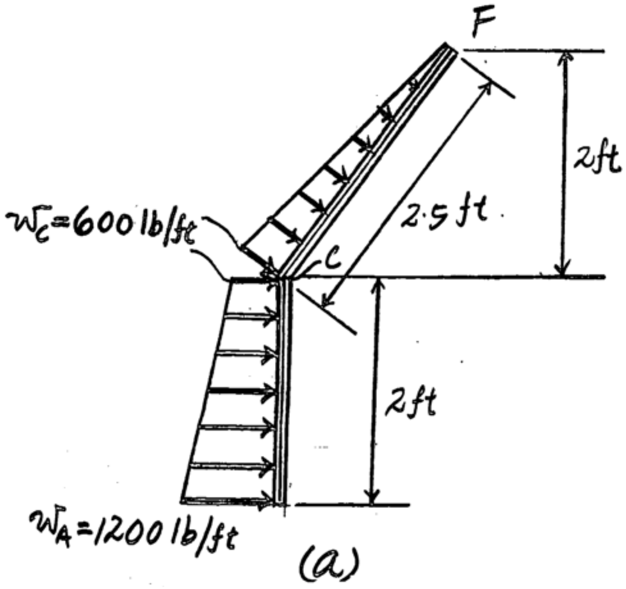
Loading: Since walls *CDEF* and *ABDC* have a constant width, the loading due to the fluid pressure on the walls can be represented by a two dimensional distributed loading. The intensity of the distributed load at points F, C, and A are given by

$$w_F = \gamma h_F b = 60(0)(5) = 0$$
$$w_C = \gamma h_C b = 60(2)(5) = 600 \text{ lb/ft}$$
$$w_A = \gamma h_A b = 60(4)(5) = 1200 \text{ lb/ft}$$

Resultant Force: The distributed loading acting on walls *CDEF* and *ABDC* is shown in Fig. *a*. Thus, the magnitude of the hydrostatic force on these two walls are

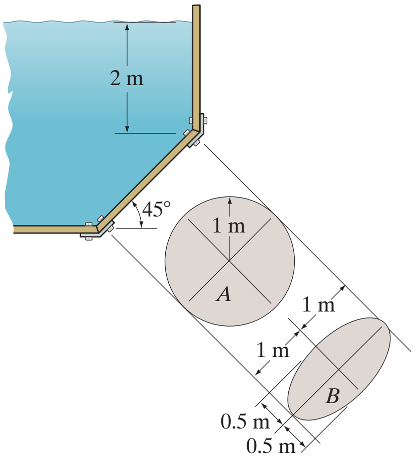
$$F_{CDEF} = \frac{1}{2}(600)(2.5) = 750 \text{ lb} \qquad \textbf{Ans.}$$

$$F_{ABDC} = \frac{1}{2}(600 + 1200)(2) = 1800 \text{ lb} \qquad \textbf{Ans.}$$



(a)

***9–108.** The circular steel plate *A* is used to seal the opening on the water storage tank. Determine the magnitude of the resultant hydrostatic force that acts on it. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth *h* expressed in terms of *y* is

$$h = \left(\frac{2}{\cos 45^\circ} + 1 - y\right)\sin 45^\circ = 2.7071 - 0.7071y$$

Thus, the water pressure at the depth *h* is

$$p = \rho_w g h = \frac{1000(9.81)(2.7071 - 0.7071y)}{1000} = (26.5567 - 6.9367y)\ \text{kN/m}^2$$

The differential force dF_R acting on the differential area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = (26.5567 - 6.9367y)\left(2\sqrt{1-y^2}\right)dy$$

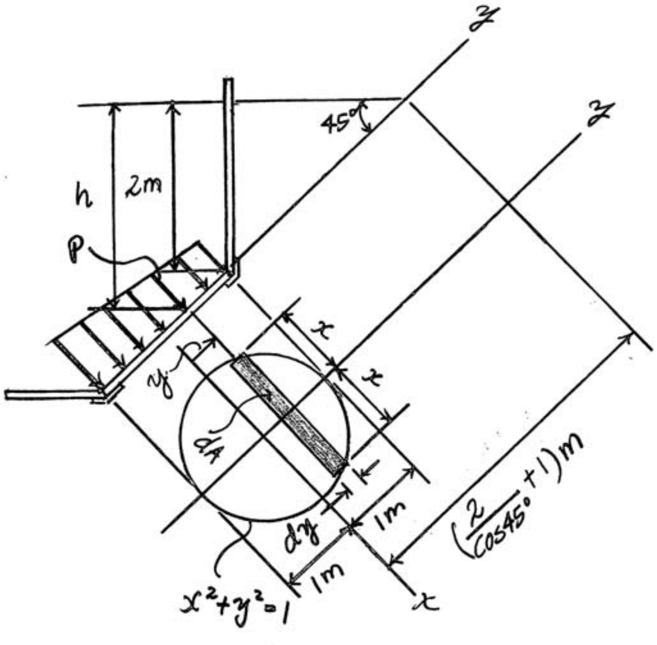
$$= \left(53.1134\sqrt{1-y^2} - 13.8734y\sqrt{1-y^2}\right)dy$$

Resultant Force: Integrating dF_R from $y = -1$ m to $y = 1$ m,

$$F_R = \int dF_R = \int_{-1\text{ m}}^{1\text{ m}}\left(53.1134\sqrt{1-y^2} - 13.8734y\sqrt{1-y^2}\right)dy$$

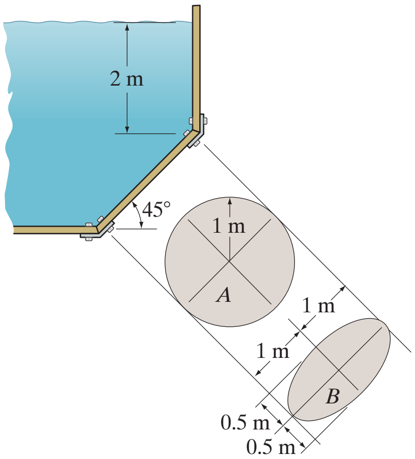
$$= \left[26.5567\left(y\sqrt{1-y^2} + \sin^{-1} y\right) + 4.6245\sqrt{(1-y^2)^3}\right]_{-1\text{ m}}^{1\text{ m}}$$

$$= 83.4\ \text{kN}$$



(a)

•9–109. The elliptical steel plate *B* is used to seal the opening on the water storage tank. Determine the magnitude of the resultant hydrostatic force that acts on it. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth *h* expressed in terms of *y* is

$$h = \left(\frac{2}{\cos 45°} + 1 - y\right)\sin 45° = 2.7071 - 0.7071y$$

Thus, the water pressure at the depth *h* is

$$p = \rho_w g h = \frac{1000(9.81)(2.7071 - 0.7071y)}{1000} = (26.5567 - 6.9367y)\ \text{kN/m}^2$$

The differential force $d\mathbf{F}_R$ acting on the differential area *dA* shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = (26.5567 - 6.9367y)\left[2\left(0.5\sqrt{1-y^2}\right)\right]dy$$

$$= \left(26.5567\sqrt{1-y^2} - 6.9367y\sqrt{1-y^2}\right)dy$$

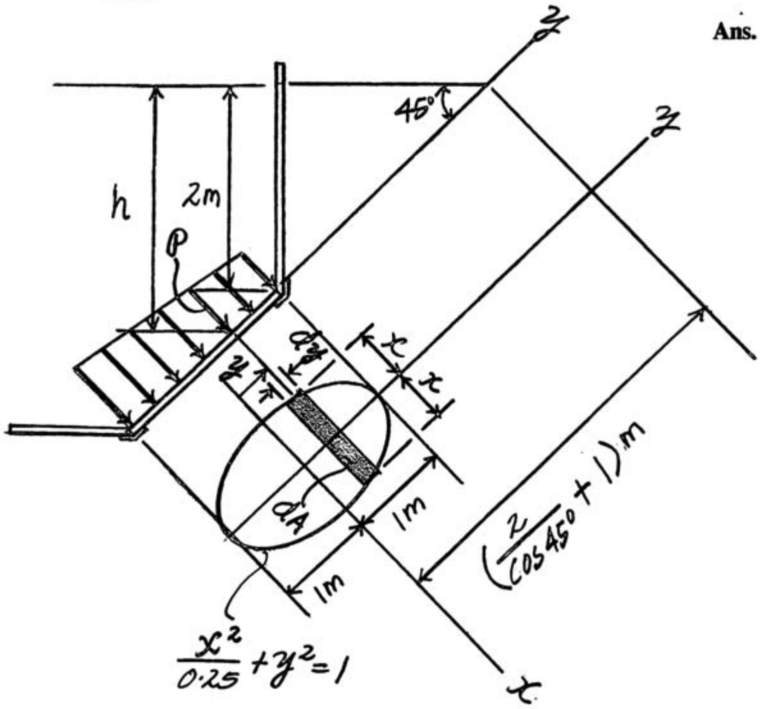
Resultant Force: Integrating $d\mathbf{F}_R$ from $y = -1$ m to $y = 1$ m,

$$F_R = \int dF_R = \int_{-1\text{ m}}^{1\text{ m}}\left(26.5567\sqrt{1-y^2} - 6.9367y\sqrt{1-y^2}\right)dy$$

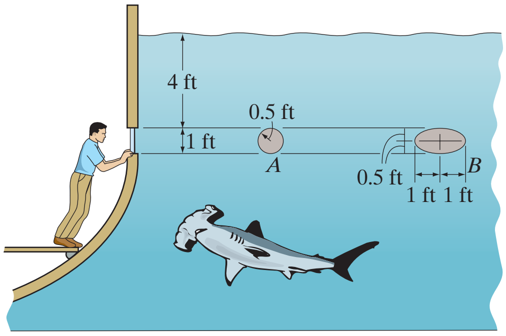
$$= \left[13.2784\left(y\sqrt{1-y^2} + \sin^{-1}y\right) + 2.3122\sqrt{(1-y^2)^3}\right]_{-1\text{ m}}^{1\text{ m}}$$

$$= 41.7\ \text{kN}$$

Ans.


y
45°
y
h
2m
p
dy
x
x
y
dA
1m
1m
$\left(\frac{2}{\cos 45°} + 1\right)$ m
$\frac{x^2}{0.25} + y^2 = 1$
x

9–110. Determine the magnitude of the hydrostatic force acting on the glass window if it is circular, A. The specific weight of seawater is $\gamma_w = 63.6\ \text{lb/ft}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth h expressed in terms of y is

$$h = 4 + 0.5 - y = (4.5 - y)\ \text{ft}$$

Thus, the water pressure at the depth h is

$$p = \gamma_w h = 63.6(4.5 - y)\ \text{lb/ft}^2$$

Resultant Force: The differential force $d\mathbf{F}_R$ acting on the differential area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = 63.6(4.5 - y)\left(2\sqrt{0.25 - y^2}\right)dy$$

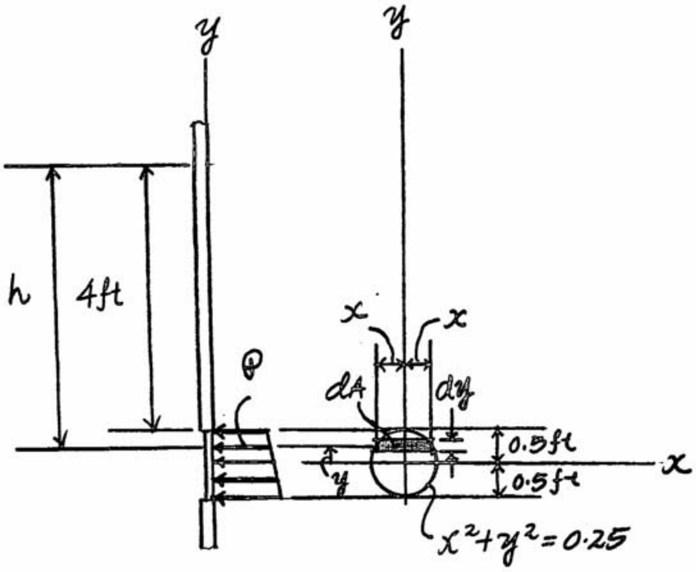
$$= \left(572.4\sqrt{0.25 - y^2} - 127.2y\sqrt{0.25 - y^2}\right)dy$$

Integrating $d\mathbf{F}_R$ from $y = -0.5$ ft to $y = 0.5$ ft,

$$F_R = \int dF_R = \int_{-0.5\ \text{ft}}^{0.5\ \text{ft}} \left(572.4\sqrt{0.25 - y^2} - 127.2y\sqrt{0.25 - y^2}\right)dy$$

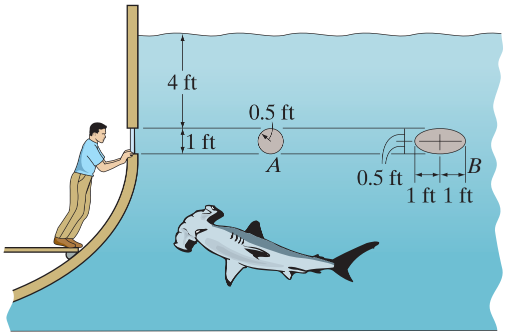
$$= \left[286.2\left(y\sqrt{0.25 - y^2} + 0.25\sin^{-1}\frac{y}{0.5}\right) + 42.4\sqrt{(0.25 - y^2)^3}\,\right]_{-0.5\ \text{ft}}^{0.5\ \text{ft}}$$

$$= 224.78\ \text{lb} = 225\ \text{lb}$$ **Ans.**



(a)

9–111. Determine the magnitude and location of the resultant hydrostatic force acting on the glass window if it is elliptical, *B*. The specific weight of seawater is $\gamma_w = 63.6\ \text{lb/ft}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth *h* expressed in terms of *y* is

$$h = 4 + 0.5 - y = (4.5 - y)\ \text{ft}$$

Thus, the water pressure at the depth *h* is

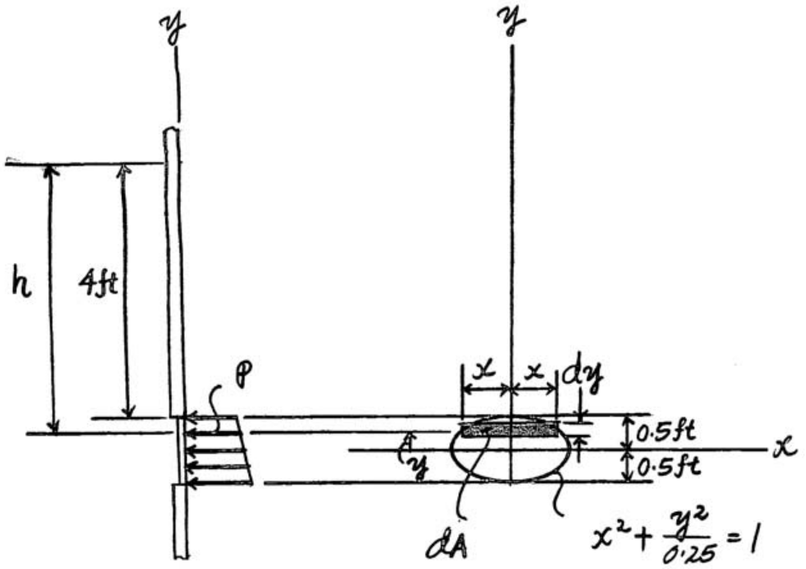
$$p = \gamma_w h = 63.6(4.5 - y)\ \text{lb/ft}^2$$

Resultant Force: The differential force $d\mathbf{F}_R$ acting on the area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = 63.6(4.5 - y)\left[2\left(2\sqrt{0.25 - y^2}\right)\right]dy$$
$$= \left(1144.8\sqrt{0.25 - y^2} - 254.4y\sqrt{0.25 - y^2}\right)dy$$

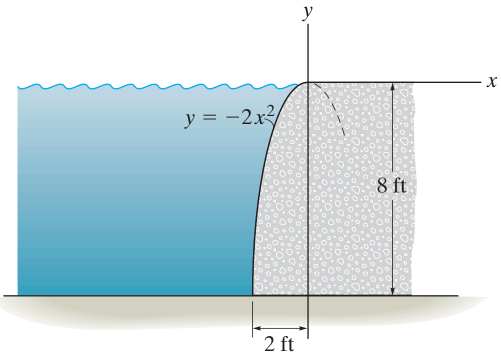
Integrating $d\mathbf{F}_R$ from $y = -0.5$ ft to $y = 0.5$ ft,

$$F_R = \int dF_R = \int_{-0.5\ \text{ft}}^{0.5\ \text{ft}} \left(1144.8\sqrt{0.25 - y^2} - 254.4y\sqrt{0.25 - y^2}\right)dy$$
$$= \left[572.4\left(y\sqrt{0.25 - y^2} + 0.25\sin^{-1}\frac{y}{0.5}\right) + 84.8\sqrt{(0.25 - y^2)^3}\right]_{-0.5\ \text{ft}}^{0.5\ \text{ft}}$$
$$= 449.56\ \text{lb} = 450\ \text{lb}$$ **Ans.**



(a)

***9–112.** Determine the magnitude of the hydrostatic force acting per foot of length on the seawall. $\gamma_w = 62.4\ \text{lb/ft}^3$.



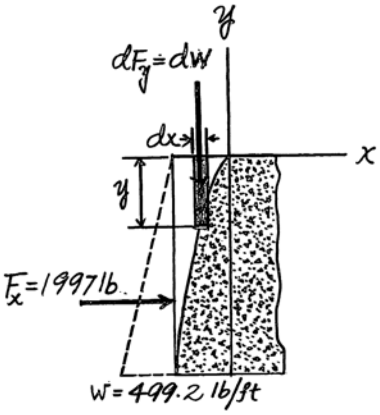
$$A = \int_A dA = \int_{-2}^{0} -y\,dx = \int_{-2}^{0} 2x^2\,dx = \frac{2}{3}x^3\Big|_{-2}^{0} = 5.333\ \text{ft}^2$$

$$w = b\gamma h = 1(62.4)(8) = 499.2\ \text{lb/ft}$$

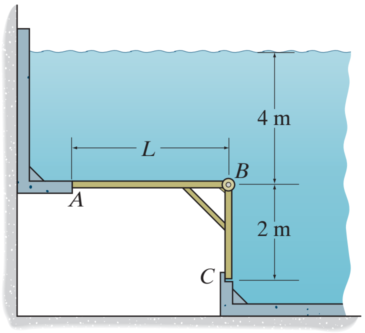
$$F_y = 5.333(1)(62.4) = 332.8\ \text{lb}$$

$$F_x = \frac{1}{2}(499.2)(8) = 1997\ \text{lb}$$

$$F_R = \sqrt{(332.8)^2 + (1997)^2} = 2024\ \text{lb} = 2.02\ \text{kip} \qquad \text{Ans}$$



•9–113. If segment *AB* of gate *ABC* is long enough, the gate will be on the verge of opening. Determine the length *L* of this segment in order for this to occur. The gate is hinged at *B* and has a width of 1 m. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points *B* and *C* are

$$w_B = \rho_w g h_B b = 1000(9.81)(4)(1) = 39\ 240\ \text{N} = 39.24\ \text{kN}$$

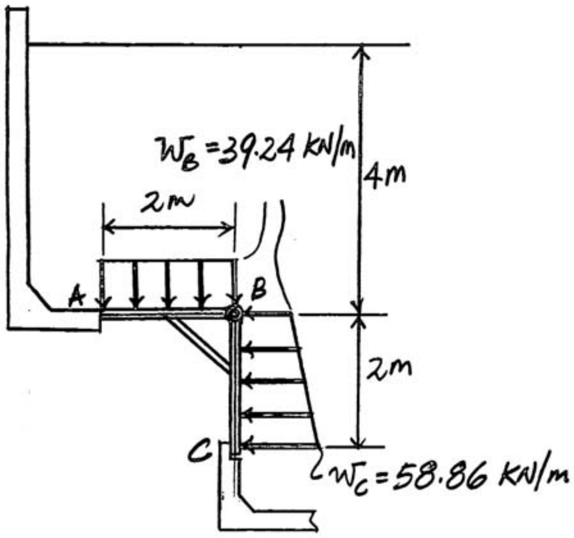
$$w_C = \rho_w g h_C b = 1000(9.81)(6)(1) = 58\ 860\ \text{N} = 58.86\ \text{kN}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

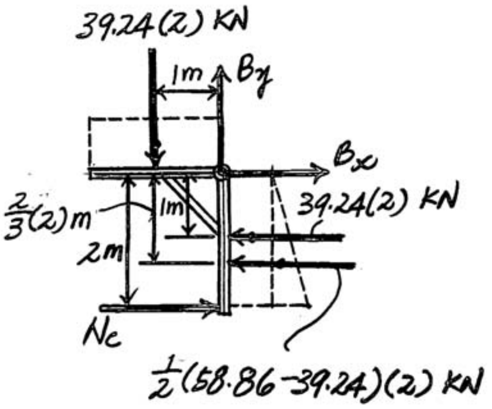
Equations of Equilibrium: Writing the moment equation of equilibrium about point *B*,

$$\curvearrowleft +\Sigma M_B = 0; \qquad N_C(2) + 39.24(2)(1) - 39.24(2)(1) - \frac{1}{2}(58.86 - 39.24)(2)\left(\frac{2}{3}\right)(2) = 0$$

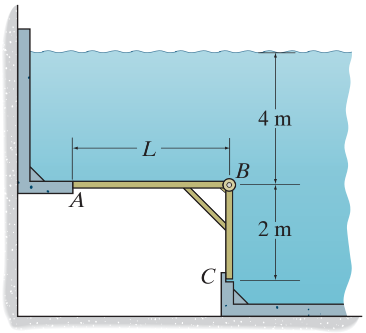
$$N_C = 13.08\ \text{kN} = 13.1\ \text{kN} \qquad \textbf{Ans.}$$



(a)



9–114. If $L = 2$ m, determine the force the gate *ABC* exerts on the smooth stopper at *C*. The gate is hinged at *B*, free at *A*, and is 1 m wide. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points *B* and *C* are

$$w_B = \rho_w g h_B b = 1000(9.81)(4)(1) = 39\ 240\ \text{N} = 39.24\ \text{kN}$$

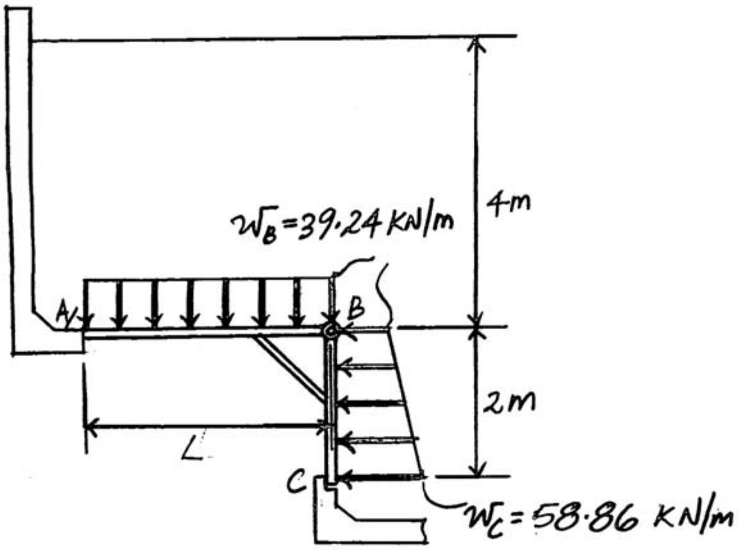
$$w_C = \rho_w g h_C b = 1000(9.81)(6)(4+2) = 58\ 860\ \text{N} = 58.86\ \text{kN}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

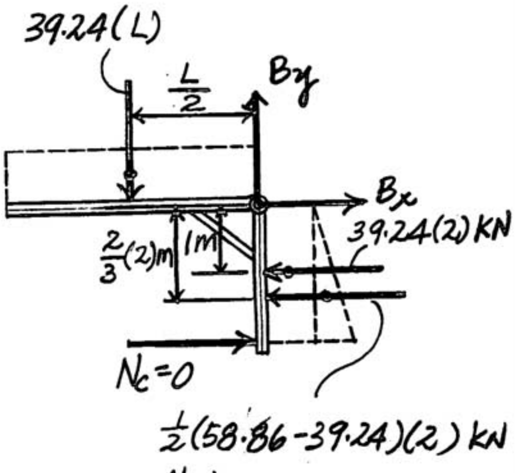
Equations of Equilibrium: Since the gate is on the verge of opening, $N_C = 0$. Writing the moment equation of equilibrium about point *B*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 39.24(L)\left(\frac{L}{2}\right) - 39.24(2)(1) - \frac{1}{2}(58.86 - 39.24)(2)\left(\frac{2}{3}\right)(2) = 0$$

$$L = 2.31\ \text{m} \qquad \textbf{Ans.}$$

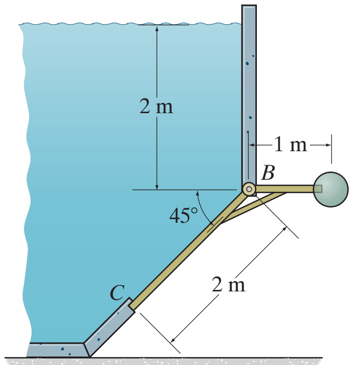


(a)



(b)

9–115. Determine the mass of the counterweight *A* if the 1-m-wide gate is on the verge of opening when the water is at the level shown. The gate is hinged at *B* and held by the smooth stop at *C*. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points *B* and *C* are

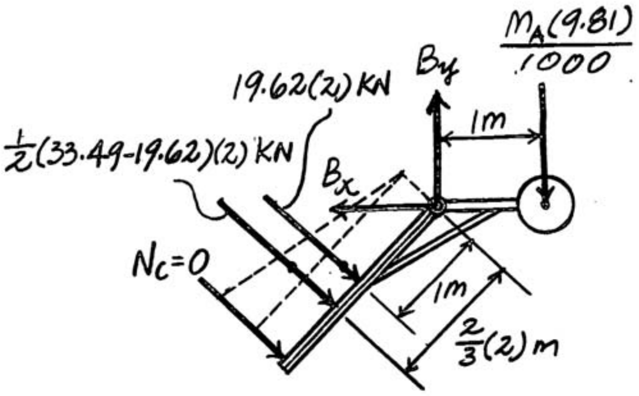
$$w_B = \rho_w g h_B b = 1000(9.81)(2)(1) = 19620\ \text{N/m} = 19.62\ \text{kN/m}$$
$$w_C = \rho_w g h_C b = 1000(9.81)(2 + 2\sin 45°)(1) = 33493.44\ \text{N/m} = 33.49\ \text{kN/m}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This distributed loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

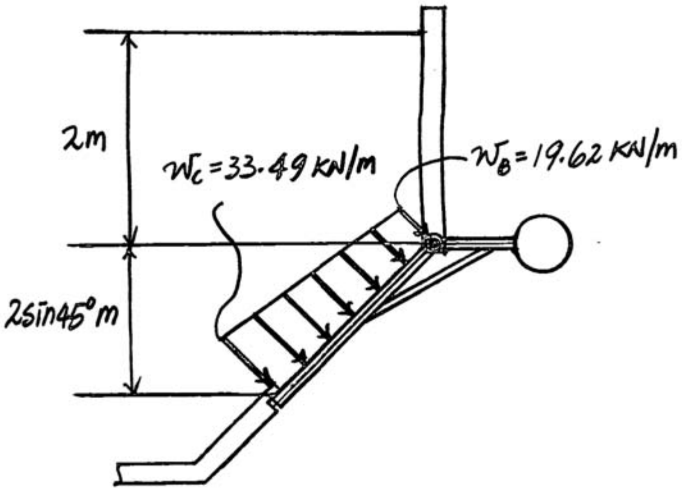
Equations of Equilibrium: Since the gate is on the verge of opening, $N_C = 0$. Writing the moment equation of equilibrium about point *B*,

$$\circlearrowleft +\Sigma M_B = 0; \qquad 19.62(2)(1) + \frac{1}{2}(33.49 - 19.62)(2)\left(\frac{2}{3}\right)(2) - \frac{m_A(9.81)(1)}{1000} = 0$$

$$m_A = 5885.62\ \text{kg} = 5.89\ \text{Mg} \qquad \textbf{Ans.}$$

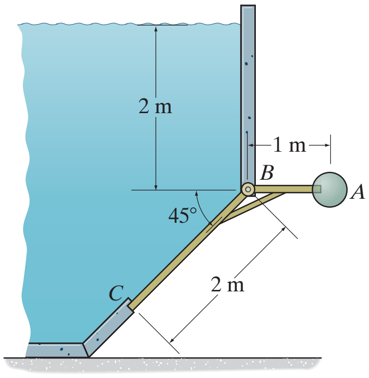


(b)



(a)

***9–116.** If the mass of the counterweight at *A* is 6500 kg, determine the force the gate exerts on the smooth stop at *C*. The gate is hinged at *B* and is 1-m wide. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points *B* and *C* are

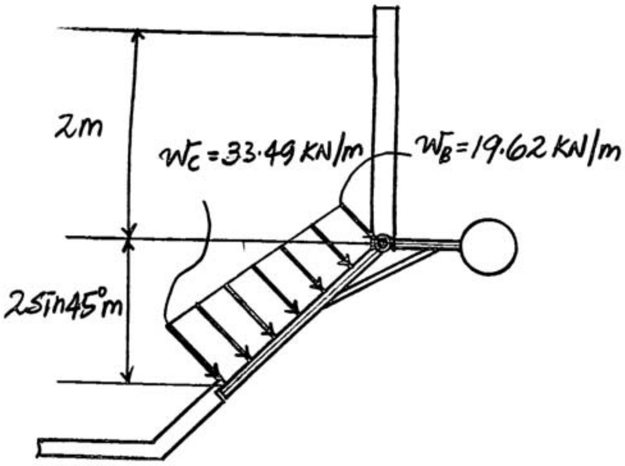
$$w_B = \rho_w g h_B b = 1000(9.81)(2)(1) = 19620\ \text{N/m} = 19.62\ \text{kN/m}$$
$$w_C = \rho_w g h_C b = 1000(9.81)(2 + 2\sin 45^\circ)(1) = 33493.44\ \text{N/m} = 33.49\ \text{kN/m}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This distributed loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

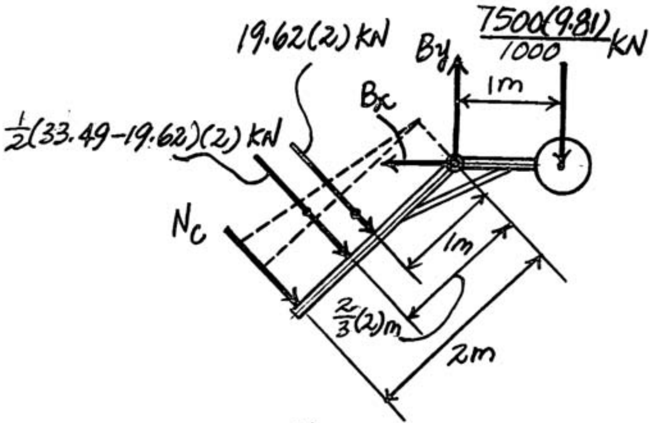
Equations of Equilibrium: Writing the moment equation of equilibrium about point *B*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad N_C(2) + \frac{1}{2}(33.49 - (19.62)(2)\left(\frac{2}{3}\right)(2) + 19.62(2)(1) - \frac{6500(9.81)}{1000}(1) = 0$$

$$N_C = 3.02\ \text{kN}$$ **Ans.**

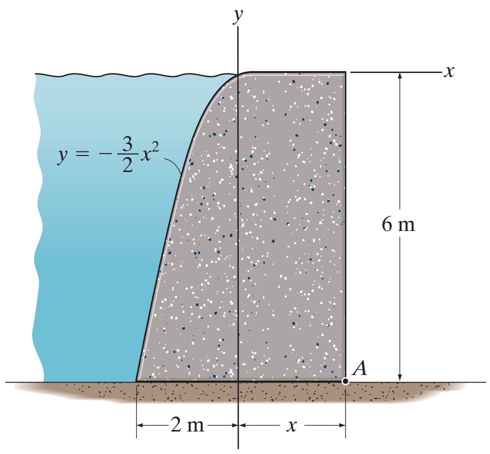


(a)


19.62(2) kN
B_y
$\frac{7500(9.81)}{1000}$ kN
1m
B_x
$\frac{1}{2}(33.49-19.62)(2)$ kN
N_C
1m
$\frac{2}{3}(2)$m
2m

(b)

•9–117. The concrete gravity dam is designed so that it is held in position by its own weight. Determine the factor of safety against overturning about point A if $x = 2$ m. The factor of safety is defined as the ratio of the stabilizing moment divided by the overturning moment. The densities of concrete and water are $\rho_{\text{conc}} = 2.40\ \text{Mg/m}^3$ and $\rho_w = 1\ \text{Mg/m}^3$, respectively. Assume that the dam does not slide.



Resultant Force Component: The analysis of this problem will be based on a per meter width of the dam. The hydrostatic force acting on the parabolic surface of the dam consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*. The vertical component $\mathbf{F}_v$ consists of the weight of water contained in the shaded area shown in Fig. *a*.

$$F_v = \rho_w g A_{BCD}(1) = (1000)(9.81)\left[\frac{1}{3}(2)(6)(1)\right] = 39240\ \text{N} = 39.24\ \text{kN}$$

The horizontal component $\mathbf{F}_h$ consists of the horizontal hydrostatic pressure, which can be represented by a triangular distributed loading shown in Fig. *a*. The intensity of the distributed loading at point B is $w_B = \rho_w g h_B b = 1000(9.81)(6)(1) = 58860\ \text{N/m} = 58.86\ \text{kN/m}$. Thus,

$$F_h = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

The weight of the parabolic shaped concrete dam is

$$(W_{\text{con}})_p = \rho_{\text{con}} g V = 2400(9.81)\left[\frac{2}{3}(2)(6)(1)\right] = 188\,352\ \text{N} = 188.352\ \text{kN}$$

The weight of the rectangular shaped concrete dam is

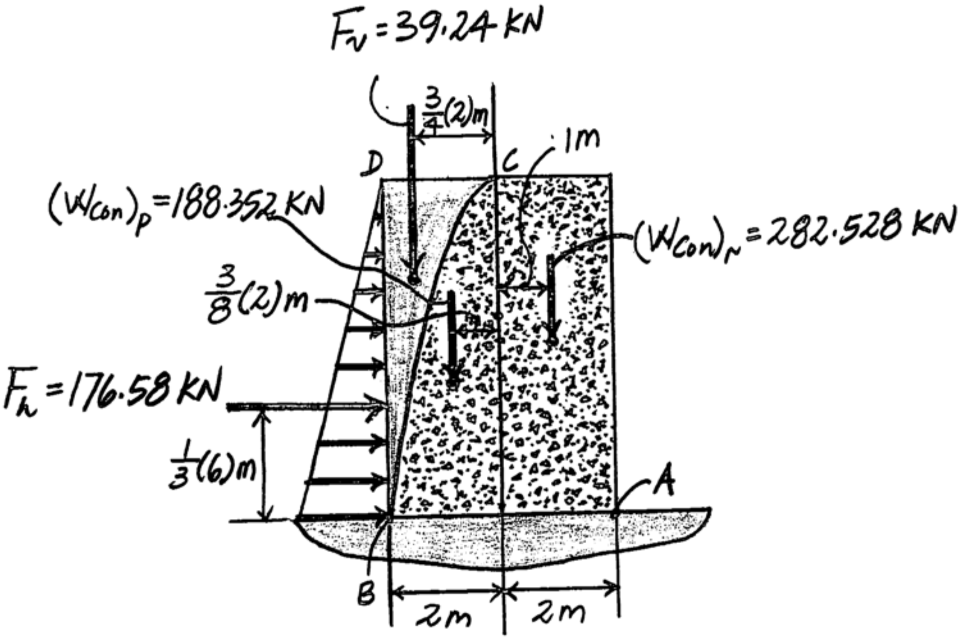
$$(W_{\text{con}})_r = \rho_{\text{con}} g V = 2400(9.81)[2(6)(1)] = 282\,528\ \text{N} = 282.528\ \text{kN}$$

Location: The location of each of the above forces are indicated in Fig. *a*. Here, $\mathbf{F}_h$ creates the overturning moment M_t about point A, while $\mathbf{F}_v$, $(\mathbf{W}_{\text{con}})_p$, and $(\mathbf{W}_{\text{con}})_r$ contribute to the stabilizing moment M_s about point A. Thus

$$\text{F.S.} = \frac{M_s}{M_t}$$

$$= \frac{282.528(1) + 188.352\left[2 + \frac{3}{8}(2)\right] + 39.24\left[2 + \frac{3}{4}(2)\right]}{176.58\left[\frac{1}{3}(6)\right]}$$

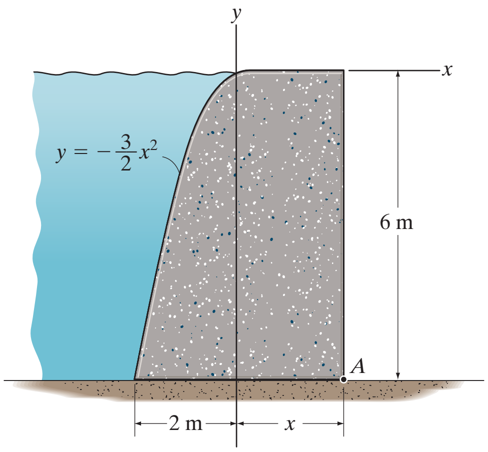
$$= 2.66 \qquad \textbf{Ans.}$$



$F_v = 39.24$ kN
$\frac{3}{4}(2)$m
D
C
1m
$(W_{con})_p = 188.352$ kN
$(W_{con})_r = 282.528$ kN
$\frac{3}{8}(2)$m
$F_h = 176.58$ kN
$\frac{1}{3}(6)$m
A
B
2m
2m

(a)

9–118. The concrete gravity dam is designed so that it is held in position by its own weight. Determine the minimum dimension x so that the factor of safety against overturning about point A of the dam is 2. The factor of safety is defined as the ratio of the stabilizing moment divided by the overturning moment. The densities of concrete and water are $\rho_{\text{conc}} = 2.40\ \text{Mg/m}^3$ and $\rho_w = 1\ \text{Mg/m}^3$, respectively. Assume that the dam does not slide.



Resultant Force Component: The analysis of this problem will be based on a per meter width of the dam. The hydrostatic force acting on the parabolic surface of the dam consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*. The vertical component $\mathbf{F}_v$ consists of the weight of water contained in the shaded area shown in Fig. *a*.

$$F_v = \rho_w g A_{BCD} b = (1000)(9.81)\left[\frac{1}{3}(2)(6)\right](1) = 39240\ \text{N} = 39.24\ \text{kN}$$

The horizontal component $\mathbf{F}_h$ consists of the horizontal hydrostatic pressure, which can be represented by a triangular distributed loading shown in Fig. *a*. The intensity of the distributed loading at point B is $w_B = \rho_w g h_B b = 1000(9.81)(6)(1) = 58860\ \text{N/m} = 58.86\ \text{kN/m}$. Thus,

$$F_h = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

The weight of the parabolic shaped concrete dam is

$$(W_{\text{con}})_p = \rho_{\text{con}} g V = 2400(9.81)\left[\frac{2}{3}(2)(6)(1)\right] = 188\,352\ \text{N} = 188.352\ \text{kN}$$

The weight of the rectangular shaped concrete dam is

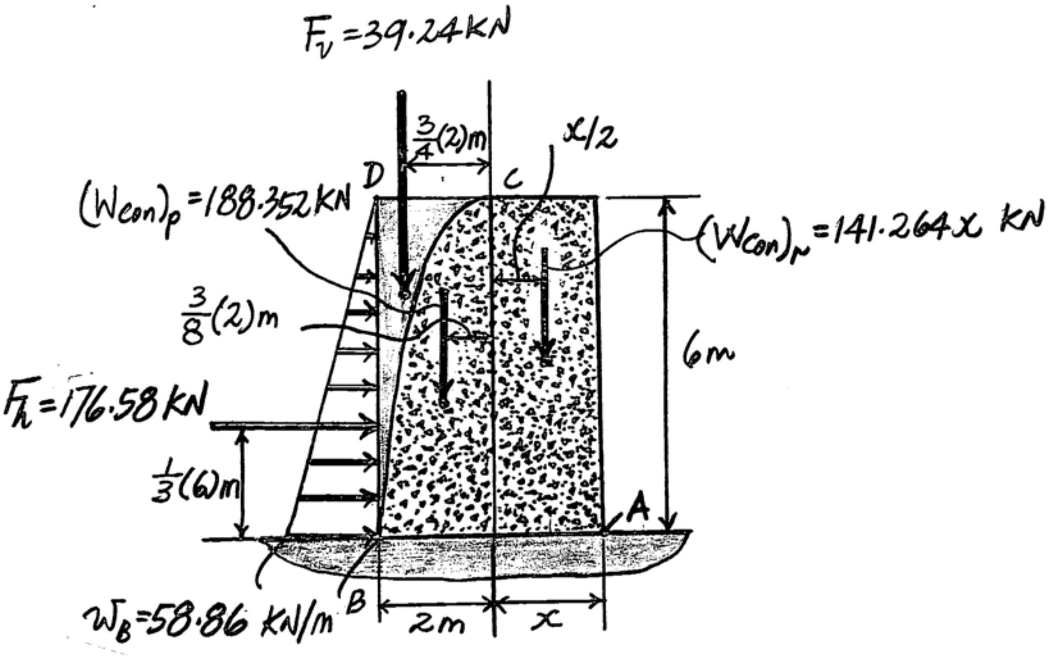
$$(W_{\text{con}})_r = \rho_{\text{con}} g V = 2400(9.81)(6)(x)(1) = 141\,264x\ \text{N} = 141.264x\ \text{kN}$$

Location: The location of each of the above forces are indicated in Fig. *a*. Here, $\mathbf{F}_h$ creates the overturning moment M_t about point A, while $\mathbf{F}_v$, $(\mathbf{W}_{\text{con}})_p$, and $(\mathbf{W}_{\text{con}})_r$ contribute to the stabilizing moment M_s about point A. Thus

$$\text{F.S.} = \frac{M_s}{M_t}$$

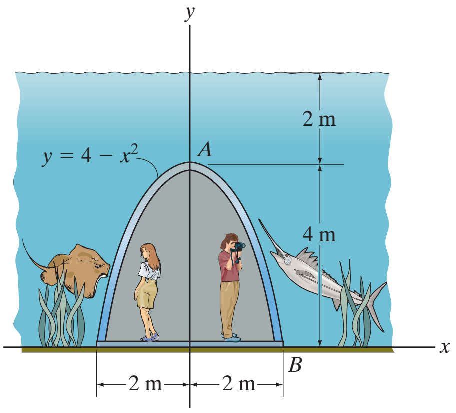
$$2 = \frac{141.264(x)\left(\frac{x}{2}\right) + 188.352\left[\frac{3}{8}(2) + x\right] + 39.24\left[\frac{3}{4}(2) + x\right]}{176.58\left[\frac{1}{3}(6)\right]}$$

$x = 1.51\ \text{m}$ **Ans.**


$F_v = 39.24$ kN
$\frac{3}{4}(2)$m
$x/2$
D
C
$(W_{con})_p = 188.352$ kN
$(W_{con})_r = 141.264x$ kN
$\frac{3}{8}(2)$m
6m
$F_h = 176.58$ kN
$\frac{1}{3}(6)$m
A
$w_B = 58.86$ kN/m
B
2m
x

(a)

9–119. The underwater tunnel in the aquatic center is fabricated from a transparent polycarbonate material formed in the shape of a parabola. Determine the magnitude of the hydrostatic force that acts per meter length along the surface *AB* of the tunnel. The density of the water is $\rho_w = 1000\ \text{kg/m}^3$.



Resultant Force Component: The hydrostatic force acting on the parabolic surface *AB* of the tunnel consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*. The vertical component $\mathbf{F}_v$ represents the weight of water contained in the shaded area shown in Fig. *a*

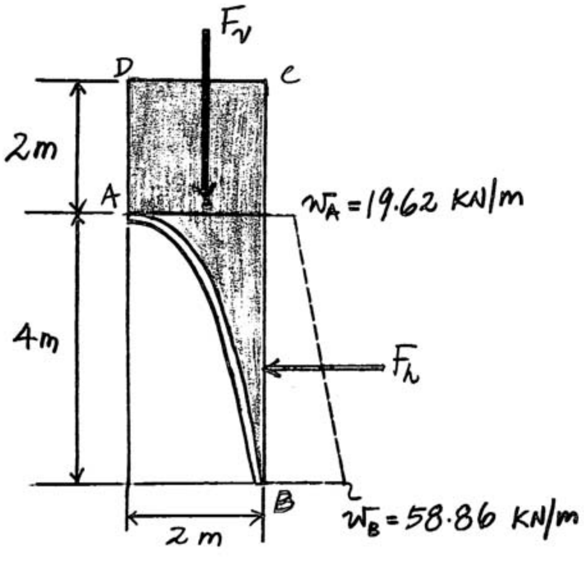
$$F_v = \rho_w g A_{ABCD} b = (1000)(9.81)\left[2(2)+\frac{1}{3}(2)(4)\right](1) = 65\,400\ \text{N} = 65.4\ \text{kN}$$

The horizontal component $\mathbf{F}_h$ represents the horizontal hydrostatic pressure. Since the width of the tunnel is constant (1 m), this horizontal loading can be represented by a trapezoidal distributed loading shown in Fig. *a*. The intensity of this distributed loading at points *A* and *B* are $w_A = \rho_w g h_A b = 1000(9.81)(2)(1) = 19\,620\ \text{N/m}$ and $w_B = \rho_w g h_B b = 1000(9.81)(6)(1) = 58860\ \text{N/m} = 58.86\ \text{kN/m}$. Thus,

$$F_h = \frac{1}{2}(19.62 + 58.86)(4) = 156.96\ \text{kN}$$

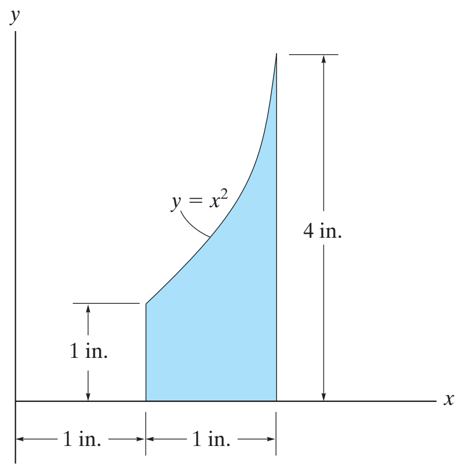
Resultant: The resultant hydrostatic force acting on the surface *AB* of the tunnel is therefore

$$F_R = \sqrt{F_h{}^2 + F_v{}^2} = \sqrt{156.96^2 + 65.4^2} = 170\ \text{kN}$$ **Ans.**



(a)

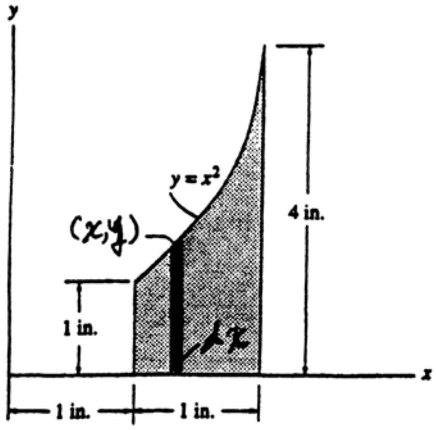
***9–120.** Locate the centroid $\bar{x}$ of the shaded area.



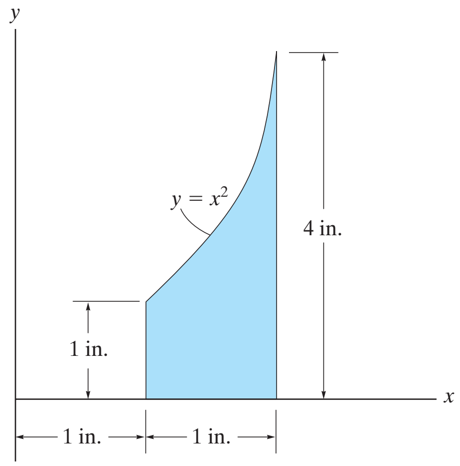
$dA = y\,dx = x^2\,dx$

$\tilde{x} = x$

$\bar{x} = \dfrac{\int_A \tilde{x}\,dA}{\int_A dA} = \dfrac{\int_1^2 x^3\,dx}{\int_1^2 x^2\,dx} = 1.61 \text{ in.}$ **Ans**



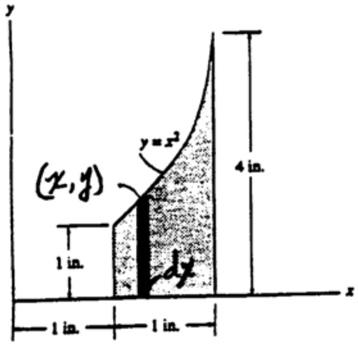
•9–121. Locate the centroid $\bar{y}$ of the shaded area.



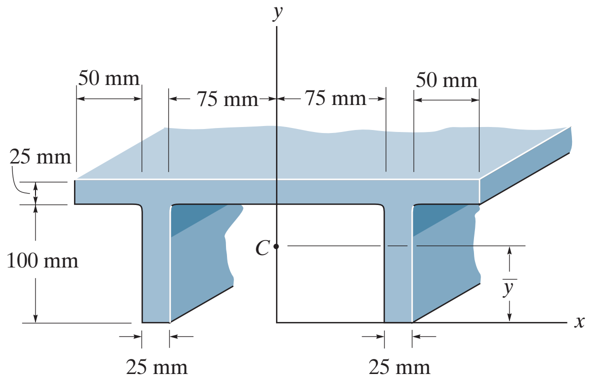
$dA = y\,dx = x^2\,dx$

$\tilde{y} = \dfrac{y}{2} = \dfrac{x^2}{2}$

$\bar{y} = \dfrac{\int_A \tilde{y}\,dA}{\int_A dA} = \dfrac{\frac{1}{2}\int_1^2 x^4\,dx}{\int_1^2 x^2\,dx} = \dfrac{\frac{1}{10}\left[(2)^5 - (1)^5\right]}{\frac{1}{3}\left[(2)^3 - (1)^3\right]} = 1.33 \text{ in.}$ **Ans**



9–122. Locate the centroid $\bar{y}$ of the beam's cross-sectional area.

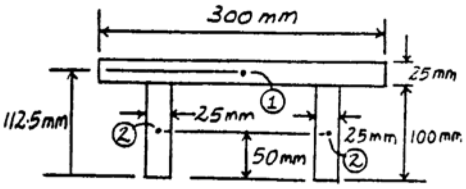


Centroid : The area of each segment and its respective centroid are tabulated below.

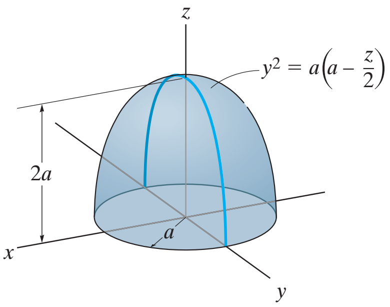
Segment	$A\,(\text{mm}^2)$	$\tilde{y}\,(\text{mm})$	$\tilde{y}A\,(\text{mm}^3)$
1	300(25)	112.5	843 750
2	100(50)	50	250 000
Σ	12 500		1 093 750

Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{1\,093\,750}{12\,500} = 87.5 \text{ mm} \qquad \textbf{Ans}$$



9–123. Locate the centroid $\bar{z}$ of the solid.



Volume and Moment Arm : The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi\left[a\left(a - \frac{z}{2}\right)\right]dz = \pi a\left(a - \frac{z}{2}\right)dz$ and its centroid is at $\tilde{z} = z$.

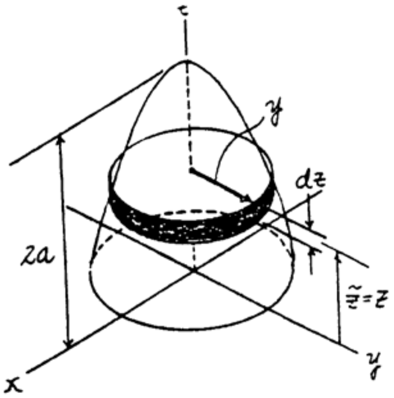
Centroid : Due to symmetry about the z axis

$$\bar{x} = \bar{y} = 0 \qquad \textbf{Ans}$$

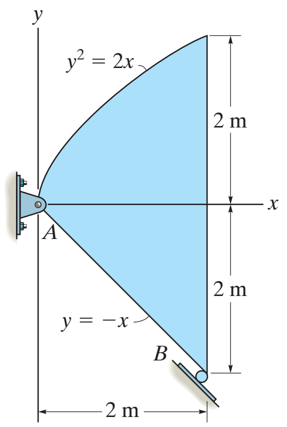
Applying Eq. 9-3 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{2a} z\left[\pi a\left(a - \frac{z}{2}\right)dz\right]}{\int_0^{2a} \pi a\left(a - \frac{z}{2}\right)dz}$$

$$= \frac{\pi a\left(\frac{az^2}{2} - \frac{z^3}{6}\right)\Big|_0^{2a}}{\pi a\left(az - \frac{z^2}{4}\right)\Big|_0^{2a}} = \frac{2}{3}a \qquad \textbf{Ans}$$



***9–124.** The steel plate is 0.3 m thick and has a density of 7850 kg/m^3. Determine the location of its center of mass. Also compute the reactions at the pin and roller support.



Fluid Pressure : The fluid pressure at the toe of the dam can be determined using Eq. $\overset{4}{9}-1\overset{9}{5}$, $p = \gamma z$.

$$p = 62.4(8) = 499.2 \text{ lb/ft}^2$$

Thus,

$$w = 499.2(1) = 499.2 \text{ lb/ft}$$

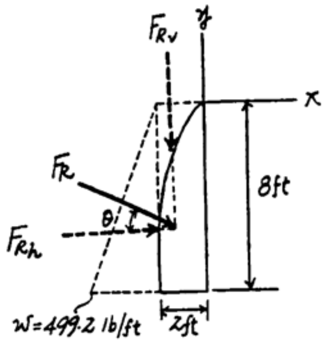
Resultant Forces : From the inside back cover of the text, the exparabolic area is $A = \frac{1}{3}ab = \frac{1}{3}(8)(2) = 5.333 \text{ ft}^2$. Then, the vertical and horizontal components of the resultant force are

$$F_{R_v} = \gamma V = 62.4[5.333(1)] = 332.8 \text{ lb}$$

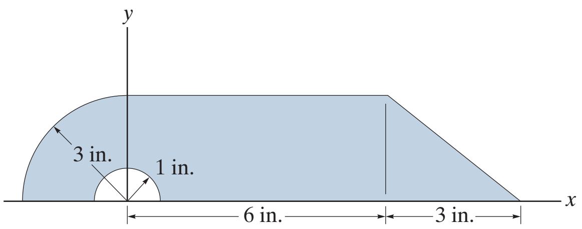
$$F_{R_h} = \frac{1}{2}(499.2)(8) = 1996.8 \text{ lb}$$

The resultant force and is

$$F_R = \sqrt{F_{R_v}^2 + F_{R_h}^2} = \sqrt{332.8^2 + 1996.8^2}$$
$$= 2024.34 \text{ lb} = 2.02 \text{ kip} \qquad \textbf{Ans}$$



•9–125. Locate the centroid $(\overline{x}, \overline{y})$ of the area.



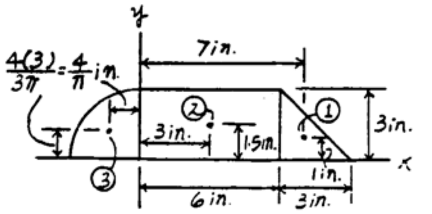
Centroid : The area of each segment and its respective centroid are tabulated below.

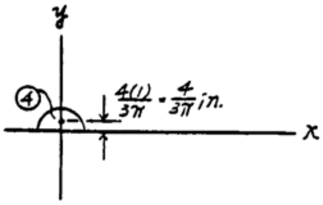
Segment	A (in^2)	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$ (in^3)	$\tilde{y}A$ (in^3)
1	$\frac{1}{2}(3)(3)$	7	1	31.5	4.50
2	$6(3)$	3	1.5	54.0	27.0
3	$\frac{\pi}{4}(3^2)$	$-\frac{4}{\pi}$	$\frac{4}{\pi}$	−9.00	9.00
4	$-\frac{\pi}{2}(1^2)$	0	$\frac{4}{3\pi}$	0	−0.667
Σ	27.998			76.50	39.833

Thus,

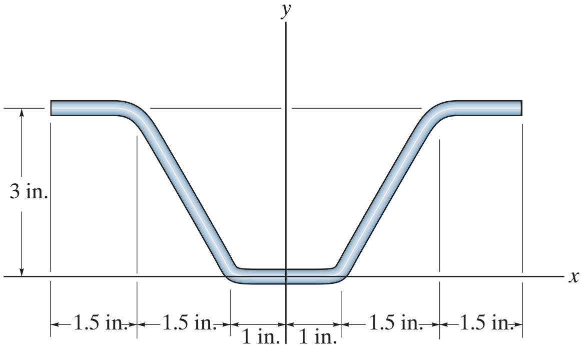
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{76.50}{27.998} = 2.73 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{39.833}{27.998} = 1.42 \text{ in.} \qquad \textbf{Ans}$$





9–126. Determine the location ($\bar{x}$, $\bar{y}$) of the centroid for the structural shape. Neglect the thickness of the member.

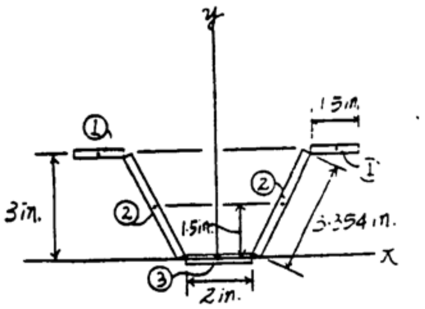


Centroid : The length of each segment and its respective centroid are tabulated below.

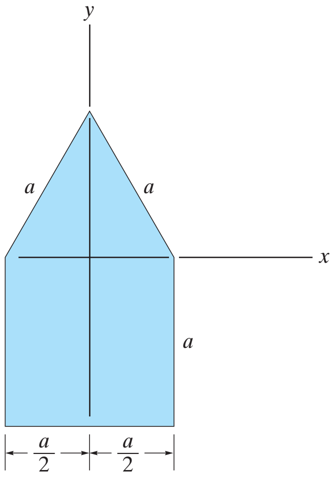
Segment	L(in.)	$\tilde{y}$(in.)	$\tilde{y}L$(in^2)
1	2(1.5)	3	9.00
2	2(3.354)	1.5	10.06
3	2	0	0
Σ	11.71		19.06

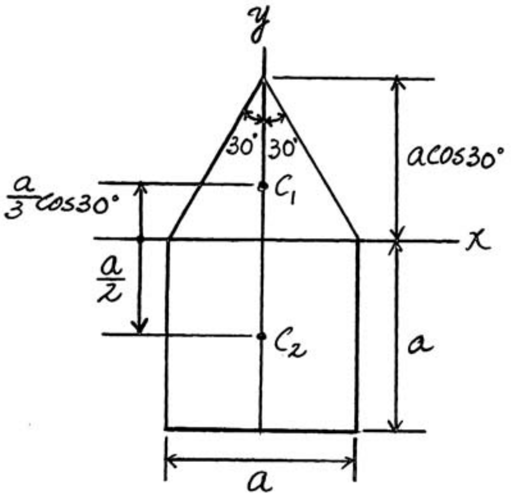
Due to symmetry about y axis, $\bar{x} = 0$ **Ans**

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{19.06}{11.71} = 1.628 \text{ in.} = 1.63 \text{ in.} \quad \textbf{Ans}$$



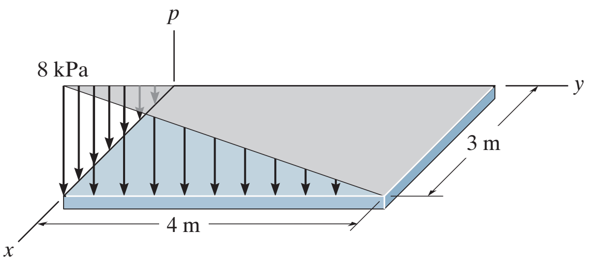
9–127. Locate the centroid $\bar{y}$ of the shaded area.



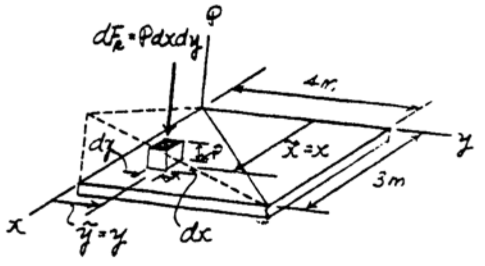


$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{a}{3}\cos 30^\circ\left[\frac{1}{2}(a)(a\cos 30^\circ)\right] - \frac{a}{2}[a(a)]}{\frac{1}{2}(a)(a\cos 30^\circ) + [a(a)]} = -0.262a$$

***9–128.** The load over the plate varies linearly along the sides of the plate such that $p = \frac{2}{3}[x(4 - y)]$ kPa. Determine the resultant force and its position $(\bar{x}, \bar{y})$ on the plate.



Resultant Force and its Location : The volume of the differential element is $dV = dF_R = p\,dx\,dy = \frac{2}{3}(x\,dx)[(4-y)\,dy]$ and its centroid are $\tilde{x} = x$ and $\tilde{y} = y$.



$$F_R = \int_{F_R} dF_R = \int_0^{3\text{m}} \frac{2}{3}(x\,dx)\int_0^{4\text{m}}(4-y)\,dy$$
$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{m}}\right] = 24.0\text{ kN} \qquad \textbf{Ans}$$

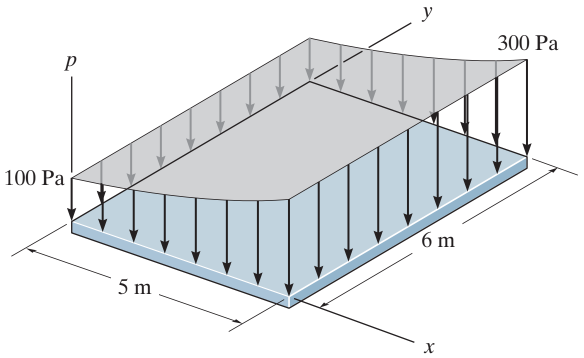
$$\int_{F_R} \tilde{x}\,dF_R = \int_0^{3\text{m}} \frac{2}{3}(x^2\,dx)\int_0^{4\text{m}}(4-y)\,dy$$
$$= \frac{2}{3}\left[\left(\frac{x^3}{3}\right)\Big|_0^{3\text{m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{m}}\right] = 48.0\text{ kN}\cdot\text{m}$$

$$\int_{F_R} \tilde{y}\,dF_R = \int_0^{3\text{m}} \frac{2}{3}(x\,dx)\int_0^{4\text{m}} y(4-y)\,dy$$
$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{m}}\left(2y^2 - \frac{y^3}{3}\right)\Big|_0^{4\text{m}}\right] = 32.0\text{ kN}\cdot\text{m}$$

$$\bar{x} = \frac{\int_{F_R}\tilde{x}\,dF_R}{\int_{F_R} dF_R} = \frac{48.0}{24.0} = 2.00\text{ m} \qquad \textbf{Ans}$$

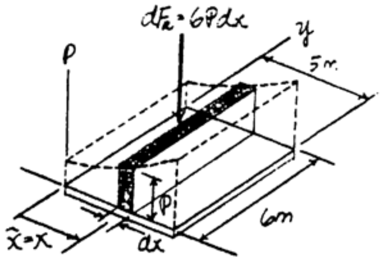
$$\bar{y} = \frac{\int_{F_R}\tilde{y}\,dF_R}{\int_{F_R} dF_R} = \frac{32.0}{24.0} = 1.33\text{ m} \qquad \textbf{Ans}$$

•9–129. The pressure loading on the plate is described by the function $p = \{-240/(x + 1) + 340\}$ Pa. Determine the magnitude of the resultant force and coordinates of the point where the line of action of the force intersects the plate.



Resultant Force and its Location : The volume of the differential element is $dV = dF_R = 6p\,dx = 6\left(-\frac{240}{x+1} + 340\right)dx$ and its centroid is $\tilde{x} = x$.

$$F_R = \int_{F_R} dF_R = \int_0^{5\text{m}} 6\left(-\frac{240}{x+1} + 340\right)dx$$
$$= 6[-240\ln(x+1) + 340x]\Big|_0^{5\text{m}}$$
$$= 7619.87\text{ N} = 7.62\text{ kN} \qquad \textbf{Ans}$$



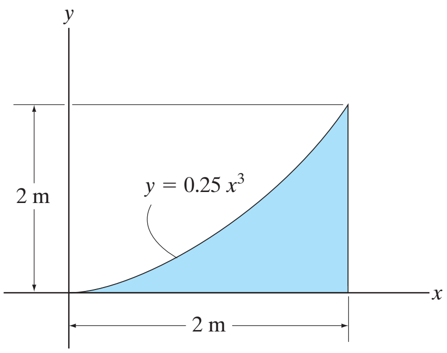
$$\int_{F_R} \tilde{x}\,dF_R = \int_0^{5\text{m}} 6x\left(-\frac{240}{x+1} + 340\right)$$
$$= \left[-1440[x - \ln(x+1)] + 1020x^2\right]\Big|_0^{5\text{m}}$$
$$= 20880.13\text{ N}\cdot\text{m}$$

$$\bar{x} = \frac{\int_{F_R}\tilde{x}\,dF_R}{\int_{F_R} dF_R} = \frac{20880.13}{7619.87} = 2.74\text{ m} \qquad \textbf{Ans}$$

Due to symmetry,

$$\bar{y} = 3.00\text{ m} \qquad \textbf{Ans}$$

•10–1. Determine the moment of inertia of the area about the x axis.



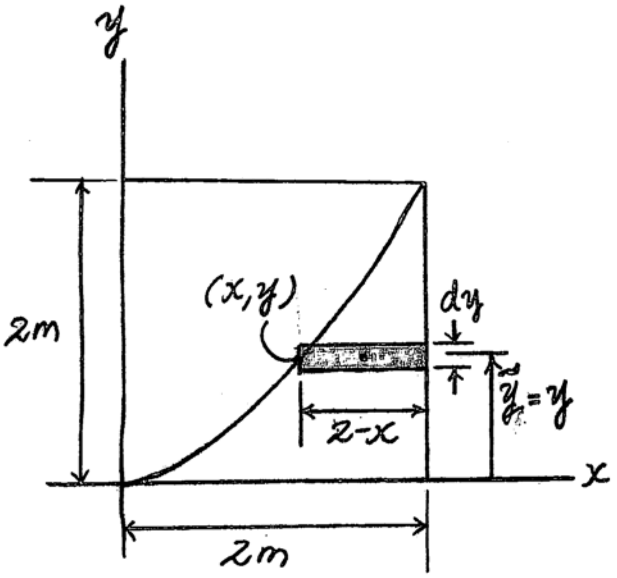
The area of the rectangular differential element in Fig. a is $dA = (2-x)\,dy$. Since $x = (4y)^{1/3}$ then $dA = \left[2-(4y)^{1/3}\right]dy$.

$$I_x = \int_A y^2\,dA$$

$$= \int_0^{2\text{ m}} y^2\left[2-(4y)^{1/3}\right]dy$$

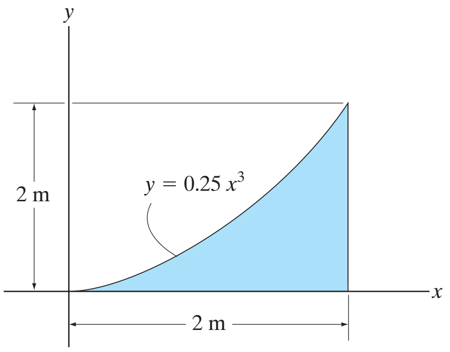
$$= \int_0^{2\text{ m}} \left(2y^2 - 4^{1/3}y^{7/3}\right)dy$$

$$= \left[\frac{2y^3}{3} - \frac{3}{10}\left(4^{1/3}\right)y^{10/3}\right]_0^{2\text{ m}} = 0.533\text{ m}^4$$ **Ans.**



(a)

10–2. Determine the moment of inertia of the area about the *y* axis.



The area of the rectangular differential element in Fig. *a* is $dA = y\,dx = \dfrac{x^3}{4}\,dx$.

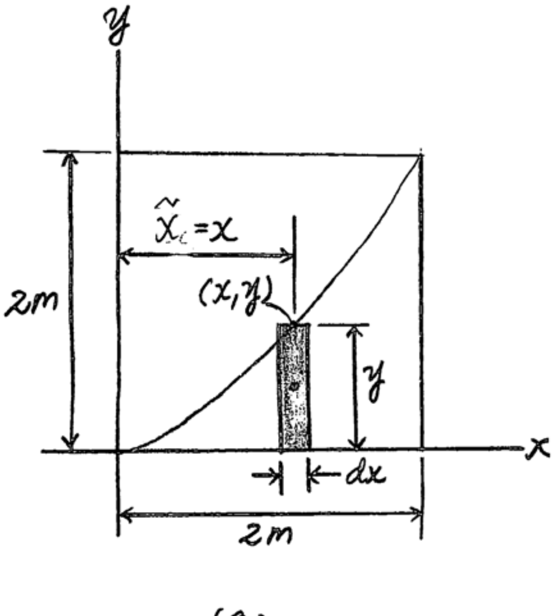
$$I_y = \int_A x^2 dA$$

$$= \int_0^{2\text{ m}} x^2\left(\frac{x^3}{4}\right)dx$$

$$= \int_0^{2\text{ m}} \frac{x^5}{4}\,dx$$

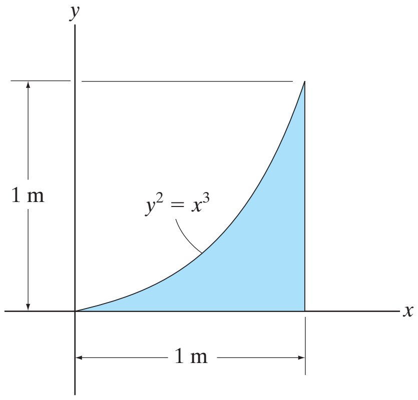
$$= \left(\frac{x^6}{24}\right)\Bigg|_0^{2\text{ m}} = 2.67\text{ m}^4$$

Ans.



(a)

10–3. Determine the moment of inertia of the area about the x axis.



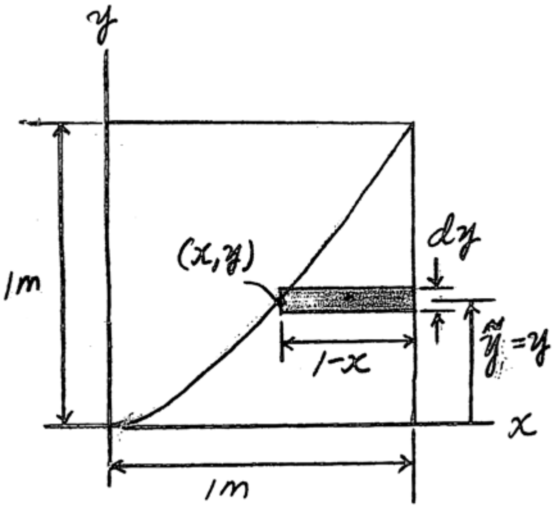
The area of the rectangular differential element in Fig. a is $dA = (1-x)\,dy$. Since $x = y^{2/3}$, then $dA = (1-y^{2/3})\,dy$.

$$I_x = \int_A y^2 dA$$

$$= \int_0^{1\text{ m}} y^2\left[1-y^{2/3}\right]dy$$

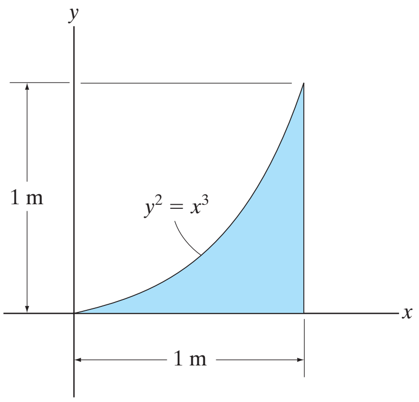
$$= \int_0^{1\text{ m}} (y^2 - y^{8/3})\,dy$$

$$= \left(\frac{y^3}{3} - \frac{3}{11}y^{11/3}\right)\Big|_0^{1\text{ m}} = 0.0606\text{ m}^4 \qquad \textbf{Ans.}$$



(a)

***10–4.** Determine the moment of inertia of the area about the y axis.



The area of the rectangular differential element in Fig. a is $dA = y\,dx = x^{3/2}\,dx$.

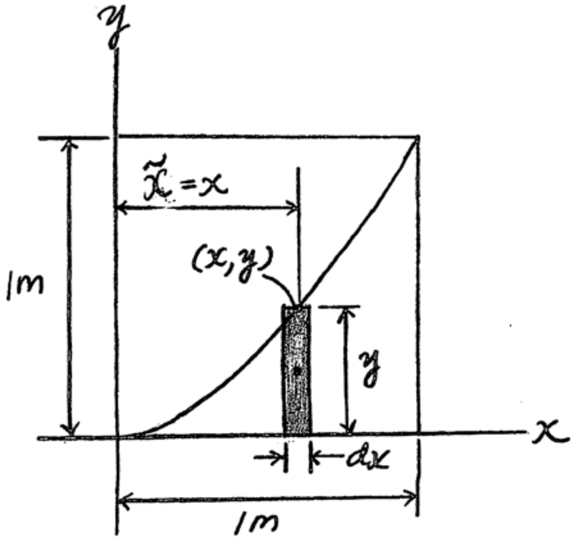
$$I_y = \int_A x^2 dA$$

$$= \int_0^{1\text{ m}} x^2\left(x^{3/2}\right) dx$$

$$= \int_0^{1\text{ m}} x^{7/2}\,dx$$

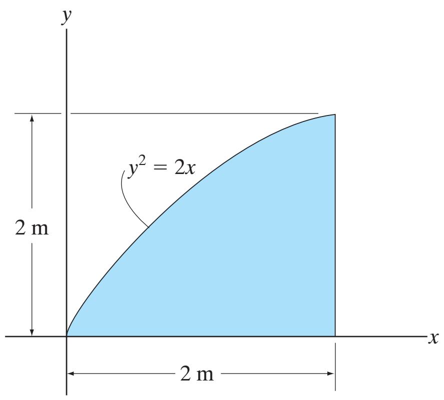
$$= \left(\frac{2}{9}x^{9/2}\right)\Bigg|_0^{1\text{ m}} = 0.222\text{ m}^4$$

Ans.



(a)

•10–5. Determine the moment of inertia of the area about the *x* axis.



The area of the rectangular differential element in Fig. *a* is $dA = (2 - x)\,dy$. Since $x = \frac{y^2}{2}$, then $dA = \left(2 - \frac{y^2}{2}\right)dy$.

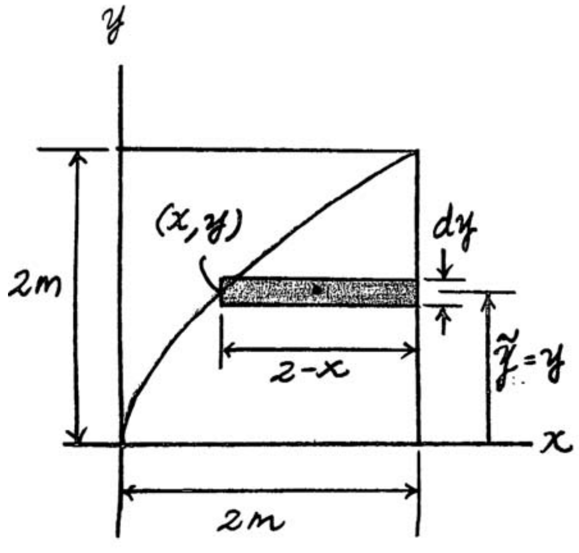
$$I_x = \int_A y^2\,dA$$

$$= \int_0^{2\,\text{m}} y^2\left(2 - \frac{y^2}{2}\right)dy$$

$$= \int_0^{2\,\text{m}} \left(2y^2 - \frac{y^4}{2}\right)dy$$

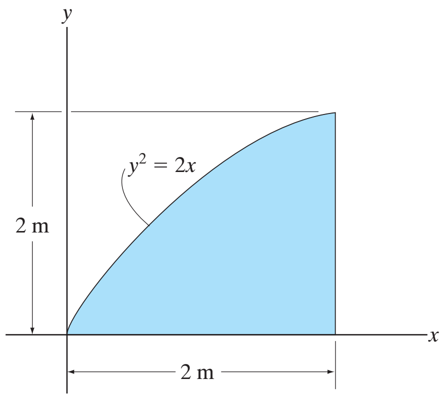
$$= \left(\frac{2}{3}y^3 - \frac{y^5}{10}\right)\Bigg|_0^{2\,\text{m}} = 2.13\ \text{m}^4$$

Ans.



(a)

10–6. Determine the moment of inertia of the area about the *y* axis.



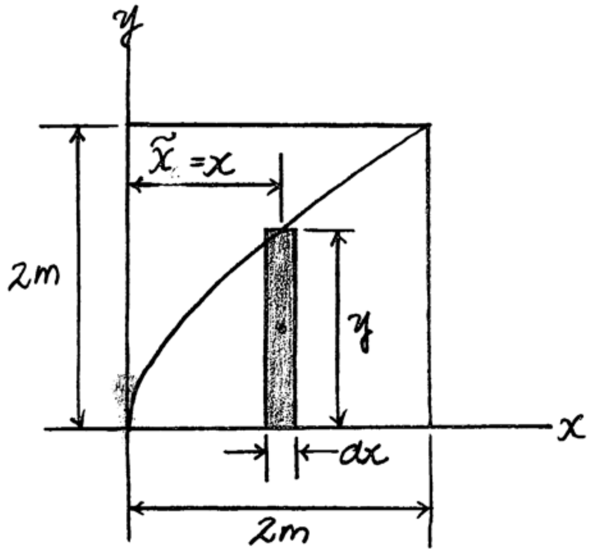
The area of the rectangular differential element in Fig. *a* is $dA = y\,dx = (2x)^{1/2}\,dx$.

$$I_y = \int_A x^2 dA$$

$$= \int_0^{2\text{ m}} x^2(2x)^{1/2}\,dA$$

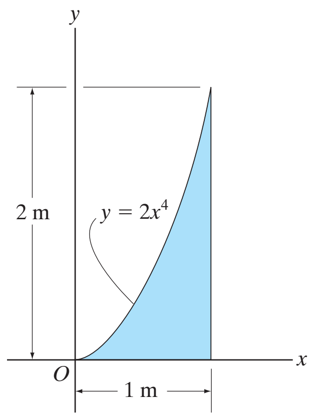
$$= \int_0^{2\text{ m}} \sqrt{2}x^{5/2}\,dx$$

$$= \left[\sqrt{2}\left(\frac{2}{7}x^{7/2}\right)\right]_0^{2\text{m}} = 4.57\text{ m}^4 \qquad \textbf{Ans.}$$



(a)

10–7. Determine the moment of inertia of the area about the *x* axis.

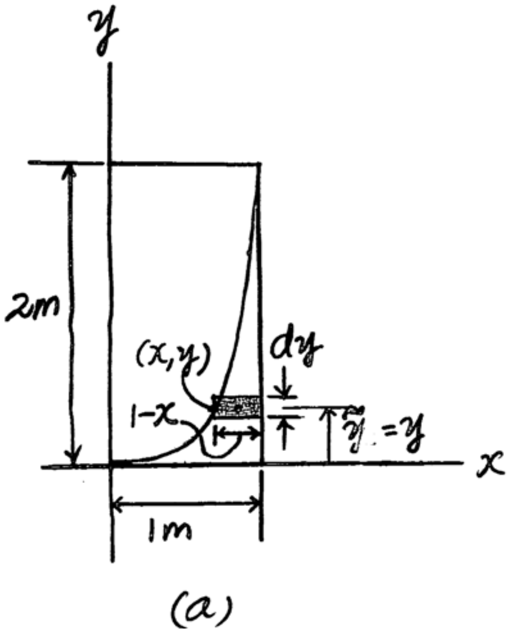


The moment of inertia of the area about the x axis will be determined using the rectangular differential element in Fig. *a*. This area is

$$dA = (1-x)\,dy = \left[1-\left(\frac{y}{2}\right)^{1/4}\right]dy$$

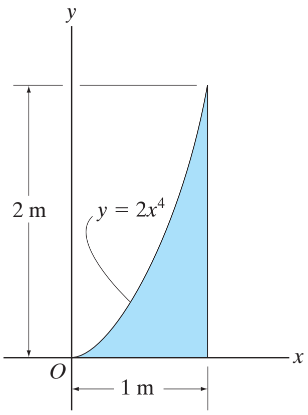
$$I_x = \int_A y^2 dA = \int_0^{2\,\mathrm{m}} y^2\left[1-\left(\frac{y}{2}\right)^{1/4}\right]dy = \int_0^{2\,\mathrm{m}}\left[y^2-\left(\frac{1}{2}\right)^{1/4}y^{9/4}\right]dy$$

$$= \left[\frac{y^3}{3}-\left(\frac{1}{2}\right)^{1/4}\left(\frac{4}{13}\right)y^{13/4}\right]\Bigg|_0^{2\,\mathrm{m}} = 0.205\ \mathrm{m}^4 \qquad \textbf{Ans.}$$



(a)

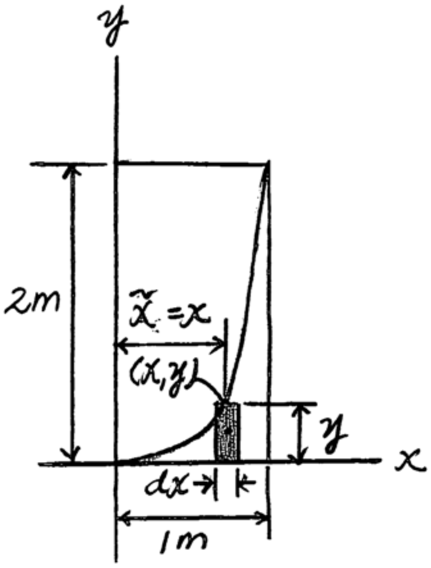
***10–8.** Determine the moment of inertia of the area about the y axis.



The moment of inertia of the area about the y axis will be determined using the rectangular differential element in Fig. *a*. This area is

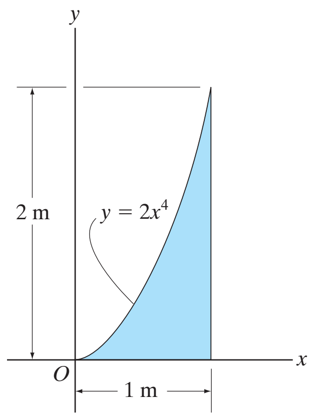
$dA = y\,dx = 2x^4\,dx$

$$I_y = \int_A x^2 dA = \int_0^{1\text{ m}} x^2\left(2x^4 dx\right) = \int_0^{1\text{ m}} 2x^6\,dx = \left(\frac{2}{7}x^7\right)\Big|_0^{1\text{ m}} = 0.286\text{ m}^4 \quad \textbf{Ans.}$$



(b)

•10–9. Determine the polar moment of inertia of the area about the z axis passing through point O.



The moment of inertia of the area about the x and y axes will be determined using the rectangular differential element in Figs. a and b. The area of these two elements are

$dA = (1 - x)\,dy = \left[1 - \left(\frac{y}{2}\right)^{1/4}\right] dy$ and $dA = y\,dx = 2x^4\,dx$.

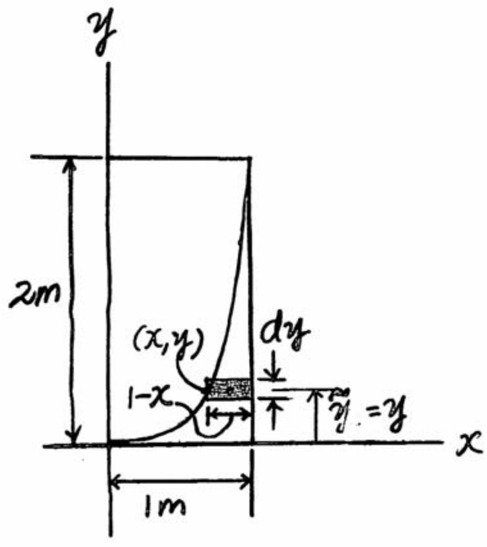
$$I_x = \int_A y^2 dA = \int_0^{2\,\text{m}} y^2\left[1-\left(\frac{y}{2}\right)^{1/4}\right]dy = \int_0^{2\,\text{m}}\left[y^2 - \left(\frac{1}{2}\right)^{1/4} y^{9/4}\right]dy$$

$$= \left[\frac{y^3}{3} - \left(\frac{1}{2}\right)^{1/4}\left(\frac{4}{13}\right)y^{13/4}\right]\Bigg|_0^{2\,\text{m}} = 0.2051\ \text{m}^4$$

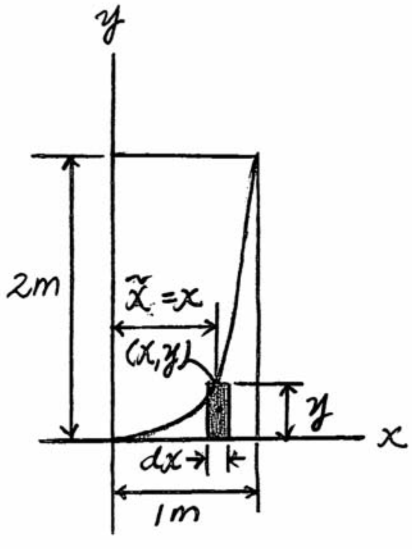
$$I_y = \int_A x^2 dA = \int_0^{1\,\text{m}} x^2\left(2x^4 dx\right) = \int_0^{1\,\text{m}} 2x^6\,dx = \left(\frac{2}{7}x^7\right)\Bigg|_0^{1\,\text{m}} = 0.2857\ \text{m}^4$$

Thus, the polar moment of inertia of the area about the z axis is

$$J_O = I_x + I_y = 0.2051 + 0.2857 = 0.491\ \text{m}^4$$ **Ans.**

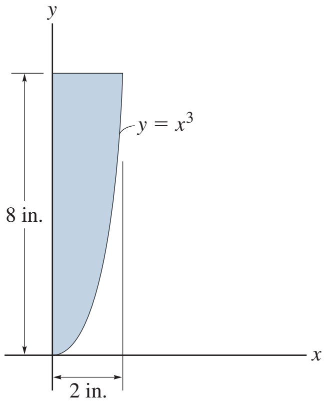


(a)



(b)

10–10. Determine the moment of inertia of the area about the *x* axis.



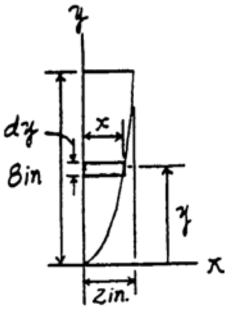
Differential Element : Here, $x = y^{\frac{1}{3}}$. The area of the differential element parallel to *x* axis is $dA = x\,dy = y^{\frac{1}{3}}dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

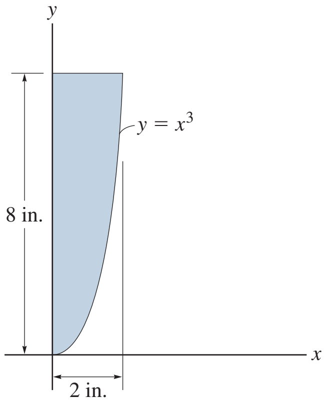
$$I_x = \int_A y^2\,dA = \int_0^{8\text{in.}} y^2\left(y^{\frac{1}{3}}\right)dy$$

$$= \left[\frac{3}{10}y^{\frac{10}{3}}\right]\Bigg|_0^{8\text{in.}}$$

$$= 307\text{ in}^4$$ **Ans**



10–11. Determine the moment of inertia of the area about the *y* axis.



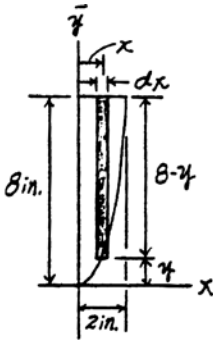
Differential Element : The area of the differential element parallel to *y* axis is $dA = (8-y)\,dx = \left(8-x^3\right)dx$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

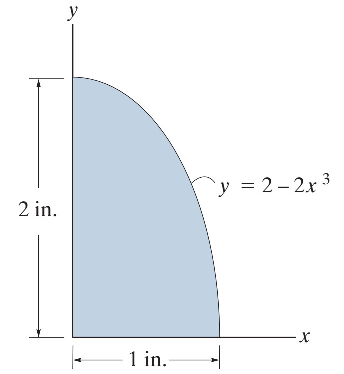
$$I_y = \int_A x^2\,dA = \int_0^{2\text{in.}} x^2\left(8-x^3\right)dx$$

$$= \left(\frac{8}{3}x^3 - \frac{1}{6}x^6\right)\Bigg|_0^{2\text{in.}}$$

$$= 10.7\text{ in}^4$$ **Ans**

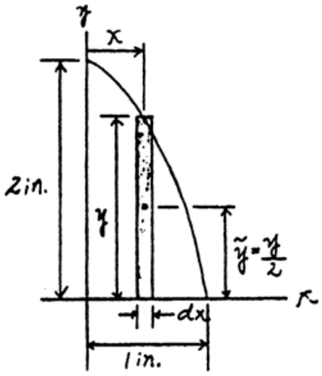


***10–12.** Determine the moment of inertia of the area about the *x* axis.



Differential Element : The area of the differential element parallel to *y* axis is $dA = ydx$. The moment of inertia of this element about *x* axis is

$$\begin{aligned} dI_x &= d\bar{I}_{x'} + dA\tilde{y}^2 \\ &= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 \\ &= \frac{1}{3}\left(2-2x^3\right)^3 dx \\ &= \frac{1}{3}\left(-8x^9+24x^6-24x^3+8\right) dx \end{aligned}$$

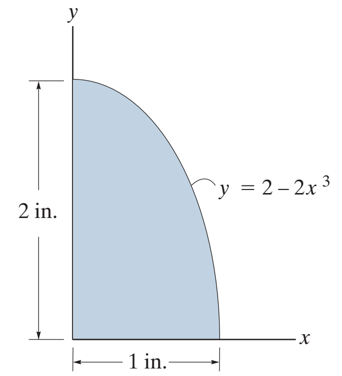


Moment of Inertia : Performing the integration, we have

$$\begin{aligned} I_x = \int dI_x &= \frac{1}{3}\int_0^{1\text{ in.}}\left(-8x^9+24x^6-24x^3+8\right) dx \\ &= \frac{1}{3}\left(-\frac{4}{5}x^{10}+\frac{24}{7}x^7-6x^4+8x\right)\Bigg|_0^{1\text{ in.}} \\ &= 1.54\text{ in}^4 \end{aligned}$$

Ans

•10–13. Determine the moment of inertia of the area about the *y* axis.

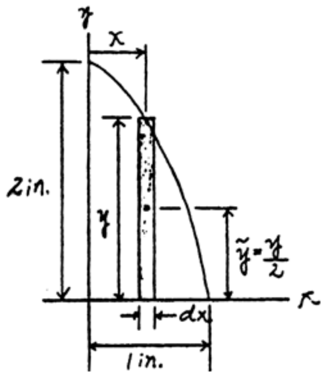


Differential Element : The area of the differential element parallel to yaxis is $dA = ydx = \left(2-2x^3\right)dx$.

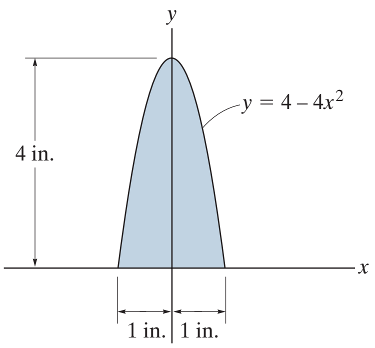
Moment of Inertia : Applying Eq. 10– 1and performing the integration, we have

$$\begin{aligned} I_y = \int_A x^2 dA &= \int_0^{1\text{ in.}} x^2\left(2-2x^3\right) dx \\ &= \left[\frac{2}{3}x^3 - \frac{1}{3}x^6\right]\Bigg|_0^{1\text{ in.}} \\ &= 0.333\text{ in}^4 \end{aligned}$$

Ans



10–14. Determine the moment of inertia of the area about the x axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of dx, and (b) having a thickness of dy.



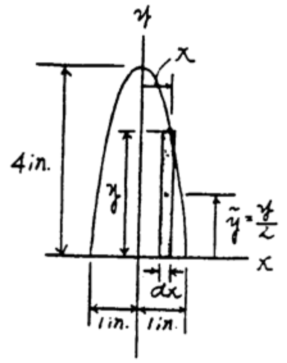
a)*Differential Element :* The area of the differential element parallel to y axis is $dA = ydx$. The moment of inertia of this element about x axis is

$$\begin{aligned} dI_x &= d\bar{I}_{x'} + dA\,\tilde{y}^2 \\ &= \frac{1}{12}(dx)\,y^3 + ydx\left(\frac{y}{2}\right)^2 \\ &= \frac{1}{3}\left(4-4x^2\right)^3 dx \\ &= \frac{1}{3}\left(-64x^6+192x^4-192x^2+64\right)dx \end{aligned}$$

Moment of Inertia : Performing the integration, we have

$$\begin{aligned} I_x = \int dI_x &= \frac{1}{3}\int_{-1\text{in.}}^{1\text{in.}} \frac{1}{3}\left(-64x^6+192x^4-192x^2+64\right)dx \\ &= \frac{1}{3}\left(-\frac{64}{7}x^7+\frac{192}{5}x^5-\frac{192}{3}x^3+64x\right)\Bigg|_{-1\text{in.}}^{1\text{in.}} \\ &= 19.5\ \text{in}^4 \end{aligned}$$

Ans



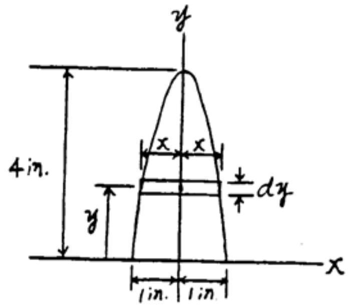
(a)

b)*Differential Element :* Here, $x = \frac{1}{2}\sqrt{4-y}$. The area of the differential element parallel to x axis is $dA = 2xdy = \sqrt{4-y}\,dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

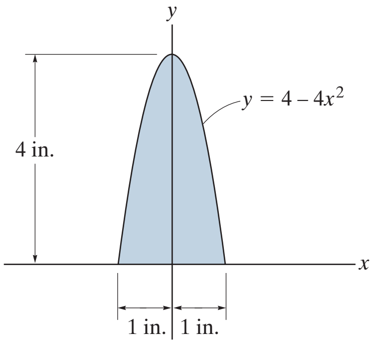
$$\begin{aligned} I_x &= \int_A y^2\,dA \\ &= \int_0^{4\text{in.}} y^2\sqrt{4-y}\,dy \\ &= \left[-\frac{2y^2}{3}(4-y)^{\frac{3}{2}}-\frac{8y}{15}(4-y)^{\frac{5}{2}}-\frac{16}{105}(4-y)^{\frac{7}{2}}\right]\Bigg|_0^{4\text{in.}} \\ &= 19.5\ \text{in}^4 \end{aligned}$$

Ans



(b)

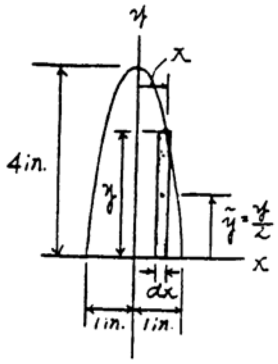
10–15. Determine the moment of inertia of the area about the y axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of dx, and (b) having a thickness of dy.



a)***Differential Element :*** The area of the differential element parallel to yaxis is $dA = y dx = \left(4 - 4x^2\right) dx$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$\begin{aligned} I_y &= \int_A x^2 dA = \int_{-1\text{in.}}^{1\text{in.}} x^2 \left(4 - 4x^2\right) dx \\ &= \left[\frac{4}{3}x^3 - \frac{4}{5}x^5\right]\Big|_{-1\text{in.}}^{1\text{in.}} \\ &= 1.07 \text{ in}^4 \qquad \textbf{Ans} \end{aligned}$$



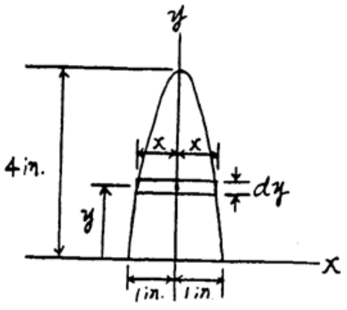
(a)

b)***Differential Element :*** Here, $x = \frac{1}{2}\sqrt{4 - y}$. The moment of inertia of the differential element about y axis is

$$dI_y = \frac{1}{12}(dy)\left(2x\right)^3 = \frac{2}{3}x^3 dy = \frac{1}{12}(4 - y)^{\frac{3}{2}} dy$$

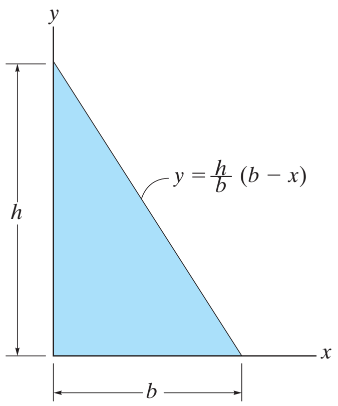
Moment of Inertia : Performing the integration, we have

$$\begin{aligned} I_y &= \int dI_y = \frac{1}{12}\int_0^{4\text{in.}} (4 - y)^{\frac{3}{2}} dy \\ &= \frac{1}{12}\left[-\frac{2}{5}(4 - y)^{\frac{5}{2}}\right]\Big|_0^{4\text{in.}} \\ &= 1.07 \text{ in}^4 \qquad \textbf{Ans} \end{aligned}$$



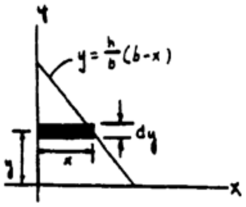
(b)

***10–16.** Determine the moment of inertia of the triangular area about the *x* axis.



Area of the differential element (shaded) $dA = x\,dy$ **where** $x = b - \frac{b}{h}y$**, hence,**
$dA = x\,dy = \left(b - \frac{b}{h}y\right)dy$.

$$I_x = \int_A y^2\,dA = \int_0^h y^2\left(b - \frac{b}{h}y\right)dy$$

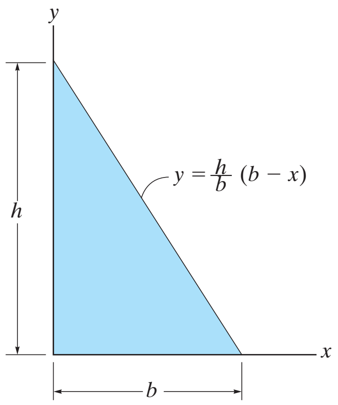


$$= \int_0^h \left(by^2 - \frac{b}{h}y^3\right)dy$$

$$= \frac{b}{3}y^3 - \frac{b}{4h}y^4\Big|_0^h$$

$$= \frac{1}{12}bh^3 \qquad \textbf{Ans}$$

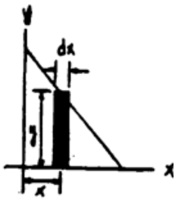
•10–17. Determine the moment of inertia of the triangular area about the *y* axis.



Area of the differential element (shaded) $dA = y\,dx$ **where** $y = h - \frac{h}{b}x$**, hence,**
$dA = y\,dx = \left(h - \frac{h}{b}x\right)dx$.

$$I_y = \int_A x^2\,dA = \int_0^b x^2\left(h - \frac{h}{b}x\right)dx$$

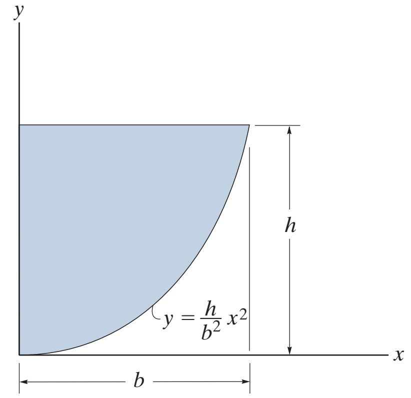
$$= \int_0^b \left(hx^2 - \frac{h}{b}x^3\right)dx$$



$$= \frac{h}{3}x^3 - \frac{h}{4b}x^4\Big|_0^b$$

$$= \frac{1}{12}hb^3 \qquad \textbf{Ans}$$

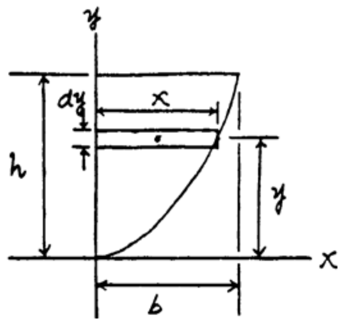
10–18. Determine the moment of inertia of the area about the x axis.



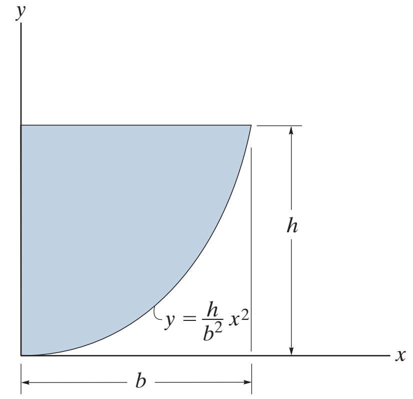
Differential Element : Here, $x = \frac{b}{\sqrt{h}}y^{\frac{1}{2}}$. The area of the differential element parallel to $x$axis is $dA = x\,dy = \left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^h y^2\left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$$
$$= \frac{b}{\sqrt{h}}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Bigg|_0^h$$
$$= \frac{2}{7}bh^3 \qquad \textbf{Ans}$$



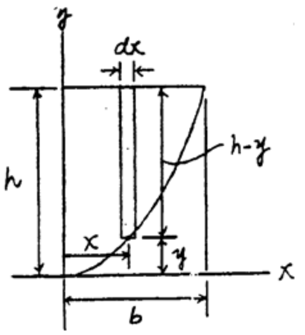
10–19. Determine the moment of inertia of the area about the y axis.



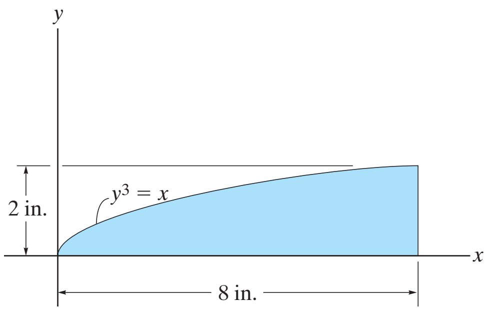
Differential Element : The area of the differential element parallel to $y$axis is $dA = (h - y)\,dx = \left(h - \frac{h}{b^2}x^2\right)dx$.

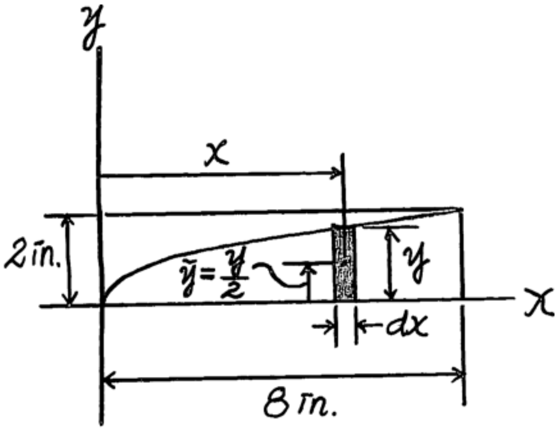
Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^b x^2\left(h - \frac{h}{b^2}x^2\right)dx$$
$$= \left(\frac{h}{3}x^3 - \frac{h}{5b^2}x^5\right)\Bigg|_0^b$$
$$= \frac{2}{15}hb^3 \qquad \textbf{Ans}$$



***10–20.** Determine the moment of inertia of the area about the x axis.





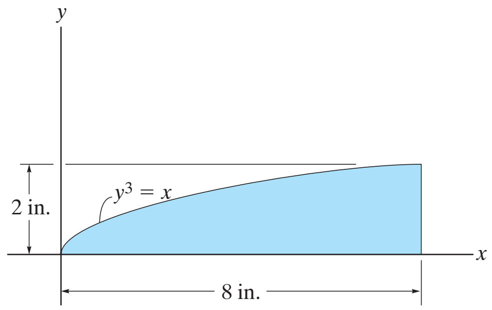
$$dI_x = dI_{\bar{x}'} + dA\,\bar{y}^2$$

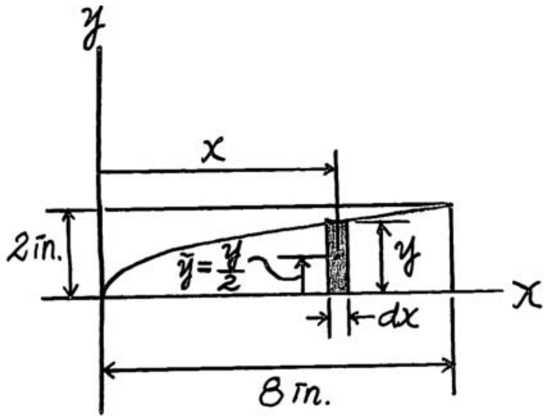
$$= \frac{1}{12} dx\, y^3 + y\, dx \left(\frac{y}{2}\right)^2$$

$$= \frac{1}{3} y^3\, dx$$

$$I_x = \int_A dI_x = \int_0^8 \frac{1}{3} y^3\, dx = \int_0^8 \frac{1}{3} x\, dx = \frac{x^2}{6}\Big]_0^8 = 10.7\ \text{in}^4 \qquad \textbf{Ans}$$

•10–21. Determine the moment of inertia of the area about the *y* axis.





$$dI_y = dI_{\bar{y}} + dA\,\bar{x}^2$$

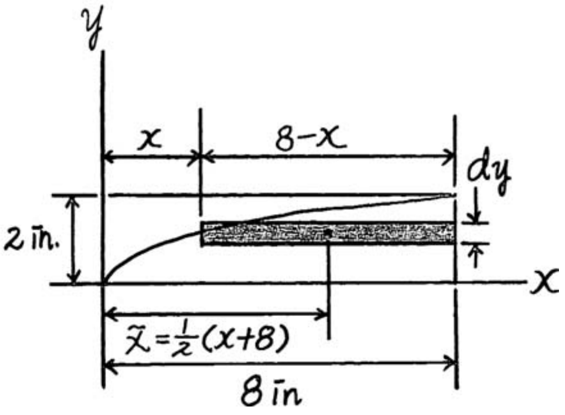
$$= \frac{1}{12}dy\,(8 - y^3)^3 + (8 - y^3)\,dy\left(y^3 + \frac{1}{2}(8 - y^3)\right)^2$$

$$= \left[\frac{1}{12}(8 - y^3)^3 + (8 - y^3)\left(\frac{1}{4}\right)(y^3 + 8)^2\right]dy$$

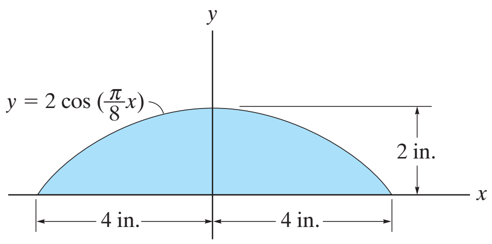
$$I_y = \int_A dI_y = \int_0^2 \left[\frac{1}{12}(8 - y^3)^3 + (8 - y^3)\left(\frac{1}{4}\right)(y^3 + 8)^2\right] dy = 307 \text{ in}^4 \qquad \text{Ans}$$

Also,

$$I_y = \int_A x^2\,dA = \int_A x^2\,y\,dx = \int_0^8 x^{\frac{7}{3}}\,dx = \frac{3}{10}x^{\frac{10}{3}}\Big]_0^8 = 307 \text{ in}^4 \qquad \text{Ans}$$



10–22. Determine the moment of inertia of the area about the x axis.

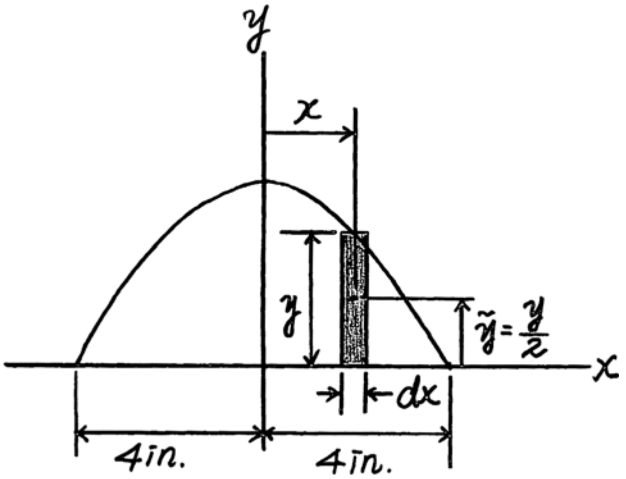


$$dI_x = dI_{\bar{x}'} + dA\,\bar{y}^2$$

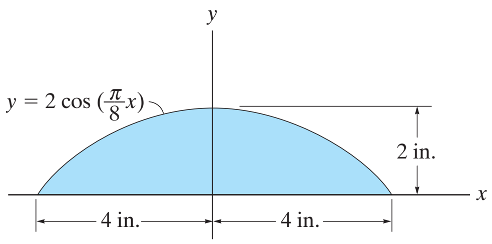
$$= \frac{1}{12}dx\,y^3 + y\,dx\left(\frac{y}{2}\right)^2 = \frac{1}{3}y^3\,dx$$

$$I_x = \int_A dI_x = \int_{-4}^{4} \frac{8}{3}\cos^3\left(\frac{\pi}{8}x\right)dx$$

$$= \frac{8}{3}\left[\frac{\sin\left(\frac{\pi}{8}x\right)}{\frac{\pi}{8}} - \frac{\sin^3\left(\frac{\pi}{8}x\right)}{\frac{3\pi}{8}}\right]_{-4}^{4} = \frac{256}{9\pi} = 9.05\text{ in}^4 \qquad \text{Ans}$$



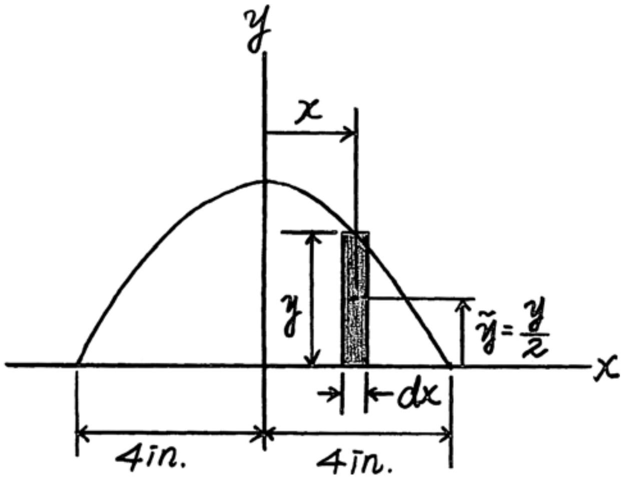
10–23. Determine the moment of inertia of the area about the y axis.



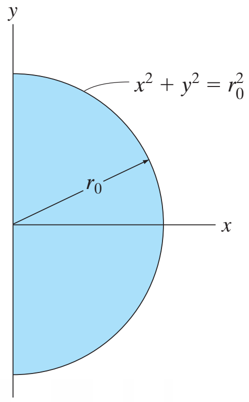
$$I_y = \int_A x^2\,dA = \int_{-4}^{4} x^2\,2\cos\left(\frac{\pi}{8}x\right)dx$$

$$= 2\left[\frac{x^2\sin\left(\frac{\pi}{8}x\right)}{\frac{\pi}{8}} + \frac{2x\cos\left(\frac{\pi}{8}x\right)}{\left(\frac{\pi}{8}\right)^2} - \frac{2\sin\left(\frac{\pi}{8}x\right)}{\left(\frac{\pi}{8}\right)^3}\right]_{-4}^{4}$$

$$= 4\left(\frac{128}{\pi} - \frac{1024}{\pi^3}\right) = 30.9\text{ in}^4 \qquad \text{Ans}$$



***10–24.** Determine the moment of inertia of the area about the x axis.



Differential Element: The area of the differential element shown shaded in Fig. *a* is $dA = \left(r d\theta\right) dr$.

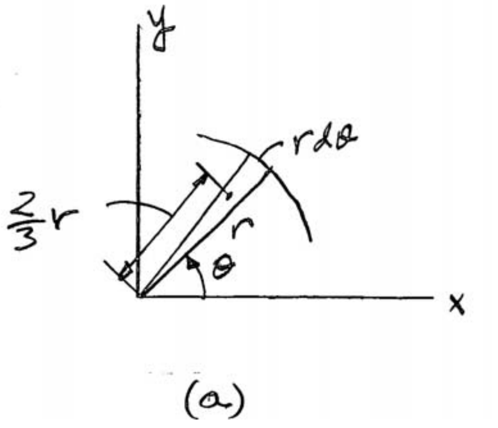
Moment of Inertia:

$$\begin{aligned}
I_x &= \int_A y^2 dA = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^2 \sin^2\theta \,(r d\theta) dr \\
&= \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3 \sin^2\theta \, dr d\theta \\
&= \int_{-\pi/2}^{\pi/2}\left(\frac{r^4}{4}\right)\Bigg|_0^{r_0} \sin^2\theta d\theta \\
&= \int_{-\pi/2}^{\pi/2} \frac{r_0{}^4}{4} \sin^2\theta d\theta
\end{aligned}$$

However, $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$. Thus,

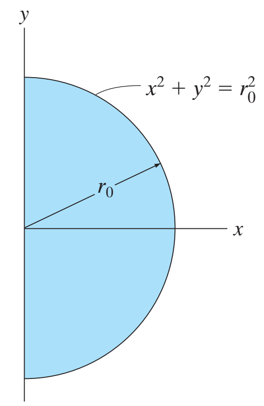
$$\begin{aligned}
I_x &= \int_{-\pi/2}^{\pi/2} \frac{r_0{}^4}{4}(1 - \cos 2\theta) d\theta \\
&= \frac{r_0{}^4}{8}\left[\theta - \frac{1}{2}\sin 2\theta\right]\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi r_0{}^4}{8}
\end{aligned}$$

Ans.



(a)

•10–25. Determine the moment of inertia of the area about the *y* axis.



Differential Element: The area of the differential element shown shaded in Fig. *a* is $dA = (rd\theta)\,dr$.

Moment of Inertia:

$$I_y = \int_A x^2 dA = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^2\cos^2\theta(rd\theta)dr$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3\cos^2\theta drd\theta$$

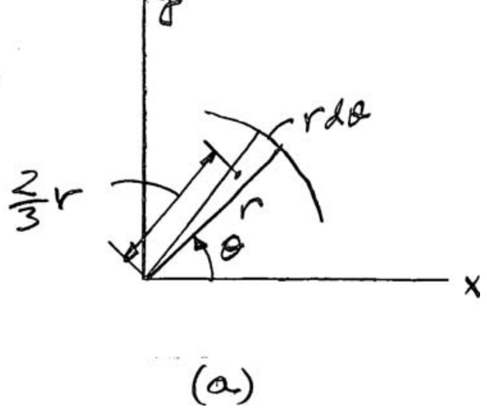
$$= \int_{-\pi/2}^{\pi/2}\left(\frac{r^4}{4}\right)\Bigg|_0^{r_0}\cos^2\theta d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\frac{r_0{}^4}{4}\cos^2\theta d\theta$$

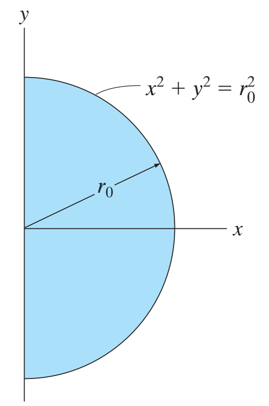
However, $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$. Thus,

$$I_y = \int_{-\pi/2}^{\pi/2}\frac{r_0{}^4}{8}(\cos 2\theta + 1)d\theta$$

$$= \frac{r_0{}^4}{8}\left[\frac{1}{2}\sin 2\theta + \theta\right]\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi r_0{}^4}{8}$$ **Ans.**

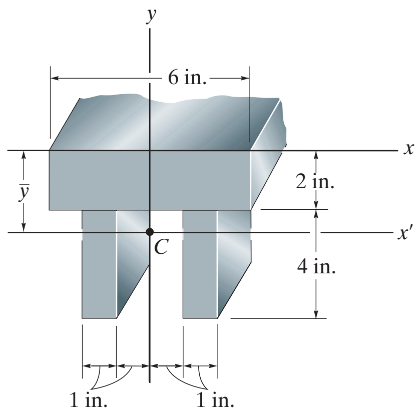


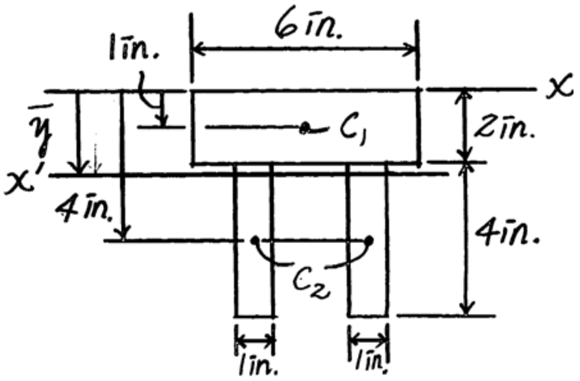
10–26. Determine the polar moment of inertia of the area about the z axis passing through point O.



$$J_0 = I_x + I_y = \frac{\pi r_0^4}{8} + \frac{\pi r_0^4}{8} = \frac{\pi r_0^4}{4}$$

10–27. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x' axis.





Centroid :

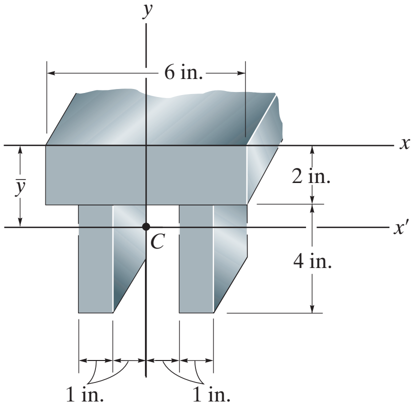
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{1(6)(2) + 2\left[4(4)(1)\right]}{6(2) + 2\left[4(1)\right]} = 2.20 \text{ in.} \qquad \text{Ans}$$

Moment inertia :

$$I_{x'} = \frac{1}{12}(6)(2)^3 + 6(2)(2.20 - 1)^2 + 2\left[\frac{1}{12}(1)(4)^3 + 1(4)(4 - 2.20)^2\right]$$

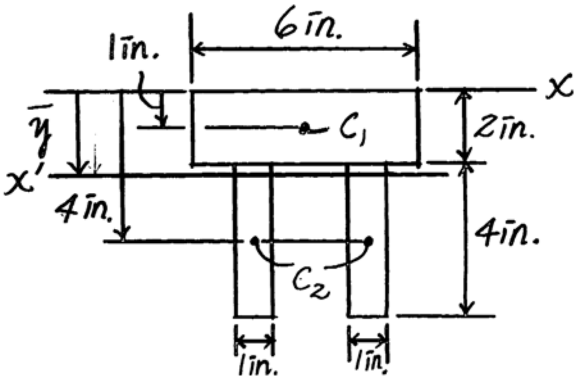
$$= 57.9 \text{ in}^4 \qquad \text{Ans}$$

***10–28.** Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.

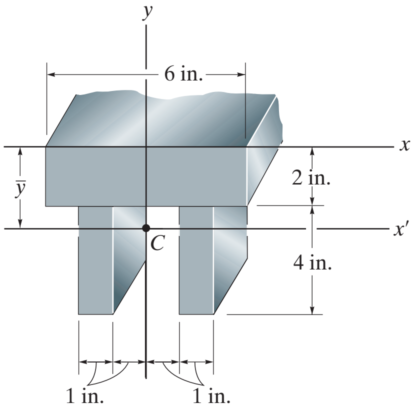


$$I_x = \left[\frac{1}{12}(6)(2)^3 + (6)(2)(1)^2\right] + 2\left[\frac{1}{12}(1)(4)^3 + (4)(1)(4)^2\right]$$

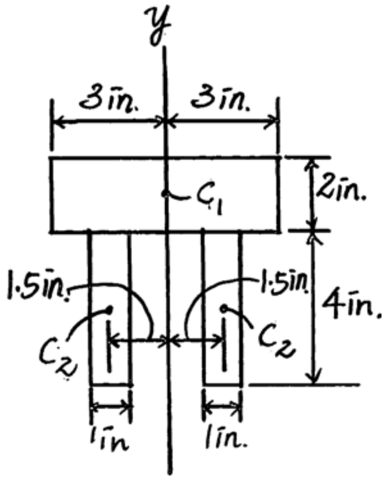
$$= 155 \text{ in.}^4 \quad \textbf{Ans}$$



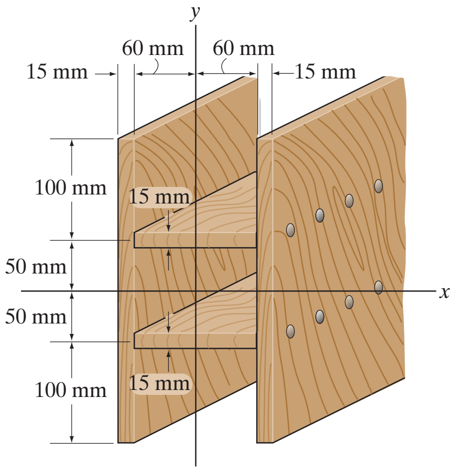
•10–29. Determine the moment of inertia of the beam's cross-sectional area about the *y* axis.



$$I_y = \frac{1}{12}(2)(6)^3 + 2\left[\frac{1}{12}(4)(1)^3 + 1(4)(1.5)^2\right] = 54.7 \text{ in}^4 \qquad \textbf{Ans}$$



10–30. Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.



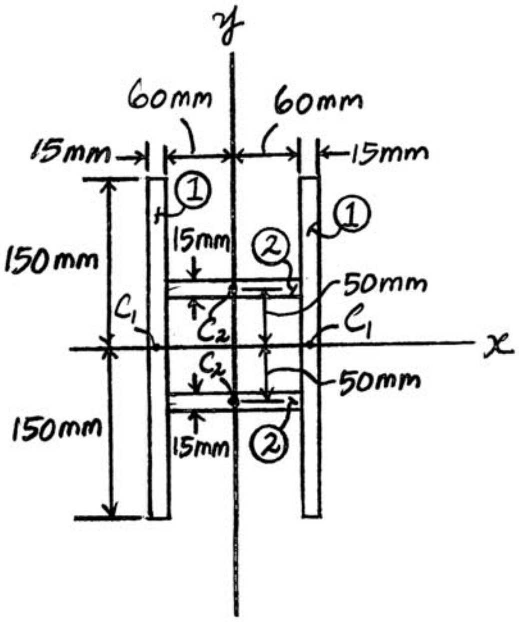
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the *x* axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the *x* axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

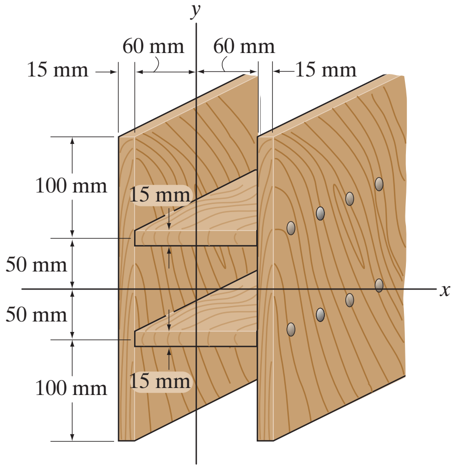
$$= \left[2\left(\frac{1}{12}(15)(300^3)\right) + 2(15)(300)(0)^2\right] + \left[2\left(\frac{1}{12}(120)(15^3)\right) + 2(120)(15)(50)^2\right]$$

$$= 67.5(10^6) + 9.0675(10^6) = 76.6(10^6)\ \text{mm}^4$$ **Ans.**



(a)

10–31. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



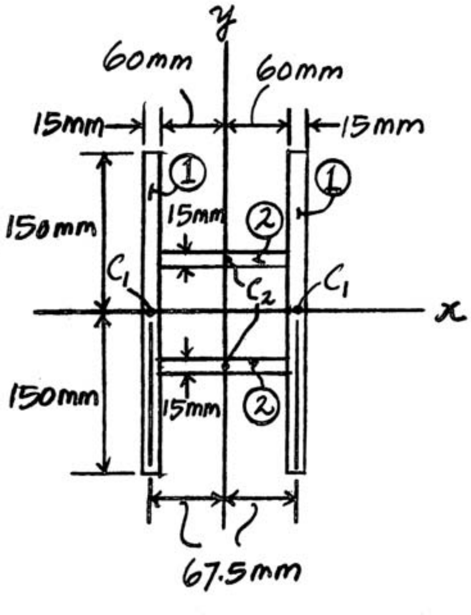
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

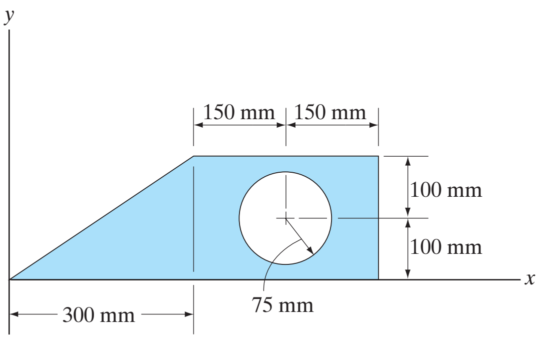
$$= \left[2\left(\frac{1}{12}(300)(15^3)\right) + 2(300)(15)(67.5)^2\right] + \left[2\left(\frac{1}{12}(15)(120^3)\right) + 2(120)(15)(0)^2\right]$$

$$= 41.175(10^6) + 4.32(10^6) = 45.5(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$



(a)

***10–32.** Determine the moment of inertia of the composite area about the x axis.



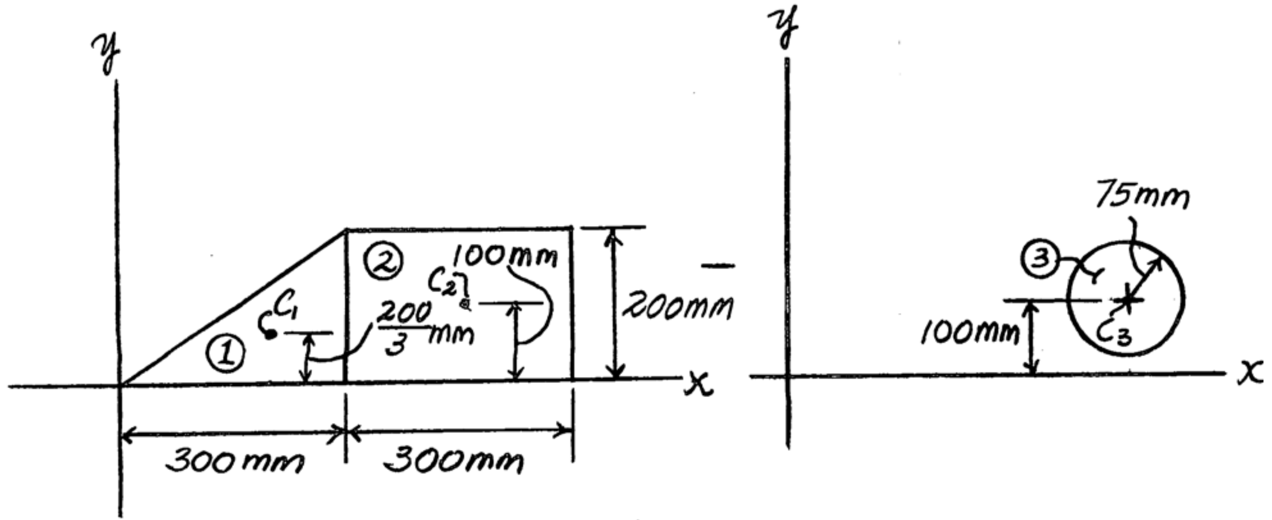
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

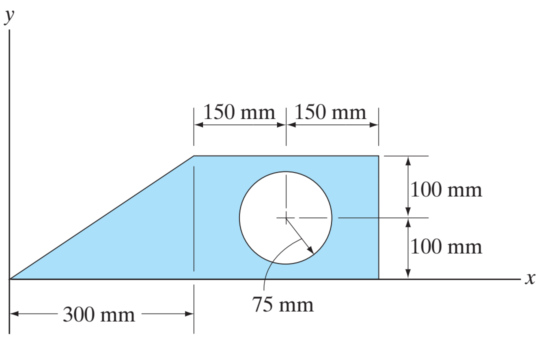
$$= \left[\frac{1}{36}(300)(200^3) + \frac{1}{2}(300)(200)\left(\frac{200}{3}\right)^2\right] + \left[\frac{1}{12}(300)(200^3) + 300(200)(100)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(100)^2\right]$$

$$= 798(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

•10–33. Determine the moment of inertia of the composite area about the y axis.



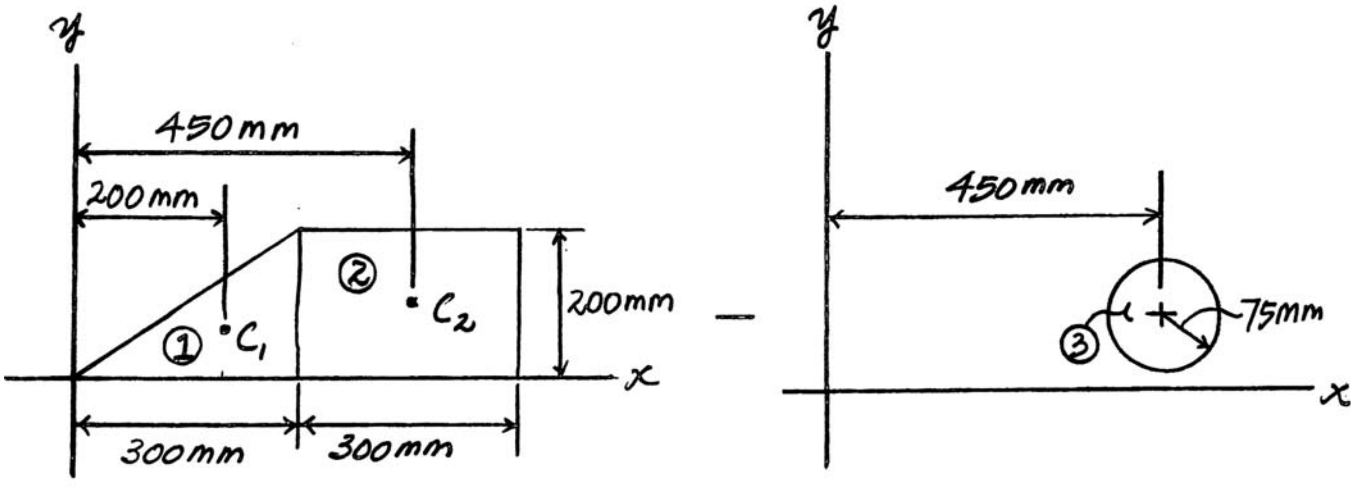
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. a. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

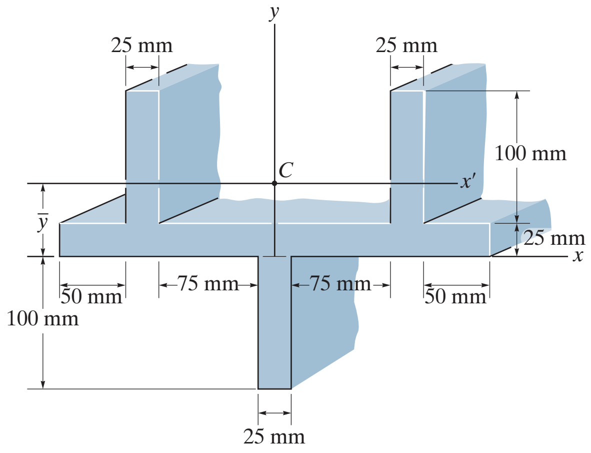
$$= \left[\frac{1}{36}(200)(300^3) + \frac{1}{2}(200)(300)(200)^2\right] + \left[\frac{1}{12}(200)(300^3) + 200(300)(450)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(450)^2\right]$$

$$= 10.3(10^9)\ \text{mm}^4$$ **Ans.**



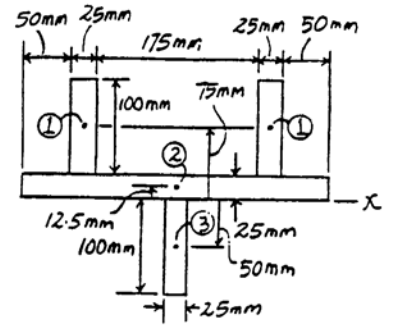
(a)

10–34. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then determine the moment of inertia about the x' axis.



Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	$A\,(\text{mm}^2)$	$\tilde{y}\,(\text{mm})$	$\tilde{y}A\,(\text{mm}^3)$
1	$50(100)$	75	$375(10^3)$
2	$325(25)$	12.5	$101.5625(10^3)$
3	$25(100)$	-50	$-125(10^3)$
Σ	$15.625(10^3)$		$351.5625(10^3)$

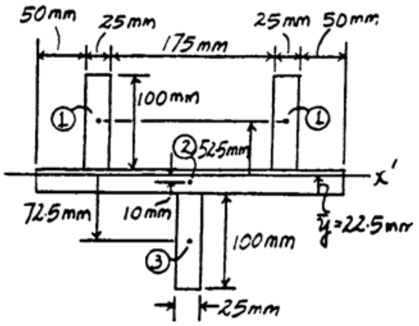


Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{351.5625(10^3)}{15.625(10^3)} = 22.5\text{ mm} \qquad \textbf{Ans}$$

Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

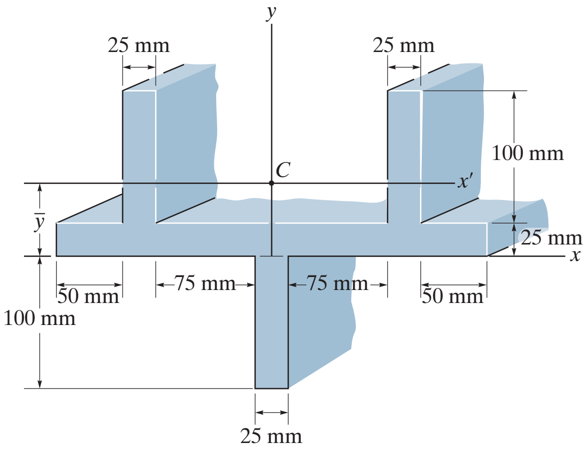
Segment	$A_i\,(\text{mm}^2)$	$(d_y)_i\,(\text{mm})$	$(\bar{I}_{x'})_i\,(\text{mm}^4)$	$(Ad_y^2)_i\,(\text{mm}^4)$	$(I_{x'})_i\,(\text{mm}^4)$
1	$50(100)$	52.5	$\frac{1}{12}(50)(100^3)$	$13.781(10^6)$	$17.948(10^6)$
2	$325(25)$	10	$\frac{1}{12}(325)(25^3)$	$0.8125(10^6)$	$1.236(10^6)$
3	$25(100)$	72.5	$\frac{1}{12}(25)(100^3)$	$13.141(10^6)$	$15.224(10^6)$



Thus,

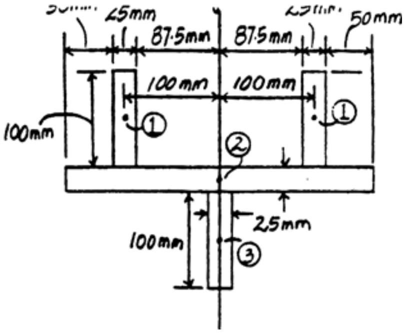
$$I_{x'} = \Sigma(I_{x'})_i = 34.41\left(10^6\right)\text{ mm}^4 = 34.4\left(10^6\right)\text{ mm}^4 \qquad \textbf{Ans}$$

10–35. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



Moment of Inertia : The moment of inertia about the y' axis for each segment can be determined using the parallel – axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

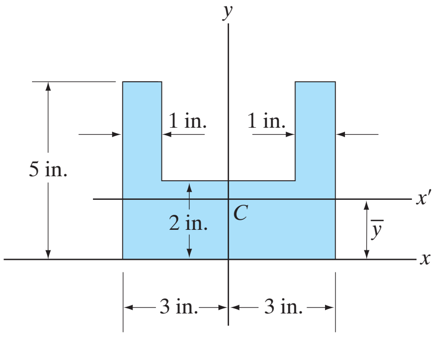
Segment	$A_i\ (\text{mm}^2)$	$(d_x)_i\ (\text{mm})$	$(\bar{I}_{y'})_i\ (\text{mm}^4)$	$(Ad_x^2)_i\ (\text{mm}^4)$	$(I_{y'})_i\ (\text{mm}^4)$
1	$2[100(25)]$	100	$2\left[\frac{1}{12}(100)(25^3)\right]$	$50.0(10^6)$	$50.130(10^6)$
2	$25(325)$	0	$\frac{1}{12}(25)(325^3)$	0	$71.519(10^6)$
3	$100(25)$	0	$\frac{1}{12}(100)(25^3)$	0	$0.130(10^6)$



Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 121.78(10^6)\ \text{mm}^4 = 122(10^6)\ \text{mm}^4 \quad \textbf{Ans}$$

***10–36.** Locate the centroid $\bar{y}$ of the composite area, then determine the moment of inertia of this area about the centroidal x' axis.



Composite Parts: The composite area can be subdivided into three segments. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Centroid: The perpendicular distances measured from the centroid of each segment to the x axis are indicated in Fig. a.

$$\bar{y} = \frac{\Sigma \tilde{y}_C A}{\Sigma A} = \frac{(1)(6)(2) + 2[3.5(3)(1)]}{(6)(2) + 2[(3)(1)]} = 1.833\ \text{in.} = 1.83\ \text{in.} \qquad \textbf{Ans.}$$

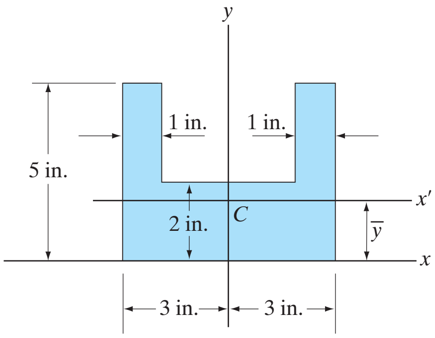
Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. Thus,

$$I_{x'} = \bar{I}_x + A(d_y)^2$$

$$= \left[\frac{1}{12}(6)(2^3) + 6(2)(1.833 - 1)^2\right] + 2\left[\frac{1}{12}(1)(3^3) + 1(3)(3.5 - 1.833)^2\right]$$

$$= 33.5\ \text{in}^4 \qquad \textbf{Ans.}$$

•10–37. Determine the moment of inertia of the composite area about the centroidal y axis.



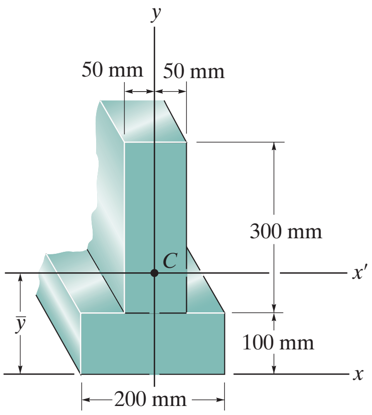
Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_y + A(d_x)^2$$

$$= \left[\frac{1}{12}(2)(6^3)\right] + 2\left[\frac{1}{12}(3)(1^3) + 3(1)(2.5)^2\right]$$

$$= 74 \text{ in}^4 \qquad \textbf{Ans.}$$

10–38. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x' axis.



Centroid :

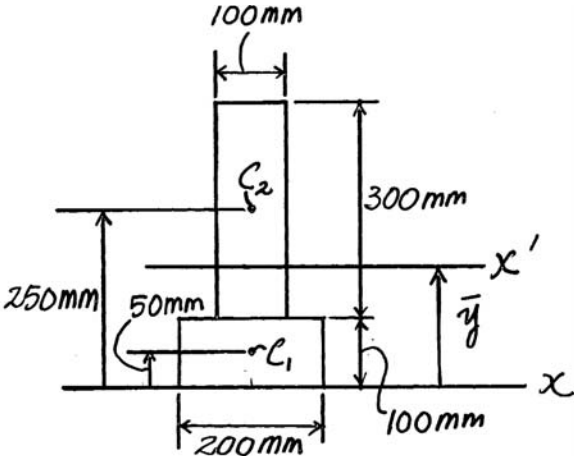
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{50(100)(200) + 250(100)(300)}{100(200) + 100(300)} = 170 \text{ mm} \qquad \textbf{Ans}$$

Moment of inertia :

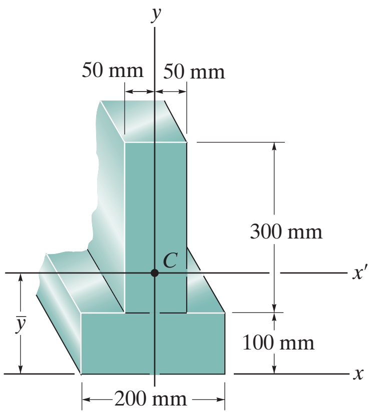
$$I_{x'} = \frac{1}{12}(200)(100)^3 + 200(100)(170-50)^2$$

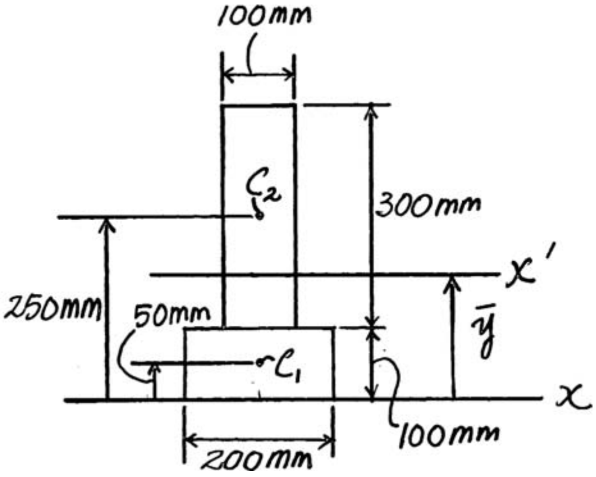
$$+ \frac{1}{12}(100)(300)^3 + 100(300)(250-170)^2$$

$$= 722(10)^6 \text{ mm}^4 \qquad \textbf{Ans}$$



10–39. Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.

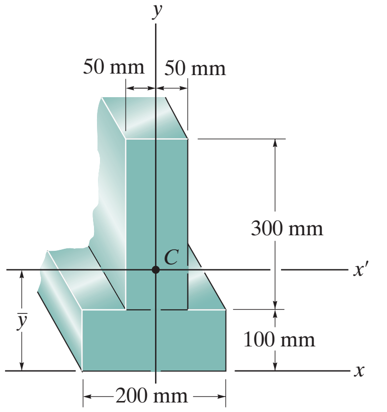


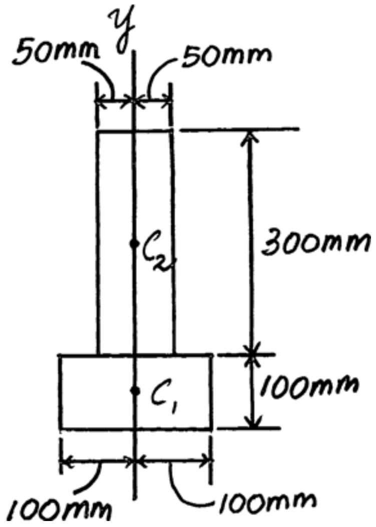


$$I_x = \left[\frac{1}{12}(0.2)(0.1)^3 + (0.2)(0.1)(0.05)^2\right]$$

$$+\left[\frac{1}{12}(0.1)(0.3)^3 + (0.1)(0.3)(0.25)^2\right] = 2.17(10^{-3})\text{ m}^4 \qquad \textbf{Ans}$$

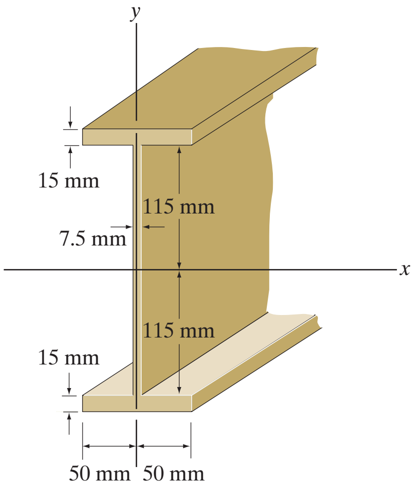
***10–40.** Determine the moment of inertia of the beam's cross-sectional area about the y axis.





$$I_y = \frac{1}{12}(100)(200)^3 + \frac{1}{12}(300)(100)^3 = 91.7(10)^6 \text{ mm}^4 \qquad \text{Ans}$$

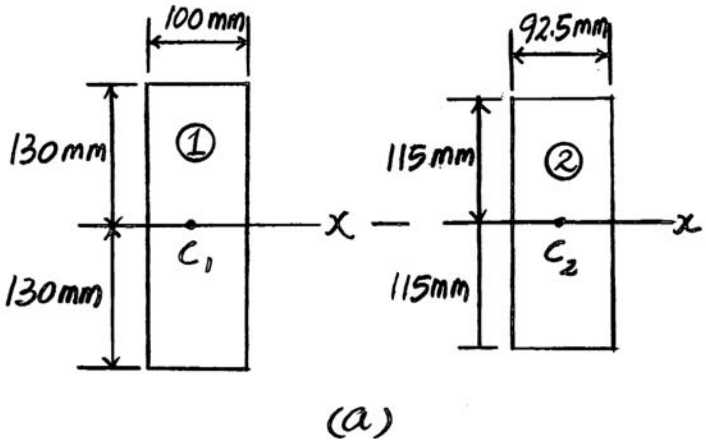
•10–41. Determine the moment of inertia of the beam's cross-sectional area about the x axis.



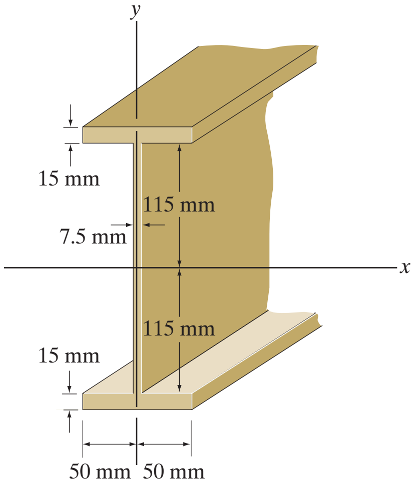
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. *a*. Here, segment (2) is a hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$
$$= \frac{1}{12}(100)(260^3) - \frac{1}{12}(92.5)(230^3)$$
$$= 52.7(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



10–42. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



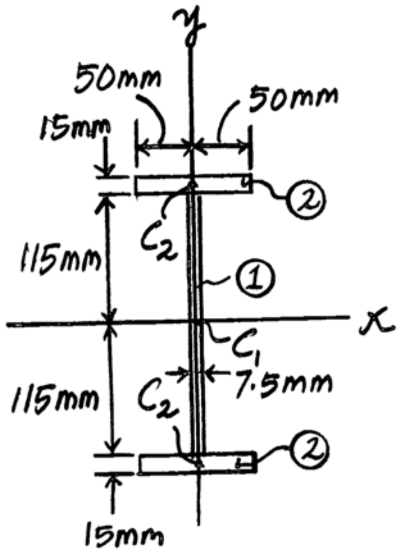
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two similar segments (2) and one segment (1) as shown in Fig. *a*. The location of the centroid of each segment is also indicated.

Moment of Inertia: Since the y axis passes through the centroid of each segment,

$$I_y = \Sigma\left(I_y\right)_i$$

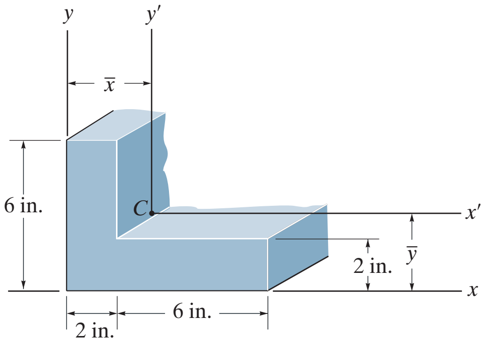
$$= 2\left[\frac{1}{12}(15)(100^3)\right] + \frac{1}{12}(230)(7.5^3)$$

$$= 2.51(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$



(a)

10–43. Locate the centroid $\bar{y}$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{x'}$ about the x' centroidal axis.

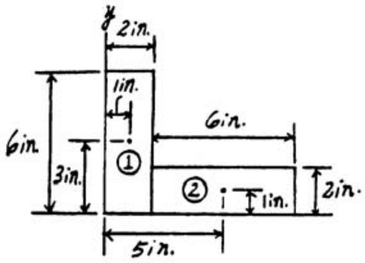


Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (in²)	$\tilde{y}$ (in.)	$\tilde{y}A$ (in³)
1	6(2)	3	36.0
2	6(2)	1	12.0
Σ	24.0		48.0

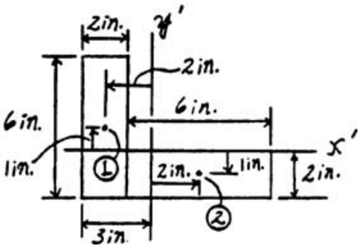
Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{48.0}{24.0} = 2.00 \text{ in.} \qquad \textbf{Ans}$$



Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

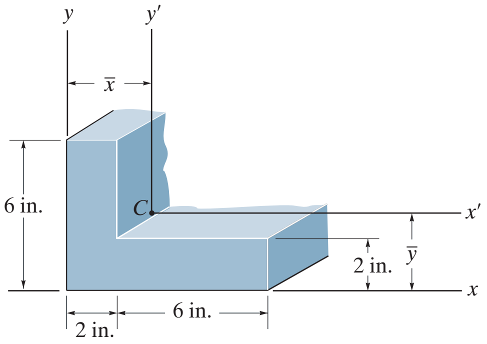
Segment	A_i (in²)	$(d_y)_i$ (in.)	$(\bar{I}_{x'})_i$ (in⁴)	$(Ad_y^2)_i$ (in⁴)	$(I_{x'})_i$ (in⁴)
1	2(6)	1	$\frac{1}{12}(2)(6^3)$	12.0	48.0
2	6(2)	1	$\frac{1}{12}(6)(2^3)$	12.0	16.0



Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 64.0 \text{ in}^4 \qquad \textbf{Ans}$$

***10–44.** Locate the centroid $\bar{x}$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{y'}$ about the y' centroidal axis.



Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (in²)	$\tilde{x}$ (in.)	$\tilde{x}A$ (in³)
1	6(2)	1	12.0
2	6(2)	5	60.0
Σ	24.0		72.0

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{72.0}{24.0} = 3.00 \text{ in.} \qquad \textbf{Ans}$$

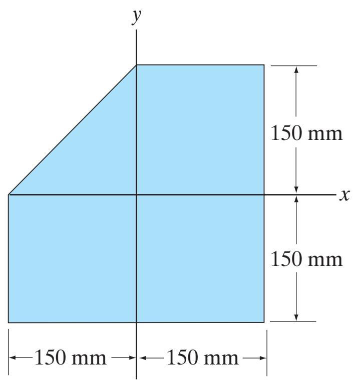
Moment of Inertia : The moment of inertia about the y' axis for each segmentcan be determined using the parallel – axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

Segment	A_i (in²)	$(d_x)_i$ (in.)	$(\bar{I}_{y'})_i$ (in⁴)	$(Ad_x^2)_i$ (in⁴)	$(I_{y'})_i$ (in⁴)
1	6(2)	2	$\frac{1}{12}(6)(2^3)$	48.0	52.0
2	2(6)	2	$\frac{1}{12}(2)(6^3)$	48.0	84.0

Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 136 \text{ in}^4 \qquad \textbf{Ans}$$

•10–45. Determine the moment of inertia of the composite area about the *x* axis.



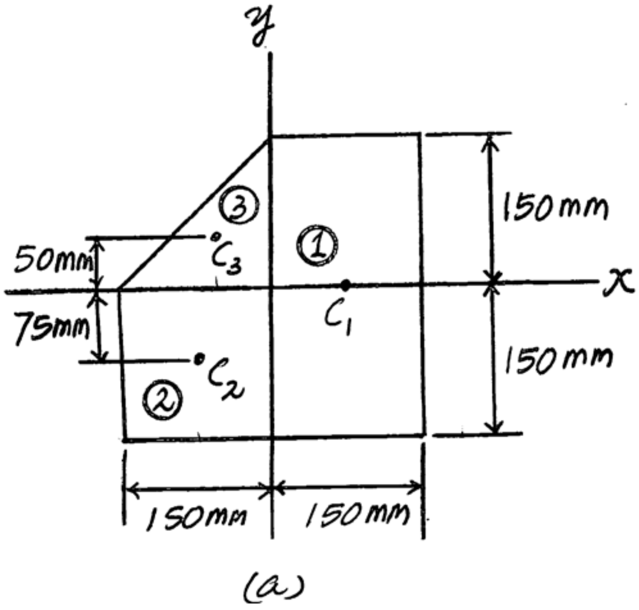
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the *x* axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the *x* axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

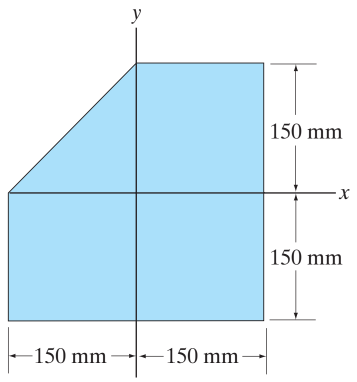
$$= \left[\frac{1}{12}(150)(300^3) + 300(150)(0)^2\right] + \left[\frac{1}{12}(150)(150^3) + 150(150)(75)^2\right] + \left[\frac{1}{36}(150)(150^3) + \frac{1}{2}(150)(150)(50)^2\right]$$

$$= 548(10^6)\text{ mm}^4$$ **Ans.**



(a)

10–46. Determine the moment of inertia of the composite area about the y axis.



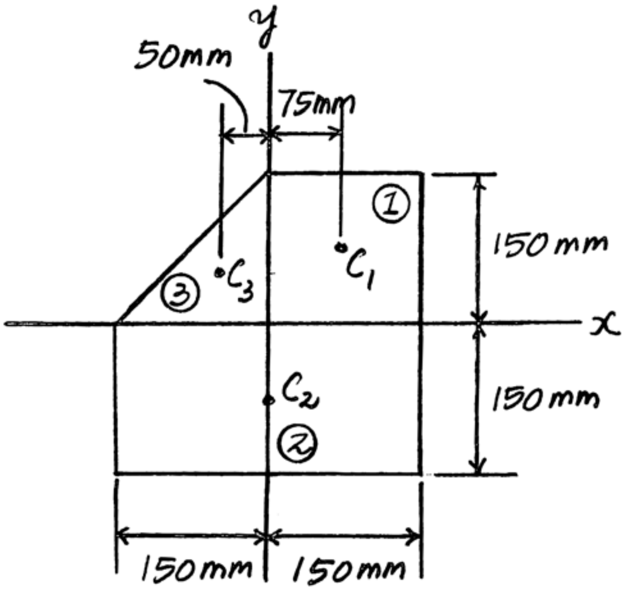
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

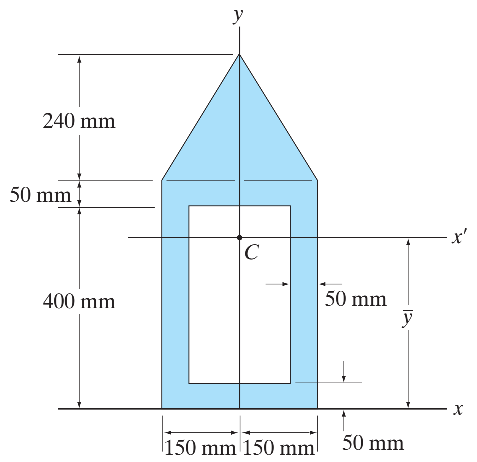
$$= \left[\frac{1}{12}(150)(150^3) + 150(150)(75)^2\right] + \left[\frac{1}{12}(150)(300^3) + 15(300)(0)^2\right] + \left[\frac{1}{36}(150)(150^3) + \frac{1}{2}(150)(150)(50)^2\right]$$

$$= 548(10^6)\ \text{mm}^4$$ **Ans.**



(a)

10–47. Determine the moment of inertia of the composite area about the centroidal y axis.



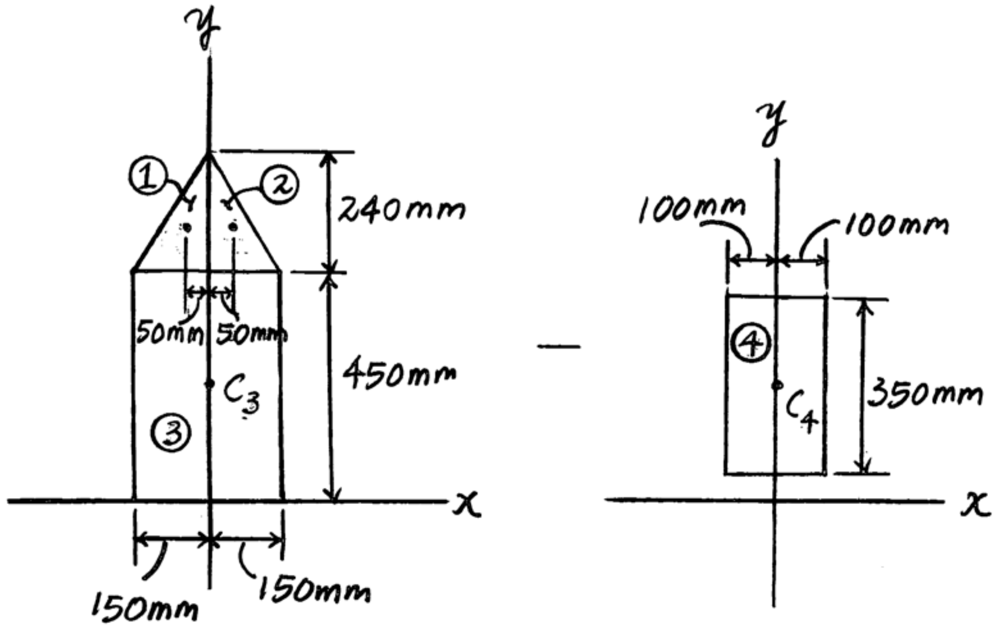
Composite Parts: The composite area can be subdivided into four segments as shown in Fig. a. Since segment (4) is a hole, it contributes a negative moment of inertia. The location of the centroid for each segment is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

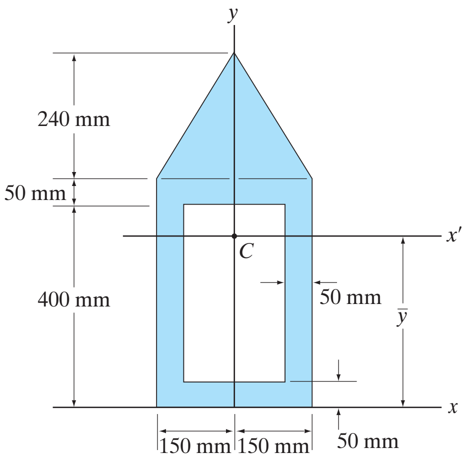
$$= \left[2\left(\frac{1}{36}(240)(150^3)\right) + 2\left(\frac{1}{2}(240)(150)\right)(50)^2\right] + \left[\frac{1}{12}(450)(300^3) + 450(300)(0)^2\right] + \left[-\frac{1}{12}(350)(200^3) + (-350(200))(0)^2\right]$$

$$= 914(10^6)\ \text{mm}^4$$ **Ans.**



(a)

***10–48.** Locate the centroid $\bar{y}$ of the composite area, then determine the moment of inertia of this area about the x' axis.



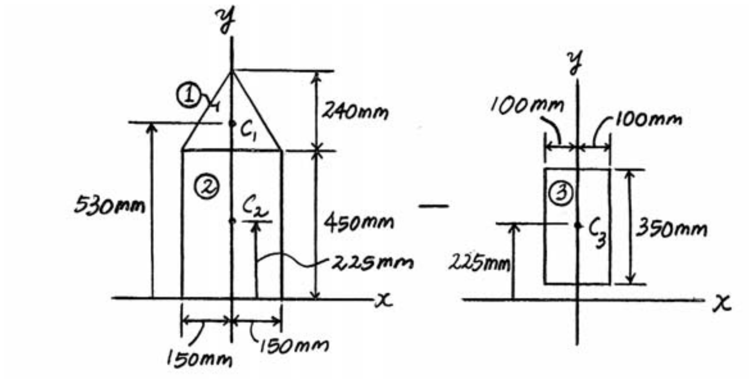
Composite Parts: The composite area can be subdivided into three segments as shown in Figs. *a* and *b*. Since segment (3) is a hole, it should be considered a negative part.

Centroid: The perpendicular distances measured from the centroid of each segment to the *x* axis are indicated in Fig. *a*.

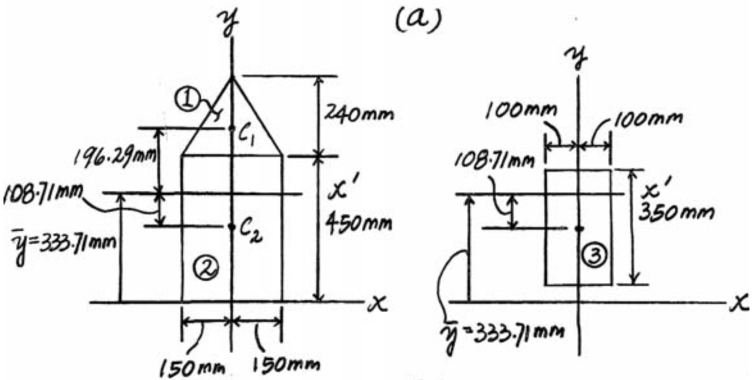
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{530\left(\frac{1}{2}(300)(240)\right) + 225(300(450)) + 225(-200(350))}{\frac{1}{2}(300)(240) + 300(450) - 200(350)} = \frac{33.705(10^6)}{101(10^3)} = 333.71\text{ mm} = 334\text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each segment to the x' axis is indicated in Fig. *b*.

$$\begin{aligned} I_{x'} &= \bar{I}_{x'} + A(d_{x'})^2 \\ &= \left[\frac{1}{36}(300)(240^3) + \frac{1}{2}(300)(240)(196.29)^2\right] + \left[\frac{1}{12}(300)(450^3) + 300(450)(108.71)^2\right] \\ &\quad + \left[-\frac{1}{12}(200)(350^3) + (-200(350))(108.71)^2\right] \\ &= 3.83(10^9)\text{ mm}^4 \qquad \textbf{Ans.} \end{aligned}$$

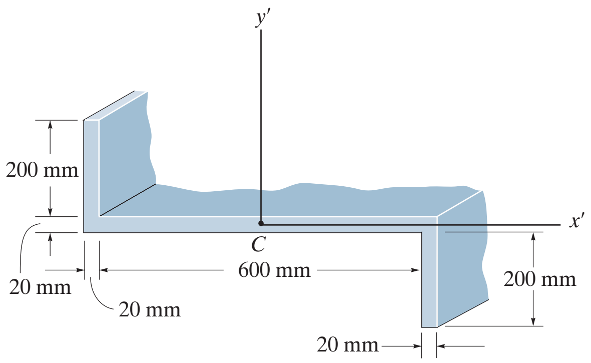


(a)



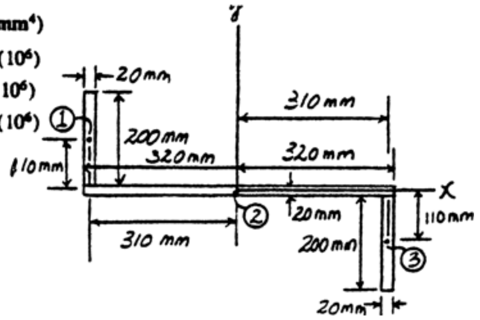
(b)

•10–49. Determine the moment of inertia $I_{x'}$ of the section. The origin of coordinates is at the centroid C.



Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

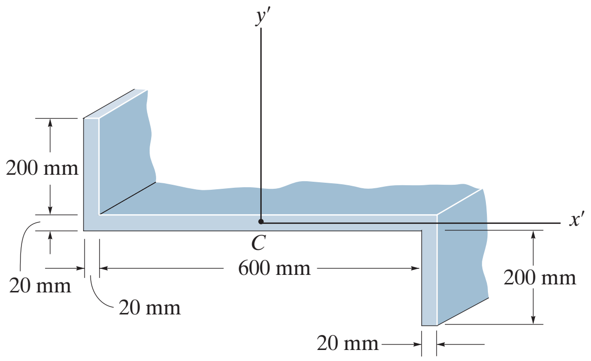
Segment	A_i (mm²)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm⁴)	$(Ad_y^2)_i$ (mm⁴)	$(I_{x'})_i$ (mm⁴)
1	200(20)	110	$\frac{1}{12}(20)(200^3)$	$48.4(10^6)$	$61.733(10^6)$
2	640(20)	0	$\frac{1}{12}(640)(20^3)$	0	$0.427(10^6)$
3	200(20)	110	$\frac{1}{12}(20)(200^3)$	$48.4(10^6)$	$61.733(10^6)$



Thus,

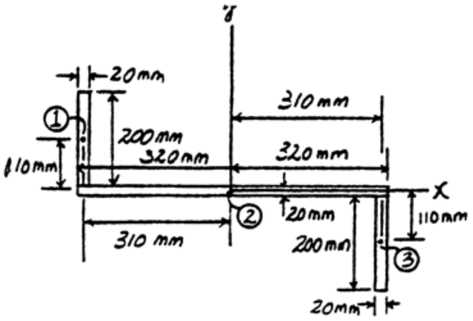
$$I_{x'} = \Sigma(I_{x'})_i = 123.89(10^6)\ \text{mm}^4 = 124(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

10–50. Determine the moment of inertia $I_{y'}$ of the section. The origin of coordinates is at the centroid C.



Moment of Inertia : The moment of inertia about the y' axis for each segment can be determined using the parallel – axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

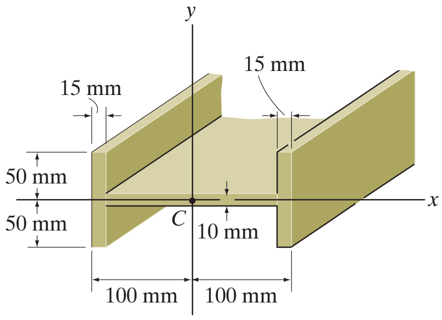
Segment	A_i (mm²)	$(d_x)_i$ (mm)	$(\bar{I}_{y'})_i$ (mm⁴)	$(Ad_x^2)_i$ (mm⁴)	$(I_{y'})_i$ (mm⁴)
1	200(20)	310	$\frac{1}{12}(200)(20^3)$	$384.4(10^6)$	$384.53(10^6)$
2	640(20)	0	$\frac{1}{12}(20)(640^3)$	0	$436.91(10^6)$
3	200(20)	310	$\frac{1}{12}(200)(20^3)$	$384.4(10^6)$	$384.53(10^6)$



Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 1.206(10^9)\ \text{mm}^4 = 1.21(10^9)\ \text{mm}^4 \qquad \textbf{Ans}$$

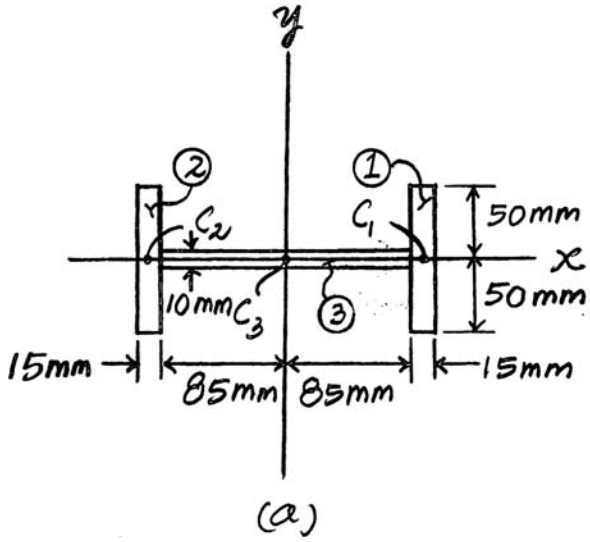
10–51. Determine the beam's moment of inertia I_x about the centroidal x axis.



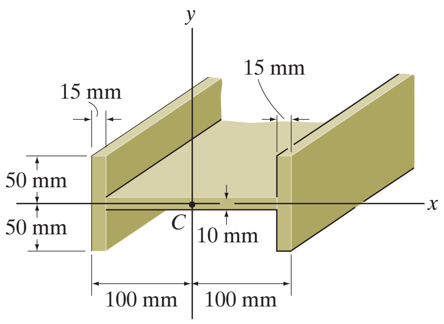
Composite Parts: The composite cross - sectioned area of the beam can be subdivided into three segments as shown in Fig. *a*. The locations of the centroid for each segment is also indicated.

Moment of Inertia: Since the centroid of each segment is located about the x axis then

$$I_x = \frac{1}{12}(15)(100^3) + \frac{1}{12}(15)(100^3) + \frac{1}{12}(170)(10^3)$$
$$= 2.51(10^6)\,\text{mm}^4$$ **Ans.**



***10–52.** Determine the beam's moment of inertia I_y about the centroidal y axis.



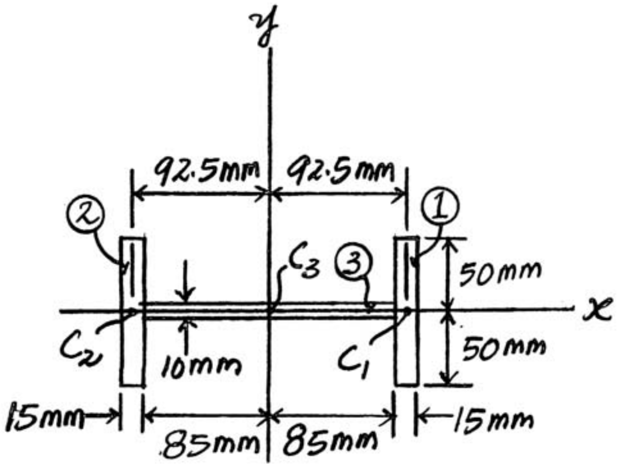
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

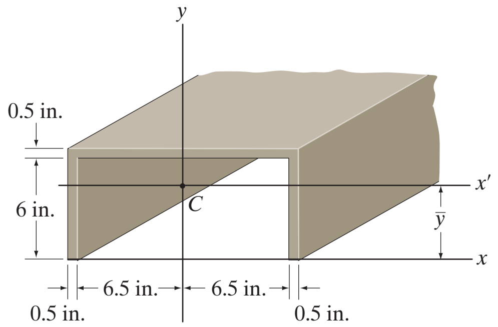
$$= \left[\frac{1}{12}(100)(15^3) + 100(15)(92.5)^2\right] + \left[\frac{1}{12}(100)(15^3) + 100(15)(92.5)^2\right] + \left[\frac{1}{12}(10)(170^3) + 170(10)(0)^2\right]$$

$$= 29.8(10^6)\ \text{mm}^4 \quad \textbf{Ans.}$$



(a)

•10–53. Locate the centroid $\bar{y}$ of the channel's cross-sectional area, then determine the moment of inertia of the area about the centroidal x' axis.



Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Figs. *a* and *b*.

Centroid: The perpendicular distances measured from the centroid of each type of segment to the x axis are also indicated in Fig. *a*. Thus,

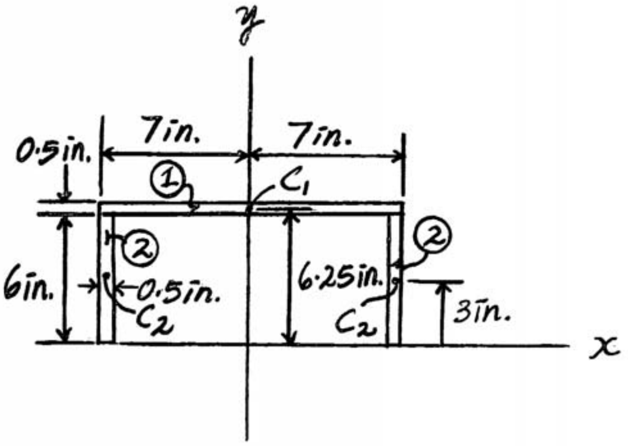
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{6.25(14(0.5)) + (3)(2)(6)(0.5)}{14(0.5) + (2)(6)(0.5)} = \frac{61.75}{13} = 4.75 \text{ in.} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each type of segment to the x' axis is indicated in Fig. *b*.

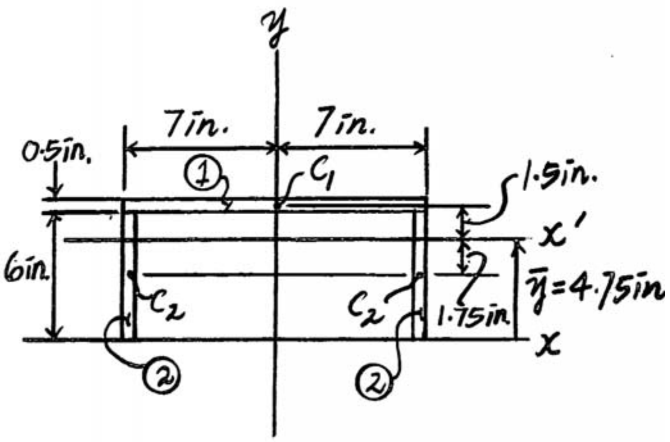
$$I_{x'} = \bar{I}_{x'} + A(d_{y'})^2$$

$$= \left[\frac{1}{12}(14)(0.5^3) + 14(0.5)(1.5)^2\right] + \left[2\left(\frac{1}{12}(0.5)(6^3)\right) + 2(6)(0.5)(1.75)^2\right]$$

$$= 15.896 + 36.375 = 52.3 \text{ in}^4 \qquad \textbf{Ans.}$$

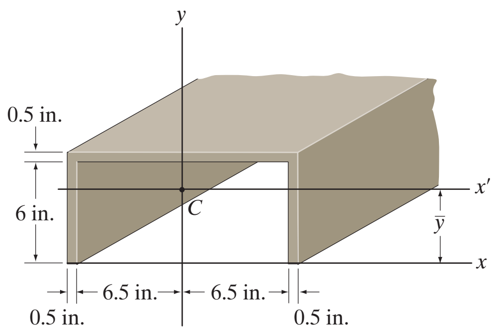


(a)



(b)

10–54. Determine the moment of inertia of the area of the channel about the y axis.



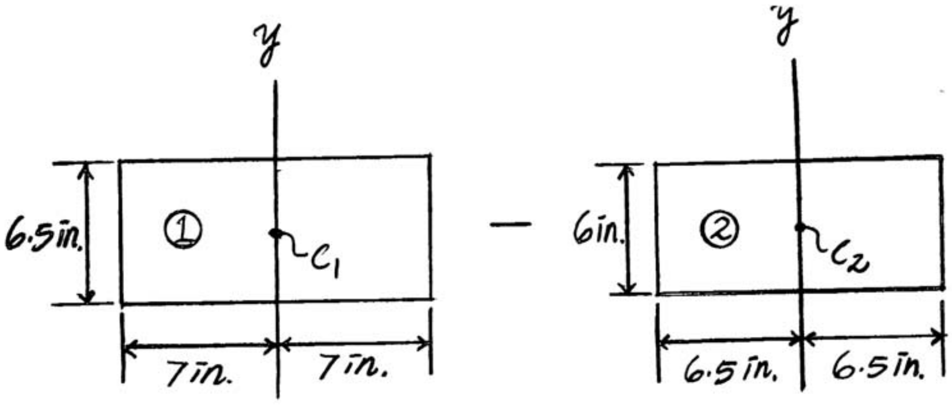
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. a. Here, segment (2) is a hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$

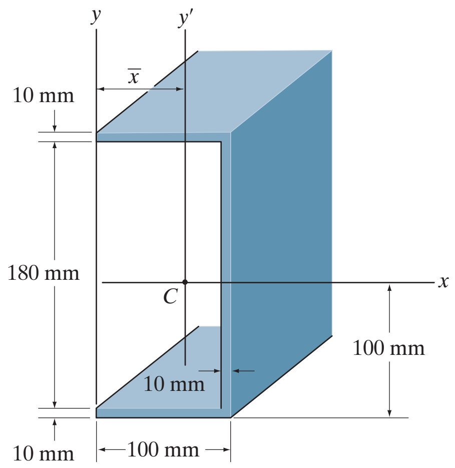
$$= \frac{1}{12}(6.5)(14^3) - \frac{1}{12}(6)(13^3)$$

$$= 388 \text{ in}^4 \qquad \textbf{Ans.}$$



(a)

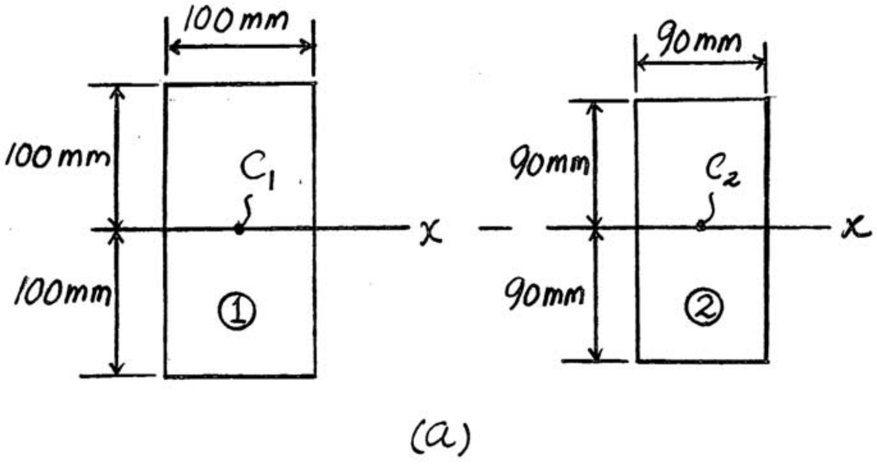
10–55. Determine the moment of inertia of the cross-sectional area about the x axis.



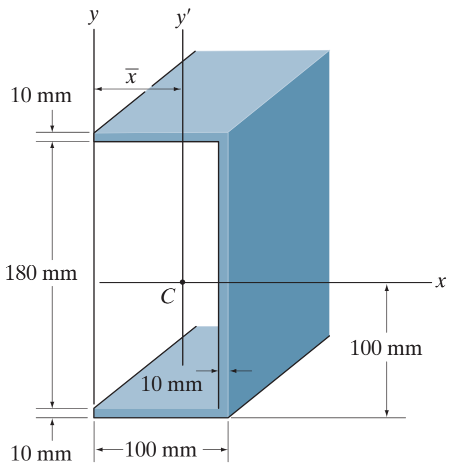
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. *a*. Here, segment (2) is a hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$
$$= \frac{1}{12}(100)(200^3) - \frac{1}{12}(90)(180^3)$$
$$= 22.9(10^6)\ \text{mm}^4$$ **Ans.**



***10–56.** Locate the centroid $\bar{x}$ of the beam's cross-sectional area, and then determine the moment of inertia of the area about the centroidal y' axis.



Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*.

Centroid: The perpendicular distance measured from the centroid of each type of segment to the *y* axis is also indicated in Fig. *a*. Thus,

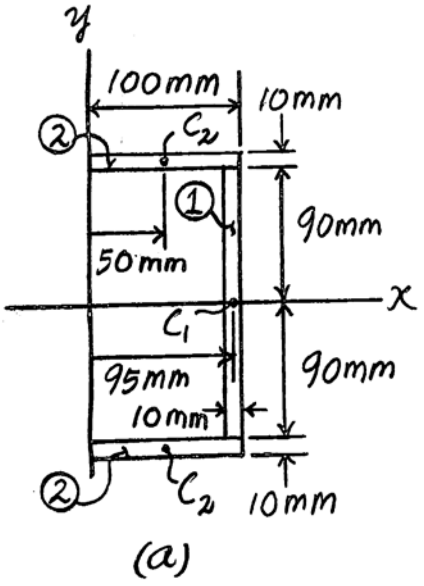
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{95(10(180)) + 50(2(100)(10))}{10(180) + 2(100)(10)} = \frac{271(10^3)}{3.8(10^3)} = 71.32 \text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia of each segment about the y' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each type of segment to the y' axis is indicated in Fig. *b*.

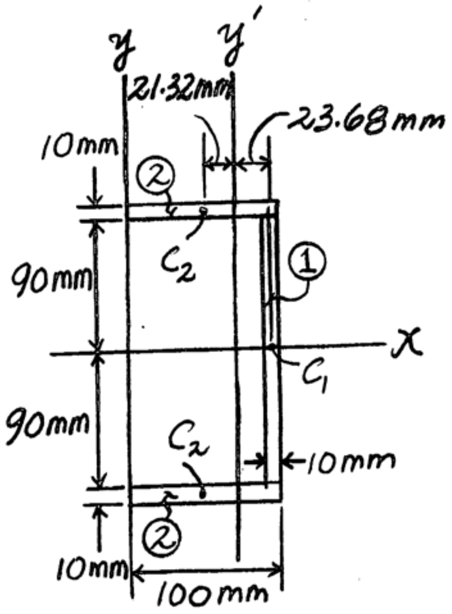
$$I_{y'} = \bar{I}_{y'} + A(d_x{}')^2$$

$$= \left[\frac{1}{12}(180)(10^3) + 180(10)(23.68)^2\right] + \left[2\left(\frac{1}{12}(10)(100^3)\right) + 2(100)(10)(21.32)^2\right]$$

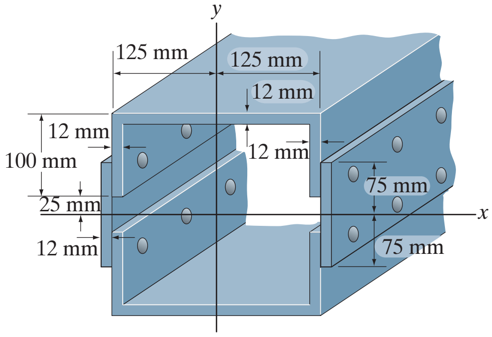
$$= 3.60(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$



(a)



•10–57. Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.

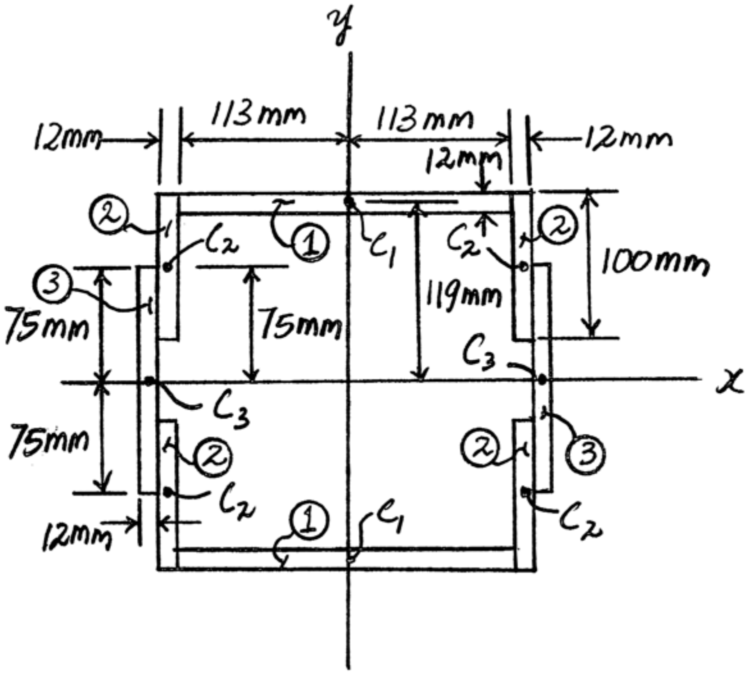


Composite Parts: The composite area can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the *x* axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the *x* axis can be determined using the parallel - axis theorem. Thus,

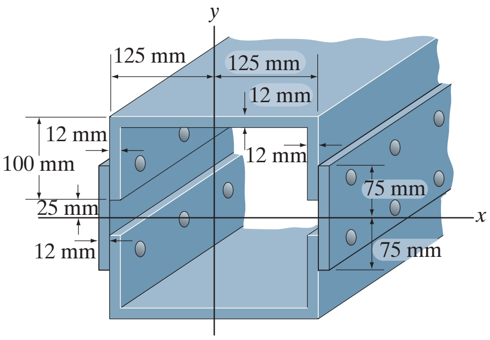
$$
\begin{aligned}
I_x &= \bar{I}_{x'} + A(d_y)^2 \\
&= \left[2\left(\frac{1}{12}(226)(12^3)\right) + 2(226)(12)(119)^2\right] + \left[4\left(\frac{1}{12}(12)(100^3)\right) + 4(12)(100)(75)^2\right] + \left[2\left(\frac{1}{12}(12)(150^3)\right) + 2(12)(150)(0)^2\right] \\
&= 114.62(10^6)\,\text{mm}^4 = 115(10^6)\,\text{mm}^4
\end{aligned}
$$

Ans.



(a)

10–58. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



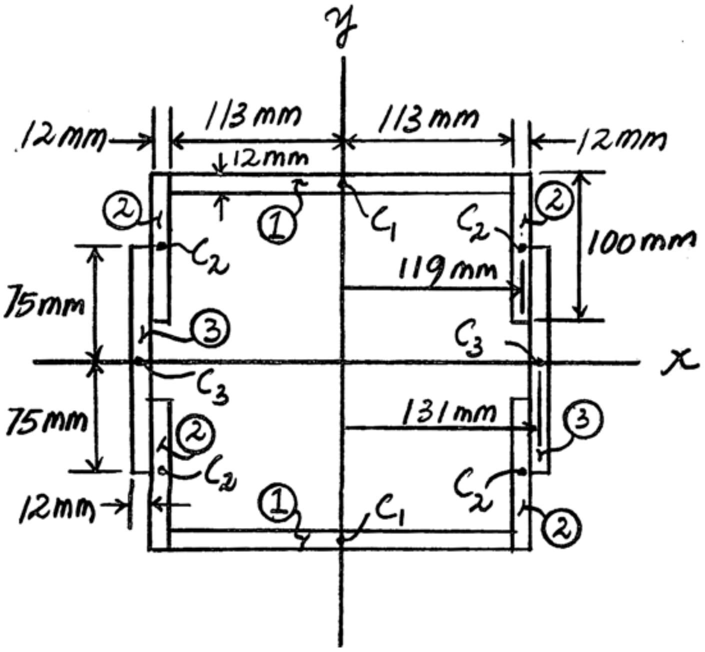
Composite Parts: The composite cross-sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel-axis theorem. Thus,

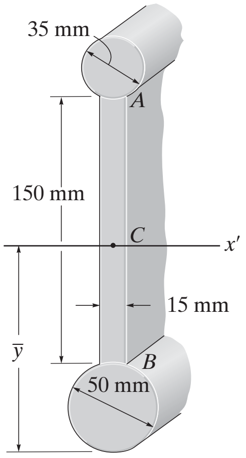
$$I_y = \bar{I}_{x'} + A(d_x)^2$$

$$= \left[2\left(\frac{1}{12}(12)(226^3)\right) + 2(226)(12)(0)^2\right] + \left[4\left(\frac{1}{12}(100)(12^3)\right) + 4(100)(12)(119)^2\right] + \left[2\left(\frac{1}{12}(150)(12^3)\right) + 2(150)(12)(131)^2\right]$$

$$= 152.94(10^6)\ \text{mm}^4 = 153(10^6)\ \text{mm}^4$$ **Ans.**



10–59. Determine the moment of inertia of the beam's cross-sectional area with respect to the x' axis passing through the centroid C of the cross section. $\bar{y} = 104.3$ mm.

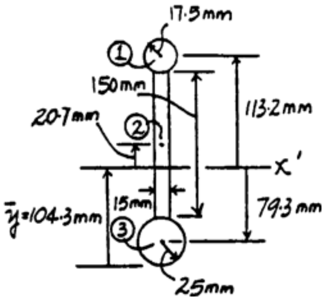


Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

Segment	A_i (mm^2)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm^4)	$(Ad_y^2)_i$ (mm^4)	$(I_{x'})_i$ (mm^4)
1	$\pi(17.5^2)$	113.2	$\frac{\pi}{4}(17.5^4)$	$12.329(10^6)$	$12.402(10^6)$
2	$15(150)$	20.7	$\frac{1}{12}(15)(150^3)$	$0.964(10^6)$	$5.183(10^6)$
3	$\pi(25^2)$	79.3	$\frac{\pi}{4}(25^4)$	$12.347(10^6)$	$12.654(10^6)$

Thus,

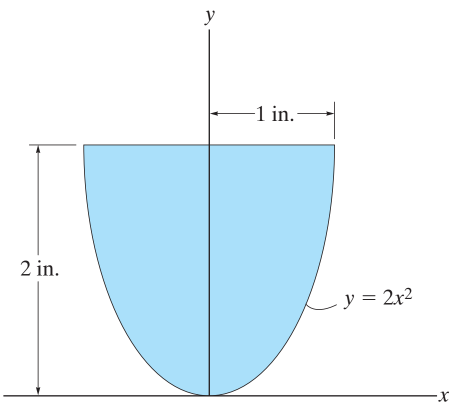
$$I_{x'} = \Sigma(I_{x'})_i = 30.24(10^6)\ \text{mm}^4 = 30.2(10^6)\ \text{mm}^4 \quad \textbf{Ans}$$



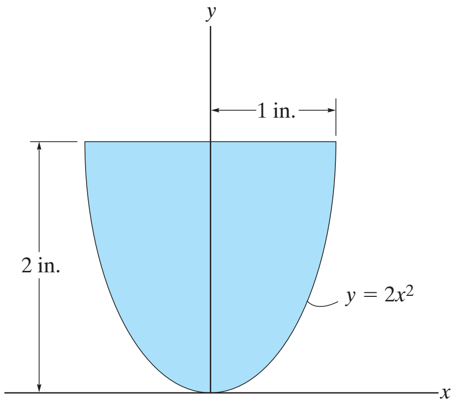
***10–60.** Determine the product of inertia of the parabolic area with respect to the x and y axes.

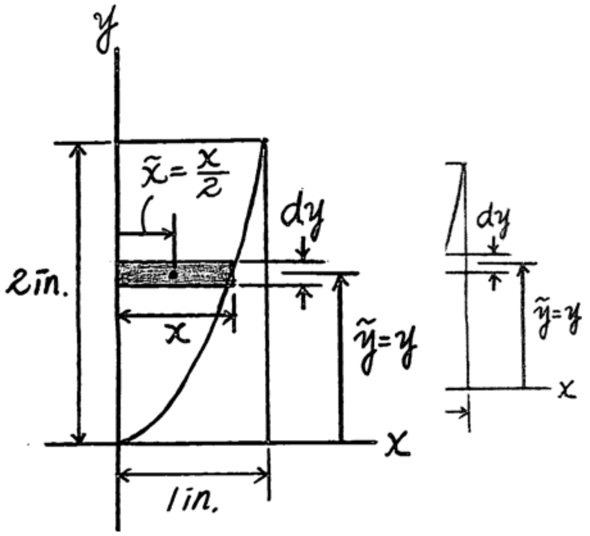
Due to symmetry about y axis

$I_{xy} = 0$ **Ans**



•10–61. Determine the product of inertia I_{xy} of the right half of the parabolic area in Prob. 10–60, bounded by the lines $y = 2$ in. and $x = 0$.





$$\tilde{x} = \frac{x}{2}$$

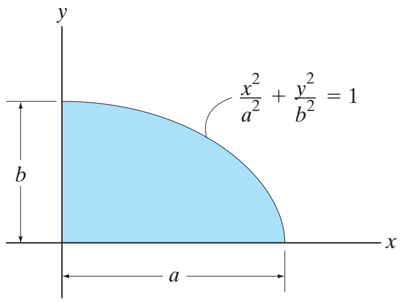
$$\tilde{y} = y$$

$$dA = x\,dy$$

$$I_{xy} = \int_A \tilde{x}\,\tilde{y}\,dA = \int_A \left(\frac{x}{2}\right)(y)(x\,dy)$$

$$= \int_0^2 \frac{1}{2}\left(\frac{1}{2}y^2\right)dy = \frac{1}{12}y^3\Big]_0^2 = 0.667\text{ in}^4 \qquad \textbf{Ans}$$

10–62. Determine the product of inertia of the quarter elliptical area with respect to the x and y axes.

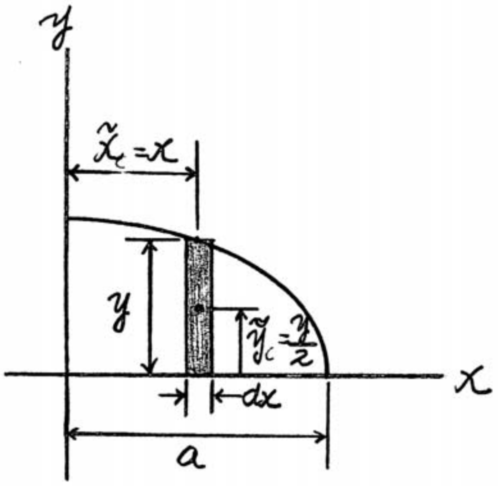


Differential Element: The area of the differenial element parallel to the y axis shown shaded in Fig. a is $dA = y\,dx$. Here, $y = \frac{b}{a}\sqrt{a^2 - x^2}$. Thus, $dA = \frac{b}{a}\sqrt{a^2 - x^2}\,dx$. The coordinates of the centroid of this element are $x_c = x$ and $y_c = \frac{y}{2} = \frac{b}{2a}\sqrt{a^2 - x^2}$. Thus, the product of inertia of this element with respect to the x and y axes is

$$
\begin{aligned}
dI_{xy} &= d\bar{I}_{x'y'} + dA\,x_c y_c \\
&= 0 + \left(\frac{b}{a}\sqrt{a^2 - x^2}\,dx\right)(x)\left(\frac{b}{2a}\sqrt{a^2 - x^2}\right) \\
&= \frac{b^2}{2a^2}\left(a^2 x - x^3\right)dx
\end{aligned}
$$

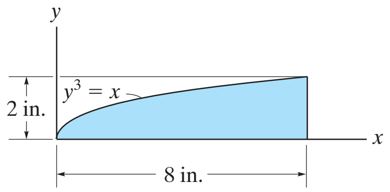
Product of Inertia: Performing the integration,

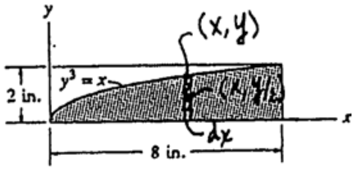
$$I_{xy} = \int dI_{xy} = \int_0^a \frac{b^2}{2a^2}\left(a^2 x - x^3\right)dx = \frac{b^2}{2a^2}\left(\frac{a^2}{2}x^2 - \frac{x^4}{4}\right)\Bigg|_0^a = \frac{a^2 b^2}{8}$$ **Ans.**



(a)

10–63. Determine the product of inertia for the area with respect to the x and y axes.





$\tilde{x} = x$

$\tilde{y} = \dfrac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \dfrac{xy^2}{2}\,dx$

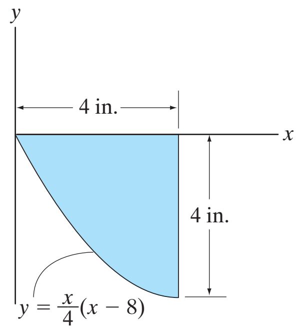
$I_{xy} = \int dI_{xy}$

$= \dfrac{1}{2}\int_0^8 x^{5/3}\,dx$

$= \dfrac{1}{2}\left(\dfrac{3}{8}\right)\left[x^{8/3}\right]_0^8$

$= 48 \text{ in}^4$ **Ans**

***10–64.** Determine the product of inertia of the area with respect to the x and y axes.



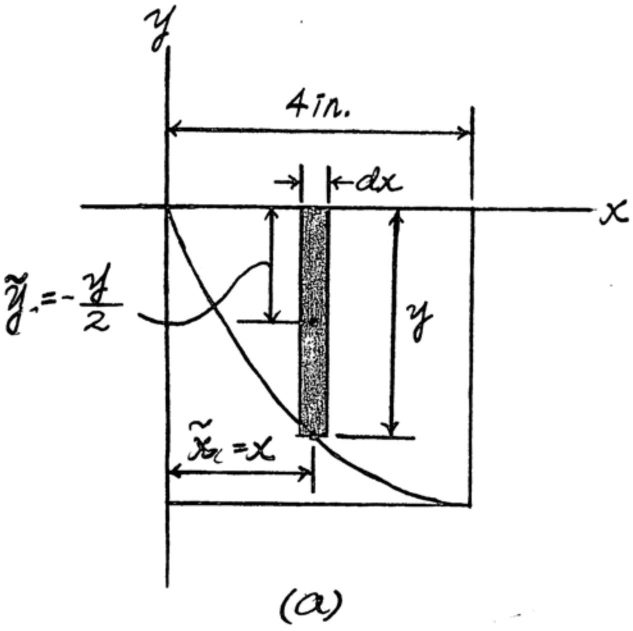
Differential Element: The area of the differential element parallel to the y axis shown shaded in Fig. *a* is $dA = y\,dx = \frac{x}{4}(x-8)\,dx = \left(\frac{x^2}{4} - 2x\right)dx$. The coordinates of the centroid of this element are $\tilde{x} = x$ and $\tilde{y} = -\frac{y}{2} = -\frac{1}{2}\left(\frac{x^2}{4} - 2x\right)$.

Thus, the product of inertia of this element with respect to the x and y axes is

$$\begin{aligned} dI_{xy} &= d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y} \\ &= 0 + \left(\frac{x^2}{4} - 2x\right)dx(x)\left[-\frac{1}{2}\left(\frac{x^2}{4} - 2x\right)\right] \\ &= \left(-\frac{x^5}{32} - 2x^3 + \frac{x^4}{2}\right)dx \end{aligned}$$

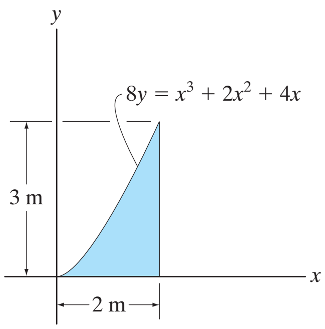
Product of Inertia: Performing the integration,

$$I_{xy} = \int dI_{xy} = \int_0^{4\text{ in.}} \left(-\frac{x^5}{32} - 2x^3 + \frac{x^4}{2}\right)dx = \left[-\frac{x^6}{192} - \frac{x^4}{2} + \frac{x^5}{10}\right]_0^{4\text{ in.}} = -46.9\text{ in}^4 \quad \textbf{Ans.}$$



(a)

•10–65. Determine the product of inertia of the area with respect to the x and y axes.



Differential Element: The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = y\,dx = \frac{1}{8}(x^3 + 2x^2 + 4x)\,dx$. The coordinates of the centroid of this element are $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2}$

Thus, the product of inertia of this element with respect to the x and y axes is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y}$$

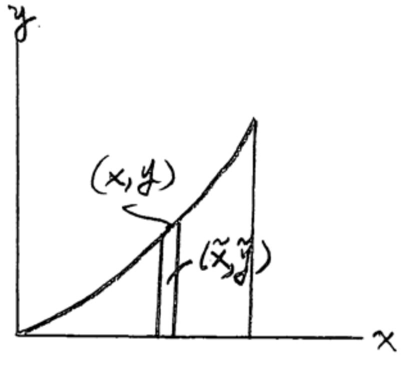
$$= 0 + \left(\frac{1}{8}(x^3 + 2x^2 + 4x)\,dx\right)(x)\left[\frac{1}{16}(x^3 + 2x^2 + 4x)\right]$$

$$= \frac{1}{128}(x^3 + 2x^2 + 4x)^2 x\,dx$$

Product of Inertia: Performing the integration,

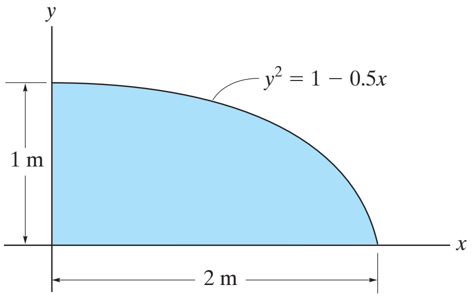
$$I_{xy} = \int dI_{xy} = \int_0^{4\text{ in.}} \frac{1}{128}(x^7 + 4x^6 + 12x^5 + 16x^4 + 16x^3)dx$$

$$= \frac{1}{128}\left[\frac{x^8}{8} + \frac{4x^7}{7} + \frac{12x^6}{6} + \frac{16x^5}{5} + \frac{16x^4}{4}\right]\Bigg|_0^{2\text{ in.}} = 3.12\text{ m}^4$$ **Ans.**



(a)

10–66. Determine the product of inertia for the area with respect to the *x* and *y* axes.

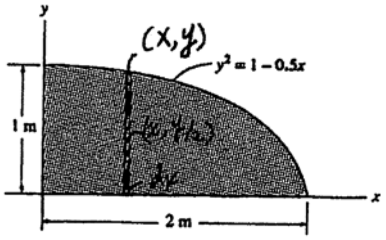


$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \frac{xy^2}{2}\,dx$



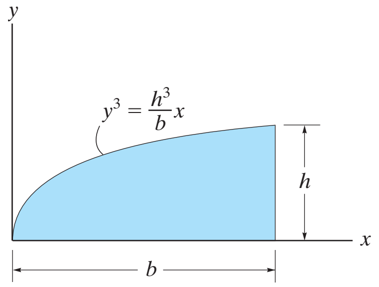
$I_{xy} = \int dI_{xy}$

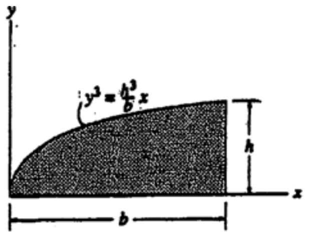
$= \int_0^2 \frac{1}{2}(x - 0.5x^2)\,dx$

$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2$

$= 0.333\ \text{m}^4$ **Ans**

10–67. Determine the product of inertia for the area with respect to the *x* and *y* axes.



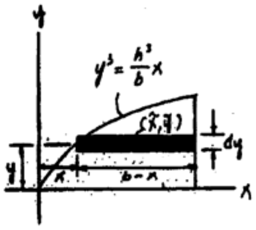


The product of inertia of the element (shaded) is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$

$$= 0 + (b-x)(dy)\left(x + \frac{b-x}{2}\right)(y) = \frac{1}{2}(b^2 - x^2)\,y\,dy \qquad \text{Where } x^2 = \frac{b^2}{h^6}y^6$$

$$= \frac{1}{2}\left(b^2 y - \frac{b^2}{h^6}y^7\right)dy$$



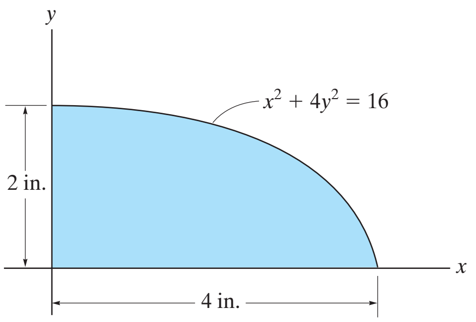
Integrating

$$\int dI_{xy} = I_{xy} = \frac{1}{2}\int_0^h \left(b^2 y - \frac{b^2}{h^6}y^7\right)dy$$

$$= \left[\frac{1}{2}\left(\frac{b^2}{2}y^2 - \frac{b^2}{8h^6}y^8\right)\right]\Bigg|_0^h$$

$$= \frac{3}{16}b^2h^2 \qquad \textbf{Ans}$$

***10–68.** Determine the product of inertia for the area of the ellipse with respect to the *x* and *y* axes.



$$I_{xy} = \int_A \tilde{x}\,\tilde{y}\,dA = \int_0^4 \left(\frac{y}{2}\right)(xy)\,dx$$

$$= \frac{1}{2}\int_0^4 y^2 x\,dx$$

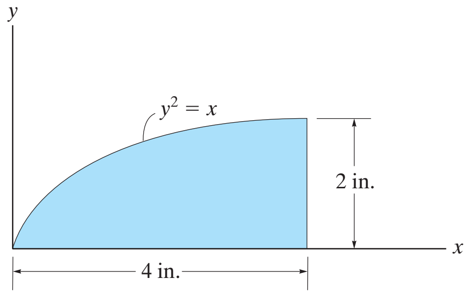
$$= \frac{1}{2}\int_0^4 \frac{1}{4}(16 - x^2)x\,dx$$

$$= \frac{1}{8}\int_0^4 (16x - x^3)\,dx$$

$$= \frac{1}{8}\left[8x^2 - \frac{1}{4}x^4\right]_0^4$$

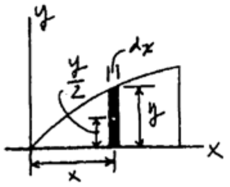
$I_{xy} = 8\ \text{in}^4$ **Ans**

•10–69. Determine the product of inertia for the parabolic area with respect to the *x* and *y* axes.



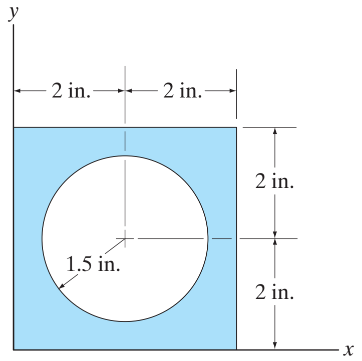
$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\,\tilde{y}$$

$$I_{xy} = 0 + \int_A x\left(\frac{y}{2}\right)y\,dx$$



$$= \frac{1}{2}\int_0^4 x^2\,dx = \frac{1}{6}x^3\Big|_0^4 = 10.6667 = 10.7\ \text{in}^4$$ **Ans**

10–70. Determine the product of inertia of the composite area with respect to the x and y axes.

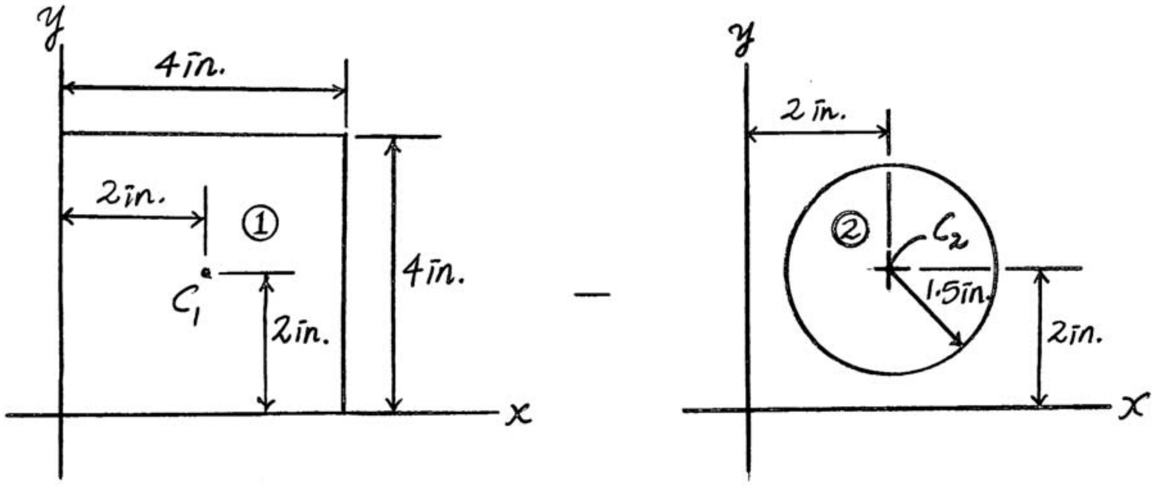


Composite Parts: The composite area can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered a negative area. The perpendicular distances measured from the centroid of each segment to the x and y axes are also indicated.

Product of Inertia: Since the centroidal axes are the axes of symmetry for both segments, then $\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{x'y'}\right)_2 = 0$. The product of inertia of each segment with respect to the x and y axes can be determined using the parallel - axis theorem.

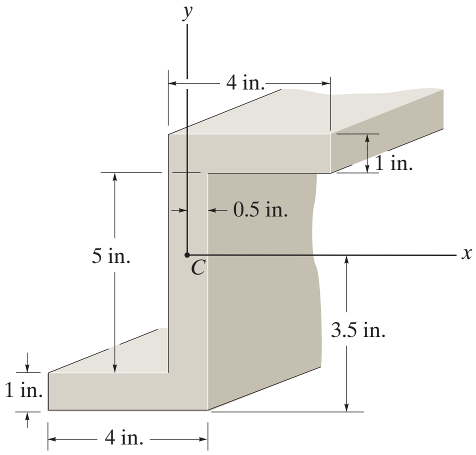
$$I_{xy} = \left(\bar{I}_{x'y'}\right)_i + Ad_x d_y = Ad_x d_y$$

$$= 4(4)(2)(2) + \left(-\pi(1.5^2)\right)(2)(2) = 64 + (-9\pi) = 35.7\ \text{in}^4 \qquad \textbf{Ans.}$$



(a)

10–71. Determine the product of inertia of the cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.

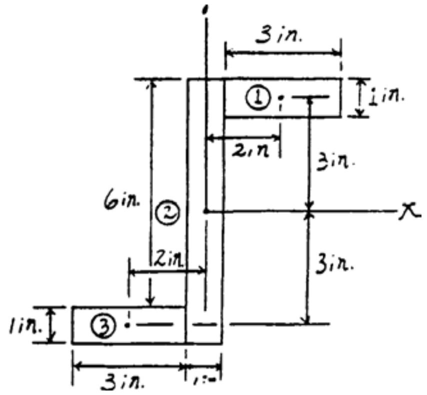


Product of Inertia : The area for each segment, its centroid and product of inertia with respect to x and y axes are tabulated below.

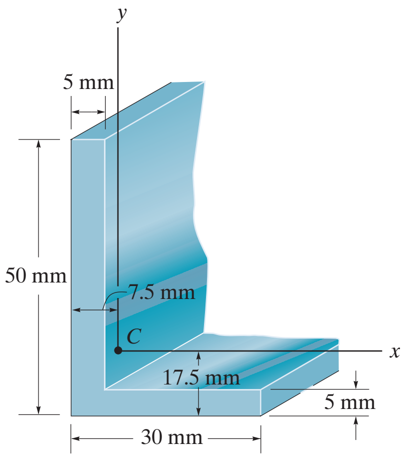
Segment	A_i (in^2)	$(d_x)_i$ (in.)	$(d_y)_i$ (in.)	$(I_{xy})_i$ (in^4)
1	3(1)	2	3	18.0
2	7(1)	0	0	0
3	3(1)	−2	−3	18.0

Thus,

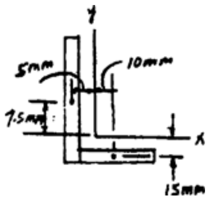
$$I_{xy} = \Sigma(I_{xy})_i = 36.0 \text{ in}^4 \qquad \textbf{Ans}$$



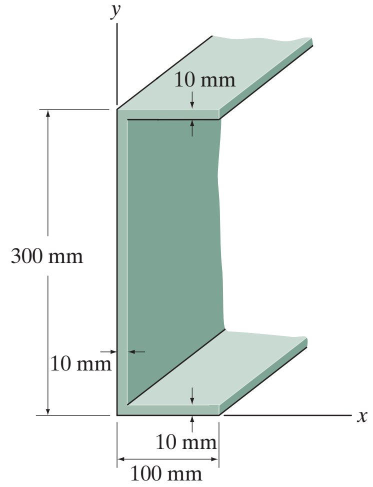
***10–72.** Determine the product of inertia for the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



$$I_{xy} = 25(5)(10)(-15) + 50(5)(-5)(7.5) = -28.1(10^3) \text{ mm}^4 \qquad \textbf{Ans}$$



•10–73. Determine the product of inertia of the beam's cross-sectional area with respect to the x and y axes.



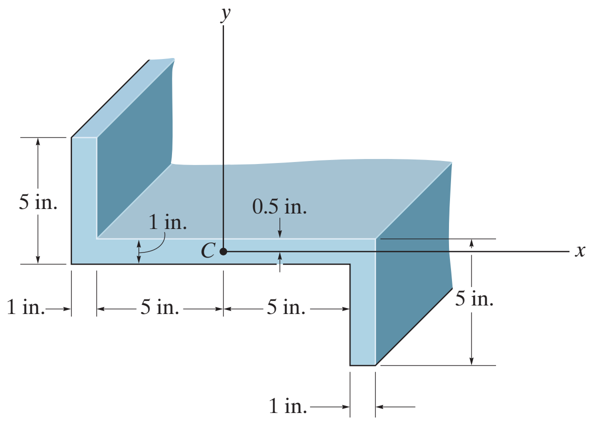
Composite Parts: The composite cross - sectional area of the beam can be subdivided into three segments. The perpendicular distances measured from the centroid of each element to the x and y axes are also indicated.

Product of Inertia: Since the centroidal axes are the axes of all the segments are the axes of symmetry, then $\bar{I}_{x'y'} = 0$. Thus, the product of inertia of each segment with respect to the x and y axes can be determined using the parallel - axis theorem.

$$I_{xy} = \bar{I}_{x'y'} + Ad_x d_y = Ad_x d_y$$

$$= 90(10)(55)(295) + 300(10)(5)(150) + 90(10)(55)(5) = \Sigma I_{xy} = 17.1(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

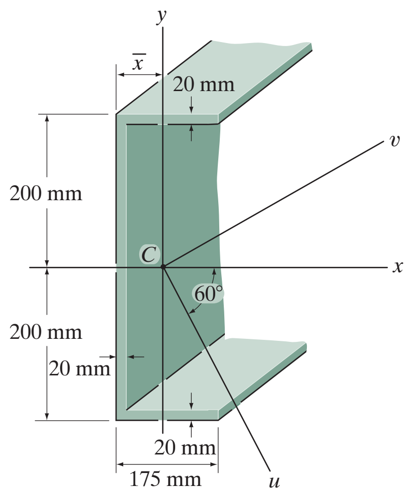
10–74. Determine the product of inertia for the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.

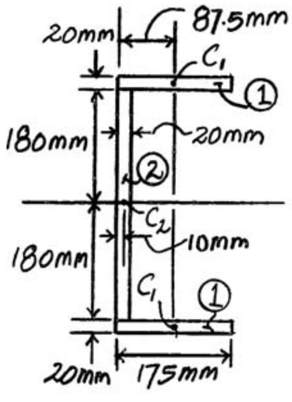


$$I_{xy} = 5(1)(5.5)(-2) + 5(1)(-5.5)(2)$$

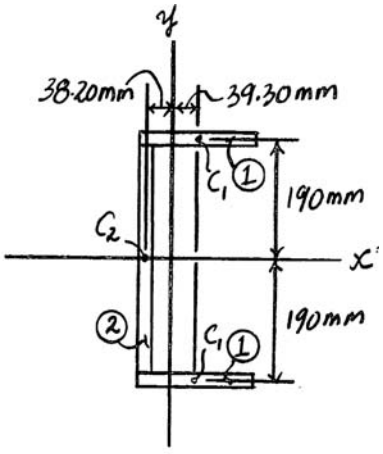
$$= -110\ \text{in}^4 \quad \textbf{Ans}$$

10–75. Locate the centroid $\bar{x}$ of the beam's cross-sectional area and then determine the moments of inertia and the product of inertia of this area with respect to the u and v axes. The axes have their origin at the centroid C.





(a)



(b)

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left of the beam's cross-sectional area leads to

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{2[(87.5)(175)(20)] + 10(360)(20)}{2(175)(20) + 360(20)} = 48.204 \text{ mm} = 48.2 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from the centroid of each segment to the x and y axes with the parallel-axis theorem gives

$$I_x = 2\left[\frac{1}{12}(175)(20^3) + 175(20)(190)^2\right] + \frac{1}{12}(20)(360^3)$$

$$= 330.69(10^6) \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(175^3) + 20(175)(39.30^2)\right] + \left[\frac{1}{12}(360)(20^3) + 360(20)(38.20^2)\right]$$

$$= 39.42(10^6) \text{ mm}^4$$

Since the cross-sectional area is symmetrical about the x axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the u and v Axes: With $\theta = -60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{330.69 + 39.42}{2} + \frac{330.69 - 39.42}{2}\cos(-120°) - 0\sin(-120°)\right](10^6)$$

$$= 112.25(10^6) \text{ mm}^4 = 112(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \left[\frac{330.69 + 39.42}{2} - \frac{330.69 - 39.42}{2}\cos(-120°) + 0\sin(-120°)\right](10^6)$$

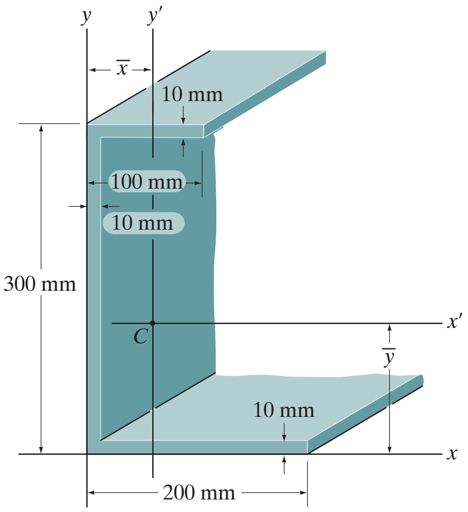
$$= 257.88(10^6) \text{ mm}^4 = 258(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \left[\frac{330.69 - 39.42}{2}\sin(-120°) + 0\cos(-120°)\right](10^6)$$

$$= -126.12(10^6) \text{ mm}^4 = -126(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

***10–76.** Locate the centroid ($\bar{x}$, $\bar{y}$) of the beam's cross-sectional area, and then determine the product of inertia of this area with respect to the centroidal x' and y' axes.



Composite Parts: The composite cross - sectional area of the beam can be subdivided into three segments as shown in Figs. *a* and *b*.

Centroid: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *a*.

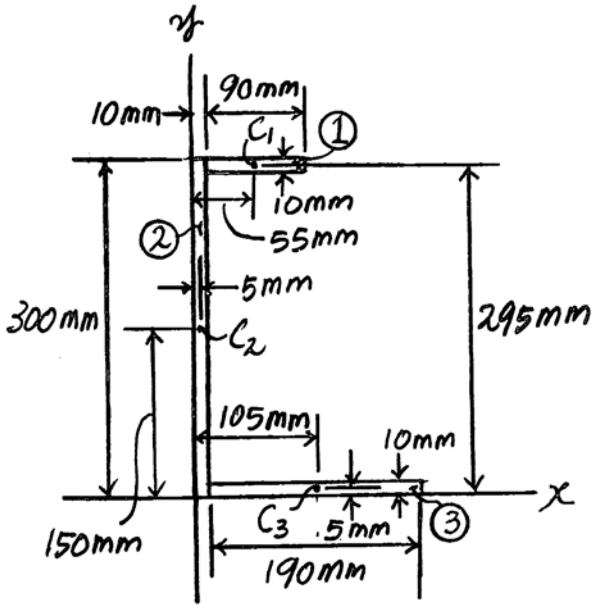
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{55(90(10)) + 5(300(10)) + 105(190(10))}{90(10) + 300(10) + 190(10)} = \frac{264(10^3)}{5.8(10^3)} = 45.52 \text{ mm} = 45.5 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{295(90(10)) + 150(300(10)) + 5(190(10))}{90(10) + 300(10) + 190(10)} = \frac{725(10^3)}{5.8(10^3)} = 125 \text{ mm} \qquad \textbf{Ans.}$$

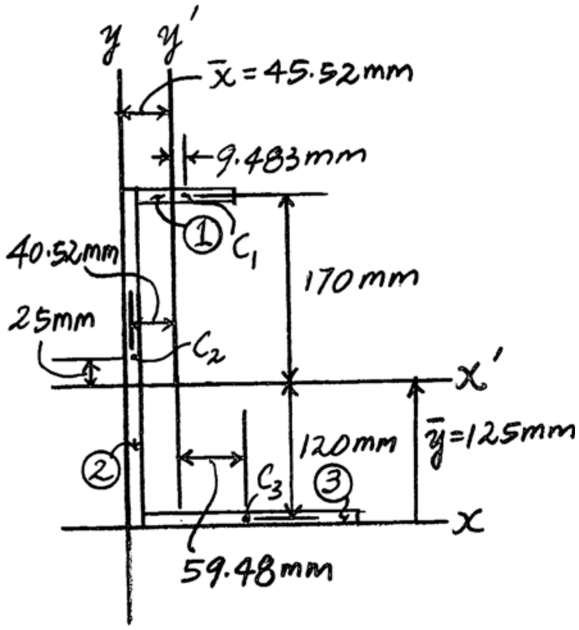
Product of Inertia: Since the centroidal axes are the axes of all the segments are the axes of symmetry, then $\bar{I}_{x'y'} = 0$. Thus, the product of inertia of each segment with respect to the x' and y' axes can be determined using the parallel - axis theorem. The perpendicular distances measured from the centroid of each segment to the x' and y' axes are indicated in Fig. *b*.

$$I_{x'y'} = \bar{I}_{x'y'} + A d_{x'} d_{y'} = A d x' d y'$$

$$= 90(10)(9.483)(170) + 300(10)(-40.52)(25) + 190(10)(59.48)(-120) = \Sigma I_{x'y'} = -15.15(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

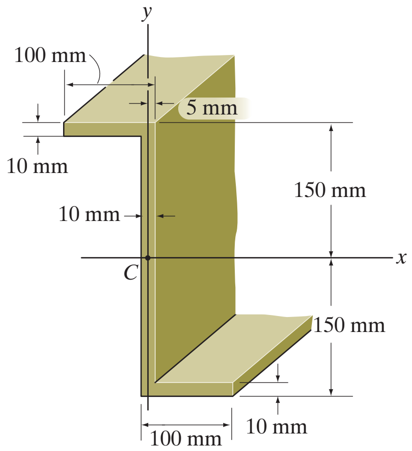


(a)



(b)

•10–77. Determine the product of inertia of the beam's cross-sectional area with respect to the centroidal x and y axes.

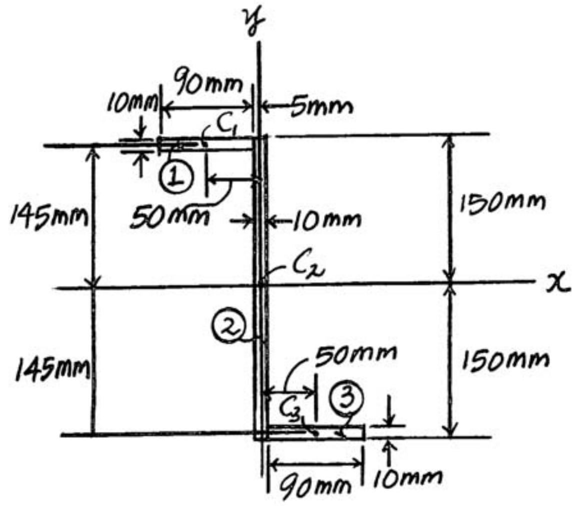


Composite Parts: The composite cross - sectional area of the beam can be subdivided into three segments as shown in Fig. *a*. The perpendicular distances measured from the centroid of each segment to the x and y axes are also indicated.

Product of Inertia: The product of inertia of segment (2) is equal to zero, $\left(\bar{I}_{x'y'}\right)_2 = 0$ since the x and y axes are axes of symmetry. Also, the centroidal axes of segments (1) and (3) are axes of symmetry. Thus, $\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{x'y'}\right)_3 = 0$. Using the parallel - axis theorem, the product of inertia of the two segments can be determined from

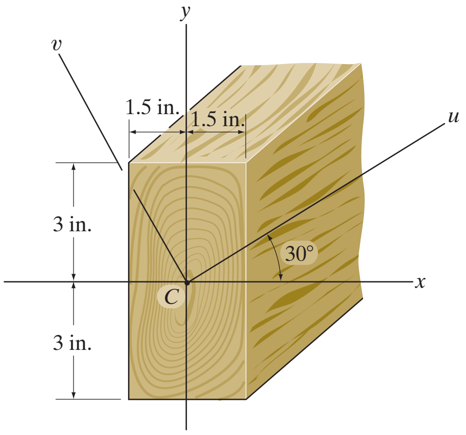
$$I_{xy} = \bar{I}_{x'y'} + Ad_x d_y = Ad_x d_y$$

$$= 90(10)(-50)(145) + 90(10)(50)(-145) = -13.05(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

10–78. Determine the moments of inertia and the product of inertia of the beam's cross-sectional area with respect to the u and v axes.



Moment and Product of Inertia with Respect to the x and y Axes: Since the rectangular beam cross-sectional area is symmetrical about the x and y axes, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4 \qquad I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

Moment and Product of Inertia with Respect to the u and v Axes: With $\theta = 30°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{54 + 13.5}{2} + \frac{54 - 13.5}{2}\cos 60° - 0 \sin 60°$$

$$= 43.9 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{54 + 13.5}{2} - \frac{54 - 13.5}{2}\cos 60° + 0 \sin 60°$$

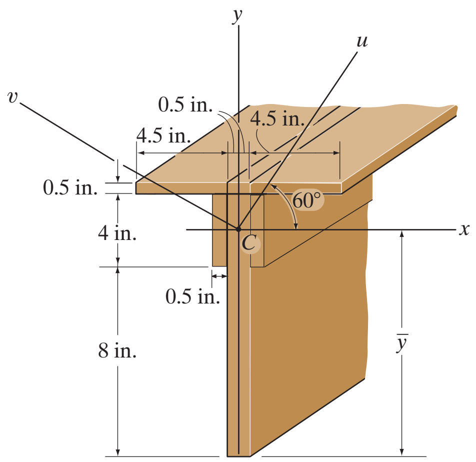
$$= 23.6 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{54 - 13.5}{2}\sin 60° + 0 \cos 60°$$

$$= 17.5 \text{ in}^4 \qquad \textbf{Ans.}$$

10–79. Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia and the product of inertia of this area with respect to the u and v axes.



Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma y_C A}{\Sigma A} = \frac{12.25(10)(0.5) + 2[10(4)(0.5)] + 6(12)(1)}{10(0.5) + 2(4)(0.5) + 12(1)} = 8.25 \text{ in.} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from the centroid of each segment to the x and y axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = \left[\frac{1}{12}(10)(0.5^3) + 10(0.5)(4)^2\right] + 2\left[\frac{1}{12}(0.5)(4^3) + 0.5(4)(1.75)^2\right] + \left[\frac{1}{12}(1)(12^3) + 1(12)(2.25)^2\right]$$

$$= 302.44 \text{ in}^4$$

$$I_y = \frac{1}{12}(0.5)(10^3) + 2\left[\frac{1}{12}(4)(0.5^3) + 4(0.5)(0.75)^2\right] + \frac{1}{12}(12)(1^3)$$

$$= 45 \text{ in}^4$$

Since the cross-sectional area is symmetrical about the y axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the u and v Axes: With $\theta = 60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{302.44 + 45}{2} + \frac{302.44 - 45}{2}\cos 120° - 0 \sin 120°$$

$$= 109.36 \text{ in}^4 = 109 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

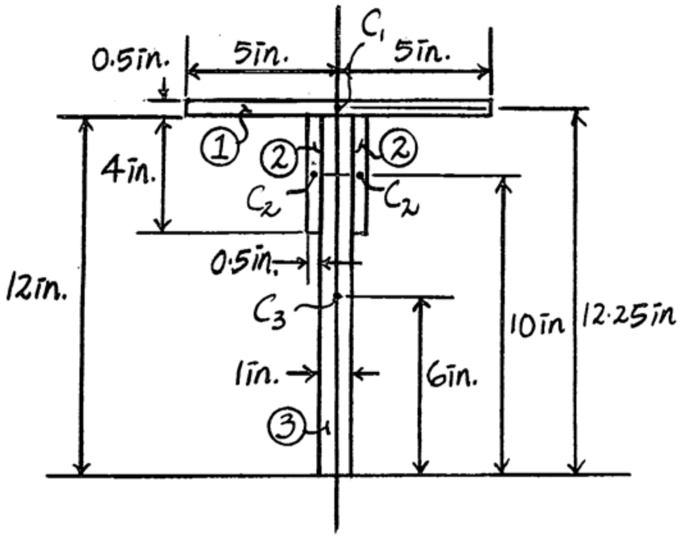
$$= \frac{302.44 + 45}{2} - \frac{302.44 - 45}{2}\cos 120° + 0 \sin 120°$$

$$= 238.08 \text{ in}^4 = 238 \text{ in}^4 \qquad \textbf{Ans.}$$

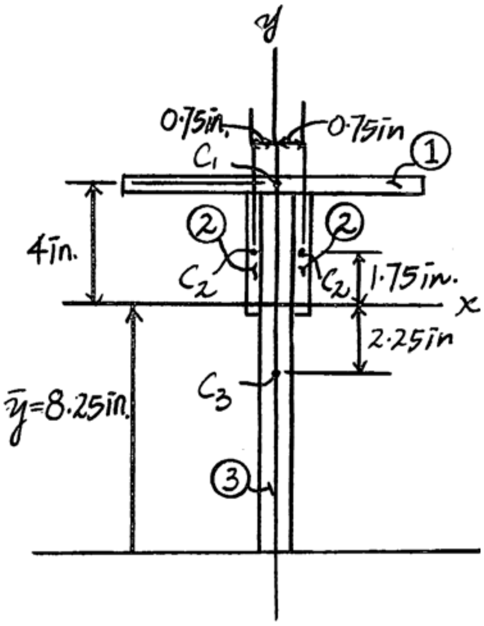
$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{302.44 - 45}{2}\sin 120° + 0 \cos 120°$$

$$= 111.47 \text{ in}^4 = 111 \text{ in}^4 \qquad \textbf{Ans.}$$

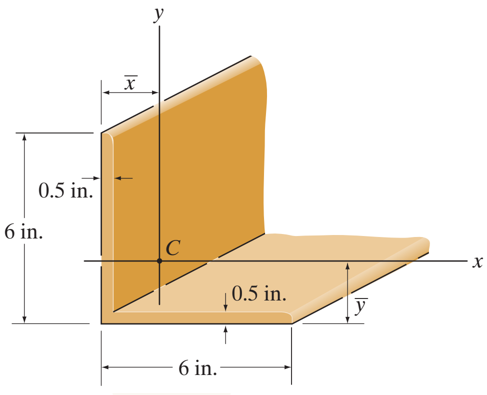

0.5in.
5in.
5in.
C_1
①
②
②
4in.
C_2
C_2
0.5in.
12in.
C_3
10in
12.25in
1in.
6in.
③

(a)


y
0.75in.
0.75in
①
C_1
4in.
②
②
C_2
C_2
1.75in.
x
2.25in
$\bar{y}=8.25in.$
C_3
③

(b)

***10–80.** Locate the centroid $\bar{x}$ and $\bar{y}$ of the cross-sectional area and then determine the orientation of the principal axes, which have their origin at the centroid C of the area. Also, find the principal moments of inertia.



Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left and bottom of the beam's cross - sectional area are indicated in Fig. *a*. Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{0.25(6)(0.5) + 3.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{3(6)(0.5) + 0.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*.

$$I_x = \left[\frac{1}{12}(5.5)(0.5^3) + 5.5(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(6^3) + 0.5(6)(1.315^2)\right]$$

$$= 19.908 \text{ in}^4$$

$$I_y = \left[\frac{1}{12}(6)(0.5^3) + 6(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(5.5^3) + 0.5(5.5)(1.565^2)\right]$$

$$= 19.908 \text{ in}^4$$

$$I_{xy} = 6(0.5)(-1.435)(1.315) + 5.5(0.5)(1.565)(-1.435)$$

$$= -11.837 \text{ in}^4$$

Principal Moment of Inertia:

$$I_{\substack{\max \\ \min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{19.908 + 19.908}{2} \pm \sqrt{\left(\frac{19.908 - 19.908}{2}\right)^2 + (-11.837)^2}$$

$$= 19.908 \pm 11.837$$

$$I_{\max} = 31.7 \text{ in}^4 \qquad I_{\min} = 8.07 \text{ in}^4 \qquad \textbf{Ans.}$$

Orientation of Principal Axes:

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-(-11.837)}{(19.08 - 19.08)/2} = \infty$$

$$2\theta_p = 90^\circ \text{ and } -90^\circ$$

$$\theta_p = 45^\circ \text{ and } -45^\circ \qquad \textbf{Ans.}$$

Substituting $\theta = \theta_p = 45°$

$$
\begin{aligned}
I_u &= \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta \\
&= \left(\frac{19.908 + 19.908}{2}\right) + \left(\frac{19.908 - 19.908}{2}\right)\cos 90° - (-11.837)\sin 90° \\
&= 31.7\ \text{in}^4 = I_{\max}
\end{aligned}
$$

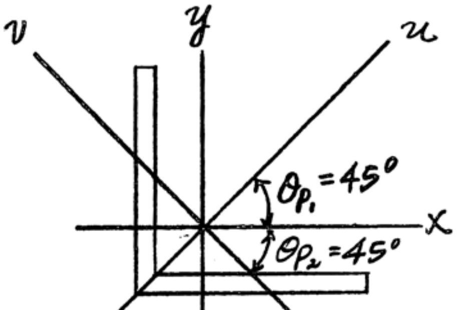
This shows that $I_{\max}$ corresponds to the principal axis orientated at

$I_{\max} = 31.7\ \text{in}^4$ $\qquad (\theta_p)_1 = 45°$ **Ans.**

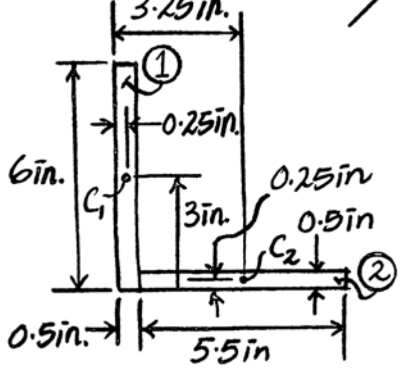
and $I_{\min}$ corresponds to the principal axis orientated at

$I_{\min} = 8.07\ \text{in}^4$ $\qquad (\theta_p)_2 = -45°$ **Ans.**

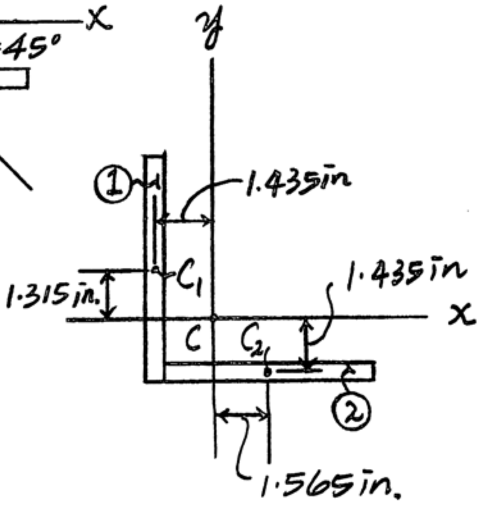
The orientation of the principal axes is shown in Fig. c.



(c)

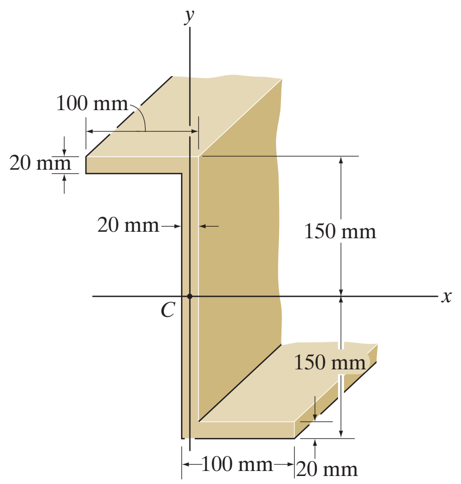


(a)



(b)

•10–81. Determine the orientation of the principal axes, which have their origin at centroid *C* of the beam's cross-sectional area. Also, find the principal moments of inertia.



Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from each subdivided segment to the *x* and *y* axes are indicated in Fig. *a*. Applying the parallel - axis theorem,

$$I_x = 2\left[\frac{1}{12}(80)(20^3)+80(20)(140^2)\right]+\frac{1}{12}(20)(300^3) = 107.83(10^6)\,\text{mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(80^3)+20(80)(50^2)\right]+\frac{1}{12}(300)(20^3) = 9.907(10^6)\,\text{mm}^4$$

$$I_{xy} = 80(20)(-50)(140)+80(20)(50)(-140) = -22.4(10^6)\,\text{mm}^4$$

Principal Moment of Inertia:

$$I_{\substack{\max\\ \min}} = \frac{I_x+I_y}{2} \pm \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}{}^2}$$

$$= \left[\frac{107.83+9.907}{2} \pm \sqrt{\left(\frac{107.83-9.907}{2}\right)^2 + (-22.4)^2}\right](10^6)$$

$$= 58.867 \pm 53.841$$

$I_{\max} = 112.71(10^6) = 113(10^6)\,\text{mm}^4$ **Ans.**

$I_{\min} = 5.026(10^6) = 5.03(10^6)\,\text{mm}^4$ **Ans.**

Orientation of Principal Axes:

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x-I_y)/2} = \frac{-(-22.4)(10^6)}{(107.83-9.907)(10^6)/2} = 0.4575$$

$$2\theta_p = 24.58° \text{ and } -155.42°$$

$$\theta_p = 12.29° \text{ and } -77.71°$$

Substituting $\theta = \theta_p = 12.29°$

$$I_u = \frac{I_x+I_y}{2} + \frac{I_x-I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{107.83+9.907}{2} + \left(\frac{107.83-9.907}{2}\right)\cos 24.58° - (-22.4)\sin 24.58°$$

$$= 112.71(10^6)\,\text{mm}^4 = I_{\max}$$

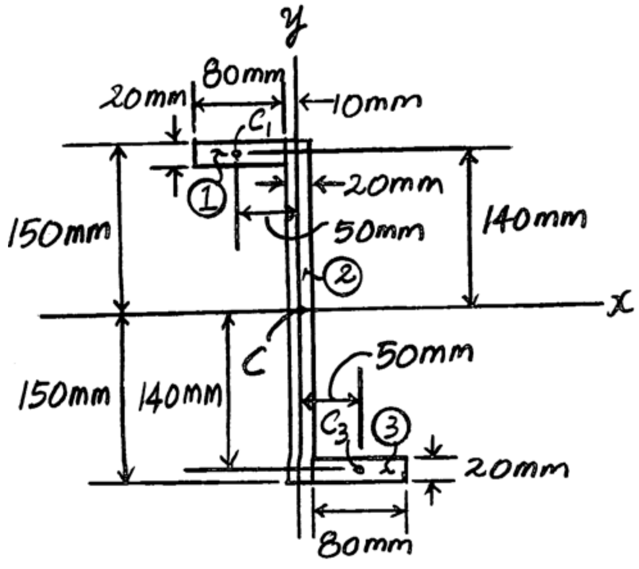
This shows that $I_{\max}$ corresponds to the principal axis orientated at

$I_{\max} = 113(10^6)\,\text{mm}^4$ $(\theta_p)_1 = 12.3°$ **Ans.**

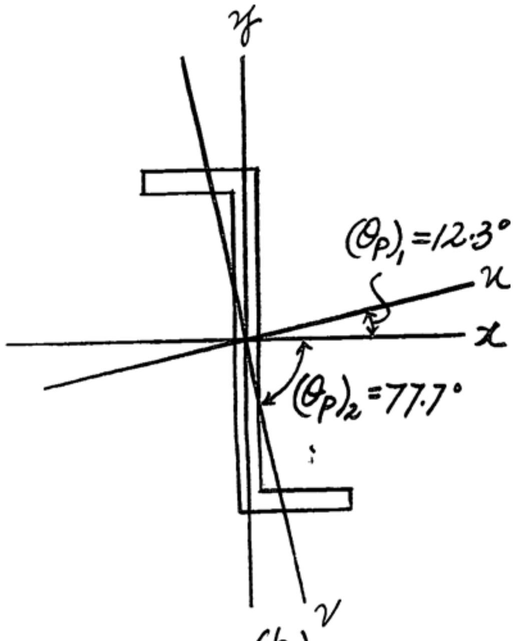
and $I_{\min}$ corresponds to the principal axis orientated at

$I_{\min} = 5.03(10^6)\,\text{mm}^4$ $(\theta_p)_2 = -77.7°$ **Ans.**

The orientation of the principal axes are shown in Fig. *b*.

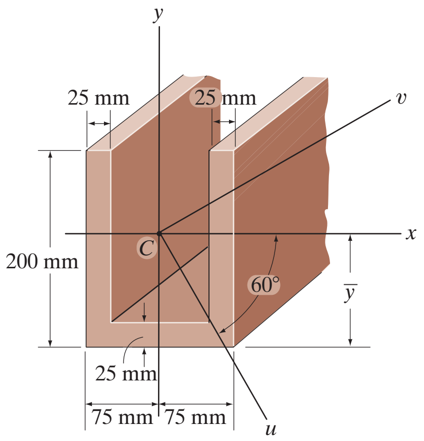

y
80mm
20mm
10mm
C1
①
20mm
50mm
140mm
150mm
②
x
C
150mm
140mm
50mm
C3
③
20mm
80mm

(a)


y
(θP)1 = 12.3°
u
x
(θP)2 = 77.7°
v

(b)

10–82. Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia of this area and the product of inertia with respect to the u and v axes. The axes have their origin at the centroid C.



Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma y_C A}{\Sigma A} = \frac{2[100(200)(25)] + 12.5(2.5)(100)}{2(200)(25) + 25(100)} = 82.5\text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the x and y axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = 2\left[\frac{1}{12}(25)(200^3) + 25(200)(17.5)^2\right] + \left[\frac{1}{12}(100)(25^3) + 100(25)(70)^2\right]$$

$$= 48.78(10^6)\text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(200)(25^3) + 200(25)(62.5)^2\right] + \frac{1}{12}(25)(100^3)$$

$$= 41.67(10^6)\text{ mm}^4$$

Since the cross-sectional area is symmetrical about the y axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the *u* and *v* Axes: With $\theta = -60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{48.78 + 41.67}{2} + \left(\frac{48.78 - 41.67}{2}\right)\cos(-120°) - 0\sin(-120°)\right](10^6)$$

$$= 43.4(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

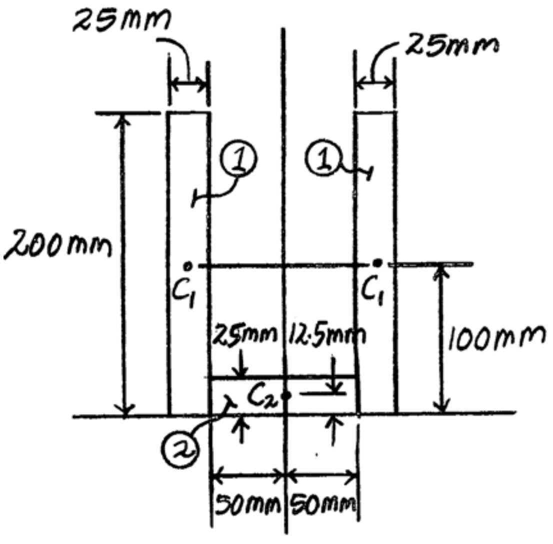
$$= \left[\frac{48.78 + 41.67}{2} - \left(\frac{48.78 - 41.67}{2}\right)\cos(-120°) + 0\sin(-120°)\right](10^6)$$

$$= 47.0(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

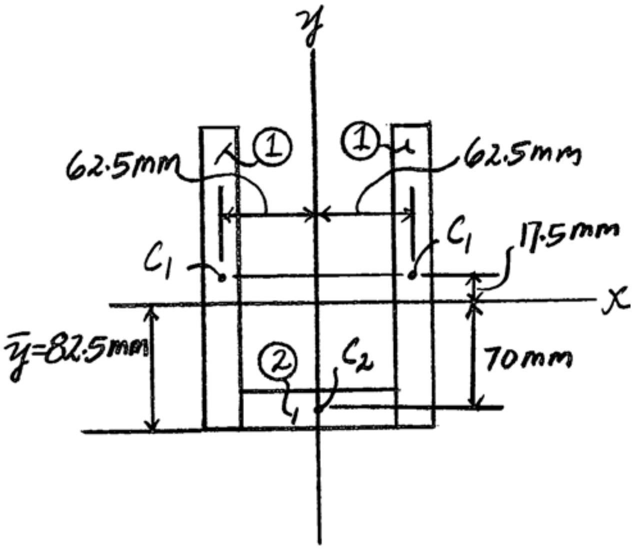
$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \left(\frac{48.78 - 41.67}{2}\right)\sin(-120°) + 0\cos(-120°)$$

$$= -3.08(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$


25mm
25mm
200mm
①
①
C_1
C_1
25mm
12.5mm
100mm
C_2
②
50mm
50mm

(a)


y
62.5mm
①
①
62.5mm
C_1
C_1
17.5mm
x
$\bar{y}$=82.5mm
②
C_2
70mm

(b)

10–83. Solve Prob. 10–75 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{2[(87.5)(175)(20)] + 10(360)(20)}{2(175)(20) + 360(20)} = 48.204 \text{ mm} = 48.2 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = 2\left[\frac{1}{12}(175)(20^3) + 175(20)(190)^2\right] + \frac{1}{12}(20)(360^3)$$

$$= 330.69(10^6) \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(175^3) + 20(175)(39.30^2)\right] + \left[\frac{1}{12}(360)(20^3) + 360(20)(38.20^2)\right]$$

$$= 39.42(10^6) \text{ mm}^4$$

Since the cross-sectional area is symmetrical about the *x* axis, $I_{xy} = 0$.

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \left(\frac{330.69 + 39.42}{2}\right)(10^6) \text{ mm}^4 = 185.06(10^6) \text{ mm}^4$$

The coordinates of the reference point *A* are $[330.69, 0](10^6)$ mm^4. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

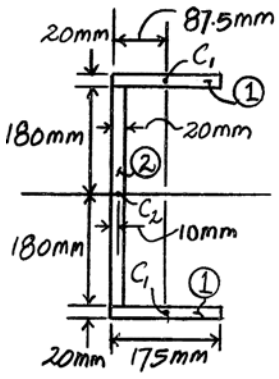
$$R = CA = (330.69 - 185.06)(10^6) = 145.64(10^6) \text{ mm}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

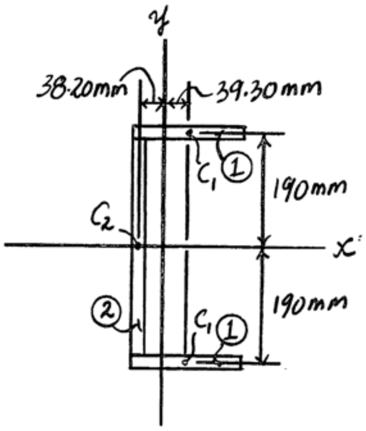
$$I_u = (185.06 - 145.64\cos 60°)(10^6) = 112(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = (185.06 + 145.64\cos 60°)(10^6) = 258(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

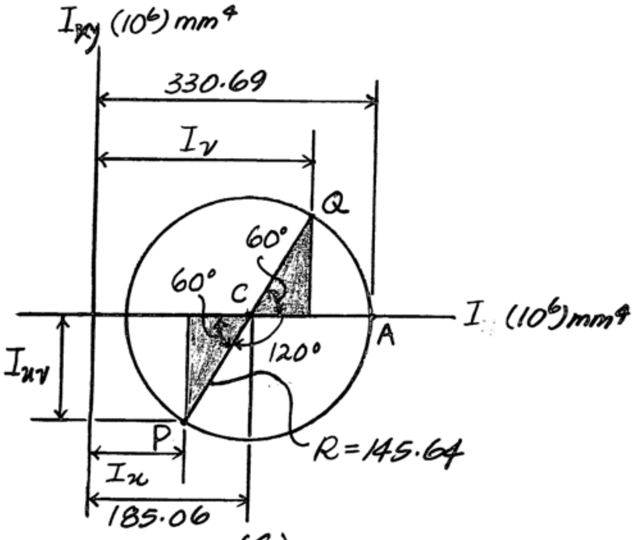
$$I_{uv} = (-145.64\sin 60°)(10^6) = -126(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$



(a)



(b)



(c)

*__10–84.__ Solve Prob. 10–78 using Mohr's circle.

Moment and Product of Inertia with Respect to the x and y Axes: Since the rectangular beam cross - sectional area is symmetrical about the x and y axes, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4 \qquad I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

Construction of Mohr's Circle: The center C of the circle lies along the u axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \frac{54 + 13.5}{2} = 33.75 \text{ in}^4$$

The coordinates of the reference point A are $(54, 0)$ in^4. The circle can be constructed as shown in Fig. a. The radius of the circle is

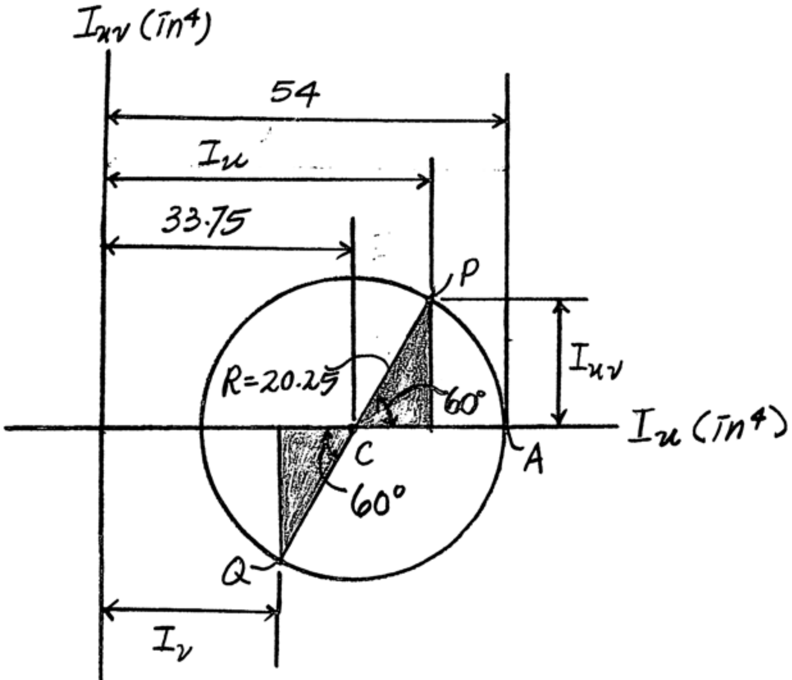
$$R = CA = 54 - 33.75 = 20.25 \text{ in}^4$$

Moment and Product of Inertia with Respect to the u and v Axes: By referring to the geometry of the circle,

$I_u = 33.75 + 20.25 \cos 60° = 43.9 \text{ in}^4$ **Ans.**

$I_v = 33.75 - 20.25 \cos 60° = 23.6 \text{ in}^4$ **Ans.**

$I_{uv} = 20.25 \sin 60° = 17.5 \text{ in}^4$ **Ans.**



(a)

•10–85. Solve Prob. 10–79 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{12.25(10)(0.5) + 2\left[10(4)(0.5)\right] + 6(12)(1)}{10(0.5) + 2(4)(0.5) + 12(1)} = 8.25\text{ in.} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = \left[\frac{1}{12}(10)(0.5^3) + 10(0.5)(4)^2\right] + 2\left[\frac{1}{12}(0.5)(4^3) + 0.5(4)(1.75)^2\right] + \left[\frac{1}{12}(1)(12^3) + 1(12)(2.25)^2\right]$$

$$= 302.44\text{ in}^4$$

$$I_y = \frac{1}{12}(0.5)(10^3) + 2\left[\frac{1}{12}(4)(0.5^3) + 4(0.5)(0.75)^2\right] + \frac{1}{12}(12)(1^3)$$

$$= 45\text{ in}^4$$

Since the cross-sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \frac{302.44 + 45}{2} = 173.72\text{ in}^4$$

The coordinates of the reference point *A* are (302.44, 0) in^4. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

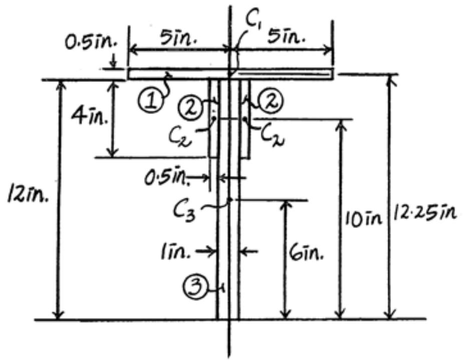
$$R = CA = (302.44 - 173.72) = 128.72\text{ in}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

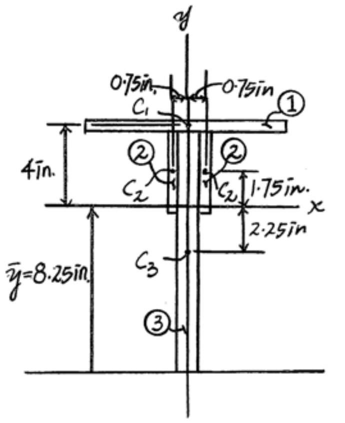
$$I_u = 173.72 - 128.72\cos 60° = 109\text{ in}^4 \qquad \textbf{Ans.}$$

$$I_v = 173.72 + 128.72\cos 60° = 238\text{ in}^4 \qquad \textbf{Ans.}$$

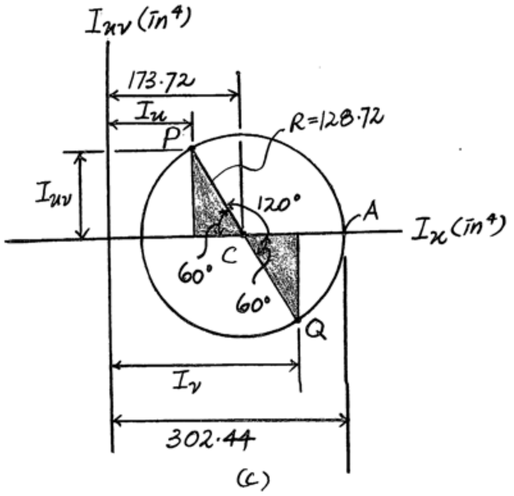
$$I_{uv} = 128.72\sin 60° = 111\text{ in}^4 \qquad \textbf{Ans.}$$



(a)



(b)



(c)

10–86. Solve Prob. 10–80 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left and bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{0.25(6)(0.5) + 3.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{3(6)(0.5) + 0.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*.

$$I_x = \left[\frac{1}{12}(5.5)(0.5^3) + 5.5(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(6^3) + 0.5(6)(1.315^2)\right]$$

$$= 19.908 \text{ in}^4$$

$$I_y = \left[\frac{1}{12}(6)(0.5^3) + 6(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(5.5^3) + 0.5(5.5)(1.565^2)\right]$$

$$= 19.908 \text{ in}^4$$

$$I_{xy} = 6(0.5)(-1.435)(1.315) + 5.5(0.5)(1.565)(-1.435)$$

$$= -11.837 \text{ in}^4$$

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \frac{19.908 + 19.908}{2} \text{ in}^4 = 19.908 \text{ in}^4$$

The coordinates of the reference point *A* are (19.908, −11.837) in^4. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

$$R = CA = 11.837 \text{ in}^4$$

Principal Moment of Inertia: By referring to the geometry of the circle,

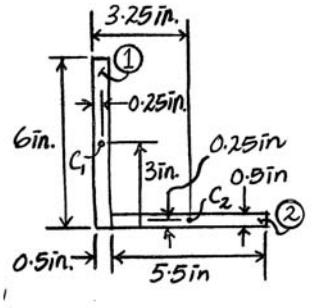
$I_{\max} = 19.908 + 11.837 = 31.7 \text{ in}^4$ **Ans.**

$I_{\min} = 19.908 - 11.837 = 8.07 \text{ in}^4$ **Ans.**

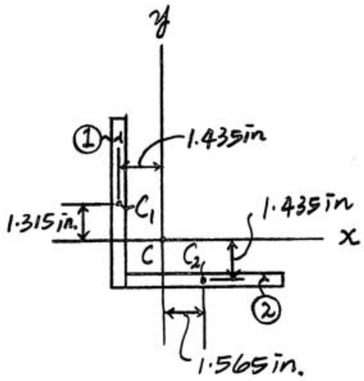
Orientation of Principal Axes: Here $(\theta_p)_1$ and $(\theta_p)_2$ are the orientation of the principal axes about which $I_{\max}$ and $I_{\min}$ occur, respectively. By observing the geometry of the circle,

$2(\theta_p)_1 = 90°$ $\quad (\theta_p)_1 = 45°$ (counterclockwise) **Ans.**

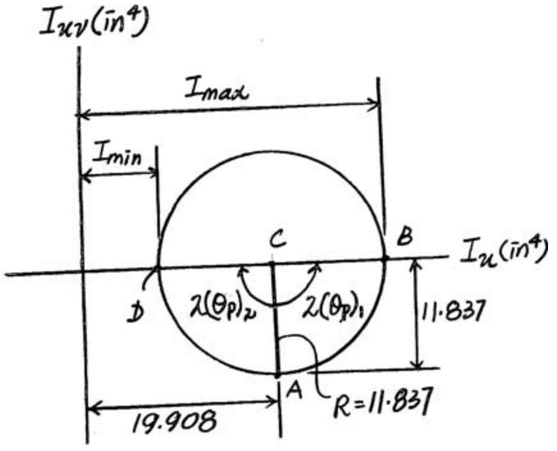
$2(\theta_p)_2 = 90°$ $\quad (\theta_p)_2 = 45°$ (clockwise) **Ans.**



(a)



(b)



(c)

10–87. Solve Prob. 10–81 using Mohr's circle.

Moment and Product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from each subdivided segment to the x and y axes are indicated in Fig. a. Applying the parallel - axis theorem,

$$I_x = \left[\frac{1}{12}(80)(20^3)+80(20)(140^2)\right]+\frac{1}{12}(20)(300^3) = 107.83(10^6)\,\text{mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(80^3)+20(80)(50^2)\right]+\frac{1}{12}(300)(20^3) = 9.907(10^6)\,\text{mm}^4$$

$$I_{xy} = 80(20)(-50)(140)+80(20)(50)(-140) = -22.4(10^6)\,\text{mm}^4$$

Construction of Mohr's Circle: The center C of the circle lies along the u axis at a distance of

$$I_{\text{avg}} = \frac{I_x+I_y}{2} = \left(\frac{107.83+9.907}{2}\right)(10^6) = 58.867(10^6)\,\text{mm}^4$$

The coordinates of the reference point A are $(107.83,\ -22.4)\,\text{mm}^4$. The circle can be constructed as shown in Fig. b. The radius of the circle is

$$R = CA = \left(\sqrt{(107.83-58.867)^2+(-22.4)^2}\right)(10^6) = 53.84(10^6)\,\text{mm}^4$$

Principal Moment of Inertia: By referring to the geometry of the circle, we obtain

$$I_{\max} = (53.84+53.84)(10^6) = 112.70(10^6) = 113(10^6)\,\text{mm}^4 \qquad \textbf{Ans.}$$

$$I_{\min} = (53.84-53.84)(10^6) = 5.026(10^6) = 5.03(10^6)\,\text{mm}^4 \qquad \textbf{Ans.}$$

Orientation of Principal Axes: Here $(\theta_p)_1$ and $(\theta_p)_2$ are the orientation of the principle axes about which $I_{\max}$ and $I_{\min}$ occur. From the geometry of the circle,

$$\tan 2(\theta_p)_1 = \frac{22.4}{107.83-58.867}$$

$$2(\theta_p)_1 = 24.58°$$

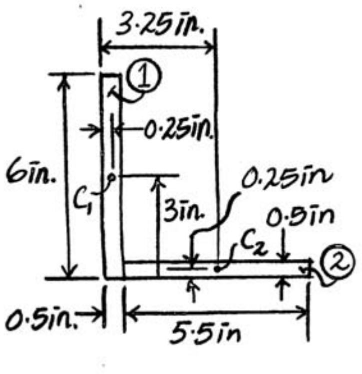
$$(\theta_p)_1 = 12.29° = 12.3°\ \text{(counterclockwise)} \qquad \textbf{Ans.}$$

Thus,

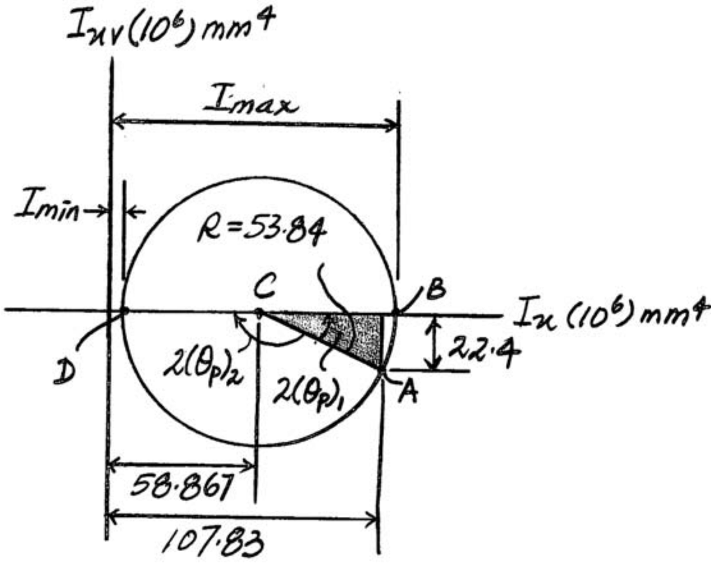
$$2(\theta_p)_2 = 180° - 2(\theta_p)_1 = 155.42°$$

$$(\theta_p)_2 = 77.7°\ \text{(clockwise)} \qquad \textbf{Ans.}$$

The orientation of the principle axes are shown in Fig. c.



(a)



(b)

*__10–88.__ Solve Prob. 10–82 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{2[100(200)(25)] + 12.5(2.5)(100)}{2(200)(25) + 25(100)} = 82.5\text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = 2\left[\frac{1}{12}(25)(200^3) + 25(200)(17.5)^2\right] + \left[\frac{1}{12}(100)(25^3) + 100(25)(70)^2\right]$$

$$= 48.78(10^6)\text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(200)(25^3) + 200(25)(62.5)^2\right] + \frac{1}{12}(25)(100^3)$$

$$= 41.67(10^6)\text{ mm}^4$$

Since the cross-sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \left(\frac{48.78 + 41.67}{2}\right)(10^6)\text{ mm}^4 = 45.22(10^6)\text{ mm}^4$$

The coordinates of the reference point *A* are $[48.78, 0](10^6)\text{ mm}^4$. The circle can be constructed as shown in Fig. *a*. The radius of the circle is

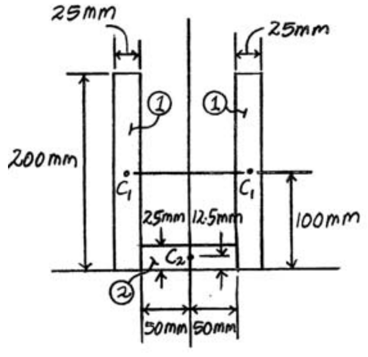
$$R = CA = (48.78 - 45.22)(10^6) = 3.56(10^6)\text{ mm}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

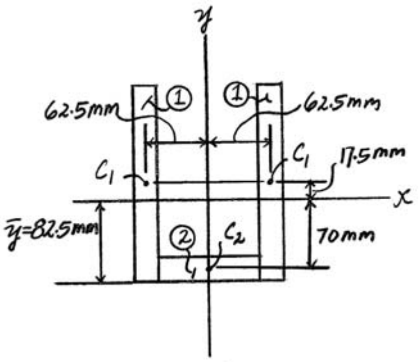
$$I_u = (45.22 - 3.56\cos 60^\circ)(10^6) = 43.4(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = (45.22 + 3.56\cos 60^\circ)(10^6) = 47.0(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

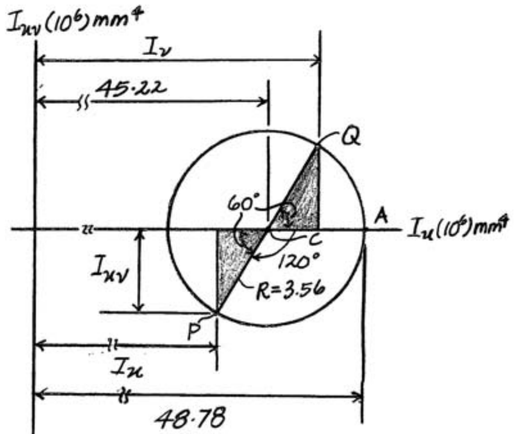
$$I_{uv} = -3.56\sin 60^\circ = -3.08(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$



(a)

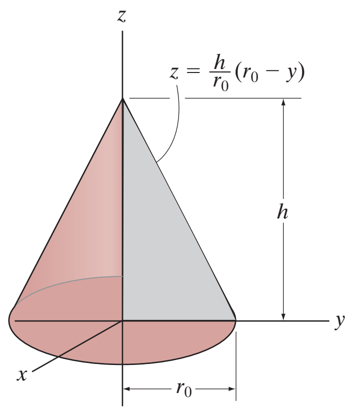


(b)



(c)

•10–89. Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \frac{r_o}{h}z$. Thus, $dm = \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is $dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2$ $= \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz.$

Mass: The mass of the cone can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_o^2 h$$

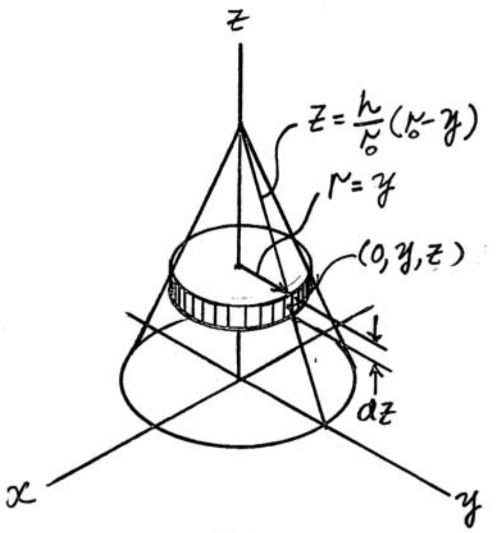
Mass Moment of Inertia: Integrating dI_z,

$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_o^4 h$$

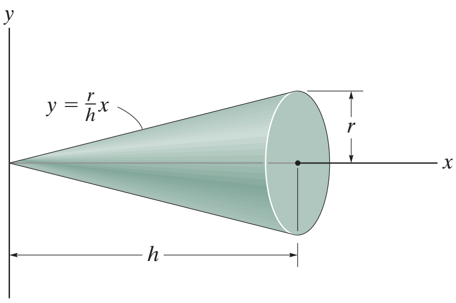
From the result of the mass, we obtain $\rho\pi r_o^2 h = 3m$. Thus, I_z can be written as

$$I_z = \frac{1}{10}\left(\rho\pi r_o^2 h\right)r_o^2 = \frac{1}{10}(3m)r_o^2 = \frac{3}{10}mr_o^2 \qquad \textbf{Ans.}$$

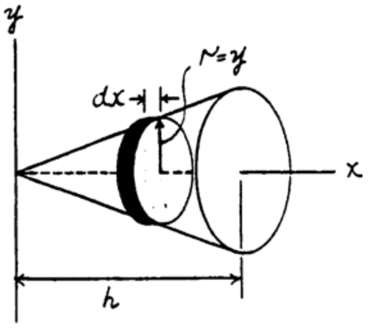


(a)

10–90. Determine the mass moment of inertia I_x of the right circular cone and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



Differential Disk Element : The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2 dx = \rho\pi\left(\frac{r^2}{h^2}x^2\right)dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\left[\rho\pi\left(\frac{r^2}{h^2}x^2\right)dx\right]\left(\frac{r^2}{h^2}x^2\right) = \frac{\rho\pi r^4}{2h^4}x^4 dx$.



Total Mass : Performing the integration, we have

$$m = \int_m dm = \int_0^h \rho\pi\left(\frac{r^2}{h^2}x^2\right)dx = \frac{\rho\pi r^2}{h^2}\left(\frac{x^3}{3}\right)\bigg|_0^h = \frac{1}{3}\rho\pi r^2 h$$

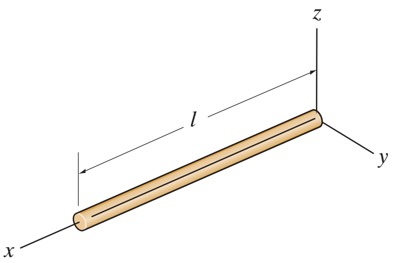
Mass Moment of Inertia : Performing the integration, we have

$$I_x = \int dI_x = \int_0^h \frac{\rho\pi r^4}{2h^4}x^4 dx = \frac{\rho\pi r^4}{2h^4}\left(\frac{x^5}{5}\right)\bigg|_0^h = \frac{1}{10}\rho\pi r^4 h$$

The mass moment of inertia expressed in terms of the total mass is

$$I_x = \frac{3}{10}\left(\frac{1}{3}\rho\pi r^2 h\right)r^2 = \frac{3}{10}mr^2 \qquad \textbf{Ans}$$

10–91. Determine the mass moment of inertia I_y of the slender rod. The rod is made of material having a variable density $\rho = \rho_0(1 + x/l)$, where ρ_0 is constant. The cross-sectional area of the rod is A. Express the result in terms of the mass m of the rod.



Differential Element: The mass of the differential element shown shaded in Fig. *a* is $dm = \rho dV = \rho_o\left(1+\frac{x}{l}\right)(Adx) = \rho_o A\left(1+\frac{x}{l}\right)dx$, where dA is the cross - sectional area of the rod. The mass moment of inertia of this element about the z axis is $dI_z = r^2 dm$. Here, $r = x$. Thus, $dI_z = \rho_o A\left[x^2\left(1+\frac{x}{l}\right)\right]dx = \rho_o A\left(x^2+\frac{x^3}{l}\right)dx$.

Mass: The mass of the rod can be determined by integrating dm. Thus,

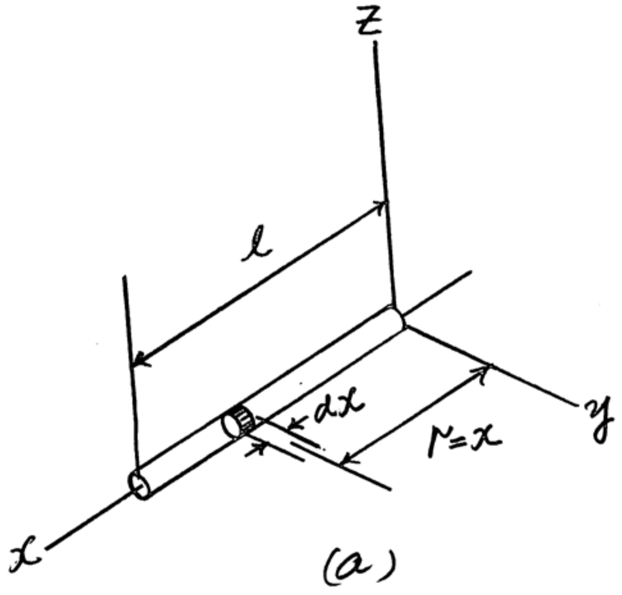
$$m = \int dm = \int_0^l \rho_o A\left[\left(1+\frac{x}{l}\right)\right]dx = \rho_o A\left(x+\frac{x^2}{2l}\right)\Big|_0^l = \frac{3}{2}\rho_o Al$$

Mass Moment of Inertia: Integrating dI_z,

$$I_z = \int dI_z = \int_0^l \rho_o A\left(x^2+\frac{x^3}{l}\right)dx = \rho_o A\left(\frac{x^3}{3}+\frac{x^4}{4l}\right)\Big|_0^l = \frac{7}{12}\rho_o Al^3$$

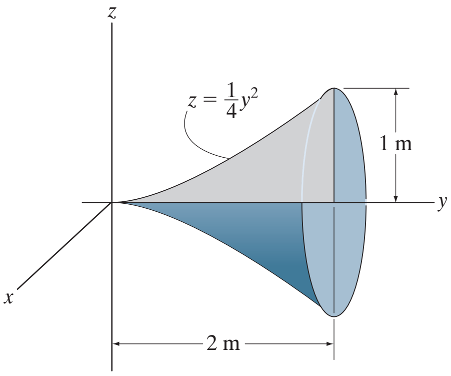
From the result of the mass, we obtain $\rho_o Al = \frac{2}{3}m$. Thus, I_z can be written as

$$I_z = \frac{7}{12}(\rho_o Al)l^2 = \frac{7}{12}\left(\frac{2}{3}m\right)l^2 = \frac{7}{18}ml^2$$ **Ans.**



(a)

***10–92.** Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the solid.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^2$. Thus, $dm = \rho\pi\left(\frac{1}{4}y^2\right)^2 dy = \frac{\rho\pi}{16}y^4 dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2$ $= \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(\frac{1}{4}y^2\right)^4 dy = \frac{\rho\pi}{512}y^8 dy$.

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{2\text{ m}} \frac{\rho\pi}{16}y^4 dy = \frac{\rho\pi}{16}\left(\frac{y^5}{5}\right)\Bigg|_0^{2\text{ m}} = \frac{2}{5}\rho\pi$$

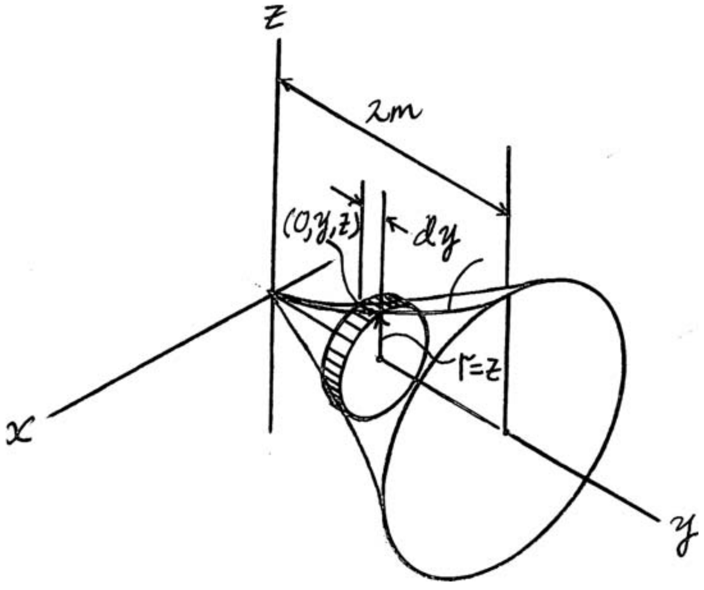
Mass Moment of Inertia: Integrating dI_y,

$$I_y = \int dI_y = \int_0^{2\text{ m}} \frac{\rho\pi}{512}y^8 dy$$

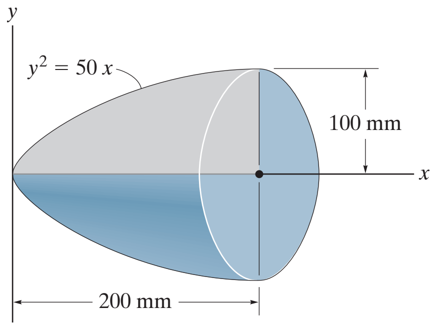
$$= \frac{\rho\pi}{512}\left(\frac{y^9}{9}\right)\Bigg|_0^{2\text{ m}} = \frac{\pi\rho}{9}$$

From the result of the mass, we obtain $\pi\rho = \frac{5m}{2}$. Thus, I_y can be written as

$$I_y = \frac{1}{9}\left(\frac{5m}{2}\right) = \frac{5}{18}m$$ **Ans.**



•10–93. The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \text{Mg/m}^3$.

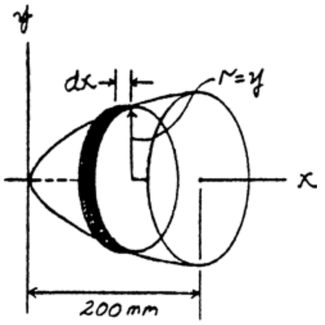


Differential Disk Element : The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2 dx = \rho\pi(50x)\,dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}[\rho\pi(50x)\,dx](50x) = \frac{\rho\pi}{2}\left(2500x^2\right)dx$.

Total Mass : Performing the integration, we have

$$m = \int_m dm = \int_0^{200\text{mm}} \rho\pi(50x)\,dx = \rho\pi\left(25x^2\right)\Big|_0^{200\text{mm}} = 1\left(10^6\right)\rho\pi$$

Mass Moment of Inertia : Performing the integration, we have

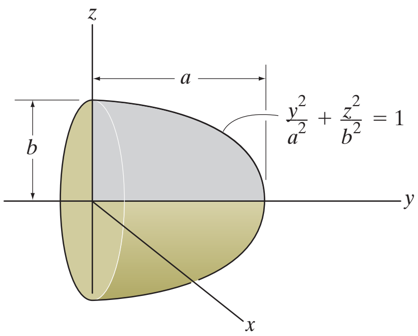


$$\begin{aligned} I_x = \int dI_x &= \int_0^{200\text{mm}} \frac{\rho\pi}{2}\left(2500x^2\right)dx \\ &= \frac{\rho\pi}{2}\left(\frac{2500x^3}{3}\right)\Big|_0^{200\text{mm}} \\ &= 3.333\left(10^9\right)\rho\pi \end{aligned}$$

The radius of gyration is

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{3.333(10^9)\rho\pi}{1(10^6)\rho\pi}} = 57.7\ \text{mm} \qquad \textbf{Ans}$$

10–94. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the semi-ellipsoid.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = b\sqrt{1+\frac{y^2}{a^2}}$.

Thus, $dm = \rho\pi\left(b\sqrt{1+\frac{y^2}{a^2}}\right)^2 dz = \rho\pi b^2\left(1-\frac{y^2}{a^2}\right)dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2$

$= \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(b\sqrt{1+\frac{y^2}{a^2}}\right)^4 dy = \frac{1}{2}\rho\pi b^4\left(1-\frac{y^2}{a^2}\right)^2 dy = \frac{1}{2}\rho\pi b^4\left(1+\frac{y^4}{a^4}-\frac{2y^2}{a^2}\right)dy.$

Mass: The mass of the semi - ellipsoid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^a \rho\pi b^2\left(1-\frac{y^2}{a^2}\right)dy = \rho\pi b^2\left(y-\frac{y^3}{3a^2}\right)\Bigg|_0^a = \frac{2}{3}\rho\pi a b^2$$

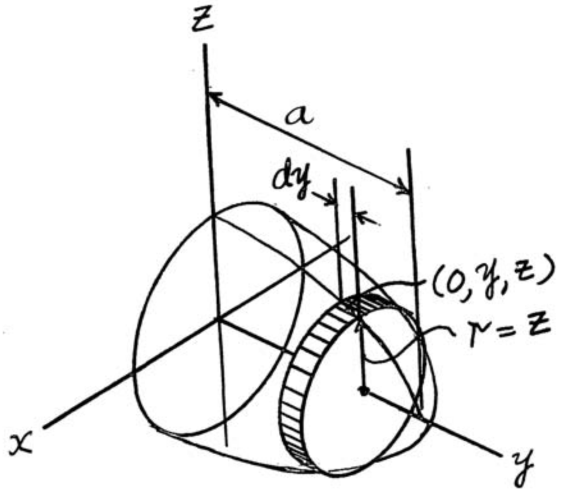
Mass Moment of Inertia: Integrating dI_y,

$$I_y = \int dI_y = \int_0^a \frac{1}{2}\rho\pi b^4\left(1+\frac{y^4}{a^4}-\frac{2y^2}{a^2}\right)dy$$

$$= \frac{1}{2}\rho\pi b^4\left(y+\frac{y^5}{5a^4}-\frac{2y^3}{3a^2}\right)\Bigg|_0^a = \frac{4}{15}\rho\pi a b^4$$

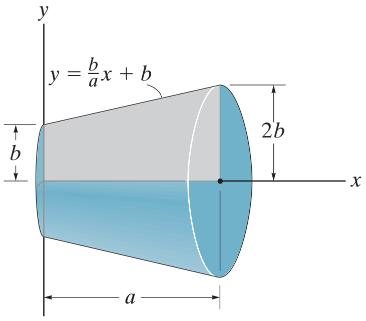
From the result of the mass, we obtain $\rho\pi a b^2 = \frac{3m}{2}$. Thus, I_y can be written as

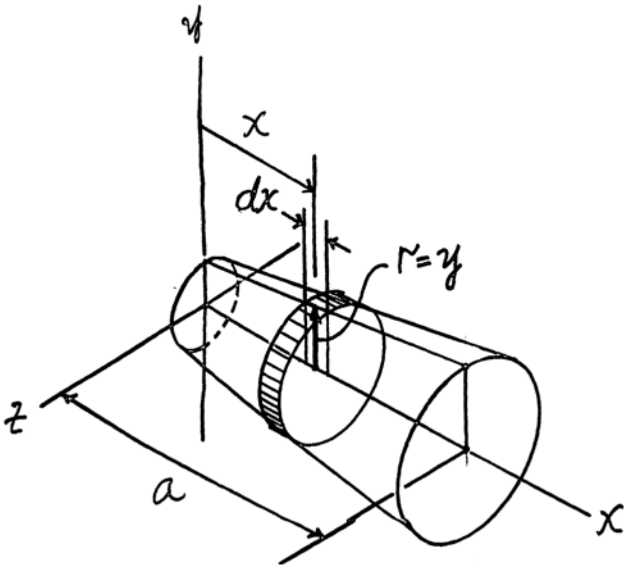
$$I_y = \frac{4}{15}\left(\rho\pi a b^2\right)b^2 = \frac{4}{15}\left(\frac{3m}{2}\right)b^2 = \frac{2}{5}mb^2$$ **Ans.**



(a)

10–95. The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The material has a constant density ρ.





$$dm = \rho dV = \rho\pi y^2 dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dm y^2 = \frac{1}{2}\rho\pi y^4 dx$$

$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

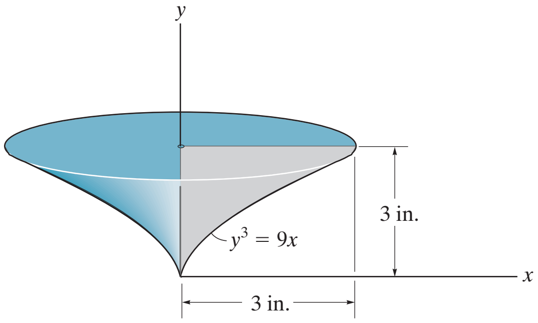
$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

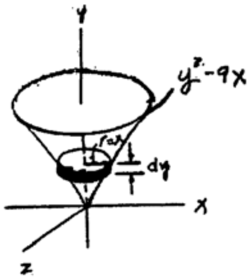
$$= \frac{31}{10}\rho\pi a b^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi a b^2$$

$I_x = \frac{93}{70}mb^2$ **Ans**

***10–96.** The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is $\gamma = 380\ \text{lb/ft}^3$.





The moment of inertia of the solid : The mass of the disk element $dm = \rho\pi x^2\,dy = \frac{1}{81}\rho\pi y^6\,dy$.

$$dI_y = \frac{1}{2}dm\,x^2$$

$$= \frac{1}{2}(\rho\pi x^2 dy)x^2$$

$$= \frac{1}{2}\rho\pi x^4 dy = \frac{1}{2(9^4)}\rho\pi y^{12} dy$$

$$I_y = \int dI_y = \frac{1}{2(9^4)}\rho\pi\int_0^3 y^{12} dy$$

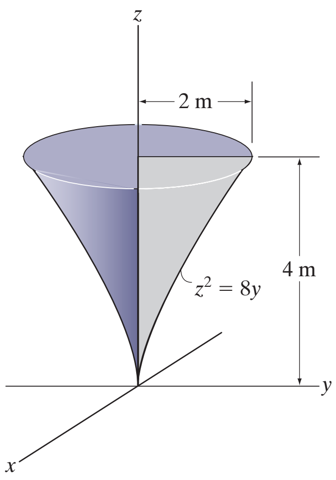
$$= 29.362\rho$$

The mass of the solid :

$$m = \int_m dm = \frac{1}{81}\rho\pi\int_0^3 y^6 dy = 12.118\rho$$

$$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.362\rho}{12.118\rho}} = 1.56\ \text{in.} \qquad \textbf{Ans}$$

•10–97. Determine the mass moment of inertia I_z of the solid formed by revolving the shaded area around the z axis. The density of the material is $\rho = 7.85\ \text{Mg/m}^3$.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. The mass moment of inertia of this element about the z axis is $dI_z = \frac{1}{2} dm r^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz$. Here, $r = z = \frac{z^2}{8}$.

Thus, $dI_z = \frac{1}{2}\rho\pi\left(\frac{z^2}{8}\right) = \frac{\rho\pi}{8192} z^8 dz$.

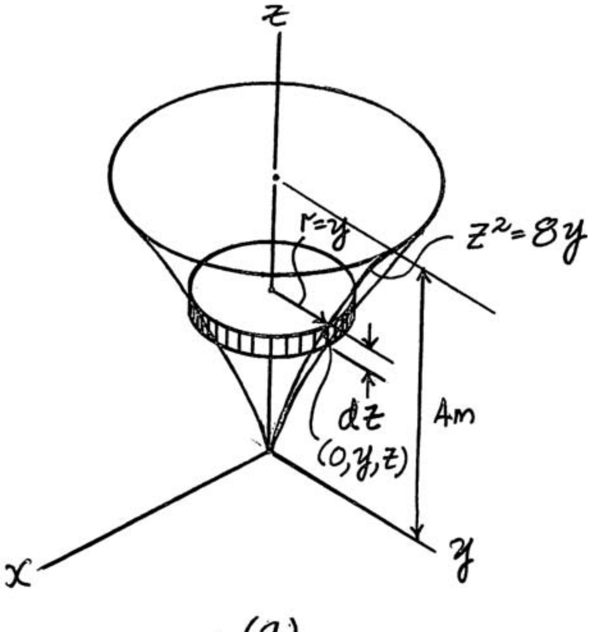
Mass Moment of Inertia: Integrating dI_z,

$$I_z = \int dI_z = \int_0^{4\text{ m}} \frac{\rho\pi}{8192} z^8 dz$$

$$= \frac{\rho\pi}{8192}\left(\frac{y^9}{9}\right)\Bigg|_0^{4\text{ m}} = \frac{32}{9}\pi\rho$$

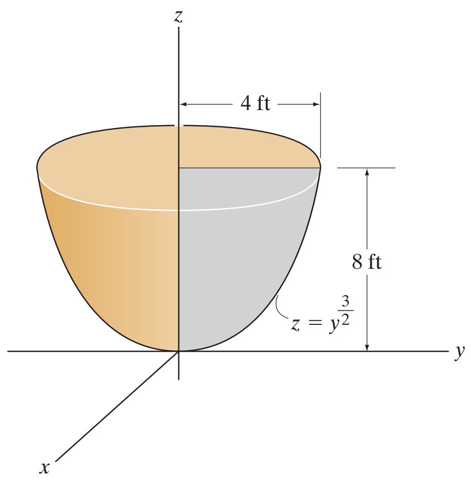
Substituting $\rho = 7.85(10^3)\ \text{kg/m}$ into I_z,

$$I_z = \frac{32}{9}\pi\rho\left[7.85(10^3)\right] = 87.7(10^3)\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

10–98. Determine the mass moment of inertia I_z of the solid formed by revolving the shaded area around the z axis. The solid is made of a homogeneous material that weighs 400 lb.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \left(\frac{\gamma}{g}\right)dV = \left(\frac{\gamma}{g}\right)\pi r^2 dz$. Here, $r = y = z^{2/3}$. Thus, $dm = \left(\frac{\gamma}{g}\right)\pi\left(z^{2/3}\right)^2 dz = \left(\frac{\gamma}{g}\right)\pi z^{4/3} dz$. The mass moment of inertia of this element about the z axis is $dI_y = \frac{1}{2} dm r^2$

$$= \frac{1}{2}\left[\left(\frac{\gamma}{g}\right)\pi r^2 dz\right] r^2 = \frac{1}{2}\left(\frac{\gamma}{g}\right)\pi r^4 dz = \frac{1}{2}\left(\frac{\gamma}{g}\right)\pi\left(z^{2/3}\right)^4 dz = \frac{\pi}{2}\left(\frac{\gamma}{g}\right) z^{8/3} dz.$$

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{8\text{ ft}} \frac{\gamma}{g}\pi z^{4/3} dz = \frac{\pi\gamma}{g}\left(\frac{3}{7} z^{7/3}\right)\Big|_0^{8\text{ ft}} = \frac{384\pi}{7g}\gamma$$

The mass of the solid is $m = \frac{400}{g}$ slug. Thus,

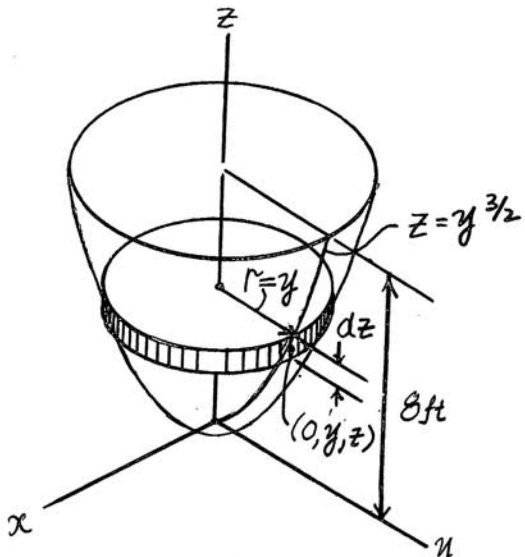
$$\frac{400}{g} = \frac{384\pi}{7g}\gamma \qquad \gamma = 2.321\ \text{lb/ft}^3$$

Mass Moment of Inertia: Integrating dI_y,

$$I_z = \int dI_z = \int_0^{8\text{ ft}} \frac{\pi}{2}\left(\frac{\gamma}{g}\right) z^{8/3} dz = \frac{\pi}{2}\left(\frac{\gamma}{g}\right)\left(\frac{3}{11} z^{11/3}\right)\Big|_0^{8\text{ ft}} = \frac{877.36\gamma}{g}$$

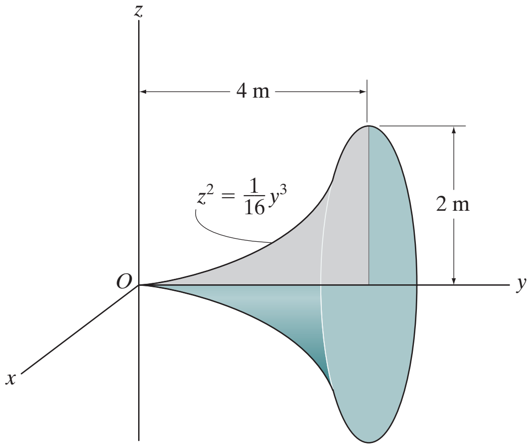
Substituting $\gamma = 2.321\ \text{lb/ft}^3$ and $g = 32.2\ \text{ft/s}^2$ into I_z,

$$I_z = \frac{877.36(2.321)}{32.2} = 63.2\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$



(a)

10–99. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The total mass of the solid is 1500 kg.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^{3/2}$.

Thus, $dm = \rho\pi\left(\frac{1}{4}y^{3/2}\right)^2 dy = \frac{\rho\pi}{16}y^3 dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2$

$= \frac{1}{2}\left(\rho\pi r^2 dy\right)r^2 = \frac{\rho\pi}{2}r^4 dy = \frac{\rho\pi}{2}\left(\frac{1}{4}y^{3/2}\right)^4 dy = \frac{\rho\pi}{512}y^6 dy.$

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{4\text{ m}} \frac{\rho\pi}{16}y^3 dy = \frac{\rho\pi}{16}\left(\frac{y^4}{4}\right)\Bigg|_0^{4\text{ m}} = 4\pi\rho$$

The mass of the solid is $m = 1500$ kg. Thus,

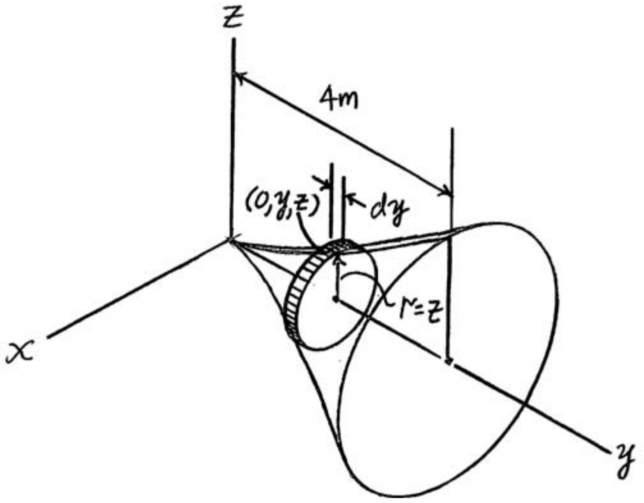
$$1500 = 4\pi\rho \qquad \rho = \frac{375}{\pi}\ \text{kg/m}^3$$

Mass Moment of Inertia: Integrating dI_y,

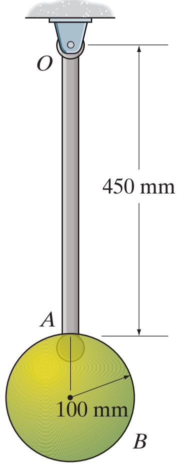
$$I_y = \int dI_y = \int_0^{4\text{ m}} \frac{\rho\pi}{512}y^6 dy = \frac{\rho\pi}{512}\left(\frac{y^7}{7}\right)\Bigg|_0^{4\text{ m}} = \frac{32\pi}{7}\rho$$

Substituting $\rho = \frac{375}{\pi}$ kg/m^3 into I_y,

$$I_y = \frac{32\pi}{7}\left(\frac{375}{\pi}\right) = 1.71(10^3)\ \text{kg}\cdot\text{m}^2$$ **Ans.**



***10–100.** Determine the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through point *O*. The slender rod has a mass of 10 kg and the sphere has a mass of 15 kg.



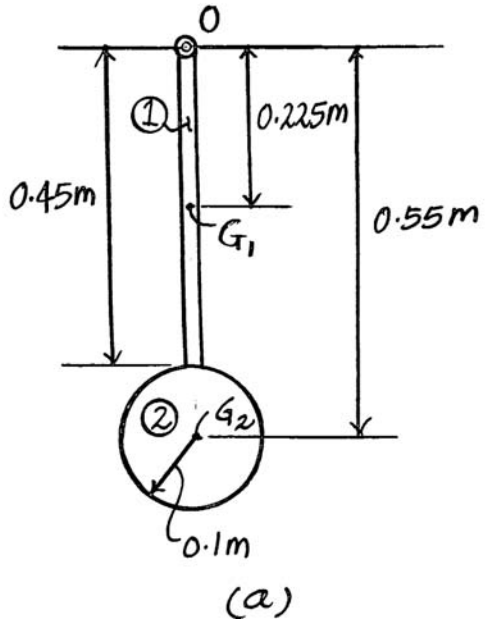
Composite Parts: The pendulum can be subdivided into two segments as shown in Fig. *a*. The perpendicular distances measured from the center of mass of each segment to the point *O* are also indicated.

Moment of Inertia: The moment of inertia of the slender rod segment (1) and the sphere segment (2) about the axis passing through their center of mass can be computed from $(I_G)_1 = \frac{1}{12}ml^2$ and $(I_G)_2 = \frac{2}{5}mr^2$. The mass moment of inertia of each segment about an axis passing through point *O* can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

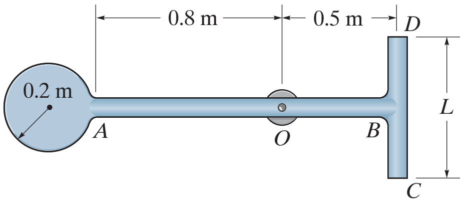
$$= \left[\frac{1}{12}(10)(0.45^2) + 10(0.225^2)\right] + \left[\frac{2}{5}(15)(0.1^2) + 15(0.55^2)\right]$$

$$= 5.27 \text{ kg}\cdot\text{m}^2$$ **Ans.**



(a)

•10–101. The pendulum consists of a disk having a mass of 6 kg and slender rods *AB* and *DC* which have a mass per unit length of 2 kg/m. Determine the length *L* of *DC* so that the center of mass is at the bearing *O*. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through point *O*?

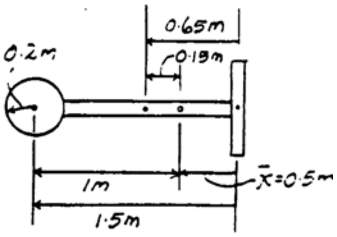


Location of Centroid : This problem requires $\bar{x} = 0.5$ m.

$$\bar{x} = \frac{\Sigma\tilde{x}m}{\Sigma m}$$

$$0.5 = \frac{1.5(6) + 0.65[1.3(2)] + 0[L(2)]}{6 + 1.3(2) + L(2)}$$

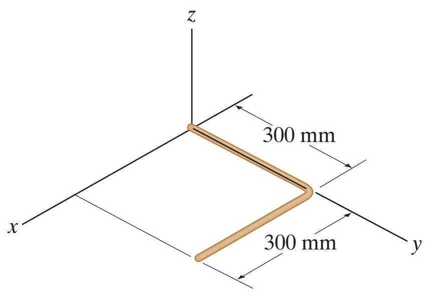
$$L = 6.39 \text{ m} \qquad \textbf{Ans}$$



Mass Moment of Inertia About an Axis Through Point O : The mass moment of inertia of each rod segment and disk about an axis passing through the center of mass can be determine using $(I_G)_i = \frac{1}{12}ml^2$ and $(I_G)_i = \frac{1}{2}mr^2$. Applying Eq. 10 – 15, we have

$$\begin{aligned} I_O &= \Sigma(I_G)_i + m_i d_i^2 \\ &= \frac{1}{12}[1.3(2)](1.3^2) + [1.3(2)](0.15^2) \\ &\qquad + \frac{1}{12}[6.39(2)](6.39^2) + [6.39(2)](0.5^2) \\ &\qquad\quad + \frac{1}{2}(6)(0.2^2) + 6(1^2) \\ &= 53.2 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans} \end{aligned}$$

10–102. Determine the mass moment of inertia of the 2-kg bent rod about the *z* axis.



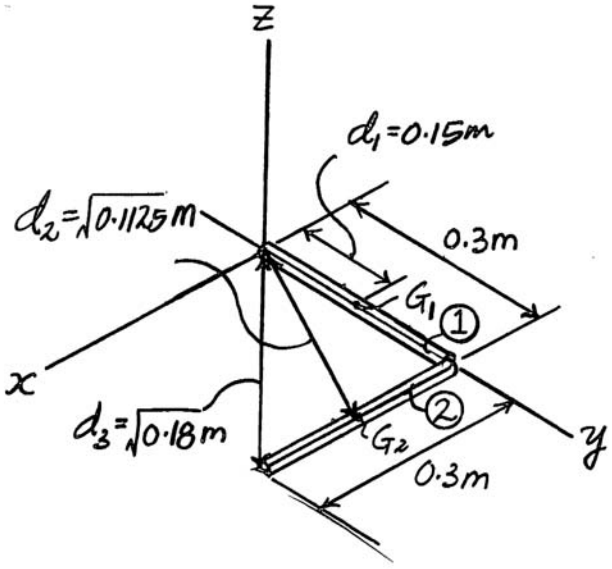
Composite Parts: The bent rod can be subdivided into two segments as shown in Fig. *a*.

Mass moment of Inertia: Here, the mass for each segment is $m_1 = m_2 = \frac{2\text{ kg}}{2} = 1$ kg. The perpendicular distances measured from the centers of mass of segments (1) and (2) are $d_1 = 0.15$ m and $d_2 = \sqrt{0.3^2 + 0.15^2} = \sqrt{0.1125}$ m, respectively. Thus, the mass moment of inertia of each segment about the *z* axis can be determined using the parallel - axis theorem.

$$I_z = \Sigma\left(I_z\right)_G + md^2$$

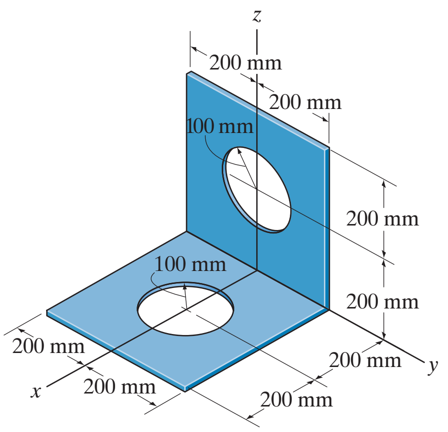
$$= \left[\frac{1}{12}(1)(0.3^2) + 1(0.15^2)\right] + \left[\frac{1}{12}(1)(0.3^2) + 1\left(\sqrt{0.1125}\right)^2\right]$$

$$= 0.150\text{ kg}\cdot\text{m}^2$$ **Ans.**



(a)

10–103. The thin plate has a mass per unit area of 10 kg/m^2. Determine its mass moment of inertia about the y axis.



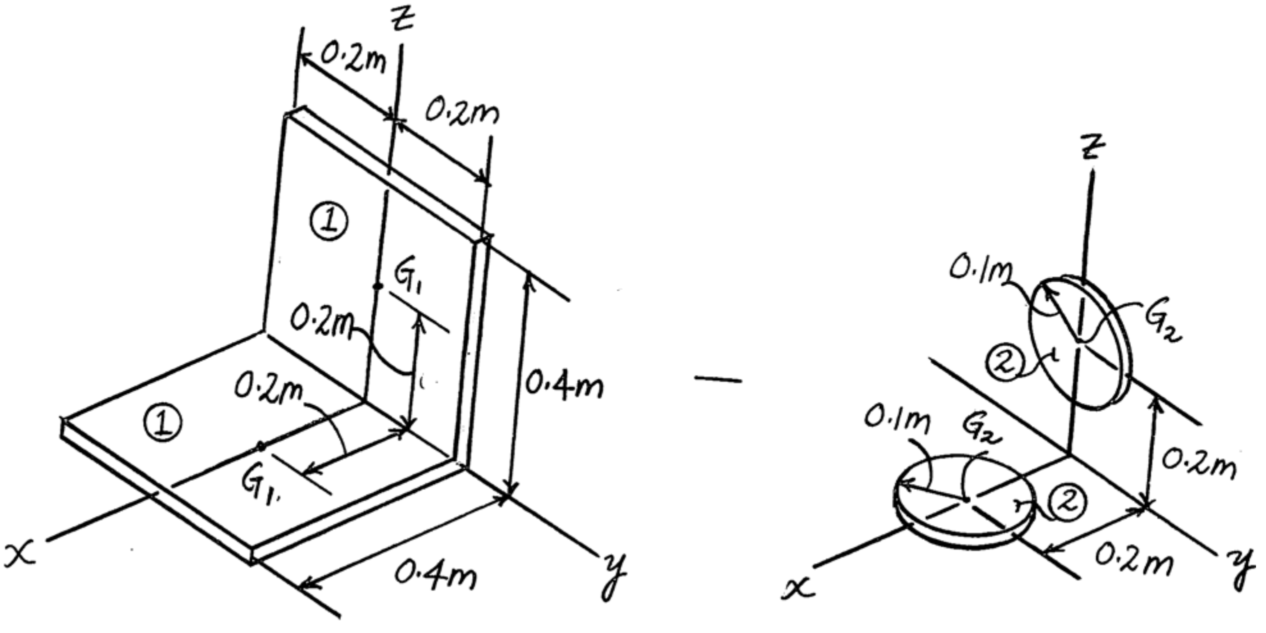
Composite Parts: The thin plate can be subdivided into segments as shown in Fig. *a*. Since the segments labeled (2) are both holes, they should be considered as negative parts.

Mass moment of Inertia: The mass of segments (1) and (2) are $m_1 = 0.4(0.4)(10) = 1.6$ kg and $m_2 = \pi(0.1^2)(10) = 0.1\pi$ kg. The perpendicular distances measured from the centroid of each segment to the y axis are indicated in Fig. *a*. The mass moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem.

$$I_y = \Sigma\left(I_y\right)_G + md^2$$

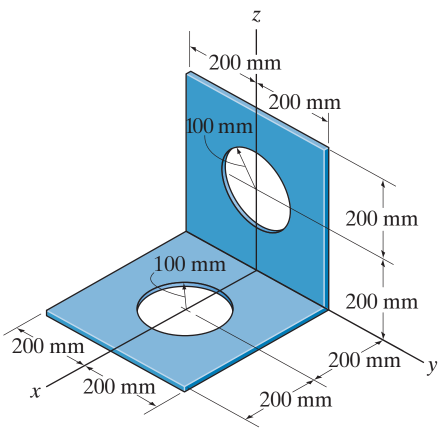
$$= 2\left[\frac{1}{12}(1.6)(0.4^2) + 1.6(0.2^2)\right] - 2\left[\frac{1}{4}(0.1\pi)(0.1^2) + 0.1\pi(0.2^2)\right]$$

$$= 0.144 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

***10–104.** The thin plate has a mass per unit area of 10 kg/m^2. Determine its mass moment of inertia about the z axis.



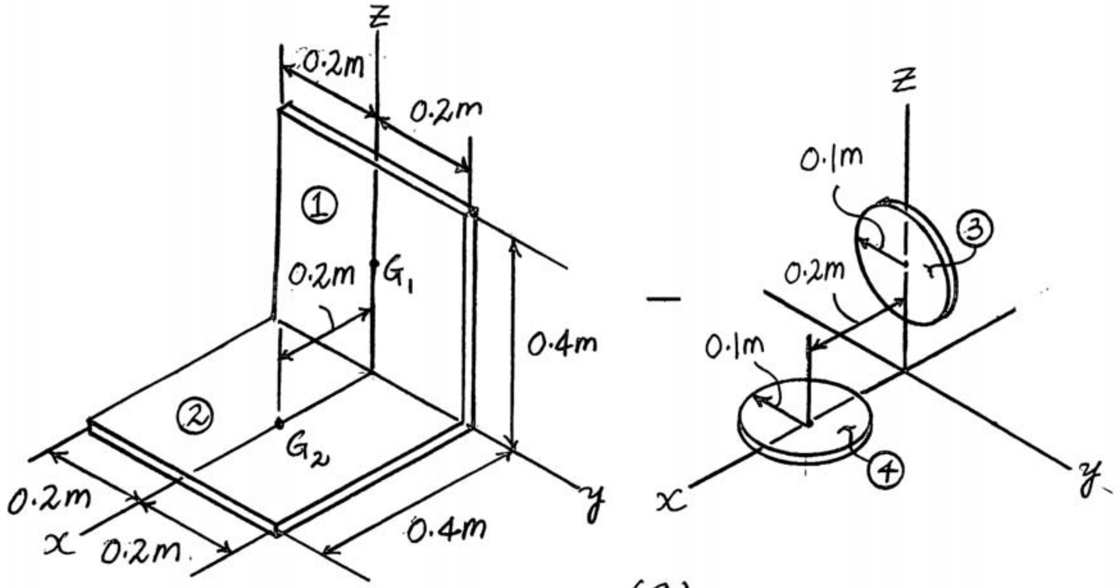
Composite Parts: The thin plate can be subdivided into four segments as shown in Fig. *a*. Since segments (3) and (4) are both holes, they should be considered as negative parts.

Mass moment of Inertia: Here, the mass for segments (1), (2), (3), and (4) are $m_1 = m_2 = 0.4(0.4)(10) = 1.6$ kg and $m_3 = m_4 = \pi(0.1^2)(10) = 0.1\pi$ kg. The mass moment of inertia of each segment about the z axis can be determined using the parallel - axis theorem.

$$I_z = \Sigma\left(I_z\right)_G + md^2$$

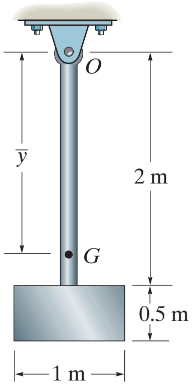
$$= \frac{1}{12}(1.6)(0.4^2) + \left[\frac{1}{12}(1.6)(0.4^2 + 0.4^2) + 1.6(0.2^2)\right] - \frac{1}{4}(0.1\pi)(0.1^2) - \left[\frac{1}{2}(0.1\pi)(0.1^2) + 0.1\pi(0.2^2)\right]$$

$$= 0.113 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

•10–105. The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then find the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.

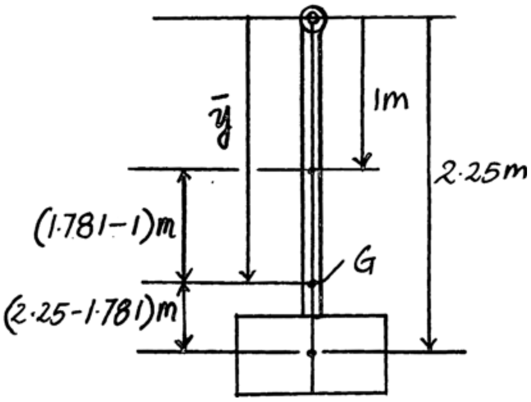


$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{1(3) + 2.25(5)}{3+5} = 1.781 \text{ m} = 1.78 \text{ m} \qquad \text{Ans}$$

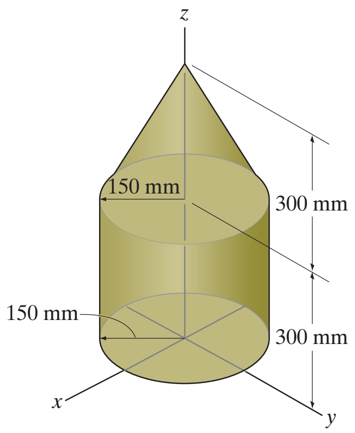
$$I_G = \Sigma \bar{I}_{G'} + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2 \qquad \text{Ans}$$



10–106. The cone and cylinder assembly is made of homogeneous material having a density of 7.85 Mg/m^3. Determine its mass moment of inertia about the z axis.



Composite Parts: The assembly can be subdivided into a circular cone segment (1) and a cylindrical segment (2) as shown in Fig. *a*.

Mass: The mass of each segment is calculated as

$$m_1 = \rho V_1 = \rho\left(\frac{1}{3}\pi r^2 h\right) = 7.85(10^3)\left[\frac{1}{3}\pi(0.15^2)(0.3)\right] = 17.6625\pi \text{ kg}$$

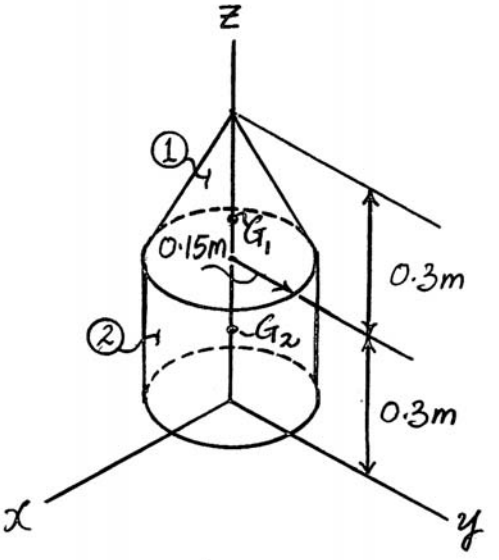
$$m_2 = \rho V_2 = \rho\left(\pi r^2 h\right) = 7.85(10^3)\left[\pi(0.15^2)(0.3)\right] = 52.9875\pi \text{ kg}$$

Mass Moment of Inertia: Since the z axis is parallel to the axis of the cone and cylinder and passes through their center of mass, their mass moment of inertia can be computed from $(I_z)_1 = \frac{3}{10}m_1 r^2$ and $(I_z)_2 = \frac{1}{2}m_2 r^2$. Thus,

$$I_z = (I_z)_1 + (I_z)_2$$

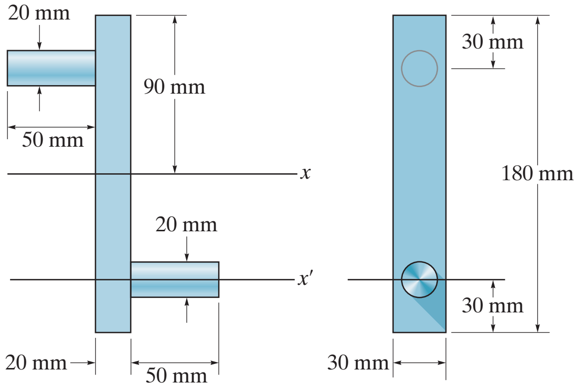
$$= \frac{3}{10}(17.6625\pi)(0.15^2) + \frac{1}{2}(52.9875\pi)(0.15^2)$$

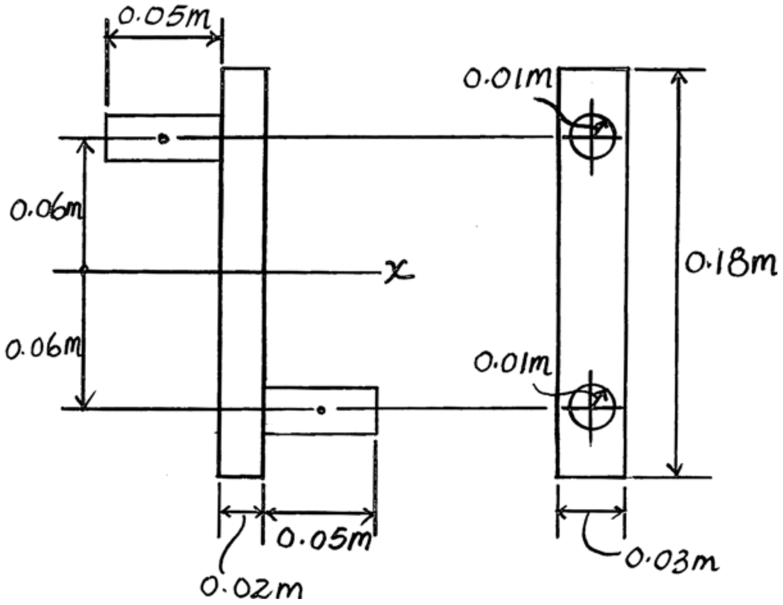
$$= 2.247 \text{ kg}\cdot\text{m}^2 = 2.25 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

10–107. Determine the mass moment of inertia of the overhung crank about the x axis. The material is steel having a density of $\rho = 7.85\ \mathrm{Mg/m^3}$.





$$m_c = 7.85(10^3)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \mathrm{kg}$$

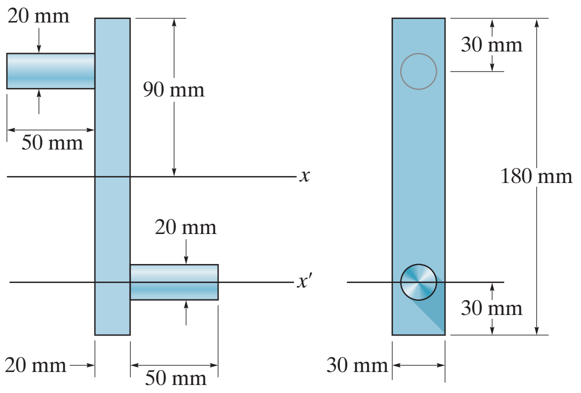
$$m_p = 7.85(10^3)\left((0.03)(0.180)(0.02)\right) = 0.8478\ \mathrm{kg}$$

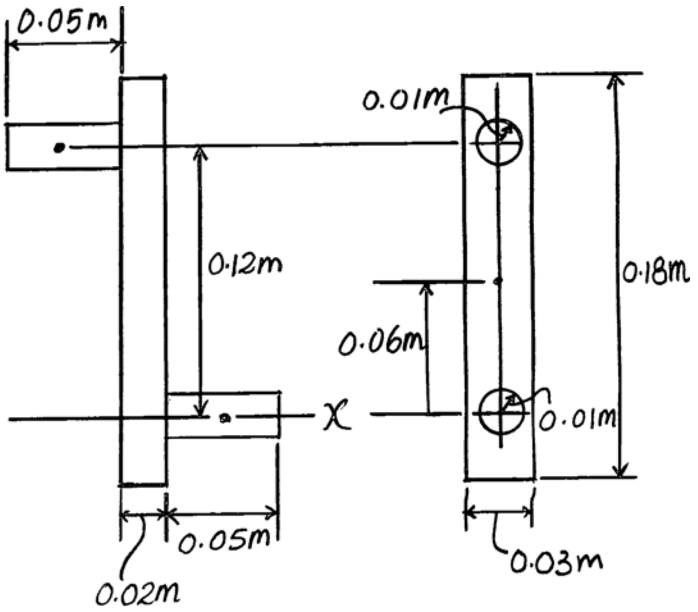
$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

$$+\left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right)\right]$$

$$= 0.00325\ \mathrm{kg \cdot m^2} = 3.25\ \mathrm{g \cdot m^2} \quad \text{Ans}$$

***10–108.** Determine the mass moment of inertia of the overhung crank about the x' axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.





$$m_c = 7.85(10^3)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

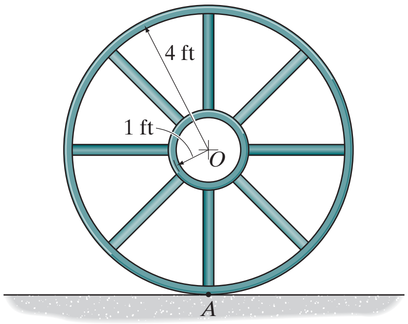
$$m_p = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_{x'} = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2 \quad \textbf{Ans.}$$

•10–109. If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A.



Composite Parts: The wheel can be subdivided into the segments shown in Fig. a. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ft $= 2.5$ ft from point O can be grouped as segment (2).

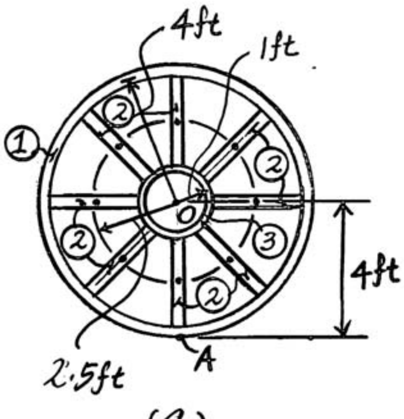
Mass Moment of Inertia: First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{1}{12}\left(\frac{20}{32.2}\right)(3^2) + \left(\frac{20}{32.2}\right)(2.5^2)\right] + \left(\frac{15}{32.2}\right)(1^2)$$

$$= 84.94 \text{ slug} \cdot \text{ft}^2$$

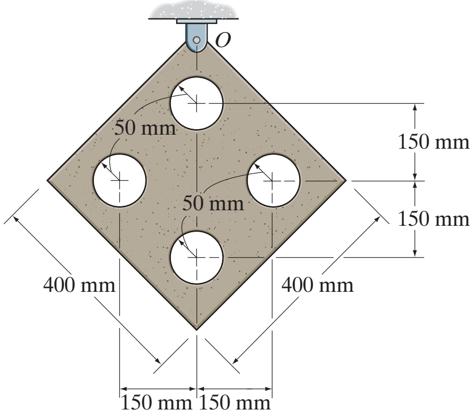
The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A can be found using the parallel - axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404$ slug and $d = 4$ ft. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58 \text{ slug} \cdot \text{ft}^2 = 222 \text{ slug} \cdot \text{ft}^2$$ **Ans.**



(a)

10–110. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point *O*. The material has a mass per unit area of 20 kg/m^2.



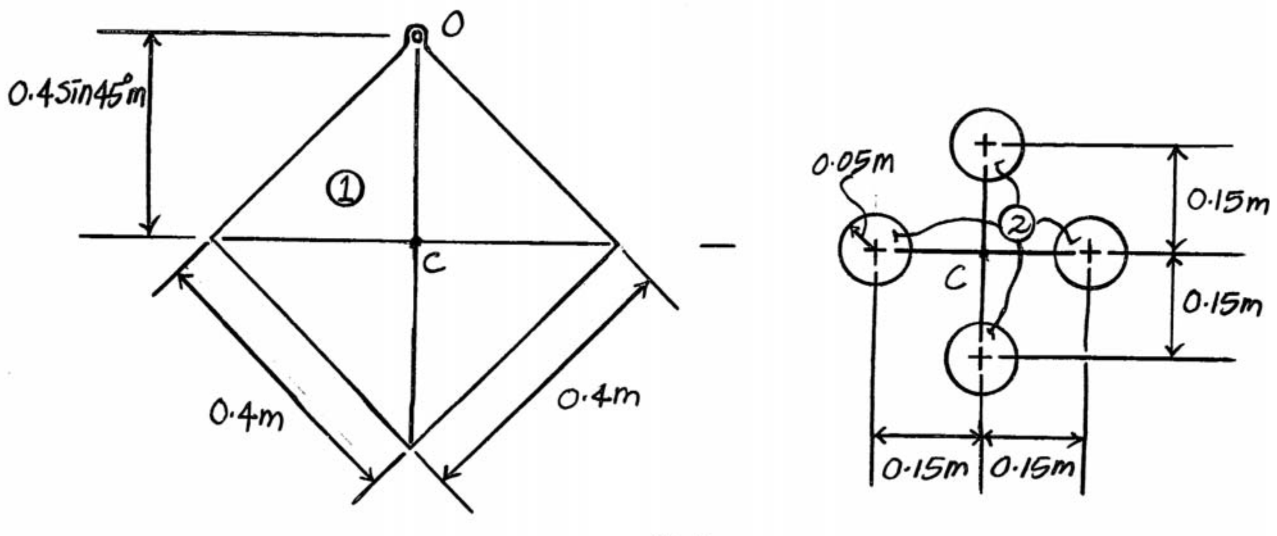
Composite Parts: The plate can be subdivided into the segments shown in Fig. *a*. Here. the four similar holes of which the perpendicular distances measured from their centers of mass to point *C* are the same and can be grouped as segment (2). This segment should be considered as a negative part.

Mass Moment of Inertia: The mass of segments (1) and (2) are $m_1 = (0.4)(0.4)(20) = 3.2$ kg and $m_2 = \pi(0.05^2)(20) = 0.05\pi$ kg, respectively. The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point *C* is

$$I_C = \frac{1}{12}(3.2)(0.4^2 + 0.4^2) - 4\left[\frac{1}{2}(0.05\pi)(0.05^2) + 0.05\pi(0.15^2)\right]$$
$$= 0.7041 \text{ kg}\cdot\text{m}^2$$

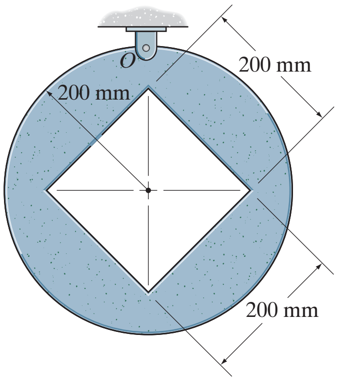
The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point *O* can be determined using the parallel - axis theorem $I_O = I_C + md^2$, where $m = m_1 - m_2 = 3.2 - 4(0.05\pi) = 2.5717$ kg and $d = 0.4\sin 45$. Thus,

$$I_O = 0.07041 + 2.5717(0.4\sin 45°)^2 = 0.276 \text{ kg}\cdot\text{m}^2$$ **Ans.**



(a)

10–111. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



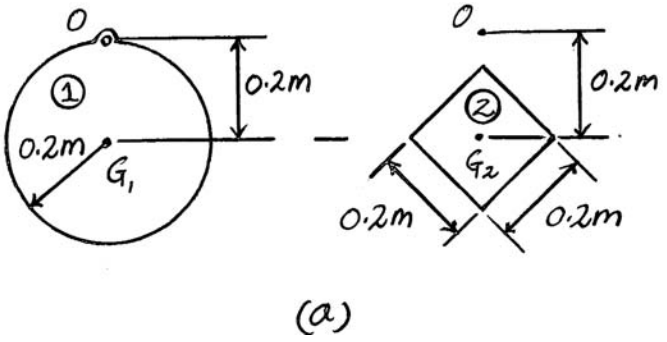
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass moment of Inertia: The masses of segmemts (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the point O for each segment can be determined using the parallel - axis theorem.

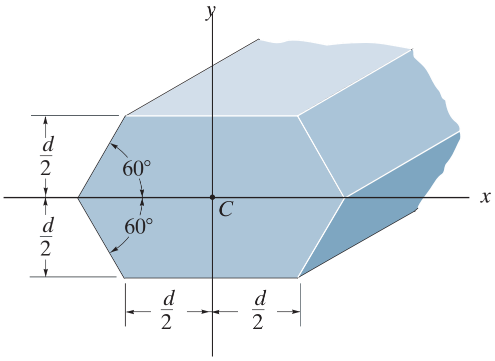
$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\,\text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



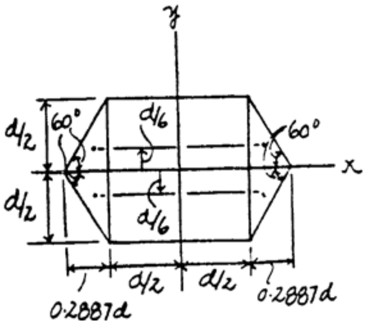
***10–112.** Determine the moment of inertia of the beam's cross-sectional area about the *x* axis which passes through the centroid *C*.



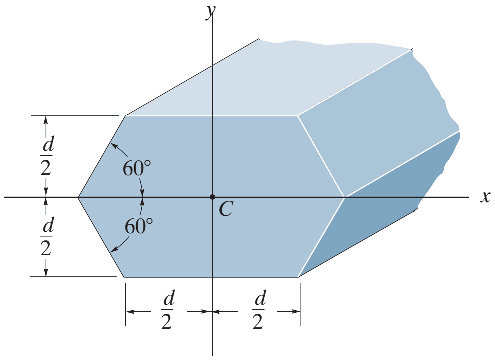
Moment of Inertia : The moment of inertia about the *x* axis for the composite beam's cross section can be determined using the parallel – axis theorem $I_x = \Sigma\left(\bar{I}_x + Ad_y^2\right)_i$.

$$I_y = \left[\frac{1}{12}(d)\left(d^3\right) + 0\right] + 4\left[\frac{1}{36}(0.2887d)\left(\frac{d}{2}\right)^3 + \frac{1}{2}(0.2887d)\left(\frac{d}{2}\right)\left(\frac{d}{6}\right)^2\right]$$

$$= 0.0954d^4 \qquad \textbf{Ans}$$



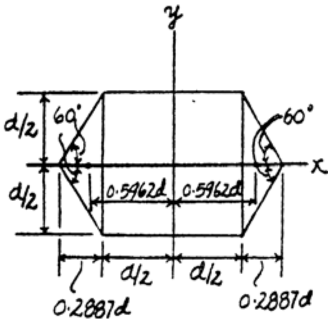
•10–113. Determine the moment of inertia of the beam's cross-sectional area about the *y* axis which passes through the centroid *C*.



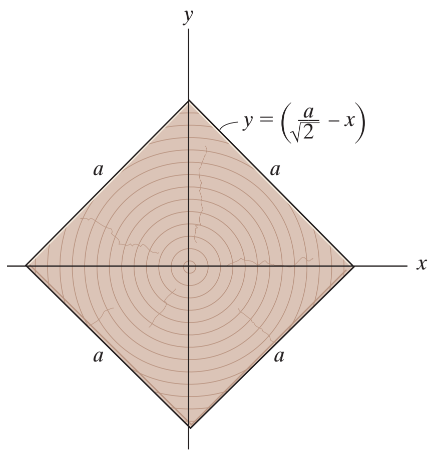
Moment of Inertia : The moment of inertia about *y*axis for the composite beam's cross section can be determined using the parallel – axis theorem $I_y = \Sigma\left(\bar{I}_y + Ad_x^2\right)_i$.

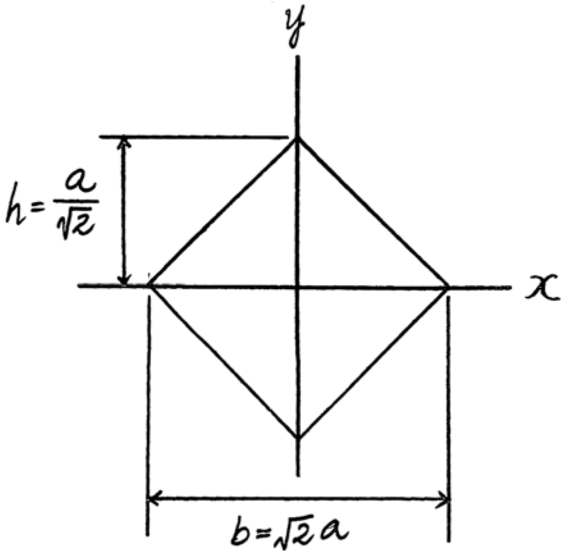
$$I_y = \left[\frac{1}{12}(d)\left(d^3\right) + 0\right] + 2\left[\frac{1}{36}(d)(0.2887d)^3 + \frac{1}{2}(d)(0.2887d)(0.5962d)^2\right]$$

$$= 0.187d^4 \qquad \textbf{Ans}$$



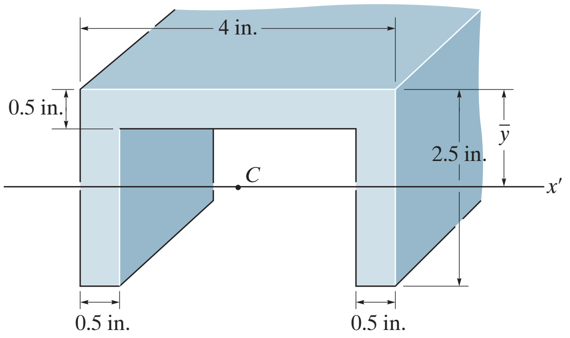
10–114. Determine the moment of inertia of the beam's cross-sectional area about the x axis.

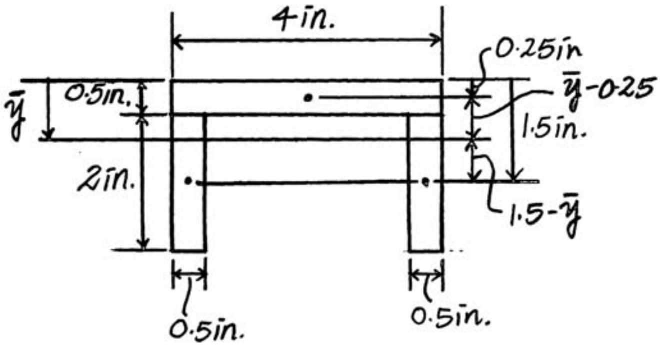




$$I_x = 2\left(\frac{bh^3}{12}\right) = 2\left(\frac{\sqrt{2}\,a\left(\frac{a}{\sqrt{2}}\right)^3}{12}\right) = \frac{1}{12}a^4 \qquad \textbf{Ans}$$

10–115. Determine the moment of inertia of the beam's cross-sectional area with respect to the *x*′ axis passing through the centroid *C*.



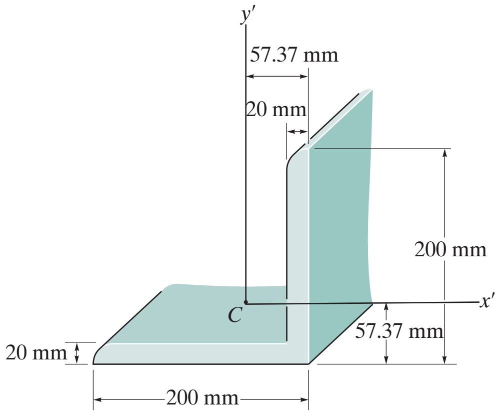


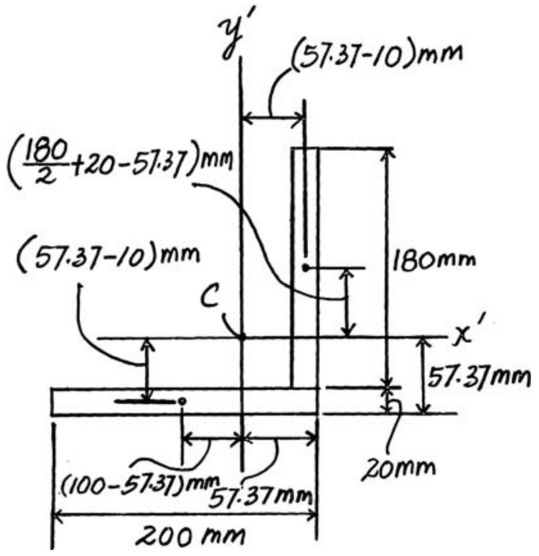
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.25(0.5)(4) + 2\left[1.5(2)(0.5)\right]}{0.5(4) + 2(2)(0.5)} = 0.875 \text{ in.}$$

$$I_{x'} = \frac{1}{12}(4)(0.5)^3 + 4(0.5)(0.875 - 0.25)^2 + 2\left[\frac{1}{12}(0.5)(2)^3 + (0.5)(2)(1.5 - 0.875)^2\right.$$

$$= 2.27 \text{ in}^4 \quad \textbf{Ans}$$

***10–116.** Determine the product of inertia for the angle's cross-sectional area with respect to the *x*′ and *y*′ axes having their origin located at the centroid *C*. Assume all corners to be right angles.



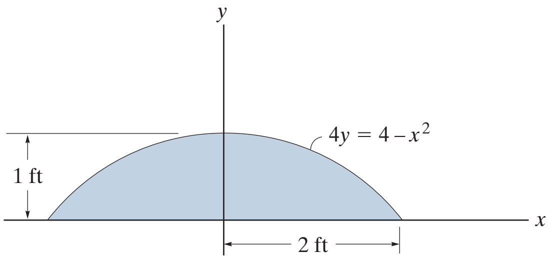


$$I_{xy} = \Sigma\tilde{x}\,\tilde{y}\,A$$

$$= \left(\frac{180}{2} + 20 - 57.37\right)(57.37 - 10)(180)(20) + (-(57.37 - 10))(-(100 - 57.37))(200)(20)$$

$$= 17.1(10)^6 \text{ mm}^4 \quad \textbf{Ans}$$

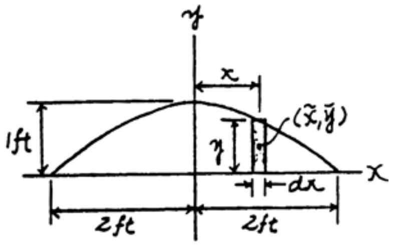
•10–117. Determine the moment of inertia of the area about the y axis.



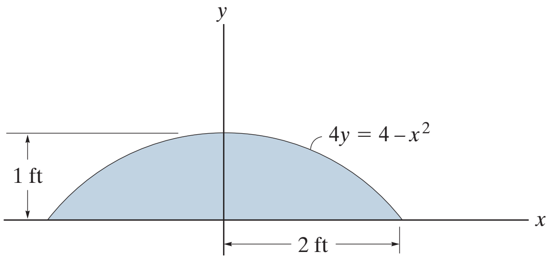
Differential Element : Here, $y = \frac{1}{4}\left(4 - x^2\right)$. The area of the differential element parallel to the y axis is $dA = y\,dx = \frac{1}{4}\left(4 - x^2\right) dx$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$
\begin{aligned}
I_y &= \int_A x^2\,dA = \frac{1}{4}\int_{-2\text{ft}}^{2\text{ft}} x^2\left(4 - x^2\right) dx \\
&= \frac{1}{4}\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]\Bigg|_{-2\text{ft}}^{2\text{ft}} \\
&= 2.13\ \text{ft}^4 \qquad \textbf{Ans}
\end{aligned}
$$



10–118. Determine the moment of inertia of the area about the x axis.

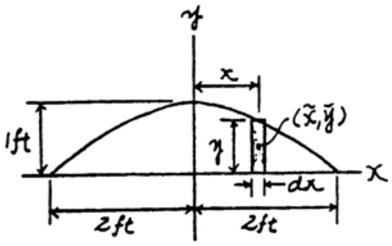


Differential Element : Here, $y = \frac{1}{4}\left(4 - x^2\right)$. The area of the differential element parallel to the y axis is $dA = y\,dx$. The moment of inertia of this differential element about the x axis is

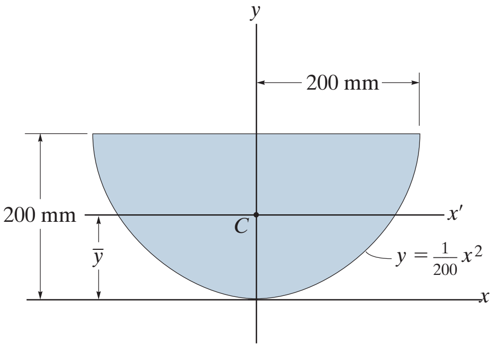
$$
\begin{aligned}
dI_x &= d\bar{I}_{x'} + dA\,\tilde{y}^2 \\
&= \frac{1}{12}(dx)y^3 + y\,dx\left(\frac{y}{2}\right)^2 \\
&= \frac{1}{3}\left[\frac{1}{4}\left(4 - x^2\right)\right]^3 dx \\
&= \frac{1}{192}\left(-x^6 + 12x^4 - 48x^2 + 64\right) dx
\end{aligned}
$$

Moment of inertia : Performing the integration, we have

$$
\begin{aligned}
I_x &= \int dI_x = \frac{1}{192}\int_{-2\text{ft}}^{2\text{ft}} \left(-x^6 + 12x^4 - 48x^2 + 64\right) dx \\
&= \frac{1}{192}\left(-\frac{1}{7}x^7 + \frac{12}{5}x^5 - 16x^3 + 64x\right)\Bigg|_{-2\text{ft}}^{2\text{ft}} \\
&= 0.610\ \text{ft}^4 \qquad \textbf{Ans}
\end{aligned}
$$



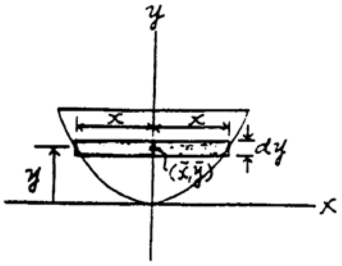
10–119. Determine the moment of inertia of the area about the x axis. Then, using the parallel-axis theorem, find the moment of inertia about the x' axis that passes through the centroid C of the area. $\bar{y} = 120$ mm.



Differential Element : Here, $x = \sqrt{200}y^{\frac{1}{2}}$. The area of the differential element parallel to the x axis is $dA = 2xdy = 2\sqrt{200}y^{\frac{1}{2}}dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^{200\text{mm}} y^2\left(2\sqrt{200}y^{\frac{1}{2}}dy\right)$$
$$= 2\sqrt{200}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Big|_0^{200\text{mm}}$$
$$= 914.29(10^6)\text{ mm}^4 = 914(10^6)\text{ mm}^4 \qquad \textbf{Ans}$$

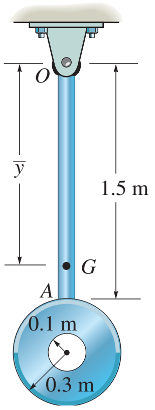


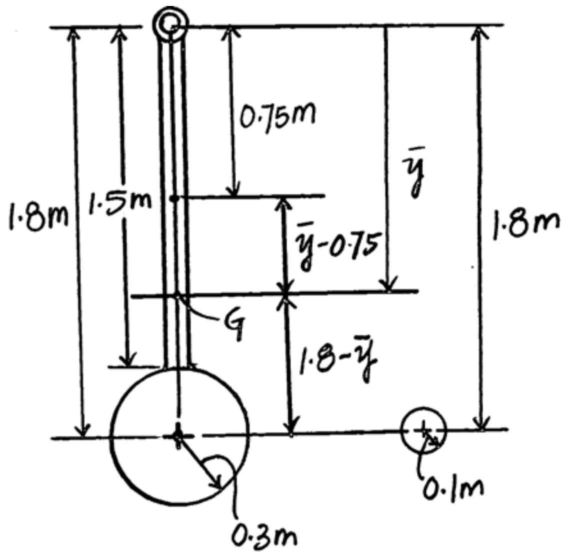
The moment of inertia about the x' axis can be determined using the parallel – axis theorem. The area is $A = \int_A dA = \int_0^{200\text{mm}} 2\sqrt{200}y^{\frac{1}{2}}dy = 53.33(10^3)\text{ mm}^2$

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$914.29(10^6) = \bar{I}_{x'} + 53.33(10^3)(120^2)$$

$$\bar{I}_{x'} = 146(10^6)\text{ mm}^4 \qquad \textbf{Ans}$$

***10–120.** The pendulum consists of the slender rod *OA*, which has a mass per unit length of 3 kg/m. The thin disk has a mass per unit area of $12\ \text{kg/m}^2$. Determine the distance $\bar{y}$ to the center of mass *G* of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through *G*.





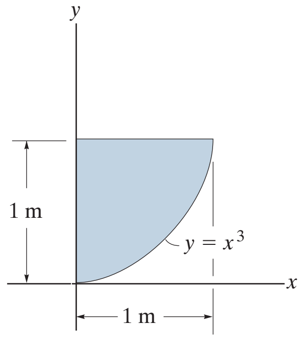
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.75[1.5(3)] + 1.8[\pi(0.3)^2(12)] - 1.8[\pi(0.1)^2(12)]}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12)}$$

$$= 1.1713\ \text{m} = 1.17\ \text{m} \qquad \textbf{Ans}$$

$$I_G = \frac{1}{12}[1.5(3)](1.5)^2 + [1.5(3)](1.1713 - 0.75)^2 + \frac{1}{2}[\pi(0.3)^2(12)](0.3)^2 + [\pi(0.3)^2(12)](1.8 - 1.1713)^2$$

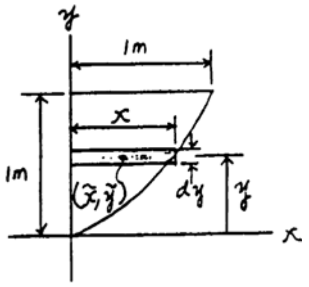
$$- \frac{1}{2}[\pi(0.1)^2(12)](0.1)^2 - [\pi(0.1)^2(12)](1.8 - 1.1713)^2 \qquad = 2.99\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

•10–121. Determine the product of inertia of the area with respect to the *x* and *y* axes.



Differential Element : Here, $x = y^{\frac{1}{3}}$. The area of the differential element parallel to the x axis is $dA = x\,dy = y^{\frac{1}{3}}dy$. The coordinates of the centroid for this element are $\tilde{x} = \frac{x}{2} = \frac{1}{2}y^{\frac{1}{3}}$, $\tilde{y} = y$. Then the product of inertia for this element i

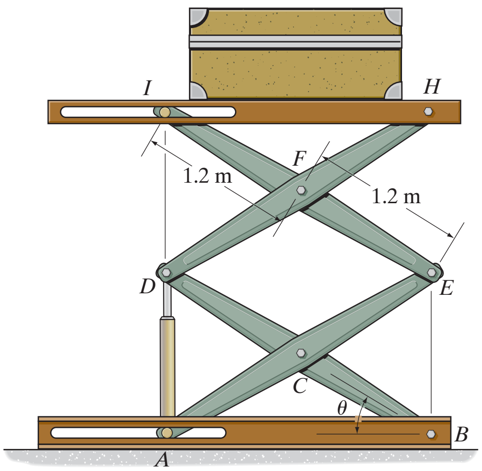
$$\begin{aligned} dI_{xy} &= d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y} \\ &= 0 + \left(y^{\frac{1}{3}}dy\right)\left(\frac{1}{2}y^{\frac{1}{3}}\right)(y) \\ &= \frac{1}{2}y^{\frac{5}{3}}dy \end{aligned}$$



Product of Inertia : Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \int_0^{1\text{m}} \frac{1}{2}y^{\frac{5}{3}}dy = \frac{3}{16}y^{\frac{8}{3}}\Big|_0^{1\text{m}} = 0.1875\ \text{m}^4 \qquad \textbf{Ans}$$

•11–1. The 200-kg crate is on the lift table at the position $\theta = 30°$. Determine the force in the hydraulic cylinder AD for equilibrium. Neglect the mass of the lift table's components.



Free - Body Diagram: When θ undergoes a positive virtual displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the force in hydraulic cylinder $\mathbf{F}_{AD}$ acting at point D and the weight of the crate $\mathbf{W}_J$ do work when the virtual displacements take place.

Virtual Displacement: The position of $\mathbf{F}_{AD}$ acting at point D and the point of application of $\mathbf{W}_J$ are specified by the position coordinates y_D and y_J, measured from the fixed point B.

$$y_D = 2.4\sin\theta \qquad \delta y_D = 2.4\cos\theta\delta\theta \qquad (1)$$

$$y_J = 2(2.4\sin\theta) + b \qquad \delta y_J = 4.8\cos\theta\delta\theta \qquad (2)$$

Virtual Work Equation: Since $\mathbf{F}_{AD}$ acts towards the positive sense of its corresponding virtual displacement, its work is positive. The work of $\mathbf{W}_J$ is negative since it acts towards the negative sense of its corresponding virtual displacement.

$$\delta U = 0; \qquad F_{AD}\,\delta y_D + [-200(9.81)\delta y_J] = 0 \qquad (3)$$

Substituting Eqs. (1) and (2) into Eq. (3),

$$F_{AD}(2.4\cos\theta\delta\theta) - 200(9.81)(4.8\cos\theta\delta\theta) = 0$$

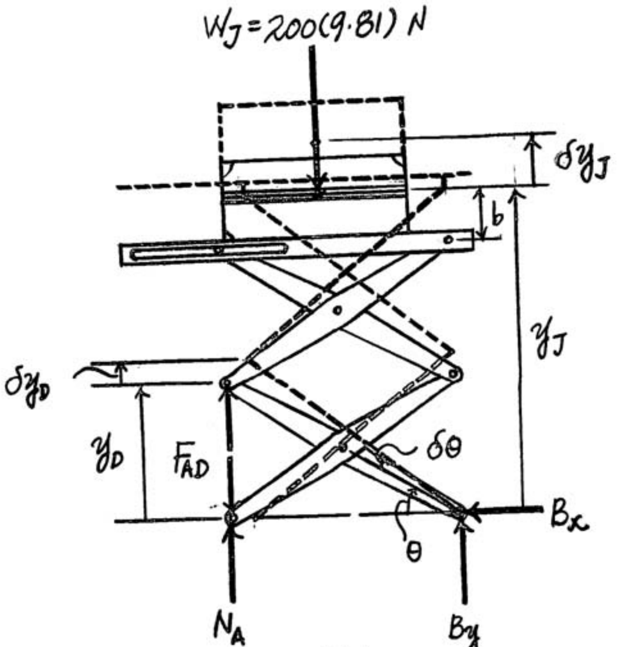
$$\cos\theta\delta\theta(2.4F_{AD} - 9417.6) = 0$$

Since $\cos\theta\delta\theta \neq 0$, then

$$2.4F_{AD} - 9417.6 = 0$$

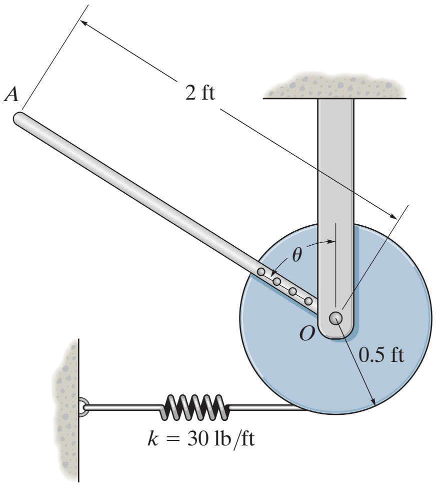
$$F_{AD} = 3924\text{ N} = 3.92\text{ kN} \qquad \textbf{Ans.}$$

Note: $\mathbf{F}_{AD}$ remains constant regardless of angle θ.



(a)

11–2. The uniform rod *OA* has a weight of 10 lb. When the rod is in a vertical position, $\theta = 0°$, the spring is unstretched. Determine the angle θ for equilibrium if the end of the spring wraps around the periphery of the disk as the disk turns.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force and the weight of rod (10 lb force) do work.

Virtual Displacements : The 10 lb force is located from the fixed point *O* using the position coordinate y_B, and the virtual displacement of point *C* is δx_C.

$$y_B = 1\cos\theta \qquad \delta y_B = -\sin\theta\,\delta\theta \qquad [1]$$

$$\delta x_C = 0.5\,\delta\theta \qquad [2]$$

Virtual - Work Equation : When points *B* and *C* undergo positive virtual displacements δy_B and δx_C, the 10 lb force and the spring force F_{sp}, do negative work.

$$\delta U = 0; \quad -10\,\delta y_B - F_{sp}\,\delta x_C = 0 \qquad [3]$$

Substituting Eqs. [1] and [2] into [3] yields

$$(-10\sin\theta - 0.5F_{sp})\,\delta\theta = 0 \qquad [4]$$

However, from the spring formula, $F_{sp} = kx = 30(0.5\theta) = 15\theta$. Substituting this value into Eq. [4] yields

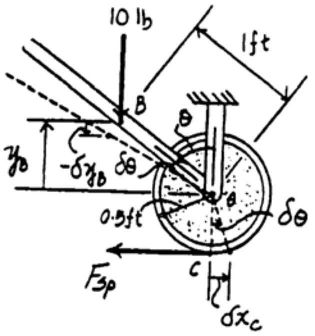
$$(-10\sin\theta - 7.5\theta)\,\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

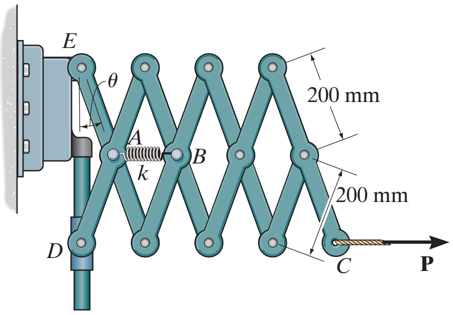
$$-10\sin\theta - 7.5\theta = 0$$

Solving by trial and error

$$\theta = 0° \qquad \text{and} \qquad \theta = 73.1° \qquad \textbf{Ans}$$



11–3. The "Nuremberg scissors" is subjected to a horizontal force of $P = 600$ N. Determine the angle θ for equilibrium. The spring has a stiffness of $k = 15$ kN/m and is unstretched when $\theta = 15°$.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$ acting at points A and B and the force $\mathbf{P}$ do work when the virtual displacements take place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula,

$$F_{sp} = kx = 15(10^3)[2(0.2\sin\theta) - 2(0.2\sin 15°)] = 6000(\sin\theta - 0.2588)\,\text{N}.$$

Virtual Displacement: The position of points A and B at which $\mathbf{F}_{sp}$ acts and point C at which force $\mathbf{P}$ acts are specified by the position coordinates y_A, y_B, and y_C, measured from the fixed point E, respectively.

$$y_A = 0.2\sin\theta \qquad \delta y_A = 0.2\cos\theta\delta\theta \qquad (1)$$

$$y_B = 3(0.2\sin\theta) \qquad \delta y_B = 0.6\cos\theta\delta\theta \qquad (2)$$

$$y_C = 8(0.2\sin\theta) \qquad \delta y_B = 1.6\cos\theta\delta\theta \qquad (3)$$

Virtual Work Equation: Since $\mathbf{F}_{sp}$ at point A and force $\mathbf{P}$ acts towards the positive sense of its corresponding virtual displacement, their work is positive. The work of $\mathbf{F}_{sp}$ at point B is negative since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad F_{sp}\delta y_A + \left(-F_{sp}\delta y_B\right) + P\delta y_C = 0 \qquad (4)$$

Substituting $F_{sp} = 6000(\sin\theta - 0.2588)$, $P = 600$ N, Eqs. (1), (2), and (3) into Eq. (4),

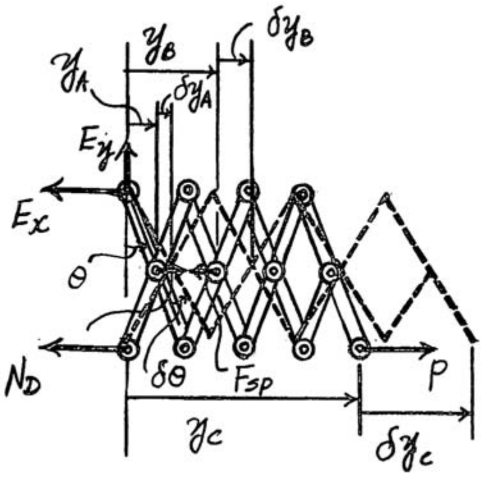
$$6000(\sin\theta - 0.2588)(0.2\cos\theta\delta\theta - 0.6\cos\theta\delta\theta) + 600(1.6\cos\theta\delta\theta) = 0$$

$$\cos\theta\delta\theta[-2400(\sin\theta - 0.2588) + 960] = 0$$

Since $\cos\theta\delta\theta \neq 0$, then

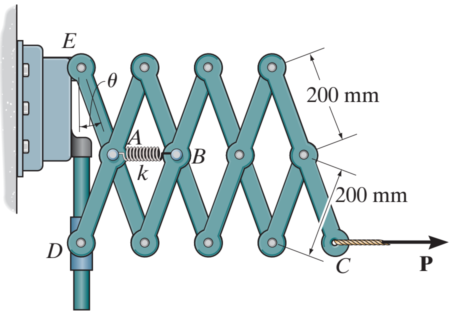
$$-2400(\sin\theta - 0.2588) + 960 = 0$$

$$\theta = 41.2° \qquad \textbf{Ans.}$$



(a)

***11–4.** The "Nuremberg scissors" is subjected to a horizontal force of $P = 600$ N. Determine the stiffness k of the spring for equilibrium when $\theta = 60°$. The spring is unstretched when $\theta = 15°$.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$ acting at points *A* and *B* and the force **P** do work when the virtual displacements take place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula.

$$F_{sp} = kx = k[2(0.2\sin\theta) - 2(0.2\sin 15°)] = (0.4)k(\sin\theta - 0.2588)\ \text{N}$$

Virtual Displacement: The position of points *A* and *B* at which $\mathbf{F}_{sp}$ acts and point *C* at which force **P** acts are specified by the position coordinates y_A, y_B, and y_C, measured from the fixed point *E*, respectively.

$$y_A = 0.2\sin\theta \qquad \delta y_A = 0.2\cos\theta\delta\theta \qquad (1)$$
$$y_B = 3(0.2\sin\theta) \qquad \delta y_B = 0.6\cos\theta\delta\theta \qquad (2)$$
$$y_C = 8(0.2\sin\theta) \qquad \delta y_B = 1.6\cos\theta\delta\theta \qquad (3)$$

Virtual Work Equation: Since $\mathbf{F}_{sp}$ at point *A* and force **P** acts towards the positive sense of its corresponding virtual displacement, their work is positive. The work of $\mathbf{F}_{sp}$ at point *B* is negative since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad F_{sp}\delta y_A + \left(-F_{sp}\delta y_B\right) + P\delta y_C = 0 \qquad (4)$$

Substituting $F_{sp} = k(\sin\theta - 0.2588)$, $P = 600$ N, Eqs. (1), (2), and (3) into Eq. (4),

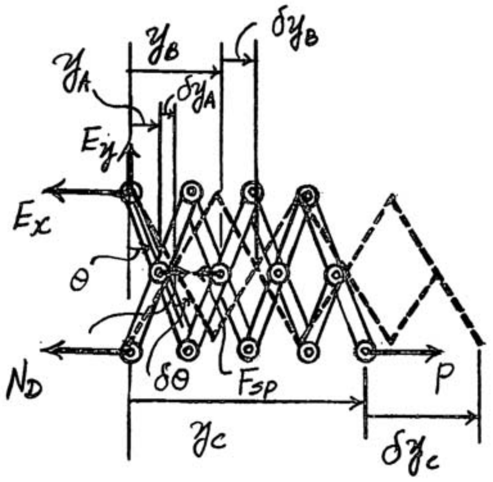
$$(0.4)k(\sin\theta - 0.2588)(0.2\cos\theta\delta\theta - 0.6\cos\theta\delta\theta) + 600(1.6\cos\theta\delta\theta) = 0$$
$$\cos\theta\delta\theta[-0.16k(\sin\theta - 0.2588) + 960] = 0$$

Since $\cos\theta\delta\theta \neq 0$, then

$$-0.16k(\sin\theta - 0.2588) + 960 = 0$$
$$k = \frac{6000}{\sin\theta - 0.2588}$$

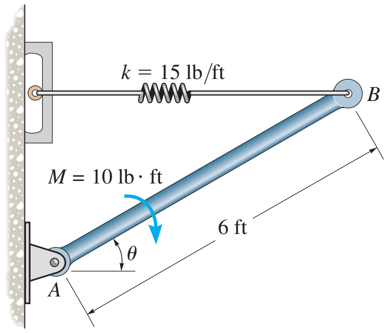
When $\theta = 60°$,

$$k = \frac{6000}{\sin 60° - 0.2588} = 9881\ \text{N/m} = 9.88\ \text{kN/m}$$ **Ans.**



(a)

•11–5. Determine the force developed in the spring required to keep the 10 lb uniform rod *AB* in equilibrium when $\theta = 35°$.



Free - Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp}, the weight of the rod (10 lb) and the 10 lb · ft couple moment do work.

Virtual Displacements : The spring force F_{sp} and the weight of the rod (10 lb) are located from the fixed point *A* using position coordinates x_B and x_C, respectively.

$$x_B = 6\cos\theta \qquad \delta x_B = -6\sin\theta\,\delta\theta \qquad [1]$$

$$y_C = 3\sin\theta \qquad \delta y_C = 3\cos\theta\,\delta\theta \qquad [2]$$

Virtual - Work Equation : When points *B* and *C* undergo positive virtual displacements δx_B and δy_C, the spring force F_{sp} and the weight of the rod (10 lb) do negative work. The 10 lb · ft couple moment does negative work when rod *AB* undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \qquad -F_{sp}\,\delta x_B - 10\delta y_C - 10\delta\theta = 0 \qquad [3]$$

Substituting Eqs. [1] and [2] into [3] yields

$$\left(6F_{sp}\sin\theta - 30\cos\theta - 10\right)\delta\theta = 0 \qquad [4]$$

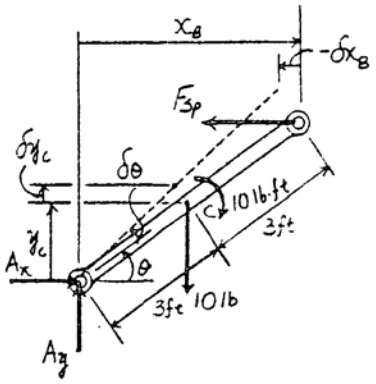
Since $\delta\theta \neq 0$, then

$$6F_{sp}\sin\theta - 30\cos\theta - 10 = 0$$

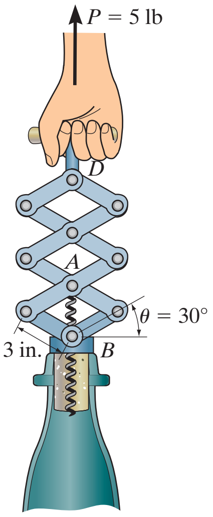
$$F_{sp} = \frac{30\cos\theta + 10}{6\sin\theta}$$

At the equilibrium position, $\theta = 35°$. Then

$$F_{sp} = \frac{30\cos 35° + 10}{6\sin 35°} = 10.0 \text{ lb} \qquad \textbf{Ans}$$



11–6. If a force of $P = 5$ lb is applied to the handle of the mechanism, determine the force the screw exerts on the cork of the bottle. The screw is attached to the pin at A and passes through the collar that is attached to the bottle neck at B.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the force in the screw $\mathbf{F}_s$ and force $\mathbf{P}$ do work when the virtual displacements take place.

Virtual Displacement: The position of the points of application for $\mathbf{F}_s$ and $\mathbf{P}$ are specified by the position coordinates y_A and y_D, measured from the fixed point B, respectively.

$y_A = 2(3\sin\theta)$ $\qquad \delta y_A = 6\cos\theta\delta\theta \qquad (1)$

$y_D = 6(3\sin\theta)$ $\qquad \delta y_D = 18\cos\theta\delta\theta \qquad (2)$

Virtual Work Equation: Since $\mathbf{P}$ acts towards the positive sense of its corresponding virtual displacement, it does positive work. However, the work of $\mathbf{F}_s$ is negative since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$\delta U = 0; \qquad P\delta y_D + (-F_s\delta y_A) = 0 \qquad (3)$

Substituting $P = 5$ lb, Eqs. (1) and (2) into Eq. (3),

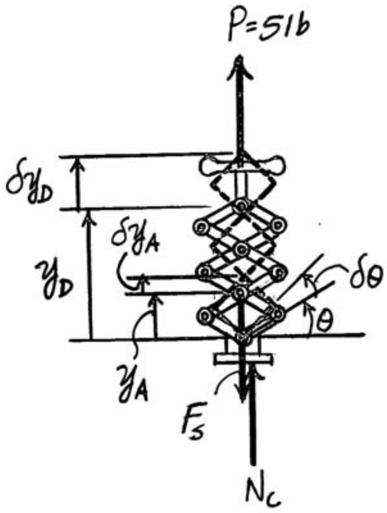
$5(18\cos\theta\delta\theta) - F_s(6\cos\theta\delta\theta) = 0$

$\cos\theta\delta\theta(90 - 6F_s) = 0$

Since $\cos\theta\delta\theta \neq 0$, then

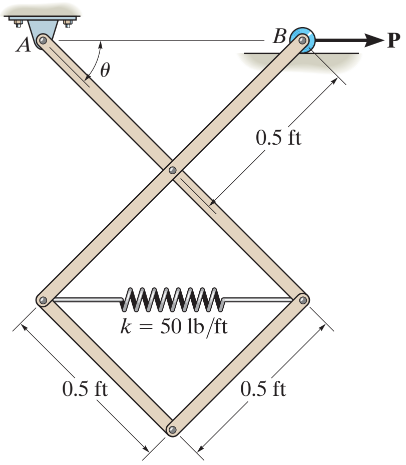
$90 - 6F_s = 0$

$F_s = 15$ lb **Ans.**



(a)

11–7. The pin-connected mechanism is constrained at A by a pin and at B by a roller. If $P = 10$ lb, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 45°$. Neglect the weight of the members.



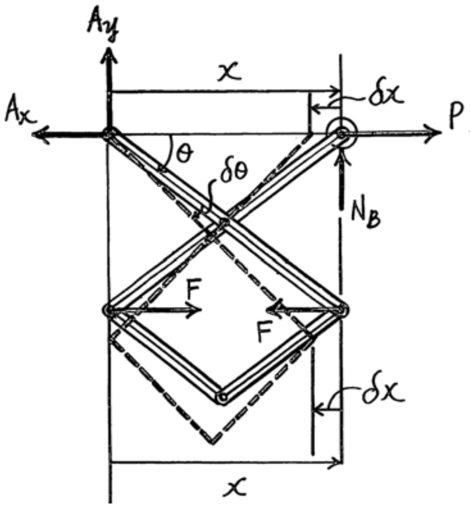
$x = 1\cos\theta$

$F_s = ks;\quad F = 50(1\cos\theta - 1\cos 45°)$

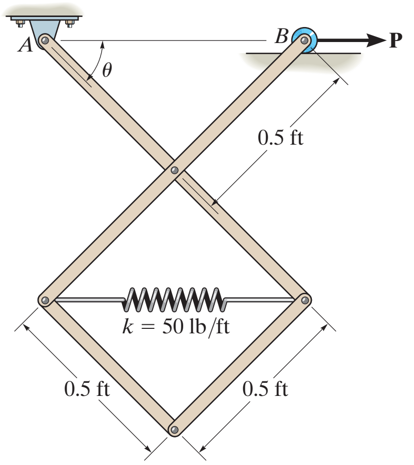
$\delta U = 0;\quad -F\,\delta x + P\,\delta x = 0$

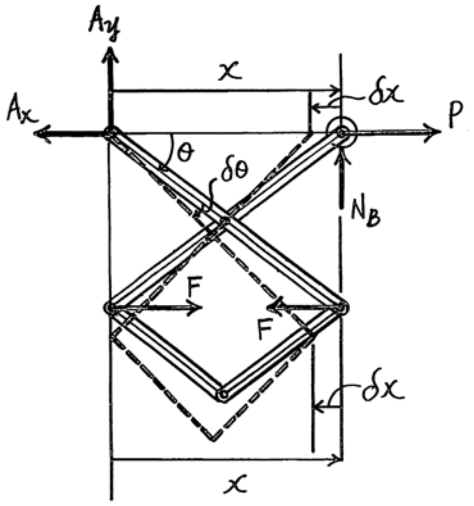
$-50(1\cos\theta - 1\cos 45°) + 10 = 0$

$\theta = 24.9°$ **Ans**



***11–8.** The pin-connected mechanism is constrained by a pin at *A* and a roller at *B*. Determine the force *P* that must be applied to the roller to hold the mechanism in equilibrium when $\theta = 30°$. The spring is unstretched when $\theta = 45°$. Neglect the weight of the members.





$x = 1\cos\theta$

$\delta U = 0; \qquad P\,\delta x - F\cdot\,\delta x = 0$

$P = F\cdot$

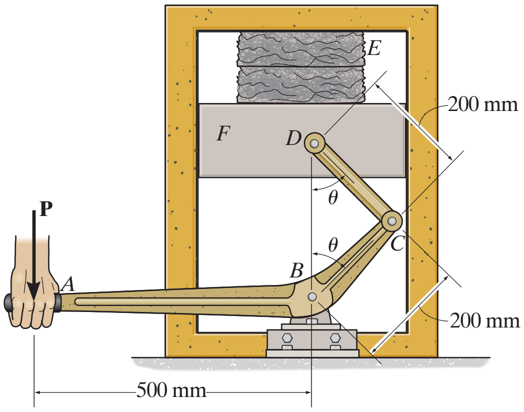
When $\theta = 45°$, $x = 1\cos 45° = 0.7071\text{ ft}$

When $\theta = 30°$, $x = 1\cos 30° = 0.86602\text{ ft}$

$F_s = ks; \qquad F = 50(0.86602 - 0.7071) = 7.95\text{ lb}$

$P = 7.95\text{ lb}$ **Ans**

•11–9. If a force $P = 100$ N is applied to the lever arm of the toggle press, determine the clamping force developed in the block when $\theta = 45°$. Neglect the weight of the block.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only force **P** and the clamping force $\mathbf{F}_E$ do work when the virtual displacement takes place.

Virtual Displacement: The position of point D at which $\mathbf{F}_E$ acts is specified by the position coordinate y_D.

$$y_D = 2(0.2\cos\theta) \qquad \delta y_D = -0.4\sin\theta\,\delta\theta \qquad (1)$$

Since $\delta\theta$ is very small, the virtual displacement of point A at which force **P** acts is

$$\delta y_A = 0.5\delta\theta \qquad (2)$$

Virtual Work Equation: Since $\mathbf{F}_E$ and **P** act towards the negative sense of their corresponding virtual displacements, their work is negative. Thus,

$$\delta U = 0; \qquad -P\delta y_A + (-F_E\delta y_D) = 0 \qquad (3)$$

Substituting $P = 100$ N, Eqs. (1) and (2) into Eq. (3),

$$-100(0.5\delta\theta) - F_E(-0.4\sin\theta\,\delta\theta) = 0$$

$$\delta\theta(-50 + 0.4F_E\sin\theta) = 0$$

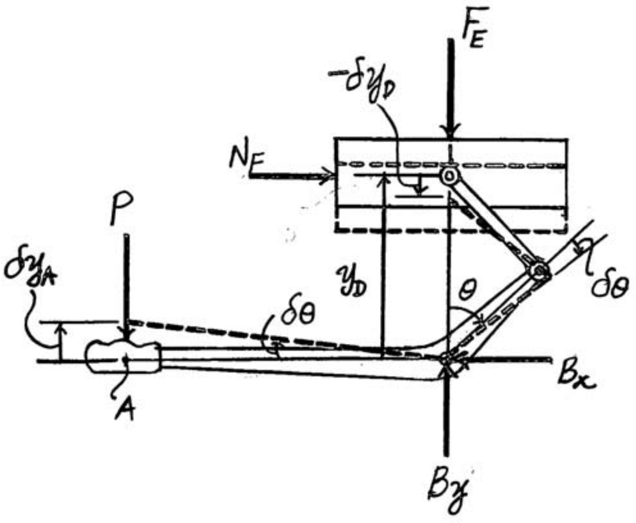
Since $\delta\theta \neq 0$, then

$$-50 + 0.4F_E\sin\theta = 0$$

$$F_E = \frac{125}{\sin\theta}$$

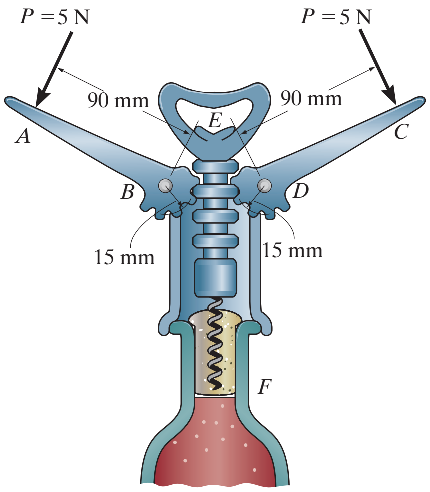
At $\theta = 45°$,

$$F_E = \frac{125}{\sin 45°} = 176.78\text{ N} = 177\text{ N} \qquad \textbf{Ans.}$$



(a)

11–10. When the forces are applied to the handles of the bottle opener, determine the pulling force developed on the cork.



Free - Body Diagram: When the handle undergoes a virtual angular displacement of $\delta\theta$, only forces **P** and **F** do work, Fig. *a*.

Virtual Displacement: Since $\delta\theta$ is very small, the virtual displacements of forces **P** and **F** can be approximated as

$$\delta_P = 0.09\delta\theta \qquad (1)$$

$$\delta_F = 0.015\delta\theta \qquad (2)$$

Virtual Work Equation: Since **P** acts towards the positive sense of its corresponding virtual displacement, its work is positive. However, force **F** does negative work since it acts towards the negative sense.

$$\delta U = 0; \qquad 2(P\delta_P) + (-F\delta F) = 0 \qquad (3)$$

Substituting $P = 5$ N and Eqs. (1) and (2) into Eq. (3),

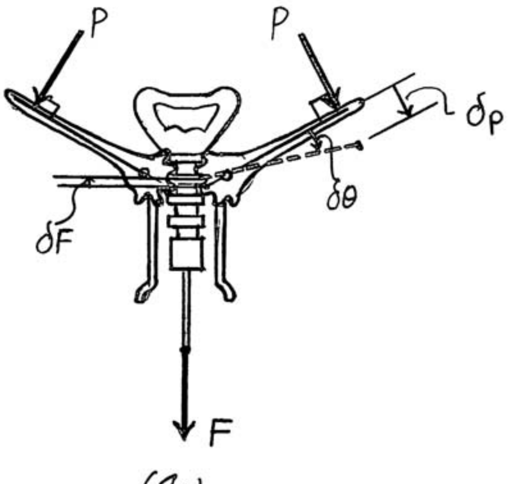
$$2[5(0.09\delta\theta)] - F(0.015\delta\theta) = 0$$

$$\delta\theta(0.9 - 0.015F) = 0$$

Since $\delta\theta \neq 0$, then

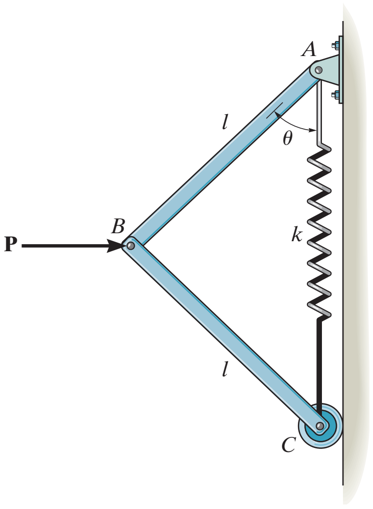
$$0.9 - 0.015F = 0$$

$$F = 60\,\text{N} \qquad \textbf{Ans.}$$



(a)

11–11. If the spring has a stiffness k and an unstretched length l_0, determine the force P when the mechanism is in the position shown. Neglect the weight of the members.



$y = 2l\cos\theta, \qquad \delta y = -2l\sin\theta\,\delta\theta$

$x = l\sin\theta, \qquad \delta x = l\cos\theta\,\delta\theta$

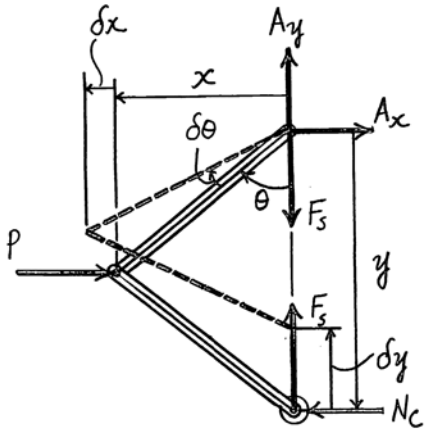
$\delta U = 0; \qquad -P\,\delta x - F_s\,\delta y = 0$

$-P(l\cos\theta\,\delta\theta) + F_s(2l\sin\theta\,\delta\theta) = 0$

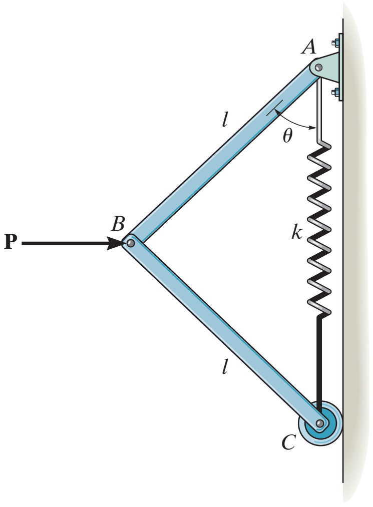
$-P\cos\theta + 2F_s\sin\theta = 0$

$F_s = k(2l\cos\theta - l_0)$

$P = 2k\tan\theta\,(2l\cos\theta - l_0)$ **Ans**



*__11–12.__ Solve Prob. 11–11 if the force **P** is applied vertically downward at B.



$y_1 = l\cos\theta, \qquad \delta y_1 = -l\sin\theta\,\delta\theta$

$y_2 = 2l\cos\theta, \qquad \delta y_2 = -2l\sin\theta\,\delta\theta$

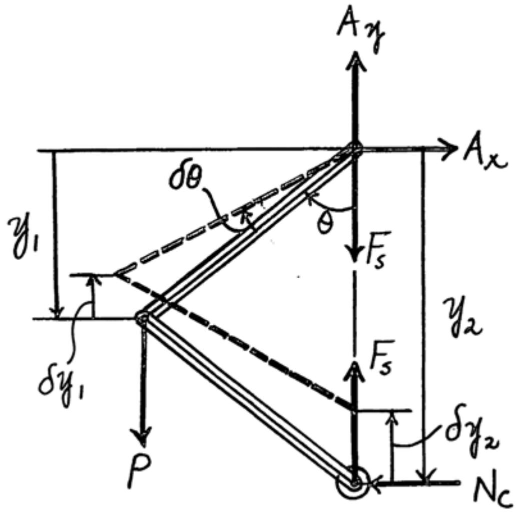
$\delta U = 0; \qquad P\,\delta y_1 - F_s\,\delta y_2 = 0$

$P(-l\sin\theta)\,\delta\theta - F_s(-2l\sin\theta)\,\delta\theta = 0$

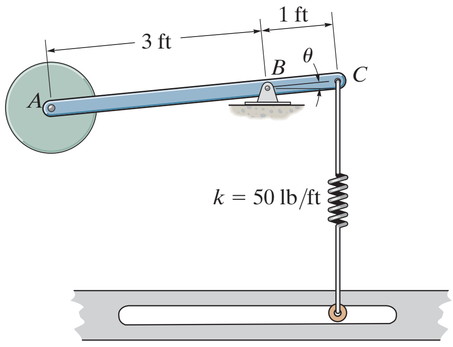
$P = 2F_s$

$F_s = ks; \qquad F_s = k(2l\cos\theta - l_0)$

$P = 2k(2l\cos\theta - l_0)$ **Ans**



•11–13. Determine the angles θ for equilibrium of the 4-lb disk using the principle of virtual work. Neglect the weight of the rod. The spring is unstretched when $\theta = 0°$ and always remains in the vertical position due to the roller guide.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and the weight of the disk (4 lb) do work.

Virtual Displacements : The spring force F_{sp} and the weight of the disk (4 lb) are located from the fixed point B using position coordinates y_C and y_A, respectively.

$$y_C = 1\sin\theta \qquad \delta y_C = \cos\theta\delta\theta \qquad [1]$$

$$y_A = 3\sin\theta \qquad \delta y_A = 3\cos\theta\delta\theta \qquad [2]$$

Virtual - Work Equation : When points C and A undergo positive virtual displacements δy_C and δy_A, the spring force F_{sp} does negative work while the weight of the disk (4 lb) do positive work.

$$\delta U = 0; \qquad 4\delta y_A - F_{sp}\delta y_C = 0 \qquad [3]$$

Substituting Eqs. [1] and [2] into [3] yields

$$\left(12 - F_{sp}\right)\cos\theta\delta\theta = 0 \qquad [4]$$

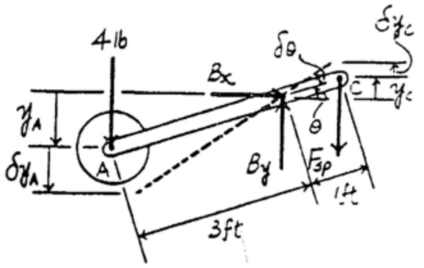
However, from the spring formula, $F_{sp} = kx = 50(1\sin\theta) = 50\sin\theta$. Substituting this value into Eq. [4] yields

$$(12 - 50\sin\theta)\cos\theta\delta\theta = 0$$

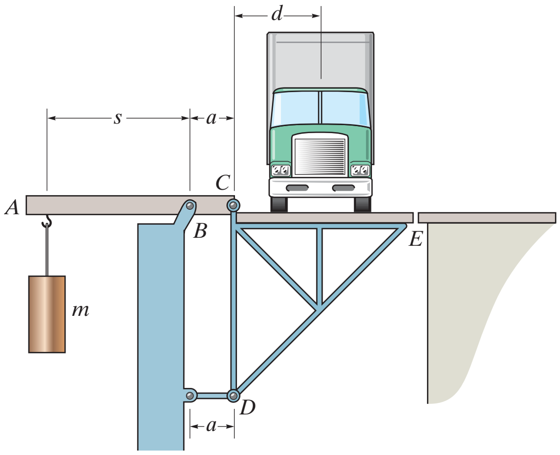
Since $\delta\theta \neq 0$, then

$$12 - 50\sin\theta = 0 \qquad \theta = 13.9° \qquad \textbf{Ans}$$

$$\cos\theta = 0 \qquad \theta = 90° \qquad \textbf{Ans}$$



11–14. The truck is weighed on the highway inspection scale. If a known mass m is placed a distance s from the fulcrum B of the scale, determine the mass of the truck m_t if its center of gravity is located at a distance d from point C. When the scale is empty, the weight of the lever ABC balances the scale CDE.



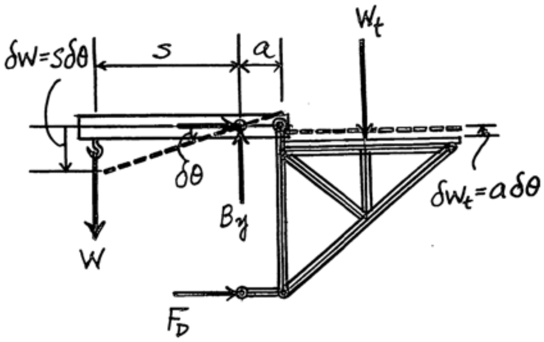
$\delta U = 0;$ $\quad (W)(s\,\delta\theta) - W_t a\,\delta\theta = 0$

$$(Ws - W_t a)\delta\theta = 0$$

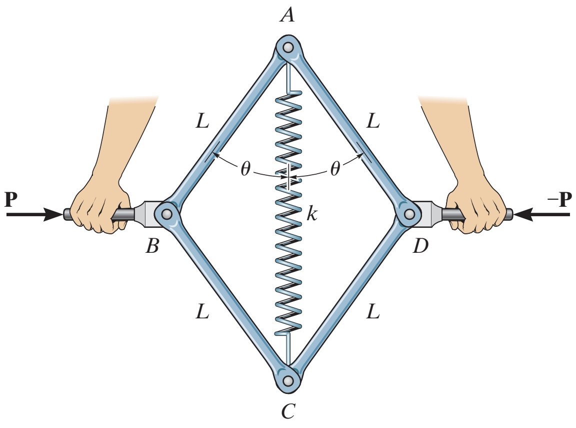
$$W_t = W\left(\frac{s}{a}\right)$$

or,

$$m_t = m\left(\frac{s}{a}\right) \quad \textbf{Ans}$$



11–15. The assembly is used for exercise. It consists of four pin-connected bars, each of length L, and a spring of stiffness k and unstretched length $a\,(< 2L)$. If horizontal forces are applied to the handles so that θ is slowly decreased, determine the angle θ at which the magnitude of **P** becomes a maximum.



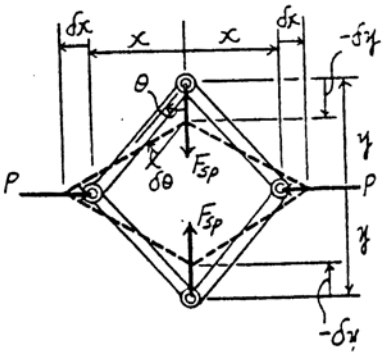
Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, the spring force F_{sp} and force **P** do work.

Virtual Displacements : The spring force F_{sp} and force **P** are located from the fixed point D and A using position coordinates y and x, respectively.

$$y = L\cos\theta \qquad \delta y = -L\sin\theta\,\delta\theta \qquad [1]$$
$$x = L\sin\theta \qquad \delta x = L\cos\theta\,\delta\theta \qquad [2]$$

Virtual - Work Equation : When points A, C, B and D undergo positive virtual displacement δy and δx, the spring force F_{sp} and force **P** do negative work.

$$\delta U = 0; \qquad -2F_{sp}\,\delta y - 2P\delta x = 0 \qquad [3]$$



Substituting Eqs. [1] and [2] into [3] yields

$$\left(2F_{sp}\sin\theta - 2P\cos\theta\right)L\delta\theta = 0 \qquad [4]$$

From the geometry, the spring stretches $s = 2L\cos\theta - a$. Then, the spring force $F_{sp} = ks = k(2L\cos\theta - a) = 2kL\cos\theta - ka$. Substituting this value into Eq. [4] yields

$$(4kL\sin\theta\cos\theta - 2ka\sin\theta - 2P\cos\theta)\,L\delta\theta = 0$$

Since $L\delta\theta \neq 0$, then

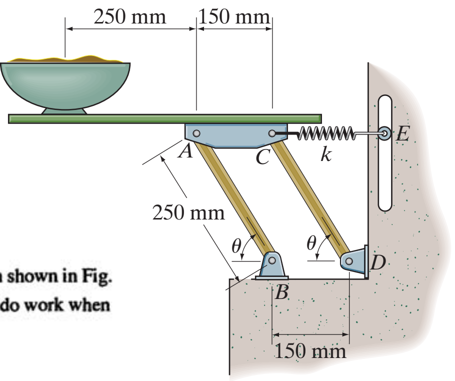
$$4kL\sin\theta\cos\theta - 2ka\sin\theta - 2P\cos\theta = 0$$
$$P = k(2L\sin\theta - a\tan\theta)$$

In order to obtain maximum P, $\dfrac{dP}{d\theta} = 0$.

$$\frac{dP}{d\theta} = k\left(2L\cos\theta - a\sec^2\theta\right) = 0$$

$$\theta = \cos^{-1}\left(\frac{a}{2L}\right)^{\frac{1}{3}} \qquad \textbf{Ans}$$

***11–16.** A 5-kg uniform serving table is supported on each side by pairs of two identical links, *AB* and *CD*, and springs *CE*. If the bowl has a mass of 1 kg, determine the angle θ where the table is in equilibrium. The springs each have a stiffness of $k = 200$ N/m and are unstretched when $\theta = 90°$. Neglect the mass of the links.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$, the weight $\mathbf{W}_t$ of the table, and the weight $\mathbf{W}_b$ of the bowl do work when the virtual displacement takes place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula,

$F_{sp} = kx = 200(0.25\cos\theta) = 50\cos\theta\,\text{N}$.

Virtual Displacement: The position of points of application of $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ are specified by the position coordinates y_{G_b}, y_{G_t}, and x_C, respectively. Here, y_{G_b} and y_{G_t} are measured from the fixed point *B* while x_C is measured from the fixed point *D*.

$$y_{G_b} = 0.25\sin\theta + b \qquad \delta y_{G_b} = 0.25\cos\theta\delta\theta \qquad (1)$$
$$y_{G_t} = 0.25\sin\theta + a \qquad \delta y_{G_t} = 0.25\cos\theta\delta\theta \qquad (2)$$
$$x_C = 0.25\cos\theta \qquad \delta x_C = -0.25\sin\theta\delta\theta \qquad (3)$$

Virtual Work Equation: Since $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ act towards the negative sense of their corresponding virtual displacement, their work is negative. Thus,

$$\delta U = 0; \qquad -W_b\delta y_{G_b} + \left(-W_t\delta y_{G_t}\right) + \left(-F_{sp}\delta x_C\right) = 0 \qquad (4)$$

Substituting $W_b = \left(\frac{1}{2}\right)(9.81) = 4.905\,\text{N}$, $W_t = \left(\frac{5}{2}\right)(9.81) = 24.525\,\text{N}$, $F_{sp} = 50\cos\theta\,\text{N}$, Eqs. (1), (2), and (3) into Eq. (4), we have

$$-4.905(0.25\cos\theta\delta\theta) - 24.525(0.25\cos\theta\delta\theta) - 50\cos\theta(-0.25\sin\theta\delta\theta) = 0$$
$$\delta\theta\left(-7.3575\cos\theta + 12.5\sin\theta\cos\theta\right) = 0$$

Since $\delta\theta \neq 0$, then

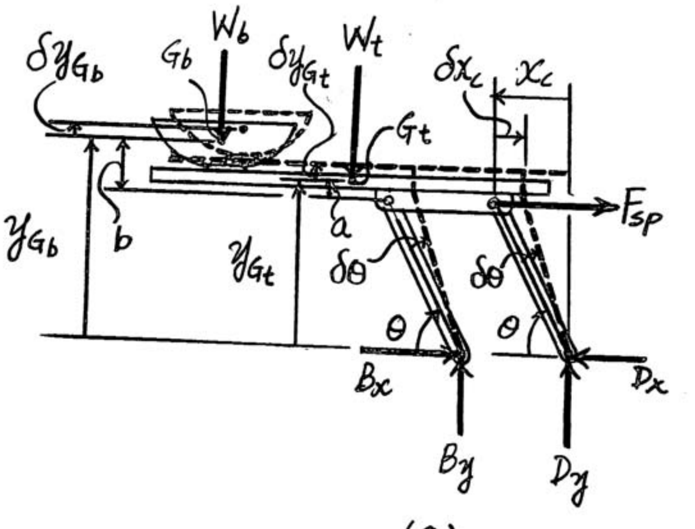
$$-7.3575\cos\theta + 12.5\sin\theta\cos\theta = 0$$
$$\cos\theta(-7.3575 + 12.5\sin\theta) = 0$$

Solving the above equation,

$\cos\theta = 0 \quad \theta = 90°$ **Ans.**

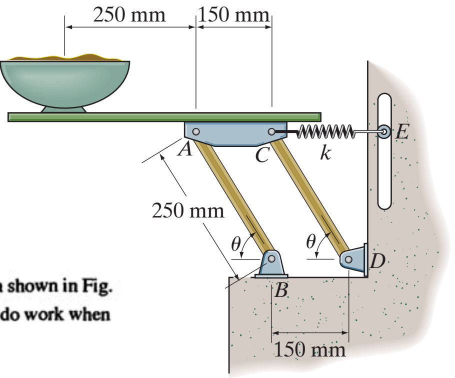
$-7.3575 + 12.5\sin\theta = 0$

$\theta = 36.1°$ **Ans.**



(a)

•11–17. A 5-kg uniform serving table is supported on each side by two pairs of identical links, AB and CD, and springs CE. If the bowl has a mass of 1 kg and is in equilibrium when $\theta = 45°$, determine the stiffness k of each spring. The springs are unstretched when $\theta = 90°$. Neglect the mass of the links.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$, the weight $\mathbf{W}_t$ of the table, and the weight $\mathbf{W}_b$ of the bowl do work when the virtual displacement takes place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula,

$F_{sp} = kx = k(0.25\cos\theta) = 0.25k\cos\theta$.

Virtual Displacement: The position of points of application of $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ are specified by the position coordinates y_{G_b}, y_{G_t}, and x_C, respectively. Here, y_{G_b} and y_{G_t} are measured from the fixed point B while x_C is measured from the fixed point D.

$$y_{G_b} = 0.25\sin\theta + b \qquad \delta y_{G_b} = 0.25\cos\theta\delta\theta \qquad (1)$$
$$y_{G_t} = 0.25\sin\theta + a \qquad \delta y_{G_t} = 0.25\cos\theta\delta\theta \qquad (2)$$
$$x_C = 0.25\cos\theta \qquad \delta x_C = -0.25\sin\theta\delta\theta \qquad (3)$$

Virtual Work Equation: Since $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ act towards the negative sense of their corresponding virtual displacement, their work is negative. Thus,

$$\delta U = 0; \qquad -W_b\delta y_{G_b} + \left(-W_t\delta y_{G_t}\right) + \left(-F_{sp}\delta x_C\right) = 0 \qquad (4)$$

Substituting $W_b = \left(\frac{1}{2}\right)(9.81) = 4.905\text{ N}$, $W_t = \left(\frac{5}{2}\right)(9.81) = 24.525\text{ N}$, $F_{sp} = 0.25k\cos\theta\text{N}$, Eqs. (1), (2), and (3) into Eq. (4), we have

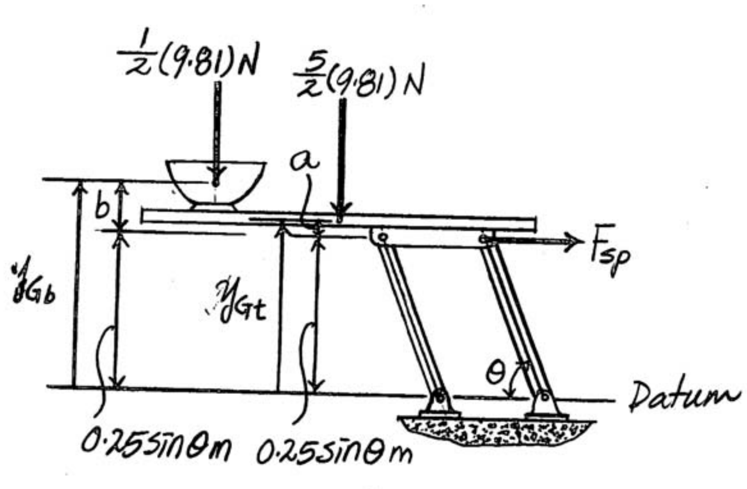
$$-4.905(0.25\cos\theta\delta\theta) - 24.525(0.25\cos\theta\delta\theta) - 0.25k\cos\theta(-0.25\sin\theta\delta\theta) = 0$$
$$\delta\theta(-7.3575\cos\theta + 0.0625k\sin\theta\cos\theta) = 0$$

Since $\delta\theta \neq 0$, then

$$-7.3575\cos\theta + 0.0625k\sin\theta\cos\theta = 0$$
$$k = \frac{117.72}{\sin\theta}$$

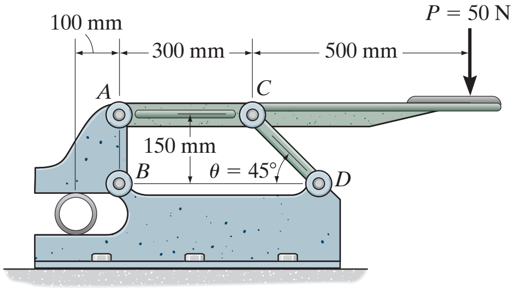
When $\theta = 45°$, then

$$k = \frac{117.72}{\sin 45°} = 166\text{ N/m}$$ **Ans.**



(a)

11–18. If a vertical force of $P = 50\text{ N}$ is applied to the handle of the toggle clamp, determine the clamping force exerted on the pipe.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only force **P** and the clamping force **F** do work when the virtual displacement takes place.

Virtual Displacement: Since $\delta\theta$ is very small, the virtual displacement of point C can be appoximated by $\delta_C = \sqrt{0.045}\delta\theta$ m. From the geometry shown in Fig. *b*, the horizontal and vertical components of δ_C are given by $(\delta_C)_x = \delta_C \sin\theta = \sqrt{0.045}\sin\theta\delta\theta$ and $(\delta_C)_y = \delta_C\cos\theta = \sqrt{0.045}\cos\theta\delta\theta$, respectively. By referring to Fig. *a*, we notice that $\delta_A = (\delta_C)_x = \sqrt{0.045}\sin\theta\delta\theta$. Also,

$$\delta\phi = \frac{\delta_F}{0.1} = \frac{\delta_A}{0.15} \text{ or } \delta_F = 0.6667\delta_A = 0.6667\sqrt{0.045}\sin\theta\delta\theta$$

and

$$\delta x = \frac{(\delta_C)_y}{0.3} = \frac{\sqrt{0.045}\cos\theta\delta\theta}{0.3} = 3.333\sqrt{0.045}\cos\theta\delta\theta$$

and

$$\delta_P = 0.5\delta x + (\delta_C)_y = 0.5\left(3.333\sqrt{0.045}\cos\theta\delta\theta\right) + \sqrt{0.045}\cos\theta\delta\theta = 2.6667\sqrt{0.045}\cos\theta\delta\theta$$

Virtual Work Equation: Since **F** acts towards the positive sense of its corresponding virtual displacements, its work is positive. However, **P** does negative work since it acts in the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad F\delta_F + \left(-P\delta_P\right) = 0$$

Substituting $P = 50\text{ N}$ and the results of δ_F and δ_P into the above equation

$$F(0.6667\sqrt{0.045}\sin\theta\delta\theta) - 50(2.6667\sqrt{0.045}\cos\theta\delta\theta) = 0$$

$$\sqrt{0.045}\delta\theta(0.6667F\sin\theta - 133.33\cos\theta) = 0$$

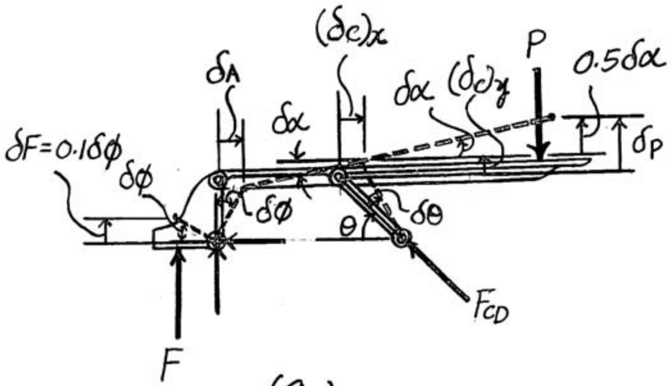
Since $\sqrt{0.045}\delta\theta \neq 0$, then

$$0.6667F\sin\theta - 133.33\cos\theta = 0$$

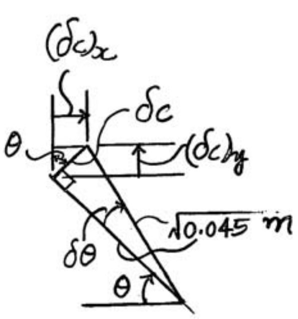
$$F = \frac{200\cos\theta}{\sin\theta}$$

At $\theta = 45°$,

$$F = \frac{200\cos 45°}{\sin 45°} = 200\text{ N} \qquad \textbf{Ans.}$$

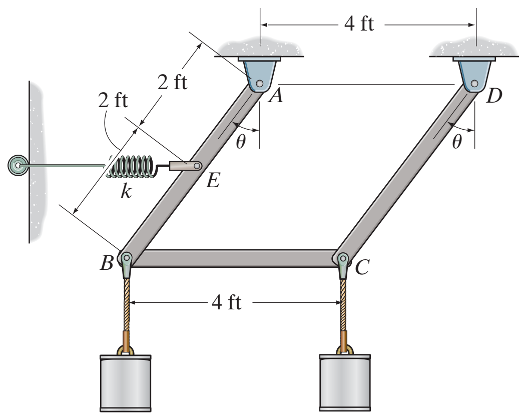


(a)



(b)

11–19. The spring is unstretched when $\theta = 45°$ and has a stiffness of $k = 1000$ lb/ft. Determine the angle θ for equilibrium if each of the cylinders weighs 50 lb. Neglect the weight of the members. The spring remains horizontal at all times due to the roller.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$ and the weight $\mathbf{W}$ of the cylinder do work when the virtual displacement takes place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula,

$F_{sp} = kx = 1000(2\sin 45° - 2\sin\theta) = 2000(0.7071 - \sin\theta)$ lb.

Virtual Displacement: The positions of the points of application of $\mathbf{W}$ and $\mathbf{F}_{sp}$ are specified by the position coordinates y_W and x_E, measured from the fixed point A.

$$y_W = 4\cos\theta + b \qquad \delta y_W = -4\sin\theta\delta\theta \qquad (1)$$

$$x_E = 2\sin\theta \qquad \delta x_E = 2\cos\theta\delta\theta \qquad (2)$$

Virtual Work Equation: Since $\mathbf{W}$ and $\mathbf{F}_{sp}$ act towards the positive sense of their corresponding virtual displacements, their work is positive. Thus,

$$\delta U = 0; \qquad 2W\delta y_W + F_{sp}\delta x_E = 0 \qquad (3)$$

Substituting $W = 50$ lb, $F_{sp} = 2000(0.7071 - \sin\theta)$, Eqs. (1), and (2) into Eq. (3),

$$2(50)(-4\sin\theta\delta\theta) + 2000(0.7071 - \sin\theta)(2\cos\theta\delta\theta) = 0$$

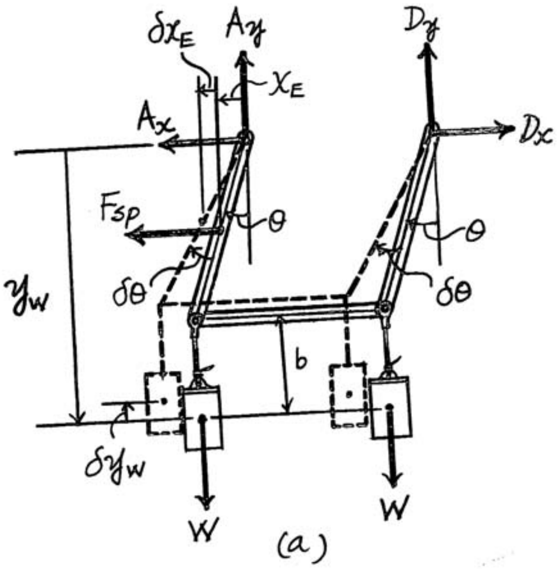
$$\delta\theta(-400\sin\theta + 2828.43\cos\theta - 4000\cos\theta\sin\theta) = 0$$

Since $\delta\theta \neq 0$, then

$$-400\sin\theta + 2828.43\cos\theta - 4000\cos\theta\sin\theta = 0$$

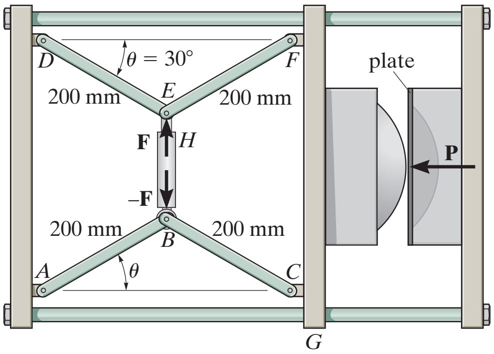
Solving by trial and error,

$\theta = 38.8°$ **Ans.**



(a)

***11–20.** The machine shown is used for forming metal plates. It consists of two toggles *ABC* and *DEF*, which are operated by the hydraulic cylinder. The toggles push the moveable bar *G* forward, pressing the plate into the cavity. If the force which the plate exerts on the head is $P = 8$ kN, determine the force *F* in the hydraulic cylinder when $\theta = 30°$.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed.

Virtual Displacement: The position of points of application of **F**, and **P** are specified by the position coordinates y_E, y_{B_t}, and x_G, respectively.

$$y_E = 0.2\sin\theta \qquad \delta y_E = 0.2\cos\theta\delta\theta \qquad (1)$$
$$y_B = 0.2\sin\theta \qquad \delta y_B = 0.2\cos\theta\delta\theta \qquad (2)$$
$$x_G = 2(0.2\cos\theta) \qquad \delta x_G = -0.4\sin\theta\delta\theta \qquad (3)$$

Virtual Work Equation: Thus,

$$\delta U = 0; \qquad -F\delta y_E + (-F\delta y_B) + (-P\delta x_G) = 0 \qquad (4)$$

Substituting Eqs. (1), (2), and (3) into Eq. (4), we have

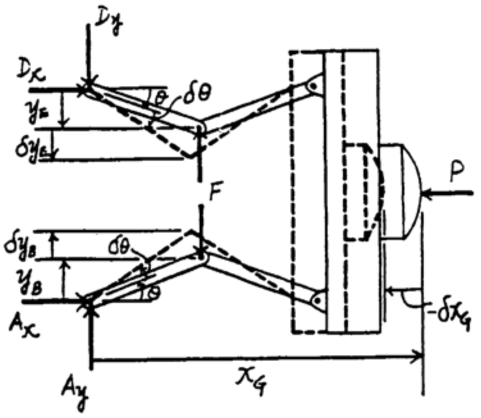
$$\delta\theta(-0.4F\cos\theta + 0.4P\sin\theta) = 0$$

Since $\delta\theta \neq 0$, then

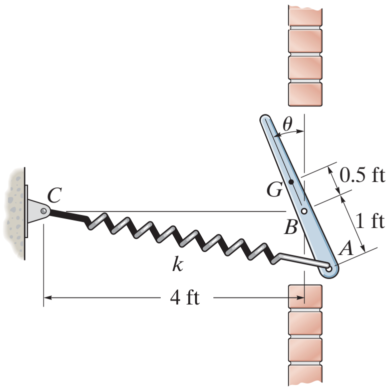
$$-0.4F\cos\theta + 0.4P\sin\theta = 0$$
$$F = P\tan\theta$$

When $\theta = 30°$, $P = 8$ kN then

$$F = 8\tan 30° = 4.62 \text{ kN} \qquad \textbf{Ans.}$$



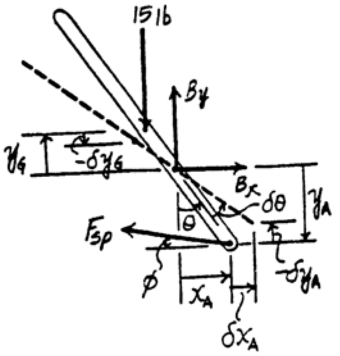
•11–21. The vent plate is supported at B by a pin. If it weighs 15 lb and has a center of gravity at G, determine the stiffness k of the spring so that the plate remains in equilibrium at $\theta = 30°$. The spring is unstretched when $\theta = 0°$.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and the weight of the vent plate (15 lb force) do work.

Virtual Displacements : The weight of the vent plate (15 lb force) is located from the fixed point B using the position coordinate y_G. The horizontal and vertical position of the spring force F_{sp} are measured from the fixed point B using the position coordinates x_A and y_A, respectively.

$$y_G = 0.5\cos\theta \qquad \delta y_G = -0.5\sin\theta\,\delta\theta \qquad [1]$$
$$y_A = 1\cos\theta \qquad \delta y_A = -\sin\theta\,\delta\theta \qquad [2]$$
$$x_A = 1\sin\theta \qquad \delta x_A = \cos\theta\,\delta\theta \qquad [3]$$



Virtual - Work Equation : When y_G, y_A and x_A undergo positive virtual displacements δy_G, δy_A and δx_A, the weight of the vent plate (15 lb force), horizontal component of F_{sp}, $F_{sp}\cos\phi$ and vertical component of F_{sp}, $F_{sp}\sin\phi$ do negative work.

$$\delta U = 0; \qquad -F_{sp}\cos\phi\,\delta x_A - F_{sp}\sin\phi\,\delta y_A - 15\delta y_G = 0 \qquad [4]$$

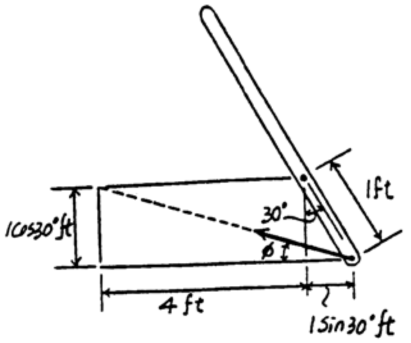
Substituting Eqs. [1], [2] and [3] into [4] yields

$$\left(-F_{sp}\cos\theta\cos\phi + F_{sp}\sin\theta\sin\phi + 7.5\sin\theta\right)\delta\theta = 0$$
$$\left(-F_{sp}\cos(\theta+\phi) + 7.5\sin\theta\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$-F_{sp}\cos(\theta+\phi) + 7.5\sin\theta = 0$$
$$F_{sp} = \frac{7.5\sin\theta}{\cos(\theta+\phi)}$$

At equilibrium position $\theta = 30°$, the angle $\phi = \tan^{-1}\left(\dfrac{1\cos 30°}{4 + 1\sin 30°}\right) = 10.89°$.

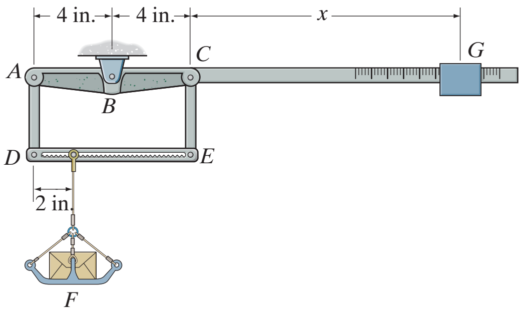


$$F_{sp} = \frac{7.5\sin 30°}{\cos(30° + 10.89°)} = 4.961 \text{ lb}$$

Spring Formula : From the geometry, the spring stretches
$$x = \sqrt{4^2 + 1^2 - 2(4)(1)\cos 120°} - \sqrt{4^2 + 1^2} = 0.4595 \text{ ft.}$$

$$F_{sp} = kx$$
$$4.961 = k(0.4595)$$
$$k = 10.8 \text{ lb/ft} \qquad \textbf{Ans}$$

11–22. Determine the weight of block G required to balance the differential lever when the 20-lb load F is placed on the pan. The lever is in balance when the load and block are not on the lever. Take $x = 12$ in.



Free - Body Diagram: When the lever undergoes a virtual angular displacement of $\delta\theta$ about point B, the dash line configuration shown in Fig. a is formed. We observe that only the weight $\mathbf{W}_G$ of block G and the weight $\mathbf{W}_F$ of load F do work when the virtual displacements take place.

Virtual Displacement: Since δy_G is very small, the vertical virtual displacement of block G and load F can be approximated as

$$\delta y_G = (12+4)\delta\theta = 16\delta\theta \qquad (1)$$

$$\delta y_F = 2\delta\theta \qquad (2)$$

Virtual Work Equation: Since $\mathbf{W}_G$ acts towards the positive sense of its corresponding virtual displacement, its work is positive. However, force $\mathbf{W}_F$ does negative work since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad W_G\delta y_G + (-W_F\delta y_F) = 0 \qquad (3)$$

Substituting $W_G = 20$ lb and Eqs. (1) and (2) into Eq. (3),

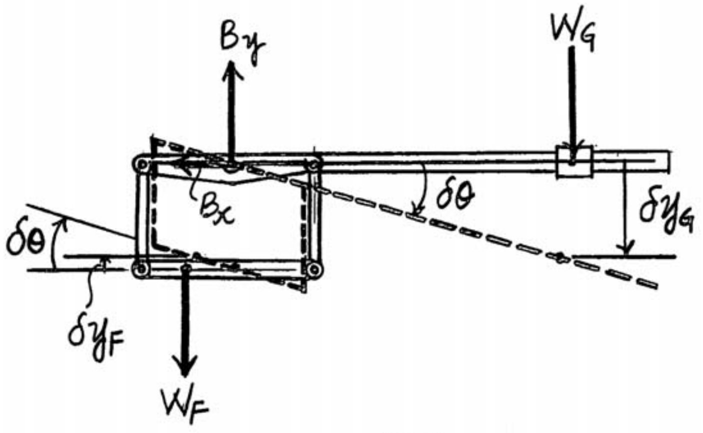
$$W_G(16\delta\theta) - 20(2\delta\theta) = 0$$

$$\delta\theta(16W_G - 40) = 0$$

Since $\delta\theta \neq 0$, then

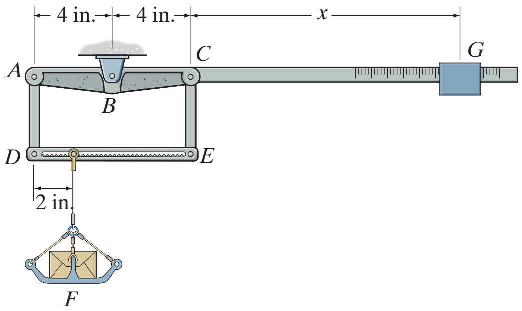
$$16W_G - 40 = 0$$

$$W_G = 2.5 \text{ lb}$$ **Ans.**



(a)

11–23. If the load F weighs 20 lb and the block G weighs 2 lb, determine its position x for equilibrium of the differential lever. The lever is in balance when the load and block are not on the lever.



Free - Body Diagram: When the lever undergoes a virtual angular displacement of $\delta\theta$ about point B, the dash line configuration shown in Fig. a is formed. We observe that only the weight $\mathbf{W}_G$ of block G and the weight $\mathbf{W}_F$ of load F do work when the virtual displacements take place.

Virtual Displacement: Since δy_G is very small, the vertical virtual displacement of block G and load F can be approximated as

$$\delta y_G = (4 + x)\delta\theta \qquad (1)$$

$$\delta y_F = 2\delta\theta \qquad (2)$$

Virtual Work Equation: Since $\mathbf{W}_G$ acts towards the positive sense of its corresponding virtual displacement, its work is positive. However, force $\mathbf{W}_F$ does negative work since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad W_G\delta y_G + (-W_F\delta y_F) = 0 \qquad (3)$$

Substituting $W_F = 20$ lb, $W_G = 2$ lb, Eqs. (1) and (2) into Eq. (3),

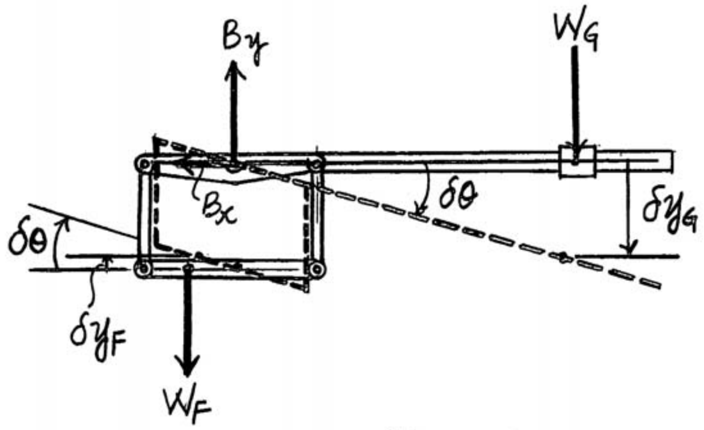
$$2(4 + x)\delta\theta - 20(2\delta\theta) = 0$$

$$\delta\theta[2(4 + x) - 40] = 0$$

Since $\delta\theta \neq 0$, then

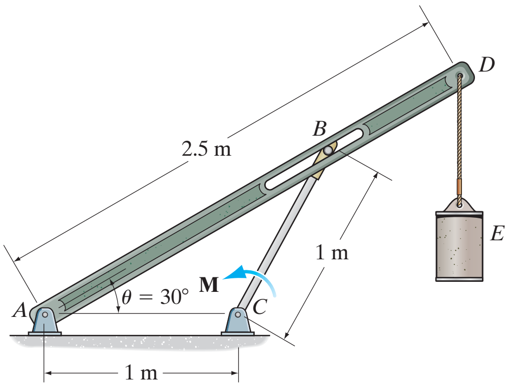
$$2(4 + x) - 40 = 0$$

$$x = 16 \text{ in.} \qquad \textbf{Ans.}$$



(a)

***11–24.** Determine the magnitude of the couple moment **M** required to support the 20-kg cylinder in the configuration shown. The smooth peg at B can slide freely within the slot. Neglect the mass of the members.



Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash - line configuration shown in Fig. a is formed. We observe that only the couple moment **M** and the weight $\mathbf{W}_E$ of the cylinder do work when the virtual displacement takes place.

Virtual Displacement: The position of the point of application of $\mathbf{W}_E$ is specified by the position coordinate y_E, measured from the fixed point C.

$$y_E = b - 2.5\sin\theta \qquad \delta y_E = -2.5\cos\theta\,\delta\theta \qquad (1)$$

From the geometry shown in Fig. b, we obtain

$$\phi = 2\theta \qquad \delta\phi = 2\delta\theta \qquad (2)$$

Virtual Work Equation: Since **M** and $\mathbf{W}_E$ act in the positive sense of their corresponding virtual displacements, their work is positive. Thus,

$$\delta U = 0; \qquad M\delta\phi + W_E\,\delta y_E = 0 \qquad (3)$$

Substituting $W_E = 20(9.81)$ N and Eqs. (1) and (2) into Eq. (3),

$$M(2\delta\theta) + 20(9.81)(-2.5\cos\theta\,\delta\theta) = 0$$

$$\delta\theta(2M - 490.5\cos\theta) = 0$$

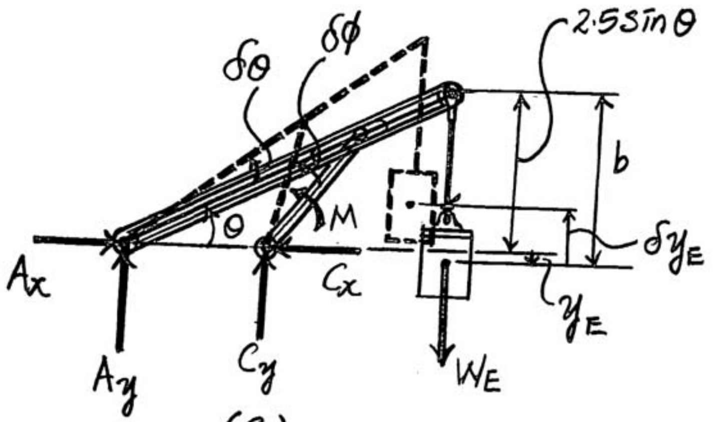
Since $\delta\theta \neq 0$, then

$$2M - 490.5\cos\theta = 0$$

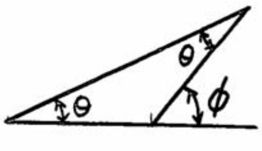
$$M = 245.25\cos\theta$$

At $\theta = 30°$,

$$M = 245.25\cos 30° = 212\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

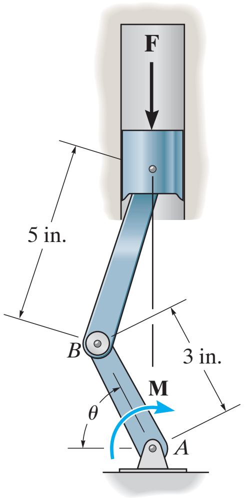


(a)



(b)

•11–25. The crankshaft is subjected to a torque of $M = 50\text{ lb}\cdot\text{ft}$. Determine the vertical compressive force **F** applied to the piston for equilibrium when $\theta = 60°$.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the force **F** and couple moment **M** do work.

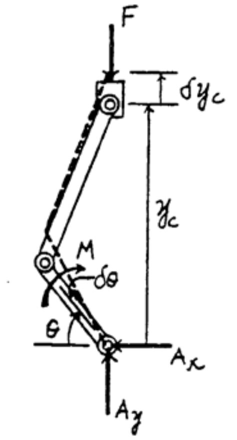
Virtual Displacements : Force **F** is located from the fixed point A using the positional coordinate y_C. Using the law of cosines.

$$5^2 = y_C^2 + 3^2 - 2(y_C)(3)\cos(90° - \theta) \qquad [1]$$

However, $\cos(90° - \theta) = \sin\theta$. Then Eq. [1] becomes $25 = y_C^2 + 9 - 6y_C\sin\theta$. Differentiating this expression, we have

$$0 = 2y_C\delta y_C - 6\delta y_C\sin\theta - 6y_C\cos\theta\,\delta\theta$$

$$\delta y_C = \frac{6y_C\cos\theta}{2y_C - 6\sin\theta}\delta\theta \qquad [2]$$



Virtual - Work Equation : When point C undergoes a positive virtual displacement δy_C, force **F** does negative work. The couple moment **M** does positive work when link AB undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \qquad -F\delta y_C + M\delta\theta = 0 \qquad [3]$$

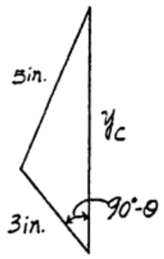
Substituting Eq. [2] into [3] yields

$$\left(-\frac{6y_C\cos\theta}{2y_C - 6\sin\theta}F + M\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$-\frac{6y_C\cos\theta}{2y_C - 6\sin\theta}F + M = 0$$

$$F = \frac{2y_C - 6\sin\theta}{6y_C\cos\theta}M \qquad [4]$$



At the equilibrium position, $\theta = 60°$. Substituting into Eq. [1], we have

$$5^2 = y_C^2 + 3^2 - 2(y_C)(3)\cos 30°$$

$$y_C = 7.368\text{ in.}$$

Substituting the above results into Eq. [4] and setting $M = 50\text{ lb}\cdot\text{ft}$, we have

$$F = \left[\frac{2(7.368) - 6\sin 60°}{6(7.368)\cos 60°}\right]50(12\text{ in.})/\text{ft} = 259\text{ lb} \qquad \textbf{Ans.}$$

11–26. If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (4x^3 - x^2 - 3x + 10)$ ft·lb, where x is given in feet, determine the equilibrium positions and investigate the stability at each position.

$$V = 4x^3 - x^2 - 3x + 10$$

Equilibrium Position :

$$\frac{dV}{dx} = 12x^2 - 2x - 3 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(12)(-3)}}{24}$$

$x = 0.590$ ft and -0.424 ft **Ans**

Stability :

$$\frac{d^2V}{dx^2} = 24x - 2$$

At $x = 0.590$ ft $\quad \frac{d^2V}{dx^2} = 24(0.590) - 2 = 12.2 > 0$ **stable** **Ans**

At $x = -0.424$ ft $\quad \frac{d^2V}{dx^2} = 24(-0.424) - 2 = -12.2 < 0$ **unstable** **Ans**

11–27. If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (24 \sin\theta + 10 \cos 2\theta)$ ft·lb, $0° \leq \theta \leq 90°$, determine the equilibrium positions and investigate the stability at each position.

$$V = 24\sin\theta + 10\cos 2\theta$$

Equilibrium Position :

$$\frac{dV}{d\theta} = 24\cos\theta - 20\sin 2\theta = 0$$

$$24\cos\theta - 40\sin\theta\cos\theta = 0$$

$$\cos\theta(24 - 40\sin\theta) = 0$$

$\cos\theta = 0 \qquad \theta = 90°$ **Ans**

$24 - 40\sin\theta = 0 \qquad \theta = 36.9°$ **Ans**

Stability :

$$\frac{d^2V}{d\theta^2} = -40\cos 2\theta - 24\sin\theta$$

At $\theta = 90°$ $\quad \frac{d^2V}{d\theta^2} = -40\cos 180° - 24\sin 90° = 16 > 0$ **stable** **Ans**

At $\theta = 36.9°$ $\quad \frac{d^2V}{d\theta^2} = -40\cos 73.7° - 24\sin 36.9° = -25.6 < 0$ **unstable** **Ans**

***11–28.** If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (3y^3 + 2y^2 - 4y + 50)$ J, where y is given in meters, determine the equilibrium positions and investigate the stability at each position.

Potential Function:

$$V = 3y^3 + 2y^2 - 4y + 50$$

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = 9y^2 + 4y - 4 = 0$$

Thus,

$y = 0.481$ m, $y = -925$ m **Ans.**

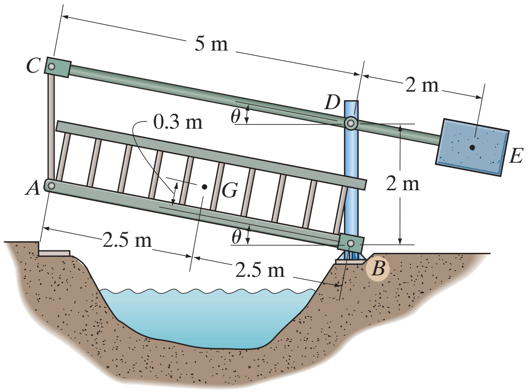
Stability: The second derivative of V is

$$\frac{d^2V}{d\theta^2} = 18y + 4$$

At $y = 0.481$ m, $\frac{d^2V}{d\theta^2} = 12.7 > 0$ Stable **Ans.**

At $y = -925$ m, $\frac{d^2V}{d\theta^2} = -12.7 < 0$ Unstable **Ans.**

•11–29. The 2-Mg bridge, with center of mass at point G, is lifted by two beams CD, located at each side of the bridge. If the 2-Mg counterweight E is attached to the beams as shown, determine the angle θ for equilibrium. Neglect the weight of the beams and the tie rods.



Potential Function: With reference to the datum, Fig. *a*, the gravitational potential energy of the bridge and counterweight are positive since their centers of gravity are located above the datum. Referring to the geometry shown in Fig. *b*,

$y_G = (0.3\cos\theta + 2.5\sin\theta)$ m and $y_E = (2 - 2\sin\theta)$ m

Thus,

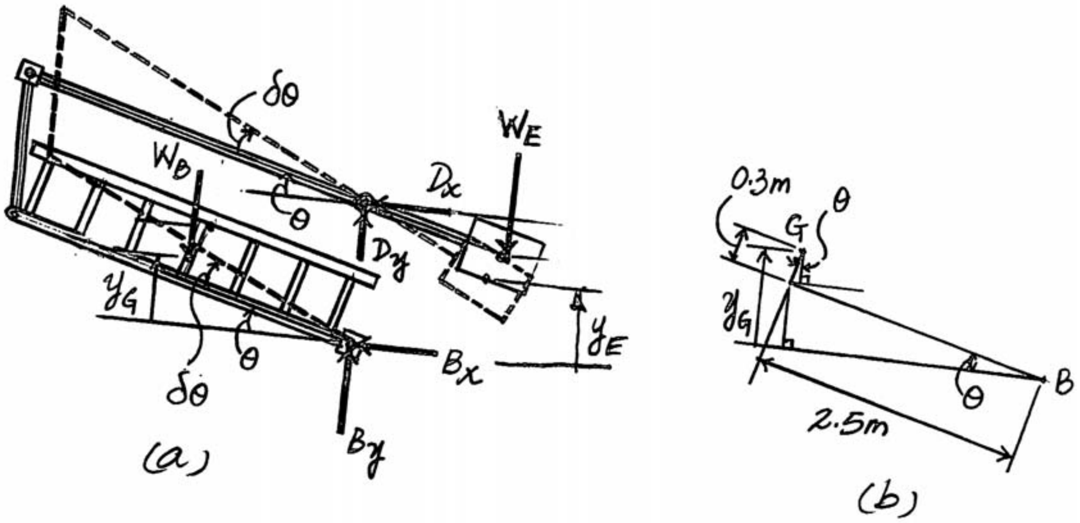
$$V = V_g = \Sigma mgy = 2000(9.81)(0.3\cos\theta + 2.5\sin\theta) + 2000(9.81)(2 - 2\sin\theta)$$
$$= 5886\cos\theta + 9810\sin\theta + 39240$$

Equilibrium Configuration: Taking the first derivative of V,

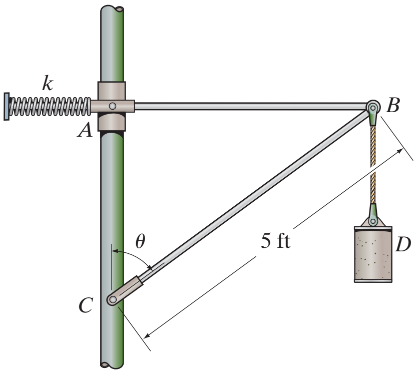
$$\frac{dV}{d\theta} = -5886\sin\theta + 9810\cos\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-5886\sin\theta + 9810\cos\theta = 0$$
$$\theta = 59.04° = 59.0° \qquad \textbf{Ans.}$$



11–30. The spring has a stiffness $k = 600\ \text{lb/ft}$ and is unstretched when $\theta = 45°$. If the mechanism is in equilibrium when $\theta = 60°$, determine the weight of cylinder D. Neglect the weight of the members. Rod AB remains horizontal at all times since the collar can slide freely along the vertical guide.



Potential Function: With reference to the datum, Fig. *a*, the gravitational potential energy of the cylinder is positive since its center of gravity is located above the datum. Here, $y = (5\cos\theta - b)$ ft. Thus,

$$V_g = Wy = W_D(5\cos\theta - b) = 5W_D\cos\theta - W_D b$$

The elastic potential energy of the spring can be computed using $V_g = \frac{1}{2}ks^2$, where $s = (5\sin\theta - 5\sin 45°)\ \text{ft} = (5\sin\theta - 3.5355)$ ft.

Thus,

$$V_e = \frac{1}{2}(600)(5\sin\theta - 3.5355)^2 = 7500\sin^2\theta - 10606.60\sin\theta + 3750$$

The total potential energy of the system is

$$V = V_g + V_e = 5W_D\cos\theta + 7500\sin^2\theta - 10606.60\sin\theta - W_D b + 3750$$

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = -5W_D\sin\theta + 1500\sin\theta\cos\theta - 10606.60\cos\theta$$

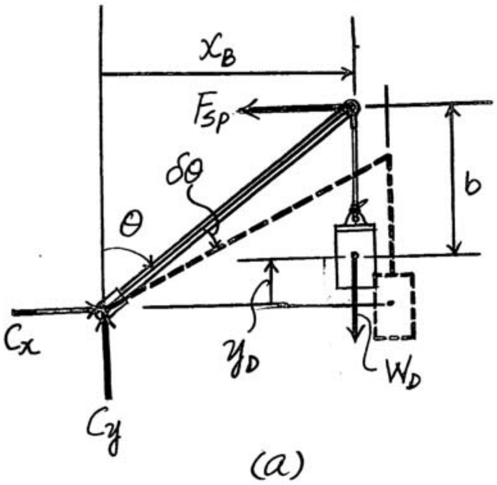
Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-5W_D\sin\theta + 1500\sin\theta\cos\theta - 10606.60\cos\theta = 0$$

$$W_D = \frac{1500\sin\theta\cos\theta - 10606.60\cos\theta}{5\sin\theta}$$

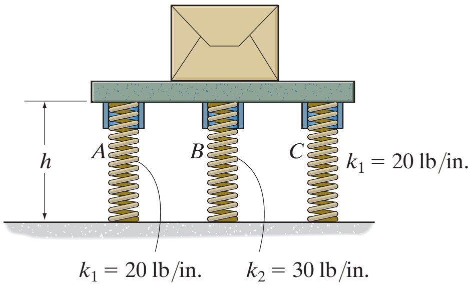
When $\theta = 60°$,

$$W_D = \frac{1500\sin 60°\cos 60° - 10606.60\cos 60°}{5\sin 60°} = 275\ \text{lb}$$ **Ans.**



(a)

11–31. If the springs at A and C have an unstretched length of 10 in. while the spring at B has an unstretched length of 12 in., determine the height h of the platform when the system is in equilibrium. Investigate the stability of this equilibrium configuration. The package and the platform have a total weight of 150 lb.



Potential Function: With reference to the datum, Fig. a, the gravitational potential energy of the package and the platform is positive since their center of gravity is located above the datum. Here, $y = h + b$ where b is a constant. Thus,

$$V_g = Wy = 150(h+b) = 150h + 150b$$

The elastic potential energy for the springs can be computed using $V_e = \frac{1}{2}ks^2$. Here, the compressions of the springs are $s_A = s_C = (10-h)$ in. and $s_B = (12-h)$ in. Thus,

$$V_e = 2\left[\frac{1}{2}(20)(10-h)^2\right] + \frac{1}{2}(30)(12-h)^2$$
$$= 35h^2 - 760h + 4160$$

The total potential energy of the system is

$$V = V_g + V_e = 35h^2 - 760h + 4160 + 150h + 150b$$

Equilibrium Configuration: Taking the first derivative of V,

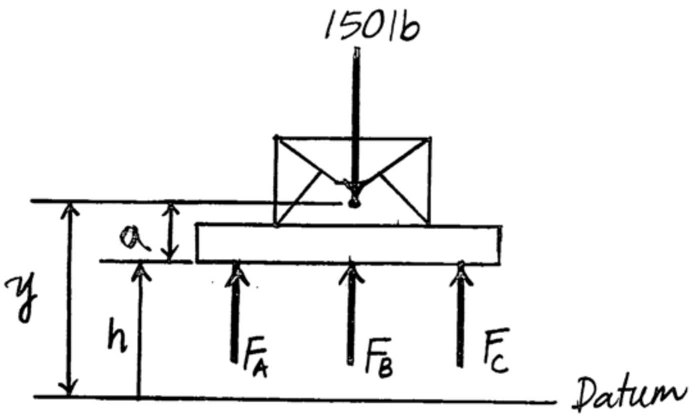
$$\frac{dV}{dh} = 70h - 610$$

Equilibrium requires $\frac{dV}{dh} = 0$. Thus,

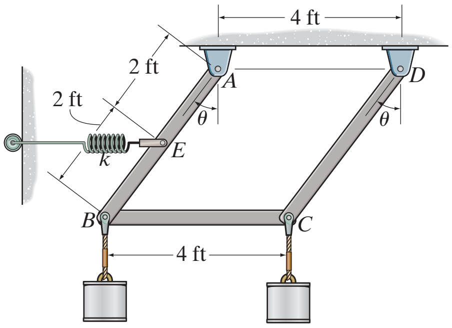
$70h - 610 = 0$ $\qquad h = 8.714 \text{ in.} = 8.71 \text{ in.}$ **Ans.**

Stability: The second derivative of V is

$\frac{d^2V}{dh^2} = 70 > 0$ *stable* **Ans.**



***11–32.** The spring is unstretched when $\theta = 45°$ and has a stiffness of $k = 1000\ \text{lb/ft}$. Determine the angle θ for equilibrium if each of the cylinders weighs 50 lb. Neglect the weight of the members.



Potential Function: With reference to the datum, Fig. *a*, the gravitational potential energy of the cylinders is negative since their centers of gravity are located below the datum. Here, $y = 4\cos\theta + b$, where b is constant. Thus,

$$V_g = \Sigma Wy = -2\left[50\left(4\cos\theta + b\right)\right] = -\left(400\cos\theta + 100b\right)$$

The elastic potential energy of the spring can be computed using $V_g = \frac{1}{2}ks^2$, where $s = 2\sin 45° - 2\sin\theta = 1.414 - 2\sin\theta$. Thus,

$$V_e = \frac{1}{2}(1000)(1.414 - 2\sin\theta)^2 = 2000\sin^2\theta - 2828.43\sin\theta + 1000$$

The total potential energy of the system is

$$V = V_g + V_e = 2000\sin^2\theta - 2828.43\sin\theta - 400\cos\theta - 100b + 1000$$

Equilibrium Configuration: Taking the first derivative of V,

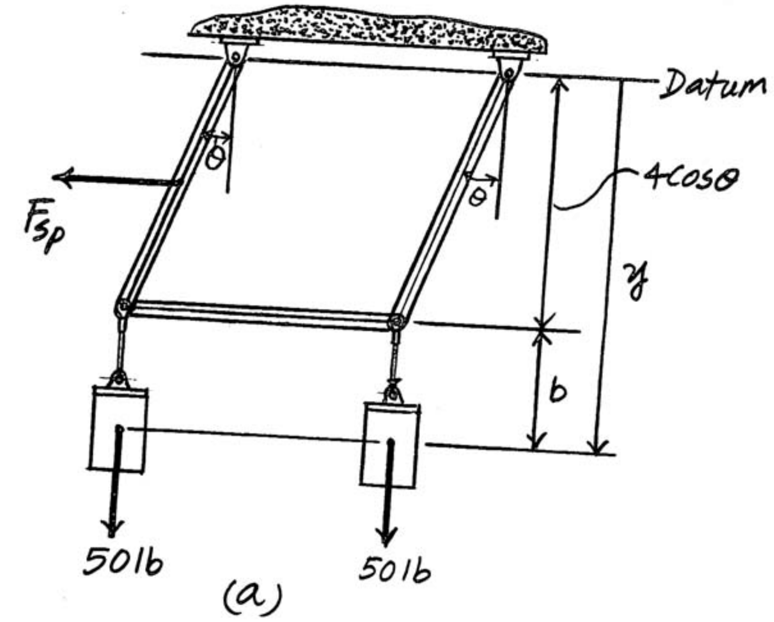
$$\frac{dV}{d\theta} = 4000\sin\theta\cos\theta - 2828.43\cos\theta + 400\sin\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$4000\sin\theta\cos\theta - 2828.43\cos\theta + 400\sin\theta = 0$$

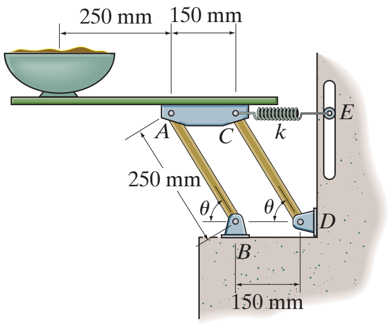
Solving by trial and error,

$$\theta = 38.8°$$ **Ans.**



(a)

•**11–33.** A 5-kg uniform serving table is supported on each side by pairs of two identical links, AB and CD, and springs CE. If the bowl has a mass of 1 kg, determine the angle θ where the table is in equilibrium. The springs each have a stiffness of $k = 200$ N/m and are unstretched when $\theta = 90°$. Neglect the mass of the links.



Potential Function: With reference to the datum, Fig. *a*, the gravitational potential energy of the bowl and the table are positive since their centers of gravity are located above the datum. Here, $y_{Gt} = (0.25\sin\theta + a)$ m and $y_{Gb} = (0.25\sin\theta + b)$ m. Thus,

$$V_g = \Sigma mgy = \frac{5}{2}(9.81)(0.25\sin\theta + a) + \frac{1}{2}(9.81)(0.25\sin\theta + b)$$
$$= 7.3575\sin\theta + 24.525a + 4.905b$$

The elastic potential energy of the spring can be computed using $V_e = \frac{1}{2}ks^2$, where $s = 0.25\cos\theta$ m. Thus,

$$V_e = \frac{1}{2}(200)(0.25\cos\theta)^2 = 6.25\cos^2\theta$$

The total potential energy of the system is

$$V = V_g + V_e = 6.25\cos^2\theta + 7.3575\sin\theta + 24.525a + 4.905b$$

Equilibrium Configuration: Taking the first derivative of V,

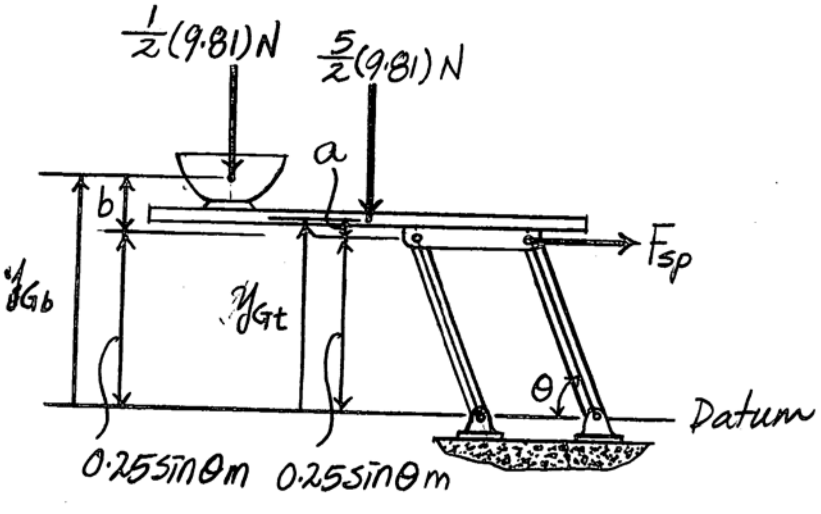
$$\frac{dV}{d\theta} = -12.5\cos\theta\sin\theta + 7.3575\cos\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-12.5\cos\theta\sin\theta + 7.3575\cos\theta = 0$$
$$\cos\theta(-12.5\sin\theta + 7.3575) = 0$$

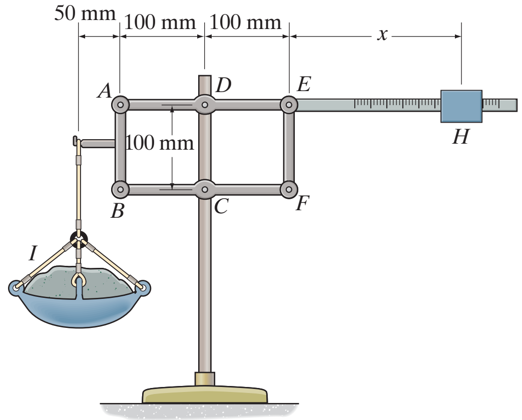
$\cos\theta = 0 \quad \theta = 90°$ **Ans.**

$-12.5\sin\theta + 7.3575 = 0 \qquad \theta = 36.1°$ **Ans.**



(a)

11–34. If a 10-kg load I is placed on the pan, determine the position x of the 0.75-kg block H for equilibrium. The scale is in balance when the weight and the load are not on the scale.



Potential Function: With reference to the datum, Fig. a, the gravitational potential energy of block H is positive since its center of gravity is located above the datum, while the gravitational potential energy of load I is negative since its center of gravity is located below the datum. Here, $y_H = [(0.1+x)\sin\theta]$ m and $y_I = (0.1\sin\theta + b)$ m where b is a constant. Thus,

$$V = V_g = \Sigma mgy = 0.75(9.81)(0.1+x)\sin\theta + [-10(9.81)(0.1\sin\theta + b)]$$
$$= 7.3575(0.1+x)\sin\theta - 9.81\sin\theta - 98.1b$$

Equilibrium Configuration: Taking the first derivative of V,

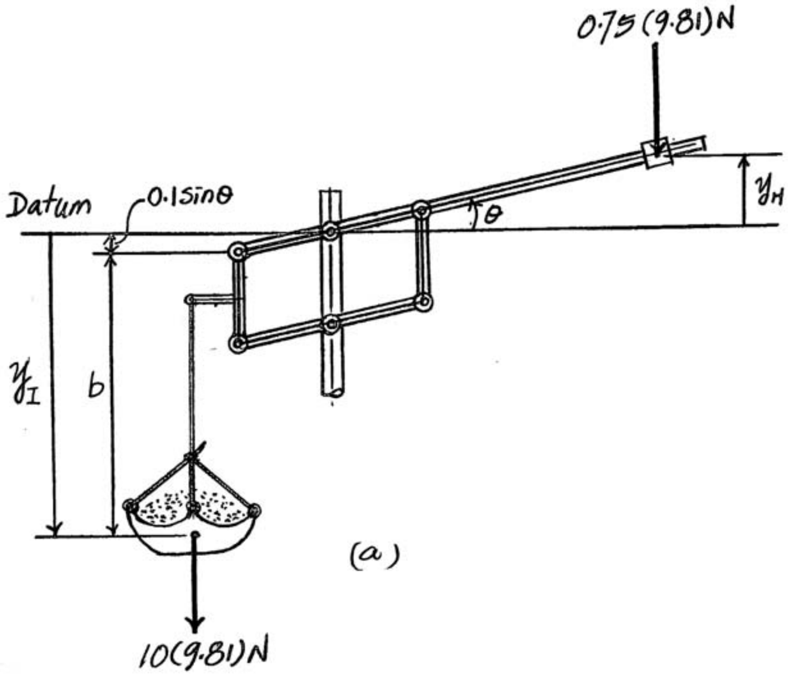
$$\frac{dV}{d\theta} = 7.3575(0.1+x)\cos\theta - 9.81\cos\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

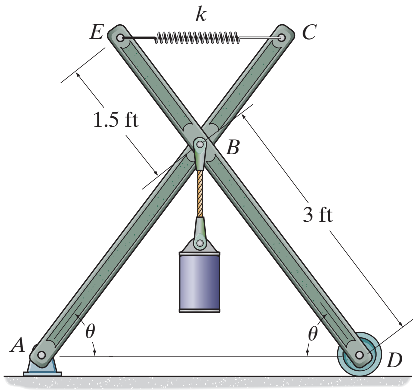
$$7.3575(0.1+x)\cos\theta - 9.81\cos\theta = 0$$
$$\cos\theta[7.3575(0.1+x) - 9.81] = 0$$
$$\cos\theta = 0 \quad \theta = 90°$$

or

$$7.3575(0.1+x) - 9.81 = 0$$
$$x = 1.23\text{ m}$$ **Ans.**



11–35. Determine the angles θ for equilibrium of the 200-lb cylinder and investigate the stability of each position. The spring has a stiffness of $k = 300\text{ lb/ft}$ and an unstretched length of 0.75 ft.



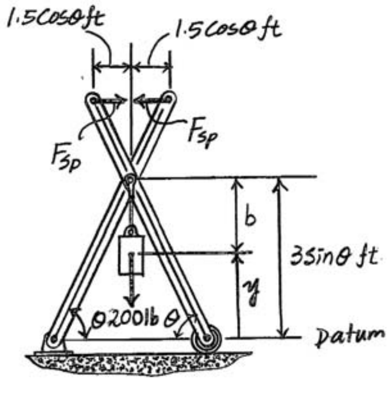
Potential Function: With reference to the datum, Fig. *a*, the gravitational potential energy of the cylinder is positive since its center of gravity is located above the datum. Here, $y = (3\sin\theta - b)$ ft, where b is a constant. Thus,

$$V_g = Wy = 200(3\sin\theta - b) = 600\sin\theta - 200b$$

The elastic potential energy of spring BC can be computed using $V_e = \frac{1}{2}ks^2$, where $s = 2(1.5\cos\theta) - 0.75 = (3\cos\theta - 0.75)$ ft. Thus,

$$V_e = \frac{1}{2}(300)(3\cos\theta - 0.75)^2$$

$$= 1350\cos^2\theta - 675\cos\theta + 84.75$$



(a)

The total potential energy of the system is

$$V = V_g + V_e = 1350\cos^2\theta - 675\cos\theta + 600\sin\theta + 84.375 - 200b$$

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = -2700\cos\theta\,\sin\theta + 675\sin\theta + 600\cos\theta$$

$$= -1375\sin 2\theta + 675\sin\theta + 600\cos\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-1375\sin 2\theta + 675\sin\theta + 600\cos\theta = 0$$

Solving by trial and error,

$\theta = 17.1°$ and $\theta = 70.9°$ **Ans.**

Stability: The second derivative of V is

$$\frac{d^2V}{d^2\theta} = -2700\cos 2\theta + 675\cos\theta - 600\sin\theta$$

At the equilibrium configuration $\theta = 17.1°$,

$$\left.\frac{d^2V}{d^2\theta}\right|_{\theta=17.1°} = -2700\cos 34.2° + 675\cos 17.1° - 600\sin 17.1°$$

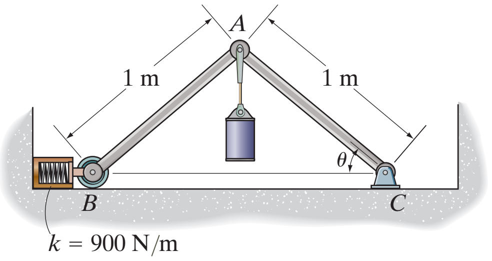
$= -1764 < 0$ *unstable* **Ans.**

At the equilibrium configuration $\theta = 70.92°$,

$$\left.\frac{d^2V}{d^2\theta}\right|_{\theta=70.92°} = -2700\cos 141.84° + 675\cos 70.92° - 600\sin 79.2°$$

$= 1776.67 > 0$ *stable* **Ans.**

***11–36.** Determine the angles θ for equilibrium of the 50-kg cylinder and investigate the stability of each position. The spring is uncompressed when $\theta = 60°$.



Potential Function: With reference to the datum, Fig. *a*, the gravitational potential energy of the cylinder is positive since its center of gravity is located above the datum. Here, $y = (1\sin\theta - b)$ m, where b is a constant. Thus,

$$V_g = mgy = 50(9.81)(\sin\theta - b) = 490.5\sin\theta - 490.5b$$

The elastic potential energy of the spring can be computed using $V_e = \frac{1}{2}ks^2$, where $s = 2(1\cos\theta - \cos 60°) = (2\cos\theta - 1)$ m. Thus,

$$V_e = \frac{1}{2}(900)(2\cos\theta - 1)^2 = 1800\cos^2\theta - 1800\cos\theta + 450$$

The total potential energy of the system is

$$V = V_g + V_e = 1800\cos^2\theta - 1800\cos\theta + 490.5\sin\theta + 450 - 490.5b$$

Equilibrium Configuration: Taking the first derivative of V,

$$\begin{aligned}\frac{dV}{d\theta} &= -3600\sin\theta\cos\theta + 1800\sin\theta + 490.5\cos\theta\\ &= -1800\sin 2\theta + 1800\sin\theta + 490.5\cos\theta\end{aligned}$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-1800\sin 2\theta + 1800\sin\theta + 490.5\cos\theta = 0$$

Solving by trial and error,

$\theta = 16.55 = 16.6°$ and $\theta = 52.9°$ **Ans.**

Stability: The second derivative of V is

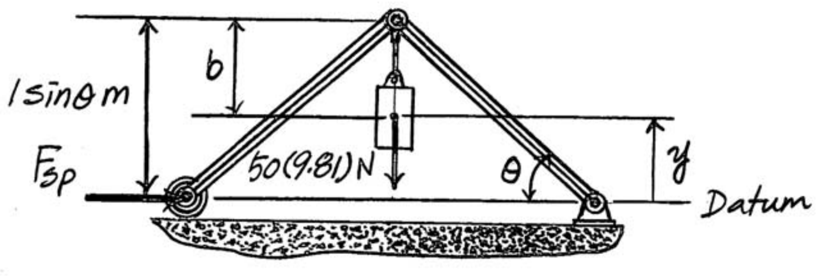
$$\frac{d^2V}{d^2\theta} = -3600\cos 2\theta + 1800\cos\theta - 490.5\sin\theta$$

At the equilibrium configuration $\theta = 16.55°$,

$$\begin{aligned}\left.\frac{d^2V}{d^2\theta}\right|_{\theta=52.916°} &= -3600\cos 33.10° + 1800\cos 16.55° - 490.5\sin 16.55°\\ &= -1430 < 0 \quad \textit{unstable}\end{aligned}$$ **Ans.**

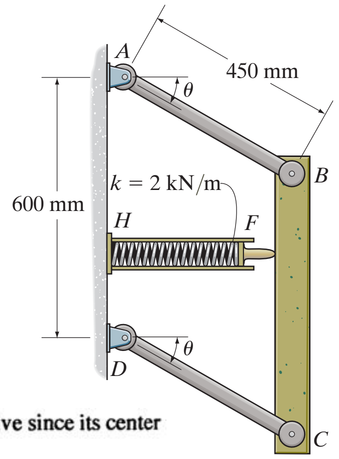
At the equilibrium configuration $\theta = 52.92°$,

$$\begin{aligned}\left.\frac{d^2V}{d^2\theta}\right|_{\theta=52.916°} &= -3600\cos 105.84° + 1800\cos 52.92° - 490.5\sin 52.92°\\ &= 1676.22 > 0 \quad \textit{stable}\end{aligned}$$ **Ans.**



(a)

•11–37. If the mechanism is in equilibrium when $\theta = 30°$, determine the mass of the bar BC. The spring has a stiffness of $k = 2$ kN/m and is uncompressed when $\theta = 0°$. Neglect the mass of the links.



Potential Function: With reference to the datum, Fig. a, the gravitational potential energy of block E is negative since its center of gravity is located below the datum. Here, $y = (0.45\sin\theta + b)$ m, where b is a constant.

$$V_g = -mgy = -m_E(9.81)(0.45\sin\theta + b) = -(4.4145m_E\sin\theta + 9.81m_E b)$$

The elastic potential energy of the spring can be computed using $V_e = \frac{1}{2}ks^2$, where $s = 0.45 - 0.45\cos\theta$. Thus,

$$V_e = \frac{1}{2}(2000)(0.45 - 0.45\cos\theta)^2$$
$$= 202.5 + 202.5\cos^2\theta - 405\cos\theta$$

The total potential energy of the system is

$$V = V_g + V_e = -4.4145m_E\sin\theta + 202.5\cos^2\theta - 405\cos\theta - 9.81m_E b + 202.5$$

Equilibrium Configuration: Taking the first derivative of V,

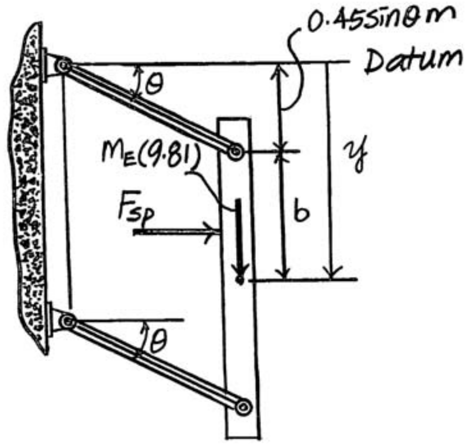
$$\frac{dV}{d\theta} = -4.4145m_E\cos\theta - 405\cos\theta\sin\theta + 405\sin\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-4.4145m_E\cos\theta - 405\cos\theta\sin\theta + 405\sin\theta = 0$$
$$m_E = \frac{405\sin\theta - 405\cos\theta\sin\theta}{4.4145\cos\theta}$$

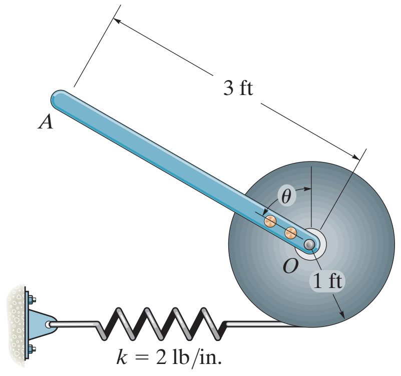
When $\theta = 30°$,

$$m_E = \frac{405\sin 30° - 405\cos 30°\sin 30°}{4.4145\cos 30°} = 7.10\text{ kg}$$ **Ans.**



(a)

11–38. The uniform rod *OA* weighs 20 lb, and when the rod is in the vertical position, the spring is unstretched. Determine the position θ for equilibrium. Investigate the stability at the equilibrium position.



Potential Function : The spring stretches $s = 12(\theta)$ in., where θ is in radians.

$$V = V_e + V_g = \frac{1}{2}(2)(12\theta)^2 + 20\left[1.5(12)\cos\theta\right]$$

$$= 144\theta^2 + 360\cos\theta$$

Equilibrium Position : $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = 288\theta - 360\sin\theta = 0$$

$\theta = 1.1311\text{ rad} = 64.8°$ **Ans**

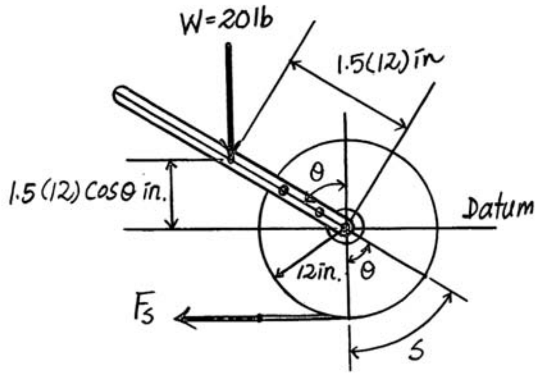
$\theta = 0°$ **Ans**

Stability :

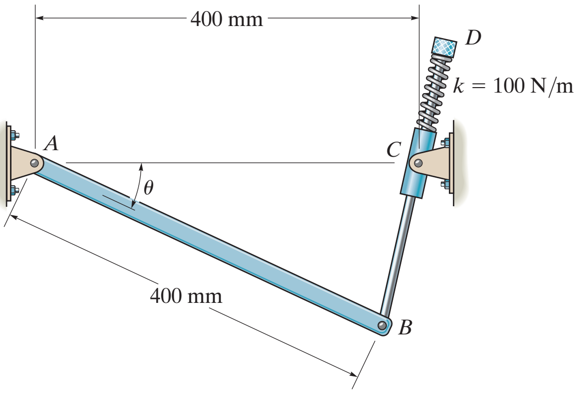
$$\frac{d^2V}{d\theta^2} = 288 - 360\cos\theta$$

At $\theta = 64.8°$, $\frac{d^2V}{d\theta^2} = 288 - 360\cos 64.8° = 135 > 0$ **stable** **Ans**

At $\theta = 0°$, $\frac{d^2V}{d\theta^2} = 288 - 360\cos 0° = -72 < 0$ **unstable** **Ans**



11–39. The uniform link *AB* has a mass of 3 kg and is pin connected at both of its ends. The rod *BD*, having negligible weight, passes through a swivel block at *C*. If the spring has a stiffness of $k = 100$ N/m and is unstretched when $\theta = 0°$, determine the angle θ for equilibrium and investigate the stability at the equilibrium position. Neglect the size of the swivel block.



$$s = \sqrt{(0.4)^2 + (0.4)^2 - 2(0.4)^2\cos\theta}$$

$$= (0.4)\sqrt{2(1 - \cos\theta)}$$

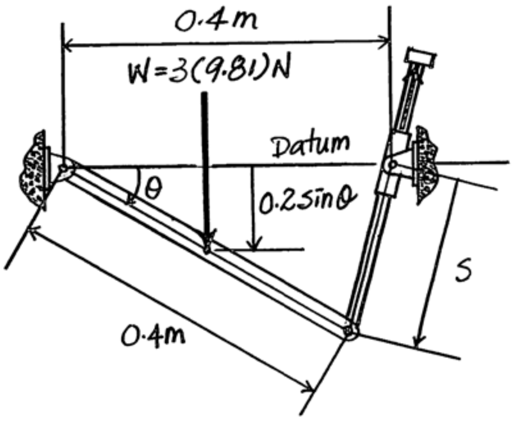
$$V = V_g + V_e$$

$$= -(0.2)(\sin\theta)3(9.81) + \frac{1}{2}(100)\left[(0.4)^2(2)(1 - \cos\theta)\right]$$

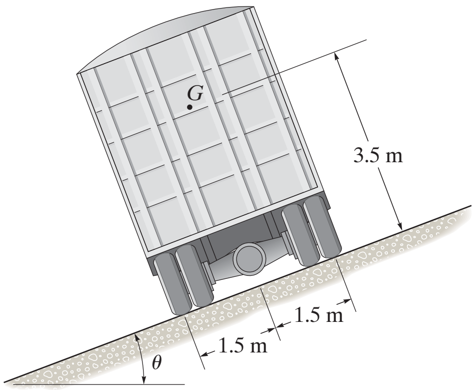
$$\frac{dV}{d\theta} = -(5.886)\cos\theta + 16(\sin\theta) = 0 \qquad (1)$$

$\theta = 20.2°$ **Ans**

$\frac{d^2V}{d\theta^2} = 5.886\sin\theta + (16)\cos\theta = 17.0 > 0$ **stable** **Ans**



***11–40.** The truck has a mass of 20 Mg and a mass center at G. Determine the steepest grade θ along which it can park without overturning and investigate the stability in this position.



Potential Function : The datum is established at point A. Since the center of gravity for the truck is above the datum, its potential energy is positive. Here, $y = (1.5\sin\theta + 3.5\cos\theta)$ m.

$$V = V_g = Wy = W(1.5\sin\theta + 3.5\cos\theta)$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = W(1.5\cos\theta - 3.5\sin\theta) = 0$$

Since $W \neq 0$,

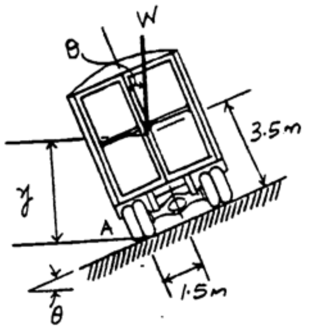
$$1.5\cos\theta - 3.5\sin\theta = 0$$
$$\theta = 23.20° = 23.2° \qquad \textbf{Ans}$$

Stability :

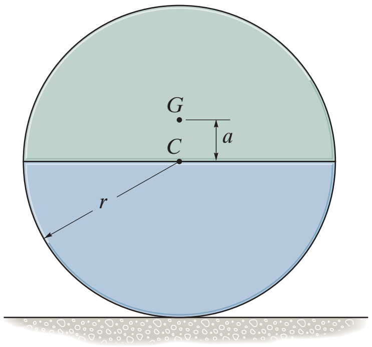
$$\frac{d^2V}{d\theta^2} = W(-1.5\sin\theta - 3.5\cos\theta)$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=23.20°} = W(-1.5\sin 23.20° - 3.5\cos 23.20°) = -3.81W < 0$$

Thus, the truck is in **unstable equilibrium** at $\theta = 23.2°$ **Ans**



•**11–41.** The cylinder is made of two materials such that it has a mass of m and a center of gravity at point G. Show that when G lies above the centroid C of the cylinder, the equilibrium is unstable.



Potential Function : The datum is established at point A. Since the center of gravity of the cylinder is above the datum, its potential energy is positive. Here, $y = r + d\cos\theta$.

$$V = V_g = Wy = mg(r + a\cos\theta)$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = -mga\sin\theta = 0$$

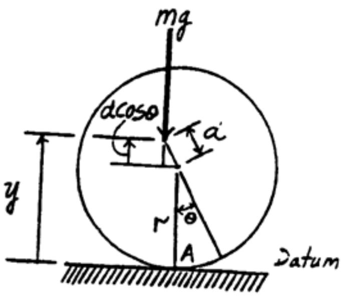
$$\sin\theta = 0 \qquad \theta = 0°.$$

Stability :

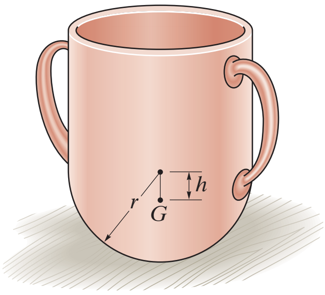
$$\frac{d^2V}{d\theta^2} = -mga\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = -mgd\cos 0° = -mga < 0$$

Thus, the cylinder is in **unstable equilibrium** at $\theta = 0°$ (*Q.E.D.*)



11–42. The cap has a hemispherical bottom and a mass *m*. Determine the position *h* of the center of mass *G* so that the cup is in neutral equilibrium.



Potential Function : The datum is established at point *A*. Since the center of gravity of the cup is above the datum, its potential energy is positive. Here, $y = r - h\cos\theta$.

$$V = V_g = Wy = mg(r - h\cos\theta)$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = mgh\sin\theta = 0$$

$$\sin\theta = 0 \qquad \theta = 0°.$$

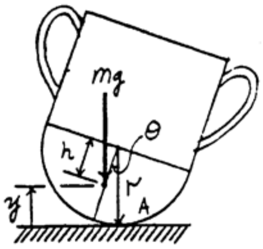
Stability : To have neutral equilibrium at $\theta = 0°$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = 0$.

$$\frac{d^2V}{d\theta^2} = mgh\cos\theta$$

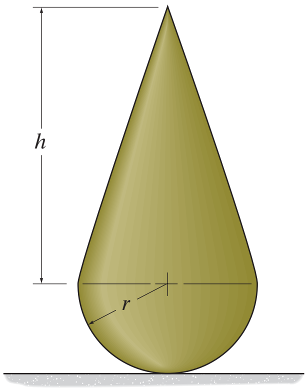
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = mgh\cos 0° = 0$$

$$h = 0 \qquad \textbf{Ans}$$

Note : Stable Equilibrium occurs if $h > 0\left(\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = mgh\cos 0° > 0\right)$.



11–43. Determine the height h of the cone in terms of the radius r of the hemisphere so that the assembly is in neutral equilibrium. Both the cone and the hemisphere are made from the same material.



Potential Function: The mass of the cone and hemisphere are $m_C = \rho\left(\frac{1}{3}\pi r^2 h\right) = \frac{1}{3}\rho\pi r^2 h$ and $m_s = \rho\left(\frac{2}{3}\pi r^3\right) = \frac{2}{3}\rho\pi r^3$, where ρ is the density of the homogeneous material. With reference to the datum, Fig. a, the gravitational potential energy of the cone and hemisphere are positive since their centers of gravity are located above the datum. Here, $y_C = r + \frac{h}{4}\cos\theta$ and $y_s = r - \frac{3}{8}r\cos\theta$. Thus,

$$V = V_g = \Sigma mgy = \left(\frac{1}{3}\rho\pi r^2 h\right)(g)\left(r + \frac{h}{4}\cos\theta\right) + \frac{2}{3}\rho\pi r^3(g)\left(r - \frac{3}{8}r\cos\theta\right)$$

$$= \frac{1}{3}\rho\pi r^2 g\left(rh + \frac{h^2}{4}\cos\theta + 2r^2 - \frac{3}{4}r^2\cos\theta\right)$$

Equilibrium Configuration: Taking the first derivative of V, we have

$$\frac{dV}{d\theta} = \frac{1}{3}\rho\pi r^2 g\left(-\frac{h^2}{4}\sin\theta + \frac{3}{4}r^2\sin\theta\right) = \frac{1}{3}\rho\pi r^2 g\sin\theta\left(-\frac{h^2}{4} + \frac{3}{4}r^2\right)$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$\frac{1}{3}\rho\pi r^2 g\sin\theta\left(-\frac{h^2}{4} + \frac{3}{4}r^2\right) = 0$$

$$\sin\theta = 0 \quad \theta = 0^\circ$$

Stability: The second derivative of V is

$$\frac{d^2V}{d\theta^2} = \frac{1}{3}\rho\pi r^2 g\cos\theta\left(-\frac{h^2}{4} + \frac{3}{4}r^2\right)$$

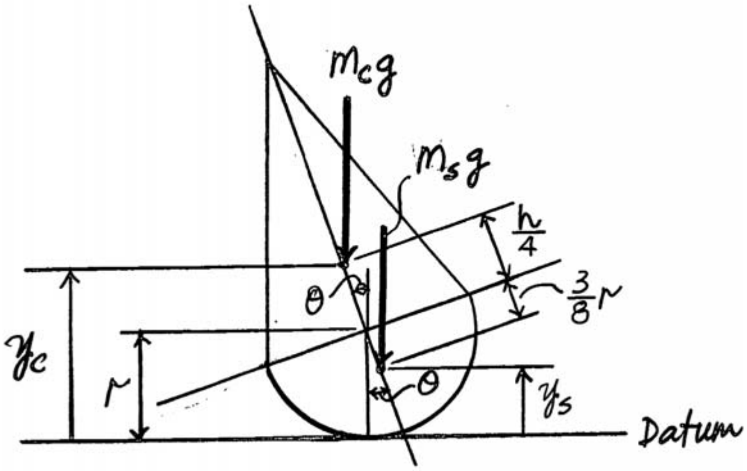
For neutral equilibrium at $\theta = 0^\circ$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = 0$. Thus,

$$\frac{1}{3}\rho\pi r^2 g\cos 0^\circ\left(-\frac{h^2}{4} + \frac{3}{4}r^2\right) = 0$$

Since $\frac{1}{3}\rho\pi r^2 g \neq 0$, then

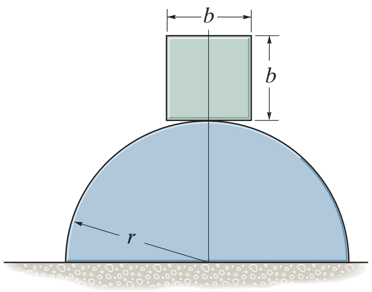
$$-\frac{h^2}{4} + \frac{3}{4}r^2 = 0$$

$$h = \sqrt{3}r \qquad \textbf{Ans.}$$



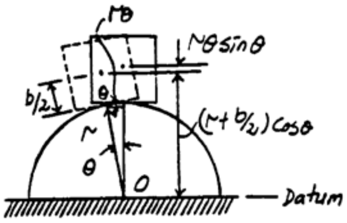
Note. The equilibrium configuration of the assembly at $\theta = 0^\circ$ is stable if $h < \sqrt{3}r\left(\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} > 0\right)$ and is unstable if $h > \sqrt{3}r\left(\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} < 0\right)$.

***11–44.** A homogeneous block rests on top of the cylindrical surface. Derive the relationship between the radius of the cylinder, r, and the dimension of the block, b, for stable equilibrium. *Hint*: Establish the potential energy function for a small angle θ, i.e., approximate $\sin\theta \approx 0$, and $\cos\theta \approx 1 - \theta^2/2$.



Potential Function : The datum is established at point O. Since the center of gravity for the block is above the datum, its potential energy is positive. Here, $y = \left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta$.

$$V = W_y = W\left[\left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta\right] \qquad [1]$$



For small angle θ, $\sin\theta = \theta$ and $\cos\theta = 1 - \frac{\theta^2}{2}$. Then Eq. [1] becomes

$$V = W\left[\left(r + \frac{b}{2}\right)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2\right]$$
$$= W\left(\frac{r\theta^2}{2} - \frac{b\theta^2}{4} + r + \frac{b}{2}\right)$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$

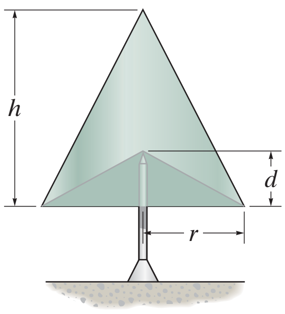
$$\frac{dV}{d\theta} = W\left(r - \frac{b}{2}\right)\theta = 0 \qquad \theta = 0^\circ$$

Stability : To have stable equilibrium, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} > 0.$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = W\left(r - \frac{b}{2}\right) > 0$$

$$\left(r - \frac{b}{2}\right) > 0$$
$$b < 2r \qquad \textbf{Ans}$$

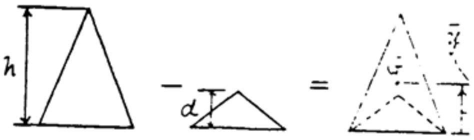
•11–45. The homogeneous cone has a conical cavity cut into it as shown. Determine the depth *d* of the cavity in terms of *h* so that the cone balances on the pivot and remains in neutral equilibrium.

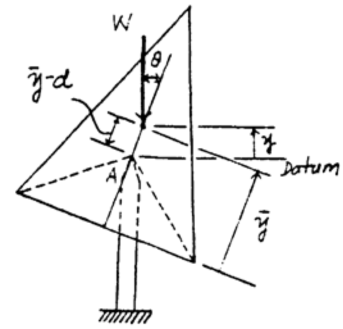


Potential Function : The datum is established at point A. Since the center of gravity of the cone is above the datum, its potential energy is positive. Here,

$y = (\bar{y} - d)\cos\theta = \left[\frac{1}{4}(h+d) - d\right]\cos\theta = \frac{1}{4}(h-3d)\cos\theta.$

$$V = W\left[\frac{1}{4}(h-3d)\cos\theta\right] = \frac{W(h-3d)}{4}\cos\theta$$





Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = -\frac{W(h-3d)}{4}\sin\theta = 0$$

$$\theta = 0 \qquad \theta = 0^\circ$$

Stability : To have neutral equilibrium at $\theta = 0^\circ$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = 0.$

$$\frac{d^2V}{d\theta^2} = -\frac{W(h-3d)}{4}\cos\theta$$

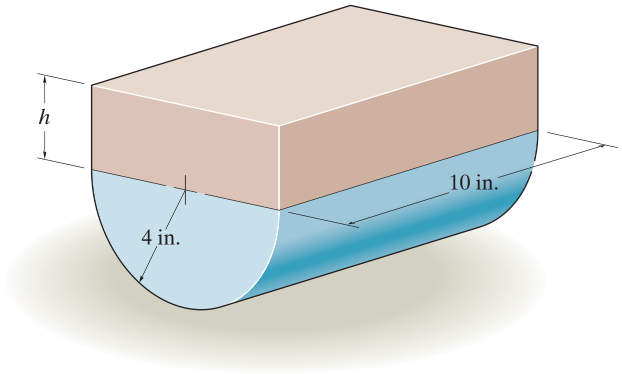
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = -\frac{W(h-3d)}{4}\cos 0^\circ = 0$$

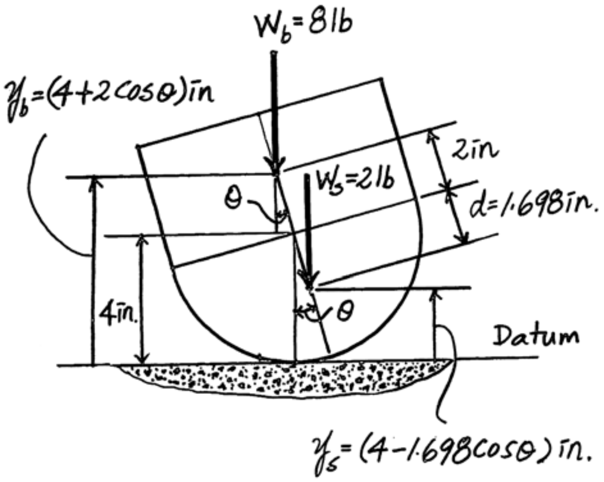
$$-\frac{W(h-3d)}{4} = 0$$

$$d = \frac{h}{3} \qquad \textbf{Ans}$$

Note : By substituting $d = \frac{h}{3}$ into Eq.[1], one realizes that the fulcrum must be at the center of gravity for neutral equilibrium.

11–46. The assembly shown consists of a semicylinder and a rectangular block. If the block weighs 8 lb and the semicylinder weighs 2 lb, investigate the stability when the assembly is resting in the equilibrium position. Set $h = 4$ in.




$W_b = 8\text{lb}$
$y_b = (4 + 2\cos\theta)\text{in}$
2in
$W_s = 2\text{lb}$
$d = 1.698\text{in.}$
θ
4in.
θ
Datum
$y_s = (4 - 1.698\cos\theta)\text{in.}$

$$d = \frac{4(4)}{3\pi} = 1.698 \text{ in.}$$

$$V = V_g = 2(4 - 1.698\cos\theta) + 8(4 + 2\cos\theta)$$

$$\frac{dV}{d\theta} = 3.395\sin\theta - 16\sin\theta = 0$$

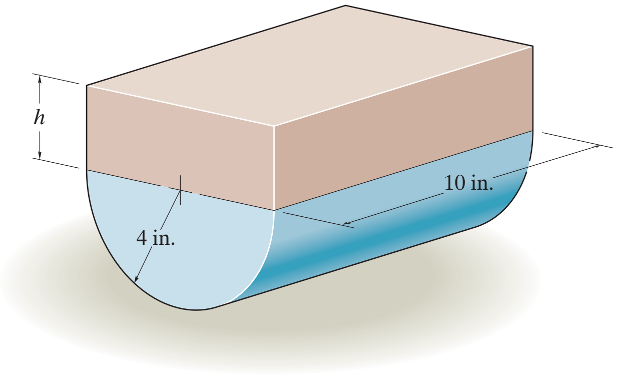
$$\sin\theta = 0$$

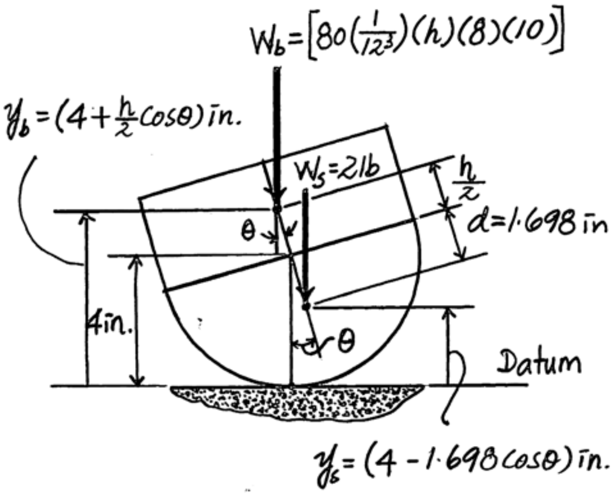
$$\theta = 0^\circ \qquad \text{(equilibrium position)}$$

$$\frac{d^2V}{d\theta^2} = 3.395\cos\theta - 16\cos\theta$$

At $\theta = 0^\circ$, $\frac{d^2V}{d\theta^2} = -12.6 < 0$ Unstable **Ans**

11–47. The 2-lb semicylinder supports the block which has a specific weight of $\gamma = 80\ \text{lb/ft}^3$. Determine the height h of the block which will produce neutral equilibrium in the position shown.





$W_b = \left[80\left(\frac{1}{12^3}\right)(h)(8)(10)\right]$
$y_b = (4 + \frac{h}{2}\cos\theta)$ in.
$W_s = 2$ lb
$\frac{h}{2}$
$d = 1.698$ in
θ
4in.
θ
Datum
$y_s = (4 - 1.698\cos\theta)$ in.

$$d = \frac{4(4)}{3\pi} = 1.698\text{ in.}$$

$$V = V_g = 2(4 - 1.698\cos\theta) + \left[80\left(\frac{1}{12^3}\right)h(8)(10)\right]\left(4 + \frac{h}{2}\cos\theta\right)$$

$$\frac{dV}{d\theta} = 3.395\sin\theta - 1.852\,h^2\sin\theta = 0$$

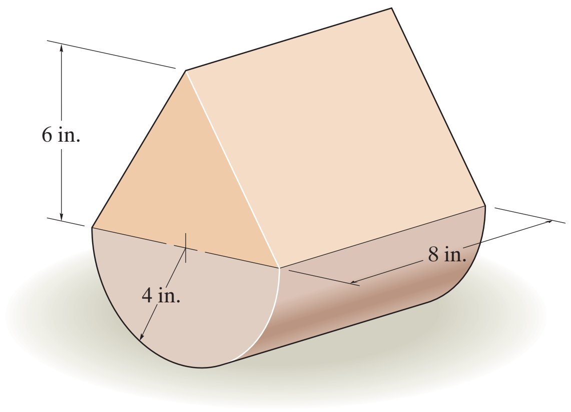
$$\sin\theta = 0$$

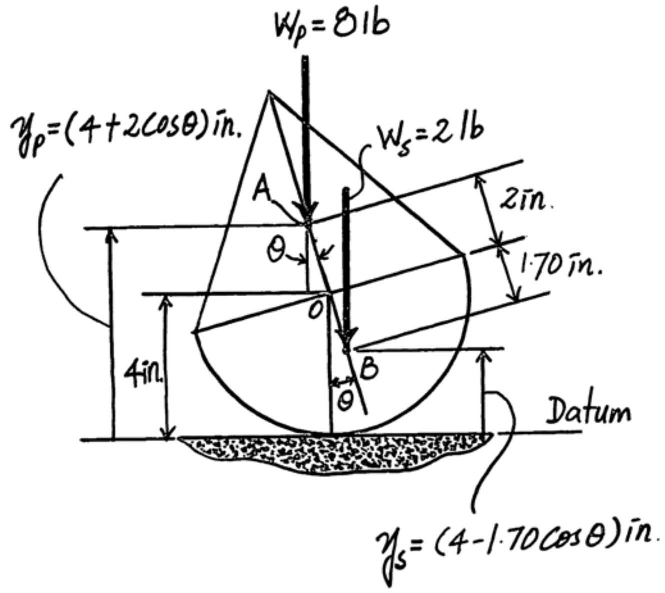
$$\theta = 0^\circ \qquad \text{(equilibrium position)}$$

$$\frac{d^2V}{d\theta^2} = 3.395\cos\theta - 1.852\,h^2\cos\theta = 0$$

$$h = \sqrt{\frac{3.395}{1.852}} = 1.35\text{ in.} \qquad \textbf{Ans}$$

***11–48.** The assembly shown consists of a semicircular cylinder and a triangular prism. If the prism weighs 8 lb and the cylinder weighs 2 lb, investigate the stability when the assembly is resting in the equilibrium position.




$W_p = 8$ lb
$y_p = (4 + 2\cos\theta)$ in.
$W_s = 2$ lb
A
θ
2 in.
1.70 in.
O
4 in.
B
θ
Datum
$y_s = (4 - 1.70\cos\theta)$ in.

$$OB = \frac{4(4)}{3\pi} = 1.70 \text{ in.}$$

$$OA = \frac{1}{3}(6) = 2 \text{ in.}$$

$$V = V_g = 8(4 + 2\cos\theta) + 2(4 - 1.70\cos\theta)$$

$$V = 40 + 12.6\cos\theta$$

$$\frac{dV}{d\theta} = -12.6\sin\theta = 0$$

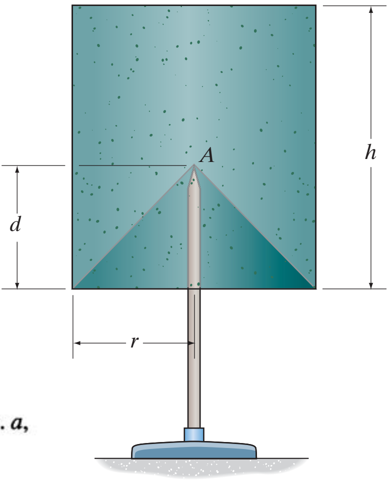
$\theta = 0°$ **Ans** (for equilibrium)

$$\frac{d^2V}{d\theta^2} = -12.6\cos\theta$$

At $\theta = 0°$,

$\dfrac{d^2V}{d\theta^2} = -12.6 < 0$ **unstable** **Ans**

•11–49. A conical hole is drilled into the bottom of the cylinder, and it is then supported on the fulcrum at *A*. Determine the minimum distance *d* in order for it to remain in stable equilibrium.



Potential Function: First, we must determine the center of gravity of the cylinder. By referring to Fig. *a*,

$$\bar{y} = \frac{\Sigma y_C m}{\Sigma m} = \frac{\frac{h}{2}(\rho\pi r^2 h) - \frac{d}{4}\left(\frac{1}{3}\rho\pi r^2 d\right)}{\rho\pi r^2 h - \frac{1}{3}\rho\pi r^2 d} = \frac{6h^2 - d^2}{4(3h-d)} \qquad (1)$$

With reference to the datum, Fig. *a*, the gravitational potential energy of the cylinder is positive since its center of gravity is located above the datum. Here,

$$y = (\bar{y} - d)\cos\theta = \left[\frac{6h^2 - d^2}{4(3h-d)} - d\right]\cos\theta = \left[\frac{6h^2 - 12hd + 3d^2}{4(3h-d)}\right]\cos\theta$$

Thus,

$$V = V_g = Wy = W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h-d)}\right]\cos\theta$$

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = -W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h-d)}\right]\sin\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h-d)}\right]\sin\theta = 0$$

$$\sin\theta = 0 \qquad \theta = 0°$$

Stability: The second derivative of V is

$$\frac{d^2V}{d^2\theta} = -W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h-d)}\right]\cos\theta$$

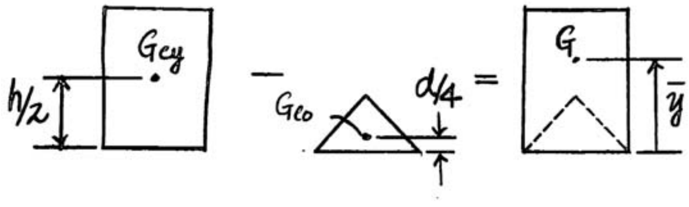
To have neutral equilibrium at $\theta = 0°$, $\left.\frac{d^2V}{d^2\theta}\right|_{\theta=0°} = 0$. Thus,

$$-W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h-d)}\right]\cos 0° = 0$$

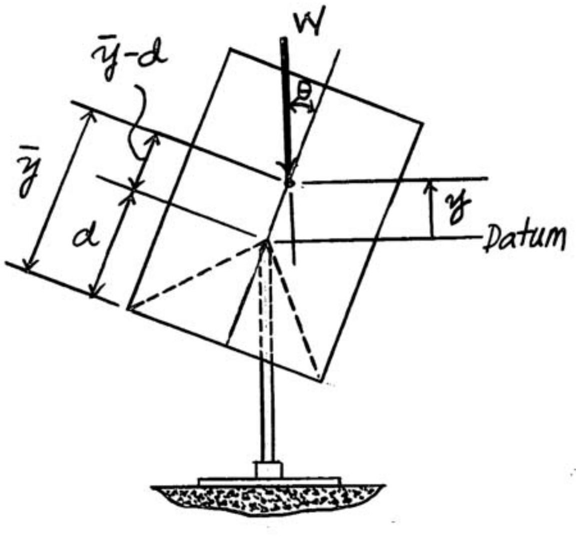
$$6h^2 - 12hd + 3d^2 = 0$$

$$d = \frac{12h \pm \sqrt{(-12h)^2 - 4(3)(6h^2)}}{2(3)} = 0.5858h = 0.586h \qquad \textbf{Ans.}$$

Note: If we substitute $d = 0.5858h$ into Eq. (1), we notice that the fulcrum must be at the center of gravity for neutral equilibrium.

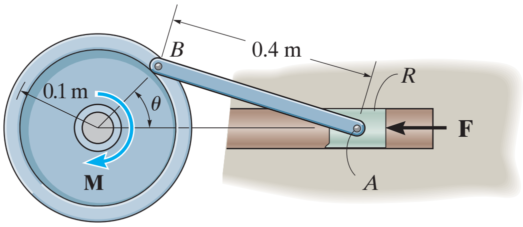


(a)



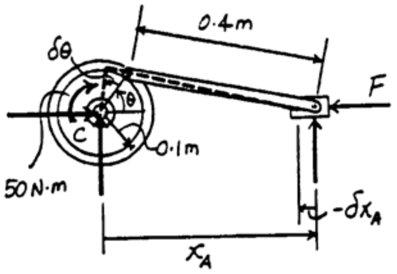
(b)

11–50. The punch press consists of the ram R, connecting rod AB, and a flywheel. If a torque of $M = 50\ \text{N}\cdot\text{m}$ is applied to the flywheel, determine the force F applied at the ram to hold the rod in the position $\theta = 60°$.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only force **F** and 50 N · m couple moment do work.

Virtual Displacements : The force **F** is located from the fixed point A using the position coordinate x_A. Using the law of cosines,



$$0.4^2 = x_A^2 + 0.1^2 - 2(x_A)(0.1)\cos\theta \qquad [1]$$

Differentiating the above expression, we have

$$0 = 2x_A\,\delta x_A - 0.2\delta x_A\cos\theta + 0.2x_A\sin\theta\,\delta\theta$$

$$\delta x_A = \frac{0.2x_A\sin\theta}{0.2\cos\theta - 2x_A}\delta\theta \qquad [2]$$

Virtual - Work Equation : When point A undergoes positive virtual displacement δx_A, force **F** does negative work. The 50 N · m couple moment does negative work when the flywheel undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \qquad -F\delta x_A - 50\delta\theta = 0 \qquad [3]$$

Substituting Eq. [2] into [3] yields

$$\left(-\frac{0.2x_A\sin\theta}{0.2\cos\theta - 2x_A}F - 50\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$-\frac{0.2x_A\sin\theta}{0.2\cos\theta - 2x_A}F - 50 = 0$$

$$F = -\frac{50(0.2\cos\theta - 2x_A)}{0.2x_A\sin\theta} \qquad [4]$$

At the equilibrium position, $\theta = 60°$. Substituting into Eq. [1], we have

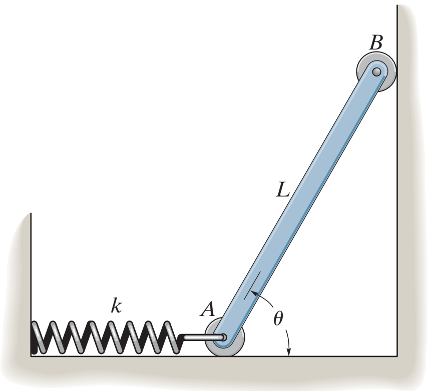
$$0.4^2 = x_A^2 + 0.1^2 - 2(x_A)(0.1)\cos 60°$$

$$x_A = 0.4405\ \text{m}$$

Substituting the above results into Eq. [4], we have

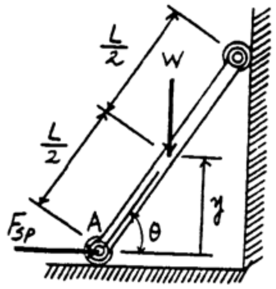
$$F = -\frac{50[0.2\cos 60° - 2(0.4405)]}{0.2(0.4405)\sin 60°} = 512\ \text{N} \qquad \textbf{Ans}$$

11–51. The uniform rod has a weight W. Determine the angle θ for equilibrium. The spring is uncompressed when $\theta = 90°$. Neglect the weight of the rollers.



Potential Function : The datum is established at point A. Since the center of gravity of the beam is above the datum, its potential energy is positive. Here, $y = \frac{L}{2}\sin\theta$ and the spring compresses $x = L\cos\theta$.

$$\begin{aligned} V &= V_e + V_g \\ &= \frac{1}{2}kx^2 + Wy \\ &= \frac{1}{2}(k)(L\cos\theta)^2 + W\left(\frac{L}{2}\sin\theta\right) \\ &= \frac{kL^2}{2}\cos^2\theta + \frac{WL}{2}\sin\theta \end{aligned}$$



Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

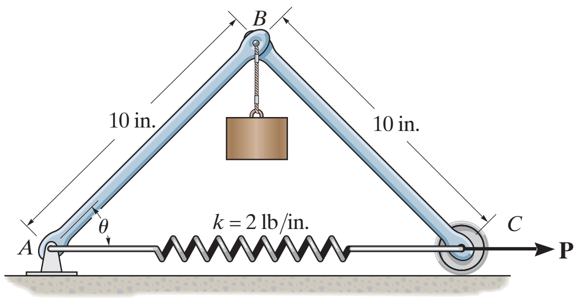
$$\frac{dV}{d\theta} = -kL^2\sin\theta\cos\theta + \frac{WL}{2}\cos\theta = 0$$

$$\cos\theta\left(-kL^2\sin\theta + \frac{WL}{2}\right) = 0$$

Solving,

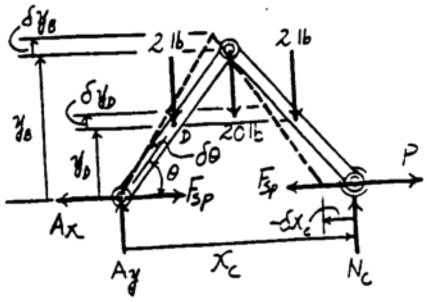
$$\theta = 90° \quad \text{or} \quad \sin^{-1}\left(\frac{W}{2kL}\right) \qquad \textbf{Ans}$$

***11–52.** The uniform links AB and BC each weigh 2 lb and the cylinder weighs 20 lb. Determine the horizontal force P required to hold the mechanism at $\theta = 45°$. The spring has an unstretched length of 6 in.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp}, the weight of links (2 lb), 20 lb force and force **P** do work.

Virtual Displacements : The positions of points B, D and C are measured from the fixed point A using position coordinates y_B, y_D and x_C respectively.



$$y_B = 10\sin\theta \qquad \delta y_B = 10\cos\theta\,\delta\theta \qquad [1]$$
$$y_D = 5\sin\theta \qquad \delta y_D = 5\cos\theta\,\delta\theta \qquad [2]$$
$$x_C = 2(10\cos\theta) \qquad \delta x_C = -20\sin\theta\,\delta\theta \qquad [3]$$

Virtual - Work Equation : When points B, D and C undergo positive virtual displacements δy_B, δy_D and δx_C, spring force F_{sp} that acts at point C, the weight of links (2 lb) and 20 lb force do negative work while force **P** does positive work.

$$\delta U = 0; \qquad -F_{sp}\,\delta x_C - 2(2\delta y_D) - 20\delta y_B + P\delta x_C = 0 \qquad [4]$$

Substituting Eqs. [1], [2] and [3] into [4] yields

$$\left(20F_{sp}\sin\theta - 20P\sin\theta - 220\cos\theta\right)\delta\theta = 0 \qquad [5]$$

However, from the spring formula, $F_{sp} = kx = 2[2(10\cos\theta) - 6] = 40\cos\theta - 12$. Substituting this value into Eq. [5] yields

$$(800\sin\theta\cos\theta - 240\sin\theta - 220\cos\theta - 20P\sin\theta)\,\delta\theta = 0$$

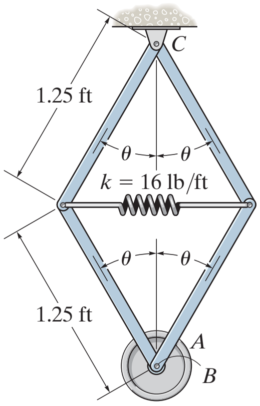
Since $\delta\theta \neq 0$, then

$$800\sin\theta\cos\theta - 240\sin\theta - 220\cos\theta - 20P\sin\theta = 0$$
$$P = 40\cos\theta - 11\cot\theta - 12$$

At the equilibrium position, $\theta = 45°$. Then

$$P = 40\cos 45° - 11\cot 45° - 12 = 5.28 \text{ lb} \qquad \textbf{Ans}$$

•11–53. The spring attached to the mechanism has an unstretched length when $\theta = 90°$. Determine the position θ for equilibrium and investigate the stability of the mechanism at this position. Disk A is pin connected to the frame at B and has a weight of 20 lb.



Potential Function : The datum is established at point C. Since the center of gravity of the disk is below the datum, its potential energy is negative. Here, $y = 2(1.25\cos\theta) = 2.5\cos\theta$ ft and the spring compresses $x = (2.5 - 2.5\sin\theta)$ ft.

$$\begin{aligned} V &= V_e + V_g \\ &= \frac{1}{2}kx^2 - Wy \\ &= \frac{1}{2}(16)(2.5 - 2.5\sin\theta)^2 - 20(2.5\cos\theta) \\ &= 50\sin^2\theta - 100\sin\theta - 50\cos\theta + 50 \end{aligned}$$

Equilibrium Position : The system is in equilibrium if $\dfrac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = 100\sin\theta\cos\theta - 100\cos\theta + 50\sin\theta = 0$$

$$\frac{dV}{d\theta} = 50\sin 2\theta - 100\cos\theta + 50\sin\theta = 0$$

Solving by trial and error,

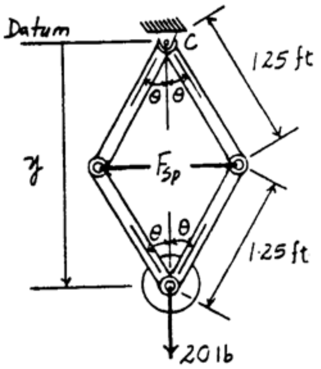
$$\theta = 37.77° = 37.8° \qquad \textbf{Ans}$$

Stability :

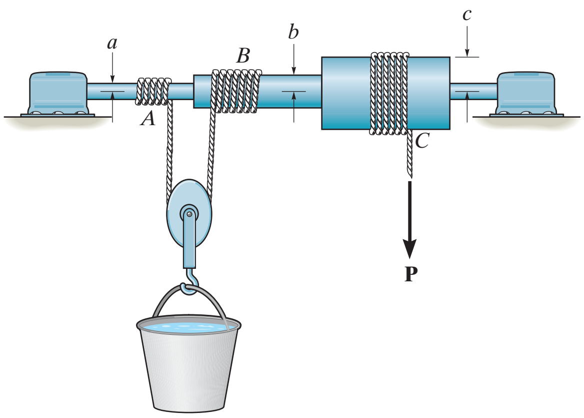
$$\frac{d^2V}{d\theta^2} = 100\cos 2\theta + 100\sin\theta + 50\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=37.77°} = 100\cos 75.54° + 100\sin 37.77° + 50\cos 37.77° = 125.7 > 0$$

Thus, the system is in **stable equilibrium** at $\theta = 37.8°$ **Ans**



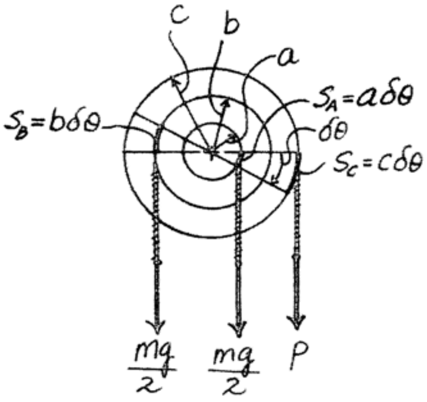
11–54. Determine the force P that must be applied to the cord wrapped around the drum at C which is necessary to lift the bucket having a mass m. Note that as the bucket is lifted, the pulley rolls on a cord that winds up on shaft B and unwinds from shaft A.



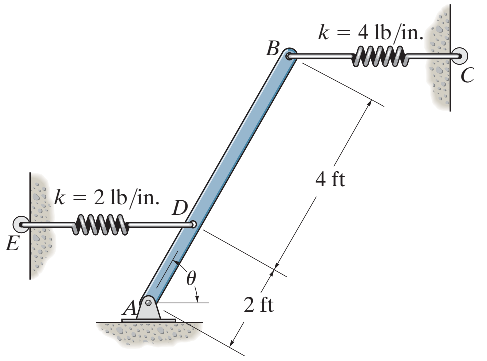
As shaft rotates $\delta\theta$

$$\delta U = 0; \quad P(c)\,\delta\theta - \frac{mg}{2}(b\,\delta\theta) + \frac{mg}{2}(a\,\delta\theta) = 0$$

$$P = \left(\frac{b-a}{2c}\right)mg \quad \text{Ans}$$

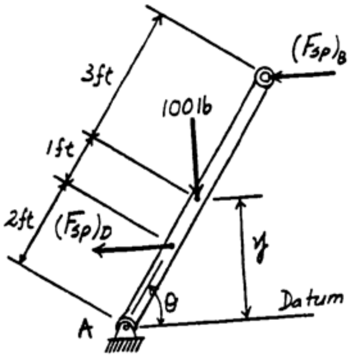


11–55. The uniform bar *AB* weighs 100 lb. If both springs *DE* and *BC* are unstretched when $\theta = 90°$, determine the angle θ for equilibrium using the principle of potential energy. Investigate the stability at the equilibrium position. Both springs always remain in the horizontal position due to the roller guides at *C* and *E*.



Potential Function : The datum is established at point *A*. Since the center of gravity of the beam is above the datum, its potential energy is positive. Here, $y = (3\sin\theta)$ ft, the spring at *D* stretches $x_D = (2\cos\theta)$ ft and the spring at *B* compresses $x = (6\cos\theta)$ ft.

$$\begin{aligned} V &= V_e + V_g \\ &= \Sigma\frac{1}{2}kx^2 + Wy \\ &= \frac{1}{2}(24)(2\cos\theta)^2 + \frac{1}{2}(48)(6\cos\theta)^2 + 100(3\sin\theta) \\ &= 912\cos^2\theta + 300\sin\theta \end{aligned}$$



Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0.$

$$\frac{dV}{d\theta} = -1824\sin\theta\cos\theta + 300\cos\theta = 0$$

$$\frac{dV}{d\theta} = -912\sin 2\theta + 300\cos\theta = 0$$

Solving,

$$\theta = 90° \quad \text{or} \quad \theta = 9.467° = 9.47° \qquad \textbf{Ans}$$

Stability :

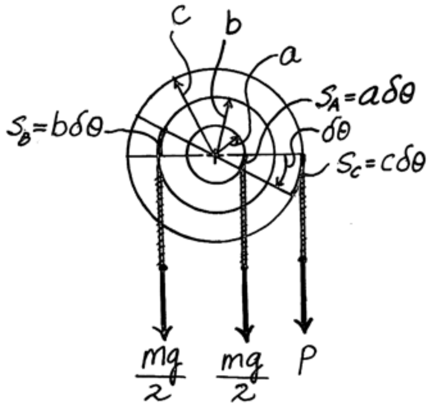
$$\frac{d^2V}{d\theta^2} = -1824\cos 2\theta - 300\sin\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90°} = -1824\cos 180° - 300\sin 90° = 1524 > 0$$

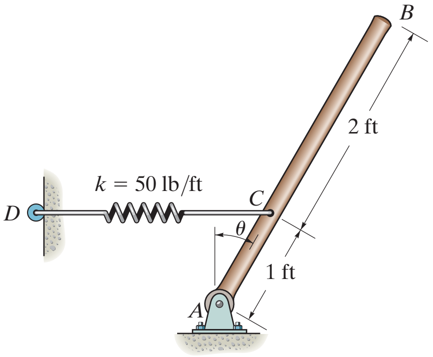
Thus, the system is in **stable equilibrium** at $\theta = 90°$ **Ans**

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=9.467°} = -1824\cos 18.933° - 300\sin 9.467° = -1774.7 < 0$$

Thus, the system is in **unstable equilibrium** at $\theta = 9.47°$ **Ans**



***11–56.** The uniform rod *AB* has a weight of 10 lb. If the spring *DC* is unstretched when $\theta = 0°$, determine the angle θ for equilibrium using the principle of virtual work. The spring always remains in the horizontal position due to the roller guide at *D*.



$y_w = 1.5\cos\theta \qquad \delta y_w = -1.5\sin\theta\,\delta\theta$

$x_F = 1\sin\theta \qquad \delta x_F = \cos\theta\,\delta\theta$

$\delta U = 0; \qquad -W\delta y_w - F_s\,\delta x_F = 0$

$-10(-1.5\sin\theta\delta\theta) - F_s(\cos\theta\,\delta\theta) = 0$

$\delta\theta(15\sin\theta - F_s\cos\theta) = 0$

Since $\delta\theta \neq 0$

$15\sin\theta - F_s\cos\theta = 0 \qquad (1)$

$F_s = kx \qquad$ where $\qquad x = 1\sin\theta \qquad (2)$

$F_s = 50(\sin\theta) = 50\sin\theta$

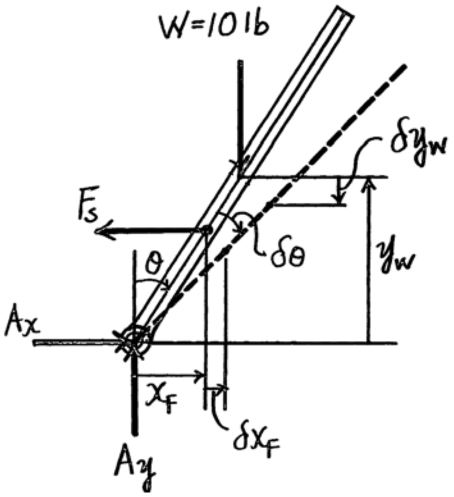
Substituting Eq. (2) into (1) yields :

$15\sin\theta - (50\sin\theta)\cos\theta = 0$

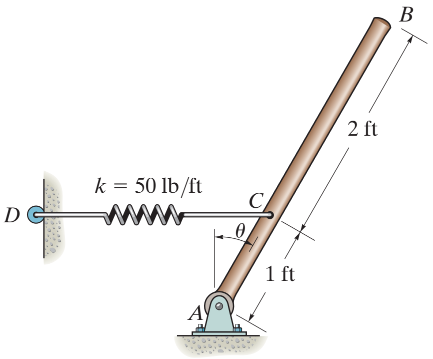
$\sin\theta(15 - 50\cos\theta) = 0$

$\sin\theta = 0 \qquad \theta = 0° \qquad$ **Ans**

$15 - 50\cos\theta = 0 \qquad \theta = 72.5° \qquad$ **Ans**

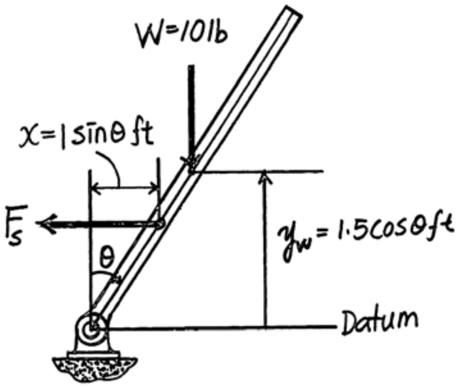


•11–57. Solve Prob. 11–56 using the principle of potential energy. Investigate the stability of the rod when it is in the equilibrium position.



$$V = V_e + V_g = \frac{1}{2}(50)(\sin\theta)^2 + 10(1.5\cos\theta)$$

$$= 25\sin^2\theta + 15\cos\theta$$



$$\frac{dV}{d\theta} = 0$$

$$\frac{dV}{d\theta} = 50\sin\theta\cos\theta - 15\sin\theta = 0$$

$$\sin\theta(50\cos\theta - 15) = 0$$

$\sin\theta = 0$ $\quad \theta = 0°$ **Ans**

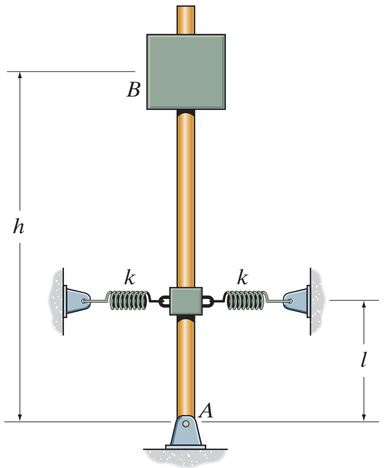
$50\cos\theta - 15 = 0$ $\quad \theta = 72.5°$ **Ans**

$$\frac{d^2V}{d\theta^2} = 50\cos 2\theta - 15\cos\theta$$

At $\theta = 0°$, $\quad \frac{d^2V}{d\theta^2} = 50\cos 0° - 15\cos 0° = 35 > 0$ **stable** **Ans**

At $\theta = 72.5°$, $\quad \frac{d^2V}{d\theta^2} = 50\cos 145° - 15\cos 72.5° = -45.5 < 0$ **unstable** **Ans**

11–58. Determine the height h of block B so that the rod is in neutral equilibrium. The springs are unstretched when the rod is in the vertical position. The block has a weight W.



Potential Function: With reference to the datum, Fig. a, the gravitational potential energy of block B is positive since its center of gravity is located above the datum. Here, the rod is tilted a small angle θ. Thus, $y = h\cos\theta$. For a small angle θ, $\cos\theta \approx 1 - \frac{\theta^2}{2}$.

Thus,

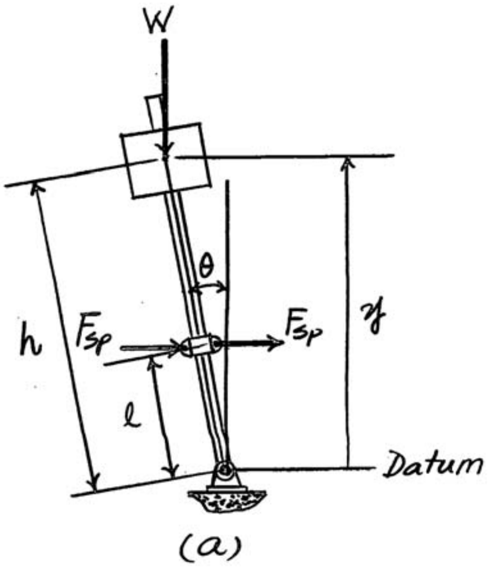
$$V_g = Wy = Wh\left(1 - \frac{\theta^2}{2}\right)$$

The elastic potential energy of each spring can be computed using $V_e = \frac{1}{2}ks^2$. Since θ is small, $s \approx l\theta$. Thus,

$$V_e = 2\left[\frac{1}{2}k(l\theta)^2\right] = kl^2\theta^2$$

The total potential energy of the system is

$$V = V_g + V_e = Wh\left(1 - \frac{\theta^2}{2}\right) + kl^2\theta^2$$



(a)

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = -Wh\theta + 2kl^2\theta = \theta(-Wh + 2kl^2)$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$\theta(-Wh + 2kl^2) = 0$$

$$\theta = 0°$$

Stability: The second derivative of V is

$$\frac{d^2V}{d\theta^2} = -Wh + 2kl^2$$

To have neutral equilibrium at $\theta = 0°$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = 0$. Thus,

$$-Wh + 2kl^2 = 0$$

$$h = \frac{2kl^2}{W} \qquad \textbf{Ans.}$$

Note: The equilibrium configuration of the system at $\theta = 0°$ is stable if $h < \frac{2kl^2}{W}\left(\frac{d^2V}{d\theta^2} > 0\right)$ and is unstable if $h > \frac{2kl^2}{W}\left(\frac{d^2V}{d\theta^2} < 0\right)$.